Dark Matter–Induced UV Radiation and the Formation of Supermassive Black Hole Seeds

Light Dark World 2026

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July 30, 2026

Outline

- **Background**

  Observations, Collapse in ΛCDM Model, Direct Collapse Black Hole (DCBH)

- **Modeling**

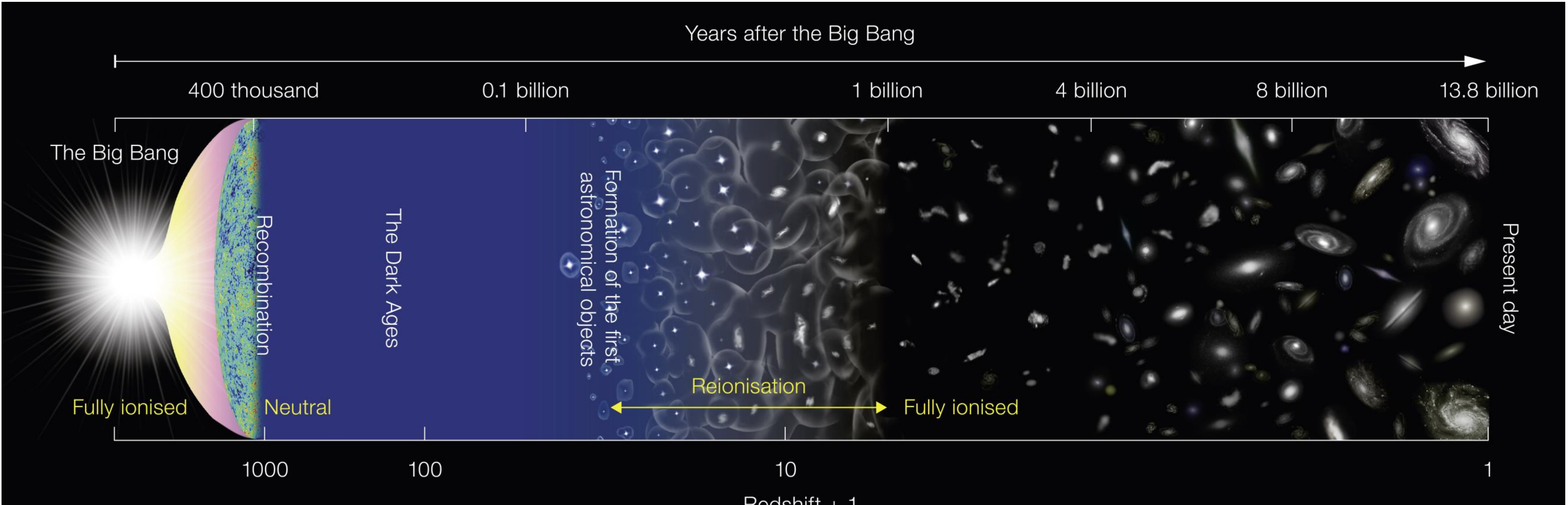
  DM Halo, Gas Cloud and DM Decay/ Annihilation, Simulation and Criteria

- **Results**

  Thermal History, Phase Diagrams

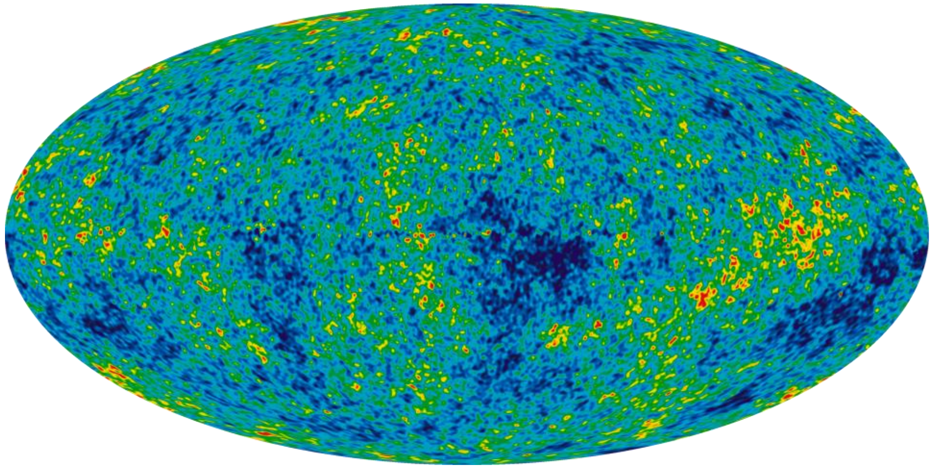
- **Takeaway Information**

Timeline of the Evolution of the Universe



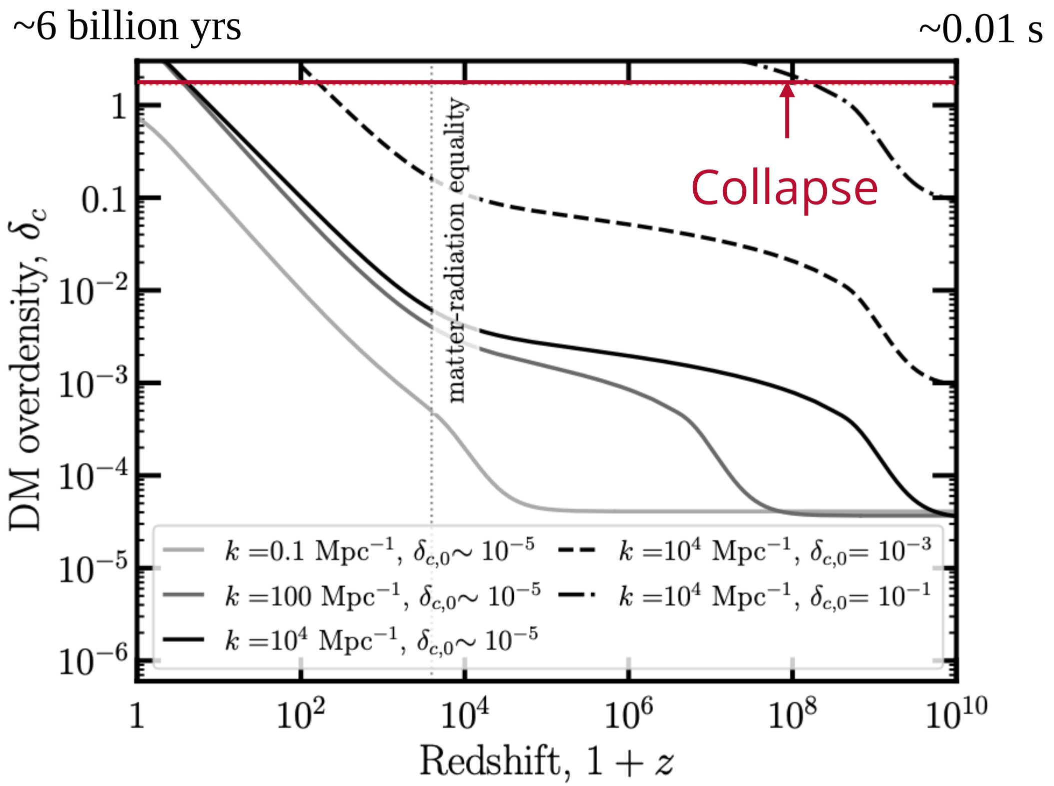
https://www.eso.org/public/images/eso1620a/

Structure Collapse in ΛCDM Model



https://science.nasa.gov/mission/wmap/wmap-overview/

https://arxiv.org/pdf/2506.13858



Initial Density Fluctuation $\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} \sim 10^{-5}$

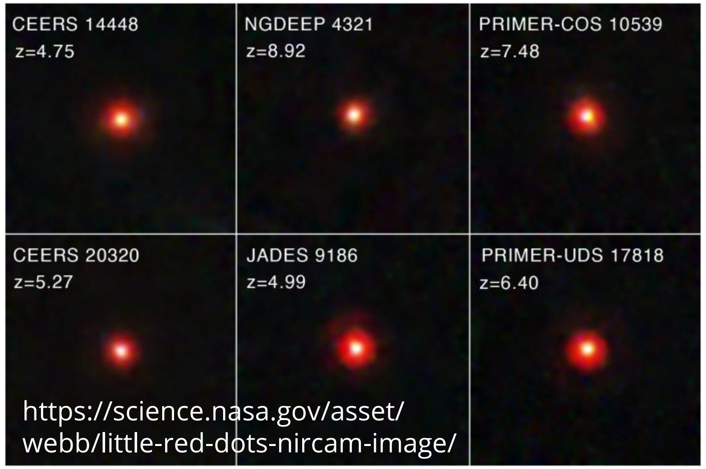
The Hubble Heritage Team (AURA/STScI/NASA)

$\delta \gg 1$

ΛCDM Model:

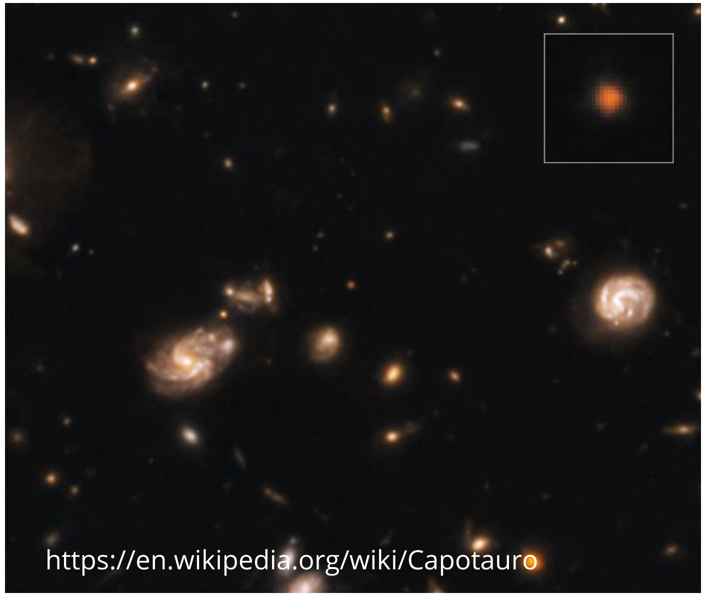
- the large-scale structure in the distribution of galaxies
- the accelerating expansion of the universe observed in the light from distant galaxies and supernovae
- …

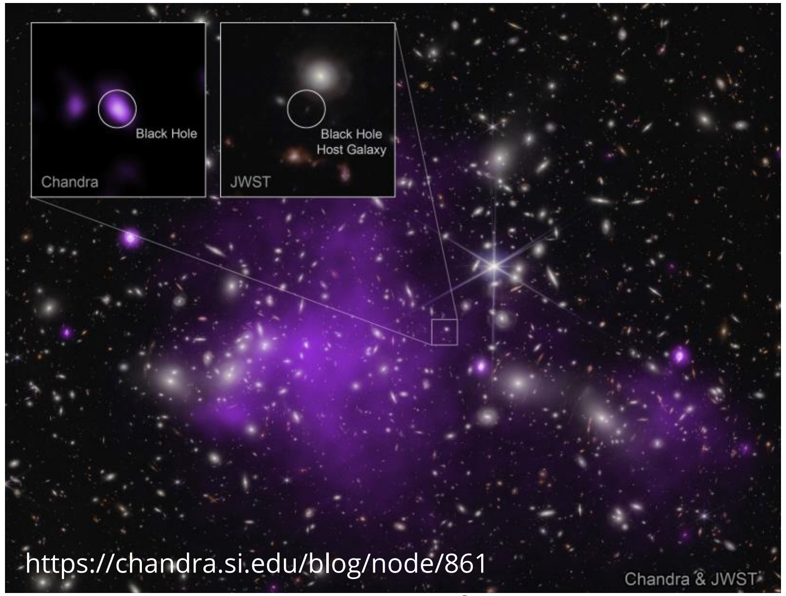
Observations



Little red dots (LRDs)

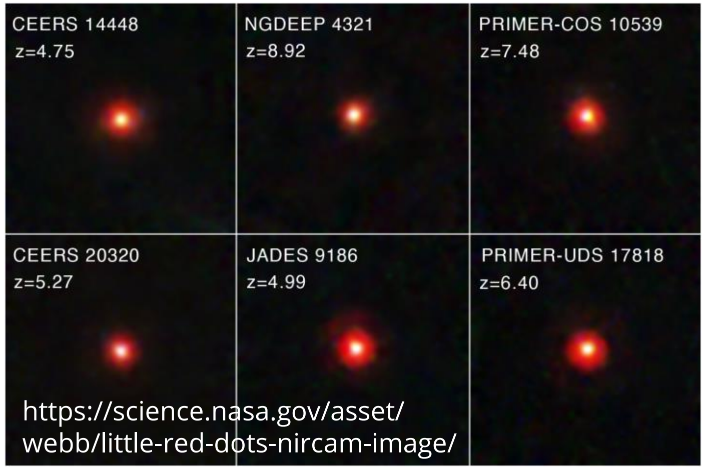
- The original reports identified LRDs as a type of early **active galactic nucleus (AGN)**
- The current leading theory is that the LRDs are a form of **primordial galaxy**
- Early galaxies as result of very slowly spinning dark matter halos
- …



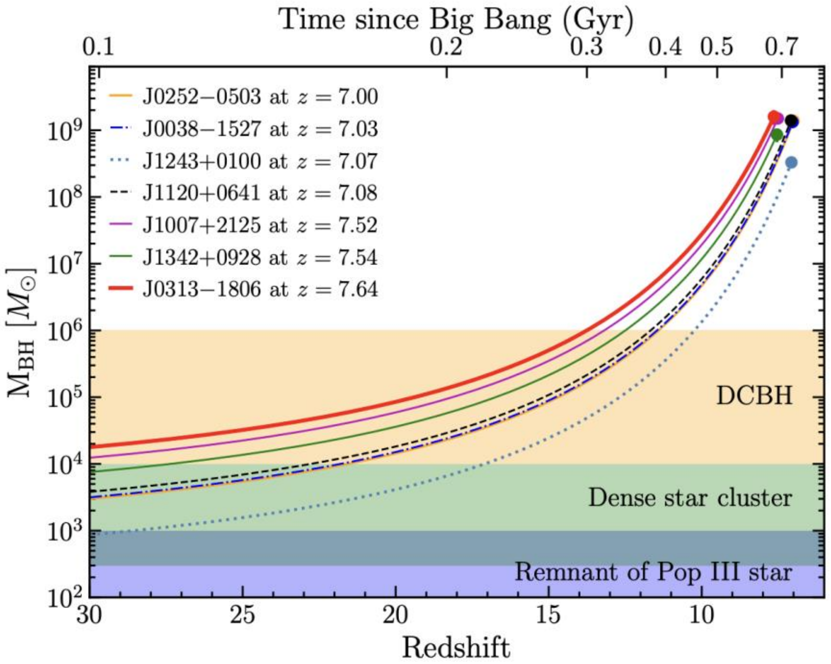


Type	Redshift
Little red dots	4.75 < z < 8.92
Galaxy candidates	z > 10
Accreting black holes	z >17

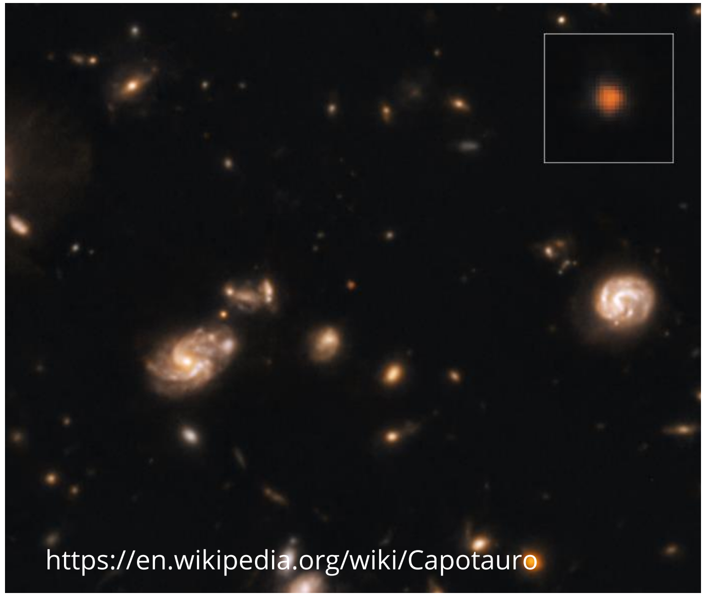
Observations

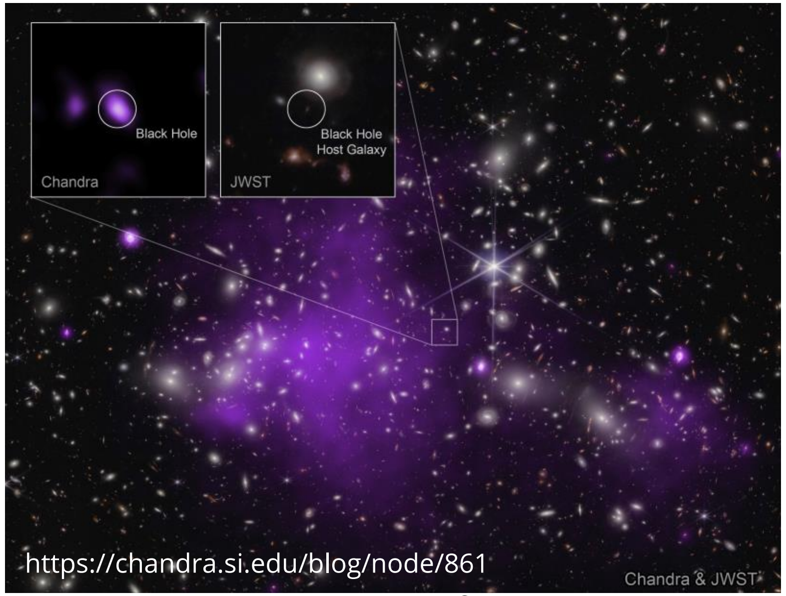


https://science.nasa.gov/asset/webb/little-red-dots-nircam-image/



https://arxiv.org/pdf/2101.03179


https://en.wikipedia.org/wiki/Capotauro



https://chandra.si.edu/blog/node/861

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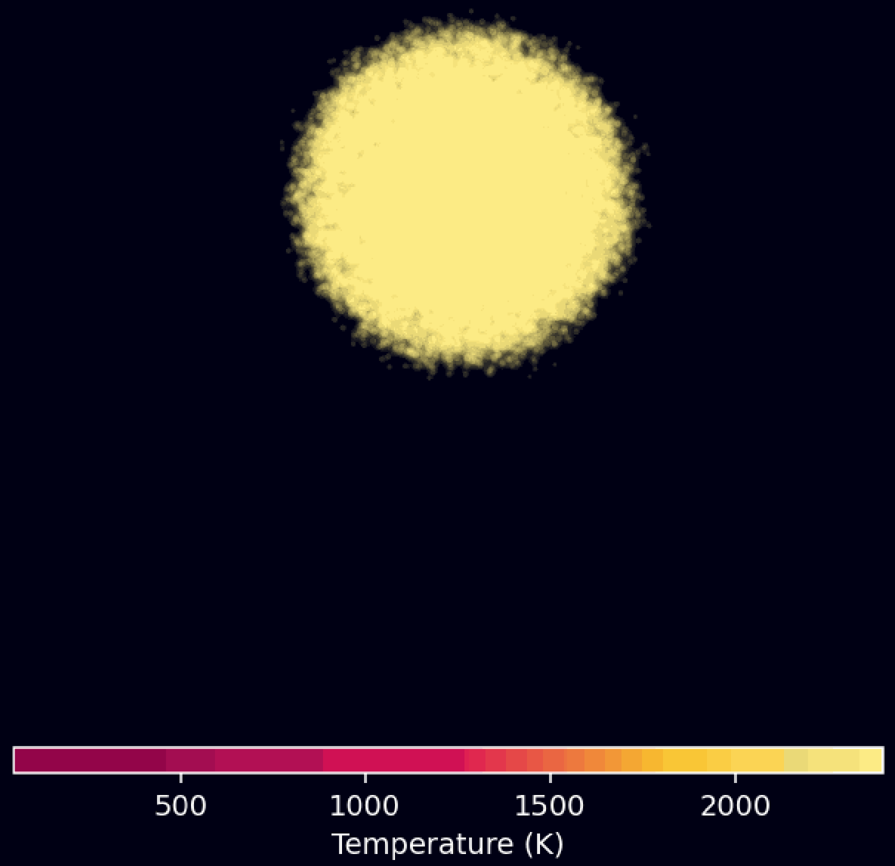
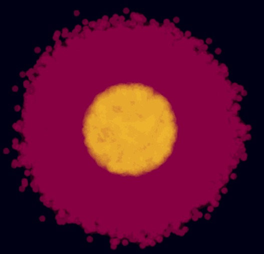
Direct Collapse Scenario



Rapid Cooling

Star Formation (Standard Scenario)

Moderate Cooling



SMBH Formation

Therefore, a moderate cooling rate is required to form a SMBH in the direct collapse black hole scenario.

