Nachtmann moment of the parity-violating structure function from Feynman-Hellmann

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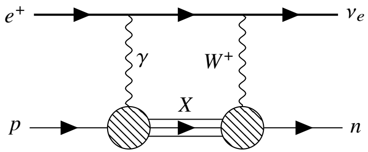
Electroweak Radiative Corrections (EWRC)

Current determinations of V_{ud} require an accurate evaluation of $\square_{\gamma W}$ radiative corrections



$$\square_{\gamma W} = \frac{3\alpha}{2\pi}\int_0^{\infty}\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_1^N[F_3^{(0)}]$$

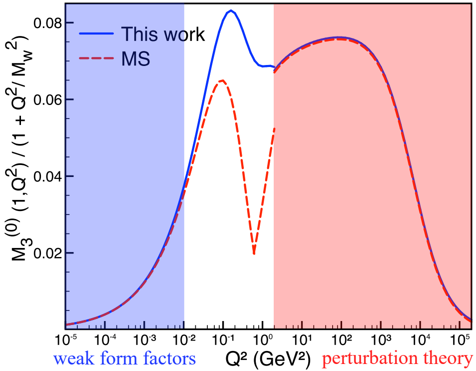
via isospin symmetry

$$F_3^{(0)} = -\frac{1}{4}(F_{3,p}^{\gamma Z} - F_{3,n}^{\gamma Z})$$

Quantity of interest

First Nachtmann moment

Seng et al., PRL 121, 241804 (2018)



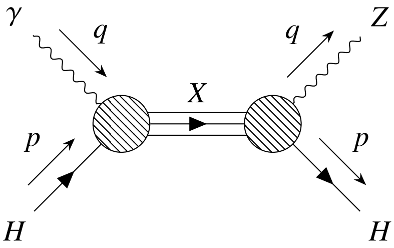
Investigate Pion for cleaner signal at low Q^2

Parity-odd structure function $\mathcal{F}_3^{\gamma Z}(\omega, Q^2)$

Compton Tensor $\quad T^{VA}_{\mu\nu} = i\int d^4z\, e^{iq\cdot z}\langle p|\mathcal{T}\{J^V_\mu(z)J^A_\nu(0)\}|p\rangle$

General Lorentz decomposition (spin-averaged)

$$T^{VA}_{\mu\nu}(p,q) = -g_{\mu\nu}\mathcal{F}_1(\omega,Q^2) + \frac{p_\mu p_\nu}{p\cdot q}\mathcal{F}_2(\omega,Q^2) + i\varepsilon^{\mu\nu\alpha\beta}\frac{p_\alpha q_\beta}{2p\cdot q}\mathcal{F}_3(\omega,Q^2) + \ldots$$



Isolate $\mathcal{F}_3$ by Setting $\mu = 1, \nu = 3$,
$q_1 = 0, q_2 \neq 0, p_1 = 0$

$$T^{VA}_{13}(p,q) = i\frac{p_4 q_2}{2p\cdot q}\mathcal{F}_3(\omega,Q^2) \quad \text{where} \quad \omega = \frac{2p\cdot q}{Q^2} = \frac{1}{x}$$

Vector $\quad J^V_\mu(z) = \sum_{f=u,d} Q_f\overline{\psi}_f(z)\gamma_\mu\psi_f(z)$

Axial Vector $\quad J^A_\nu(y) = \sum_{f=u,d} g^f_A\overline{\psi}_f(y)\gamma_5\gamma_\nu\psi_f(y)$

Feynman-Hellmann (FH) Theorem in QFT

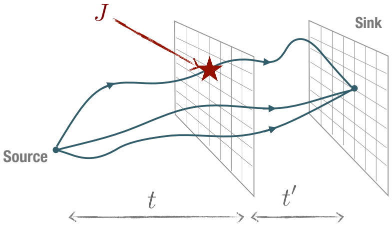
Perturb system by modifying action with some operator $\mathcal{O}$

