First constraints on the nonperturbative gluon Collins-Soper kernel

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in collaboration with
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arXiv:2607.24587

(cute)

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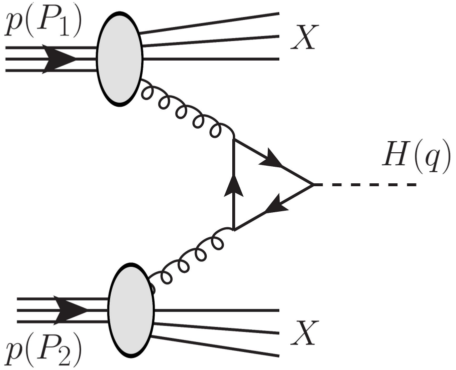
The gluon Collins-Soper (CS) kernel

- **Relevant processes:** CS kernel onnects descriptions of gluon structure across:

Higgs production



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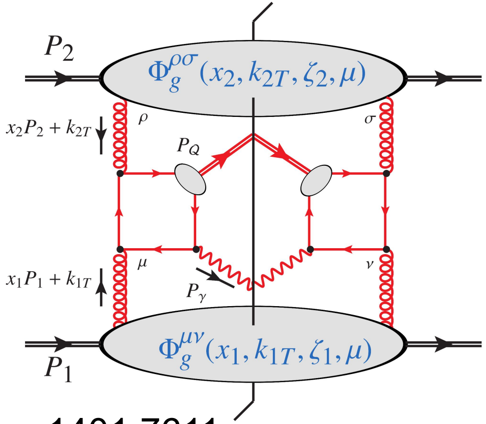
Quarkonium production

(e.g. Q + photon)



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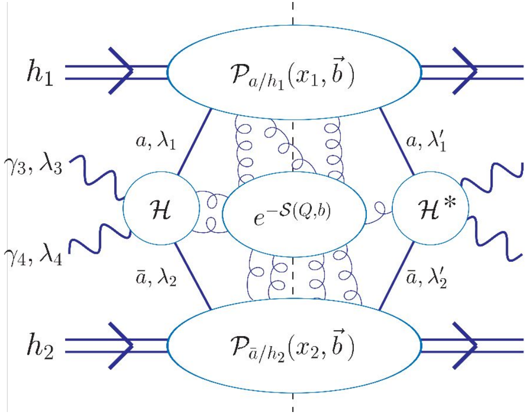
Photoproduction

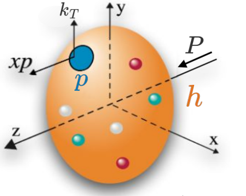
(ultra-peripheral collisions)



hep-ph/0702003

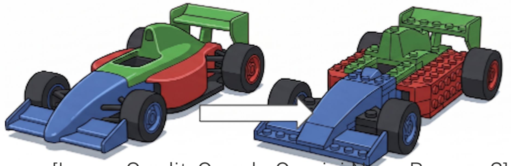
- **Framework:** description based on gluon **Transverse-Momentum-Dependent (TMD)** functions — "*TMDs*".

$$f_{p/h}(x, k_T, \mu, \zeta)$$



Based on Fig. 1.1 in TMD Handbook, 2304.03302

TMD Factorization

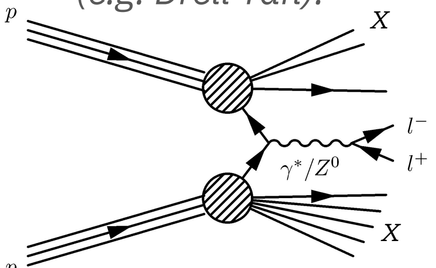


[Image Credit: Google Gemini Nano Banana 2]

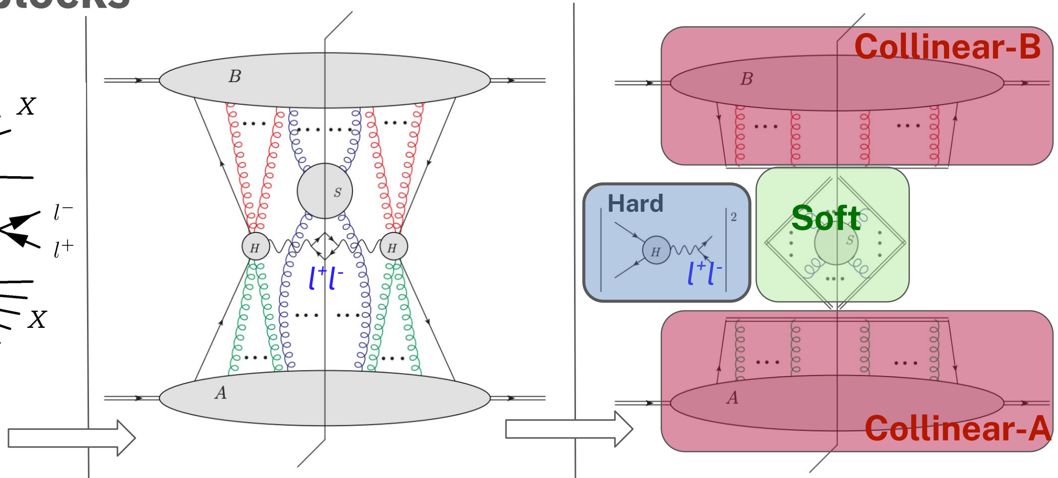
- Physical observables from **“Lego blocks”**

- Assigns dofs into regions: **hard**, **collinear**, **soft**

(e.g. Drell Yan):



$$Q^2 = m_{l\bar{l}}^2$$



R. Boussarie et al., eds., TMD Handbook, arXiv:2304.03302 [hep-ph]

- Defines **TMDs** = **collinear** x **soft** $f_{q/h}(x, b_T, \mu, \zeta) = B_{q/h}(x, b_T, \mu, \zeta \ldots) \times \sqrt{S_q(b_T, \mu, \ldots)}$

- **Factorization formula** in *bT*-space:

$$\frac{d\sigma^W}{dQ^2\, dY\, d^2q_T} = \sigma_0 \sum_{q,\bar{q}} H_{q\bar{q}}\left[Q/\mu\right] \int_0^\infty d^2\mathbf{b}_T\, e^{i\mathbf{b}_T\cdot\mathbf{q}_T} f_{q/p_a}(x_a, b_T, \mu, \zeta_a) f_{\bar{q}/p_b}(x_b, b_T, \mu, \zeta_b)$$

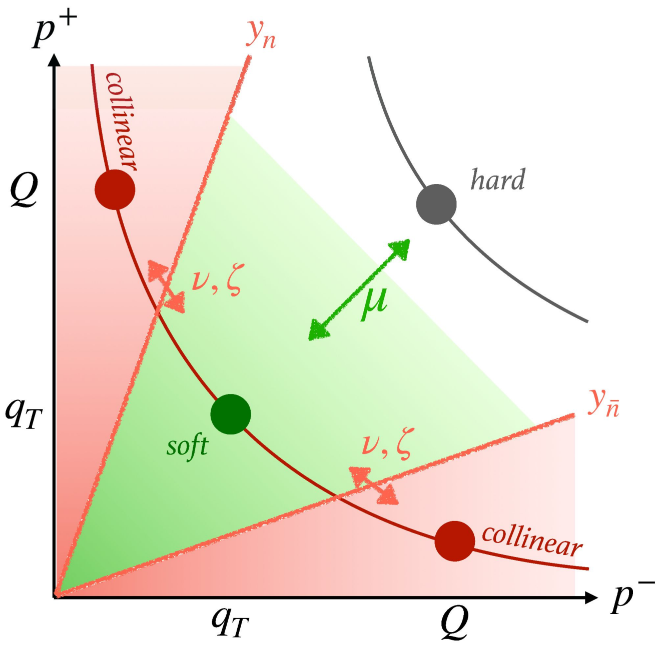
bT is Fourier-conjugate to relative transverse mtm

Evolution eqs & CS kernel

- **Cross-section**: no divergences
  $\Rightarrow$ no RG scale dependence μ, ζ
- **Individual factors**: divergences
  $\Rightarrow$ RG scale dependence μ, ζ
- Scale dependence must cancel out between factors $\Rightarrow$ **evolution equations:**

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln f_{p/h}(x, b_T, \mu, \zeta) = \gamma_\mu^{q/g}(\mu, \zeta)$$

$$\sqrt{\zeta} \frac{\partial}{\partial \sqrt{\zeta}} \ln f_{p/h}(x, b_T, \mu, \zeta) = \boxed{\gamma_\zeta^{q/g}(\mu, b_T)}$$



R. Boussarie et al., eds., TMD Handbook, arXiv:2304.03302 [hep-ph]

Collins-Soper (CS) kernel

- **Universal for all TMDs.**
  Independent of hadronic state, quark flavor, *x*
- **Differs for quarks and gluons**
- **Nonperturbative** at large *bT*, for any *μ*.

Constraints on quark & gluon CS kernels

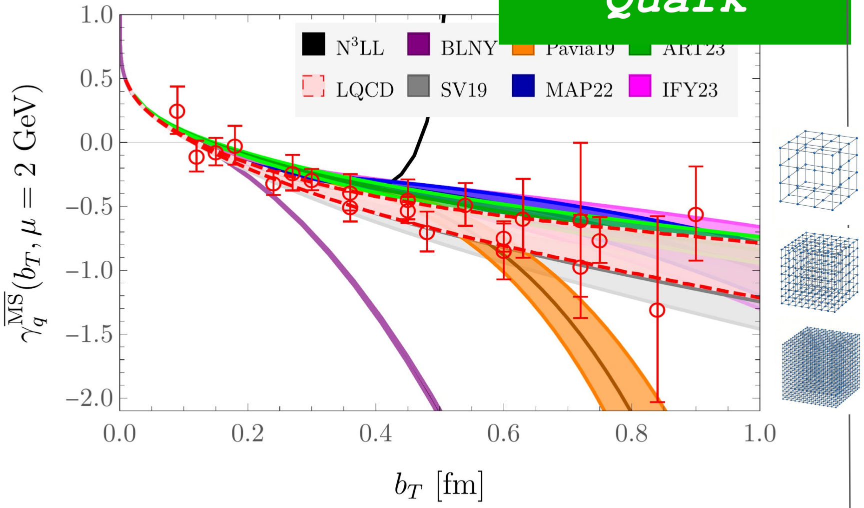
Casimir scaling assumed in gluon TMD pheno. analyses

1. **Perturbative:**

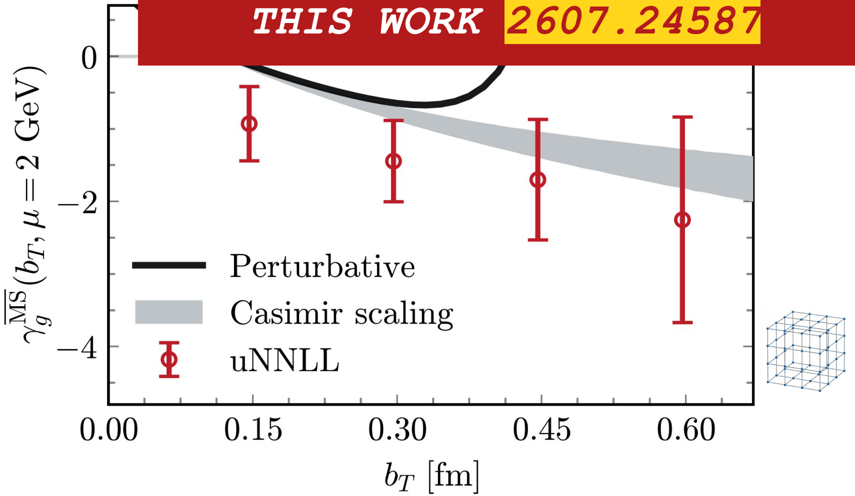
$$\gamma_\zeta(\mu, b_T) = -\frac{\alpha_s}{\pi}\ln\frac{b_T^2\mu^2}{4e^{-\gamma_E}} \times \boxed{\begin{cases} C_F\,, & \text{quark}\,, \\ C_A\,, & \text{gluon} \end{cases}} + \mathcal{O}(\alpha_s^2)$$

2. **Nonperturbative:**

Quark



Gluon — FIRST CONSTRAINTS, THIS WORK 2607.24587



Quark		Gluon	
✅	Global fits (SIDIS, DY...)	❌	Global fits
✅	Lattice (systematic control) — **SEVERAL GROUPS**	🚧	**Lattice (not fully controlled)**
✅	Joint experiment + lattice	❌	Joint experiment + lattice

CS kernel from lattice QCD + LaMET

[Large Momentum Effective Theory]

- Enters RG rapidity evolution of TMDs:

$$f_{p/h}(x, b_T, \mu, \zeta) = f_{p/h}(x, b_T, \mu, \zeta_0) \exp\left[\frac{1}{2}\gamma_p(b_T, \mu) \ln \frac{\zeta}{\zeta_0}\right]$$