Let's slow down the cooling

Cooling Channels:

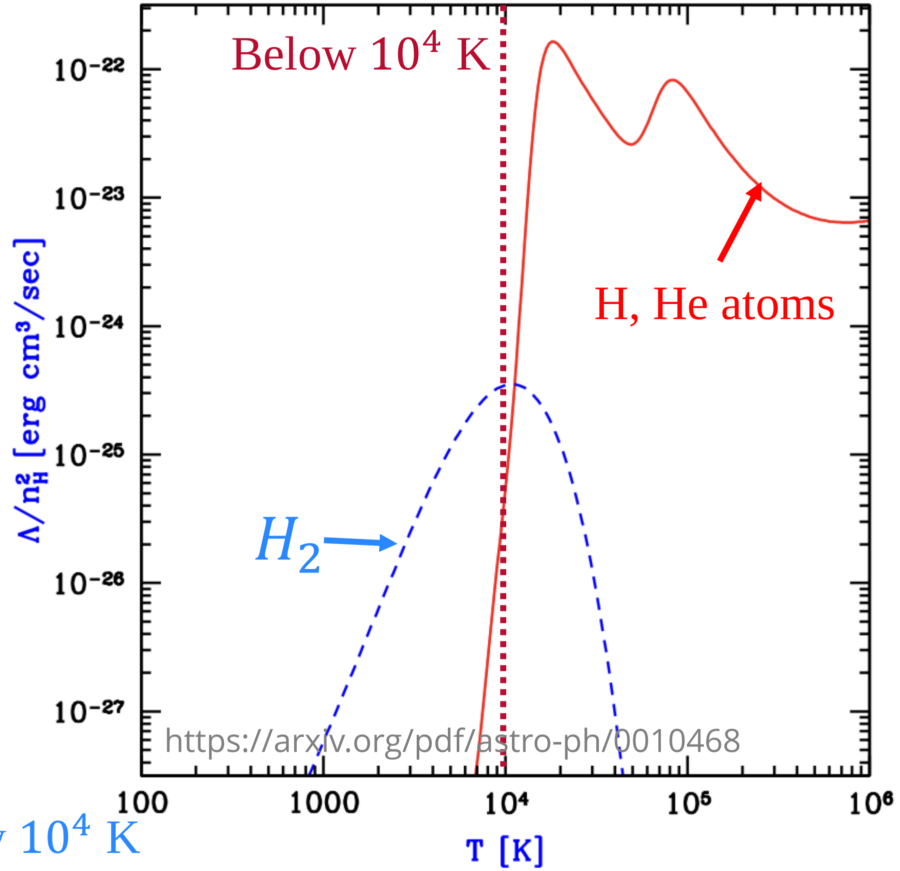
Above 10^4 K

- Collisional excitation cooling ($n_e n_H$, $n_e^2 n_{He^+}$, $n_e n_{He^+}$)
- Collisional ionization cooling ($n_e n_H$, $n_e n_{He}$, $n_e n_{He^+}$, $n_e^2 n_{He^+}$)
- Recombination cooling ($n_e n_{H^+}$, $n_e n_{He^+}$, $n_e n_{He^{++}}$)
- Bremsstrahlung cooling (All ionized species)
- Photoionization heating (n_H, n_{He}, n_{He^+})

Below 10^4 K

- H_2 cooling (Collisions to H, H_2, He, H^+ , e^-)
- H_2 radiative cooling (formation or destruction of molecular hydrogen)

- Compton cooling/heating off the CMB

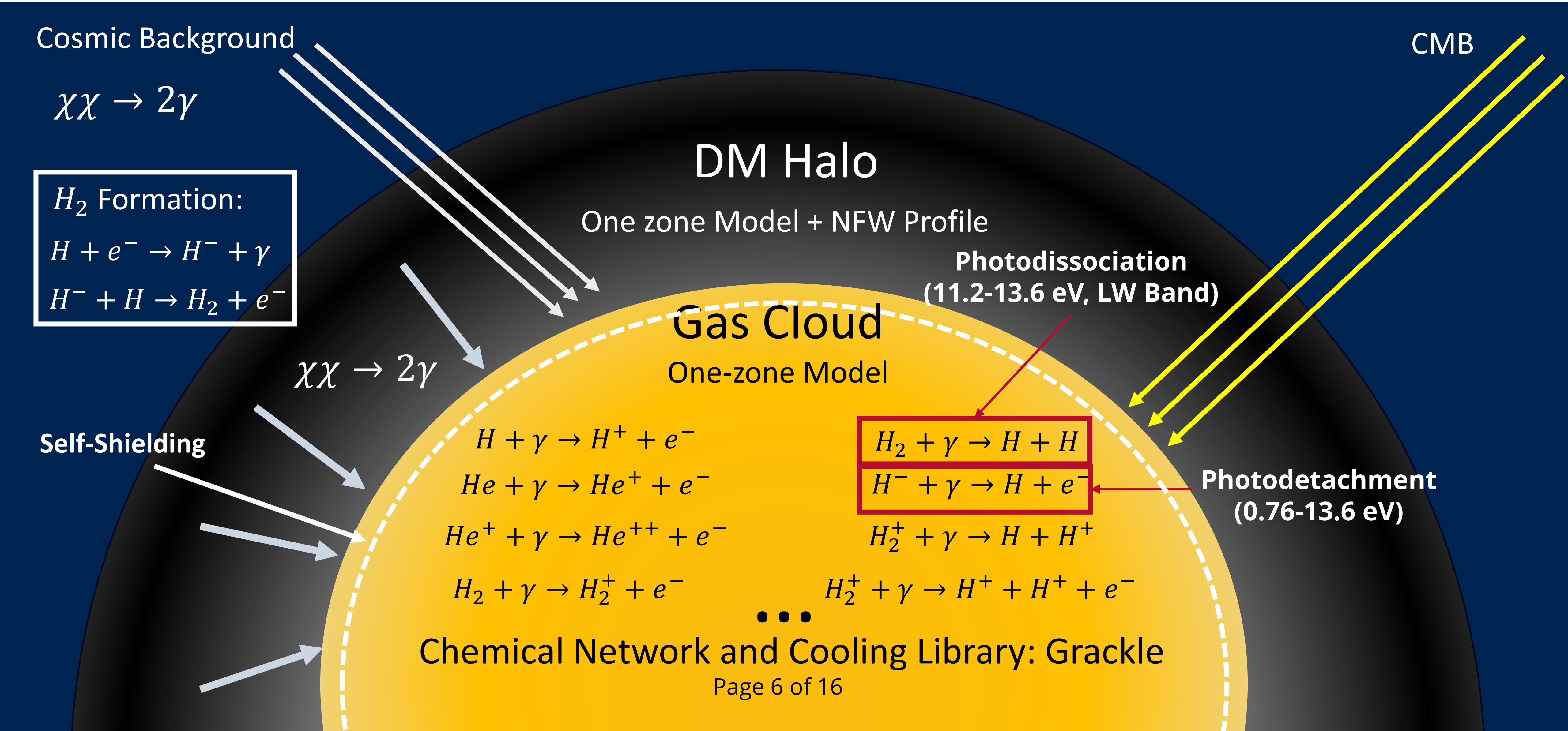


We can slow down the cooling by suppressing the formation of H_2.

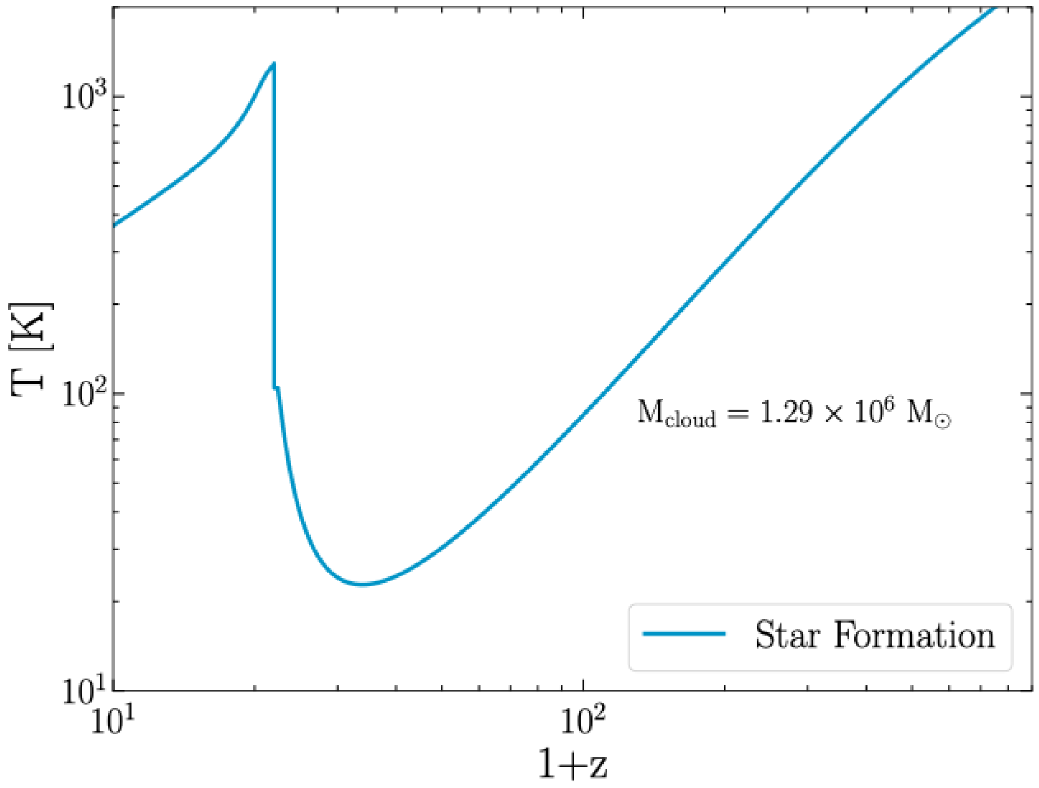
$$M_{gas\ cloud} \sim 10^6 - 10^7\ M_\odot$$

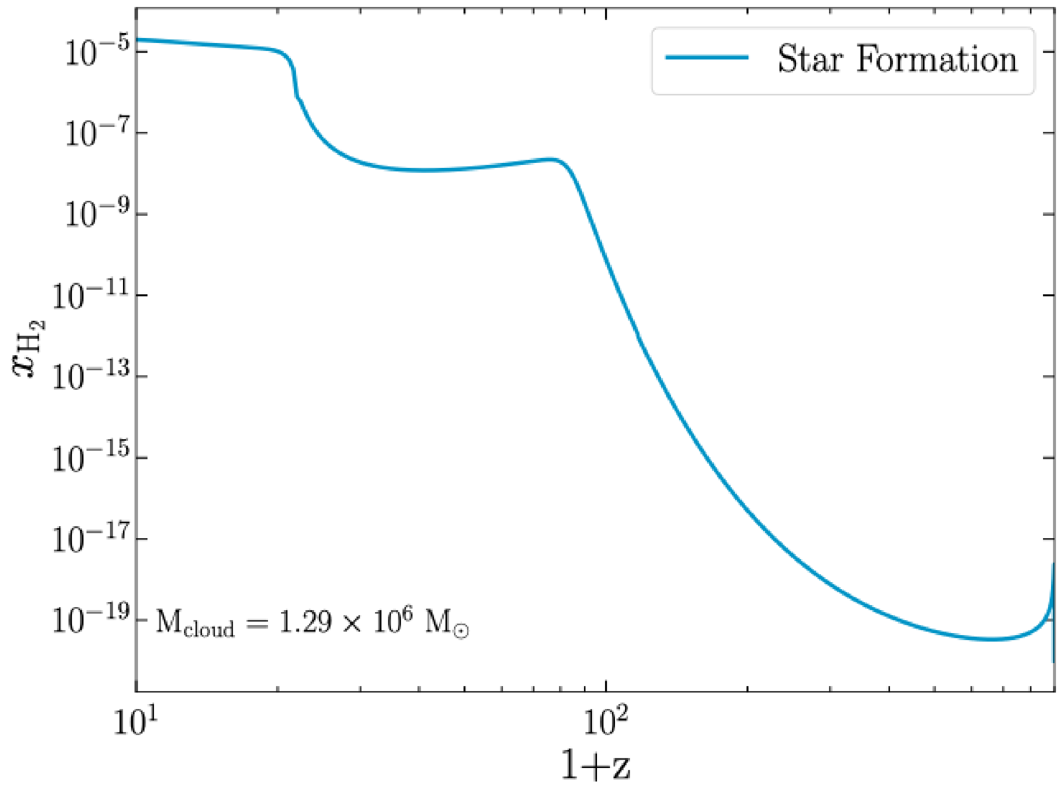
$$T_{vir} \sim 1000 - 6000\ K$$

Overview of the Model



Results: Thermal History





Present ← Past

DM Parameter:

- Star Formation: $m_\chi = 47.1 \ eV, \tau = 7.74 \times 10^{30} \ s$

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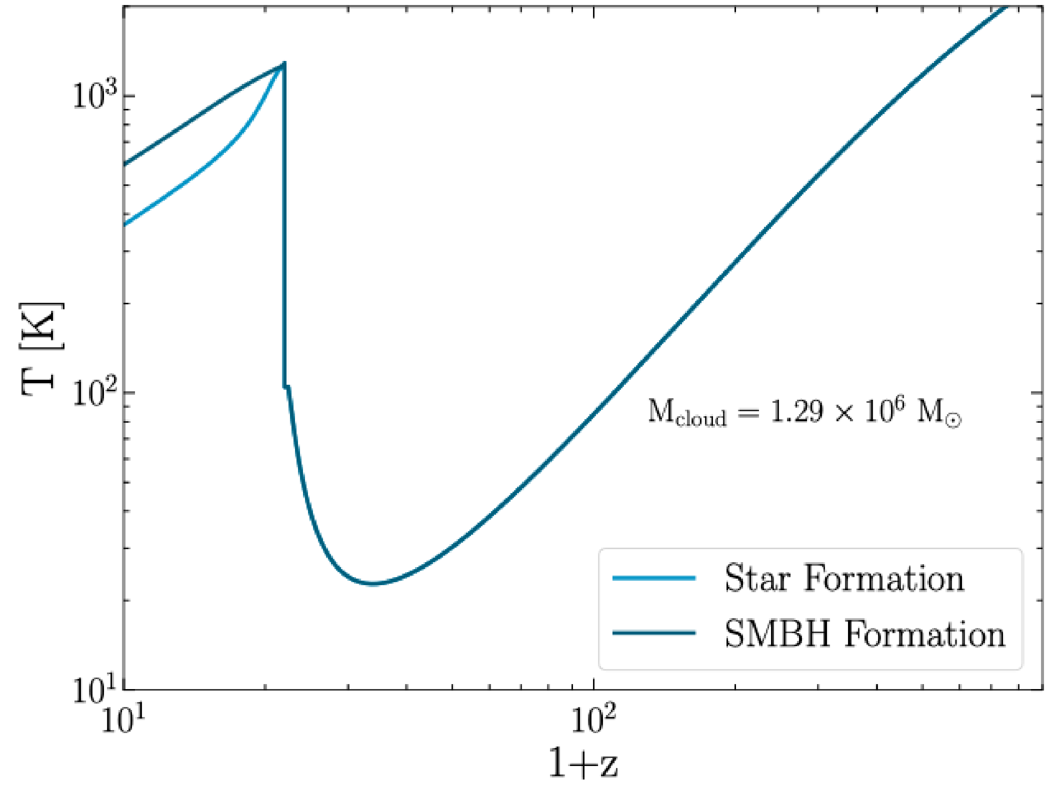

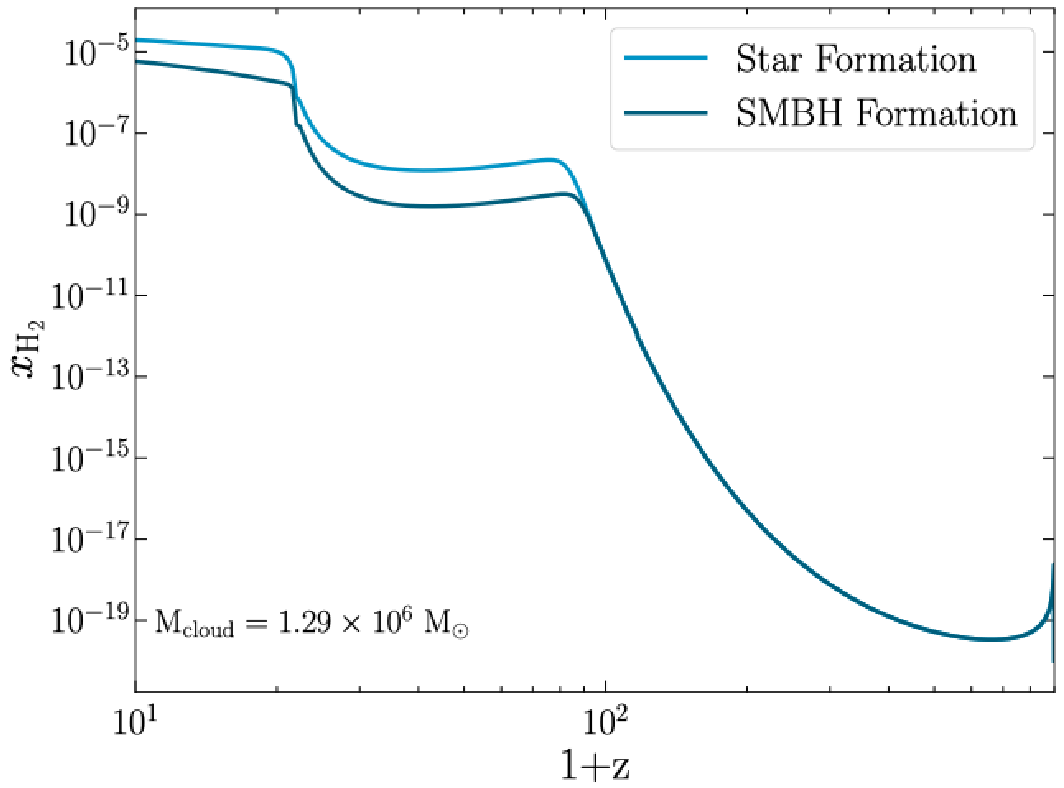

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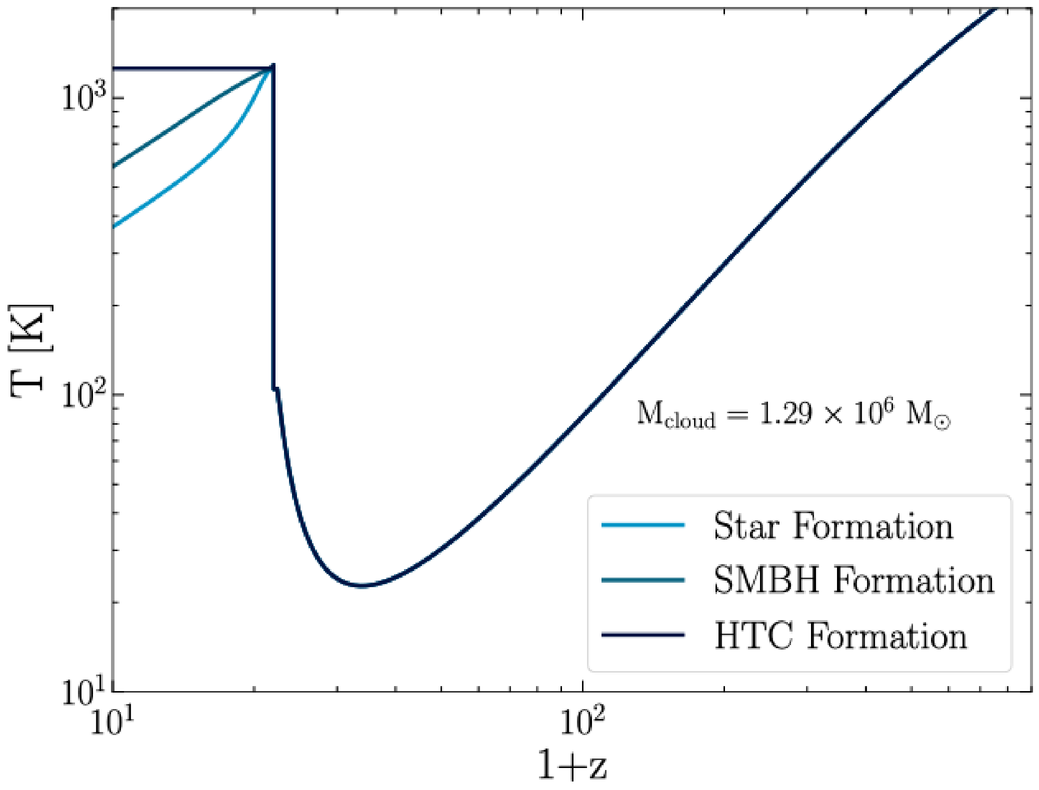

