Superradiant Interactions of Cosmic Relics

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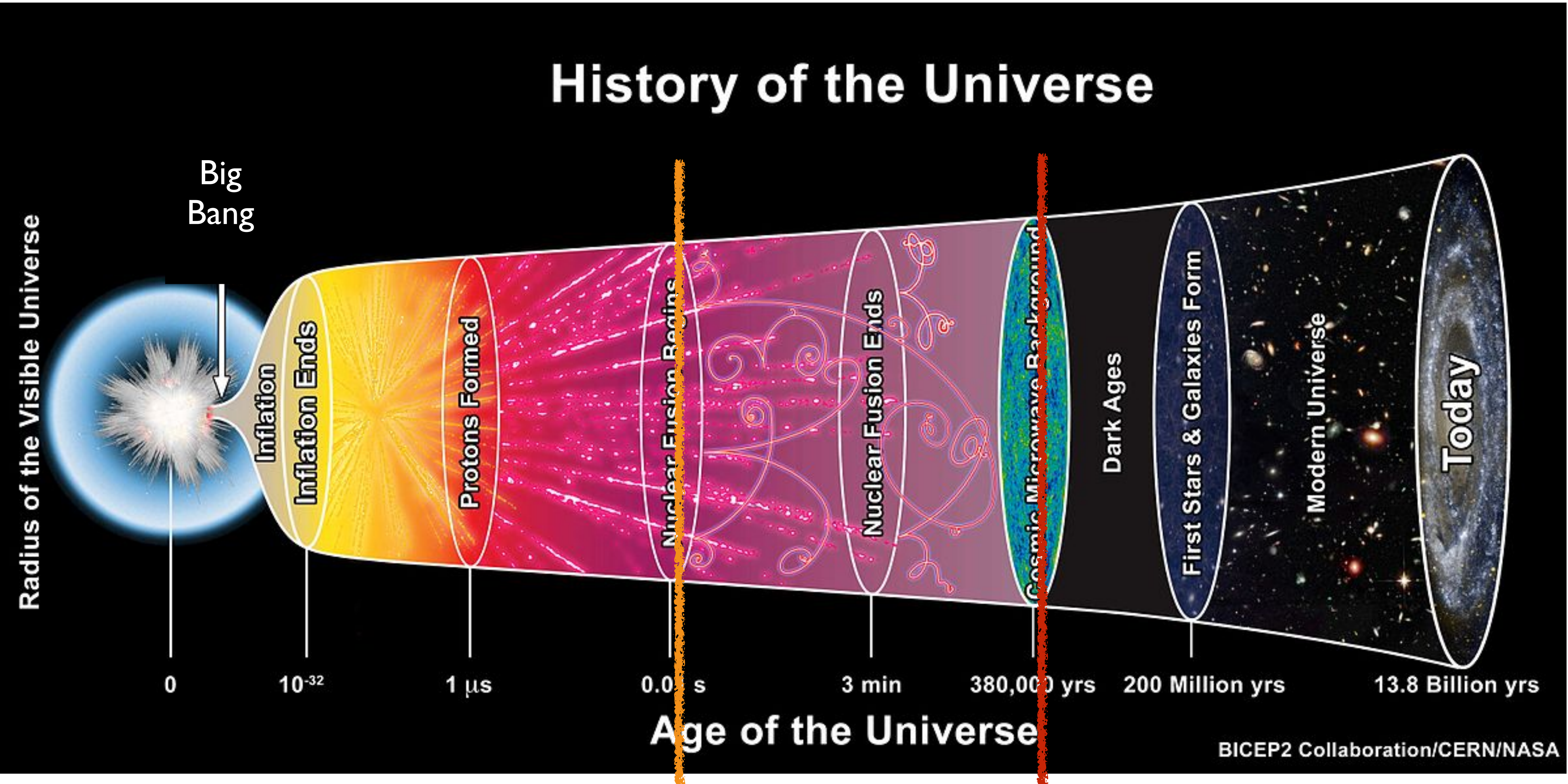
with

Savas Dimopoulos and Marios Galanis

Is it possible to have inelastic scattering processes enhanced by N^2?

N : Number of constituents of a target

A Brief History of the Universe

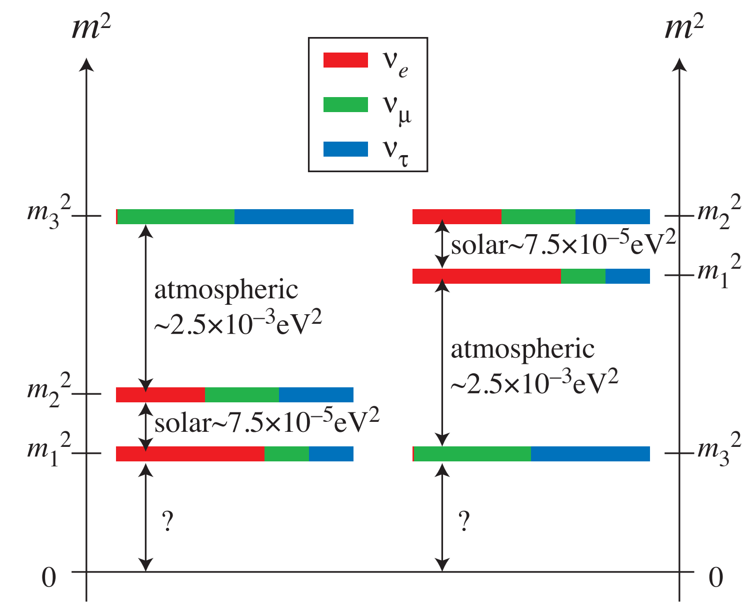


The Cosmic Neutrino Background (CνB)

The CMB

The Cosmic Neutrino Background (CνB)

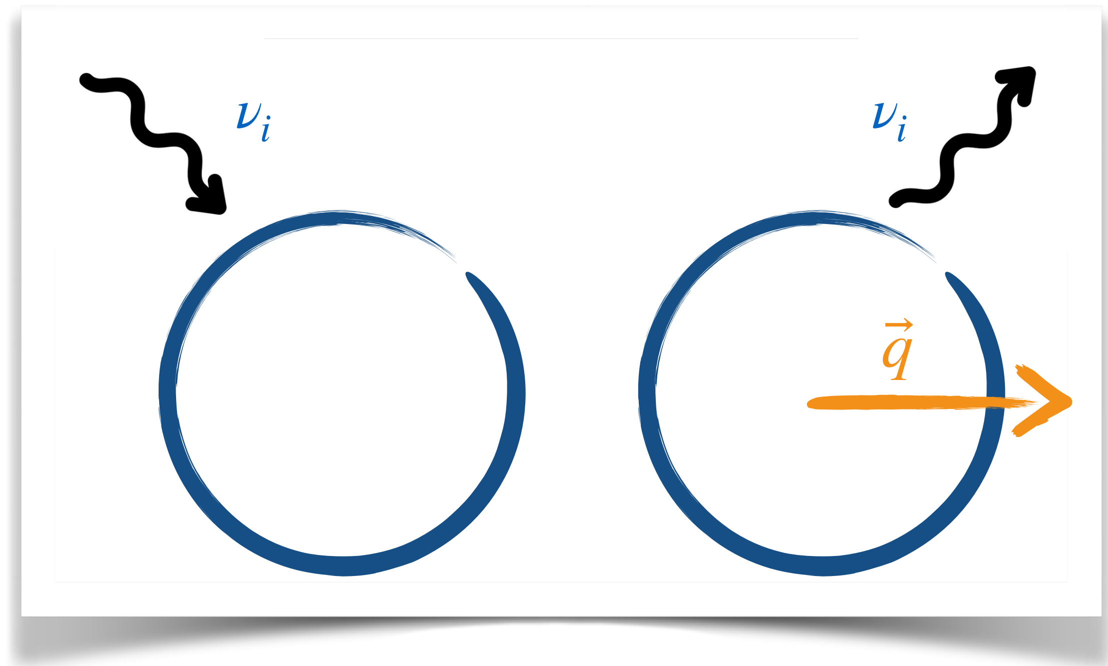
- Relic neutrinos from the pre-BBN era $\tau_{\text{universe}} \sim 0.1$ sec
- They follow a Fermi-Dirac distribution with:
  - $\langle p_\nu \rangle = 6 \times 10^{-4}$ eV
  - $\langle E_\nu \rangle = 1.6 \times 10^{-6}\ \text{eV} \left(\dfrac{0.1\ \text{eV}}{m_\nu} \right)$
  - $\langle \lambda_\nu \rangle = 2.1$ mm
  - $n_\nu = 56\ \text{cm}^{-3}$ per mass eigenstate (total of 3) , per helicity mode



Outline

- The $C\nu B$ challenge
- Superradiant interactions: Inelastic processes with N^2 rates
- Measuring N^2 rates
- A timeline

Why is the CνB hard to detect?



Smith & Lewin (1983), …

- Coherent elastic scattering from an extended object, with momentum transfer $\mathcal{O}(\langle p_\nu \rangle)$
    - 2/day on a mm size object because rate $\propto N^2$

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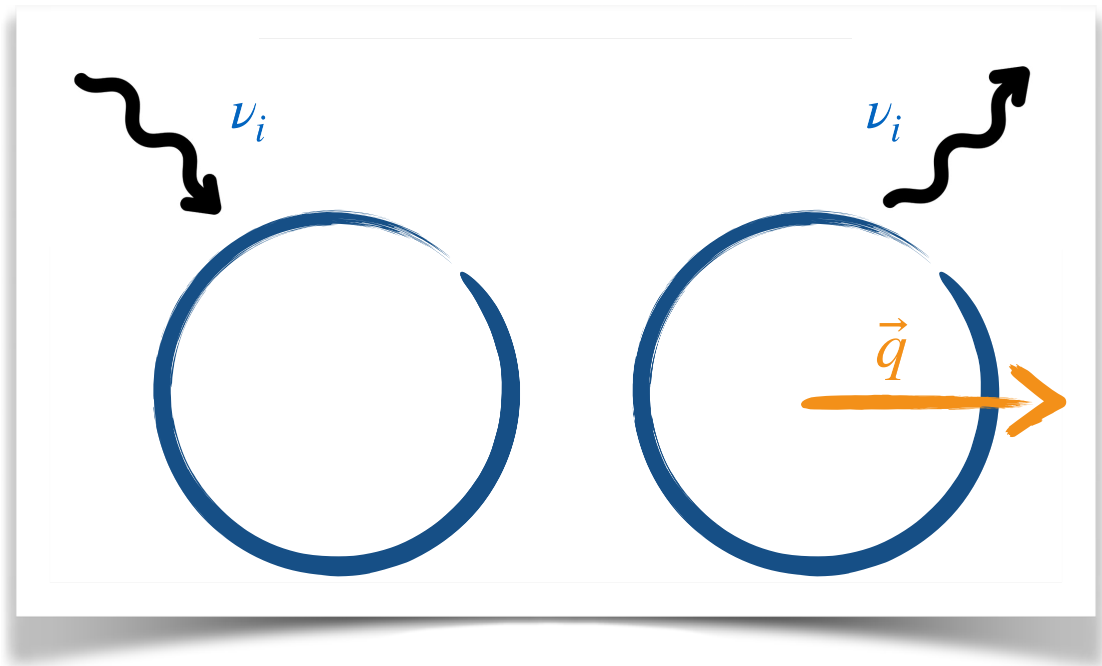

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- Coherent elastic scattering from an extended object, with momentum transfer $\mathcal{O}(\langle p_\nu \rangle)$
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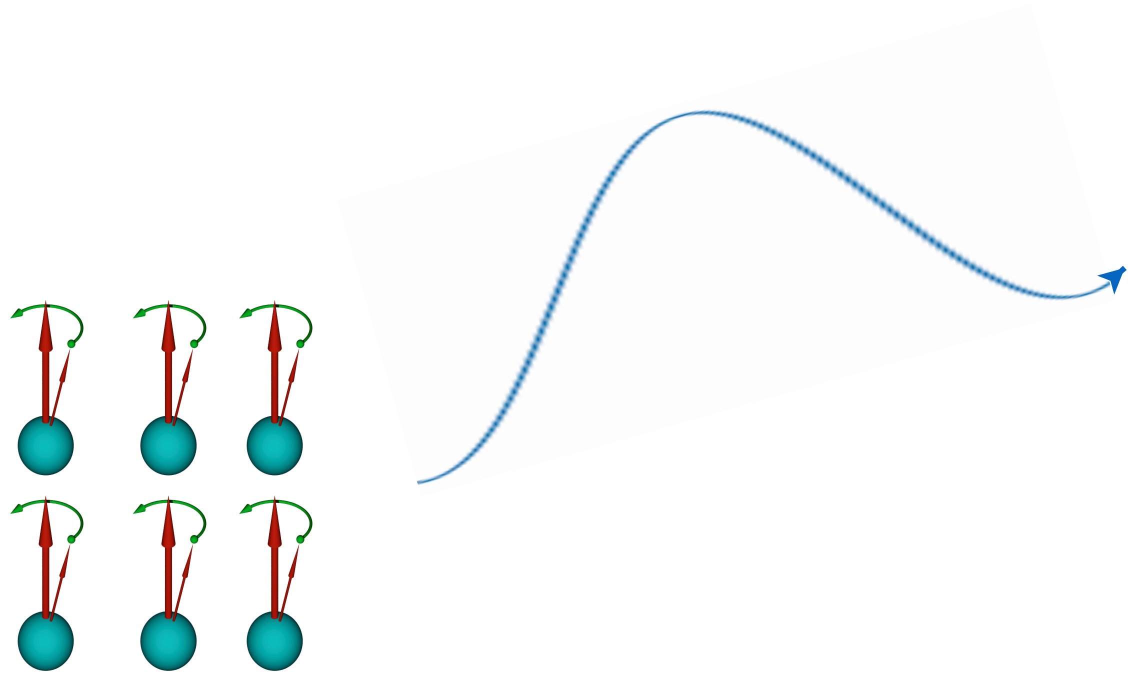
- Challenge: $\delta v_{\text{object}} \sim 10^{-33}$ assuming mm object

Can we transfer more energy while preserving interaction rates that scale like N^2?