Proportional to hadron momentum

- ⇒ Computed as a ratio of TMDs at different ζ:

$$\gamma_q(b_T, \mu) = \frac{2}{\ln(\zeta_1/\zeta_2)} \ln \frac{f_{q/h}(x, b_T, \mu, \zeta_1)}{f_{q/h}(x, b_T, \mu, \zeta_2)}$$

- ⇒ Matched onto spacelike quasi-TMDs with LaMET

$$\gamma_q(b_T, \mu) = \frac{1}{\ln(P_1/P_2)} \ln \frac{\boxed{C_q(xP_1^z, \mu)}\, \tilde{f}_{q/h}(x, b_T, \mu, P_1)}{\boxed{C_q(xP_2^z, \mu)}\, \tilde{f}_{q/h}(x, b_T, \mu, P_2)} + \text{p. c.}$$

LaMET matching coefficients Encoded by space-like matrix elements

Staple-shaped operators:

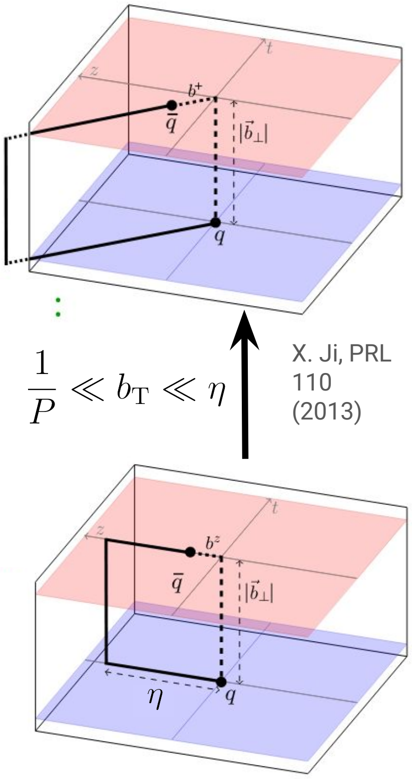


Fig. by Ebert, Stewart, Zhao, JHEP 1909 (2019)

Computational strategy

1. Quasi-TMDs in position space
2. Fourier-transform
3. LaMET matching
4. Power corrections
5. ~~Continuum limit~~ One small box

5. 2.

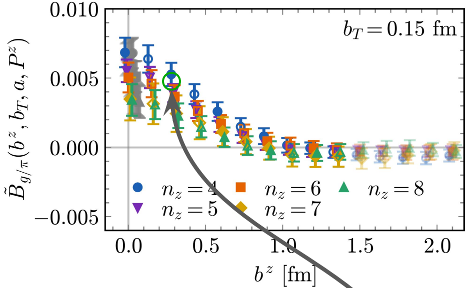
$$\gamma_\zeta^{(i)}(b_T,\mu) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{\int_{-\infty}^{\infty} \frac{dP_1^z b^z}{2\pi} e^{-ixP_1^z b^z} \lim_{\ell\to\infty} [\tilde{\mathcal{R}}_{i/h}^{(s)\ell}] (b^z, b_T, P_1^z, \mu, a)}{\int_{-\infty}^{\infty} \frac{dP_2^z b^z}{2\pi} e^{-ixP_2^z b^z} \lim_{\ell\to\infty} [\tilde{\mathcal{R}}_{i/h}^{(s)\ell}] (b^z, b_T, P_2^z, \mu, a)} + \delta\gamma_{(i)}(\mu, x, P_1^z, P_2^z) \boxed{+\,\mathrm{p.\,c.}}$$

3. 4. 1.

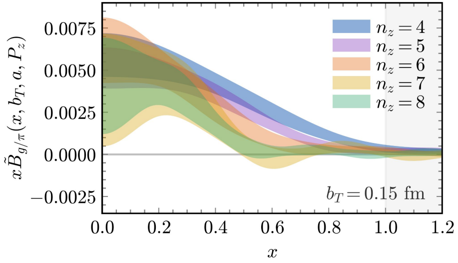
X. Ji et. al, PRD91 (2015);
Ebert et. al, PRD99 (2019), JHEP09 (2019) 037;
X. Ji et. al., Phys. Lett. B 811 [1911.03840]

Analysis summary

Calculate position-space MEs → **Fourier transform (FT)** to momentum-space MEs → Extract kernel from fits + **match** + fit in x → ...repeat for each *bT*

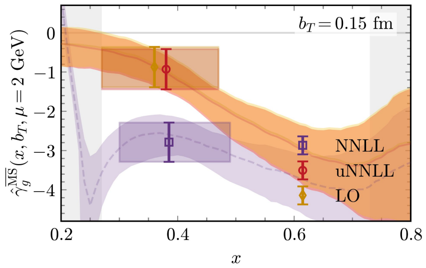


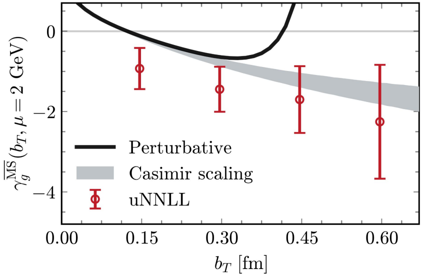
$$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z)$$



$$\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z)$$

$$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z)$$





$$\gamma_q(b_T,\mu) = \boxed{\lim_{a\to 0}} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int db^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_1^z)}{\int db^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_T, \ell, P_2^z)} \right] + \delta\gamma_q(\mu, P_1^z, P_2^z) + \boxed{\text{p. c.}}$$

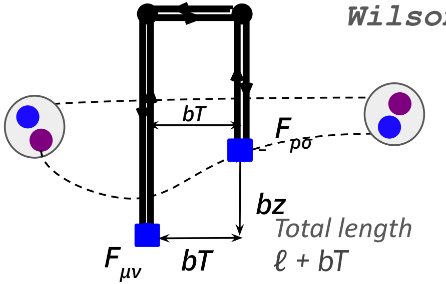
Each point is a separate matrix element calculation:

Gluon quasi-TMD beam functions in a pion state:

$$\tilde{B}^{(s)}_{g/\pi}(b^z, b_T, a, P^z) = \langle \pi(P^z) | O_g^{(s)}(b^z, \mathbf{b}_T, 0) | \pi(P^z) \rangle$$

From 3pt functions ***at each bz, bT, ℓ, and Pz***

Nonlocal gluon operator with staple-shaped Wilson lines

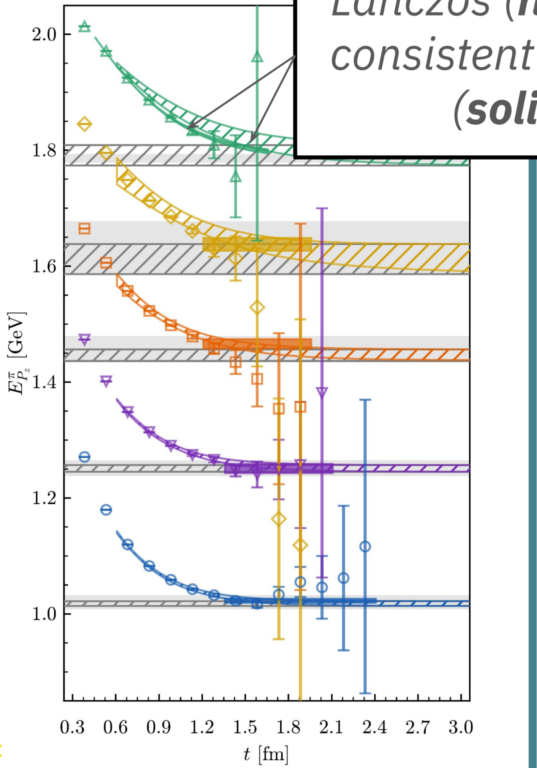


Applying Lanczos "at industrial scale"

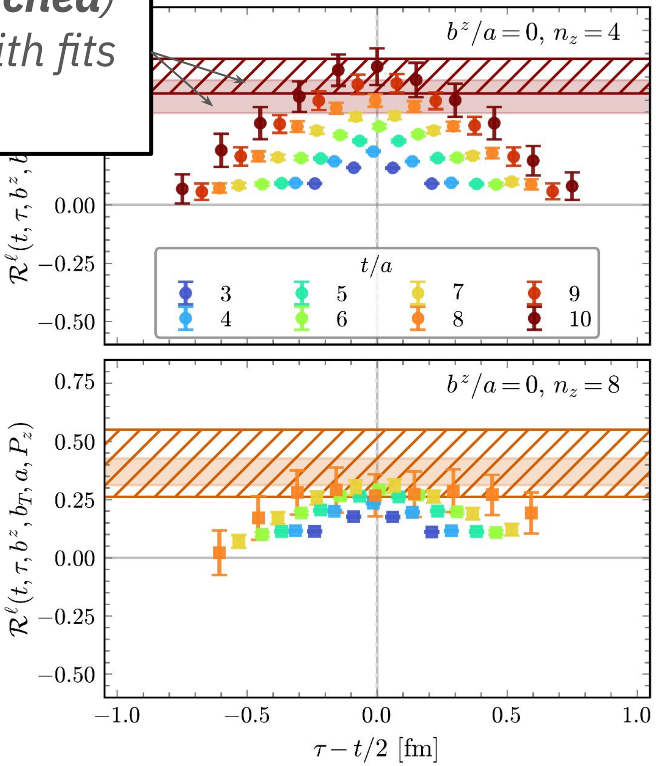
Correlate 2pts & operators cfg-by-cfg:

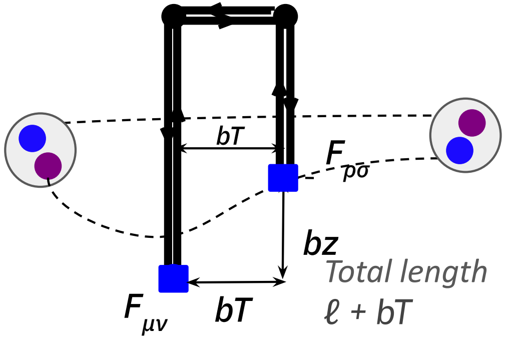
$$\boxed{C_{3\text{pt}}^{(s)}(t,\tau,b^z,b_{\mathrm{T}},a,\mathbf{P}=P_z\hat{\mathbf{z}})} = \langle \boxed{C_{2\text{pt}}(t,a,\mathbf{P})} \sum_{\mathbf{v}} O_g^{(s)}((\mathbf{y},\tau),b^z,b_{\mathrm{T}})\rangle$$

