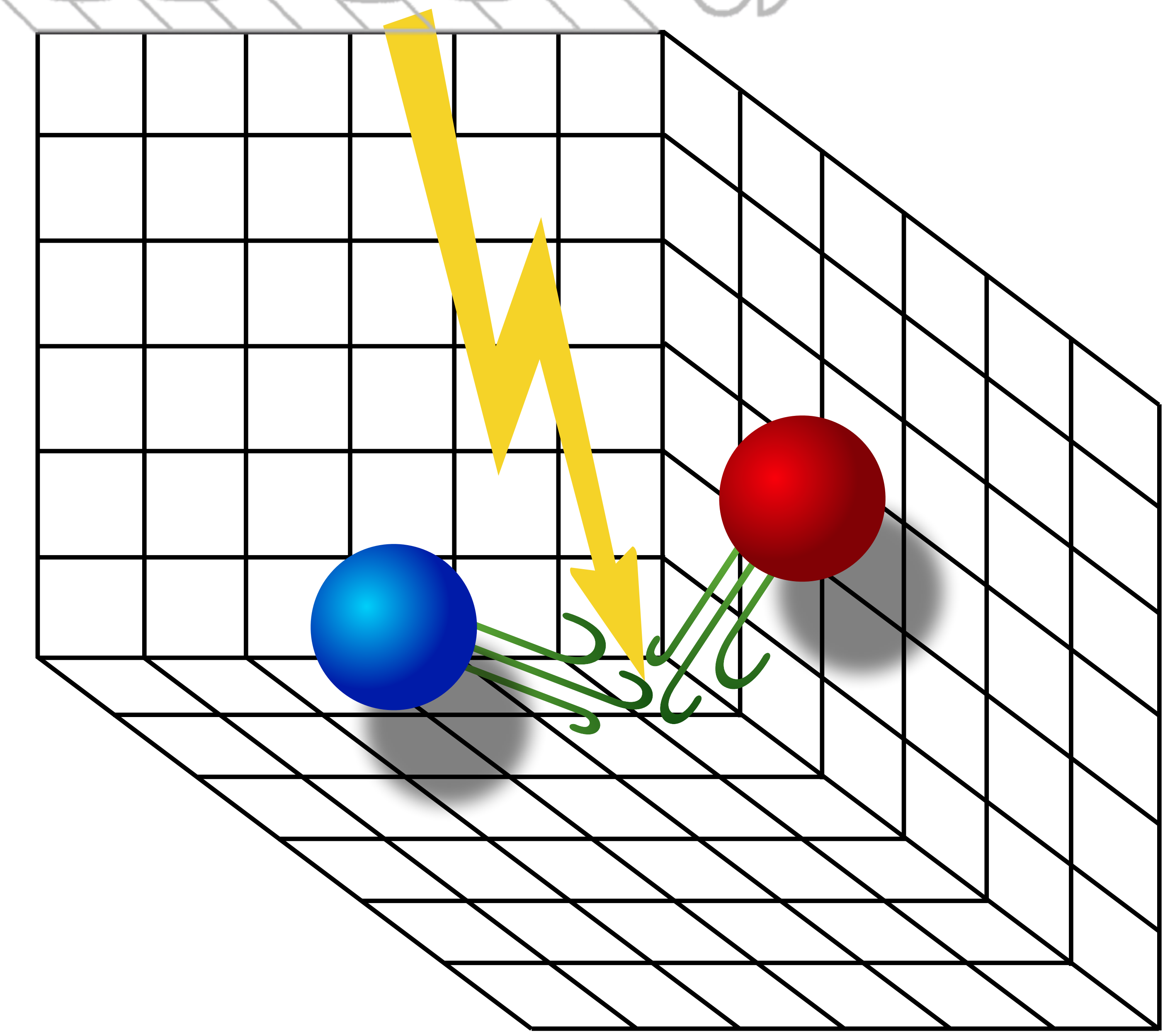
Energy dependence of two-pion transitions