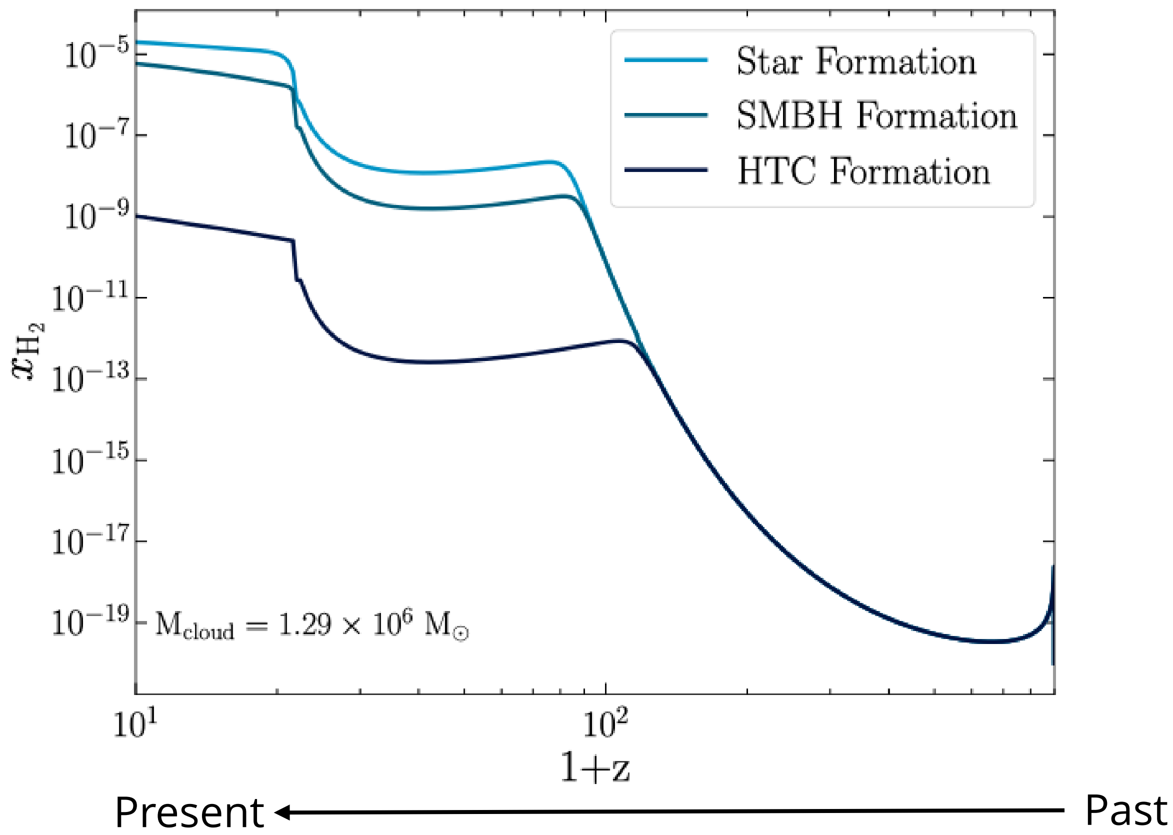

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- Supermassive Black Hole (SMBH) Formation: $m_\chi = 47.1\ eV, \tau = 1 \times 10^{30}\ s$

- High Temperature Cloud (HTC) Formation: $m_\chi = 22.86\ eV, \tau = 2.78 \times 10^{26}\ s$

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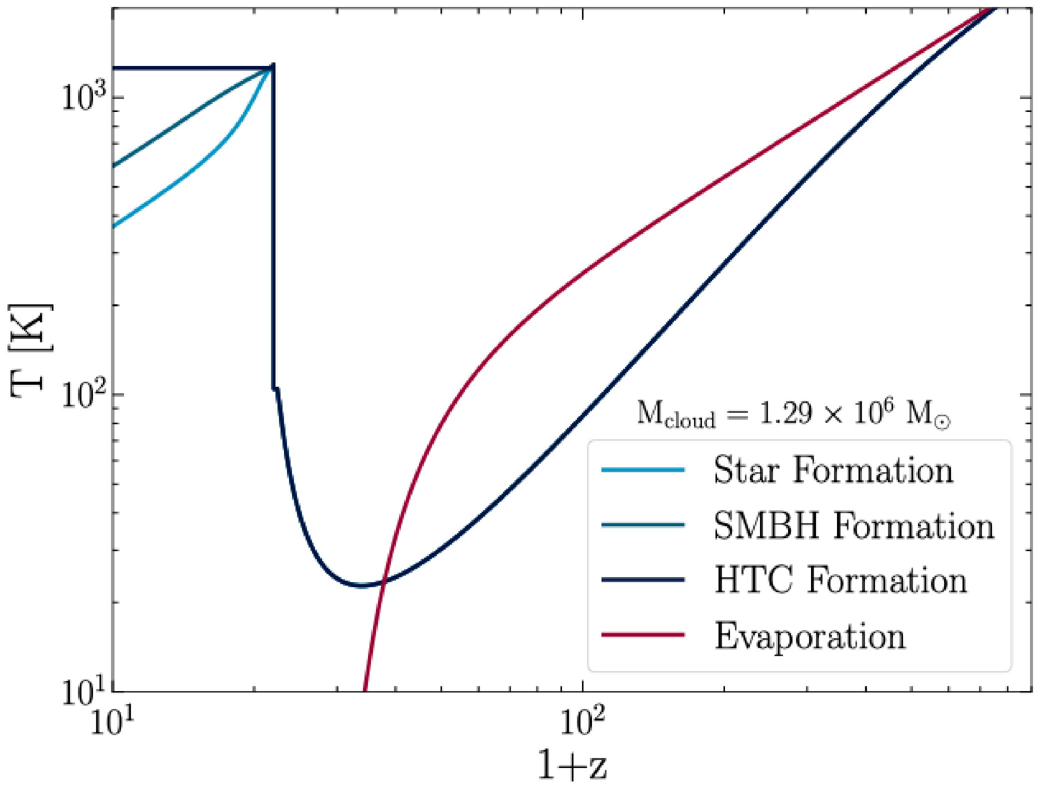

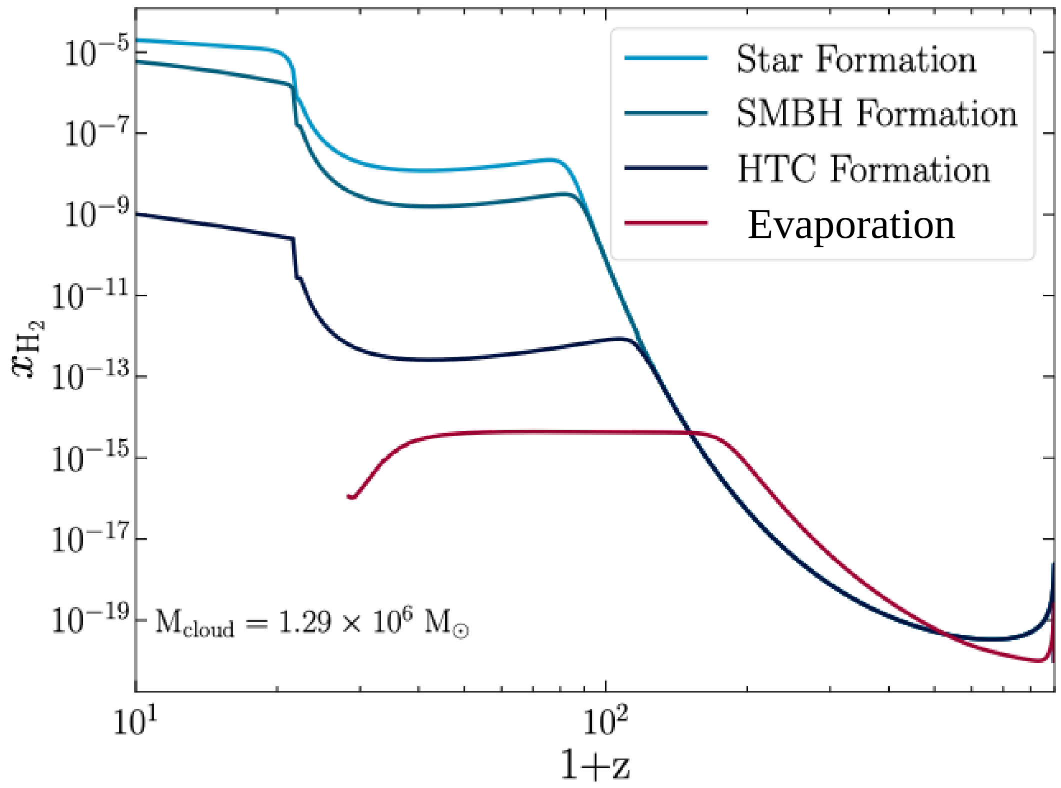

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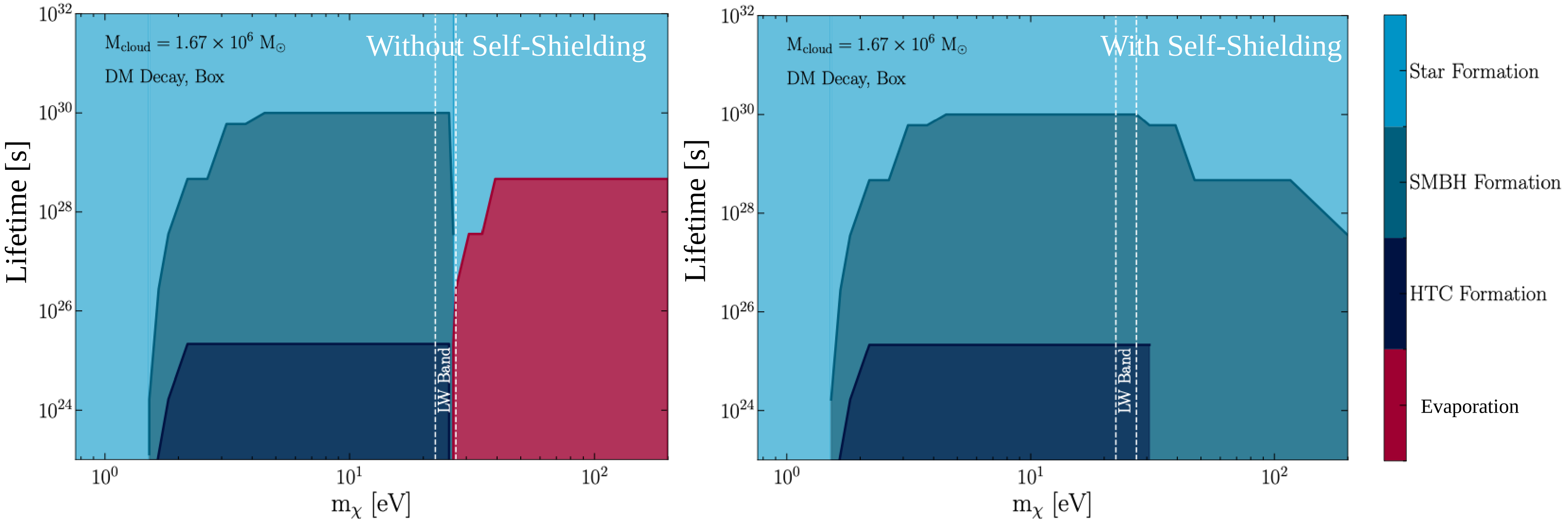
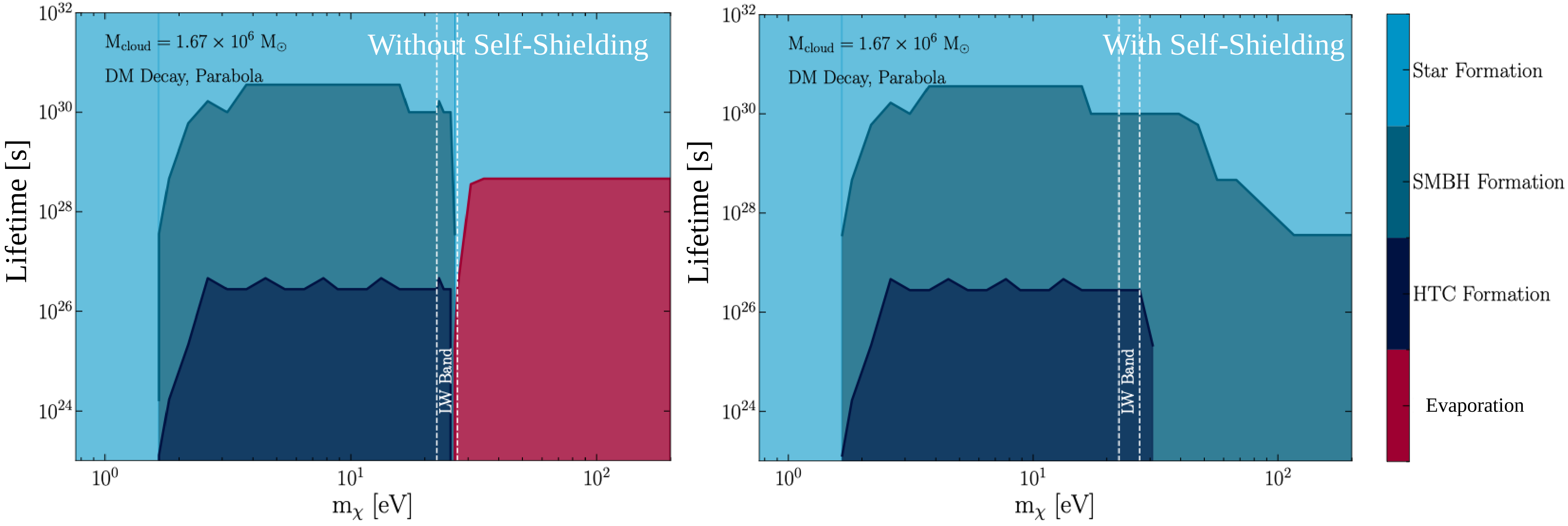
- High Temperature Cloud (HTC) Formation: $m_\chi = 22.86\ eV, \tau = 2.78 \times 10^{26}\ s$
- Evaporation Formation: $m_\chi = 47.1\ eV, \tau = 1 \times 10^{22}\ s$

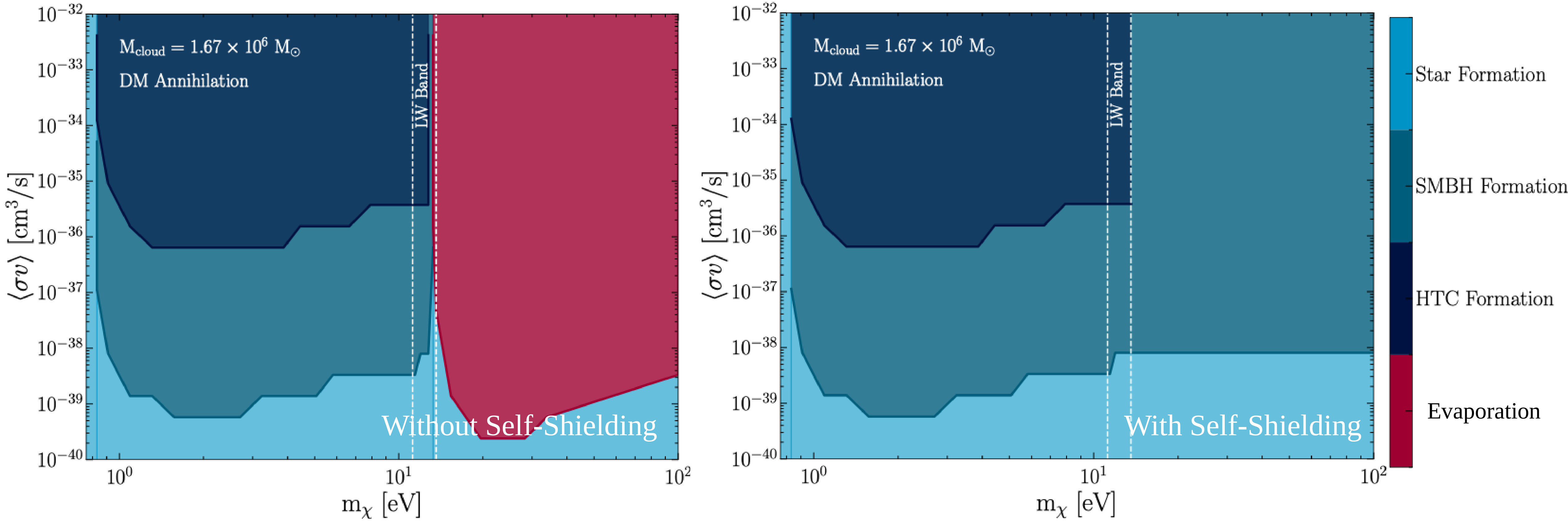
Phase Diagram: Decaying DM with Box Spectrum



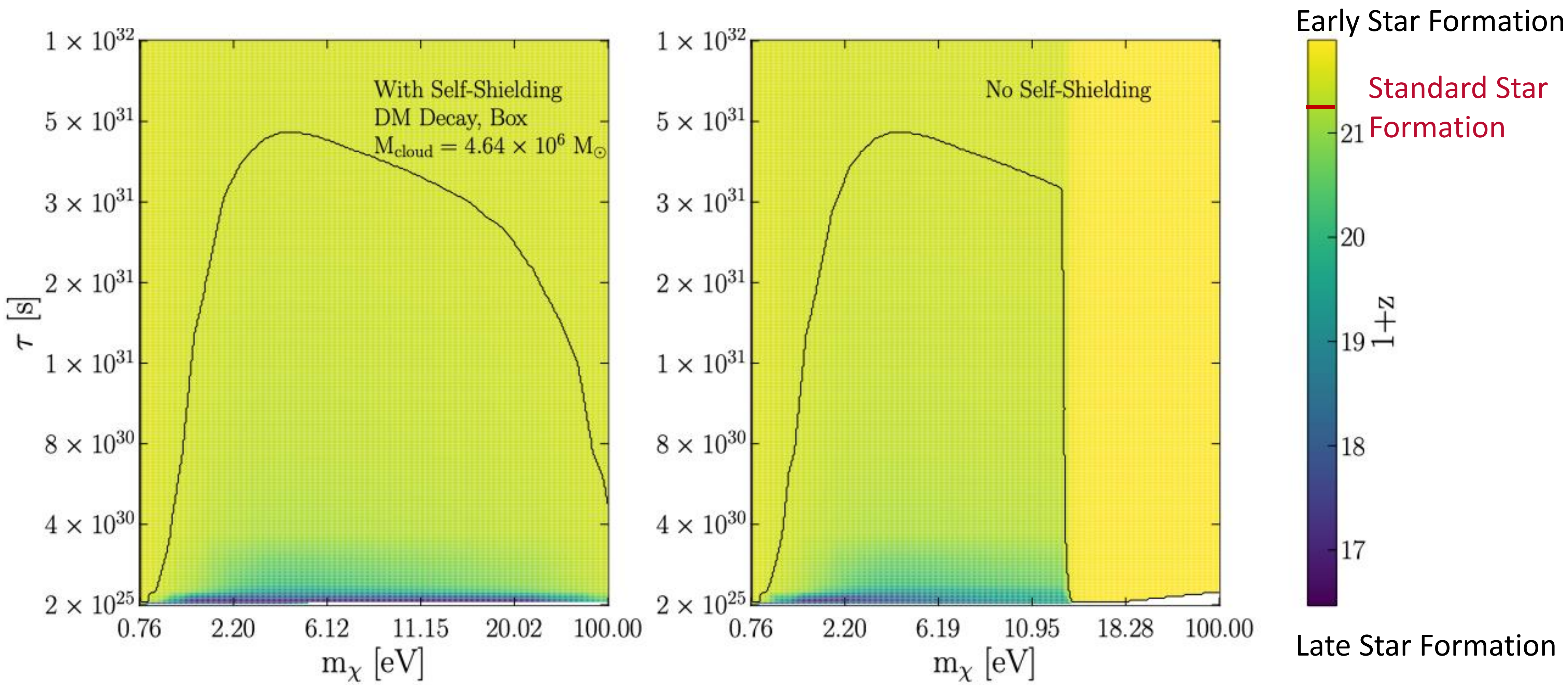
Phase Diagram: Decaying DM with Parabola Spectrum



Phase Diagram: Annihilating DM



Star Formation Time: Decaying DM with Box Spectrum



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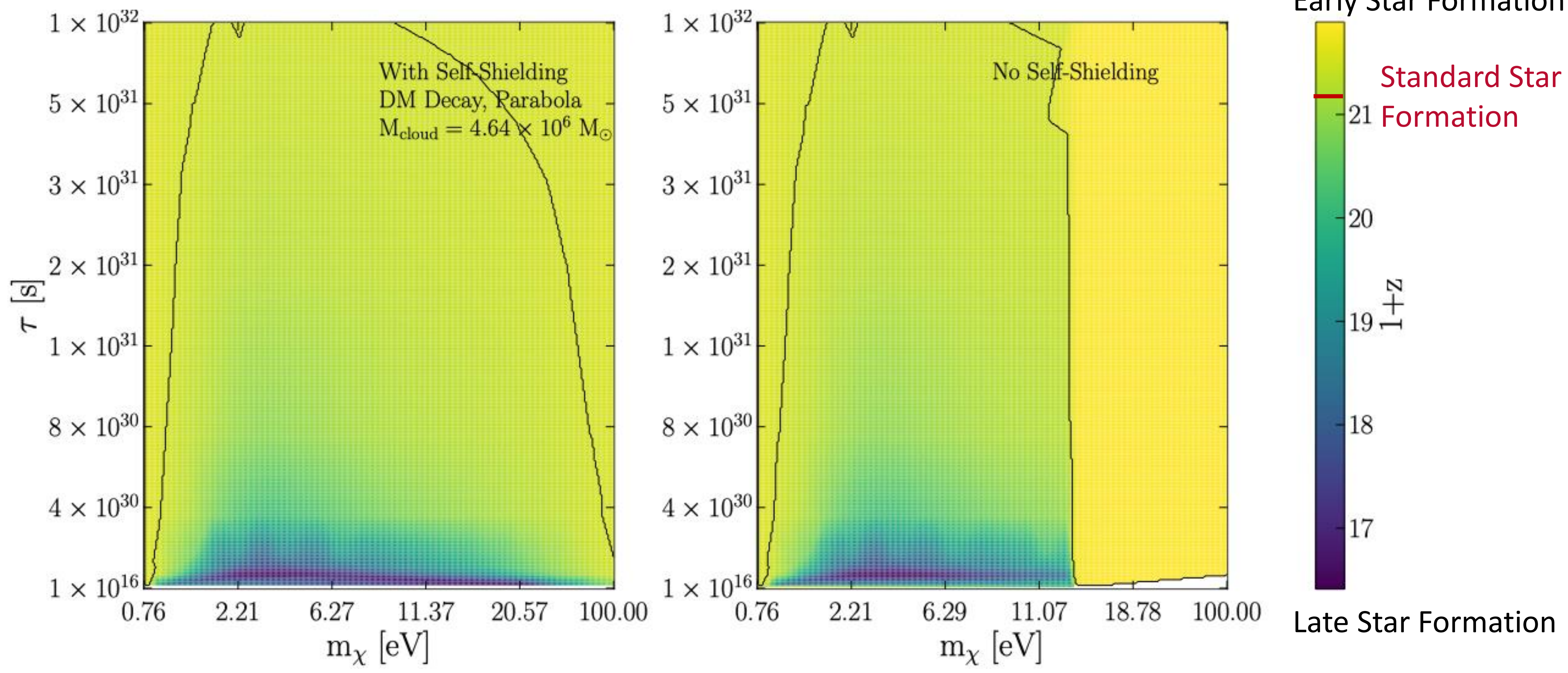