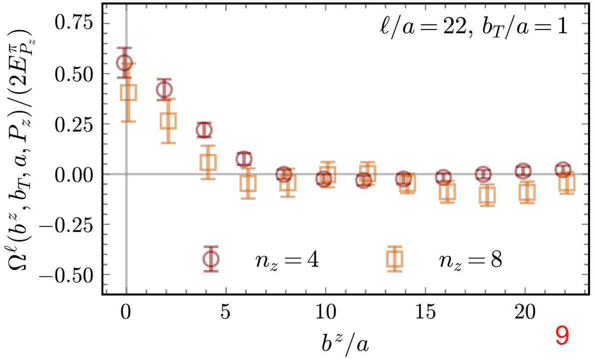
3pts ***2pts*** ***Operator VEV***



*Lanczos (**hatched**) consistent with fits (**solid**)*

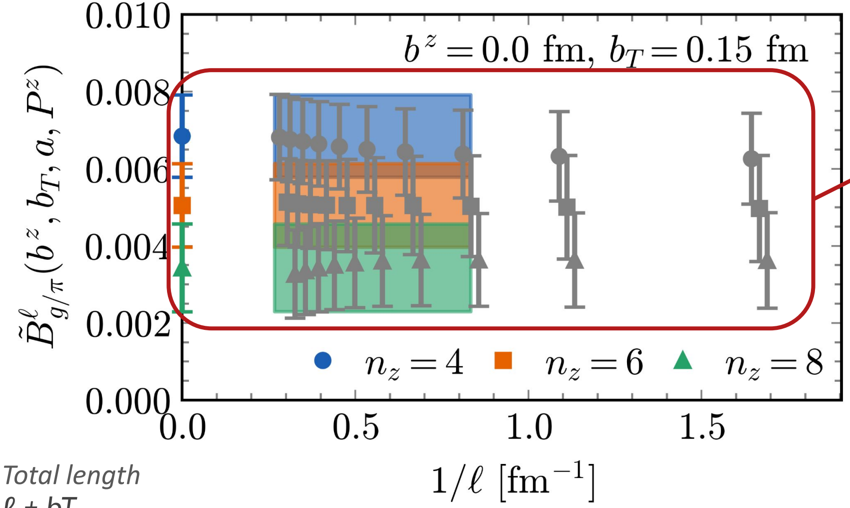


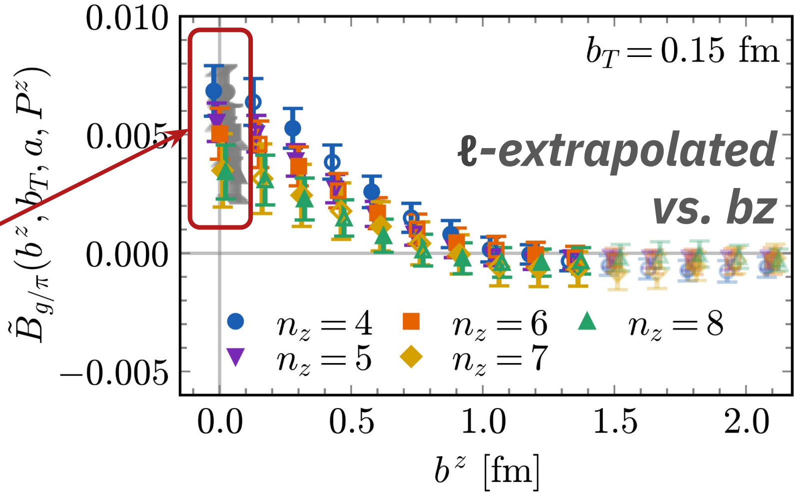


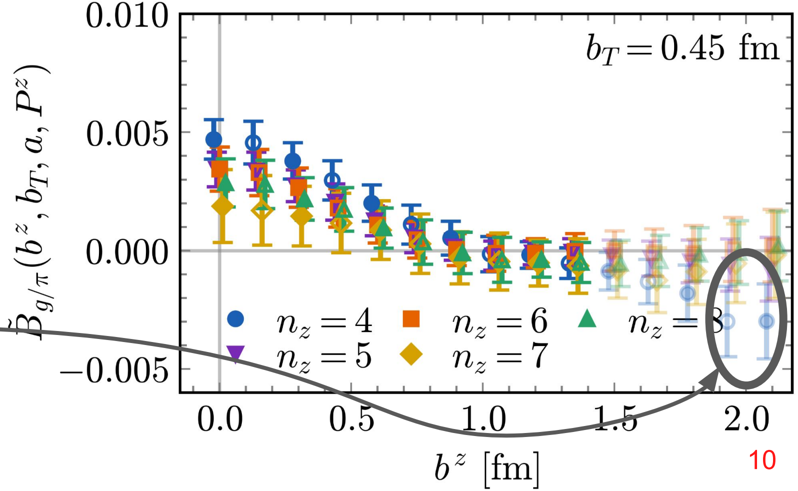


Position-space matrix elements

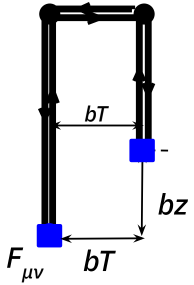
ℓ-dependent at each bz







Total length
ℓ + bT



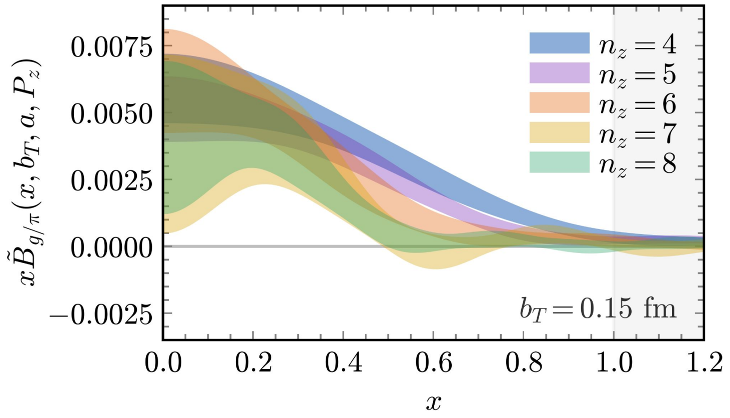
- Each point from ell-extrapolation, **noise grows w/ ℓ**, esp. at large *bT*
- **Finite-volume effects** at *bz* ~ *L*
- Symmetric in *bz* → *-bz*

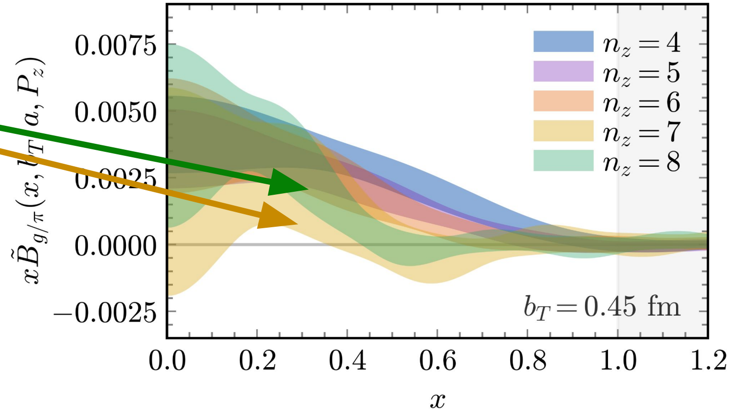
Fourier transformation

- Implemented as a **DFT** [Discrete Fourier Transform]

$$x\tilde{B}^{\mathrm{DFT}}_{g/\pi}(x, b_T, P_z) = \frac{P_z}{2\pi} \sum_{b^z \le b^z_{\mathrm{cut}}} e^{ixP_z b^z} \tilde{B}_{g/\pi}(b^z, b_T, P_z)$$

- Symmetric in *x* → -*x*
- **"Ringing" artifacts from *bz* ~ *L*** effects at large bT
- Robust to to all other systematics (DFT, analytic transforms of fits, DFT + fitting tails)
- **CS kernel encoded in *Pz*-dependence**





Estimates of CS kernel

- **Fit *Pz*-dependence** to extract CS kernel:

$$\frac{\tilde{B}^{\mathrm{DFT}}_{g/\pi}(x, b_T, P_z)}{H^{\mathrm{LAMET}}_{g}(x, b_T, P_z)} = c_{\gamma\text{-fit}}(x, b_T) \exp\left[\hat{\gamma}_g(x, b_T) \ln P_z\right]$$

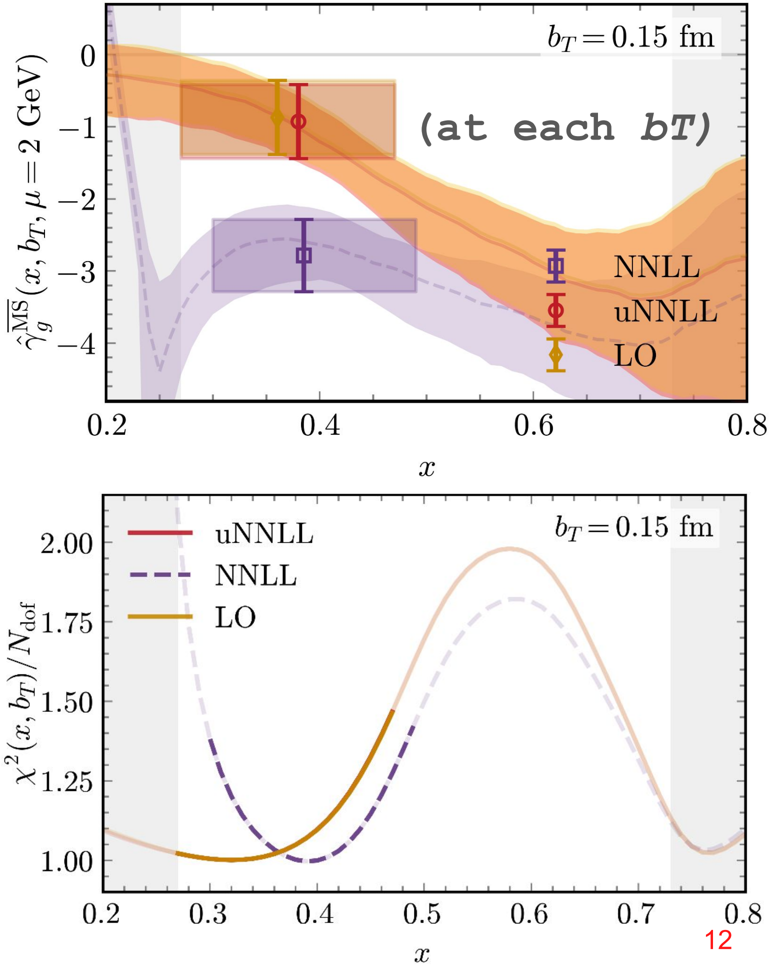
(excludes power corrections)

- Expect ***x*-independent CS kernel** + *x*-dependent p.c.
- Matching reduces corrections, shifts estimates
- Small p.c. → plateau in x + better fit quality
- Extract CS kernel at each order (**LO** / **NNLL**/ **uNNLL**) as **weighted average from limited x-range:**

(1) ***Perturbative constraint*** *to control bT-independent p.c.*

$$\alpha_s^2(2\bar{x}P_z) < \delta_{\mathrm{p.c.}}$$

(2) ***Data-driven constraint*** *for bT-dependent p.c. not yet fully controlled by LaMET*

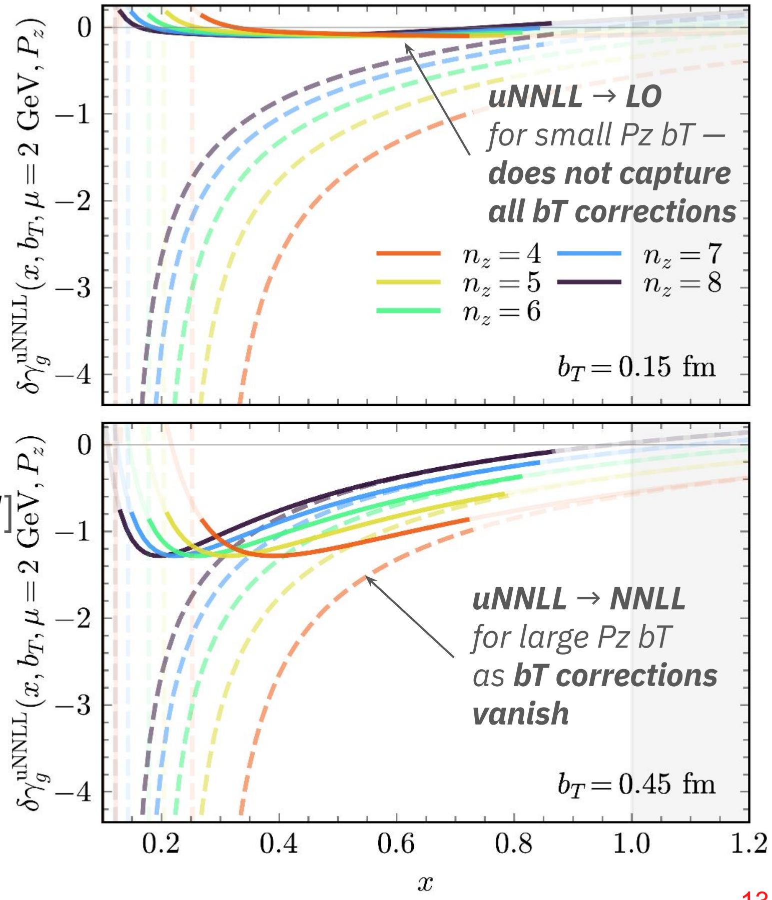


LaMET matching

- ***x*-dependent shift** of CS kernel estimates, att each bT

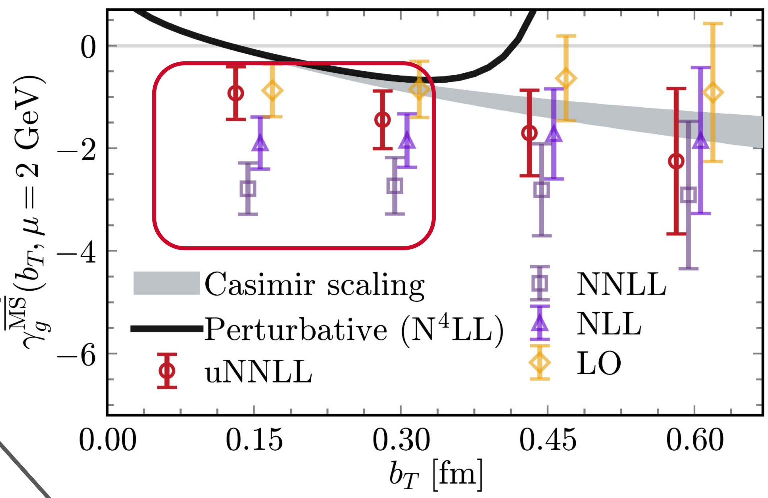
$$\delta\gamma_g^{\mathrm{LaMET}}(x, b_T, \mu, P_z) = -P_z \frac{\mathrm{d}}{\mathrm{d}P_z} \log H_g^{\mathrm{LaMET}}(x, b_T, \mu, P_z)$$

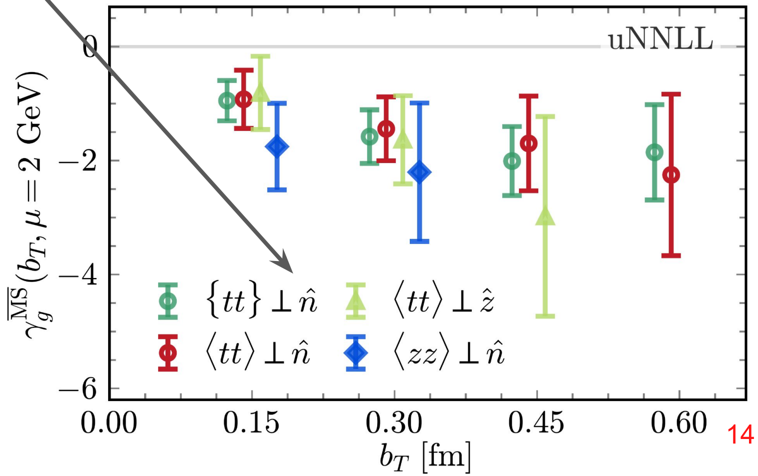
- **Pz bT ≫ 1:** *bT*-independent shift
  - *Known:* LO + resummation (NLL), NLO + resummation [**NNLL** – *dashed*]
  - *Unknown:* NNLO & higher
- **Pz bT ~ 1:** *bT*-dependent shift
  - *Known*: partial multiplicative [**uNNLL** – *solid*] ("u" = "unexpanded" in bT)
  - *Unknown*: full, convolutional matching in x.