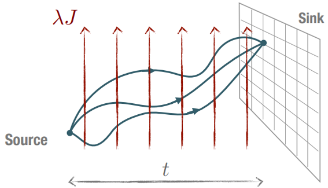
$$S \to S + \lambda \int d^4x \, \mathcal{O}(x)$$

For the physical system evaluate at $\lambda = 0$, FH provides the relation

$$\left.\frac{\partial E_{H,\lambda}(\mathbf{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E_H(\mathbf{p})} \langle H(\mathbf{p})|\mathcal{O}|H(\mathbf{p})\rangle$$

Computing the system at different λ provides a way to determine the physical quantities of interest

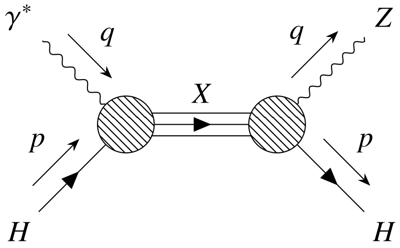




Feynman-Hellmann Implementation

Introduce background fields to fermion action

$$S \rightarrow S(\lambda) = S_0 + \lambda_V \int d^4z(e^{iq\cdot z} + e^{-iq\cdot z})\mathcal{J}_\mu^V(z) - i\lambda_A \int d^4y(e^{iq\cdot y} - e^{-iq\cdot y})\mathcal{J}_\nu^A(y)$$



Feynman-Hellmann relation provides

$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{q_2} \frac{\partial^2 E(\mathbf{p}, \mathbf{q})}{\partial\lambda_V \partial\lambda_A}\bigg|_{\lambda=0}$$

Lattice Details

Details of the gauge ensembles used in this work

N_f	β	a [fm]	κ_l	κ_s	$L^3 \times T$	m_π [MeV]	N_{cfg}
2 + 1	5.65	0.068	0.122005	0.122005	$48^3 \times 96$	412	535
2 + 1	5.95	**0.052**	0.123460	0.123460	$48^3 \times 96$	418	507
2 + 1	5.65	0.068	0.122197	0.121623	$64^3 \times 96$	**220**	420

Symanzik improved gauge, NP-improved Clover action

- FH implementation at the valence quark level (connected contribution qq)
- Access range of $\omega = \frac{2p \cdot q}{Q^2}$ for each choice of q, varying p cheap

LPT correction

$$\text{Continuum} \qquad \frac{Q^2}{q_2} \quad \to \quad \frac{\sum_i \sin^2 q_i + [\sum_i (1 - \cos q_i)]^2}{\sin q_2} \qquad \text{LQCD}$$

K. U. Can, *et al.*, Phys.Rev.D 111 (2025) 11, 114505

Gross-Llewelyn Smith (GLS) Sum Rule

$$\frac{\mathcal{F}_3(\omega,Q^2)}{\omega} = 4\int_0^1 dx \frac{1}{1-x^2\omega^2} F_3(x,Q^2) \quad \Rightarrow \quad \left.\frac{\mathcal{F}_3(\omega,Q^2)}{\omega}\right|_{\omega=0} = 4M_1^{CN}(Q^2)$$