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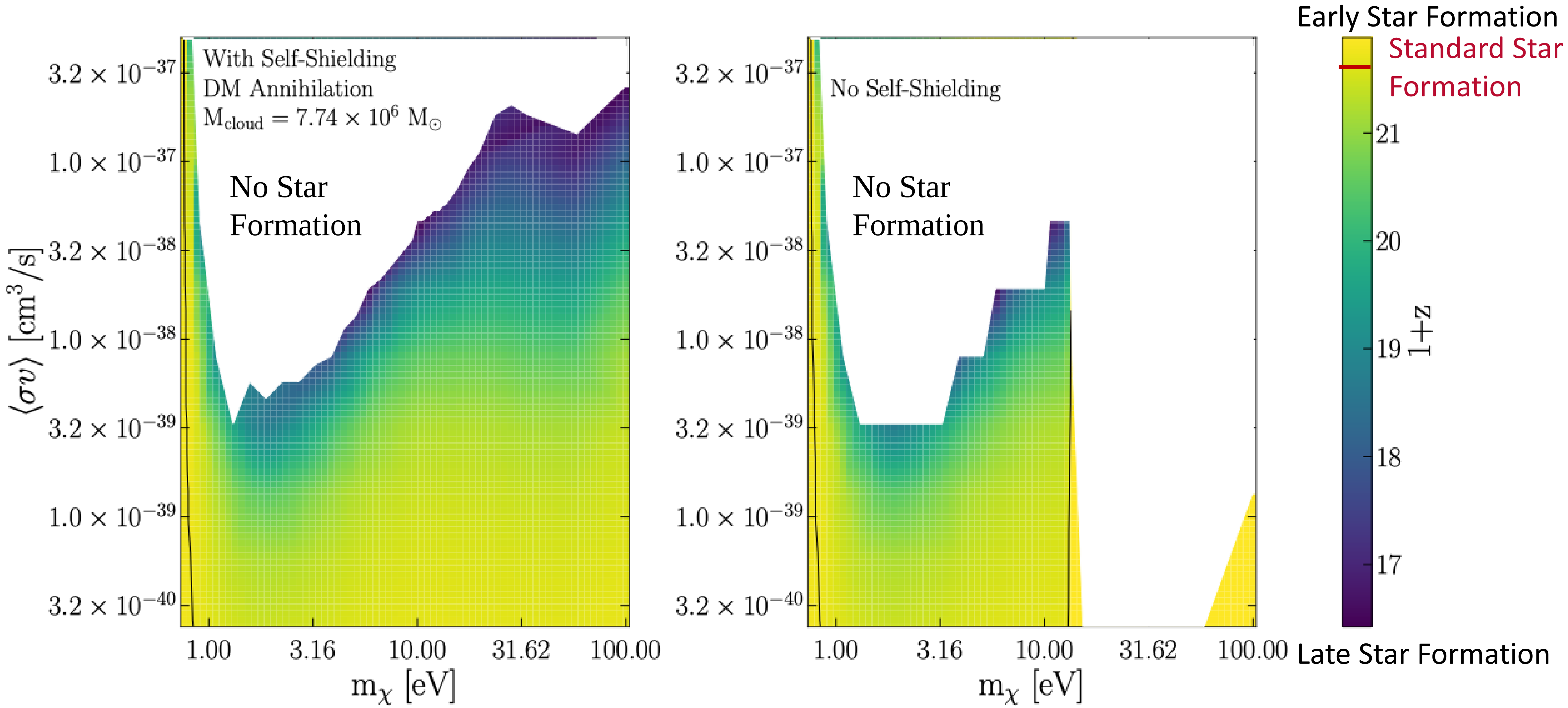
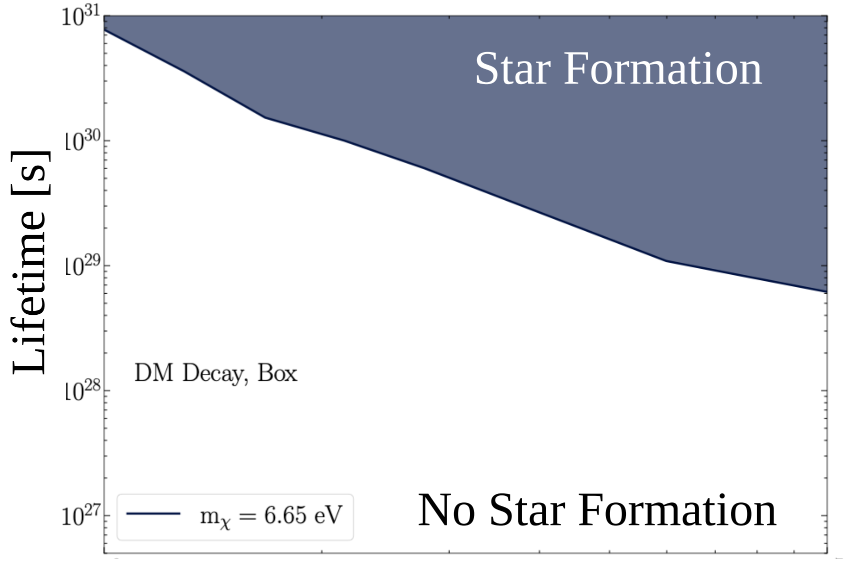
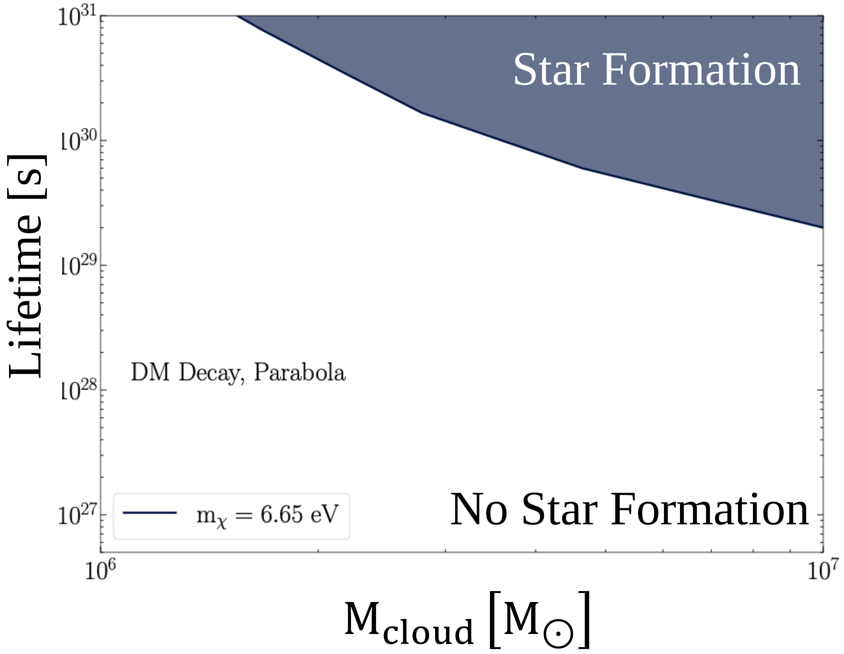
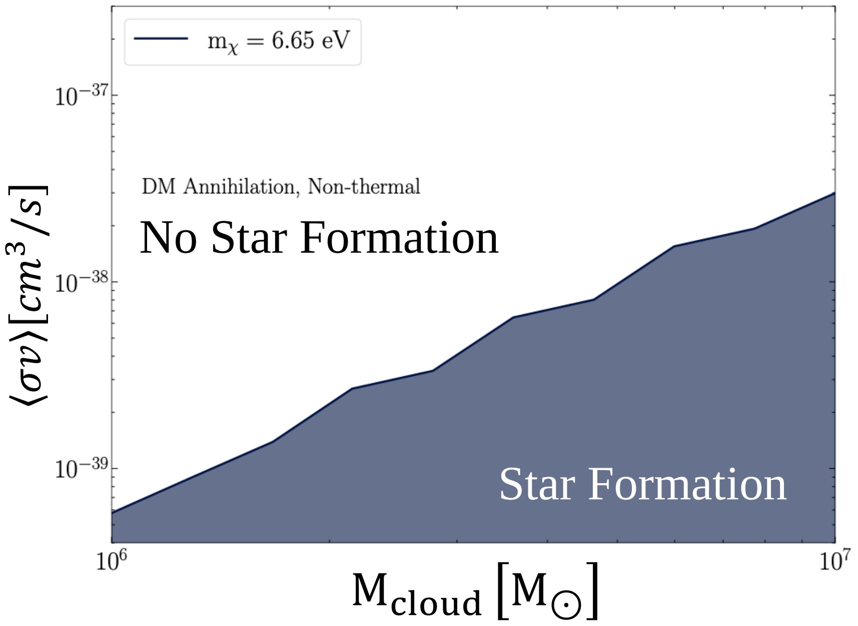


Star Formation: Gas Cloud Mass Critical Curves







$$J_{decay} \propto (\tau m_\chi)^{-1}$$

$$J_{ann} \propto \langle \sigma v \rangle m_\chi^{-2}$$

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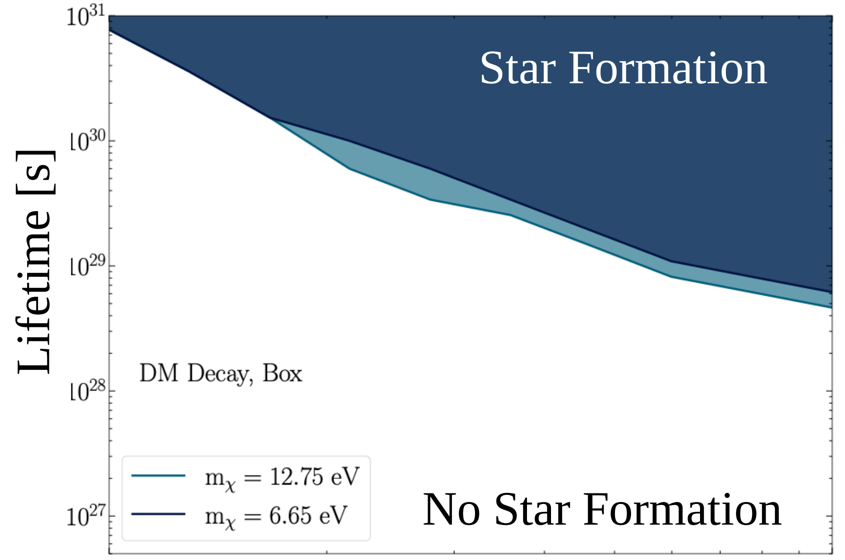

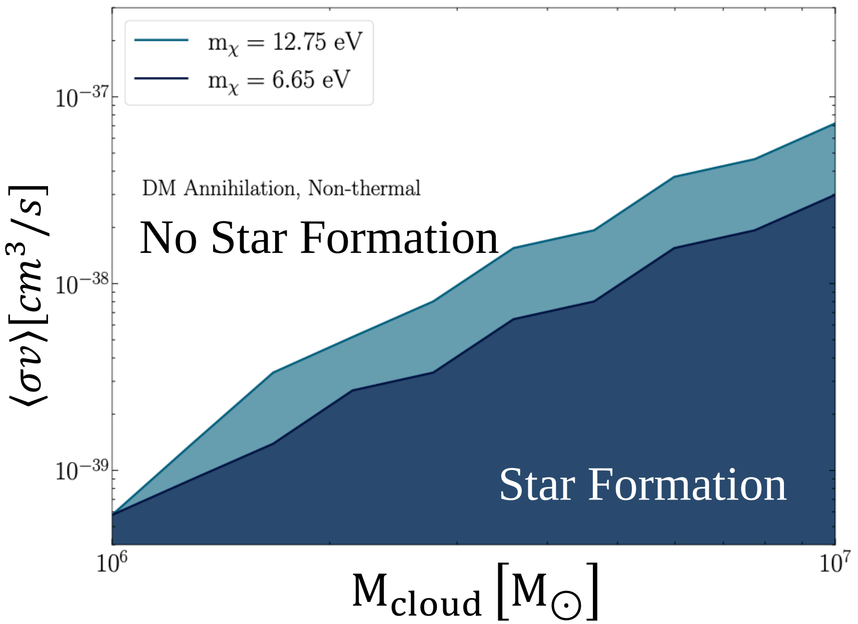

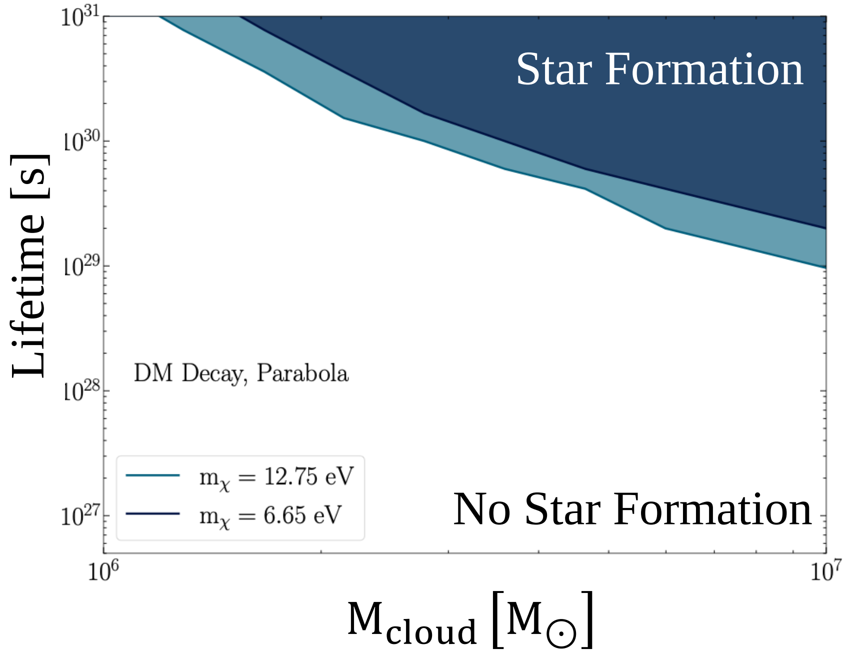

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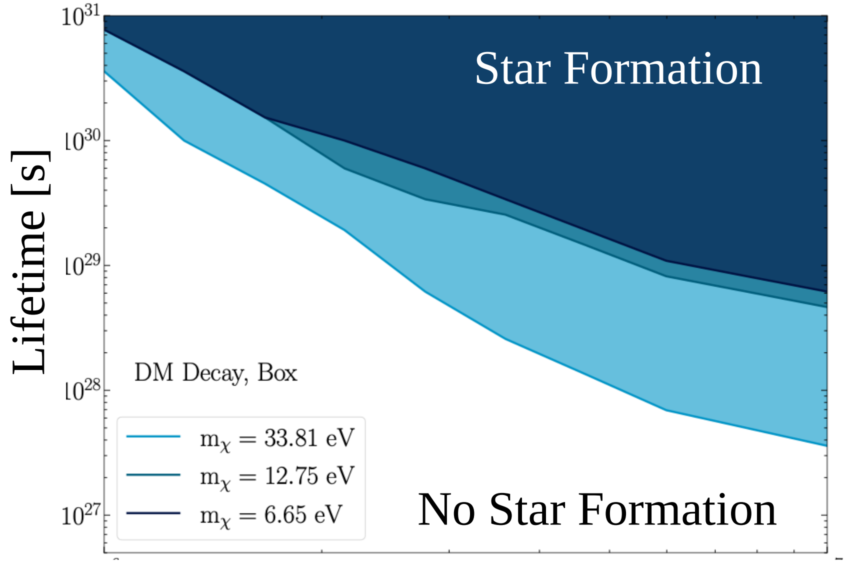

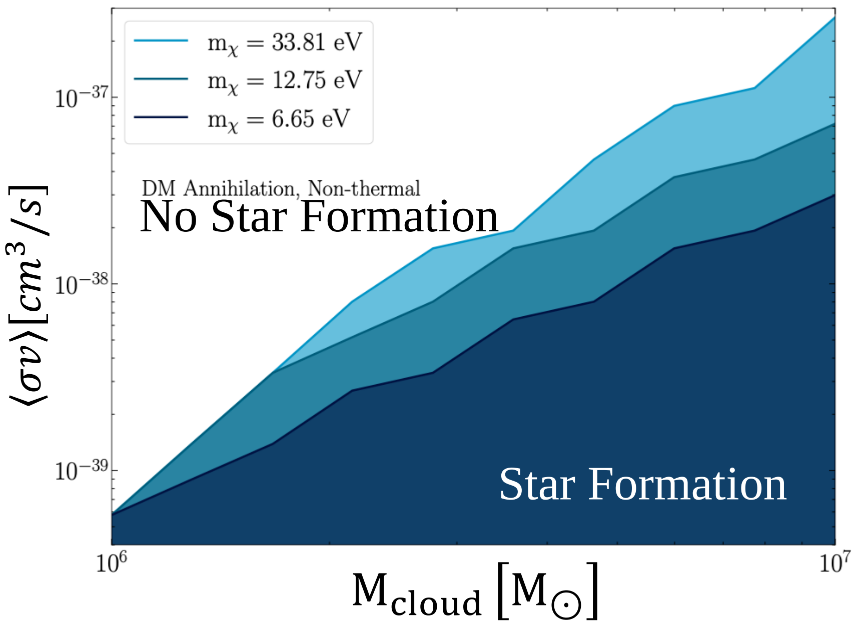

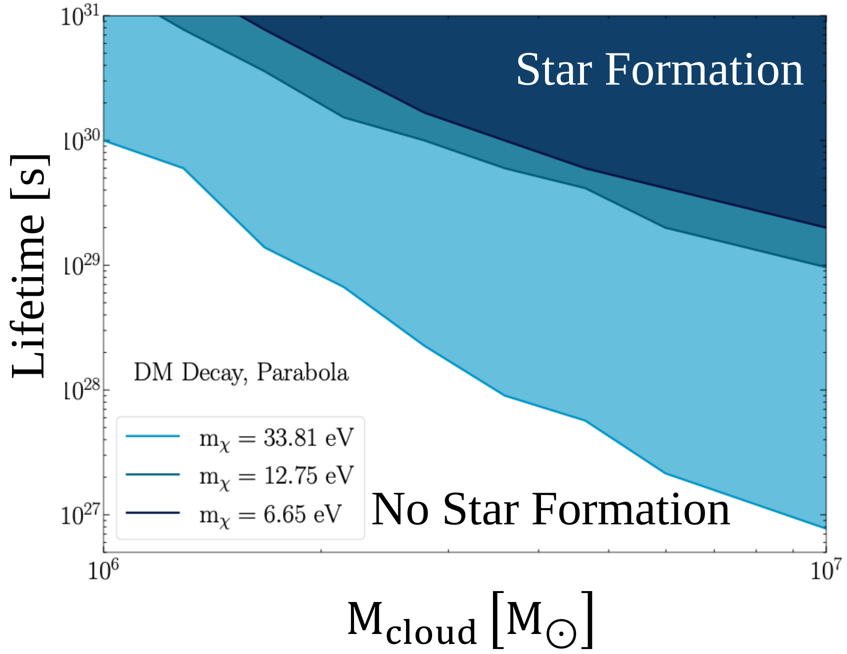

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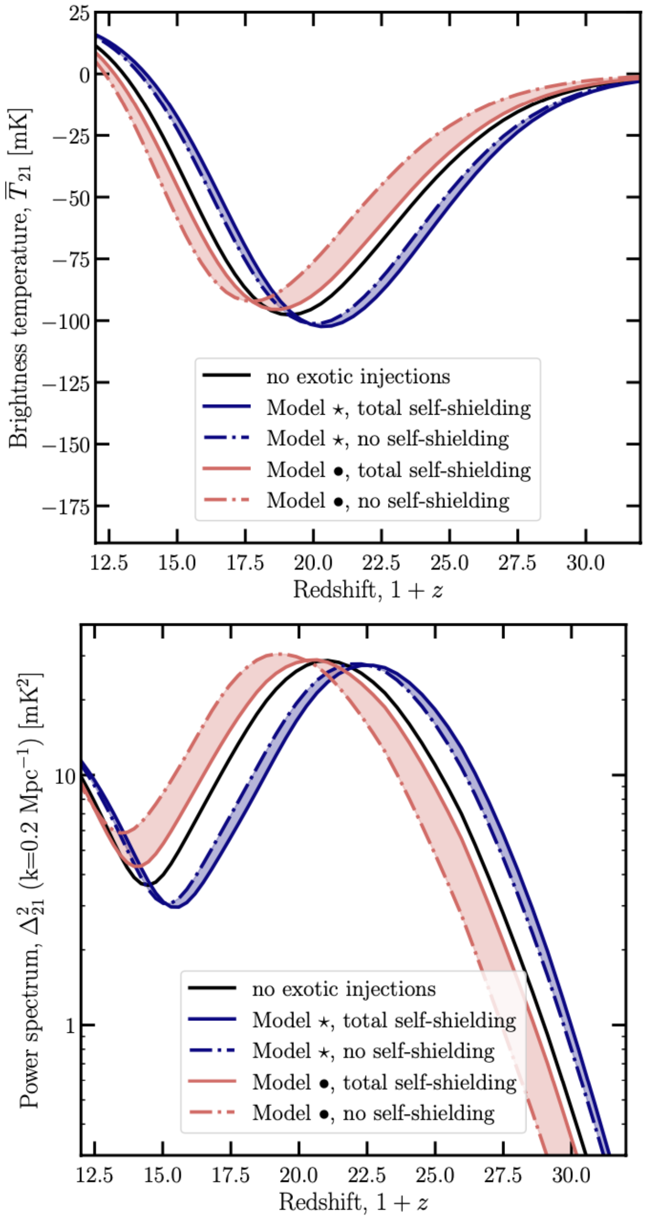
Ongoing Work

- Next Step:
  - How about WIMPs?