Dominant uncertainties

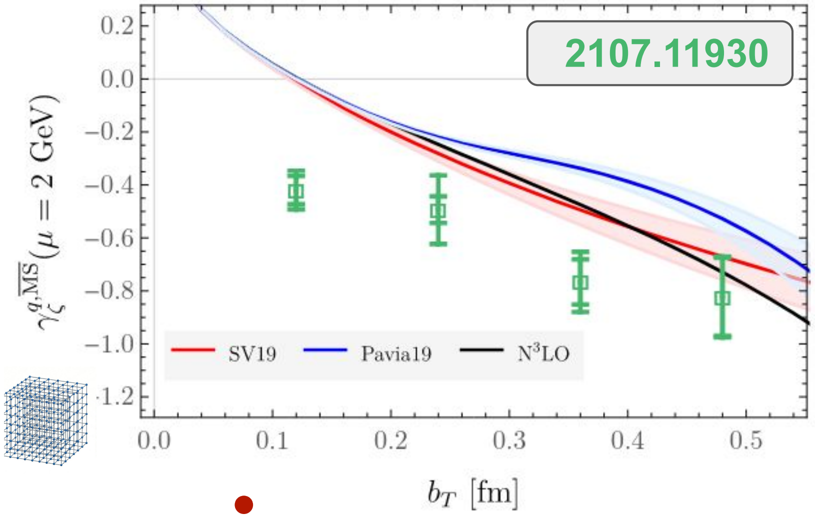
- **Small-*bT* systematics**
  - Missing bT corrections significant
    - Observed across several operator choices
    - Needs full convolutional matching
  - Need higher-order matching to test perturbative convergence
- **Large-*bT* statistics**
  - Statistical noise scaling with *bT*
  - FV effects for bz ~ L ⇒ ringing artifacts in the DFT
  - Coulomb-gauge method could help with both stats. & systematics

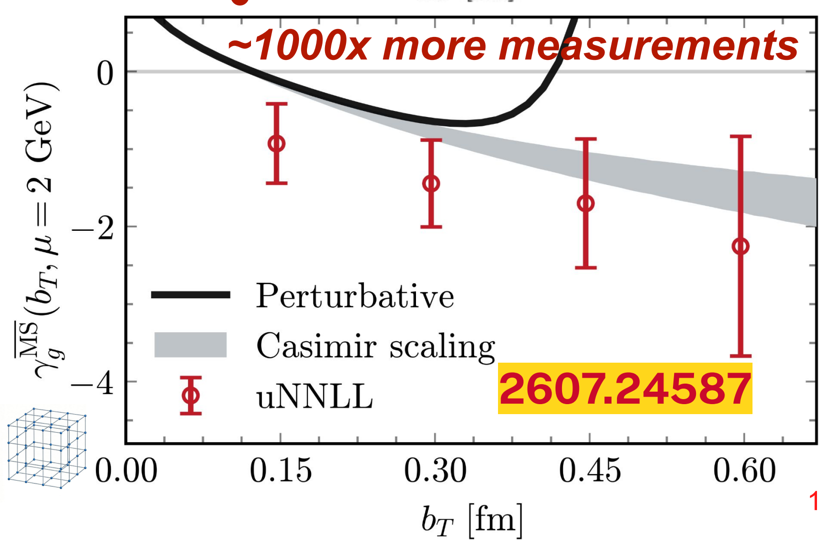




Outlook

- Similar to quark CS kernel ~5 years ago
    - Need better perturbative matching
    - Need bigger boxes to control FV effects, Fourier transform and discretization artifacts
- But:
    - Statistics more challenging for gluon
    - CG method promising
      for statistics and systematic control
    - No pheno. results yet
- **Ongoing pheno. impact of quark results provides motivation for gluon case!**





Thank you!
Q&A

Backup

Impact of quark CS kernel determination

Quark results impact: Joint fit w/ experiment + lattice

ACCEPTED PAPER

Extraction of the Collins-Soper kernel from a joint analysis of experimental and lattice data

Artur Avkhadiev and Valerio Bertone and Chiara Bissolotti and Matteo Cerutti and Yang Fu and Simone Rodini and Phiala Shanahan and Michael Wagman and Yong Zhao

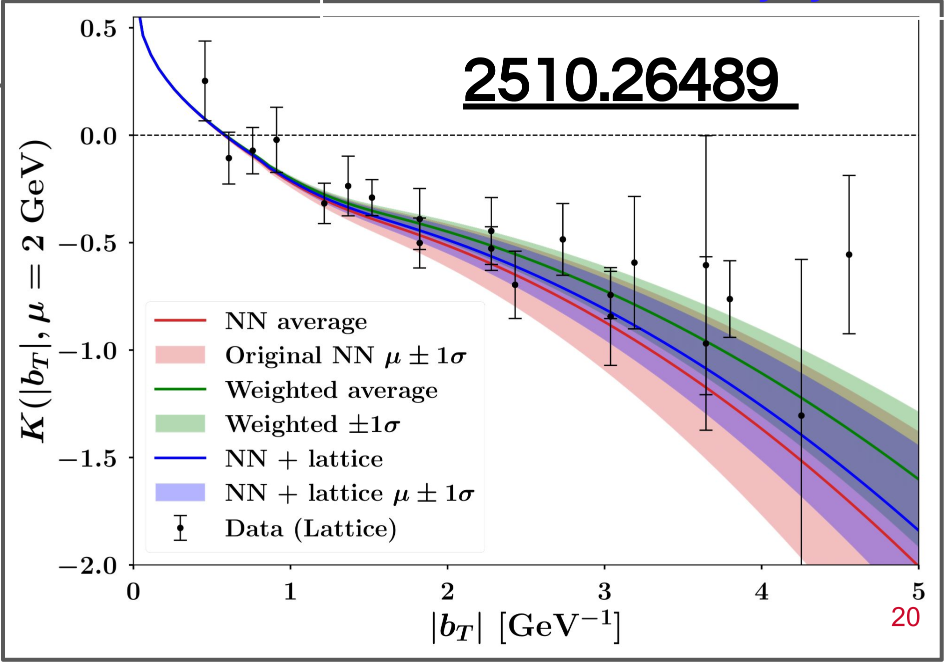
Phys. Rev. Lett. - **Accepted** 11 March, 2026

CS kernel model:

$$\boxed{\hat{\gamma}_\zeta(b_T, g_2) = -g_2 b_T^2}$$

shift g2 central value by ~10%, reduce uncertainty by ~40%

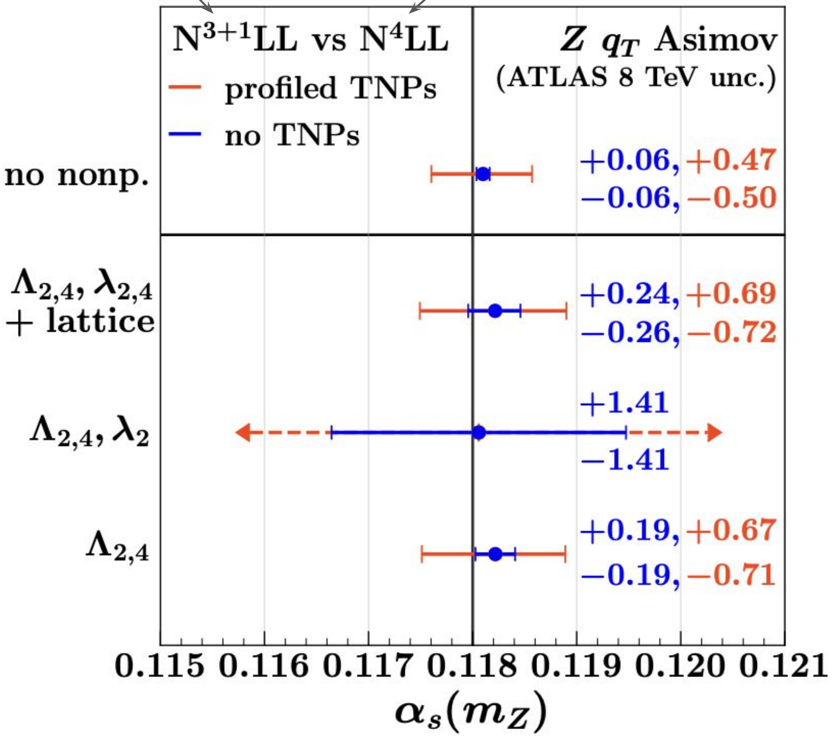
- **Experimental data [482 pts]** from full **TMD fit by MAP**
- **Lattice results [21 pts]** *from the 3 ensembles*
- Incorporate lattice results by (1) **reweighting** original replicas and (2) **refitting**:



CS kernel & α_{strong} from pT^Z

- Use small $pT^{\ell\ell}$ Drell-Yan to constrain $\alpha_{\text{strong}}(m_Z)$
- Subpercent precision at LHC
  ⇒ need reliable theory uncertainties
- Pseudo-data study: 2506.13874

theory model *pseudo-data*



Replace *exp. data w/ simulated data at $\alpha_{strong}(mZ) = 0.118$*

Keep *cov. matrix from experiment*

Fit *theory model parameters, with/without varying* ***TNPs***

$$\chi^2 = \sum_{i,j}(y_i - \hat{y}_i(\theta))^T C_{ij}(y_j - \hat{y}_j(\theta)) + \sum_i \frac{(\theta_i - \bar{\theta}_i)^2}{\Delta\theta_i^2}$$

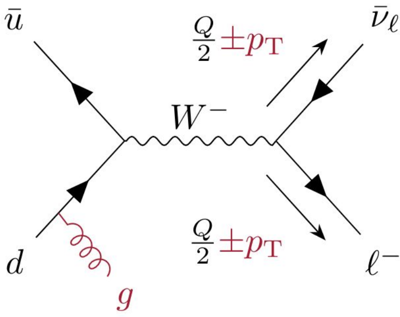
θ includes CS kernel model params $\lambda_{2,4}$:

$$\hat{\gamma}_\zeta(b_T, \lambda_2) = -\frac{\lambda_2}{2} b_T^2 - \frac{\lambda_4}{2} b_T^4$$

Interpretation: pT^Z spectrum alone insufficient to constrain both α_{strong} + NP parameters.
Need additional NP constraints.