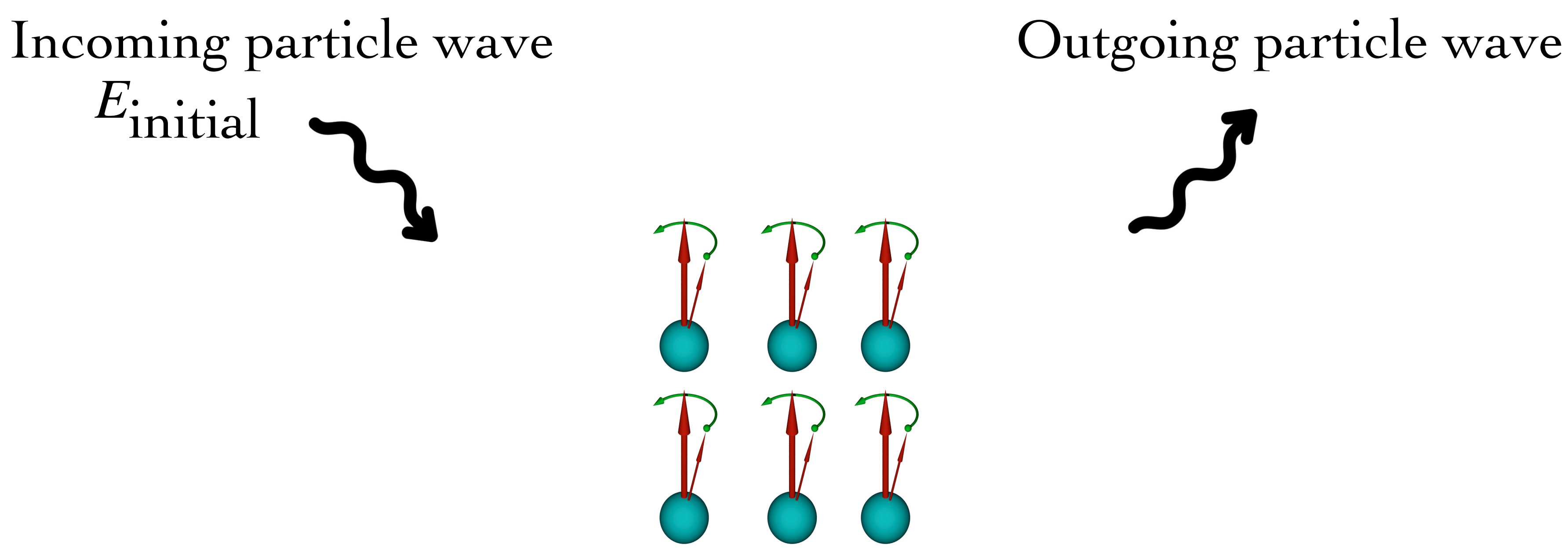
Coherence in emission and absorption of light



Power of the emitted light grows like the N^2 as long as all precessing dipoles are within the wavelength

Dicke Superradiance 1954

Coherence in inelastic scattering processes



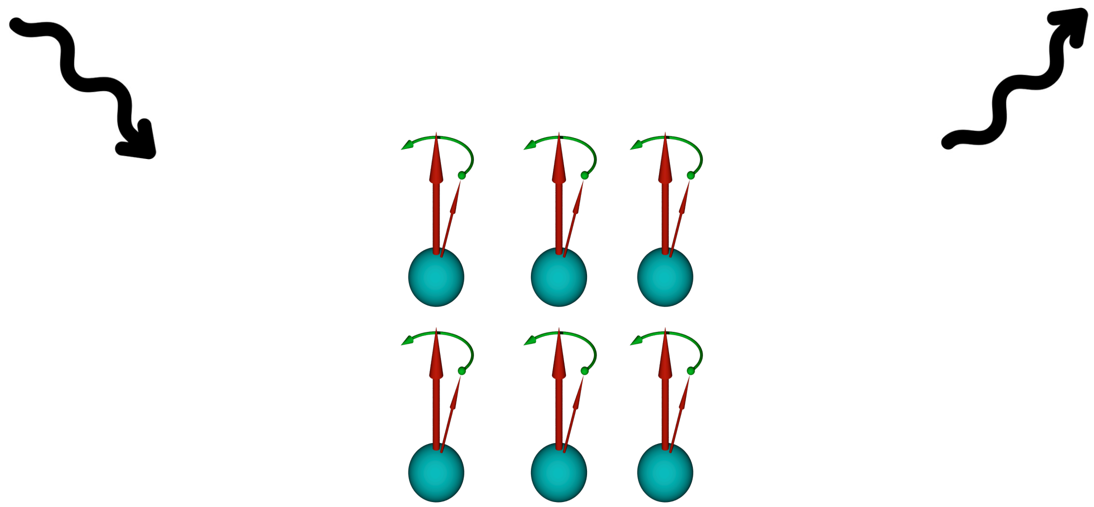
- Spin-Spin interaction results in a time-dependent potential
  $H \sim \frac{G_F}{\sqrt{2}} \delta^{(3)}(\vec{x}_\nu - \vec{x}_S) N \; \vec{\sigma}_\nu \cdot \vec{\sigma}_S \cos(\omega t)$

- At small momentum transfers, scattered outgoing neutrino energies $E_{\text{initial}} \pm \omega$ and scattering rate scales like N^2

Coherence in inelastic scattering processes

Incoming particle wave E_{initial}

Outgoing particle wave



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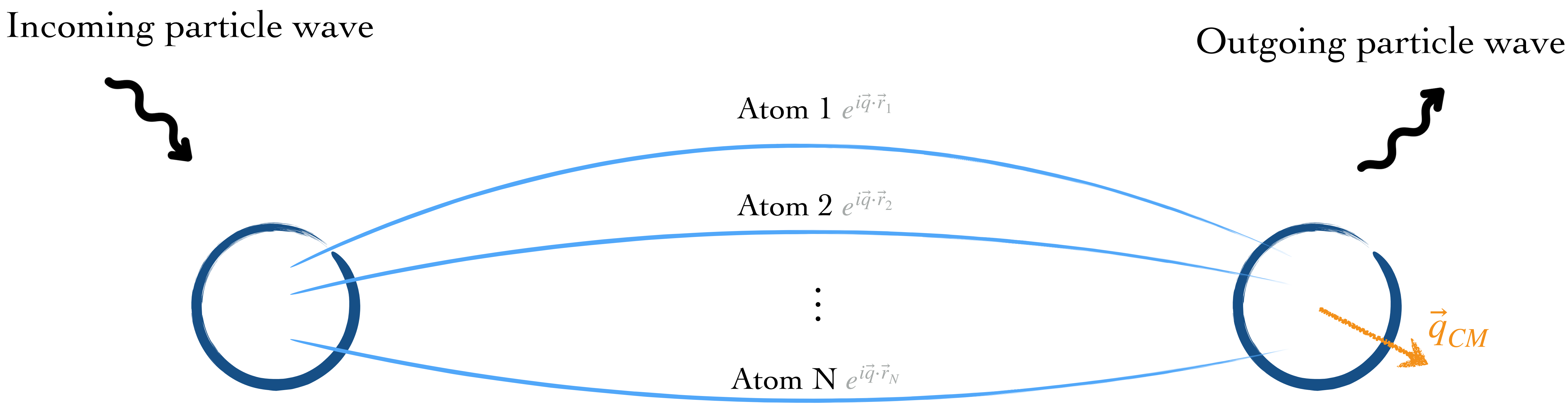
Coherence in inelastic processes shows that
what Dicke coined as supperadiance goes beyond decays

How does elastic scattering give you N^2?

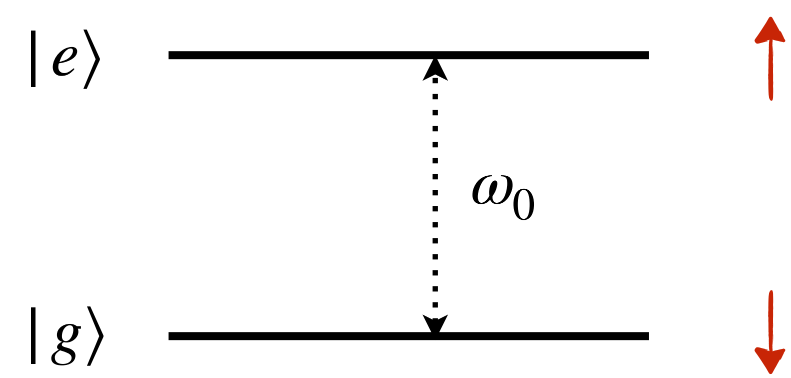


- For small enough momentum transfer, there are N paths all leading to the same final state
  - Scattering amplitude $\propto N$
  - Scattering rate $\propto N^2$
  - Momentum transfer $q_{CM} \sim 5 \times 10^{-6}$ eV
  - Energy transfer $E_{CM} \sim 10^{-49}$ eV

For $R = 10$ cm

Understanding N^2 scalings in inelastic scattering

- A target made of N two-level "atoms": nuclear and atomic transitions, spins in magnetic field, etc…

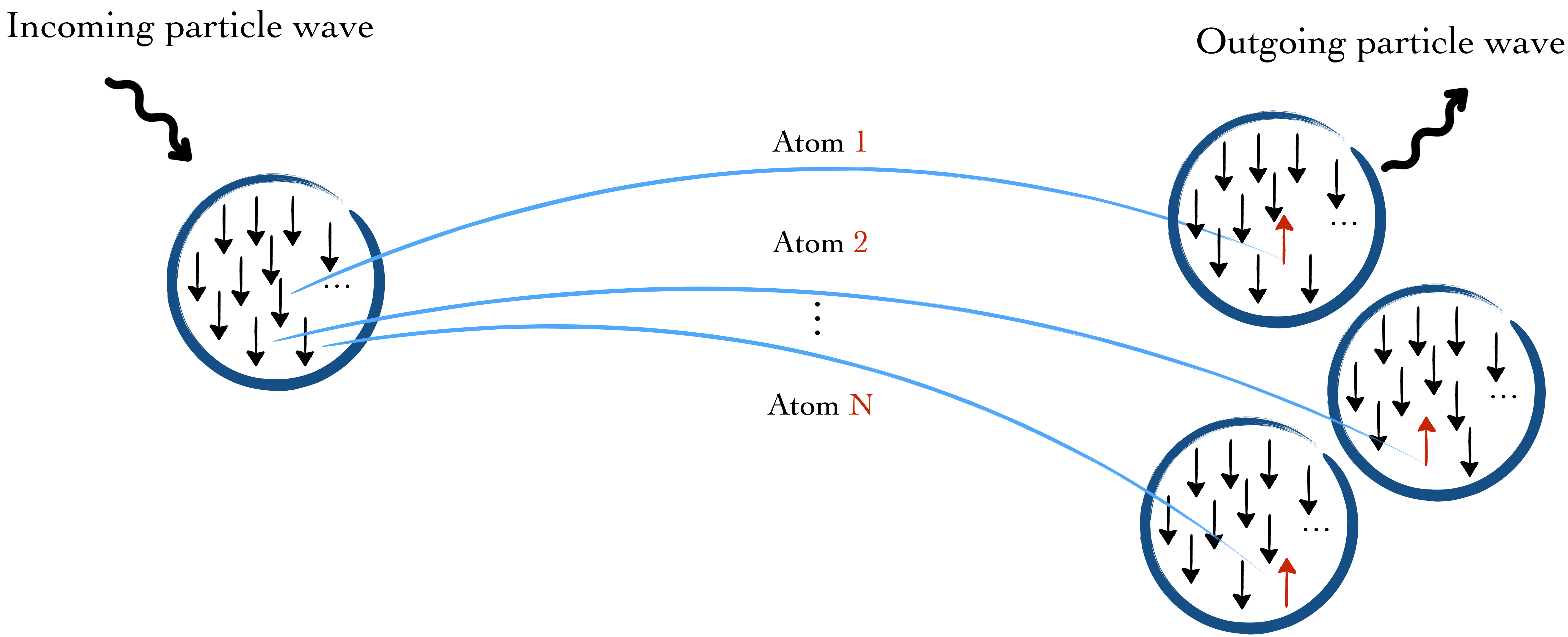


- Two extreme possibilities:
  - All atoms in the ground state $\prod |\downarrow\rangle$
  - All atoms in an equal superposition of ground and excited:

$$\prod \left(\frac{1}{\sqrt{2}} (|\downarrow\rangle + |\uparrow\rangle) \right) = \prod |\rightarrow\rangle \equiv \text{Product state}$$

Why does inelastic scattering normally $\propto N$?