  Cascade: As propagating through the gas cloud, high-energy photons from heavier DM decay/ annihilation gradually lose energy via interactions with the ambient medium.

  - Observation Signals: 21 cm signal, JWST
- Ambitions:
  - 1-D hydrodynamic simulation
  - 3-D hydrodynamic simulation



https://arxiv.org/abs/2308.12992

Takeaway Information

- Dark matter with different parameter values can alter the evolution of a gas cloud, leading to the formation of the first stars, SMBHs, a high temperature cloud, or simply a cold overdensity region in the universe.
- In the direct-collapse black hole scenario, photons produced by DM decay or annihilation can suppress H_2 formation and slow cooling, promoting SMBH formation.
- In the first-star formation scenario, these photons can either delay or accelerate star formation.
- When self-shielding is included, higher-mass DM can further contribute to H_2 destruction, thereby affecting the thermal and ionization history of the gas cloud.

Thank You!

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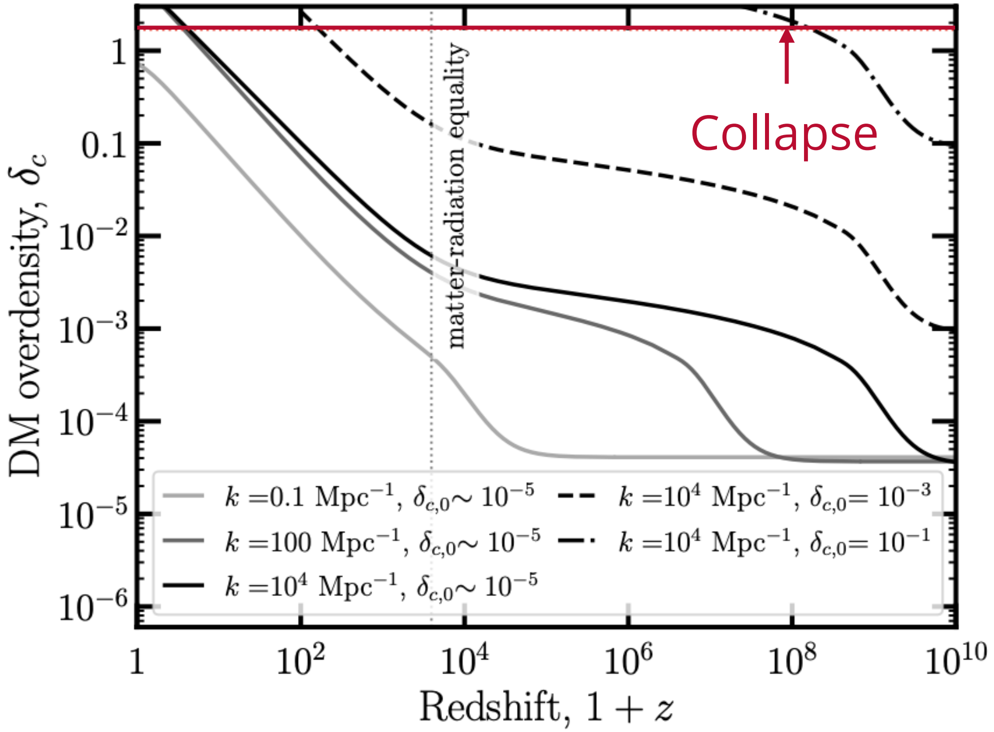
Backup

Structure Collapse in ΛCDM Model

ΛCDM Model:

- the existence and structure of the cosmic microwave background
- the large-scale structure in the distribution of galaxies
- the observed abundances of hydrogen (including deuterium), helium, and lithium
- the accelerating expansion of the universe observed in the light from distant galaxies and supernovae



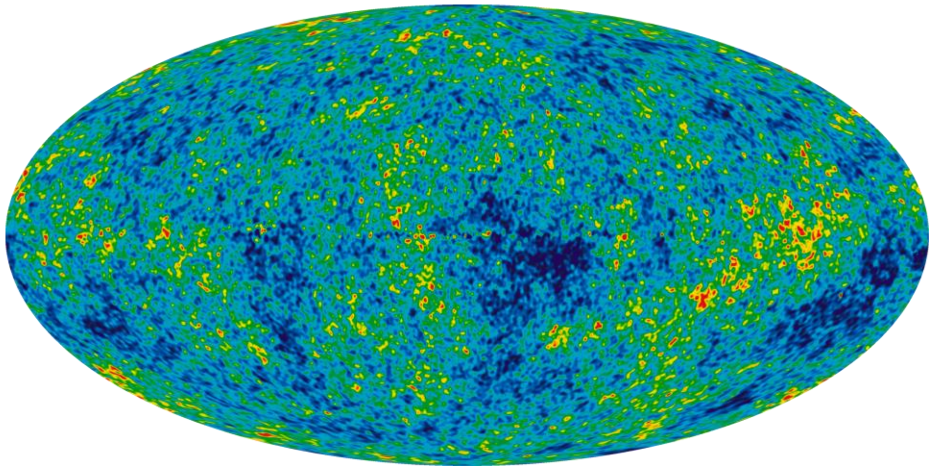
https://arxiv.org/pdf/2506.13858

Initial Density Fluctuation:

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

The typical amplitude of an initial fluctuation $\sim 10^{-5}$

All modes collapse at similar redshifts $1 + z \lesssim 10$

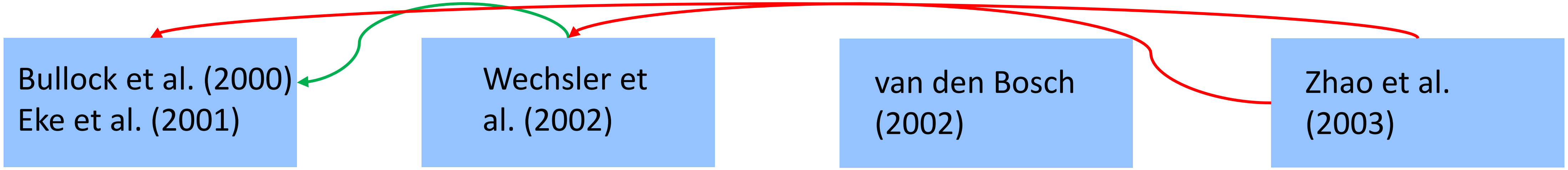


https://science.nasa.gov/mission/wmap/wmap-overview/

Backup: Mass Accretion Histories (MAH)

Questions about the fixed mass modeling:

- Bullock et al.(2000) found that the halo concentration (at fixed mass) is systematically lower at higher redshift
- haloes of fixed mass have significant scatter in their c values (Jing 2000), although there is a mean trend of c with M_h
- $c \propto (1+z)^{-1}$ corresponds to the scaling of the virial radius for an object whose mass did not increase with redshift, that is for an isolated system surrounded by a void. Haloes never exist in complete isolation, however; as the virial radius increases, it will enclose more matter, increasing both the total mass and the virial radius further.



$$c \propto \frac{(M/M_\star)^{-0.13}}{(1+z)}$$

$$M(z) = M(z_{\rm obs})\exp[-2(z - z_{\rm obs})/(1+z_f)]$$

the average MAH of CDM halos follows a simple universal function with two scaling variables.

the MAH of a halo in general consists of two distinct phases
halos which are still in the fast growth regime should all have a similar concentration

Backup: Concentration Parameter

Concentration Parameter Epoch

- **Before NFW**

  Power law density profile → Steeper profiles for large n → Galaxy cluster density profiles show no clear dependence on n → Significant departures from power-law behavior

- **NFW and later (slow phase, $c \propto (1+z)^{-1}$)**

  Halo density profiles with a universal shape that does not depend on their mass, on the power spectrum of initial fluctuations, or on the cosmological parameters

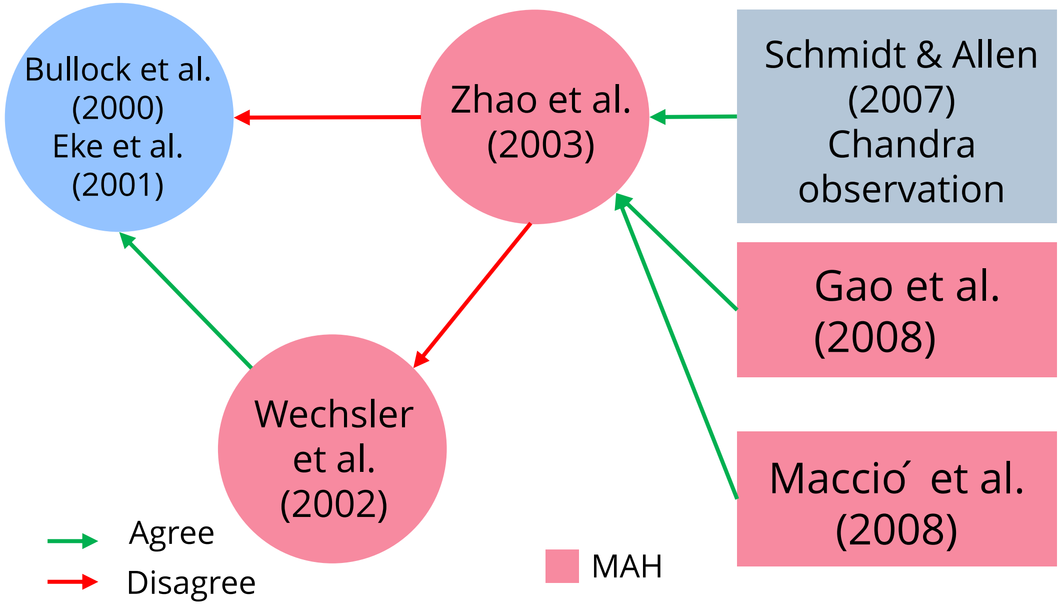
- **Mass Accretion Histories (MAH)**

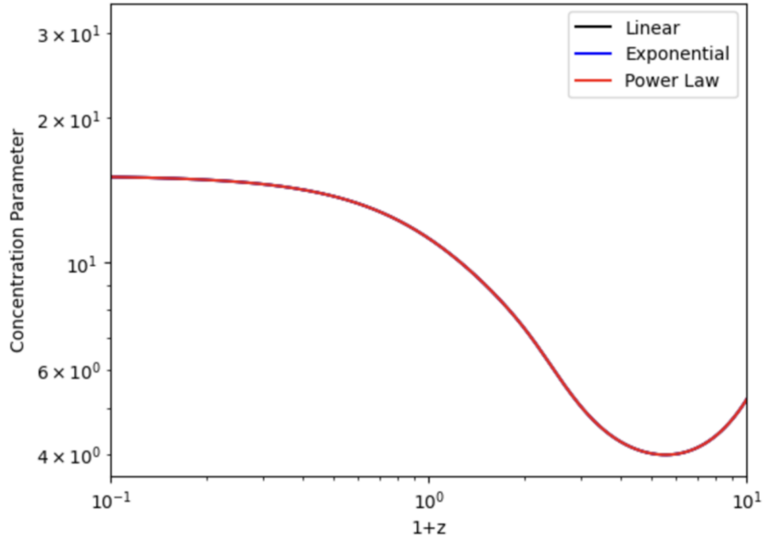
  Early rapid phase

  Late slow phase

- **Propose and confirm of Rapid Phase ($c \sim 3-4$)**

  The typical binding energy of the halo is approximately equal to that of a singular isothermal sphere with the same circular velocity





Modeling

- **Gas Cloud and DM Halo**

  One-Zone model, NFW Profile

- **DM Decay/ Annihilation**

  Photon Spectrum, Photon source

- **Chemical Reaction Rate**

  Photodissociation, Photodetachment, Photoionization, Photoheating

- **Thermal and Ionized History Simulation**

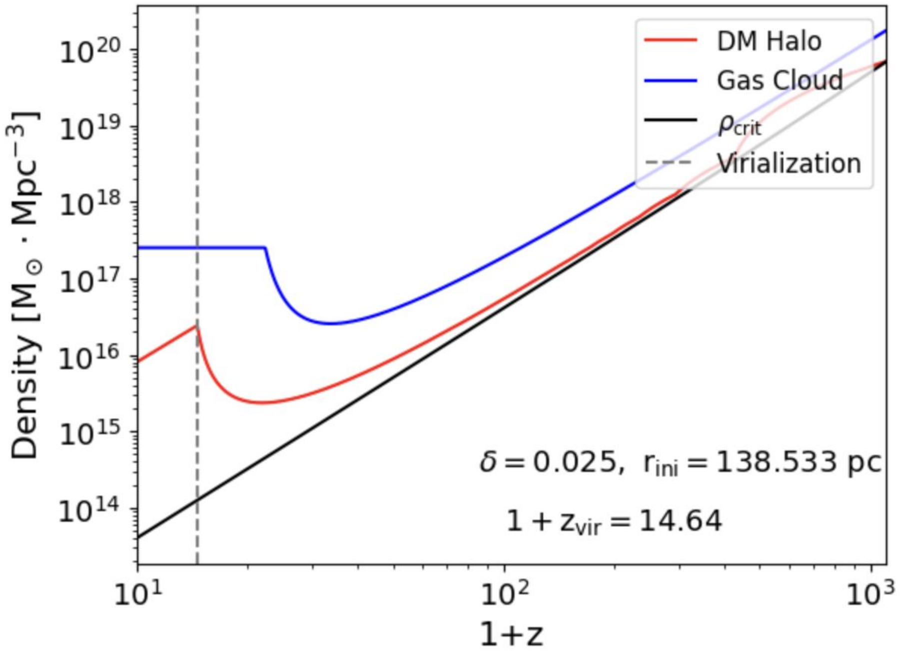
  Initial Conditions (CLASS), Grackle

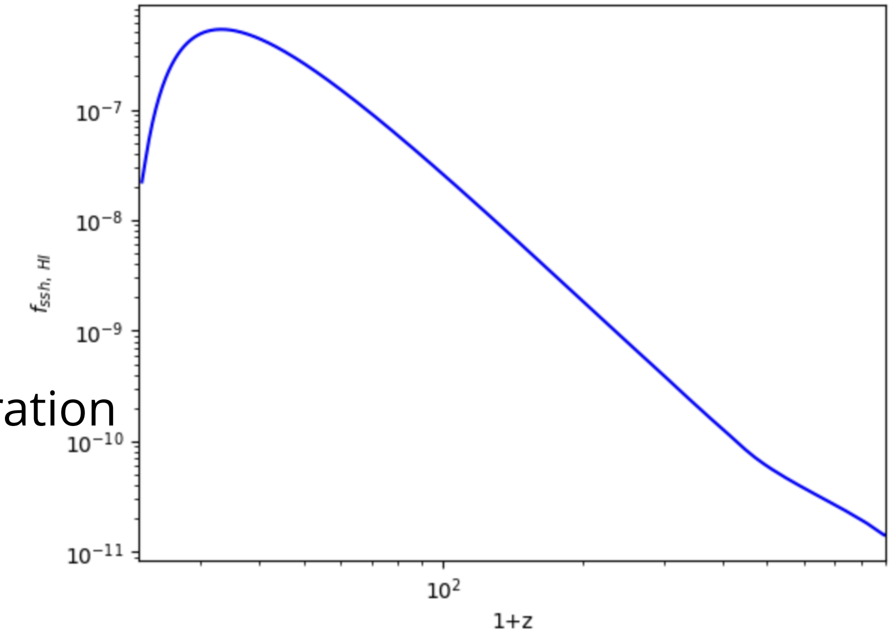
- **Criteria**

  Star, SMBH, High Temperature Cloud Formation and Evaporation

- **Self-Shielding**

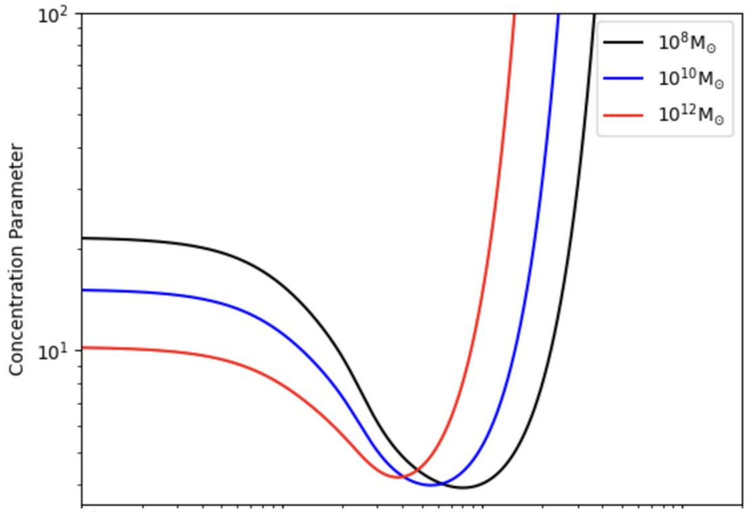
  Fitting Formula from IGM Studies





Backup: B-Factor

Owing to the limited investigation of the mass–redshift–concentration relation and the scarcity of simulations at high redshift ($z \sim 10 - 1100$), its behavior in this regime remains poorly constrained. Although Prada et al. (2012) proposed a fitting formula, it becomes unreliable for redshift above 10.



- **DM Halo**

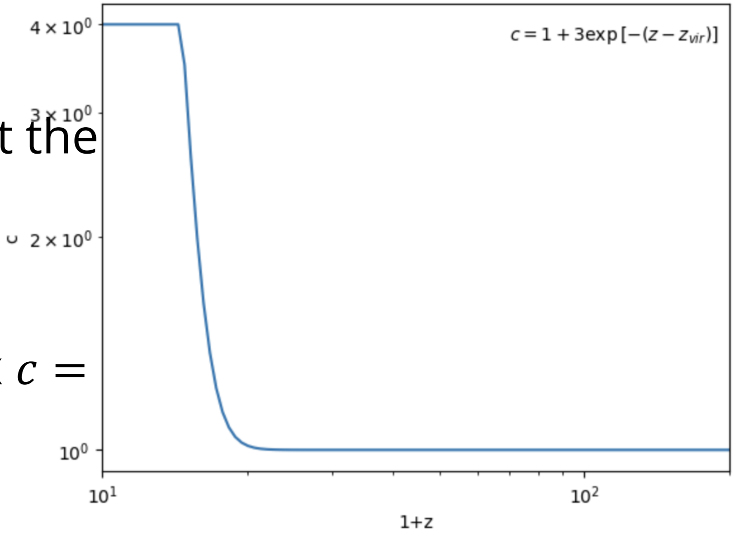
1. Solving ODE with initial conditions:

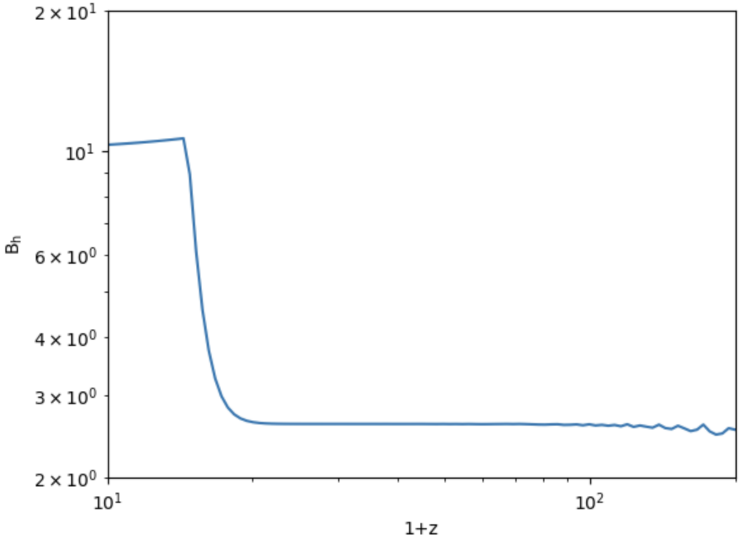
$$\frac{d^2 r}{dt^2} = -\frac{G M_{halo}}{r^2}$$

2. Using an artificial mass–redshift–concentration relation function, fit the NFW profile

$$c = 1 + e^{-(z - z_{vir})}$$

3. After halo reaches virialization, fix $c = 4$, fit the NFW profile





Modeling: Gas Cloud and DM Halo

- **DM Halo**
  - NFW Profile:

$$\rho_{NFW} = \frac{4\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

$$r_s = r_{200}/c \qquad \rho_s = \frac{c^3(1+c)M_{200}}{16\pi r_{200}^3[-c+(1+c)\ln(1+c)]}.$$

