LATTICE 2026 - PARALLEL TALK

Isospin-breaking corrections for muon g-2 hadronic vacuum polarization

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Image credit: Muon (g-2) Collaboration

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01

HVP calculation overview

^R. Aliberti et al., Phys. Rept. 1143 (2025) 1-158, arXiv:2505.21476

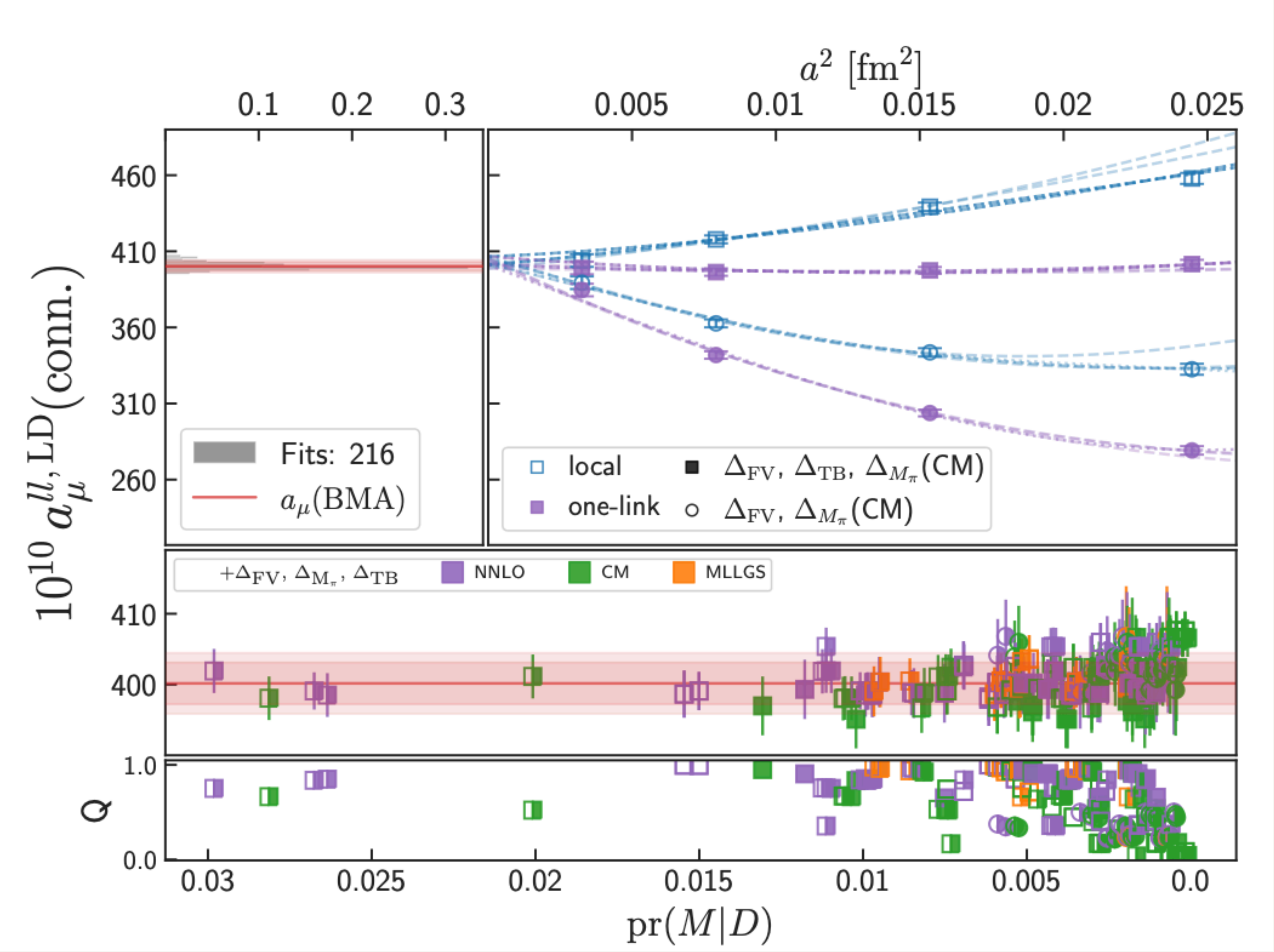
Hadronic vacuum polarization in lattice QCD

- Simple $\langle V_\mu V_\mu \rangle$ two-point correlation function! But: noisy long-distance tail, high precision demands
- Experimental result (as of WP 25^): $a_\mu^{exp} = 11659207.15(1.45) \times 10^{-10}$ [rel. error: **125 ppb**]
- HVP contribution is O(700) in 10^{-10} units —> need rel. error of **2 per mille** to match! (Our current goal for next paper is **sub-percent**.)
- Relevant sub-components for this talk: from WP25, $\boldsymbol{\Delta a_\mu^{(IB)} \sim 0(3)} \times 10^{-10}$.
  - Only really need a factor of 2 error reduction to get to experimental target
  - *But*, this is a sum of QED + SIB, conn + disc, and there are significant cancellations!

#Cross-checked using bootstrap/jackknife in previous analyses

Analysis strategy

- **Full propagation of errors** and correlations using `gvar` and `lsqfit`#
- **Noise reduction** at long-distance using fit method (replace C(t) at large t with model fit to small t.)
  - More robust than bounding in fake-data tests (see supplement of arXiv:2412.18491*)
- **Bayesian model averaging** to capture systematic errors
  - Average includes continuum extrapolation + model choices for FV/TB/mass tuning
  - Covariance computed including correlations in model average (see appendix C of arXiv:2411.09656^)