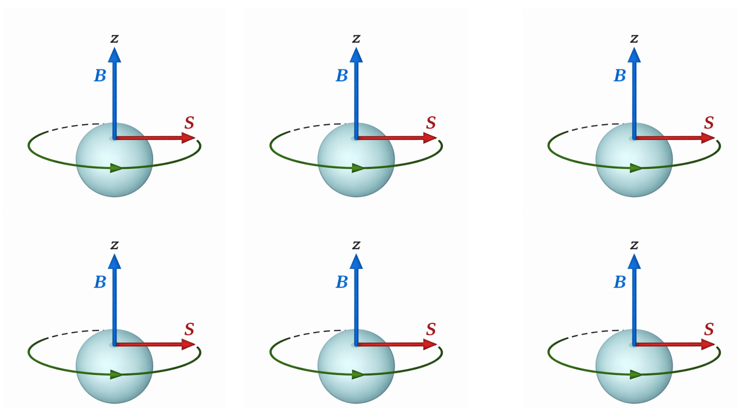
The rate of exciting a single atom



- N atoms all in the ground state $\Rightarrow$ N distinct orthogonal final states
  - Scattering amplitude $\propto 1$
  - Scattering rate $\propto N$
  - Momentum transfer $q \sim 5 \times 10^{-6}$ eV
  - Energy transfer $E \sim \omega_0 \gg 10^{-49}$ eV

For $R = 10$ cm

The coherent spin state or product state



$$\Longleftrightarrow \quad \prod\left(\frac{1}{\sqrt{2}}(|\downarrow\rangle+|\uparrow\rangle)\right)=\prod|\rightarrow\rangle \equiv \text{Product state}$$

Analogous to classical spins precessing in a magnetic field

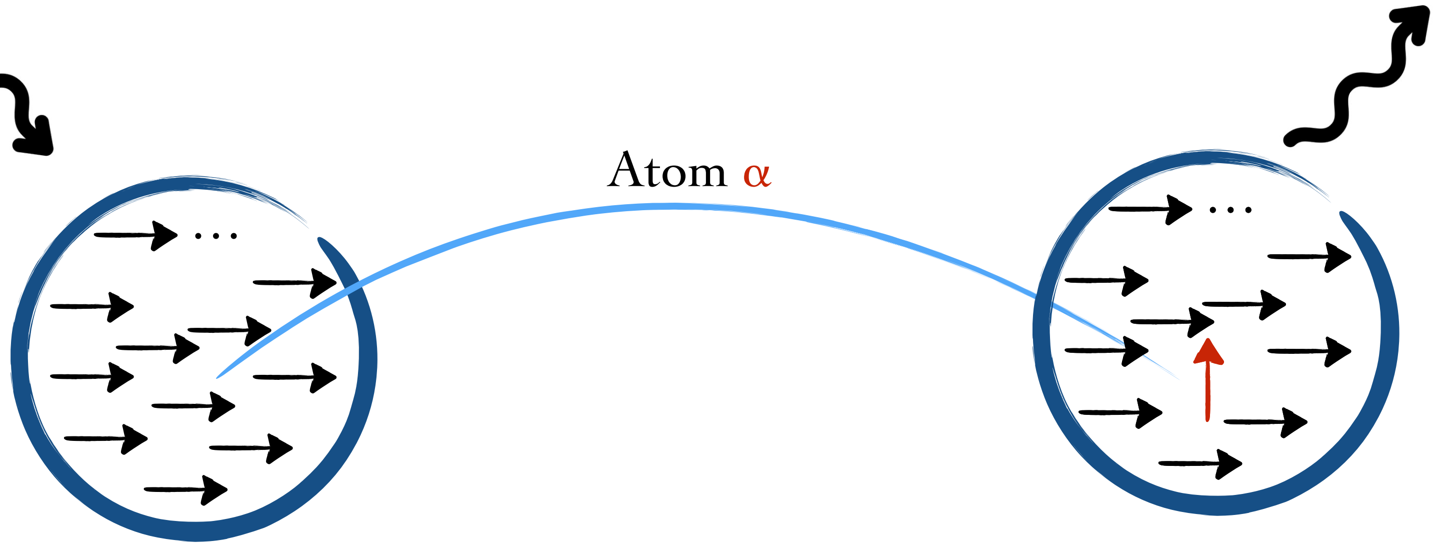
Easily prepared with a $\frac{\pi}{2}$ Rabi pulse

When does inelastic scattering $\propto N^2$?

AA, S. Dimopoulos, M. Galanis (2024)

Incoming particle wave

Outgoing particle wave



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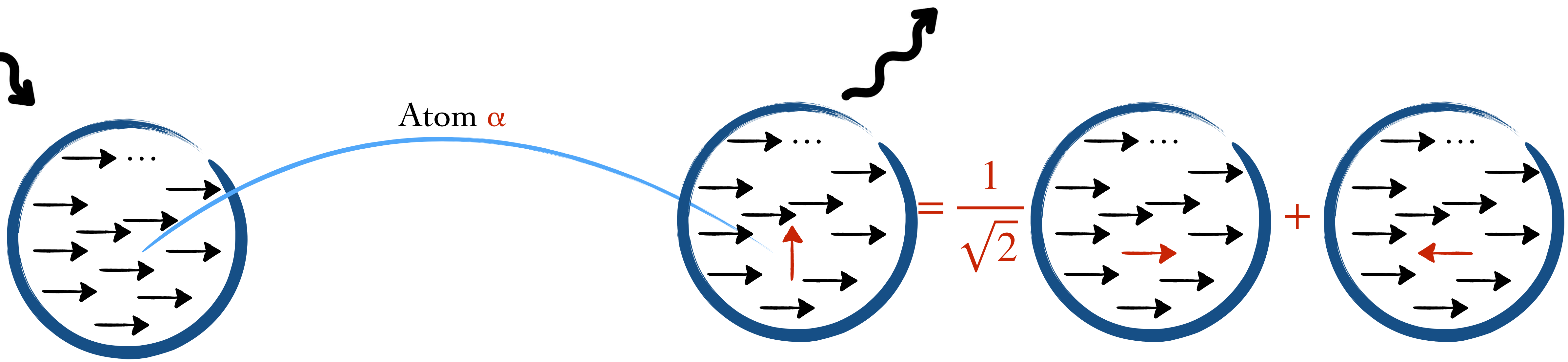

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)$$

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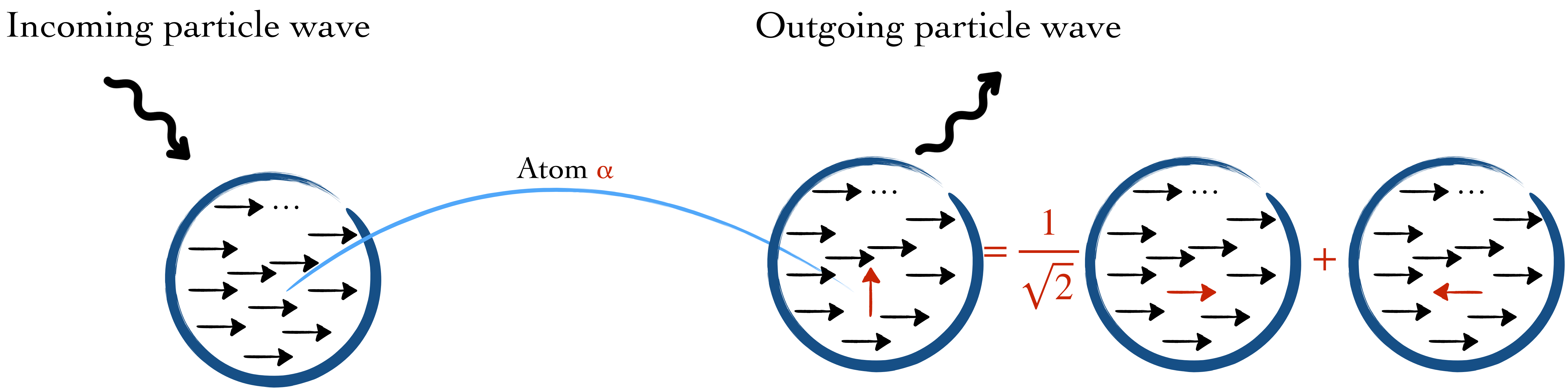


$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle + |\leftarrow\rangle\right)$$

- N atoms in an equal superposition of ground and excited, therefore there are $\sim \frac{N}{2}$ indistinguishable final states
    - Scattering amplitude $\propto N$
    - Scattering rate $\propto N^2$
    - Energy transfer still large, $E = \omega_0$

Superradiant Decays vs Superradiant Scattering

- Particle number violating process: Needs bosons
- There is always a minimum momentum transfer, $q_{\rm min} = \dfrac{2\pi}{\lambda}$
- Condition for superradiance to dominate over incoherent contribution is $n\lambda^3 \gg 1$

- Particle number is conserved: Works for both non-degenerated fermions and bosons
- As long as $\omega_0 < \dfrac{v}{R}$, $q_{\rm min} < \dfrac{1}{R}$
- Condition for superradiance to dominate over incoherent contribution is $n\lambda^2 R \gg 1$

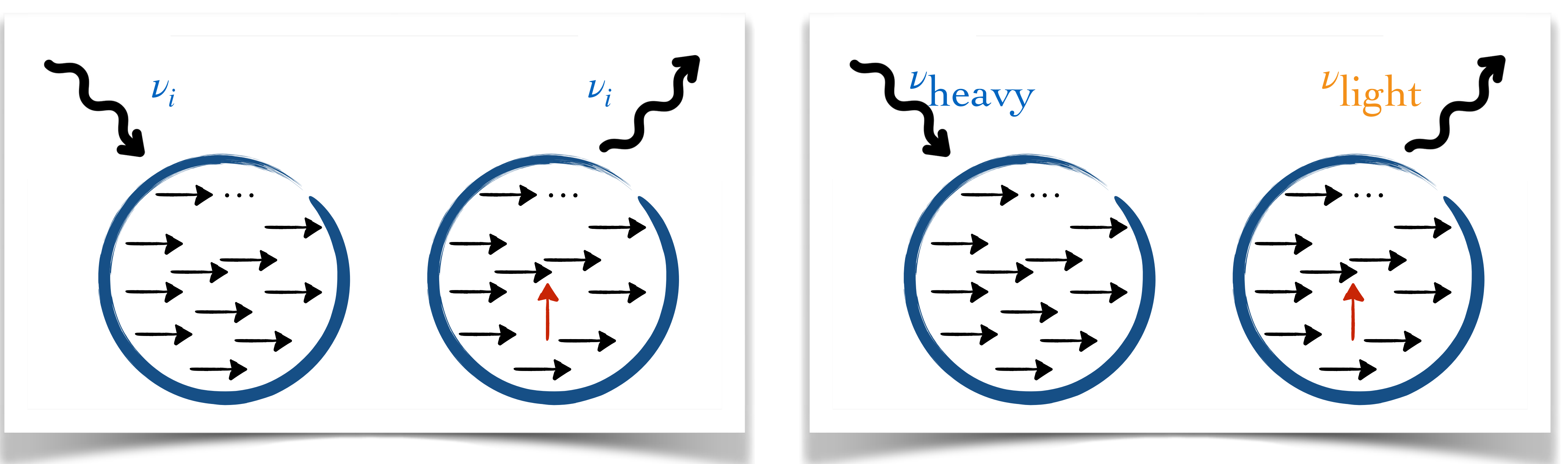
Superradiant CvB scattering

- CvB scattering from an ensemble of polarized nuclear or electron spins in a $\vec{B}_0$ magnetic field (N tunable two-level atoms) and prepared in the product state $\prod |\rightarrow\rangle$

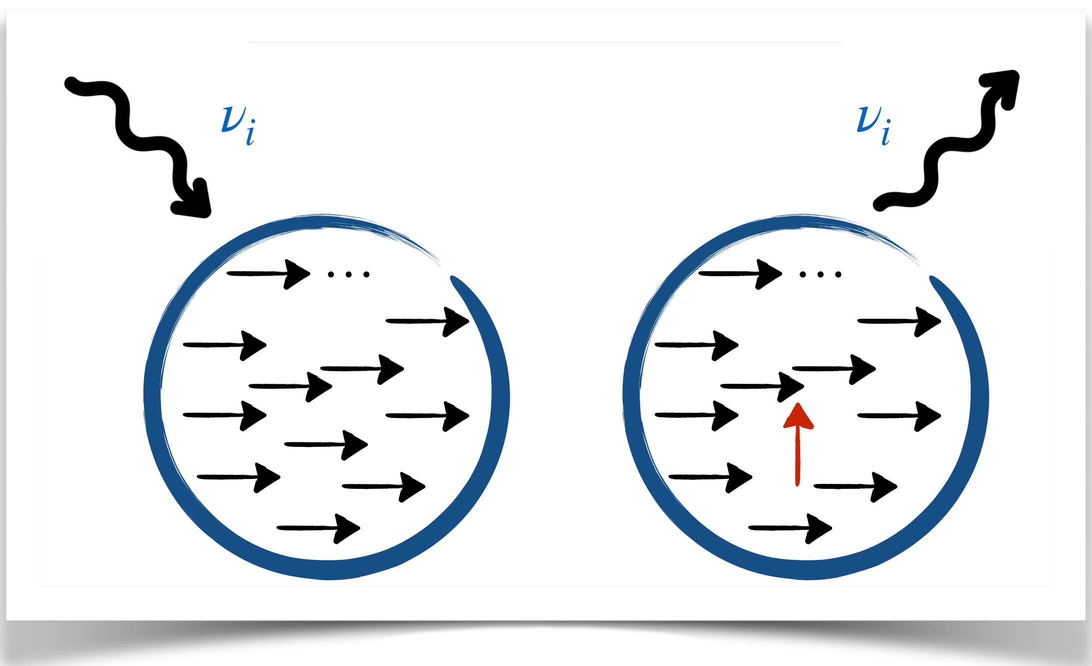
$$\mathscr{L}_I = \frac{G_F}{\sqrt{2}} \bar{\nu}_f \gamma_\mu \gamma_5 \nu_f \bar{\psi}(g_L - g_R)\gamma^\mu \gamma_5 \psi \quad , \text{where } \psi: \text{ any SM fermion}$$

In the non-relativistic limit: $H_I = \frac{G_F}{\sqrt{2}}(g_L - g_R)\ \delta^{(3)}(\vec{x}_\nu - \vec{x}_\psi)\ \vec{\sigma}_\nu \cdot \vec{\sigma}_\psi$