  - Halo Radius r_{200}
  - Halo Density ρ_{200}

$$\frac{d^2r}{dt^2} = -\frac{GM_{halo}}{r^2}$$

  - Concentration parameter c

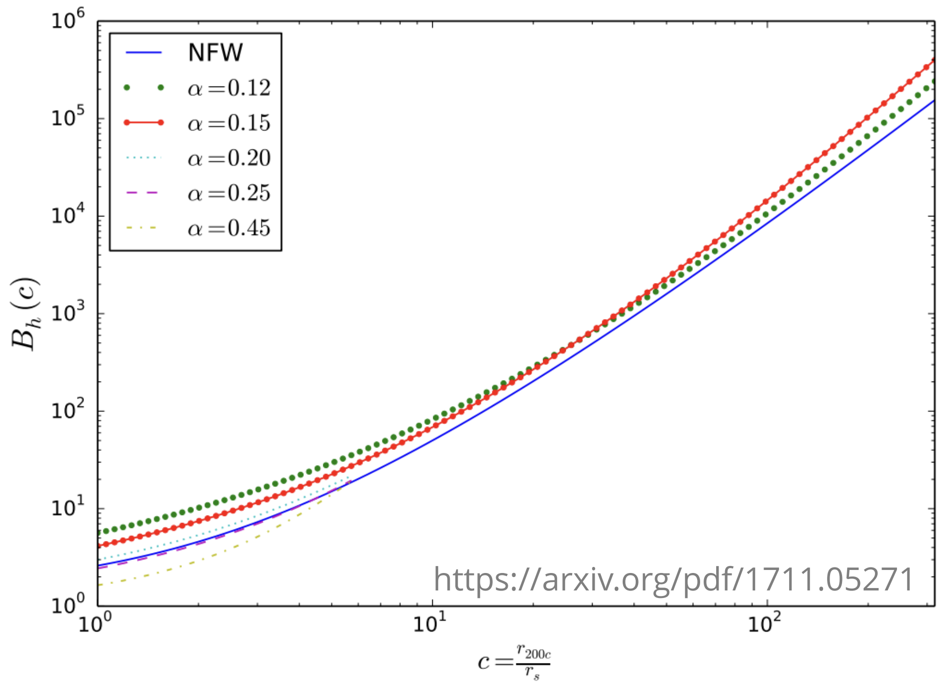
$$c \equiv \frac{r_{200}}{r_s}, \qquad c = \frac{v_{peak}}{v_{vir}}$$

A measure of the size or density of the central region relative to the size or density of the whole halo.

$$c = ?$$

Halo Boost Factor

$$B(V) \equiv \frac{1}{\bar{\rho}^2 V}\int \rho^2 dV$$



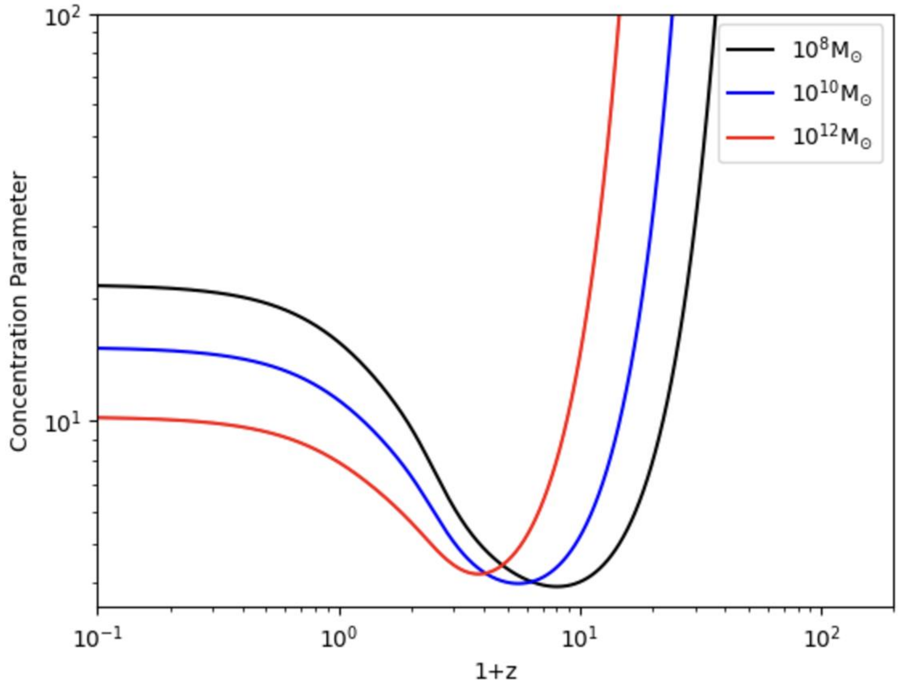
B characterizes the inhomogeneity of the particle distribution

Modeling: Gas Cloud and DM Halo

Gas Cloud DM Halo → DM Decay Annihilation → Chemical Reactions → Simulation Criteria

- **Concentration Parameter**
  - Origin Paper of NFW Profile → $\boldsymbol{c \propto (1+z)^{-1}}$
  - Later Simulations agree with $c \propto (1+z)^{-1}$
  - Simulations of fixed mass halo underestimate the c → **Mass Accretion History (MAH)**
  - MAH Halo Evolution:
    - Rapid Phase: Halos start as rare density peaks and experience fast and nearly radial infall that brings mass closer to the centre.
    - halos slide into the plateau regime where the accretion becomes less radial
    - Slow Phase: halos move into the domain of declining concentration–mass relation

Rapid Phase → ? How long it will take? → Slow Phase



Modeling: Gas Cloud and DM Halo

Gas Cloud DM Halo → DM Decay Annihilation → Chemical Reactions → Simulation Criteria

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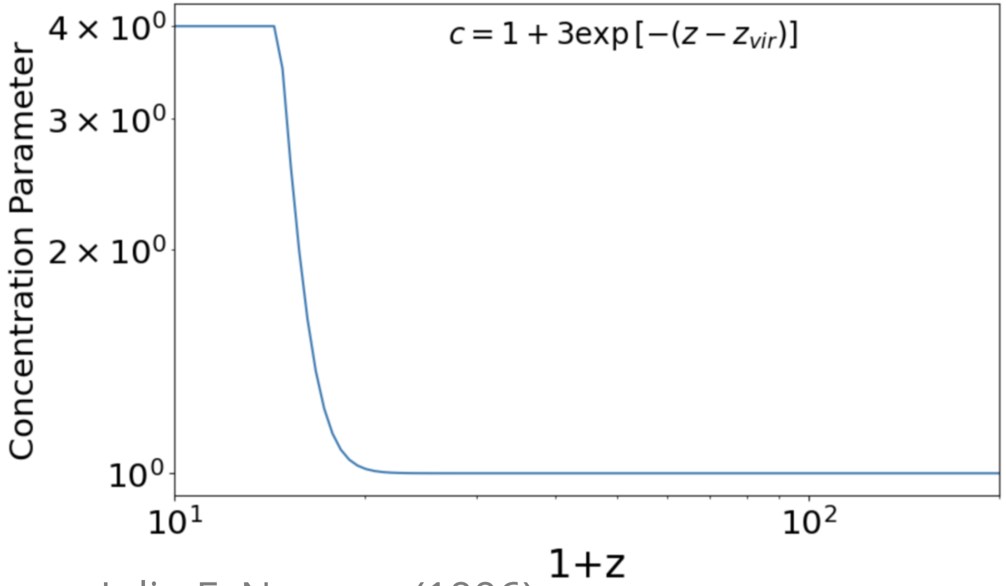
    - Slow Phase: halos move into the domain of declining concentration–mass relation
- **Interpolating Concentration Function**

$$c = \begin{cases} 1 + e^{-(z - z_{vir})}. & \text{Before virialization} \\ 4 & \text{After virialization} \end{cases}$$



Julio F. Navarro (1996)
Ewa L. Lokas (1999)
Bullock et al.(2000)
Zhao et al. (2003)
Gao et al. (2008)

Modeling: Gas Cloud and DM Halo

- **DM Halo: NFW Profile**

- Solving ODE with initial conditions:

$$\frac{d^2r}{dt^2} = -\frac{GM_{halo}}{r^2}$$

- Using an modified mass–redshift–concentration relation function, fit the NFW profile

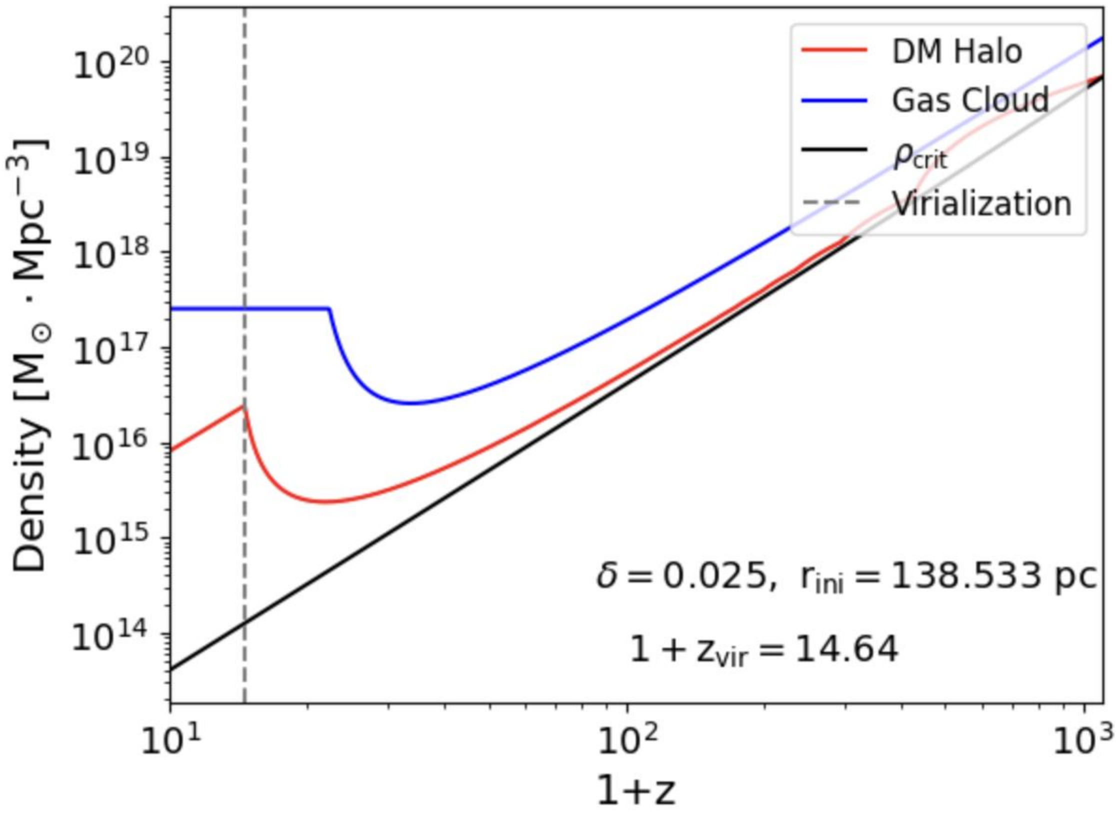
$$c = 1 + 3e^{-(z-z_{vir})}$$

- 3. After halo reaches virialization, fix $c = 4$, fit the NFW profile

- **Gas Cloud: One Zone Model**

$$\rho(z) \approx \rho_0(1+z)^3 \exp\left(\frac{1.9A}{1-0.75A^2}\right)$$

$$A = \frac{1+z_{\rm vir}}{1+z}$$



$z_{vir} = 20$

$M_{halo} = M_{cloud}/f_b$ $\quad$ $f_b = \Omega_{baryon}/\Omega_m$

Modeling: DM Decay/ Annihilation and Source of Photon

Gas Cloud DM Halo → **DM Decay Annihilation** → Chemical Reactions → Simulation Criteria

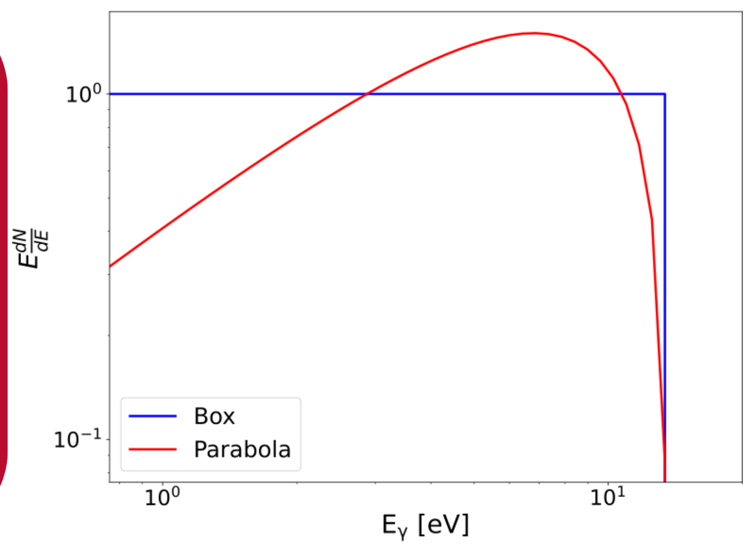
- DM Decay ($\chi \to \gamma\gamma$ ...)
  - Box Spectrum

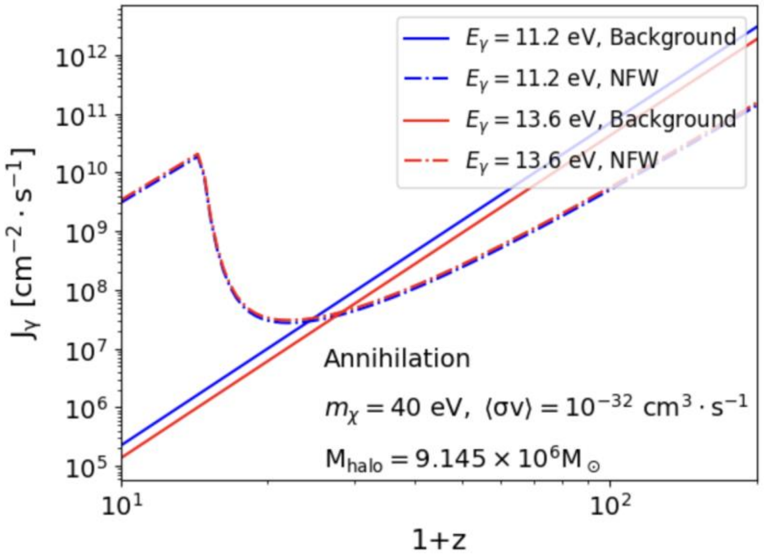
$$E\frac{dN}{dE}\bigg|_{\text{box}} = \Theta\left(1 - \frac{E}{m_{dm}}\right)$$

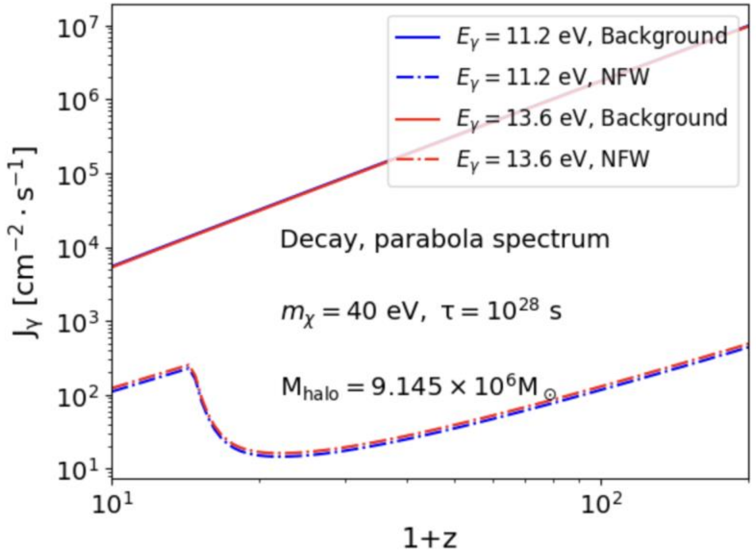
  - Parabola Spectrum

$$E\frac{dN}{dE}\bigg|_{\text{parabola}} = 6\left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}}\right)^2\right]\Theta\left(1 - \frac{E}{m_{dm}}\right)$$

- DM Annihilation ($\chi\chi \to 2\gamma$)







UV Photon Source

- CMB (high redshift)
- Background DM
- DM Halo

Gas Cloud DM Halo | DM Decay Annihilation | **Chemical Reactions** | Simulation Criteria

Modeling: Chemical Reaction Rate

Reaction		
$H + e^-$	$\rightarrow$	$H^- + \gamma$
$H^- + H$	$\rightarrow$	$H_2 + e^-$
$H + H^+$	$\rightarrow$	$H_2^+ + \gamma$
$H_2^+ + H$	$\rightarrow$	$H_2 + H^+$
$H_2 + H^+$	$\rightarrow$	$H_2^+ + H$
$H_2 + e^-$	$\rightarrow$	$H + H + e^-$
$H_2 + H$	$\rightarrow$	$H + H + H$
$H^- + e^-$	$\rightarrow$	$H + e^- + e^-$
$H^- + H$	$\rightarrow$	$H + e^- + H$
$H^- + H^+$	$\rightarrow$	$H + H$
$H^- + H^+$	$\rightarrow$	$H_2^+ + e^-$
$H_2^+ + e^-$	$\rightarrow$	$H + H$
$H_2^+ + H^-$	$\rightarrow$	$H_2 + H$
$H + H + H$	$\rightarrow$	$H_2 + H$
$H + H + H_2$	$\rightarrow$	$H_2 + H_2$
$H^- + \gamma$	$\rightarrow$	$H + e^-$
$H_2^+ + \gamma$	$\rightarrow$	$H + H^+$
$H_2 + \gamma$	$\rightarrow$	$H_2^+ + e^-$
$H_2^+ + \gamma$	$\rightarrow$	$H^+ + H^+ + e^-$
$H_2 + \gamma$	$\rightarrow$	$H + H$

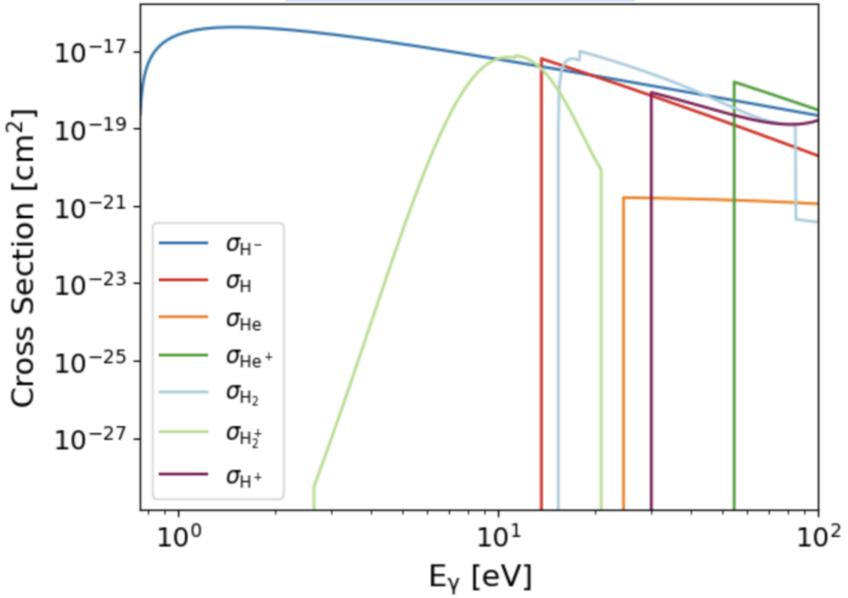
Reaction		
$H + e^-$	$\rightarrow$	$H^+ + e^- + e^-$
$H^+ + e^-$	$\rightarrow$	$H + \gamma$
$He + e^-$	$\rightarrow$	$He^+ + e^- + e^-$
$He^+ + e^-$	$\rightarrow$	$He + \gamma$
$He^+ + e^-$	$\rightarrow$	$He^{++} + e^- + e^-$
$He^{++} + e^-$	$\rightarrow$	$He^+ + \gamma$
$H + H$	$\rightarrow$	$H^+ + e^- + H$
$H + He$	$\rightarrow$	$H^+ + e^- + He$
$H + \gamma$	$\rightarrow$	$H^+ + e^-$
$He + \gamma$	$\rightarrow$	$He^+ + e^-$
$He^+ + \gamma$	$\rightarrow$	$He^{++} + e^-$

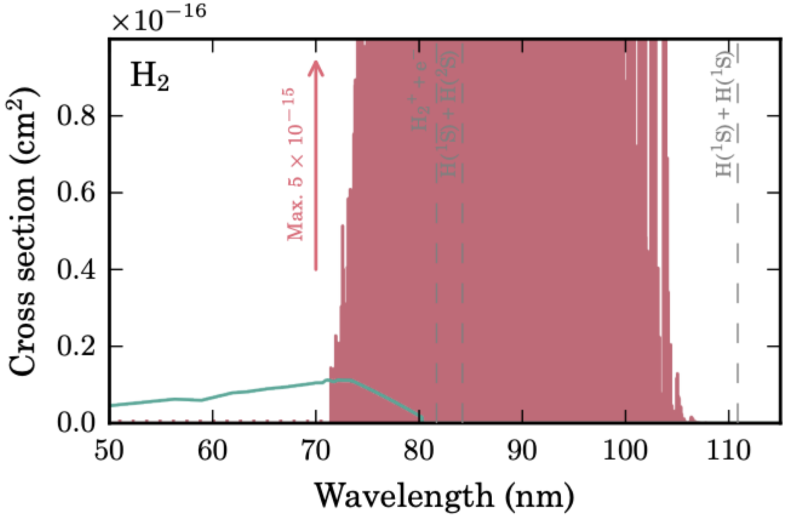
Neutralization

Photodetachment (0.76– 13.6 eV)

Photodissociation (11.2 – 13.6 eV)

31 reactions; 8 require manual input.





https://arxiv.org/pdf/1701.04459

Modeling: Thermal and Ionized Simulation and Criteria

Gas Cloud DM Halo → DM Decay Annihilation → Chemical Reactions → **Simulation Criteria**

- **Initial Conditions** (Temperature, f_X, x_e...)