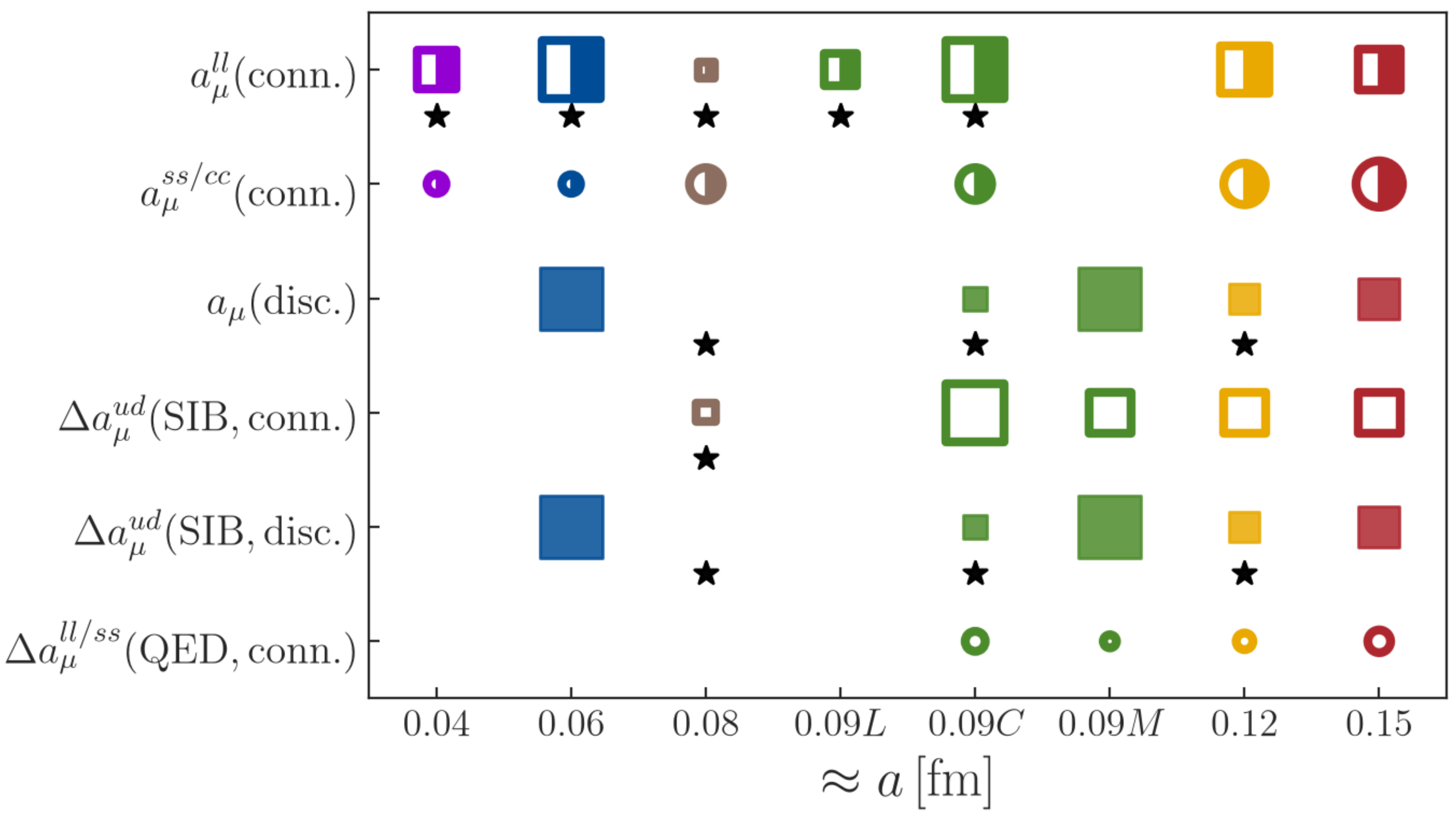


Fermilab-MILC-HPQCD Collaborations, arXiv:2412.18491, PRL 135 (2025) 1, 011901

*A. Bazavov et al (Fermilab Lattice/HPQCD/MILC), PRL 135 (2025) 1, 011901
^A. Bazavov et al (Fermilab Lattice/HPQCD/MILC), PRD 111 (2025) 9, 094508

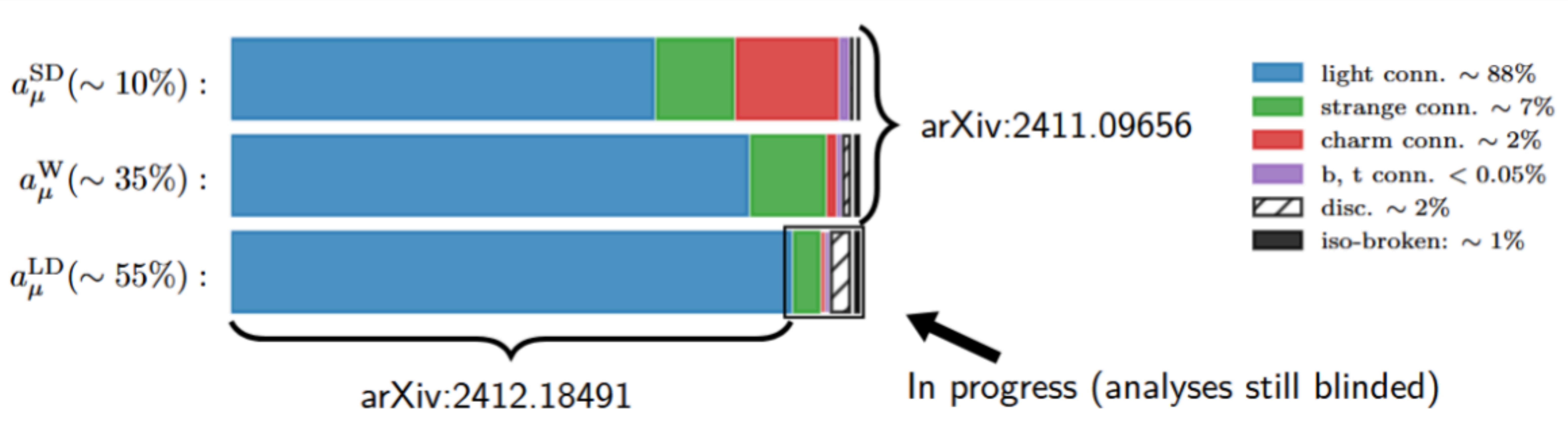
HVP correlator data and projections

- *: ensembles currently generating more data
- a~0.09fm varieties:
  - "M": sea mistuned
  - "C": re-tuned
  - "L": large-volume (5.8 fm —> 11.9 fm)
  - a~0.08, 0.04fm ensembles **new** since last published results
  - Ongoing work on other uses of HVP data: e.g. R-ratio (M. Saccardi talk, M 14:20)



Where our calculation stands

(summary plot courtesy of S. Lahert)



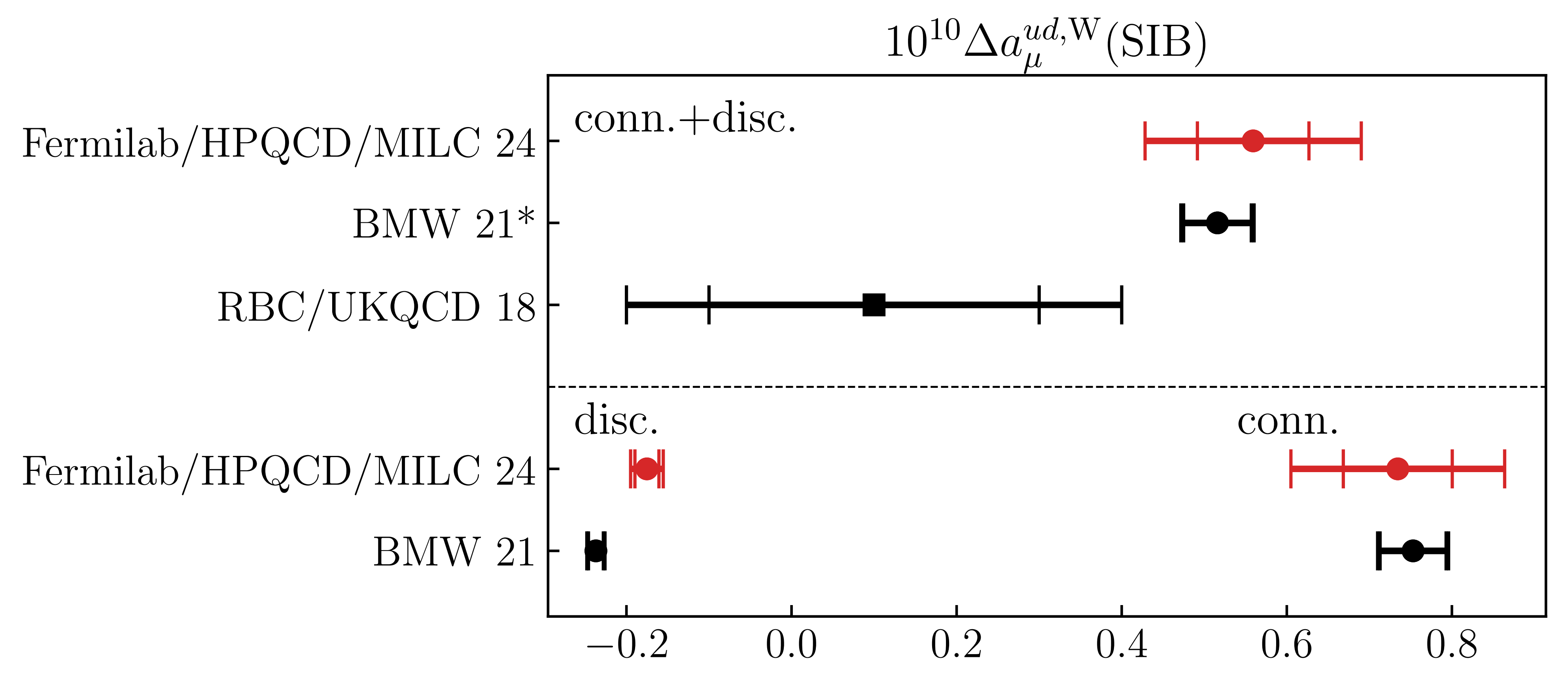
- Published: complete short-distance + intermediate windows; light-quark connected LD
- This talk: progress on full QED and strong-isospin corrections
- Next talk: (S. Lahert) LD strange, charm, disconnected; data updates for everything.