Example: CS kernel & *pT* spectra of electroweak bosons

- Ideally: compare experiment to cross sections factorized into TMDs:



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}y\,\mathrm{d}q_T} \propto \sum_{q\bar{q}} \underbrace{H(Q,\mu)}_{\text{Hard matching coefficient}} \underbrace{\int_0^\infty \mathrm{d}^2 b_T\, e^{-iq_T b_T}}_{\text{Fourier transform}} \underbrace{f_{q/p}(x, b_T, \mu, \zeta) f_{\bar{q}/\bar{p}}(x, b_T, \mu, \zeta)}_{\text{TMDs}}$$

*(**large bT is nonperturbative**)*

- Practice: TMDs unknown, need matching onto collinear PDFs at perturbative ***bT=b****

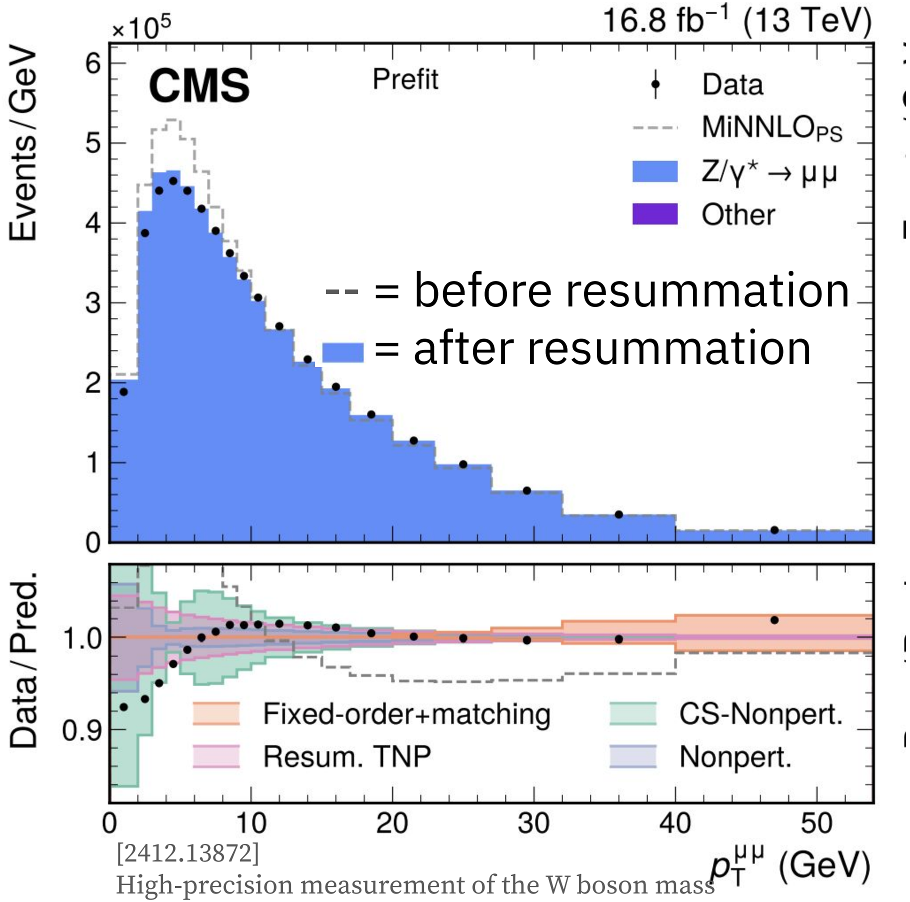
$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}y\,\mathrm{d}q_T} \propto \sum_{q\bar{q}} H_{q\bar{q}}(Q,\mu) \int_0^\infty \mathrm{d}^2 b_T\, e^{-iq_T b_T} \overbrace{\underbrace{e^{-\mathcal{S}(Q,b_T)}}_{\substack{\text{Nonperturbative piece of the Sudakov factor} \\ \textbf{sensitive to CS kernel at large bT}}} \left(C_{q'} \otimes f^{\text{coll.}}_{q'/p}\right) \left(C_{\bar{q}'} \otimes f^{\text{coll.}}_{\bar{q}'/p}\right)}^{\text{Collinear PDFs matched at perturbative } \textbf{bT=b*}}$$

- To match, need (a model of) CS kernel.
  Variations in CS kernel ⇒ variations in differential cross section.

CS kernel & measuring masses of *Z* & *W* bosons

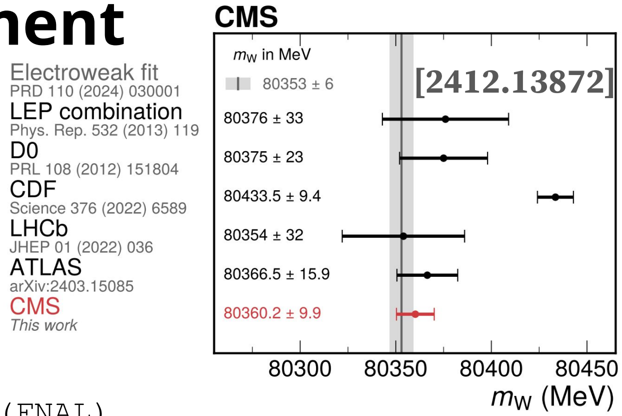
- Fit $pT^{\ell\ell}$ (pT^{ℓ}) exp. data to simulated templates
- $M^{Z/W}$ = free parameter in fit
- BUT: $pT^{\ell\ell}$ (pT^{ℓ}) affected by **both $M^{Z/W}$ and uncert. in $\mathbf{pT^{Z/W}}$**
- Uncertainty in CS kernel affects resummed $pT^{\ell\ell}$ (pT^{ℓ}) in the nonperturbative region

  Propagated as *Theory Nuisance Parameters (TNPs)* implemented in `SCETLib` [2411.18606]



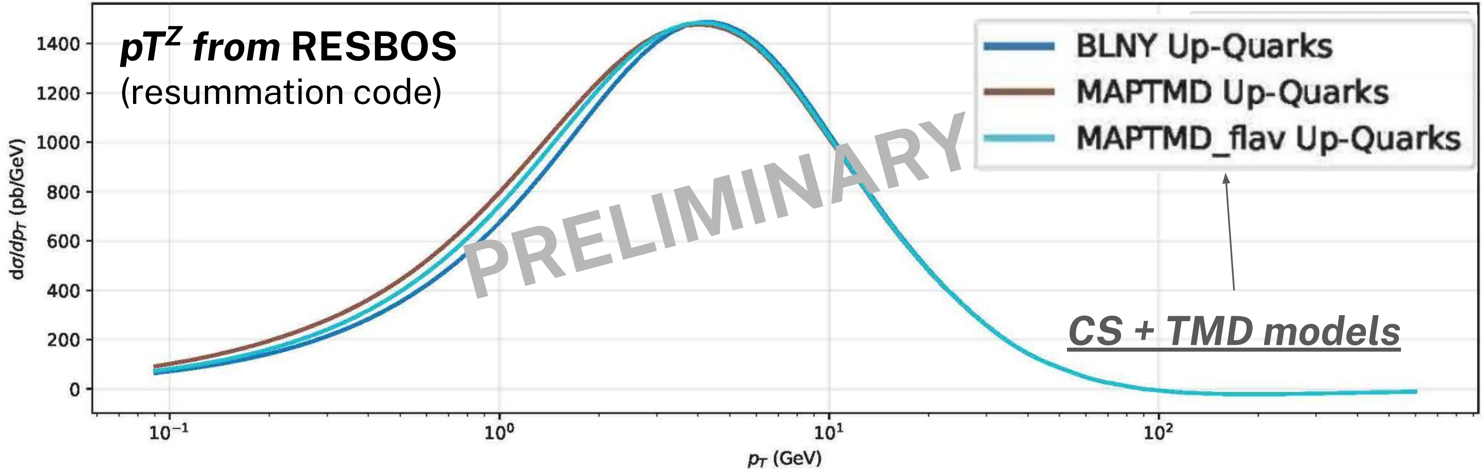
[2412.13872]
High-precision measurement of the W boson mass with the CMS experiment at the LHC

CS kernel & *W* mass from CDF measurement



- CDF measurement of M^W used Z data to fit NP params., then held params. fixed to fit M^W from *W* data.
- Would CDF analysis results change with different NP models / priors?

Ongoing w/ Yang Fu (MIT) Josh Isaacson (MSU), Mike Wagman (FNAL)



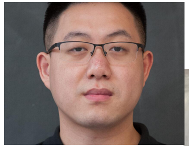
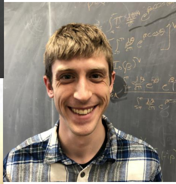
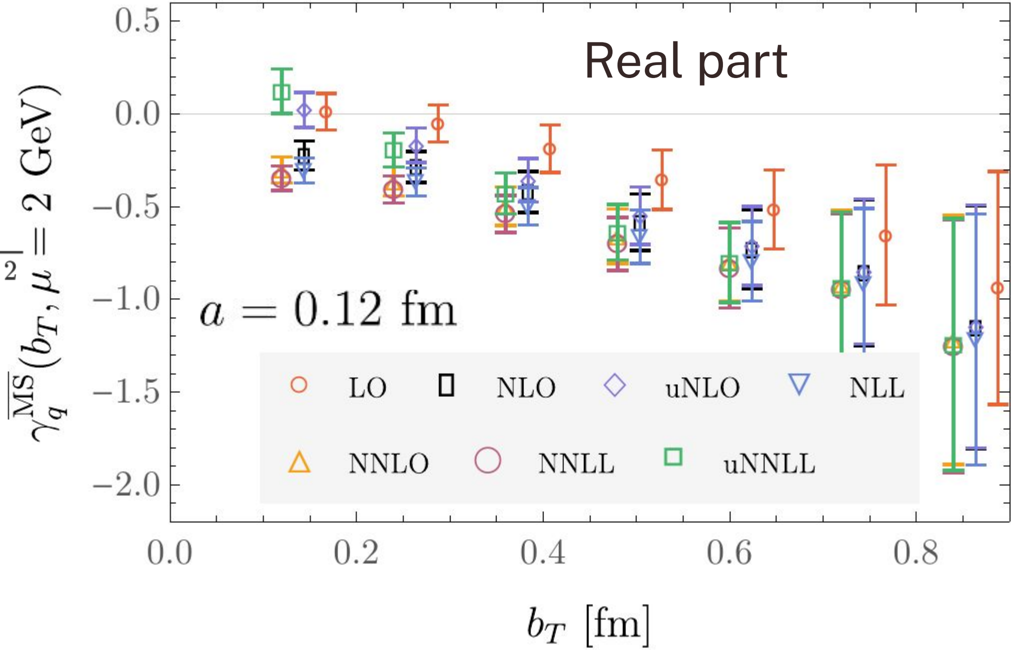




Fig. credit Josh Isaacson

bT-dependent LaMET matching

bT-dependence of power corrections (numerical evidence) [1/3]

- TMD LaMET matching needs **qT << Pz**
- Lattice results at small bT have **qT~Pz =>** power corrections $\frac{1}{(xP^z b_T)^2}$
- Full matching in this regime is
  (1) ***bT*-dependent** and
  (2) **convolutional** in ***x***:



$$\tilde{\phi}(x, b_T, \mu, P^z) = \int_0^1 \mathrm{d}y\, H_\phi(y, x, b_T, \mu, P^z)\phi(y, \mu)$$

[2307.12359], [2504.04625]

bT-dependence of power corrections (numerical evidence) [2/3]

- Full ***bT*-dependent convolutional** matching: $H_\phi(y, x, \mu, P^z)$

- So far: isolate the *bT*-dependent **multiplicative** part:

Avkhadiev, Shanahan, Wagman, Y.Zhao
PRD 108 (2023) 11, 114505 [2307.12359]
D. Bollweg, X. Gao, J. He, S. Mukherjee, Y. Zhao
[2504.04625]

$$H_\phi(y, x, b_T, \mu, P^z) \xrightarrow{P^z b_T \gg 1} \delta(y - x)\left(\mathbf{C}_\phi^{(\mathrm{u})}(xP^z, b_T, \mu)\mathbf{C}_\phi^{(\mathrm{u})}((1-x)P^z, b_T, \mu)\right)$$

$$\cancel{+\delta H_\phi(y, x, b_T, \mu, P^z)}$$

Drop convolutional piece (for now)

- Defines the **"unexpanded" matching** in this work:

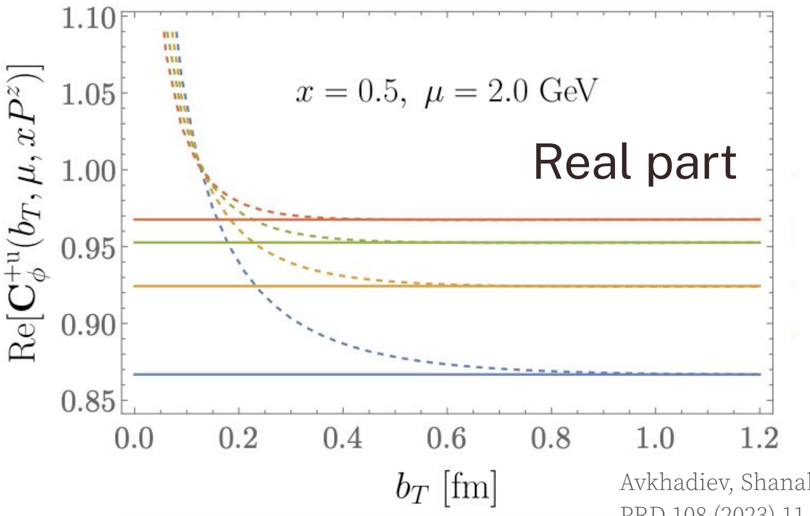
$$\mathbf{C}_\phi^{(\mathrm{u})}(xP^z, b_T, \mu) = \boxed{C_\phi(xP^z, \mu)} + \delta C_\phi(xP^z, b_T, \mu)$$