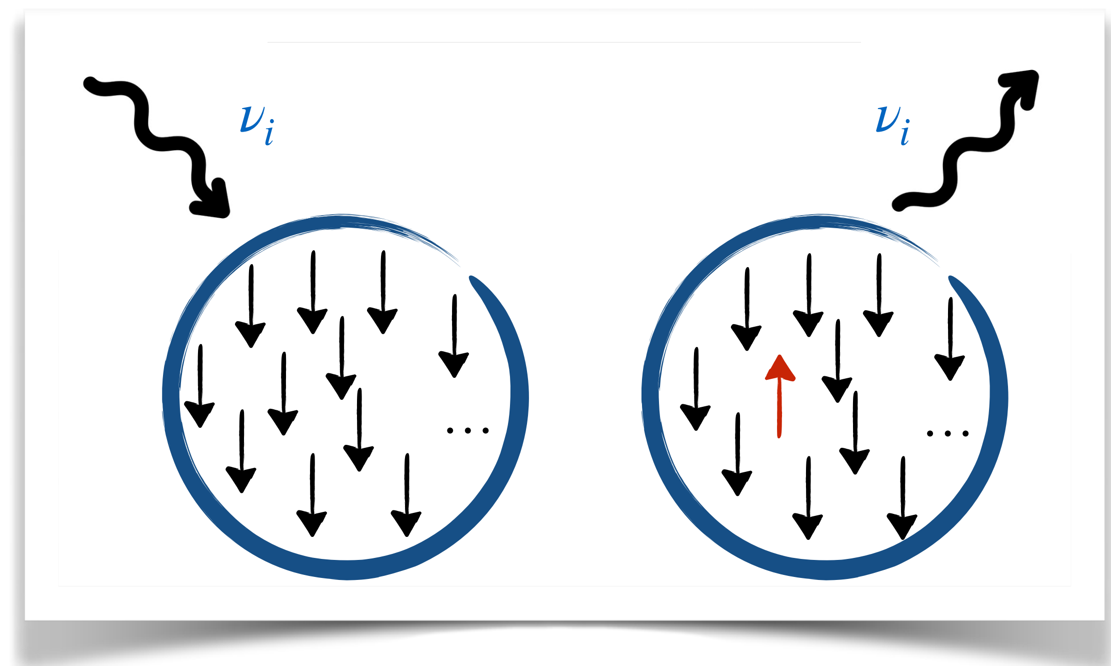
- Two possible processes:



Coherent Spin State vs Ground State Interaction Rates



versus



$$\sim 0.2\ \mathrm{Hz}\frac{n^2}{(3\times 10^{22}\ cm^{-3})^2}\frac{R^4}{(10\ \mathrm{cm})^4}$$

versus

$$\sim 10^{-22}\ \mathbf{Hz}\frac{n}{3\times 10^{22}\ \mathbf{cm}^{-3}}\frac{R^3}{(10\ \mathbf{cm})^3}$$

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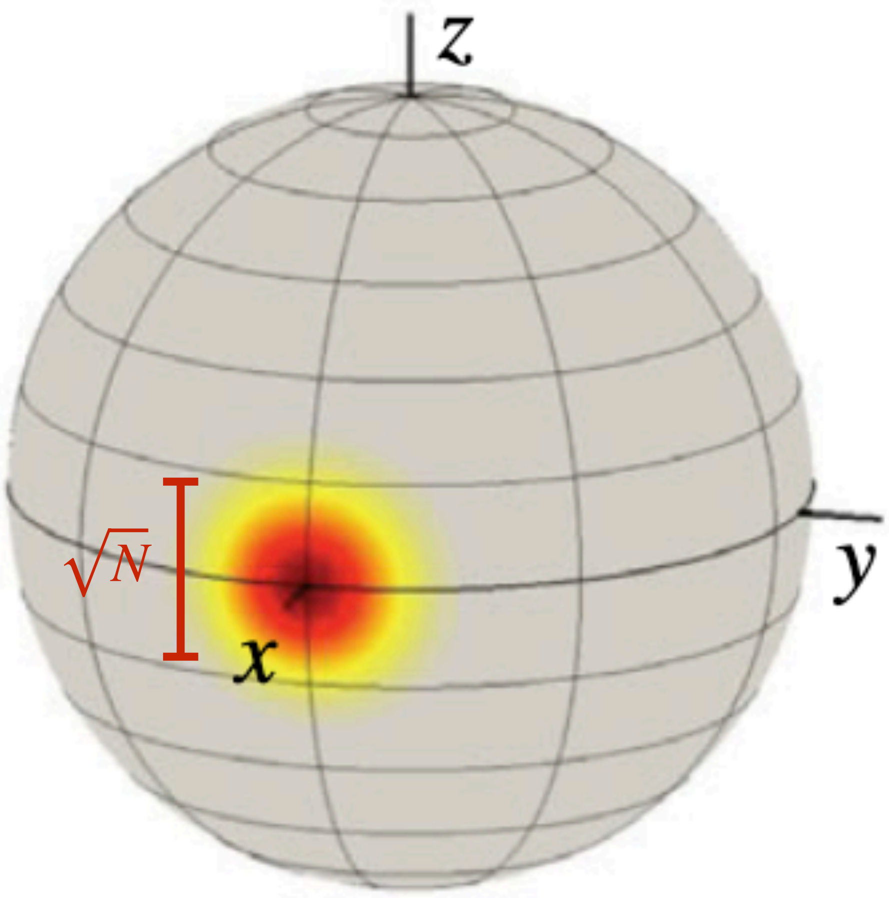
How would you measure the total N^2 superradiant rate?

The effects on an open quantum system

- Net energy exchange for scattering appears as drift

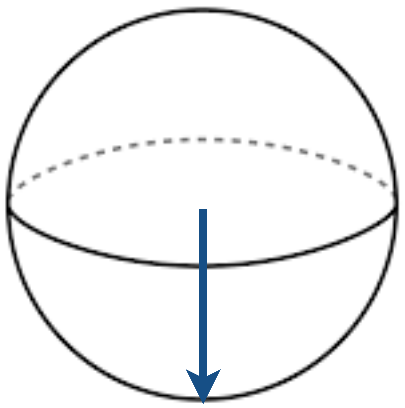
- Observables sensitive to the random walk/diffusion of the state vector on the Bloch sphere, ex. $\langle J_z^2 \rangle$



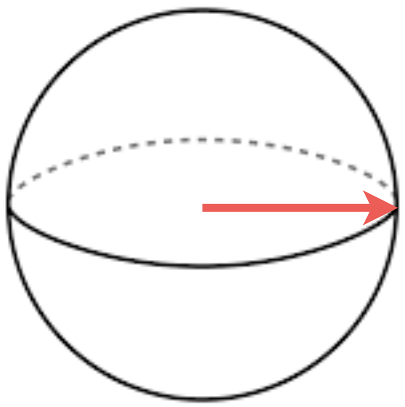
The product state on the Bloch sphere

How does one measure N^2 effects?

Net energy transfer, i.e $\langle J_z \rangle$



vs



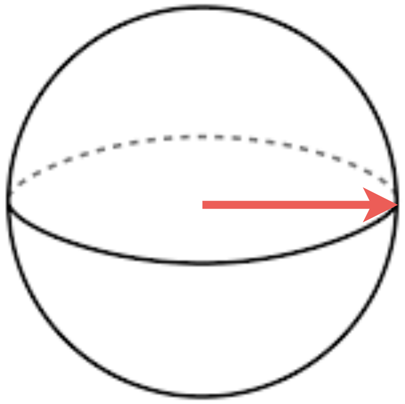
- Starting from $\prod |\downarrow\rangle$
  - $\langle J_z \rangle = -\frac{N}{2} + N\gamma_+ t$
  - $\delta J_z \approx \sqrt{N\gamma_+ t}$
  - $SNR \approx \sqrt{N\gamma_+ t}$

- Starting from $\prod |\rightarrow\rangle$
  - $\langle J_z \rangle = \frac{N^2}{4}(\gamma_+ - \gamma_-)t$
  - $\delta J_z \approx \sqrt{N}/2$
  - $SNR \approx N^{3/2}/2|\gamma_+ - \gamma_-|t$

Product state easily prepared and
may offer metrological advantage when the net rate is non-zero

Measuring net energy transfer for the CνB

vs

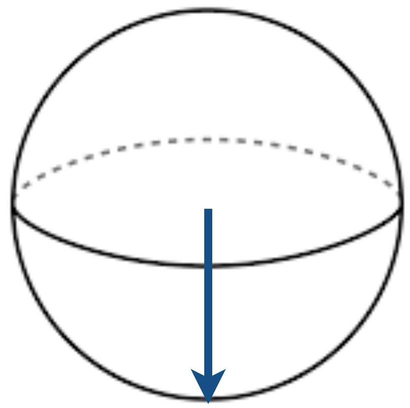


- The KATRIN experiment measures the end point of tritium decay to determine the electron neutrino mass
- Can look for the CνB being absorbed by the tritium
- Sensitivity of KATRIN comparable to a much smaller spin sample prepared in the product state*

$$C_{\text{boost}} \sim 2 \times 10^{11} \left(\frac{10 \text{ cm}}{R}\right)^{3/2} \left(\frac{3 \times 10^{22} \text{ cm}^3}{n_s}\right)^{3/2} \left(\frac{1000 \text{ sec}}{t}\right) \left(\frac{10^3}{N_{\text{shots}}}\right)^{1/2}$$

How does one measure N^2 effects?

Diffusion on the bloch sphere, i.e $\langle J_z^2 \rangle$



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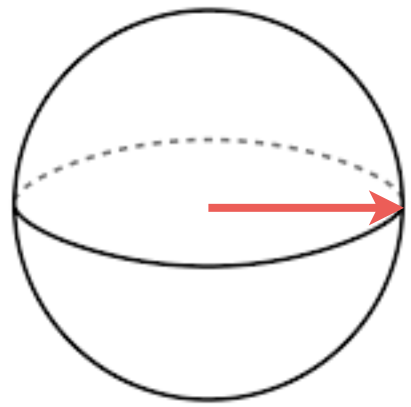


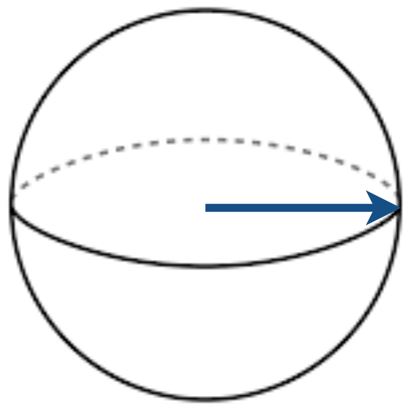
- Starting from $\prod |\downarrow\rangle$
  - $\langle J_z^2 \rangle = \frac{N^2}{4} - (N^2 - N)\gamma_+ t$
  - $\delta J_z^2 = \sqrt{N^3 \gamma_+ t}$
  - $SNR \approx \sqrt{N\gamma_+ t}$

- Starting from $\prod |\rightarrow\rangle$
  - $\langle J_z^2 \rangle = \frac{N}{4} + \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
  - $\delta J_z^2 \approx \frac{N}{2\sqrt{2}}$
  - $SNR \approx \sqrt{2} N |\gamma_+ + \gamma_-| t$

Product state sensitive to the sum of the rates when measuring $\langle J_z^2 \rangle$

How does one measure N^2 effects?

Diffusion on the bloch sphere, i.e $\langle J_z^2 \rangle$



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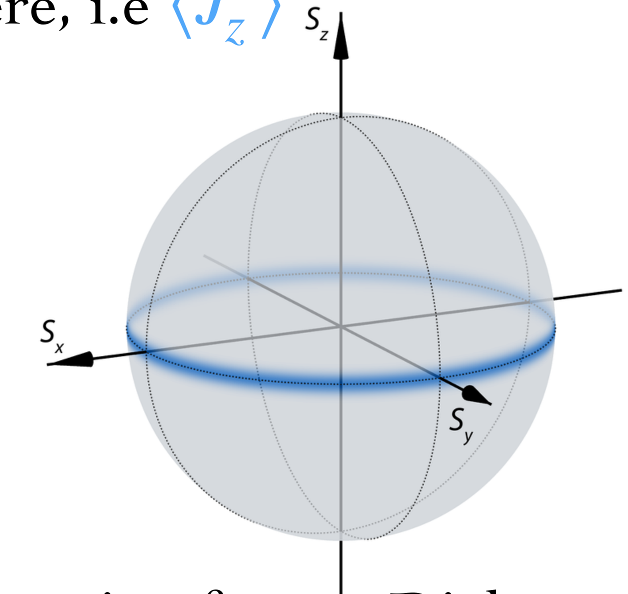


- Starting from $\prod |\rightarrow\rangle$
  - $\langle J_z^2 \rangle = \frac{N}{4} + \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
  - $\delta J_z^2 \approx \frac{N}{2\sqrt{2}}$
  - $SNR \approx \sqrt{2}N(\gamma_+ + \gamma_-)t$

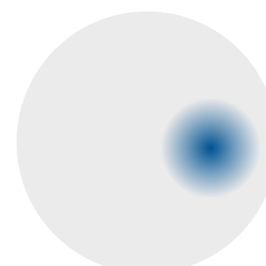
- Starting from a Dicke state
  - $\langle J_z^2 \rangle = \frac{N^2}{4}(\gamma_+ + \gamma_-)t$
  - $\delta J_z^2 \approx \frac{1}{2}\sqrt{N^2(\gamma_+ + \gamma_-)t}$
  - $SNR \approx \sqrt{2}\sqrt{N^2(\gamma_+ + \gamma_-)t}$

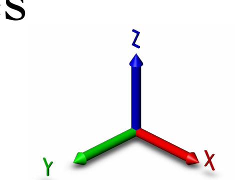
Dicke state is hard to produce

Need a special quantum protocol to maximize the potential of these states

The Middle Way

- States that exhibit N^2 interaction rates



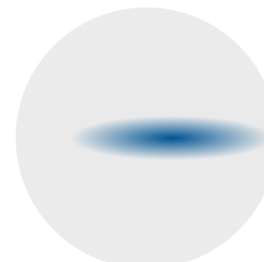


  - The product state
    - Easy to produce

  - The Dicke state $|\ell, m\rangle = |\frac{N}{2}, 0\rangle$
    - Much harder to prepare than the product state