02

Strong isospin breaking (SIB)

SIB overview and previous results

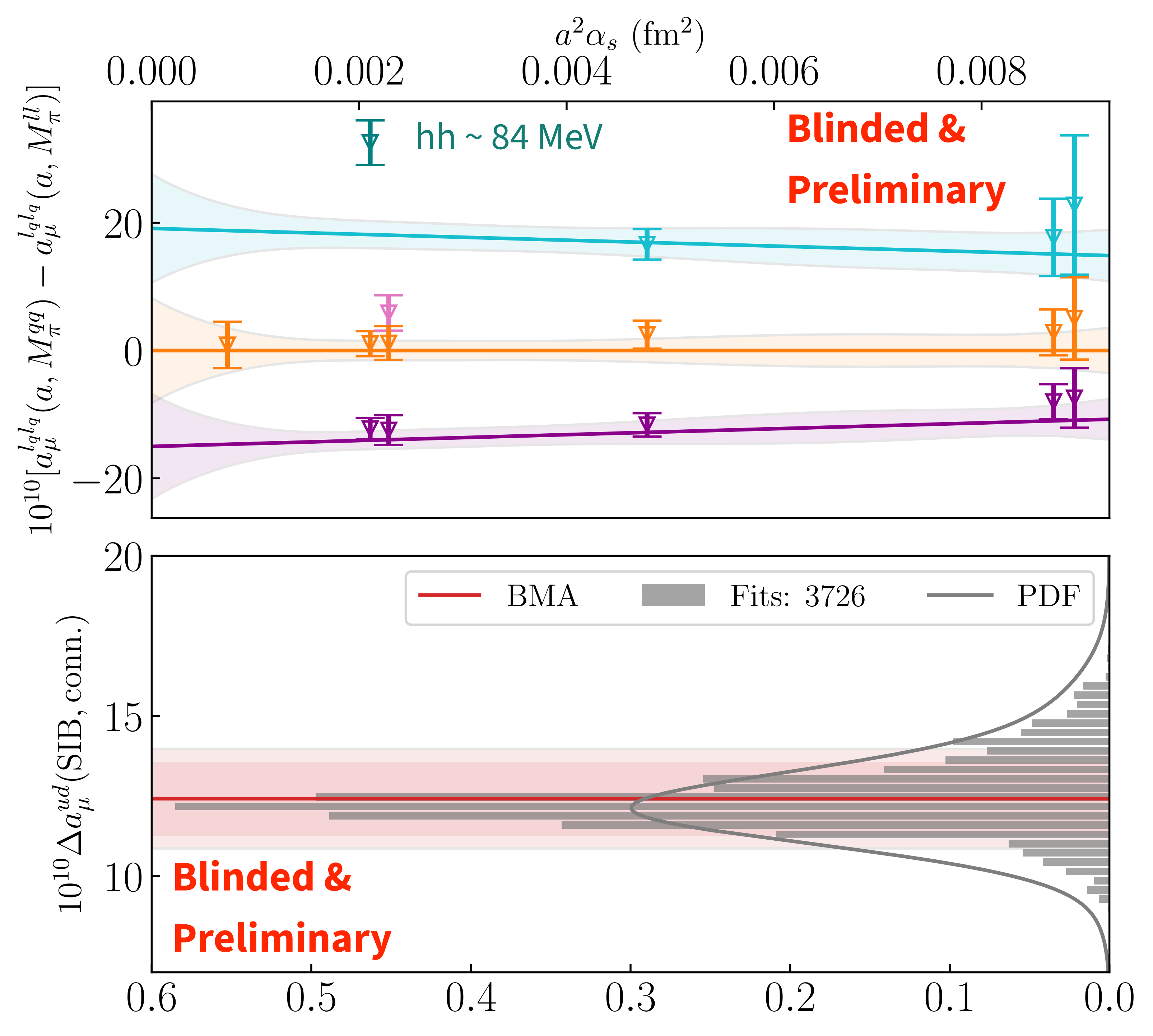
$$\Delta a_\mu^{ud}(\text{SIB, conn.}) \equiv a_\mu^{ud}(\text{conn.}) - a_\mu^{ll}(\text{conn.}),$$
$$\Delta a_\mu^{ud}(\text{SIB, disc.}) \equiv a_\mu^{ud}(\text{disc.}) - a_\mu(\text{disc.}).$$



Fermilab-MILC-HPQCD Collaborations, arXiv:2411.09656

- "Light-quark" a_μ computed in degenerate N_f=2 limit ($m_u^{(v)}=m_d^{(v)}=m_l$)
- Correct for up-down mass splitting by computing the (correlated) differences shown to the left
- Results shown for our previous calculation for intermediate window W; SD window also calculated in that work
- Current goal: extend to LD window to get "full" SIB corrections

Connected SIB



- Correlators computed with valence mass l_q={*h*,u,l,d} (+ mistuned l on 0.09M.)
- Joint chiral-continuum fit to reconstruct SIB correction:

$$\Delta a_\mu^{ud}(\mathrm{SIB}, \mathrm{conn}) = a_\mu^{ud}(\mathrm{conn}) - a_\mu^{ll}(\mathrm{conn})$$

$$a_\mu^{ud} = \frac{q_u^2}{q_l^2} a_\mu^{l_u l_u} + \frac{q_d^2}{q_l^2} a_\mu^{l_d l_d}$$

- Continuum extrapolation remains quite flat, mass dependence significant
- Projecting **>factor of 2 error reduction** (new error around $2\text{x}10^{-10}$) from previous published conn SIB correction^

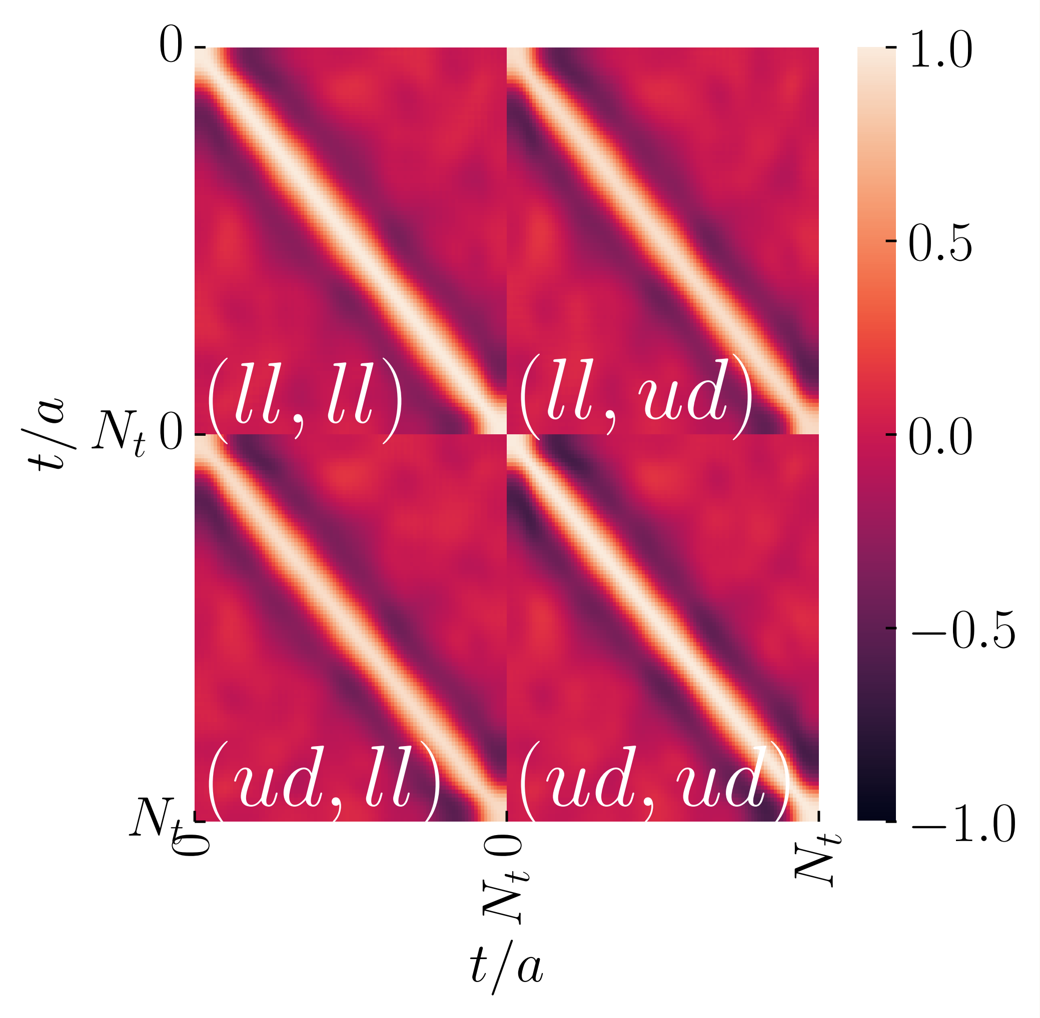
^B. Chakraborty et al (Fermilab Lattice/HPQCD/MILC), arXiv:1710.11212, PRL 120 (2018) 15, 152001