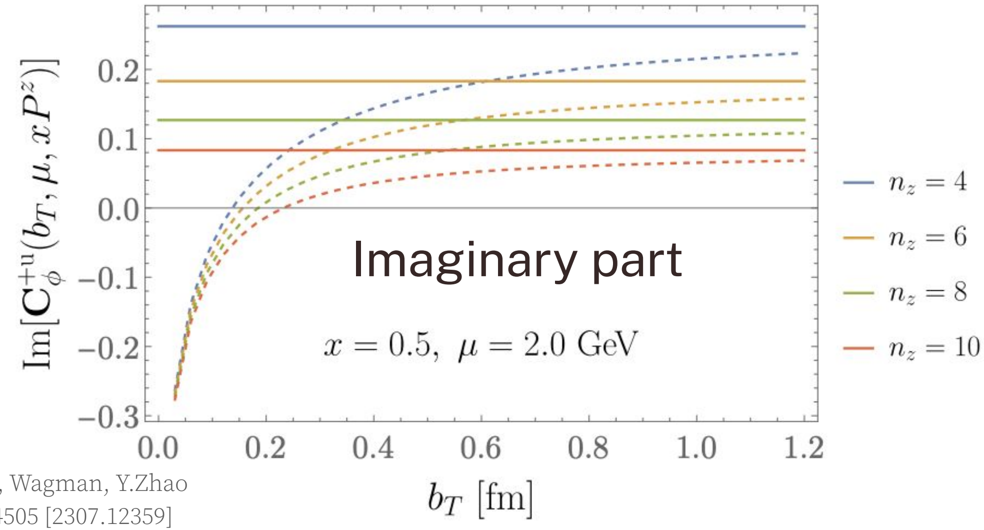
Usual, bT-independent LaMET matching

bT-dependence of power corrections (numerical evidence) [3/3]

- This "unexpanded" matching mitigates *bT*-dependent power corrections
  [(--)**dashed** = unexpanded, (–)**solid** = bT-independent]

- Higher bT sensitivity in the imaginary part of estimate — **explains imaginary part at small bT after renormalon subtraction.**

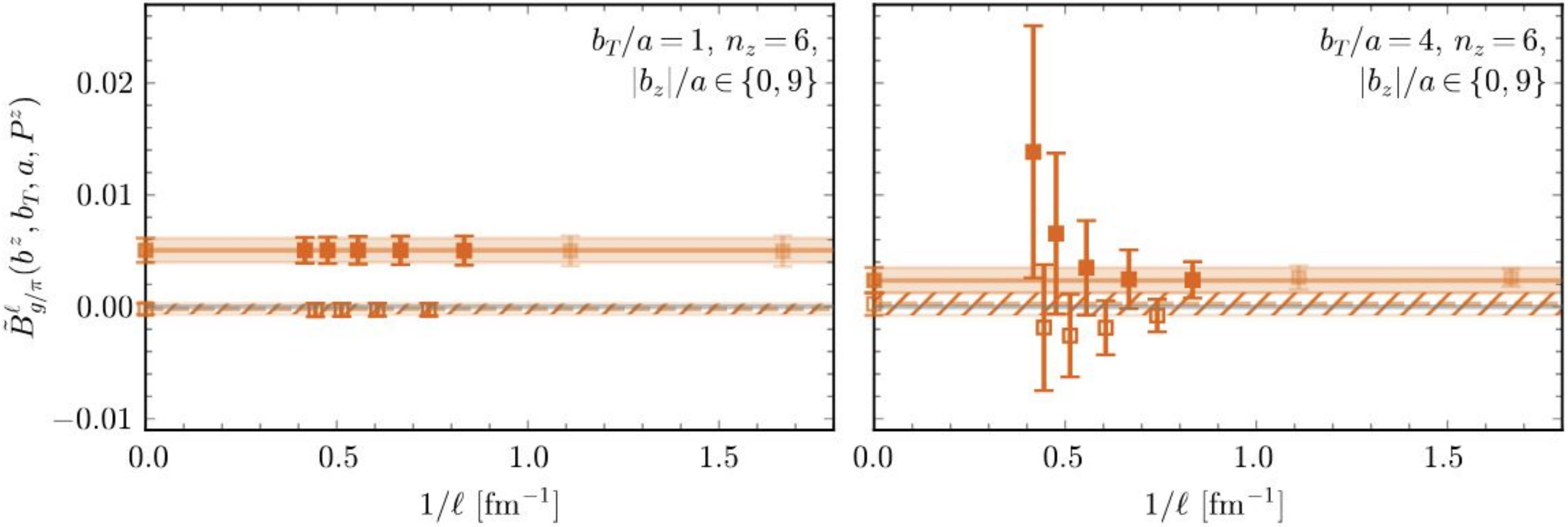




Avkhadiev, Shanahan, Wagman, Y.Zhao
PRD 108 (2023) 11, 114505 [2307.12359]

More on uncertainties

FV systematics

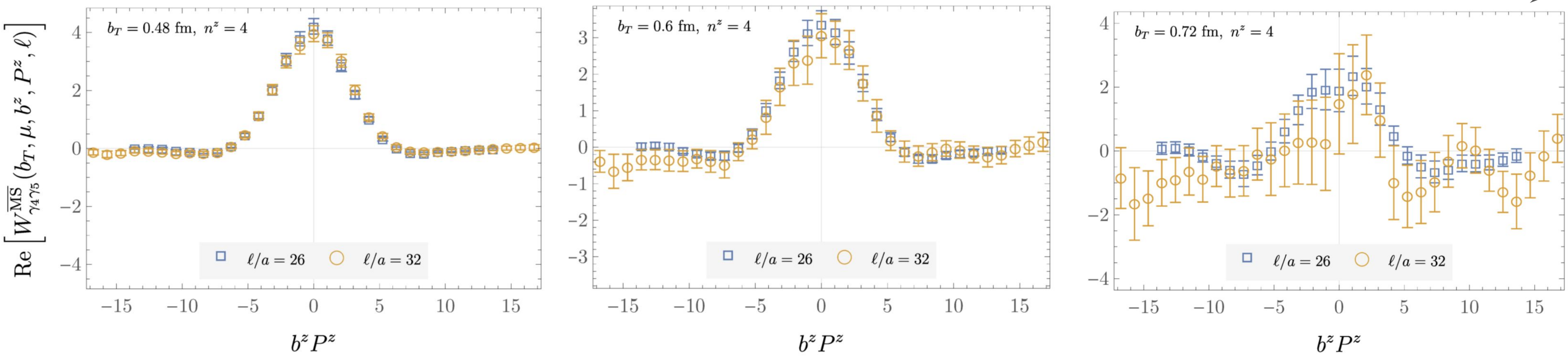


Statistical noise from Wilson lines

E.g. for quark CS kernel:

- Statistical noise grows with length of Wilson line.
- Compare *l=26* vs *l=32* with increasing *bT*:

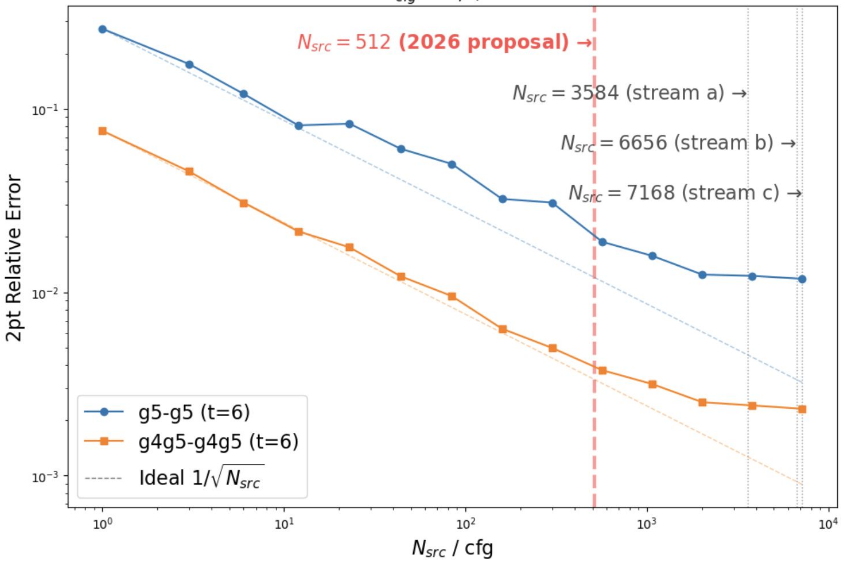
Increasing bT



Even greater challenge for gluon CS kernel calculations

1) How large are the autocorrelations among the pion two-point functions computed with different source positions on the same configuration?

Sufficiently small so sqrt(N) scaling holds *for the proposed 512 srcs / cfg with* ***kinematically enhanced interpolators****:*

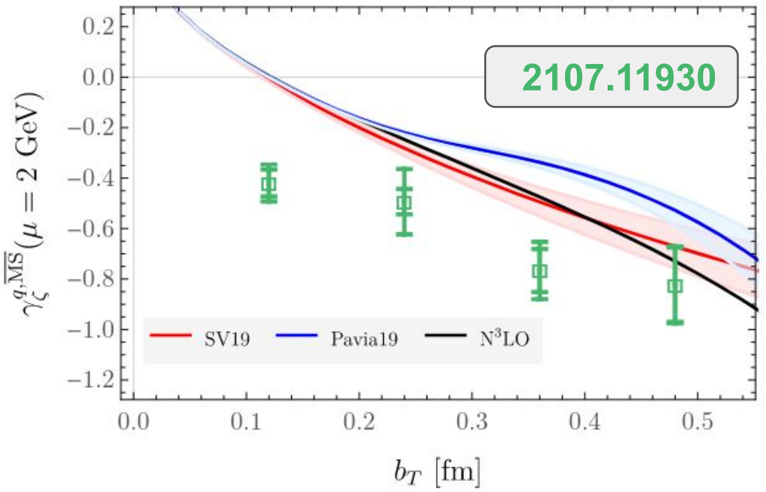


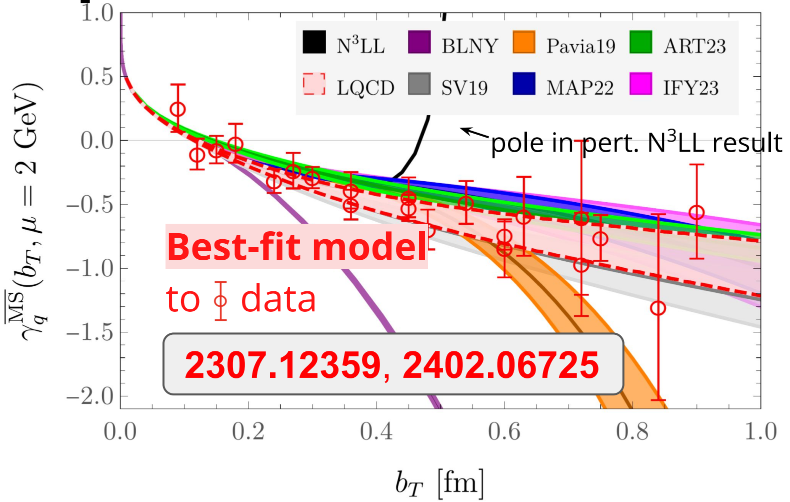
2) How do the results look when using the nucleon two-point functions instead of the pion two-point functions?