The Middle Way

- Best bet: a squeezed state

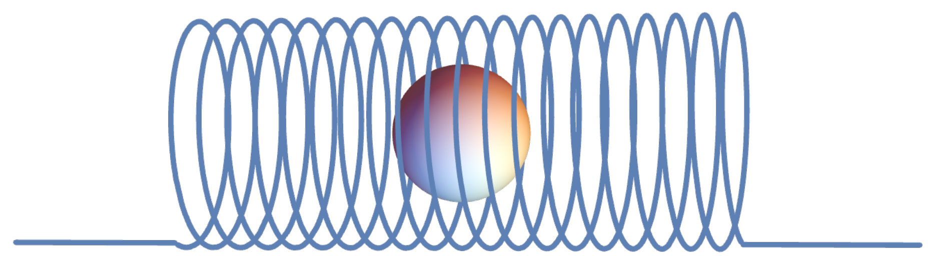


  - Somewhere in between a product state and a Dicke state
  - Preserves much of the good signal-to-noise ratio properties of the Dicke state

- Proposed a squeezing and magnification protocol and experimental setup with S. Dimopoulos, M. Galanis, O. Hosten

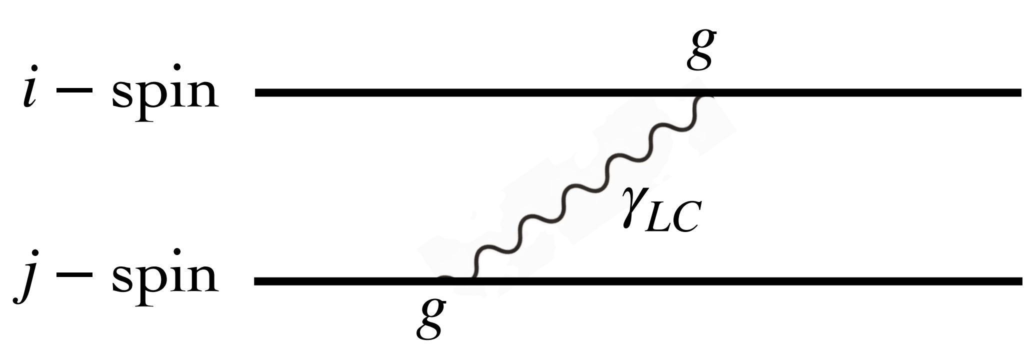
Towards measuring coherent inelastic interactions

Nuclear spin polarized sphere coupled to an LC circuit



- Quantum optics techniques to reduce the spins quantum uncertainty

Davis, Bentsen, Schleier-Smith, 2016
Hosten, Engelsen, Krishnakumar, Kasevich 2016



- The interaction between the two coupled oscillators $g(a_{LC}J_+ + a^\dagger_{LC}J_-)$ with frequencies $\omega_{LC} \neq \omega_{\rm spins}$ induces a Hamiltonian $\dfrac{g^2}{\omega_{LC}-\omega_{\rm spins}}J_z^2$

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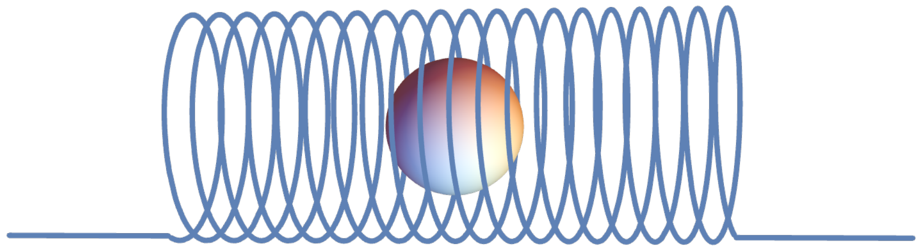

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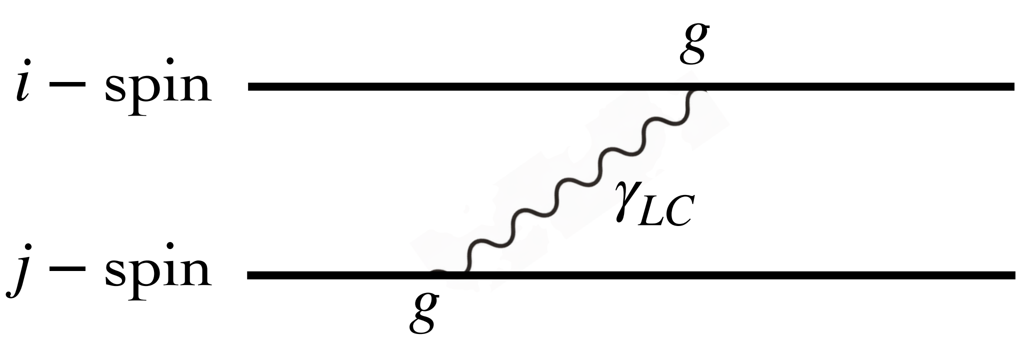
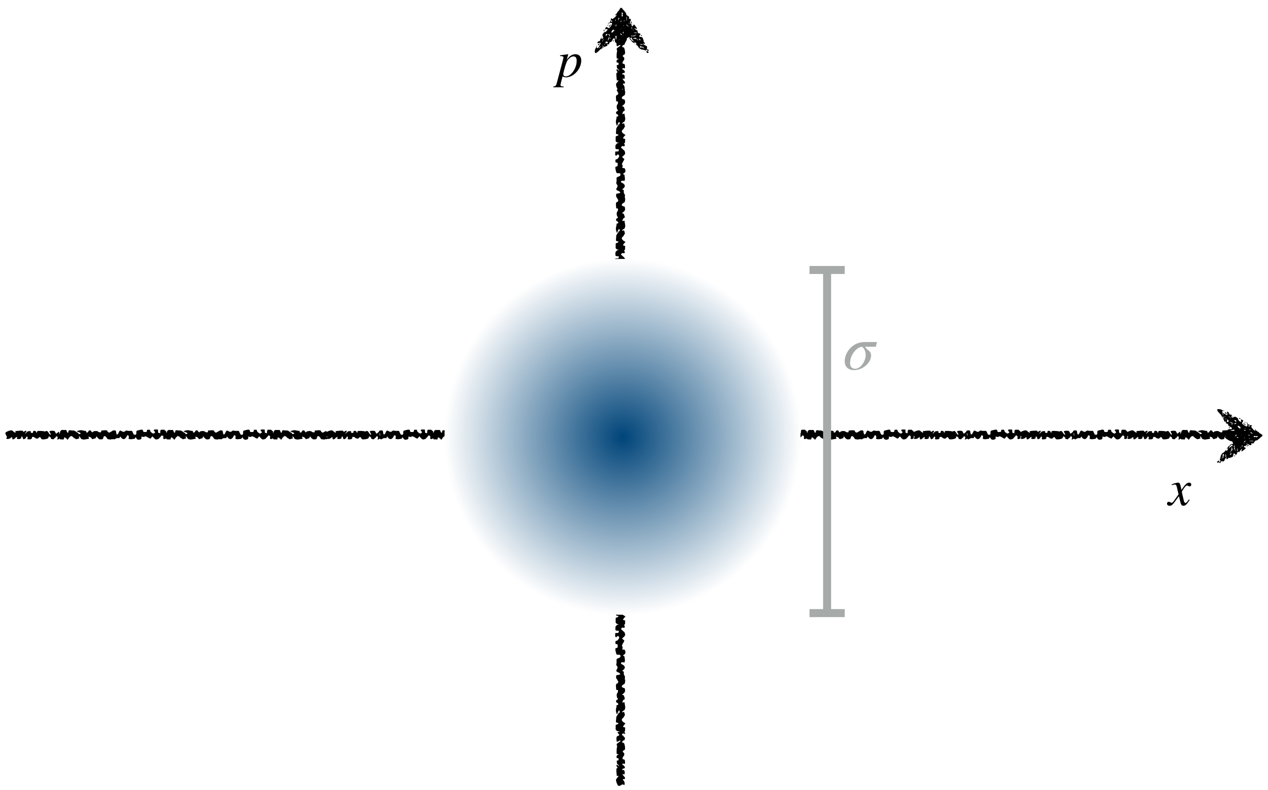


- The interaction between the two coupled oscillators $g(a_{LC}J_+ + a^\dagger_{LC}J_-)$ with frequencies $\omega_{LC} \neq \omega_{\rm spins}$ induces a Hamiltonian $\dfrac{g^2}{\omega_{LC}-\omega_{\rm spins}}J_z^2$

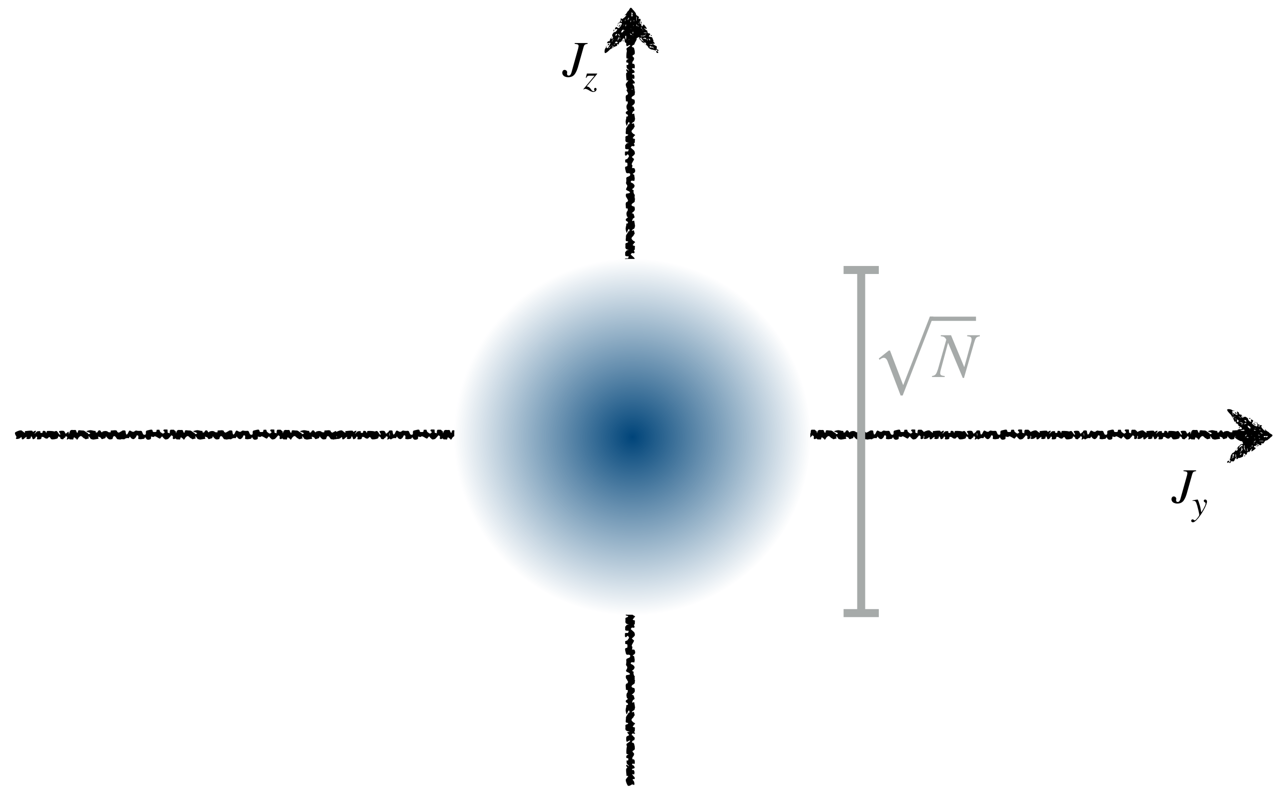
This Hamiltonian is similar to that of a free particle $H = \dfrac{p^2}{2m}$

How does squeezing work?