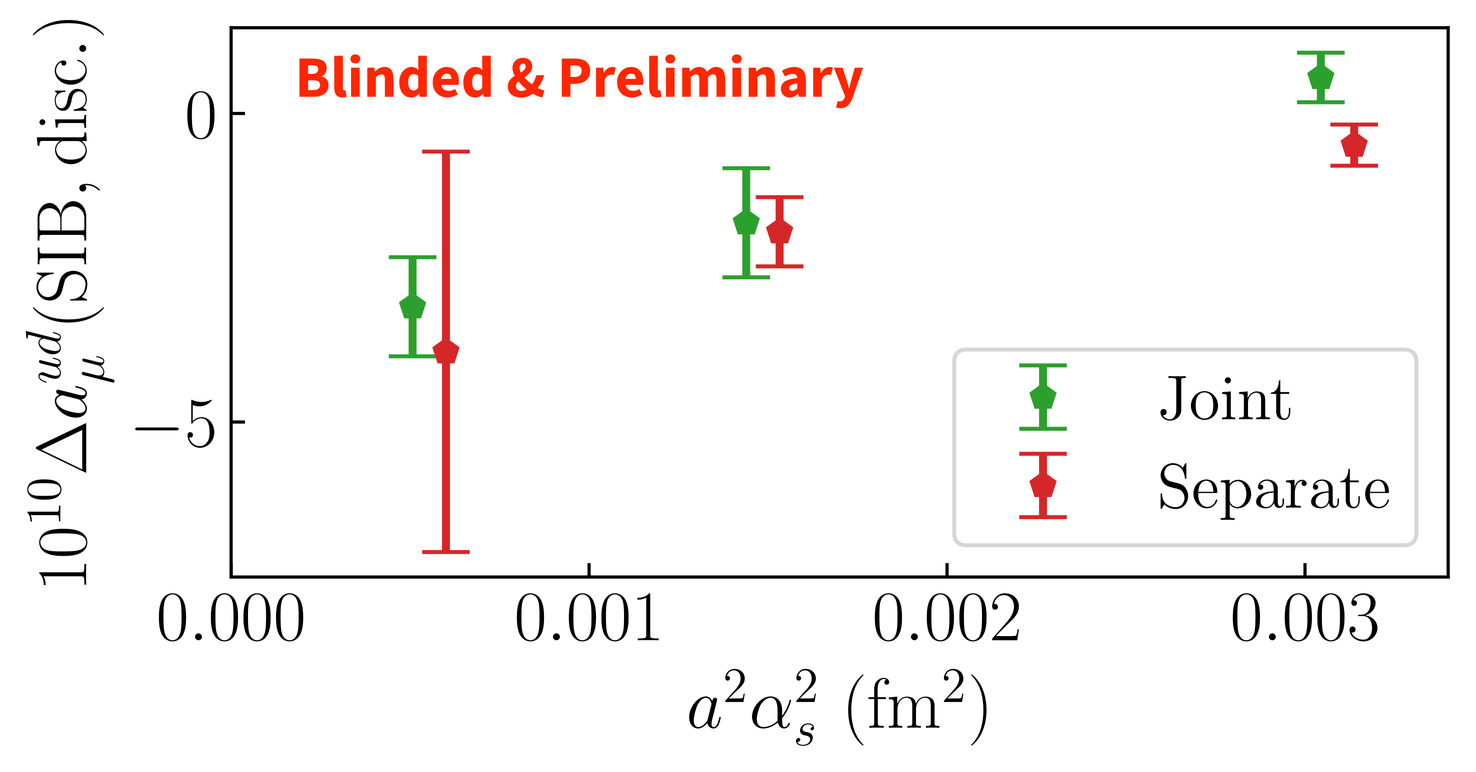
Disconnected SIB

- Disconnected correlation function is computed directly as C_{ud}
- Off-diagonal correlations with C_{ll} are strong (*top plot shows 0.09C*); must include full covariance and do joint fits
- Working on fit decomposition into isospin channels I={0,1} in Nf=2 (see Lahert talk); but isospin mixing in SIB,

$$C_{ud}^{\mathrm{disc}}(t) = C_{I=0}(t) - \frac{1}{9} C_{I=1}(t) + C_{\mathrm{mix}}(t)$$

- Can series expand Z —> Z + δZ, E —> E + δE in C_{ud} vs. C_{ll} (isospin corrections O(1%) or smaller); work in progress