  CLASS (http://class-code.net/)

(the Cosmic Linear Anisotropy Solving System)

CLASS

the Cosmic Linear Anisotropy Solving System

- **Chemistry and Cooling Library for Astrophysics**

  Grackle (https://github.com/grackle-project/grackle)

Grackle: a Chemistry and Cooling Library for Astrophysics

Criteria:

- Star Formation

$$T_{cloud}\big|_{z=\eta z_{vir}} \leq \eta\, T_{cloud}\big|_{z=z_{vir}}$$

Rapid Cooling

- SMBH Formation

$$T_{cloud}\big|_{z=\eta z_{vir}} > \eta\, T_{cloud}\big|_{z=z_{vir}}$$

$$M_{cloud} > M_{Jeans}$$

Moderate Cooling

- High Temperature Cloud (HTC) Formation

$$M_{cloud} > M_{Jeans}$$

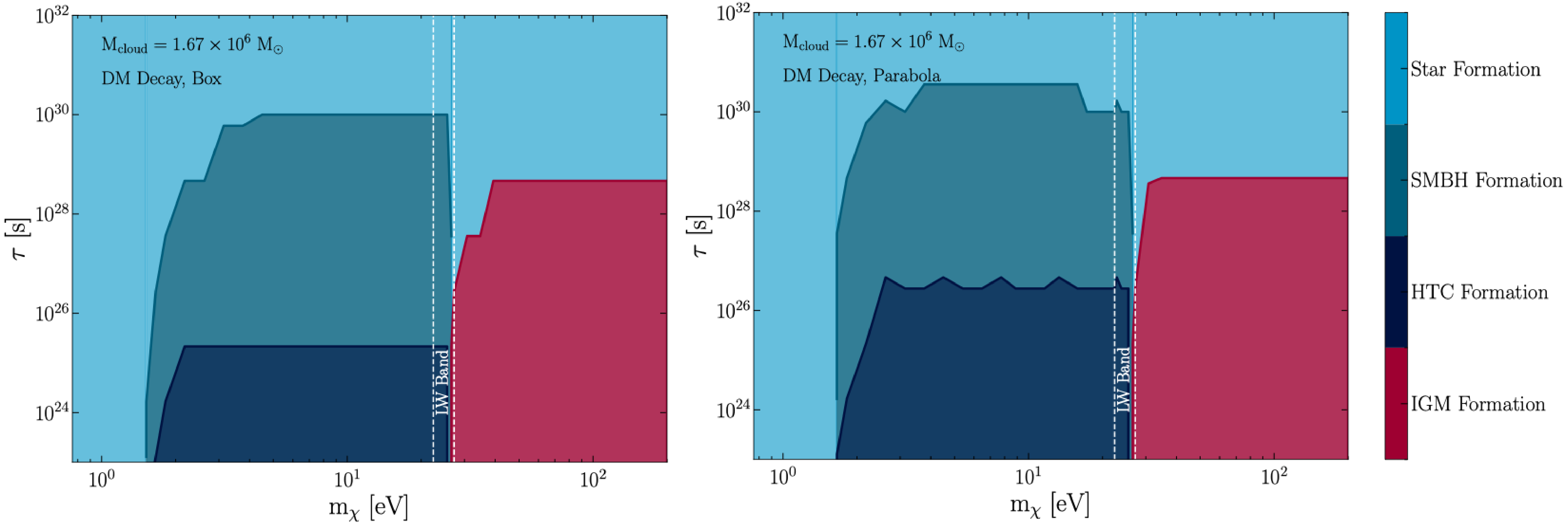
$$T_{cloud} \sim T_{cloud}\big|_{z=z_{vir}}$$

Very slow Cooling

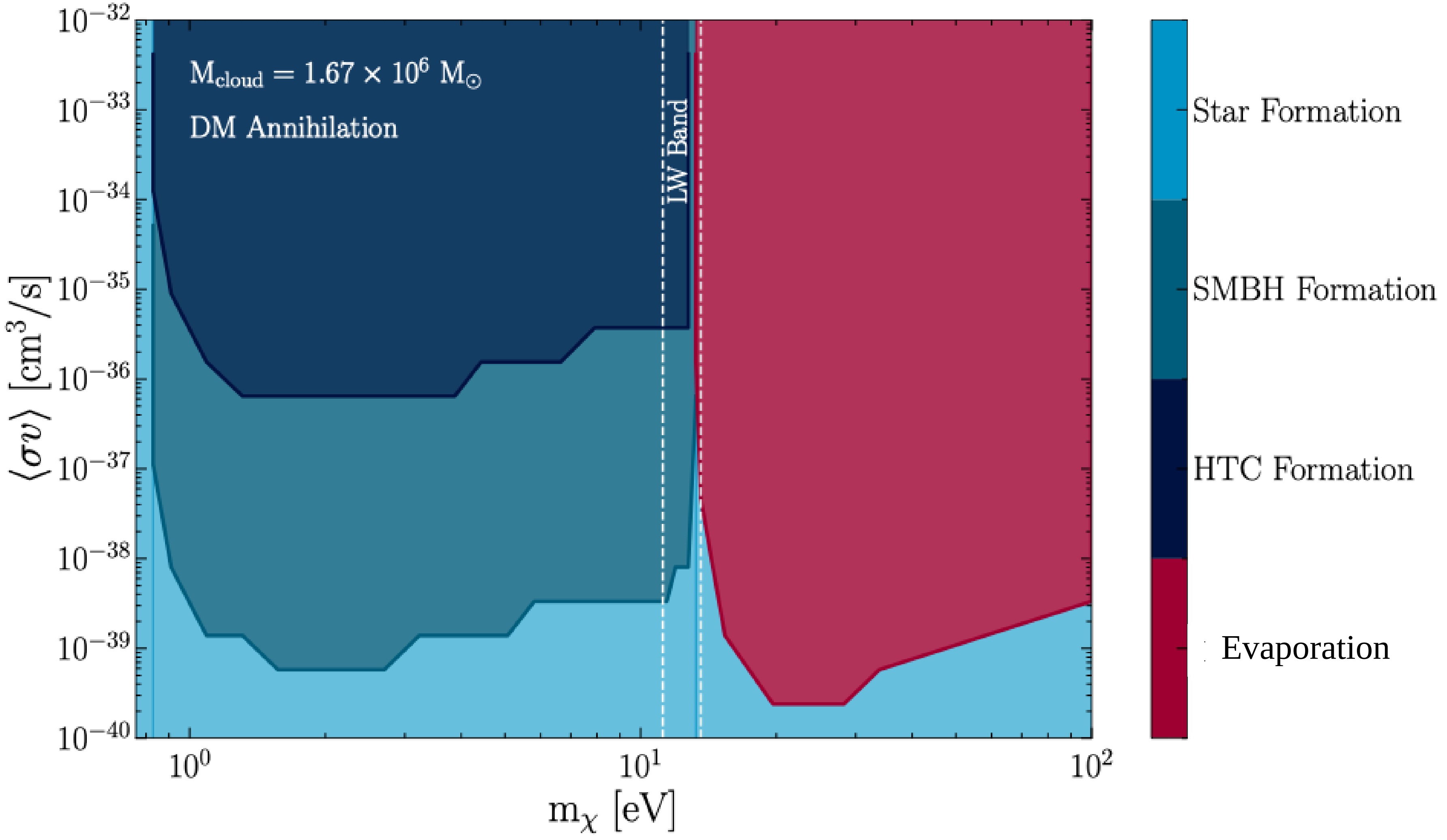
- Evaporation

  Gas cloud becomes too cold as high redshift

Phase Diagram: DM Decay



Phase Diagram: Non-Thermal DM Annihilation



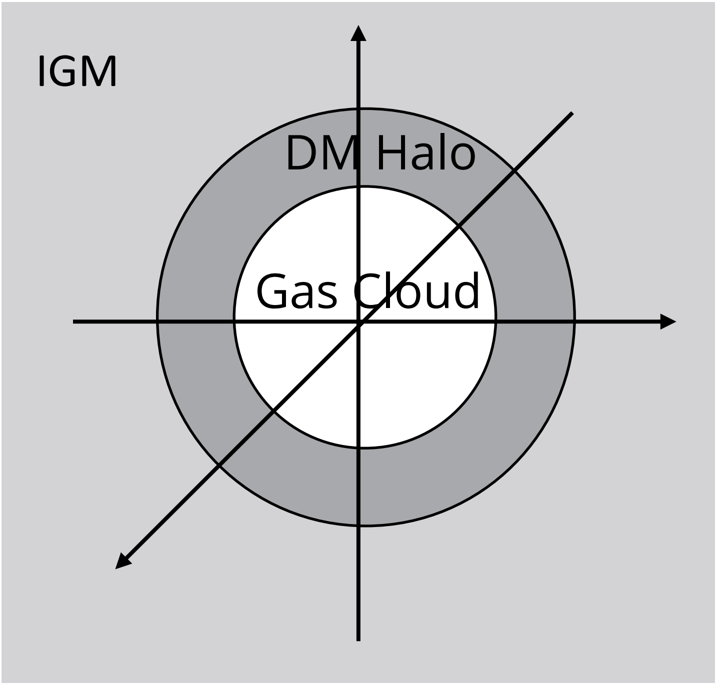
Self-Shielding

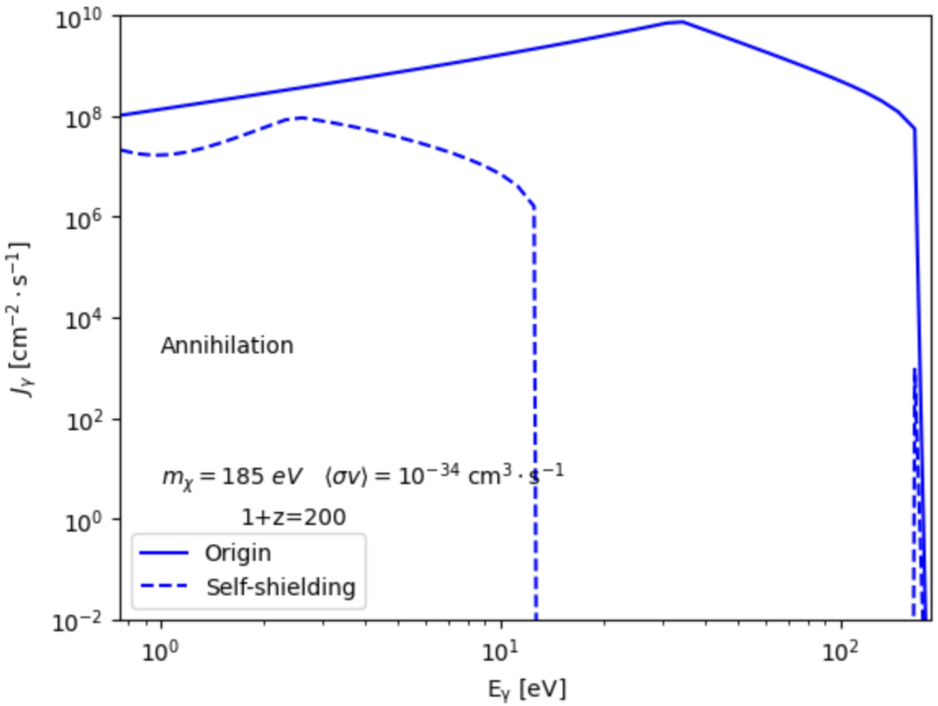
- **Distance**

$$L\left(\vec{r'},\vec{r}\right)=\begin{cases}\left|\vec{r'}-\vec{r}\right|, & \left|\vec{r'}\right|\le R_{cloud}\\ \ell\left(\vec{r'},\vec{r}\right)+s\left(\vec{r'},\vec{r}\right), & \left|\vec{r'}\right|> R_{cloud}\end{cases}$$

- **Optical Depth**

$$\tau\left(\vec{r'},\vec{r}\right)=\begin{cases}n_{cloud}\sigma_\nu\left|\vec{r'}-\vec{r}\right|, & \left|\vec{r'}\right|\le R_{cloud}\\ n_{cloud}\sigma_\nu s\left(\vec{r'},\vec{r}\right)+n_{background}\sigma_\nu\ell\left(\vec{r'},\vec{r}\right), & \left|\vec{r'}\right|> R_{cloud}\end{cases},$$





- Photons from DM Decay/ Annihilation in the DM Halo

$$J_{\text{total}}(E)=\pi E\int_0^{R_{cloud}}r^2dr\int_0^{R_{halo}}r'^2dr'\int_0^{\pi}d\theta\frac{\sin\theta}{r^2+r'^2-2rr'\cos\theta}\frac{dn}{dEdt}e^{-\tau(r,r',\theta)}.$$

- Photons from Background DM Decay/ Annihilation

$$J_{\text{eff}}(E)=(1+z_{obs})^3\int_{z_{obs}}^{\infty}\frac{dz}{H(z)(1+z)^3}E'\frac{dn'_\gamma}{dE'dt'}e^{-\tau(E,z)}$$

Self-Shielding Fitting Formula

- H_2 Self-Shielding Fraction

$$f_{sh}(N_{\rm H_2}, T) = \frac{0.965}{(1 + x/b_5)^{1.1}} + \frac{0.035}{(1+x)^{0.5}} \exp[-8.5 \times 10^{-4}(1+x)^{0.5}]$$

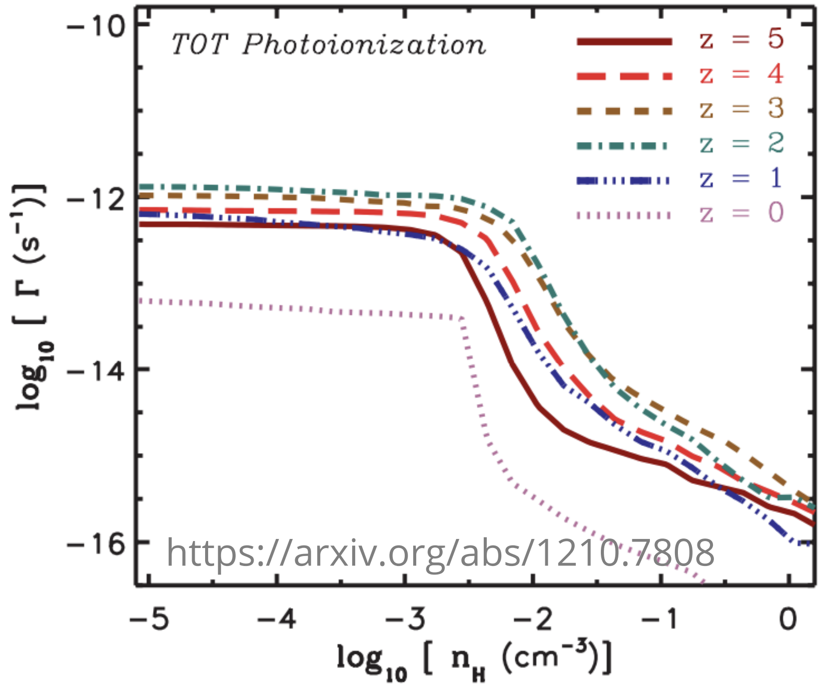
- H_2 Self-Shielding Fraction
- HI Self-Shielding Fraction

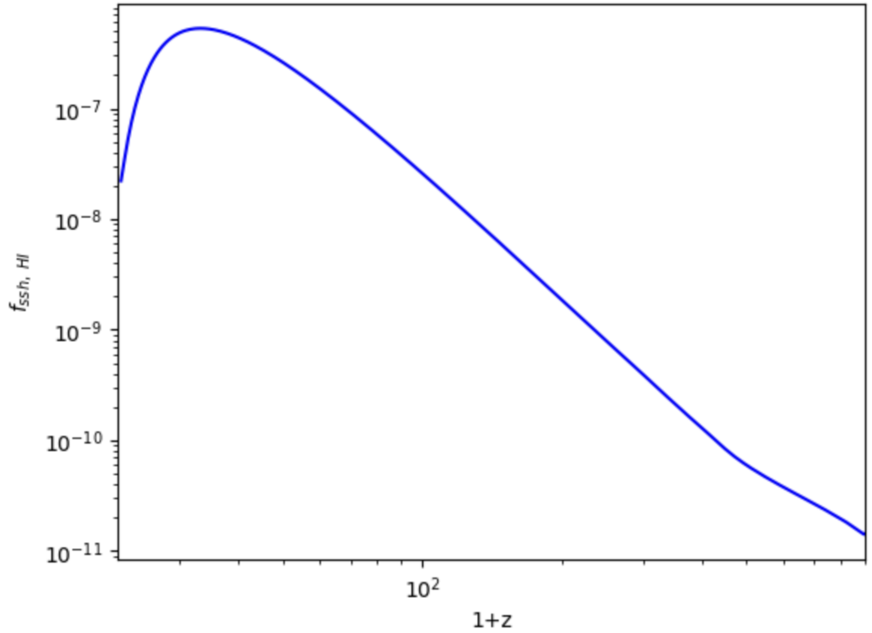
and b is the Doppler broadening parameter

- HI Self-Shielding Fraction

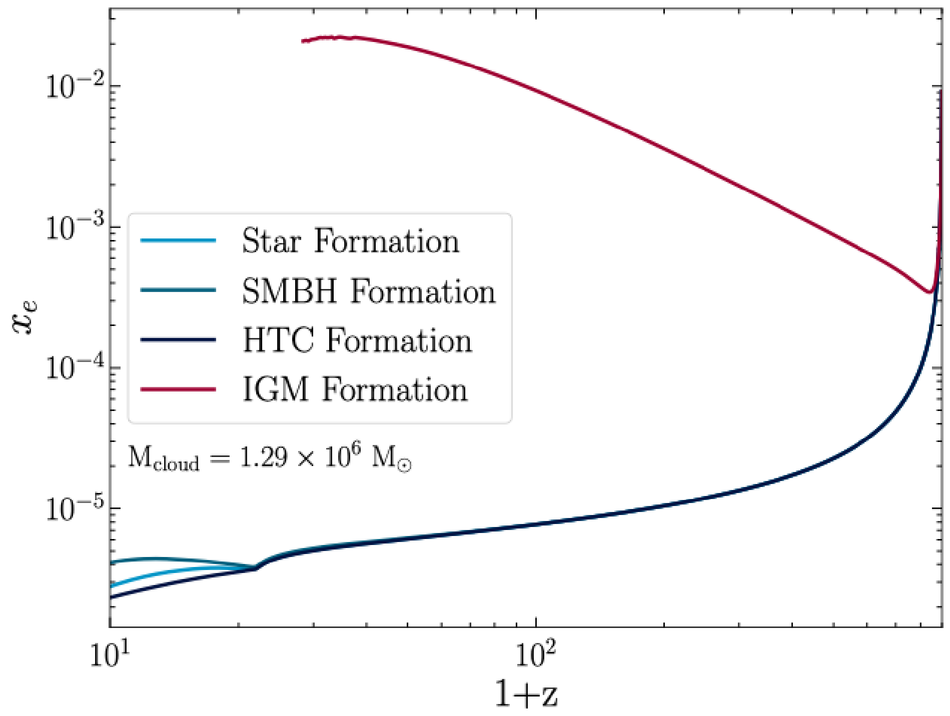
$$\frac{\Gamma}{\langle\Gamma\rangle_{\rm HII}} = (1-f)\left[1 + \left(\frac{n_H}{n_0}\right)^{\beta}\right]^{\alpha_1} + f\left[1 + \frac{n_H}{n_0}\right]^{\alpha_2}$$

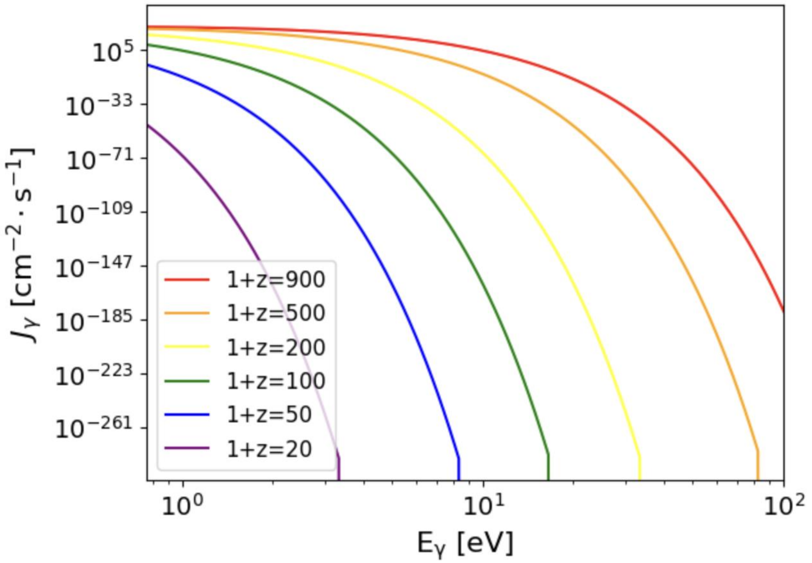
z	$\Gamma_{\rm HI}$ (10^{-12} s^{-1})	n_0 (cm^{-3})	α_1	α_2	β	f
3.0	1.13	0.0090	−1.12	−1.65	5.32	0.018
4.0	1.05	0.0093	−0.95	−1.50	5.87	0.015
5.0	0.90	0.0103	−1.29	−1.60	5.06	0.024
6.0	0.34	0.0070	−0.94	−1.51	6.11	0.029
7.0	0.07	0.0027	−0.86	−1.27	7.08	0.041
8.0	0.10	0.0040	−0.74	−1.40	7.12	0.041
9.0	0.14	0.0046	−0.64	−1.21	9.99	0.029
10.0	0.20	0.0047	−0.39	−0.86	12.94	0.006