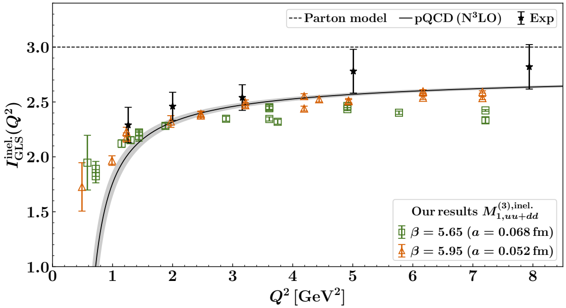
Quark-parton model

$$M_{1,qq}^{CN,\mathrm{QPM}}(Q^2) = \int_0^1 dx\,(q-\overline{q}) = N_q$$

N^{3}LO Perturbative corrections

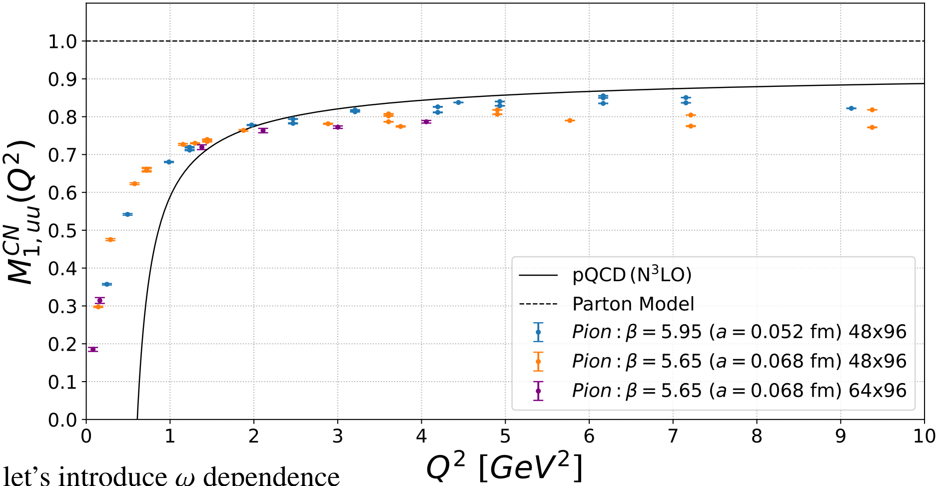
$$M_{1,qq}^{CN}(Q^2) = N_q\left[1-\sum_{i=1}^{4} C_i\left(\frac{\alpha_s(Q^2)}{\pi}\right)^i\right]$$

with known coefficients C_i



K. U. Can, *et al.*, Phys.Rev.D 111 (2025) 11, 114505

Pion GLS Results - Preliminary



Now let's introduce ω dependence

First Nachtmann Moment Approximation via Taylor series

$$M_3^N(1,Q^2) = \frac{4}{3}\int_0^1 dx\,\frac{1+2r}{(1+r)^2}F_3(x,Q^2) \quad \text{with} \quad r = \sqrt{1+\frac{4m^2x^2}{Q^2}}$$

Taylor expand kernel for large Q^2

$$\frac{1+2r}{(1+r)^2} \approx \frac{3}{4} - \frac{m^2x^2}{2Q^2} + \frac{3m^4x^4}{4Q^4} - \frac{3m^6x^6}{2Q^6} + \dots$$

Cornwall-Norton Moments $\quad M_{2n-1}^{CN}(Q^2) = \int_0^1 dx\, x^{2n-2}F_3(x,Q^2)$

$$M_3^N(1,Q^2) = M_1^{CN}(Q^2) - \frac{2m^2}{3Q^2}M_3^{CN}(Q^2) + \dots \qquad \left(\frac{m^2}{Q^2} \ll 1\right)$$

P. G. Bluden, W. Melnitchouk, and A. W. Thomas, Phys.Rev.Lett. 107, 081801 (2011)

Nachtmann Moment Approximation via Change of Basis

The Nachtmann kernel can be approximated using the FH-accessible ω-dependence.

Nachtmann moment $$M_3^N(1,Q^2) = \frac{4}{3}\int_0^1 dx\, \frac{1+2r}{(1+r)^2} F_3(x,Q^2)$$

Dispersion relation $$\frac{\mathcal{F}_3(\omega,Q^2)}{\omega} = 4\int_0^1 dx\, \frac{1}{1-x^2\omega^2} F_3(x,Q^2)$$