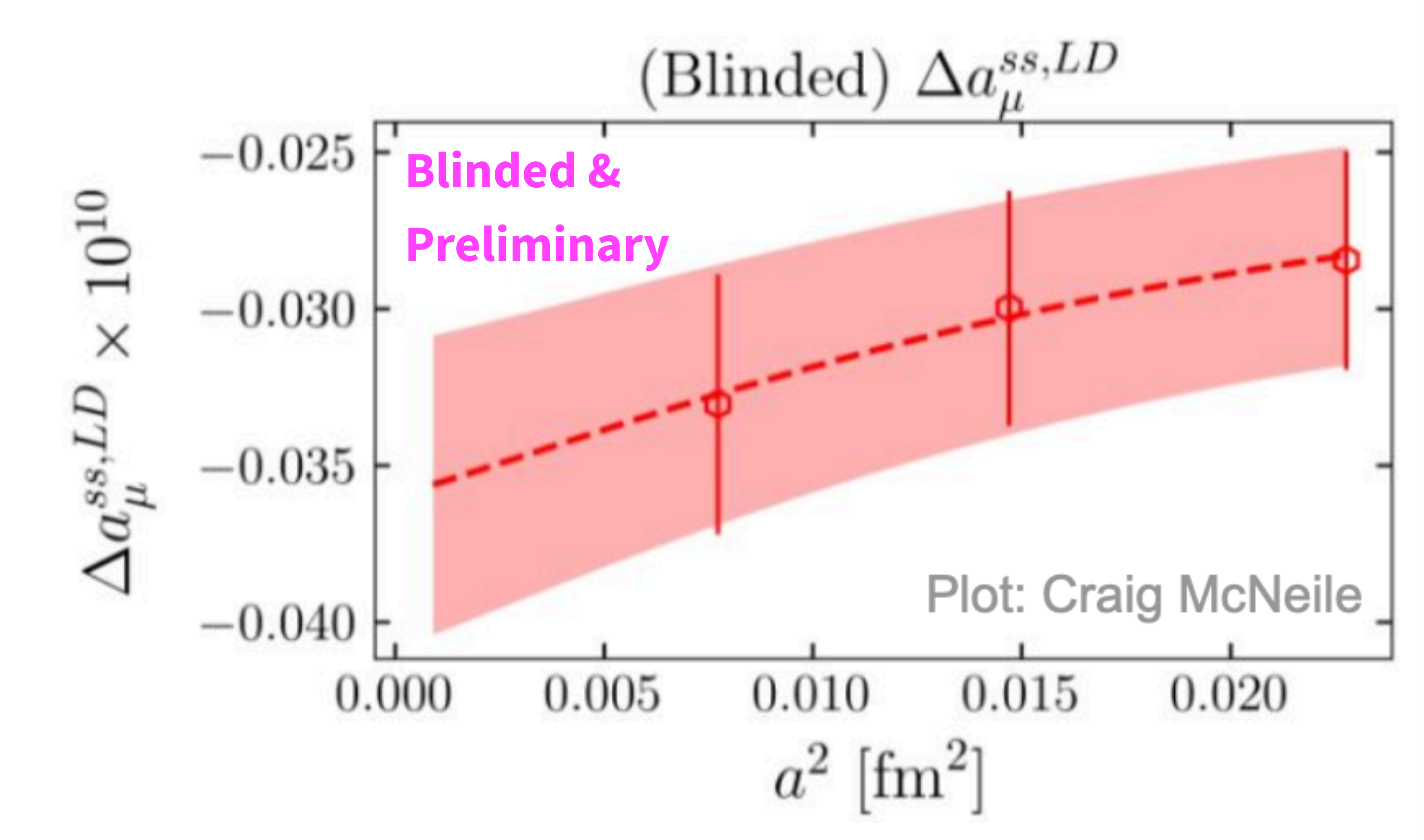
03

Electromagnetic corrections (QED)

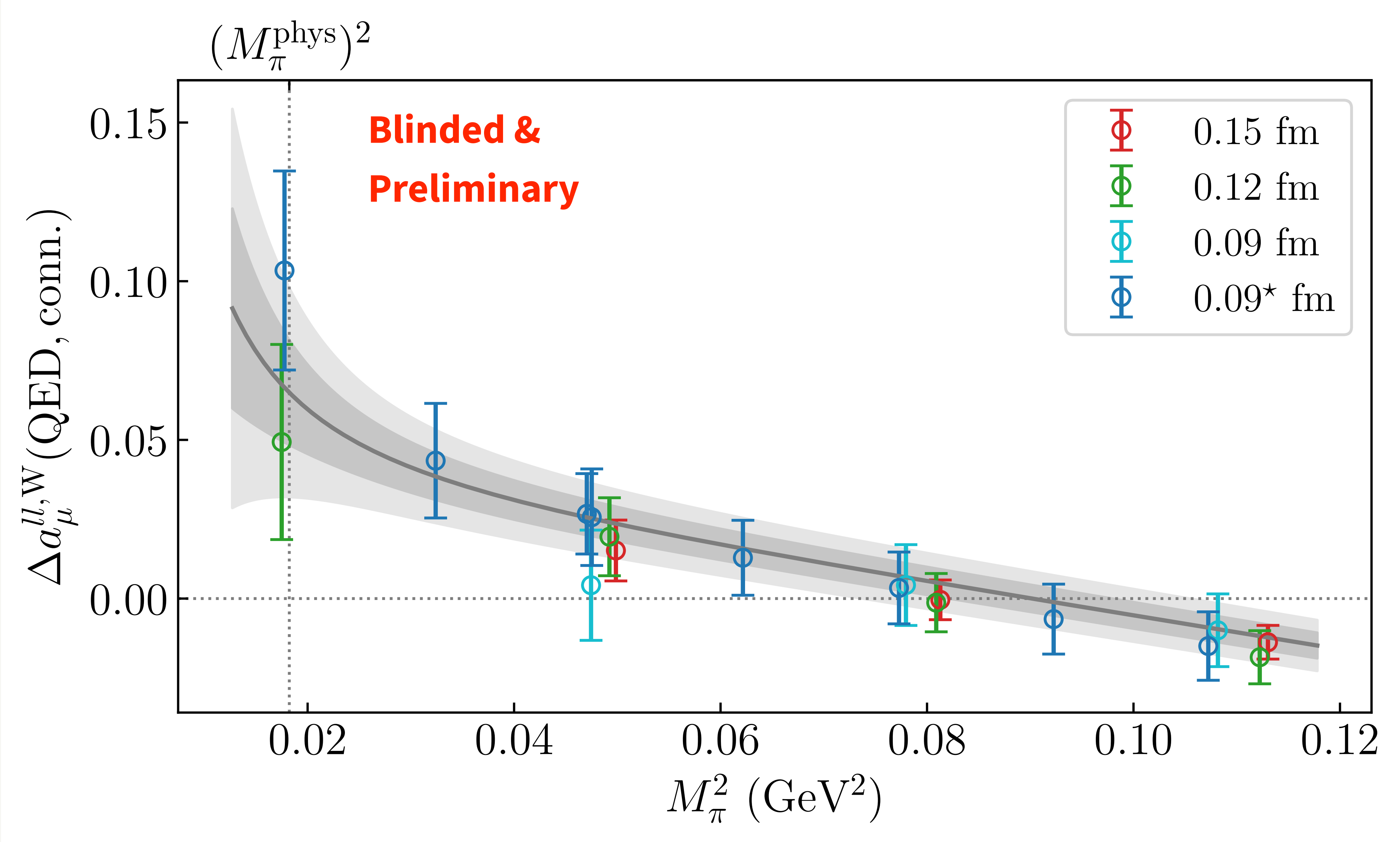
*see e.g. D. Hatton et al. (HPQCD Collaboration), arXiv:2005.01845, PRD 102 (2020) 054511 for technical setup
^M. Hayakawa and S. Uno, arXiv:0804.2044, Prog. Theor. Phys. 120:413-441, 2008

QED overview and technical details

- Electro-quenched approximation*: dynamical (Gaussian) QED gauge fields included, but not coupled to sea quarks
- QED_L formulation^ used to deal with photon zero modes
- Calculations done with quark charges set to Q~0.1, 0.2 and valence masses up to $7m_l$; connected C(t) only so far
- **Strange-quark contribution** also calculated; preliminary LD window extrapolation shown (right).