A free Gaussian wave-packet under the influence of $\frac{p^2}{2m}$



An equatorial product state evolution under the influence of a term $\frac{g^2}{\Delta} J_z^2$



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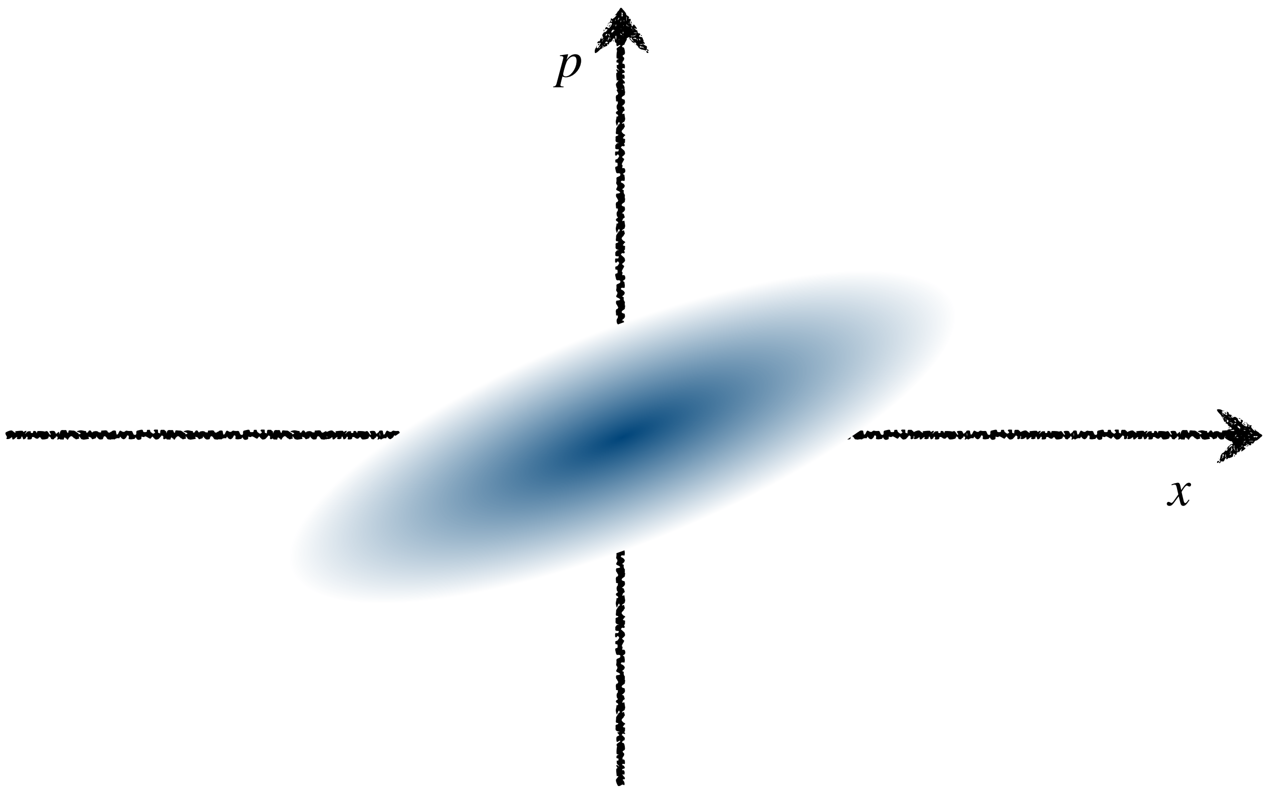

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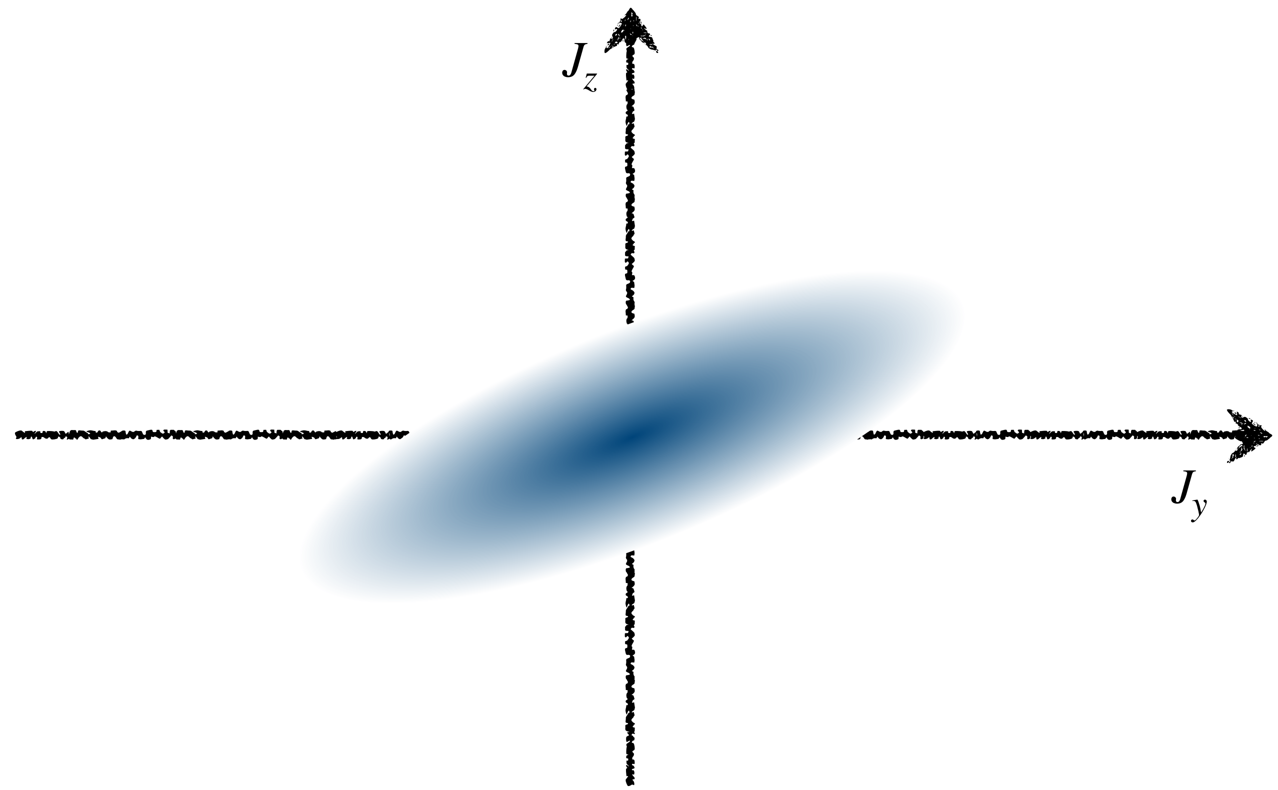
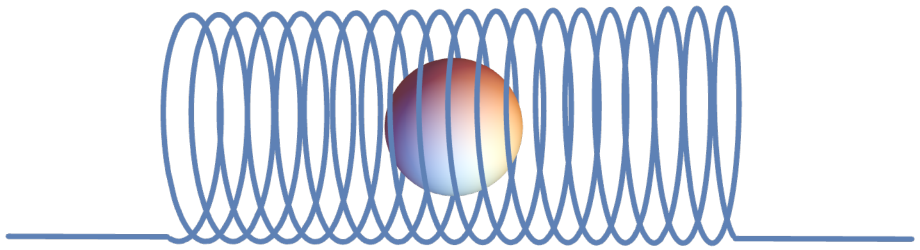


Spread in x set by the velocity $\frac{p}{m}$
Phase space conservation insures that uncertainty in Δp is reduced

Spread in J_y set by the velocity $gJ_z \times N$
because of $[J_i, J_j] = i\epsilon_{ijk}J_k$ and $\langle J_x \rangle = N$
Phase space conservation insures that uncertainty in J_z is reduced

Towards measuring coherent inelastic interactions

Nuclear spin polarized sphere coupled to an LC circuit



- Quantum optics techniques to reduce the spins quantum uncertainty

Davis, Bentsen, Schleier-Smith, 2016
Hosten, Engelsen, Krishnakumar, Kasevich 2016

- Maximum squeezing set by competition between evolution under the J_z^2 Hamiltonian and decay of spins into circuit modes

- Potentially a factor of 10^5 gain beyond the quantum limit

With S. Dimopoulos, M. Galanis, and O. Hosten (2025)

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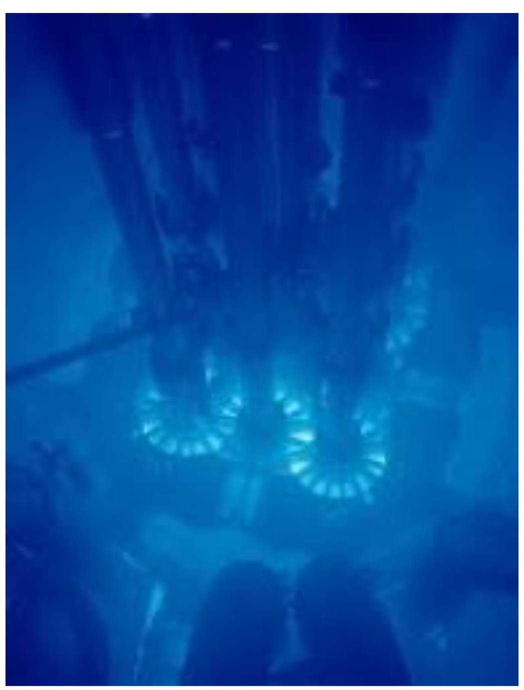
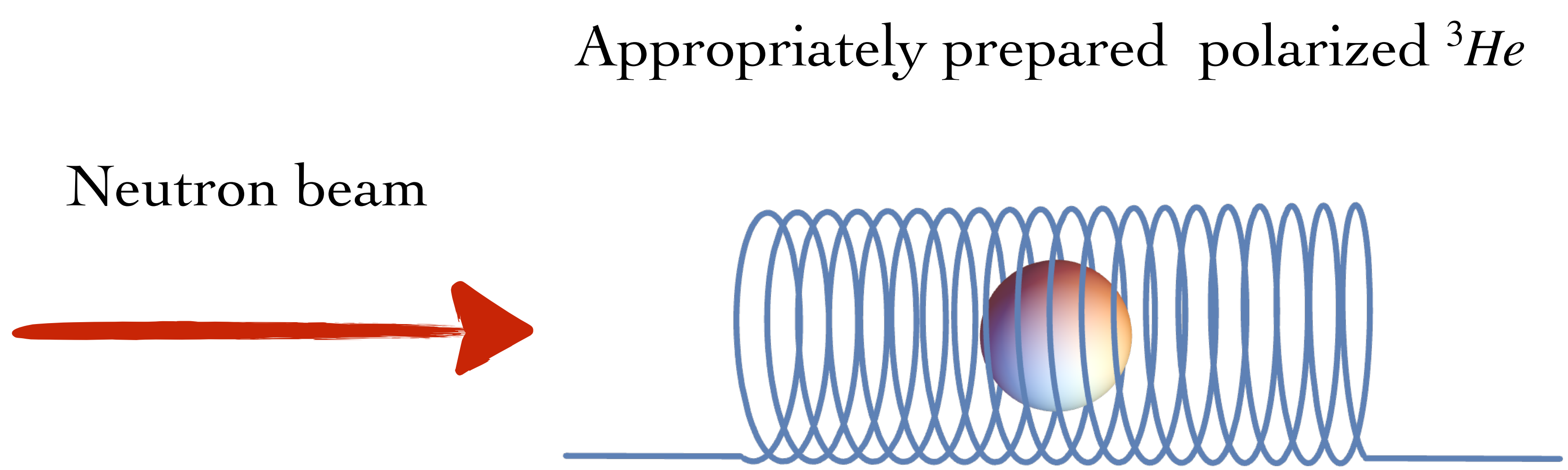
A timeline

- Thermal neutron scattering from a polarized target prepared in a product state: The SIREN experiment

  in progress with Dimopoulos, Galanis, Geraci, Irwin, Rapidis

  - Use: Test quantum protocols
  - Advantages: Thermal neutron sources have large flux, large interaction rates, can be polarized, and the physics is known





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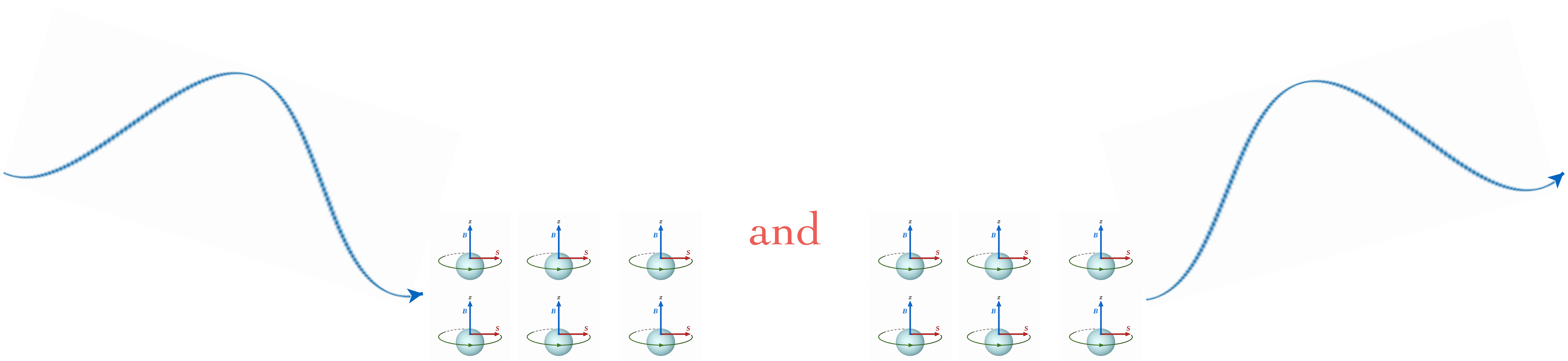
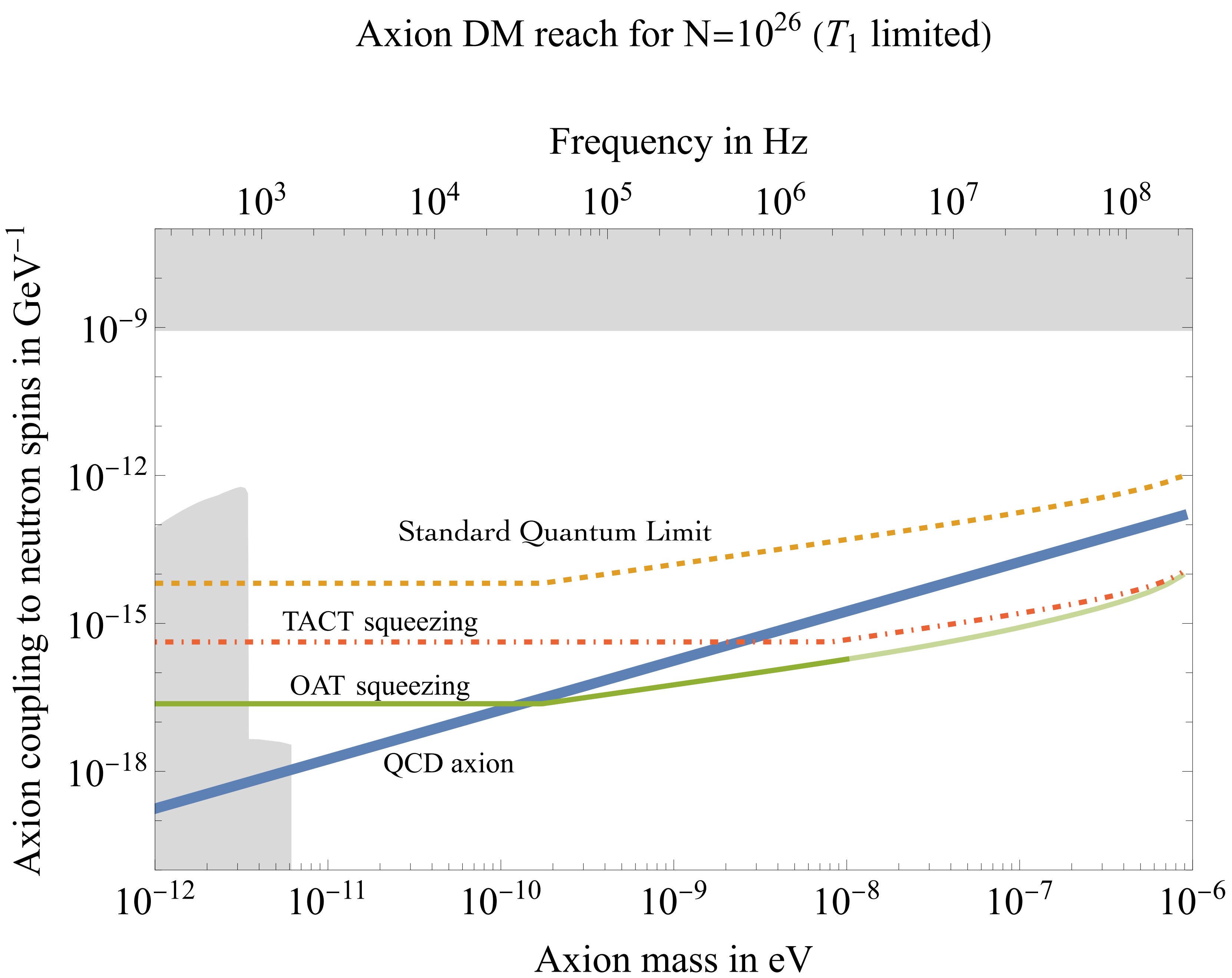
- Dark Matter searches: Axion and Dark-Photon DM, ALPs, sub-GeV, millicharged particles