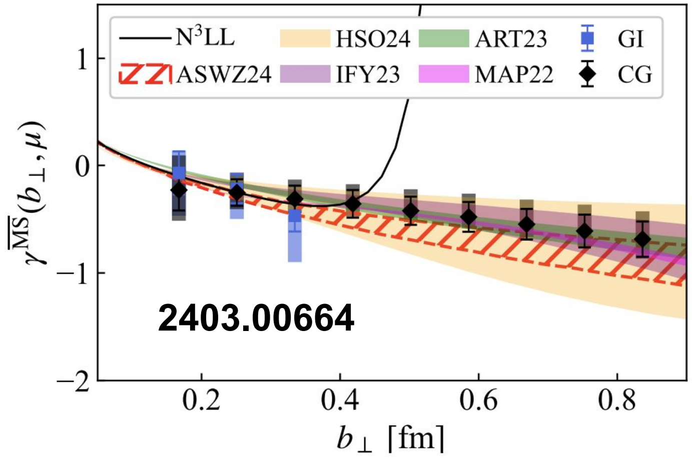
$$C_{2\mathrm{pt}}(t, P = K/\zeta) \underset{\text{reweight}}{=} \sum_x e^{i(K/\zeta)\cdot x} \langle G_K(x,0)\Gamma' G_{-K}(x,0)\Gamma\rangle$$

- *2pt calculations:*
  - **Inversions done with mtm-smeared quarks** at fixed ***K = ζ P***
  - Optimal smearing fractions: *ζ ~0.1* nucleon, *ζ ~0.9* pion
- *Nucleon 2pts:*
  - **2024: no signal in prelim. studies** with *ζ ~ 0.4* *(incl. with kinematic enhancement).* **no nucleon 2pts in full 2pt set** w/ *ζ ~ 0.9.*
  - **2026:** will include nucleon 2 pts, but keep *ζ* optimized for pions

Progress in systematic control and precision



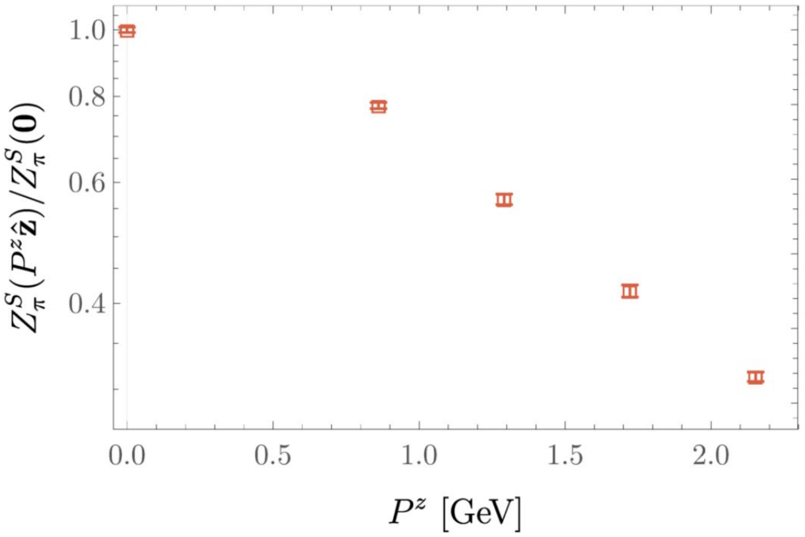




Highly encouraging results for quark CS kernel with CG calculations: extends bT range from ~0.4 fm to >1 fm on just 64 configurations (vs. ~500x3 configurations above on the right)

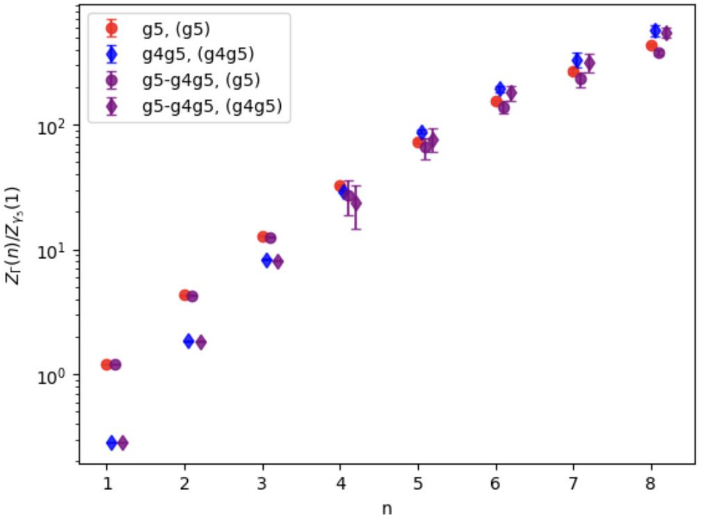
Quark smearing momenta & overlap factors

Quark kernel calculations
Smearing optimized for each mtm



(overlap factors from fits)

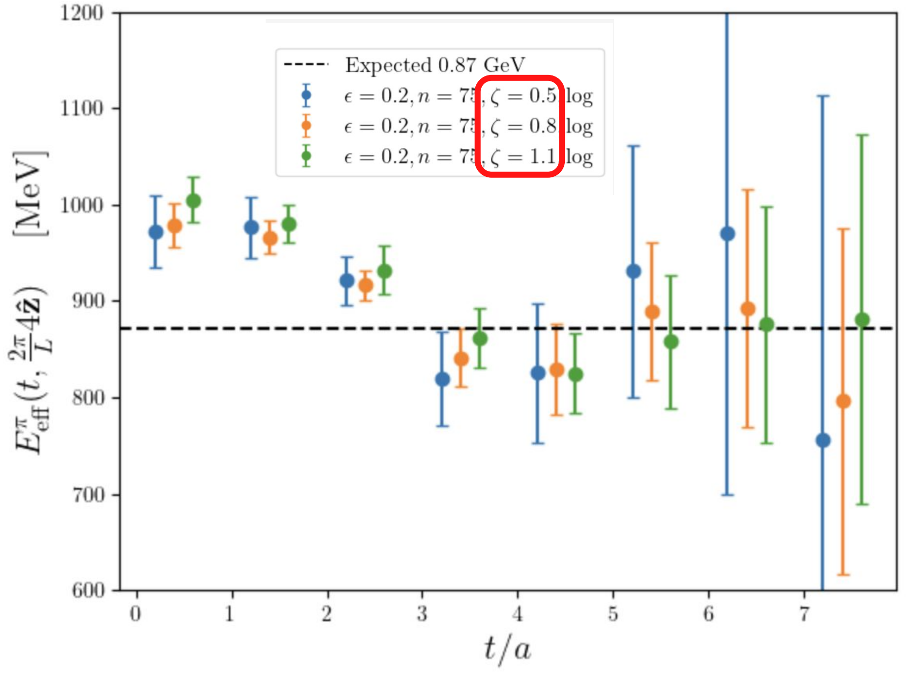
Gluon kernel calcualtions
Smearing optimized for mtm
w/ n = 9



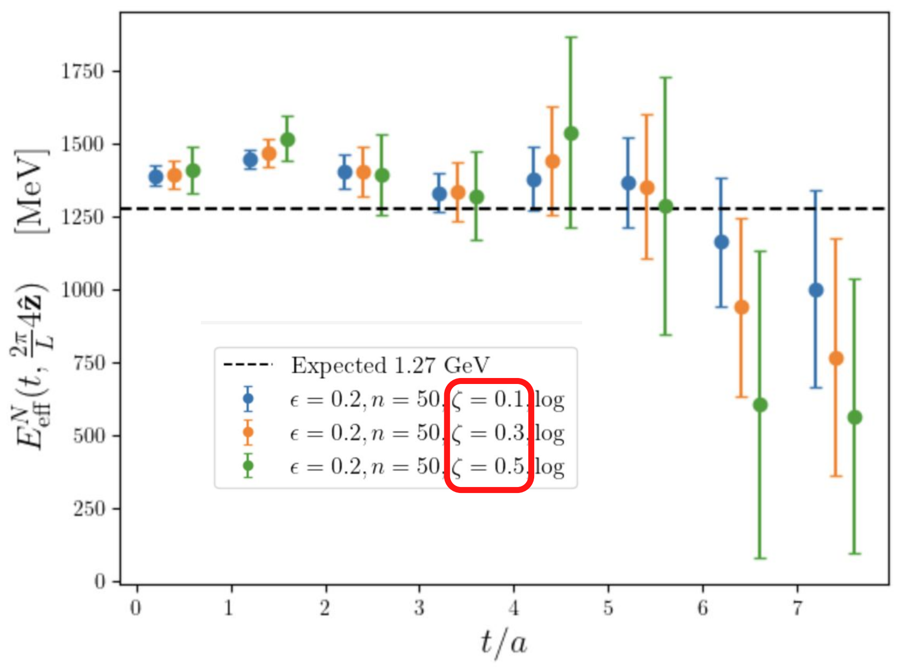
(overlap factors from Lanczos)

Momentum smearing tests

Pion

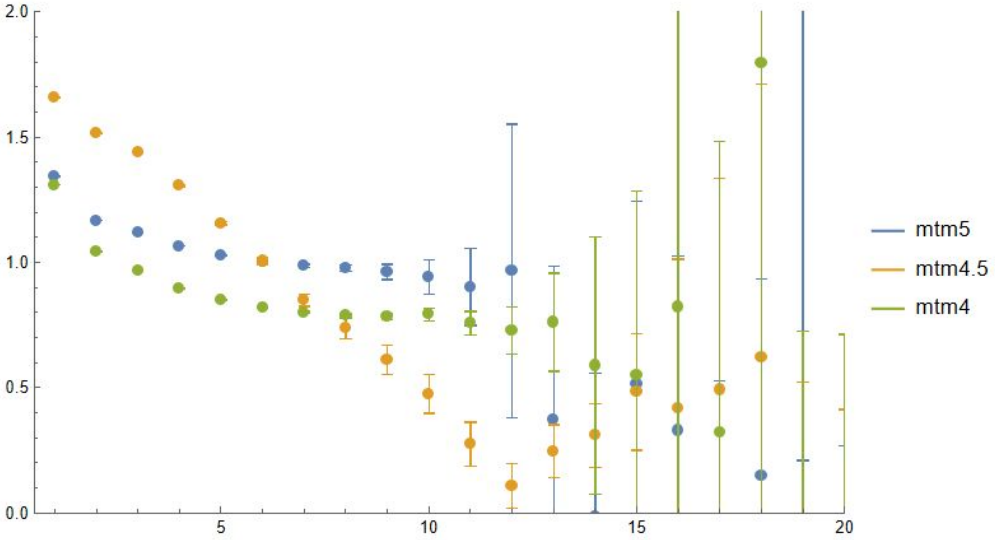


Nucleon



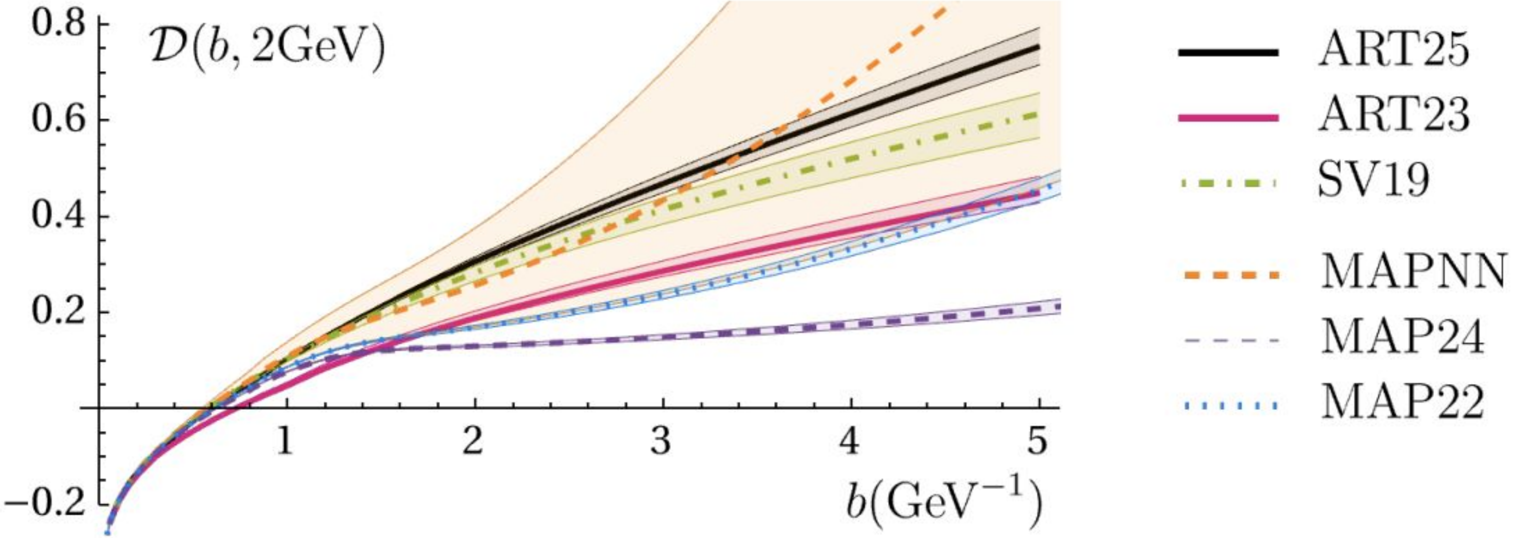
Reweighting to non-integer momenta

Noninteger momenta suffer from large finite-volume effects:



[Image credit Yang Fu]

Nonperturbative modeling challenges persist in recent analyses



This model for the CS kernel is almost identical to the one used in ART23 [21]. The only difference between the two implementations is that the value of B_{NP} is fixed here, whereas it was a fitting parameter in ART23. The ART23 fit determined $B_{\text{NP}} = 1.56^{+0.13}_{-0.09}$ GeV^{-1} and demonstrated the existence of a large correlation between this parameter and $c_{0,1}$. Therefore, we decided to keep it fixed.