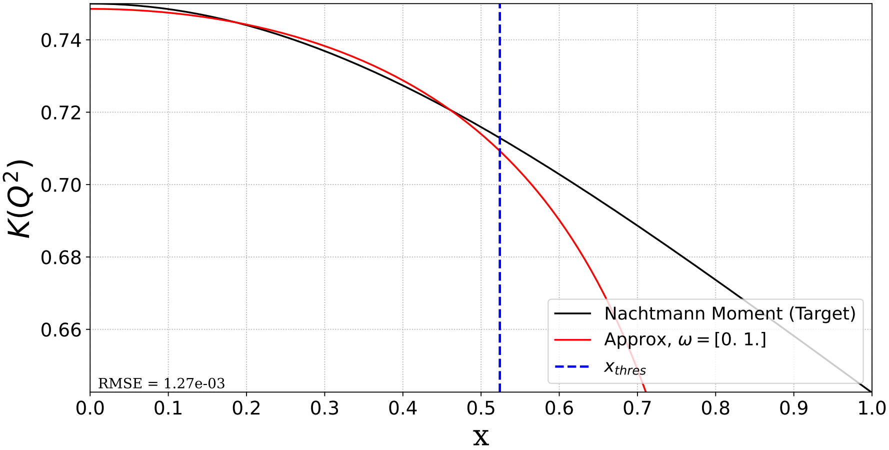
Change of kernel basis

$$\frac{1+2r}{(1+r)^2} \approx \sum_i c_i \frac{1}{1-x^2\omega_i^2} \quad \Rightarrow \quad M_3^N(1,Q^2) = \frac{1}{3}\sum_i c_i \frac{\mathcal{F}_3(\omega_i,Q^2)}{\omega_i}$$

Estimate systematic uncertainty by strictly over- or underestimating the kernal

C.-Y. Seng and U.-G. Meißner, Phys. Rev. Lett. 122, 211802 (2019)

Change of kernal basis $q = [0, 2, 0]\left(\frac{2\pi}{L}\right)$, $Q^2 = 0.58\ \text{GeV}^2$



Pion production threshold

$$x_{\text{thres}} = \frac{1}{\frac{3m_\pi^2}{Q^2} + 1}$$

$x > x_{\text{thres}}$ no contribution

Nachtmann Moment Determination $q = [0, 1, 2]\left(\frac{2\pi}{L}\right)$, $Q^2 = 0.721$ GeV^2

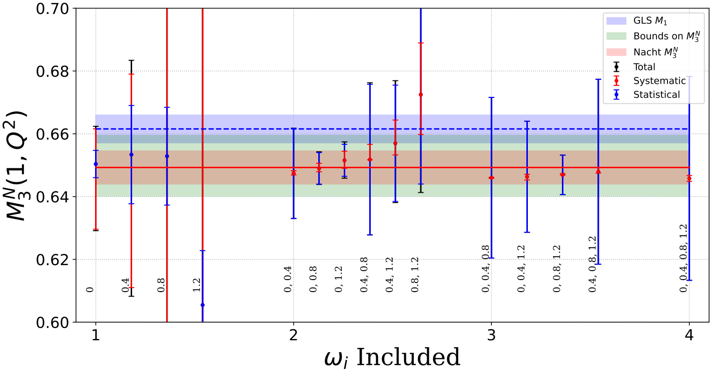
Attempt all combinations of ω_i included