Stimulated emission and absorption of the QCD axion



Axions couple to spins: $\dfrac{\vec{\nabla} a}{f_a} \cdot \vec{\sigma}$

Reach for Axion Dark Matter



[With S. Dimopoulos, M. Galanis, and O. Hosten]

A timeline

- Thermal neutron scattering from a polarized target prepared in a product state: The SIREN experiment

  in progress with Dimopoulos, Galanis, Geraci, Irwin, Rapidis

  - Use: Test quantum protocols
  - Advantages: Thermal neutron sources have large flux, large interaction rates, can be polarized, and the physics is known

- Dark Matter searches: Axion and Dark-Photon DM, ALPs, sub-GeV, millicharged particles

- CvB: The ultimate goal. Will be determined by the limitations of possible quantum protocols

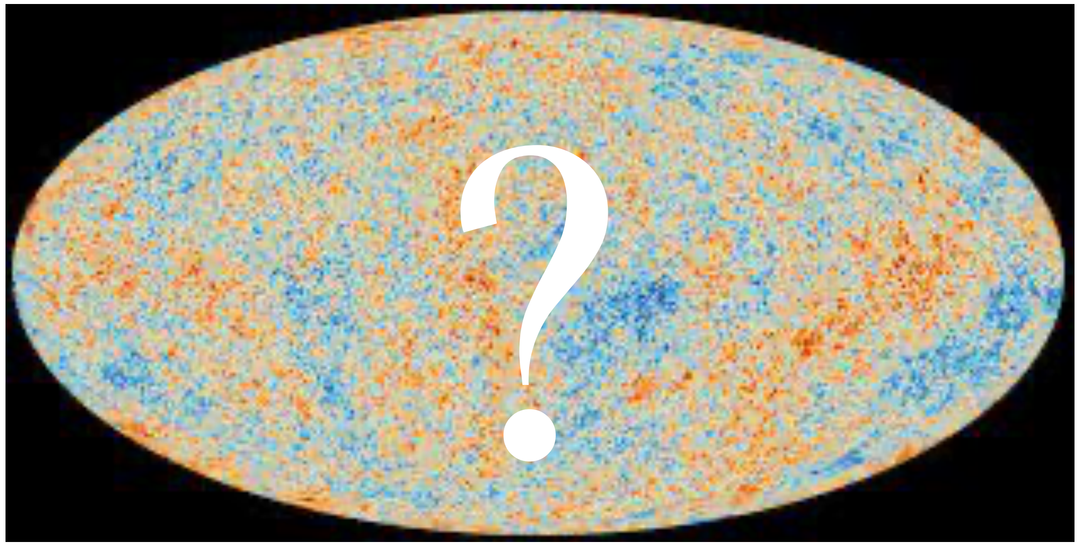
Summary

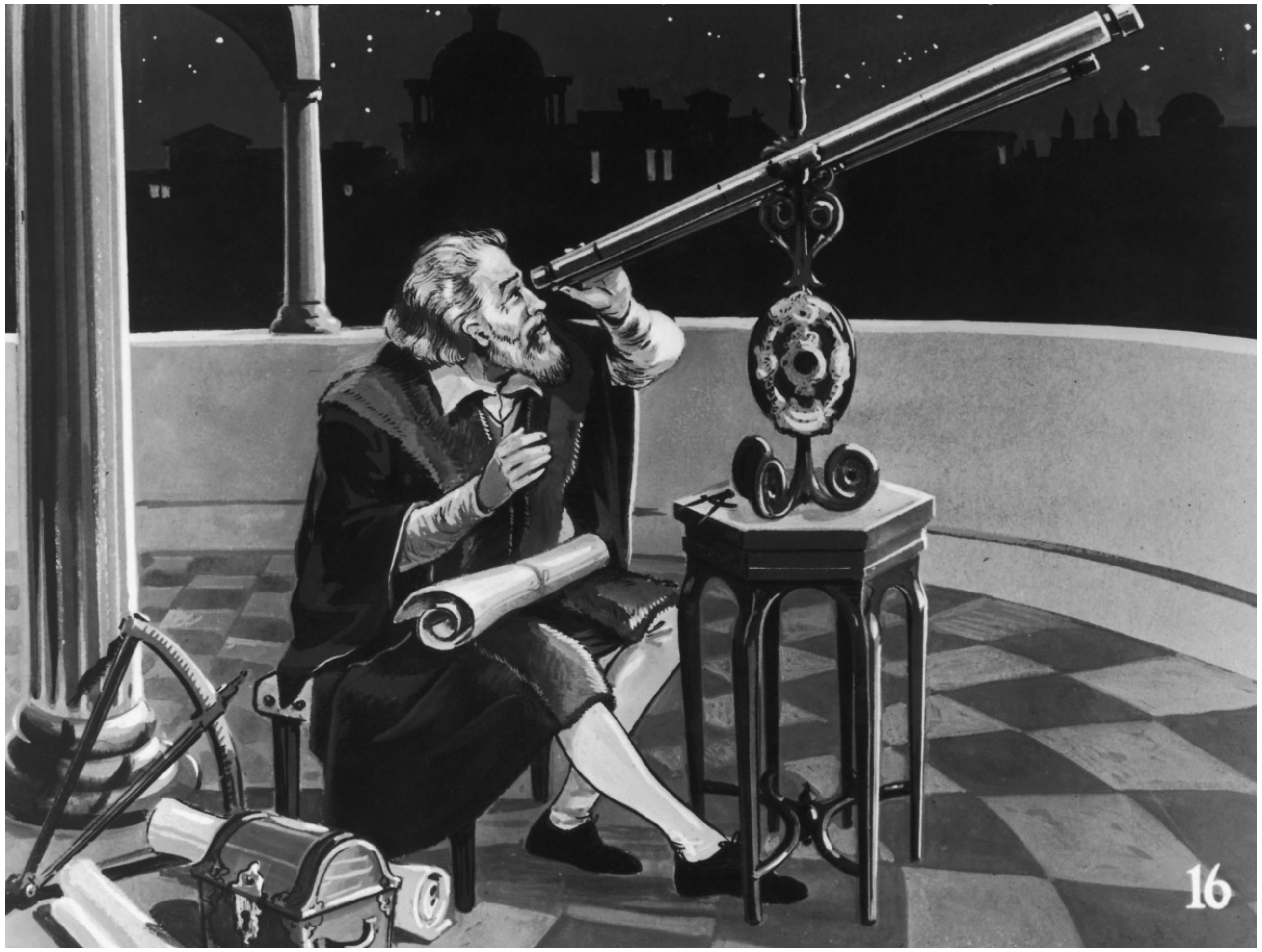
- Inelastic scattering rates can scale like N^2 — going beyond what Dicke Superradiance established for decays

- New ultra-low threshold (quantum) detectors

- This is just the beginning

A Cosmic Neutrino Background Telescope?





How did the Universe looked like when it was less than 1 second old?…

Why is the CνB important?

- Probes physics at a time much earlier than the CMB
- An entire sector of the Standard Model: 3 flavors and 7 parameters
- Are neutrinos their own antiparticle? (Dirac vs Majorana)
- Using non-relativistic particles for 3D tomography of the Universe

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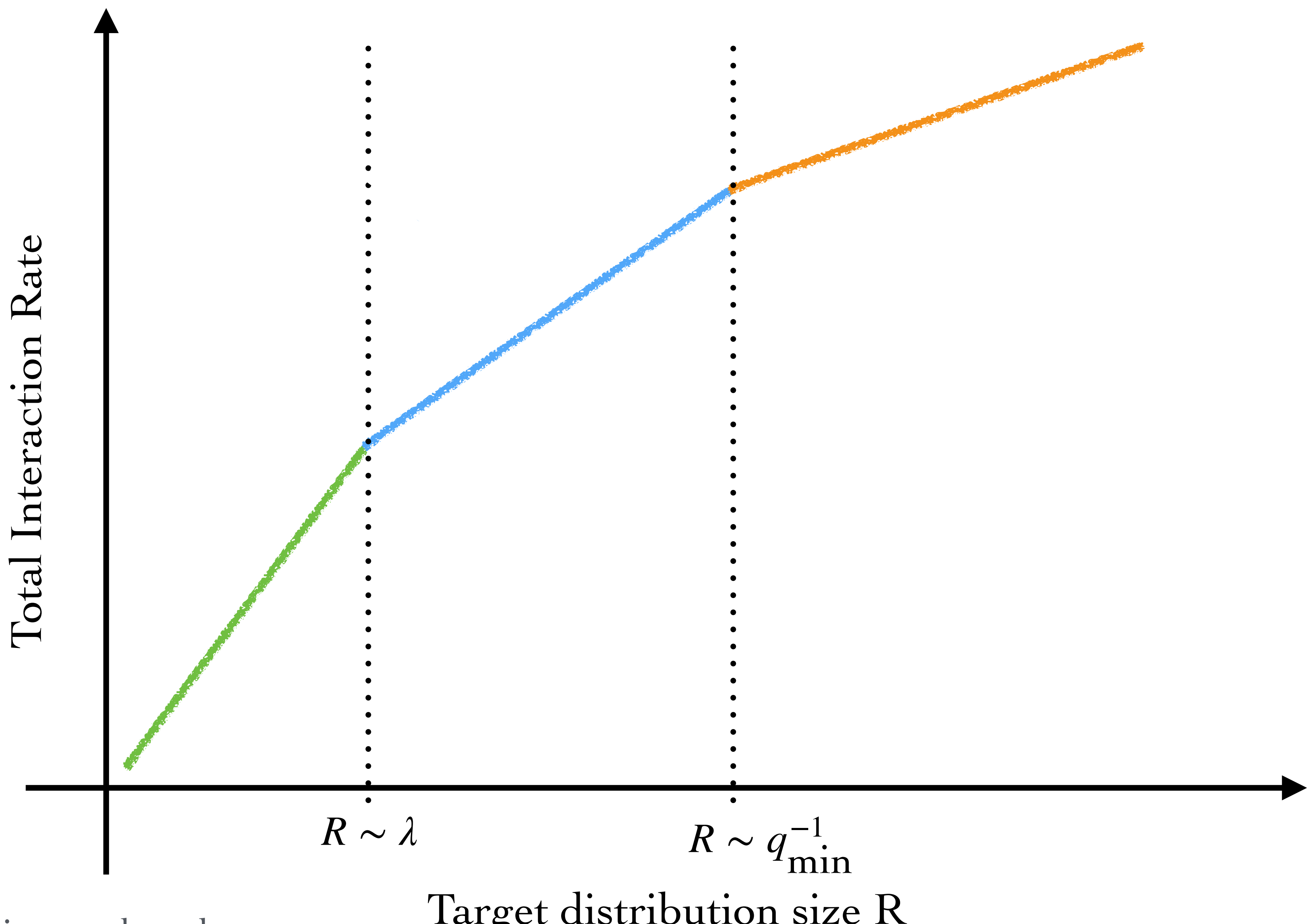
- Using non-relativistic particles for 3D tomography of the Universe
- They are predicted to exist!