Determination of unpolarized TMD distributions from the fit of Drell-Yan and SIDIS data at N4LL; 2503.11201

Quark CS kernel determination

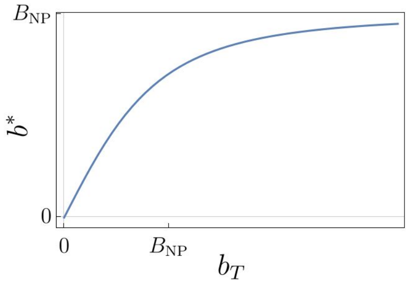
Quark CS kernel: pheno. models and input from LQCD

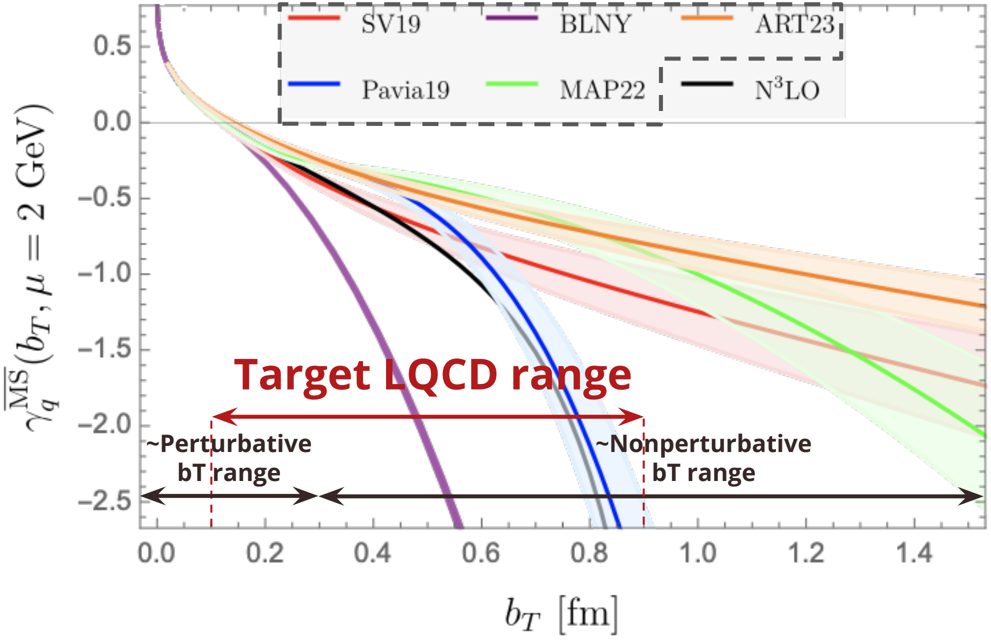
Pheno. models fit to experimental data (SIDIS, DY)

$$-\frac{1}{2}\gamma_q^{\text{param.}}(b_T,\mu;B_{\text{NP}},\{c_i\})$$ free parameters

$$= \boxed{\mathcal{D}_{\text{pert.}}(b^*;\mu)} + \boxed{\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})}$$

Perturbative term
controlled by $b^*(b_T;B_{\text{NP}})$:





Non-perturbative term
modeled b_T dependence, e.g.

$$\mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\}) = b_T b^*\left[c_0 + c_1 \ln\frac{b^*}{B_{\text{NP}}}\right]$$

$$\mathcal{D}_{\text{NP}}(b_T;g_2) = -g_2\frac{b_T^2}{2}$$

LQCD goal: reach precision + control for direct comparison

BLNY: PRD 67 (2003), [hep-ph/0212159]
SV19: JHEP 06, 137 [1912.06532]
Pavia19: JHEP 07, 117, [1912.07550]
MAP22: JHEP 10, 127, [2206.07598]
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Other recent results (not plotted):
Isaacson, Fu, Yuan [2311.09916]
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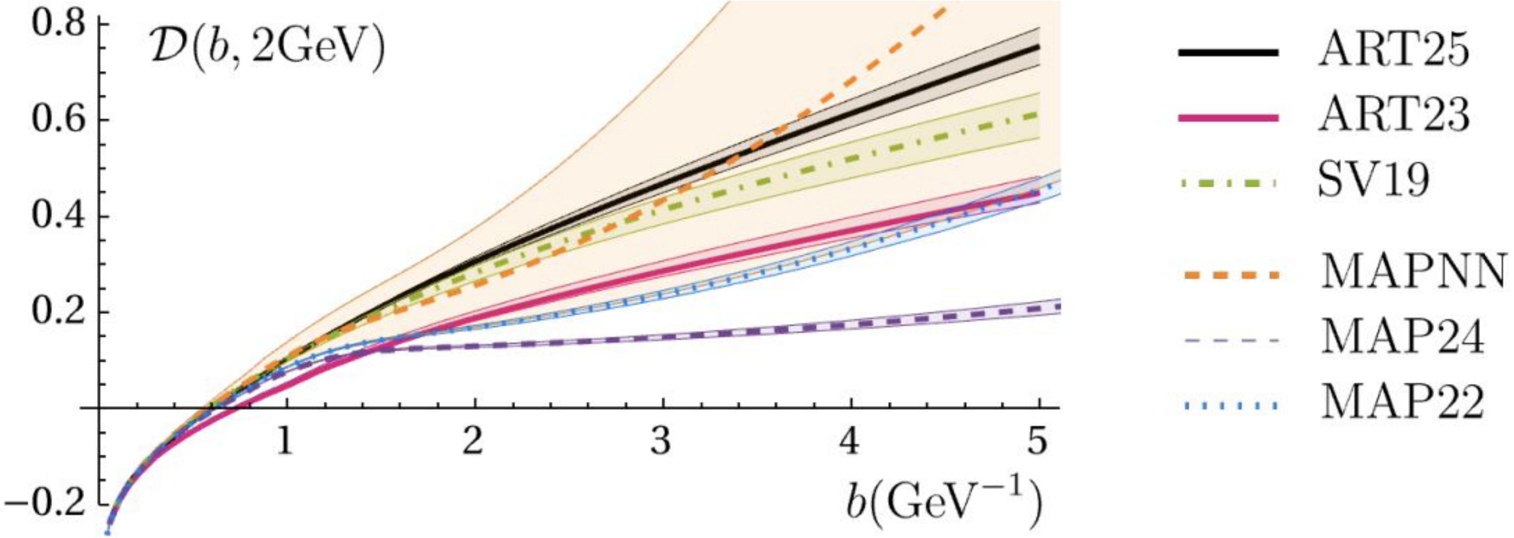

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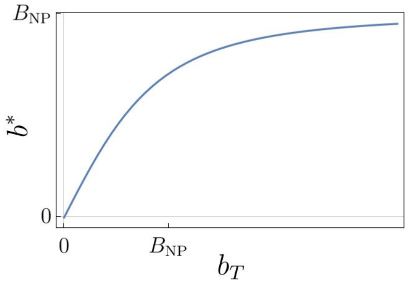

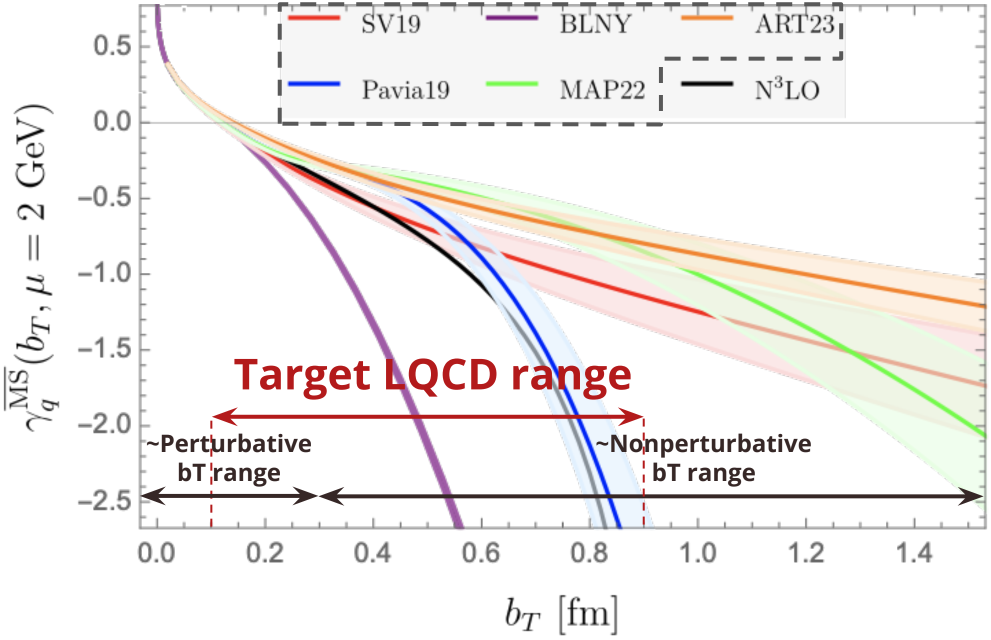

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LQCD results directly comparable with pheno. models

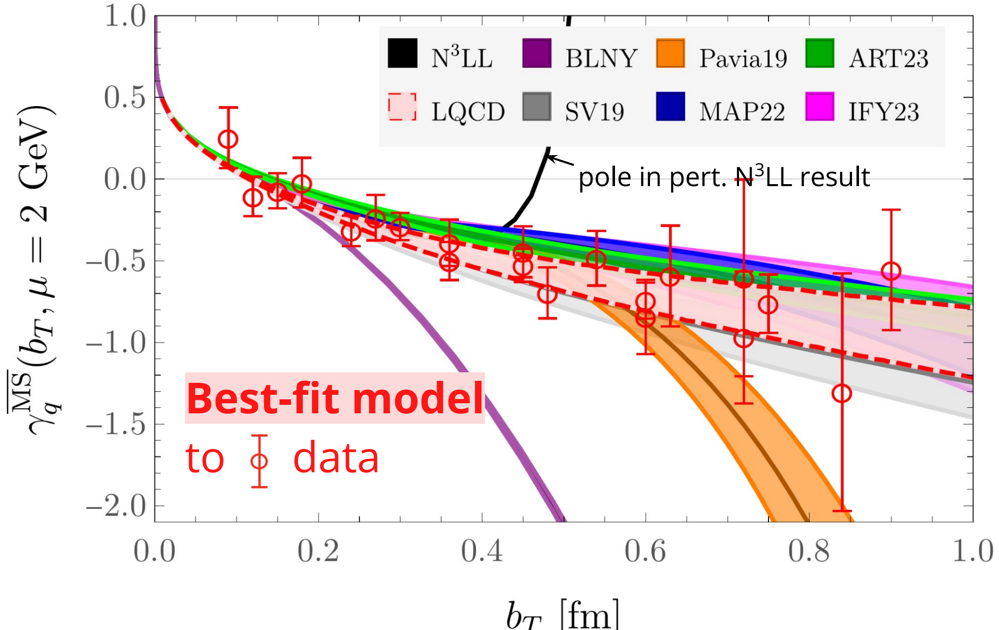
Fit CS parameterization to **LQCD data** from 3 lattice spacings

pheno. parameters

$$-\frac{1}{2}\left(\gamma_q^{\text{param.}}(b_T,\mu;B_{\text{NP}},\{c_i\}) - k_1\frac{a}{b_T} - k_2\frac{a^2}{b_T^2}\right) = \mathcal{D}_{\text{pert.}}(b^*;\mu) + \mathcal{D}_{\text{NP}}(b_T,b^*;B_{\text{NP}},\{c_i\})$$

discretization effects

Consistent with:
SV19: JHEP 06, 137, [1912.06532]
MAP22: JHEP 10, 127, [2206.07598]
ART23: [2305.07473]
IFY23: Isaacson, Fu, Yuan [2311.09916]



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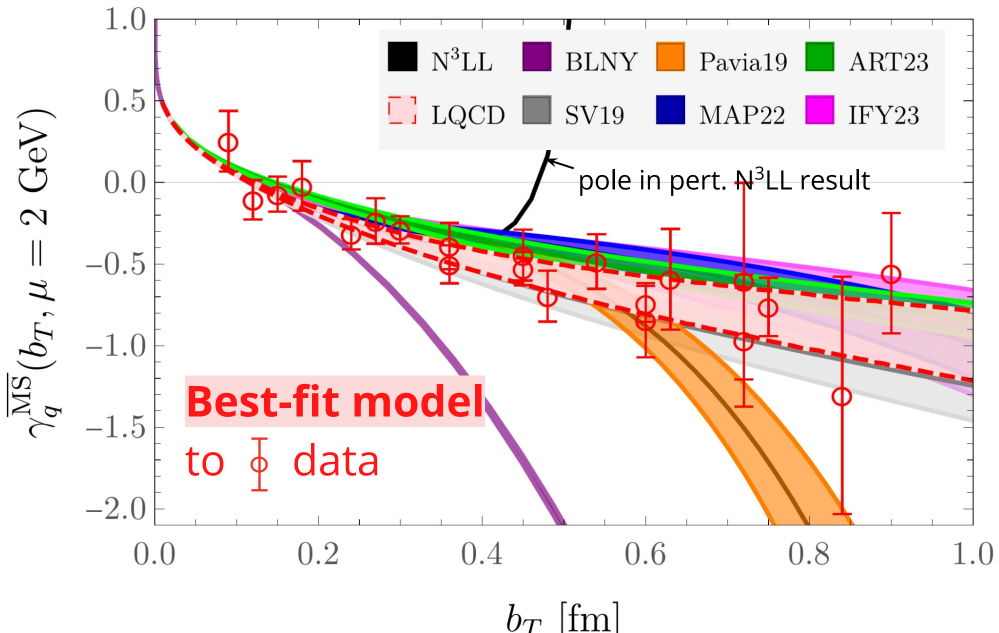

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