New connected QED data



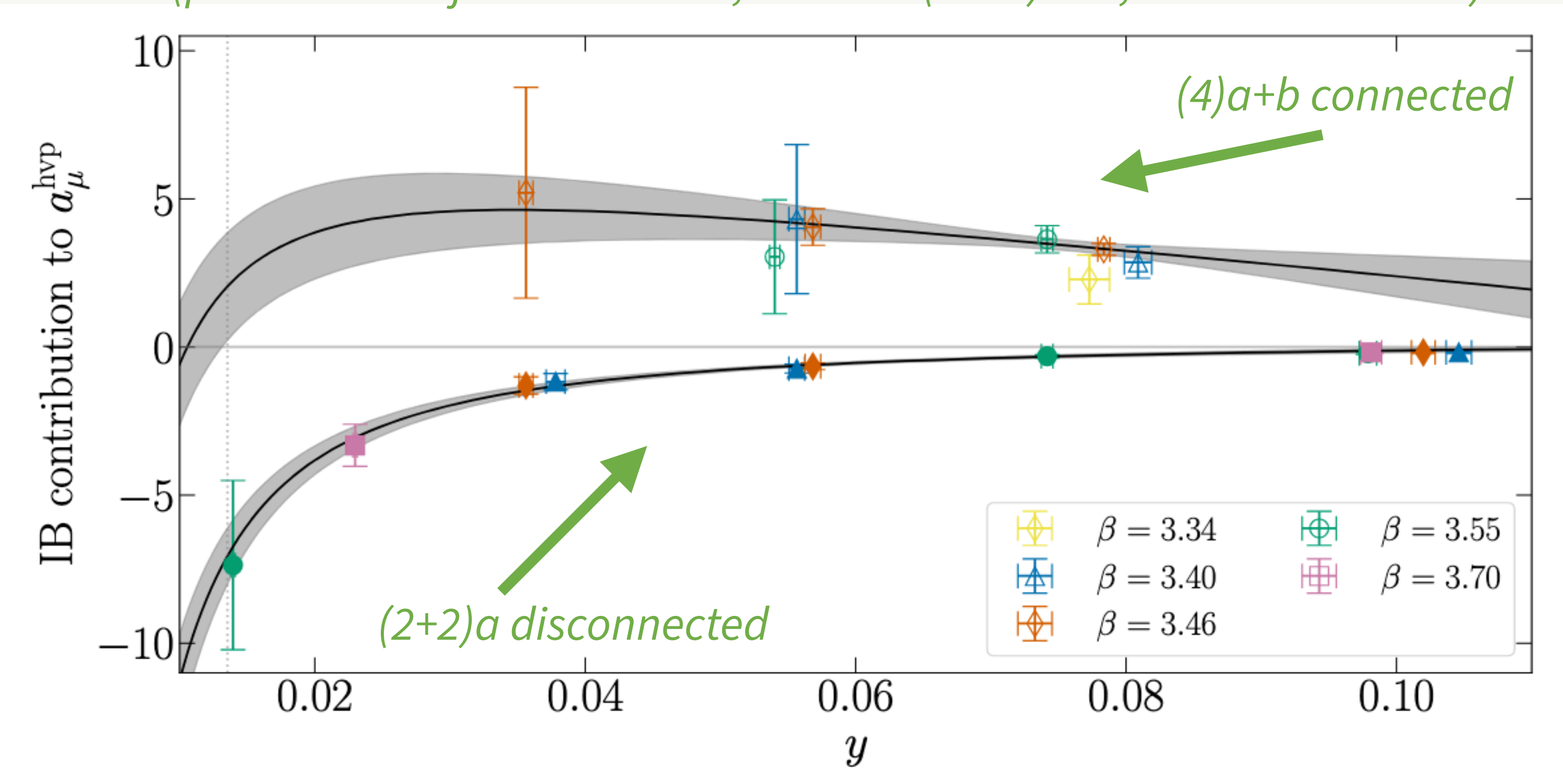
(note: error bands are 1σ, 2σ on this plot)

- Preliminary chiral-continuum extrapolation (left) describes the connected data well
- EFT analysis of leading chiral dependence: calculate HLbL in 2-loop scalar QED + neutral pion contributions, contract one photon —> HVP. (Inspired by ^ but our EFT approach differs)
- EFT matching allows decomposition by diagram, including conn vs. disc; we can reconstruct disc QED from data + model as a bonus!

^J. Parrino , V. Biloshytskyi, E.-H. Chao, H.B. Meyer, V. Pascalutsa, JHEP 07 (2025) 201, arXiv:2501.03192

Data-driven disconnected QED from EFT modeling

(plot from D. Djukanovic et al, JHEP 04 (2025) 098, arXiv:2411.07969)



- We carry out charge spurion analysis to match instead of partial quenching; worse C/D separation, but better flavor separation, no unphysical ss meson contributions
- Obtain similar joint ansatz: fit coefficients to connected QED data, predict dominant QED disc contribution (2+2)l [preliminary!]:

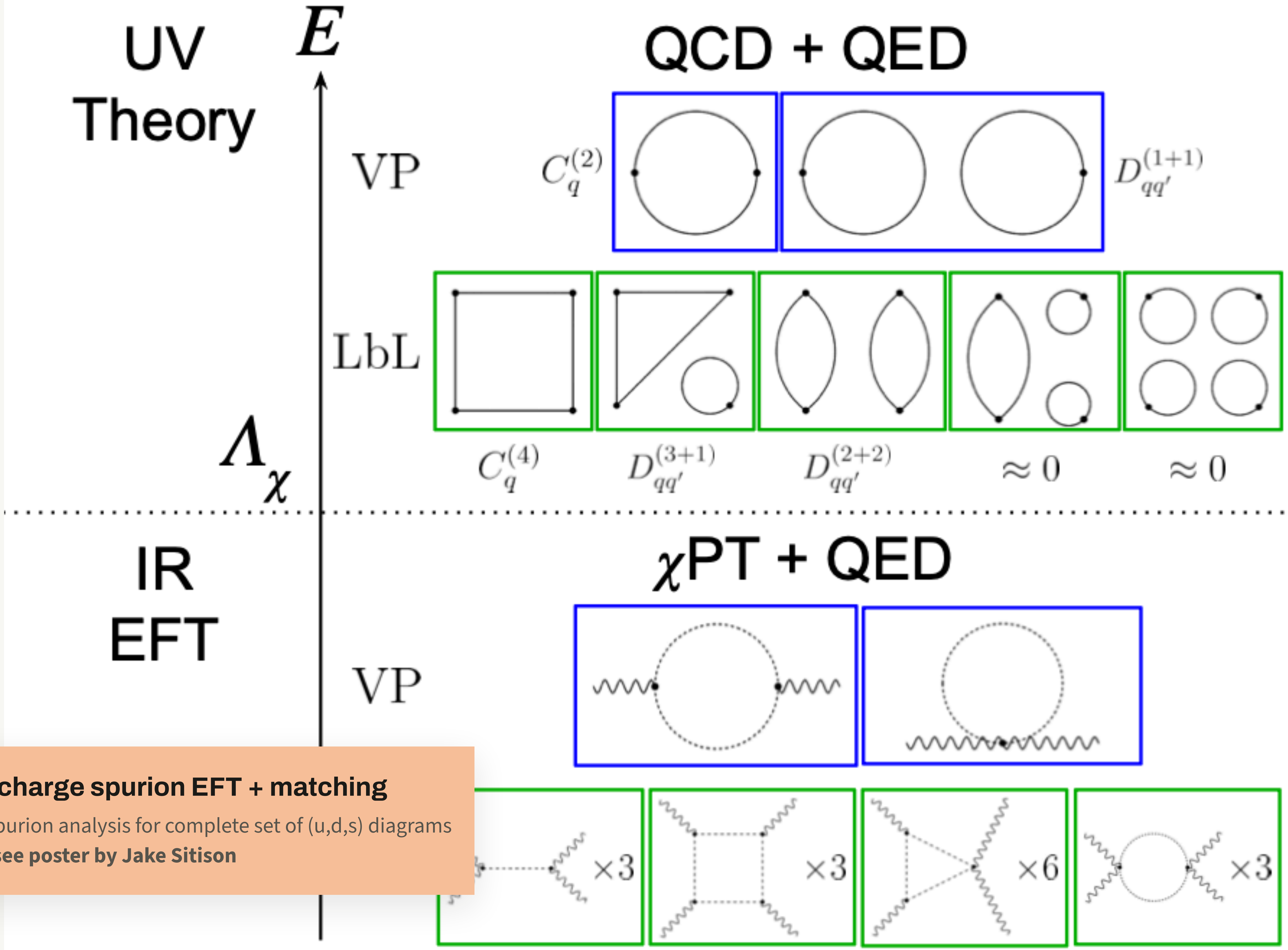
$$a_\mu^{(4)\text{-}l} = \frac{17}{81}\left[2A_1 m_{\pi^+}^{n_1} + A_2 m_{K^+}^{n_2} + 2A_e m_{\pi^0}^{n_3}\right]$$

$$a_\mu^{(2+2)\text{-}ll} = \frac{25}{81}\left[3A_1 m_{\pi^+}^{n_1} - A_3 m_{\pi^0}^{n_3} + (\eta, \eta' \text{ terms})\right]$$