Superradiant scattering from a target of size R

Scattering form factor: $\left| \sum_{\alpha=1}^{N} e^{i\vec{q}\cdot\vec{r}_a} \right|^2$



λ: particle deBroglie wavelength

$q_{\rm min}$: Minimum momentum transfer set by energy conservation

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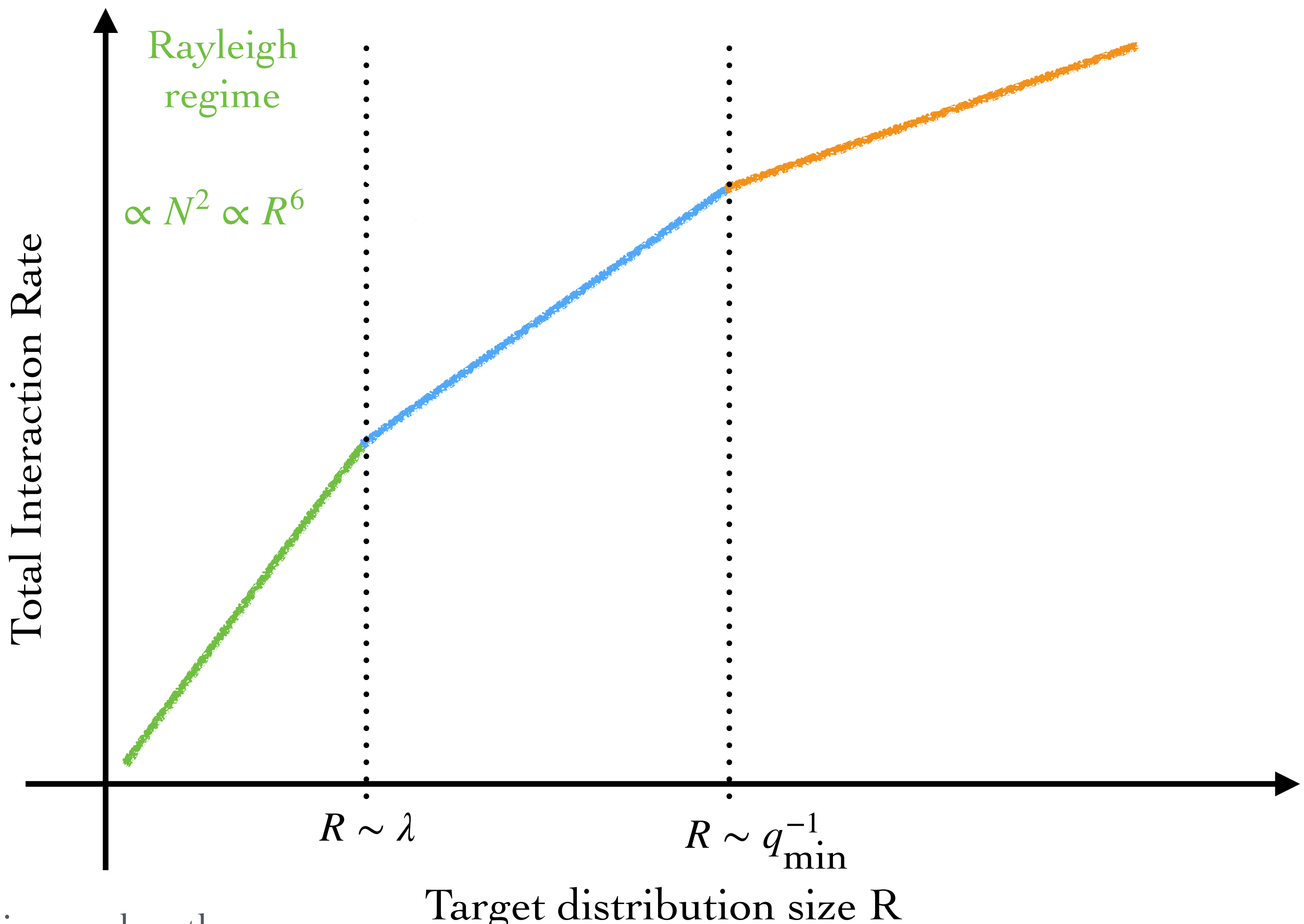

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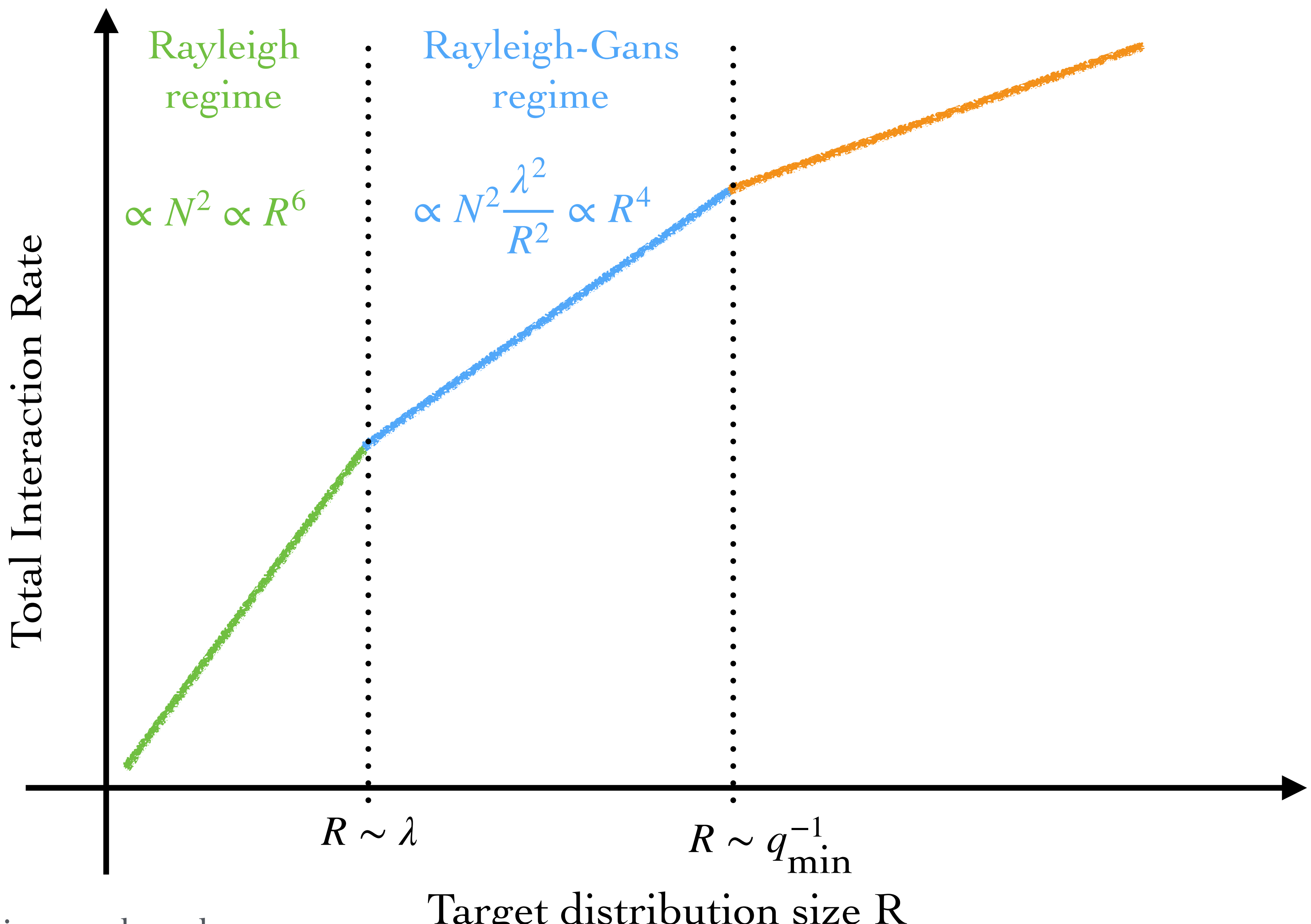

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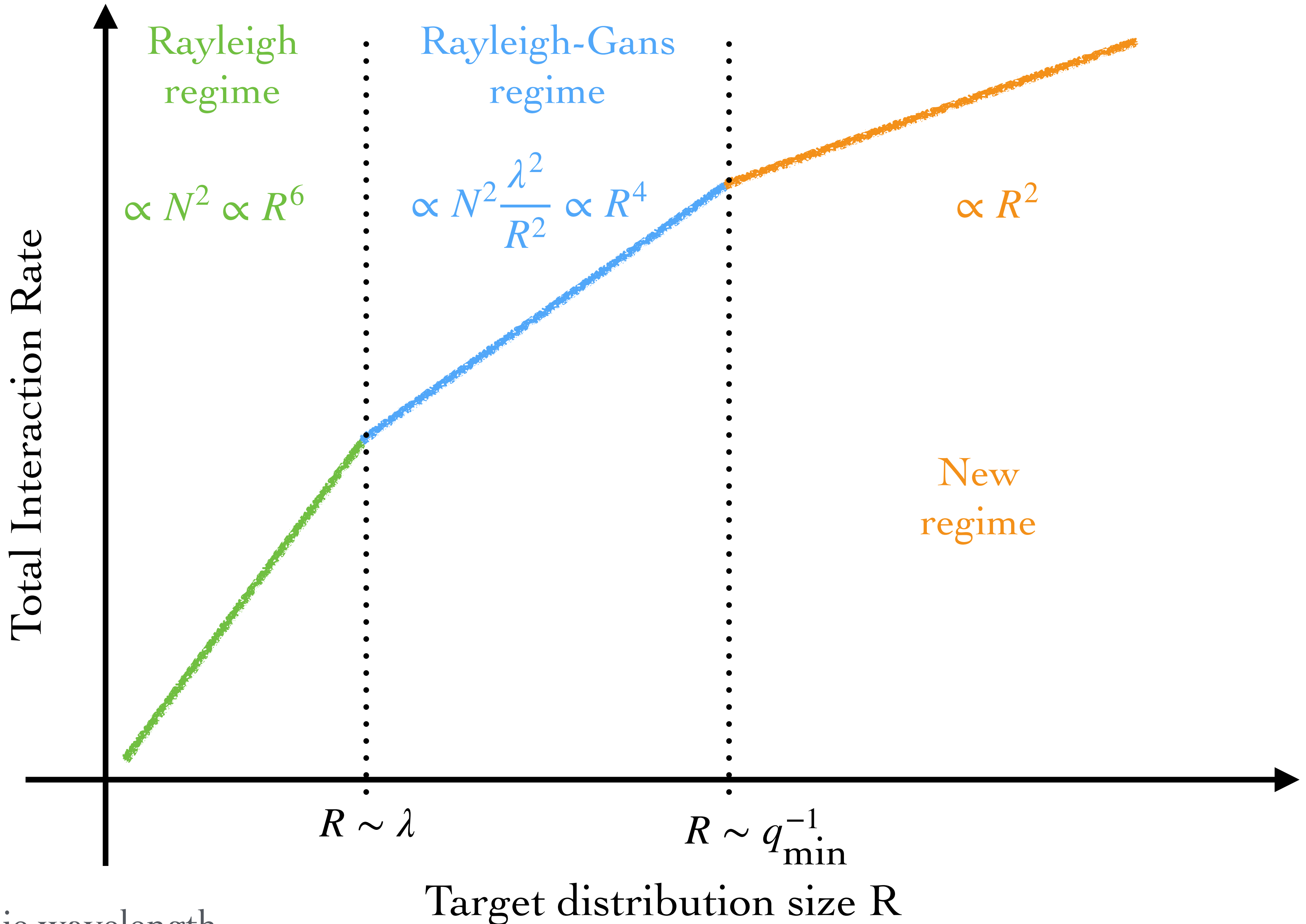

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Evolution of the quantum state due to cosmic relics

An open quantum system formalism

Ex. the cosmic neutrino background density matrix

$$\rho_\nu = \prod_{\{\mathbf{k},s\}} \left[(1 - \langle n \rangle_{\mathbf{k},s}) \, |0\rangle_{\mathbf{k},s} \langle 0|_{\mathbf{k},s} + \langle n \rangle_{\mathbf{k},s} \, |1\rangle_{\mathbf{k},s} \langle 1|_{\mathbf{k},s} \right]$$

Integrating out the cosmic relic parameters

$$\dot{\rho}_S \simeq \boxed{-i\delta\omega_S[J_z, \rho_S]} + \frac{\gamma_-}{2}\mathcal{L}_{J_-}[\rho_S(t)] + \frac{\gamma_+}{2}\mathcal{L}_{J_+}[\rho_S(t)] + \frac{\gamma_z}{2}\mathcal{L}_{J_z}[\rho_S(t)]$$

constant energy shift

$$\mathscr{L}_A[\rho_S] \equiv 2A\rho_S A^\dagger - A^\dagger A \rho_S - \rho_S A^\dagger A$$

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Cosmic relics

This is analogous to the Lindblad formalism for spins/atoms in a photon bath

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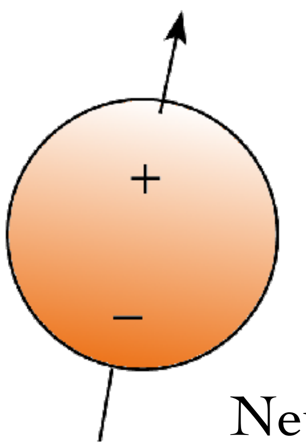
Why is the Electric Dipole Moment of the Neutron Small?

The Strong CP Problem and the QCD axion



Neutron
EDM

$$\frac{g_s^2}{32\pi^2}\theta_s \vec{E}_s \cdot \vec{B}_s$$

EDM ~ e fm θ_s

Experimental bound: $\theta_s < 10^{-10}$

Solution:
θ_s ~a(x,t) is a dynamical field, an axion

Axion mass from QCD:

$$\mu_a \sim 6 \times 10^{-11}\ \text{eV}\ \frac{10^{17}\ \text{GeV}}{f_a} \sim (3\ \text{km})^{-1}\ \frac{10^{17}\ \text{GeV}}{\mathrm{f_a}}$$

f_a : axion decay constant

Factors determining squeezing

- The effective coupling $\chi \equiv \dfrac{g^2}{\Delta}$

- The losses, in cavity decay $\eta \approx \dfrac{g^2 \kappa}{\Delta^2}$

- Everything needs to happen in a time less than the dephasing time