Uncertainties

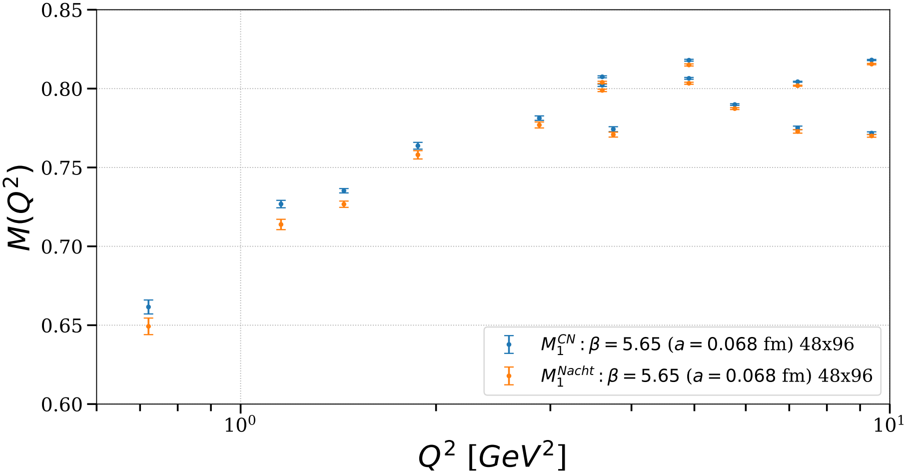
Statistical and Systematic inform average

Shaded Regions

Compare Nachtmann to GLS



Nachtmann Moment Q^2 Dependence - Preliminary



Conclusion

Summary

- Access hadronic structure via the Feynman-Hellmann approach
- Demonstrated determination of F_3 structures via GLS
- Determined the first Nachtmann moment of $F_3^{\gamma Z}$
- Extending to nucleons will provide the hadronic input for γW radiative corrections.

Future Work

- Extend M_1^N analysis to span $0.01 \lesssim Q^2 \lesssim 2\ \mathrm{GeV}^2$
- Twisted Boundary Conditions - extend accessible ω, GEVP - clean analysis
- Extending to nucleons - hadronic input for γW radiative corrections.

Thanks for listening!

Twisted Boundary Conditions

- Extend the accesible range of ω values
- Introduce phase factor $\theta_i \in [0, 2\pi]$

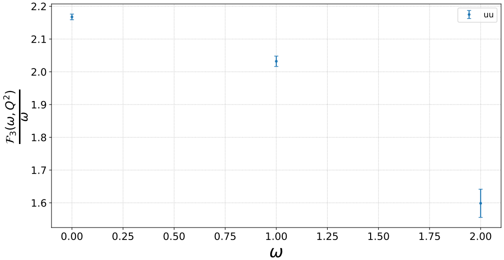
$$\psi(t, x_i = 0) = e^{i\theta_i}\psi(t, x_i = L)$$

$$\Longrightarrow \quad p_i = p_i^F + \frac{\theta_i}{L}$$

- Extending the accessible range of momenta, and thus ω

$$\omega = \frac{2p \cdot q}{Q^2}$$

Discretization limits access to ω values at low Q^2



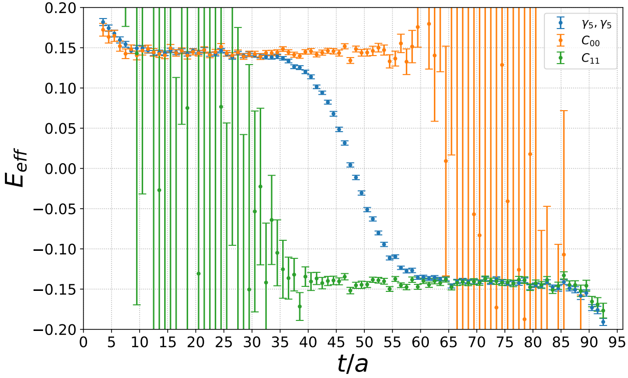
Example : $q = [0, 1, 1]\left(\frac{2\pi}{L}\right)$

Generalized Eigenvalue Problem (GEVP)

Correlator $C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle \qquad \chi_i = \bar{\psi}\Gamma_i\psi$

Variational Method

- Choose an interpolator operator basis $\Gamma_i \in \{\gamma_5, \gamma_t\gamma_5\}$
- Form the correlation matrix $C_{ij}(t)$
- Solve the GEVP $C(t)v_n = \lambda_n(t, t_0)C(t_0)v_n$
- Eigenvectors isolate the forward and backwards propagating states



Effective masses after GEVP rotation

Application of GEVP for determination of ΔE