- Example from Mainz HVP analysis above: joint fit to conn/disc data w/ansatz given by scalar QED model + EFT matching,

$$a_\mu^{\mathrm{hvp}1\gamma^*,(4)} = \frac{34}{81}\frac{A}{m_\pi^3} + bm_\pi^2 + c + 0.22\,\log\frac{m_V^2}{m_\pi^2},$$

$$a_\mu^{\mathrm{hvp}1\gamma^*,(2+2)\mathrm{a}} = \frac{50}{81}\frac{A}{m_\pi^3} + d\,,$$



Poster on charge spurion EFT + matching

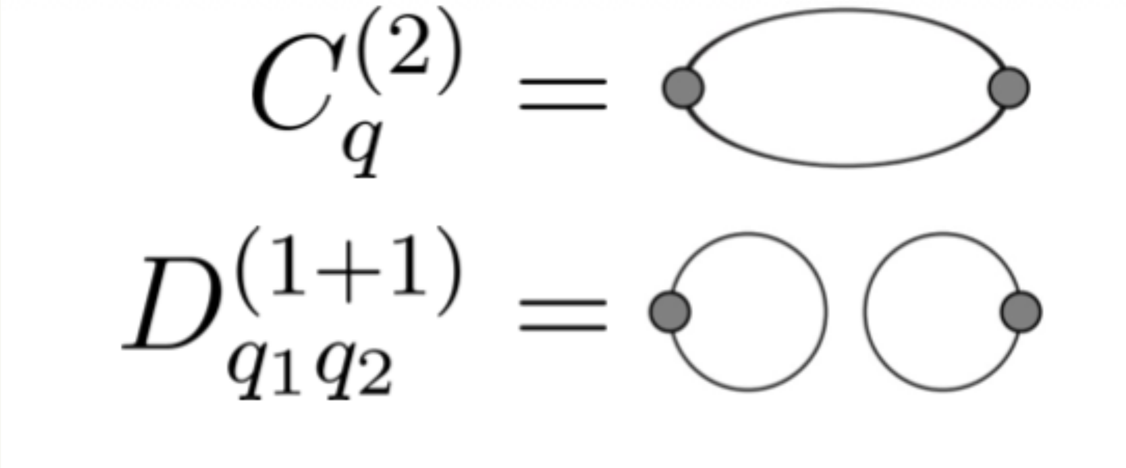
New charge-spurion analysis for complete set of (u,d,s) diagrams for QED HVP; **see poster by Jake Sitison**

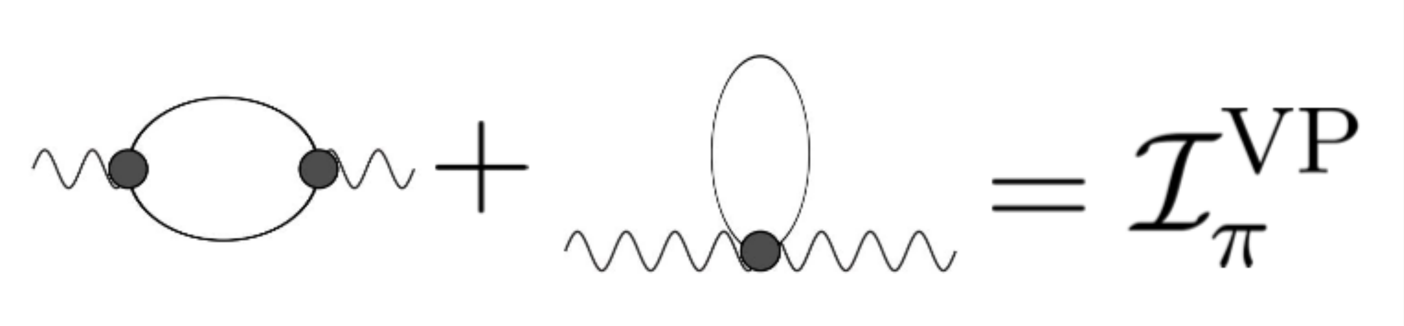
Summary & references

- SIB corrections: both conn and disc underway for full aμ, projected error **2x10^{-10}** (conn) and <**2x10^{-10}** (disc)
- QED corrections: theoretical work to develop charge spurion EFT approach and data analysis in progress, projected error of **2x10^{-10}** on QED total
- EFT approach allows data-driven estimation of disc QED; can be tested against disc QED lattice data directly in future work
- All well below the target of sub-percent (~7x10^{-10}) for our full HVP calculation in progress.
- Stay tuned for next publication!

Backup slides

EFT matching example: leading order HVP





- <u>Top:</u> in UV theory (i.e. QCD), we can classify diagrams into quark-connected $C^{(2)}$ and quark-disconnected $D^{(1+1)}$ by using quark charge. (E.g., $\langle J_l\, J_l \rangle \sim Q_l^2 \longrightarrow$ only $C_l^{(2)}$.)
- <u>Bottom:</u> in IR theory (chiral PT, or equivalently scalar QED), track the *same* external currents and calculate at leading order.

$$C_l = \frac{10}{9}\mathcal{I}_{\pi^+} + \frac{5}{9}\mathcal{I}_{K^+}$$

$$D_{ll} = -\frac{1}{9}\mathcal{I}_{\pi^+}$$

$$2D_{ls} = +\frac{2}{9}\mathcal{I}_{K^+}$$

$$C_s + D_{ss} = +\frac{2}{9}\mathcal{I}_{K^+}$$

Series expansion for disconnected SIB

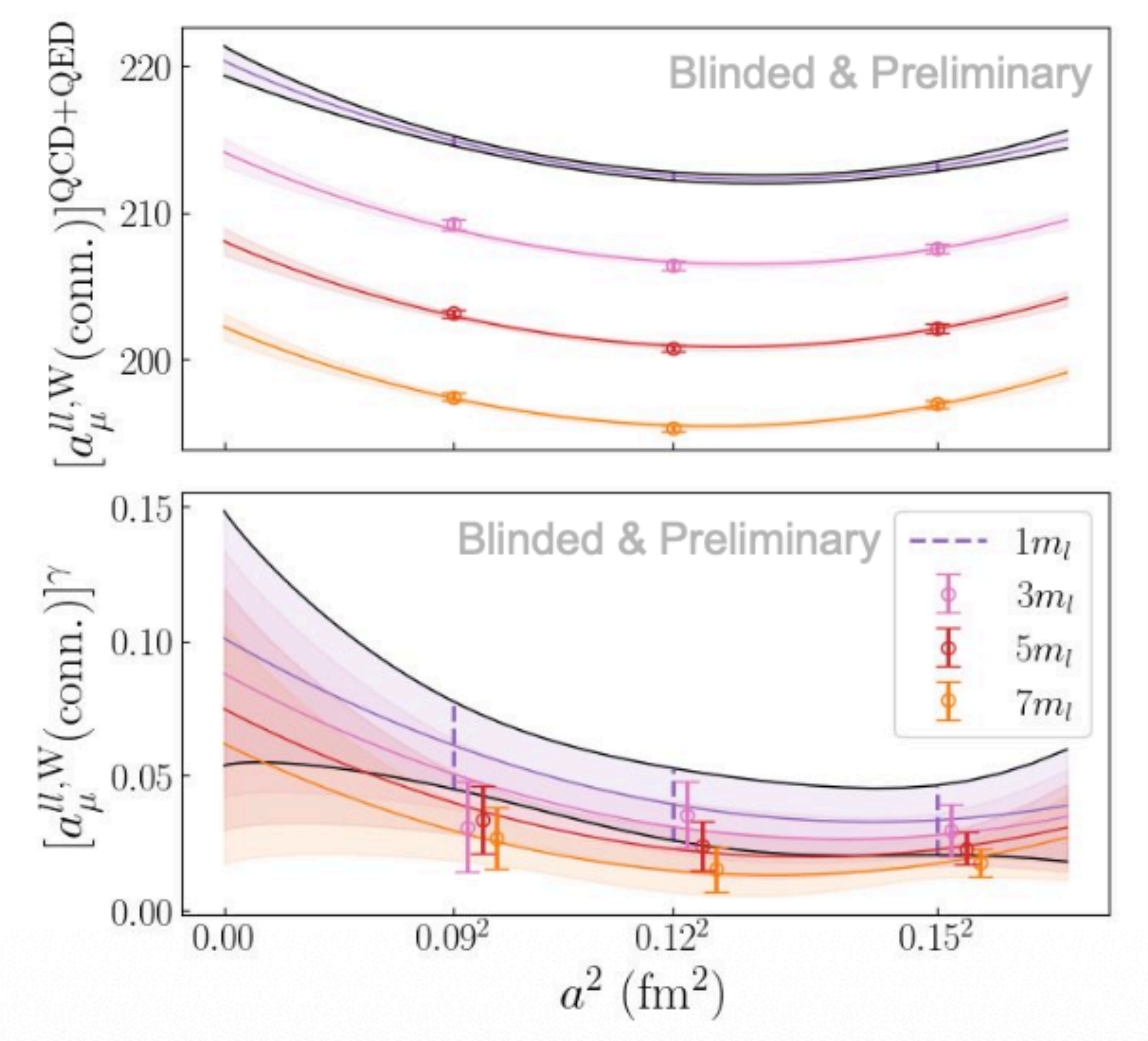
- With isospin preserved, disc HVP splits cleanly into isospin channels^:

$$C^{\mathrm{disc}}_{\mathrm{iso}}(t) = C^{I=0}_{\mathrm{iso}} - \frac{1}{9} C^{I=1}_{\mathrm{iso}}$$
$$= \sum_{j \in (I=0)} |Z_j^{I=0}|^2 e^{-E_j t} - \frac{1}{9} \sum_{k \in (I=1)} |Z_k^{I=1}|^2 e^{-E_k t}$$

- Add isospin breaking —> <$J_0 J_1$> mixed correlator, *plus* all states can mix into both channels
- Take SIB difference —> only small corrections proportional to $(m_u - m_d)/\Lambda \sim 1\%$ should remain; series expand overlaps and energies. Gives apparent t-linear behavior; joint fit with iso-sym data.

$$\Delta C_{\mathrm{SIB}} \approx \sum_{j \in (I=0)} \left(\frac{2}{|Z_j^{I=0}|^2} \delta Z_j^{I=1} - t \delta E_j \right) |Z_j^{I=0}|^2 e^{-E_j t}$$
$$- \frac{1}{9} \sum_{k \in (I=1)} \left(\frac{2}{|Z_k^{I=1}|^2} \delta Z_k^{I=0} - t \delta E_k \right) |Z_k^{I=1}|^2 e^{-E_k t}$$

Alternate QED chiral-continuum fits



- QCD+QED and pure QCD connected correlators at $m_q=\{3,5,7\}m_l$
- Average over two values of valence charge
- Significant lattice spacing and m_q dependence visible in QCD+QED data alone (top panel); mostly cancels in the difference (bottom panel)
- Preliminary study here; full analysis (see main talk) uses additional data including at physical light-quark mass

Covariance and model averaging

- General formula for BMA covariance straightforward to derive:

$$\mathrm{Cov}[a,b] = \sum_{i=1}^{N_M} \underbrace{\mathrm{Cov}_i[a,b]}_{\text{(stat. cov.)}}\mathrm{pr}(M_i|D) + \underbrace{\sum_{i=1}^{N_M} \langle a\rangle_i \langle b\rangle_i \mathrm{pr}(M_i|D) - \langle a\rangle\langle b\rangle}_{\text{(model syst. cov.)}}$$

- Common situation: model space factorizes as $\{M_i\} = \{M_{m(i)}^A\} \times \{M_{n(i)}^B\}$ (e.g. independent model choice for aμ + LQC + W and aμ + SIB + SD)
- If we assume **independence** ($\mathrm{pr}(M_i|D) = \mathrm{pr}(M_{m_i}^A|D)\, \mathrm{pr}(M_{n_i}^B|D)$), can show that *only* the stat cov term above survives. Straightforward to compute with bootstrap or error propagation.
- On the other end, can prove bound using diagonal dominance of covariance:

$$\mathrm{Cov}[a,b] \leq \mathrm{Cov}_{\mathrm{stat}}[a,b] + \sigma_a^{\mathrm{syst}} \sigma_b^{\mathrm{syst}}$$

(see appendix C of arXiv:2411.09656)

Covariance and model averaging (II)

- In practice, we take most model choices to be uncorrelated across observables and just estimate stat cov, except for finite-volume models which are significant/correlated; assume they saturate the bound above (i.e. 100% correlation in FV systematics.)
- Resulting correlation matrix for W:

	$a_\mu^{ll}(\mathrm{conn.})$	$a_\mu^{ss}(\mathrm{conn.})$	$a_\mu^{cc}(\mathrm{conn.})$	$a_\mu(\mathrm{disc.})$	$\Delta a_\mu^{ud}(\mathrm{SIB, conn.})$	$\Delta a_\mu^{ud}(\mathrm{SIB, disc.})$
$a_\mu^{ll}(\mathrm{conn.})$	1.00	0.32	0.24	0.09	0.44	0.00
$a_\mu^{ss}(\mathrm{conn.})$	0.32	1.00	0.43	0.00	0.00	0.00
$a_\mu^{cc}(\mathrm{conn.})$	0.24	0.43	1.00	0.00	0.02	0.00
$a_\mu(\mathrm{disc.})$	0.09	0.00	0.00	1.00	0.11	0.06
$\Delta a_\mu^{ud}(\mathrm{SIB, conn.})$	0.44	0.00	0.02	0.11	1.00	0.00
$\Delta a_\mu^{ud}(\mathrm{SIB, disc.})$	0.00	0.00	0.00	0.06	0.00	1.00

Connected SIB chiral-continuum fits

- Fit variations include mass dependence, lattice-spacing dependence, sea mass and mass-tuning corrections; FV choice and data subset choice also

$$a_\mu(a, M_\pi, \{M_A\}) = a_\mu(a, M_\pi)(1 + F^M(\{M_A\}))$$

$$a_\mu(a, M_\pi) = \sum_{i=-1}^{1} c_i(a)(M_\pi/\Lambda)^{2i}$$

$$c_i(a) = \sum_{j=0}^{n_i} c_{ij}(a\Lambda)^{2j}\alpha_s^{\delta_{1j}\nu}$$

$$F^M(\{M_A\}) = C_{\text{sea}} \sum_{A=\pi,K} \frac{M^2_{\pi,\text{phys.}}}{M^2_{A,\text{phys.}}}\delta M_A^2$$

$$\delta M_A^2 = \frac{M^2_{A,\text{phys.}} - M^2_{A,\text{latt.}}}{2B_0\Lambda}$$

Dependence	Variations
M_π^2	$\{M_\pi^{-2}, \log M_\pi^2, M_\pi^4\} + M_\pi^0 + M_\pi^2$, $\{M_\pi^{-2}, \log M_\pi^2, M_\pi^2\} + M_\pi^0$
a^2	$n = [0,1,0], \ldots, [2,3,2]$*
α_s	$a^2\alpha_s^\nu$, $\nu = 1, 2$
F^M	With and without
FV	CM, NNLO χPT, HP, on $[0.4, \infty]$, $[1, \infty]$, $[1.5, \infty]$ fm
Data subsets	$a \in [0.06, 0.15]$, $[0.06, 0.12]$, $[0.09, 0.15]$ fm

*Only fits with sufficient DoF included

HVP definitions

- Standard approach w/Bernecker-Meyer time-momentum representation:

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t) C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \langle J_i(x) J_i(0)\rangle$$

$$J_i(x) = \sum_f Q_{q_f} \bar{q}_f(x)\gamma_i q_f(x), \quad Q_u = +\frac{2}{3},\ Q_d = -\frac{1}{3},\ Q_s = -\frac{1}{3}, \ldots$$

- Low-mode averaging used to compute all correlation functions (some older data uses truncated solver method, mostly not used in current analyses)