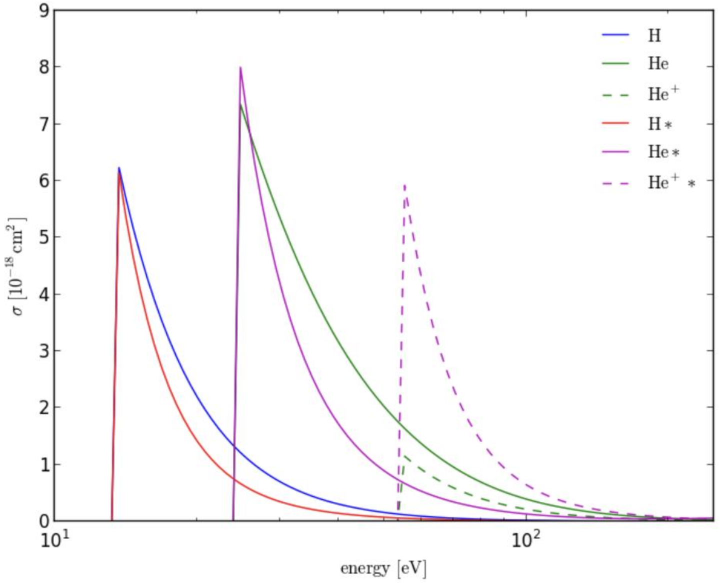




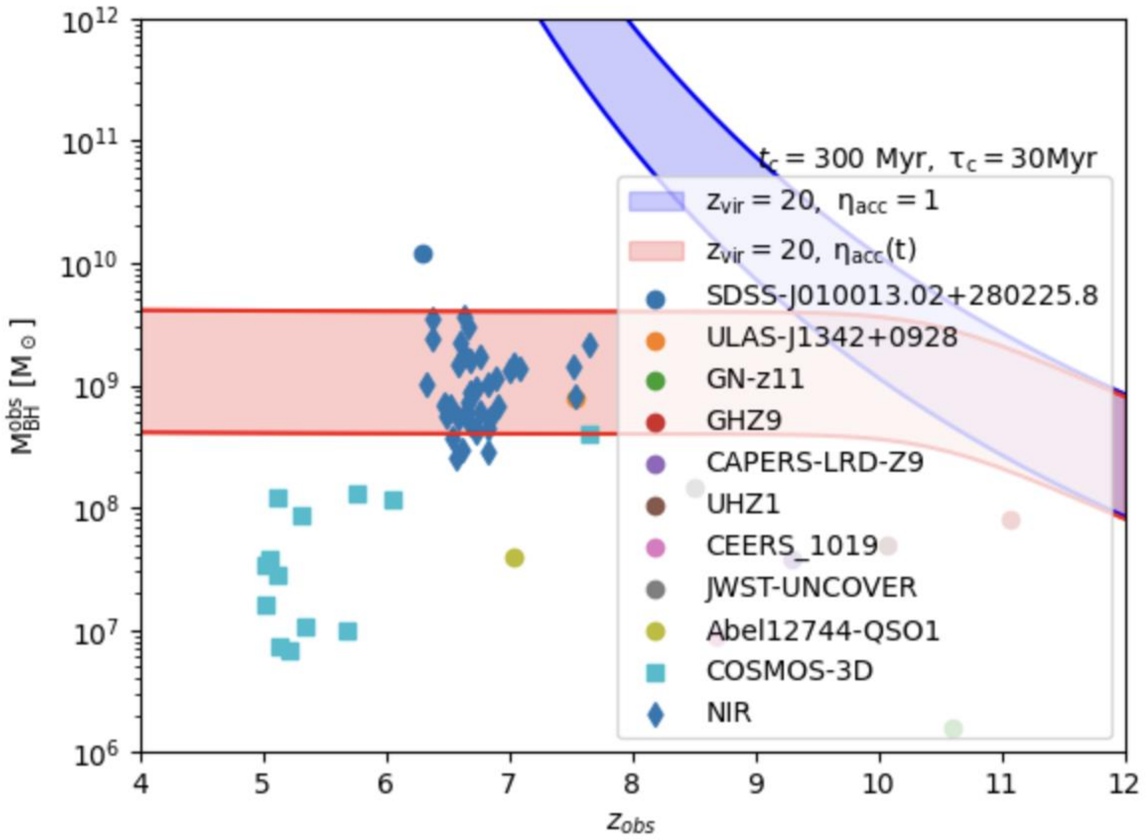
Backup: Ionized History

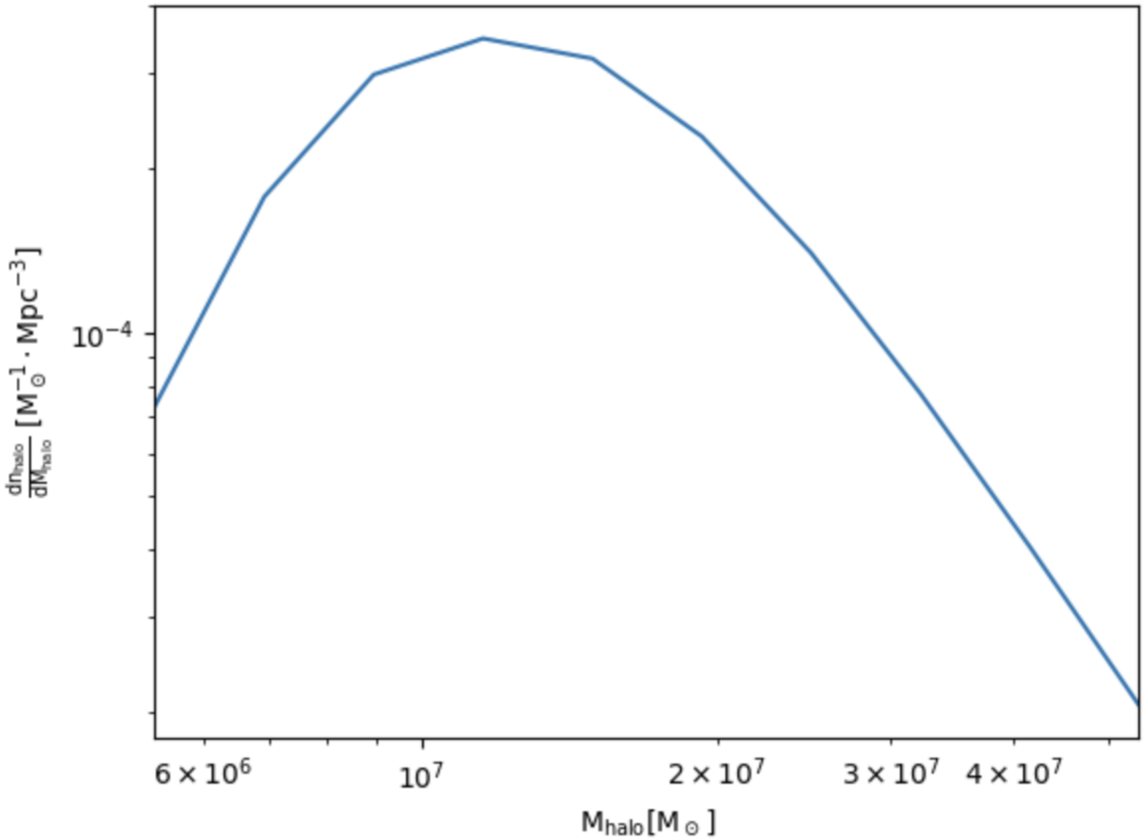




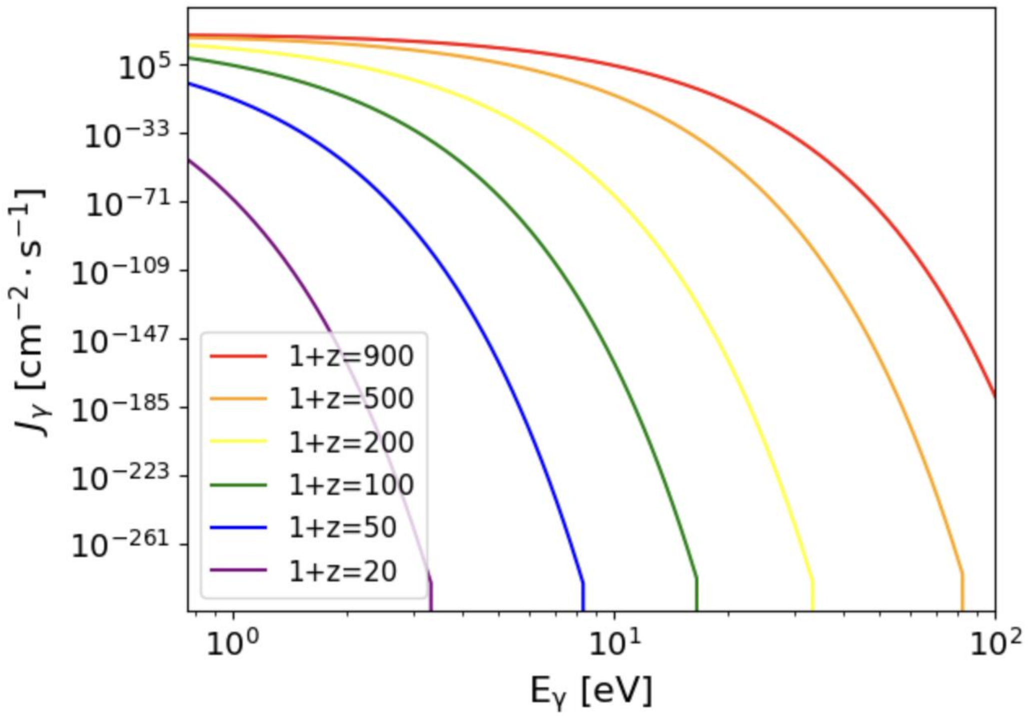


Backup: Accretion and Number Density of the Halo





Backup: CMB and Photon Flux



- **Decay DM**

$$J(E) = \frac{f_\gamma f_{dm}}{\tau m_{dm}} E\frac{dN}{dE}\int dr' \rho_{dm}(r')$$

$$E\frac{dN}{dE}\bigg|_{\text{box}} = \Theta\left(1 - \frac{E}{m_{dm}}\right)$$

$$E\frac{dN}{dE}\bigg|_{\text{parabola}} = 6\left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}}\right)^2\right]\Theta\left(1 - \frac{E}{m_{dm}}\right)$$

$$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{1}{4\pi}\frac{f_\gamma f_\chi \Omega_{DM}\rho_c}{\tau m_\chi}\int_{z_{step}}^{\infty}\frac{1}{H(z)}\frac{2}{E_\gamma}\delta\left(\frac{m_\chi}{E_\gamma} - \frac{1+z}{1+z_{step}}\right)dz$$

- **Annihilation DM**

$$\frac{d\Phi_\gamma}{dE_\gamma} = \frac{1}{4\pi}\frac{\Omega_{DM}^2\rho_c^2\langle\sigma v\rangle}{2m_\chi^2}\int_{z_{step}}^{\infty}\frac{(1+z)^3}{H(z)}\frac{2}{E_\gamma}\delta\left(\frac{m_\chi}{E_\gamma} - \frac{1+z}{1+z_{step}}\right)dz$$

$$J_{bg}(1+z, E_\gamma) = E_\gamma\frac{d\Phi_\gamma}{dR_\gamma}$$

$$J_{ann}(E_\gamma) = \frac{3\langle\sigma v\rangle}{m_\chi^2}\left[\frac{E_\gamma}{m_\chi} - \left(\frac{E_\gamma}{m_\chi}\right)^2\right]\int_0^r \rho_{dm}^2(r')dr'$$

Backup: DM Halo Modeling

At very high redshift (e.g. $z \sim 1100$), before the overdensity becomes nonlinear, the mass shell is still moving with the Hubble flow. So the initial velocity of the mass shell is

$$v_{ini} = r_{ini} \times H(z), \tag{8}$$

However, we still need to consider the correction of this initial velocity since too fast initial velocity will delay the virialization of the Halo. From the Eq.5.43 in Baumann's book

$$\nabla \cdot \mathbf{v} = -a\dot{\delta}, \tag{9}$$

where $\mathbf{v} = a\dot{\mathbf{x}}$ is the peculiar velocity, δ is the density constrast. For spherically symmetric, the peculiar velocity is radial:

$$\mathbf{v} = v(r)\hat{\mathbf{r}}, \tag{10}$$

the divergence of $\mathbf{v}$ is

$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{d}{dr}(r^2 v(r)). \tag{11}$$

Assume $v(r) = Ar$, where A is a constant, then

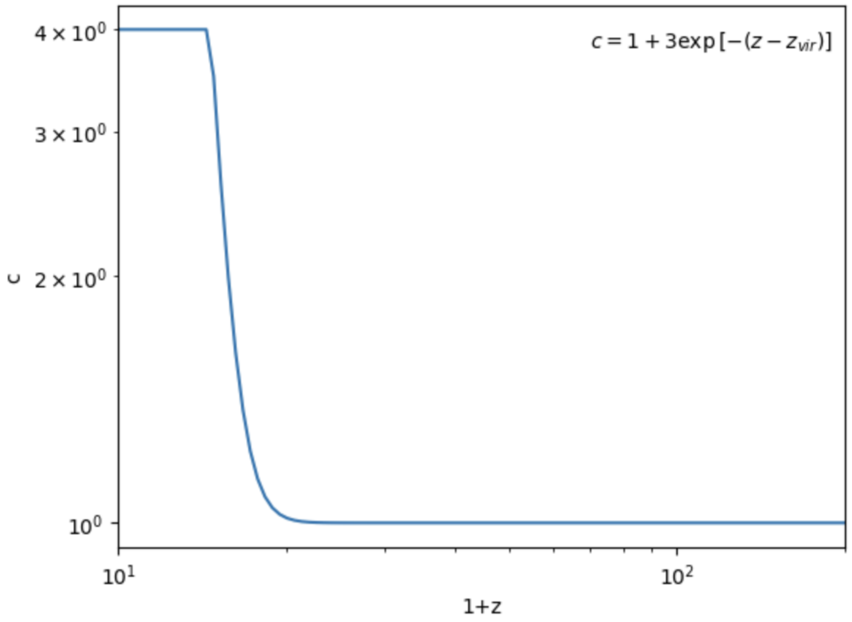
$$\nabla \cdot \mathbf{v} = \frac{1}{r^2}\frac{d}{dr}(Ar^3) = 3A \Rightarrow A = -\frac{a\dot{\delta}}{3},\ v(r) = -\frac{a\dot{\delta}}{3}r. \tag{12}$$

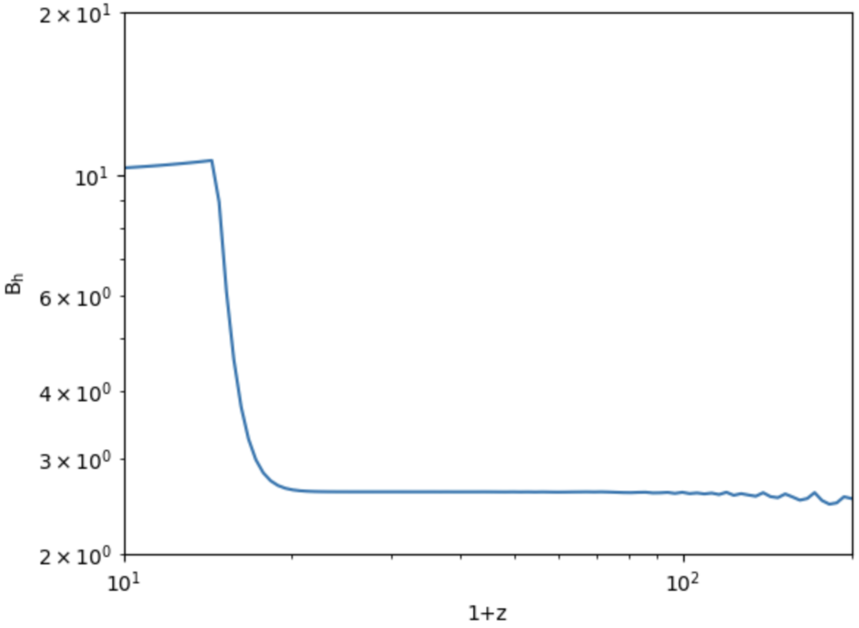
We should be careful with the notation r in the above calculation, actually it is in the comoving coordinate. Since the physical velocity is

$$\mathbf{v}_{physical} = \dot{a}r_{phy} + \dot{r}_{phy}a,$$

meanwhile, it is reasonable to estimate the time derivative of density constrast as $\dot{\delta} \approx H\delta$, therefore, the velocity of halo is

$$v_{phy} = Hr_{phy} + v = Hr_{phy}\left(1 - \frac{\delta}{3}\right). \tag{14}$$





Backup: DM Halo Modeling

the external photon flux from the background dark matter decay/annihilation is

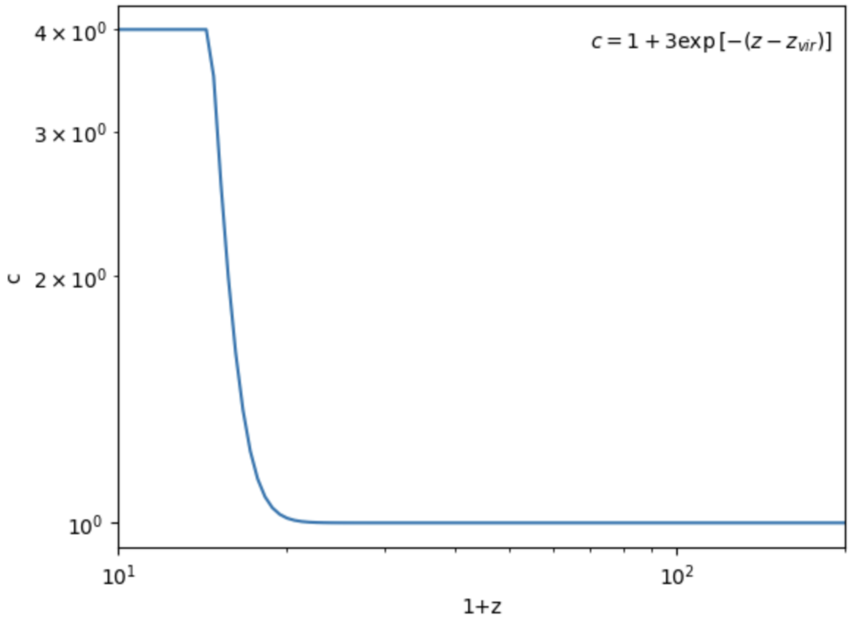
$$J(E) = (1+z_{obs})^3 \int_{z_{obs}}^{\infty} \frac{dz}{H(z)\,(1+z)^3} E' \frac{dn'_\gamma}{dE'dt'},$$

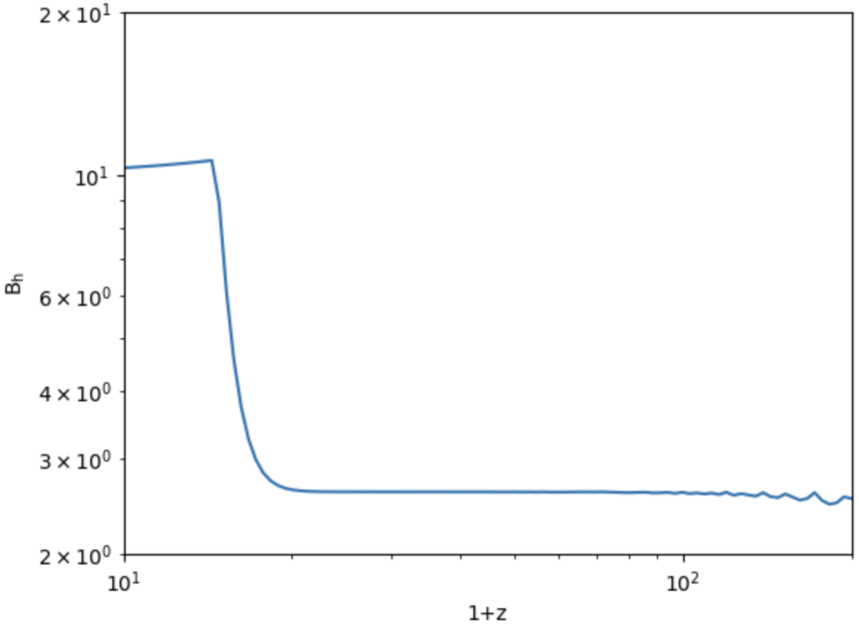
$$J_{decay,\ halo}(E,z) = \frac{f_\gamma f_{dm} \bar{\rho}_{dm}(z) R_{dm}(z)}{\tau m_{dm}} E \frac{dN}{dE}.$$

$$J_{decay,\ background}(E, z_{obs}) = \frac{(1+z_{obs})^3}{4\pi} \frac{f_\gamma f_{dm} \Omega_{dm} \rho_{crit}}{\tau m_{dm}} \int_{z_{obs}}^{\frac{m_{dm}(1+z_{obs})}{E}-1} \frac{dz}{H(z)}$$

$$J_{ann,halo}(E,z) = \frac{6 f_\gamma f_{dm}^2 \bar{\rho}_{dm}^2(z) R_{dm}(z) \langle\sigma v\rangle}{2^p m_{dm}^2} \left[\frac{E}{m_{dm}} - \left(\frac{E}{m_{dm}}\right)^2\right] \Theta\left(1 - \frac{E}{m_{dm}}\right)$$

$$J_{ann,background} = \frac{3}{2\pi} \frac{f_\gamma f_{dm}^2 \Omega_{dm}^2 \rho_{cirt}^2 \langle\sigma v\rangle}{2^p m_{dm}^2} \int_{z_{obs}}^{\frac{m_{dm}(1+z_{obs})}{E}-1} dz \left[\frac{(1+z_{obs})^2 E}{m_{dm}} \frac{(1+z)^4}{H(z)} - \frac{(1+z_{obs}) E^2}{m_{dm}^2} \frac{(1+z)^5}{H(z)}\right]$$





Backup: Gas Cloud Mass