Felipe Ortega-Gama

UC Berkeley

fgortegagama@berkeley.edu

July 29 2026

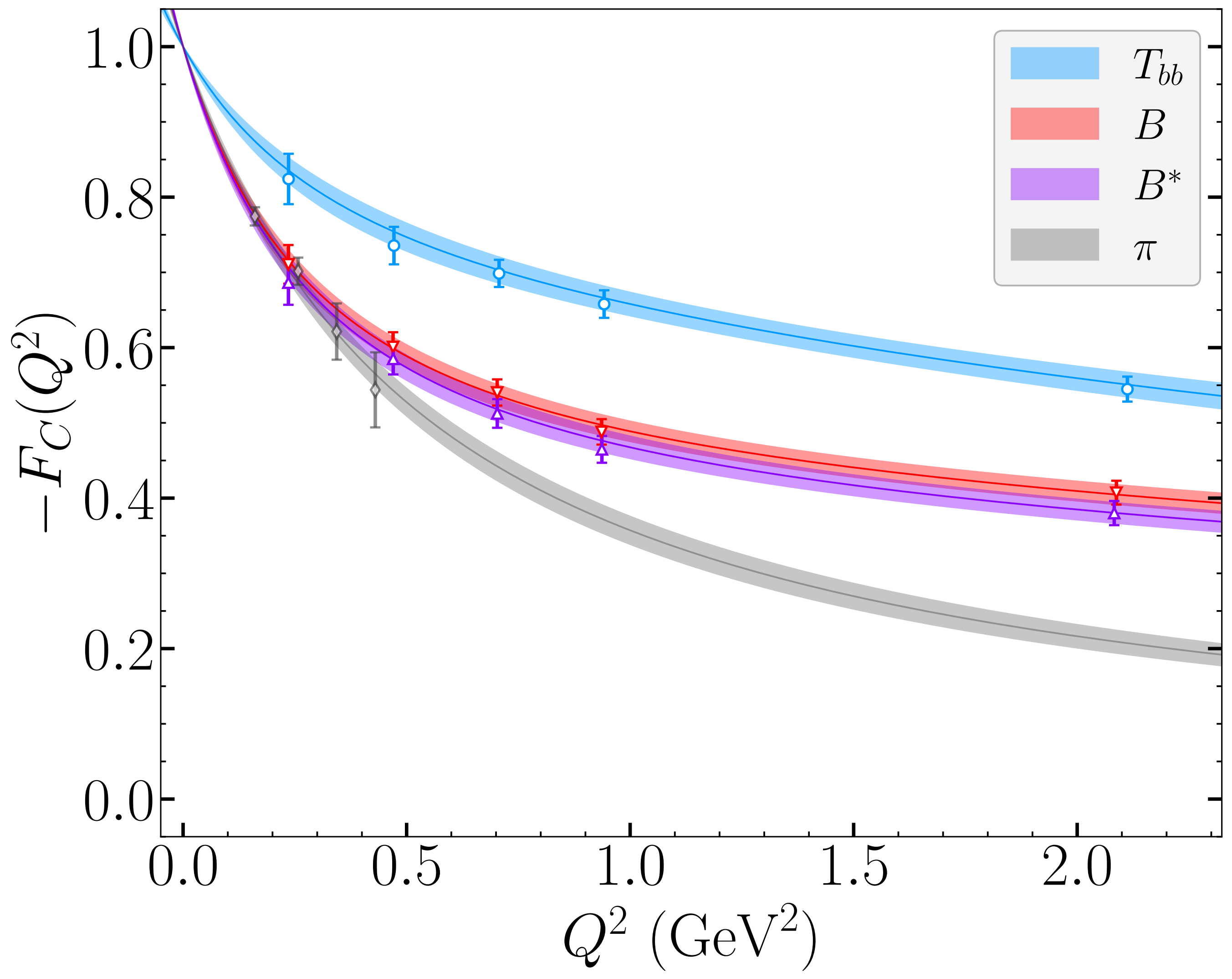
UC Berkeley

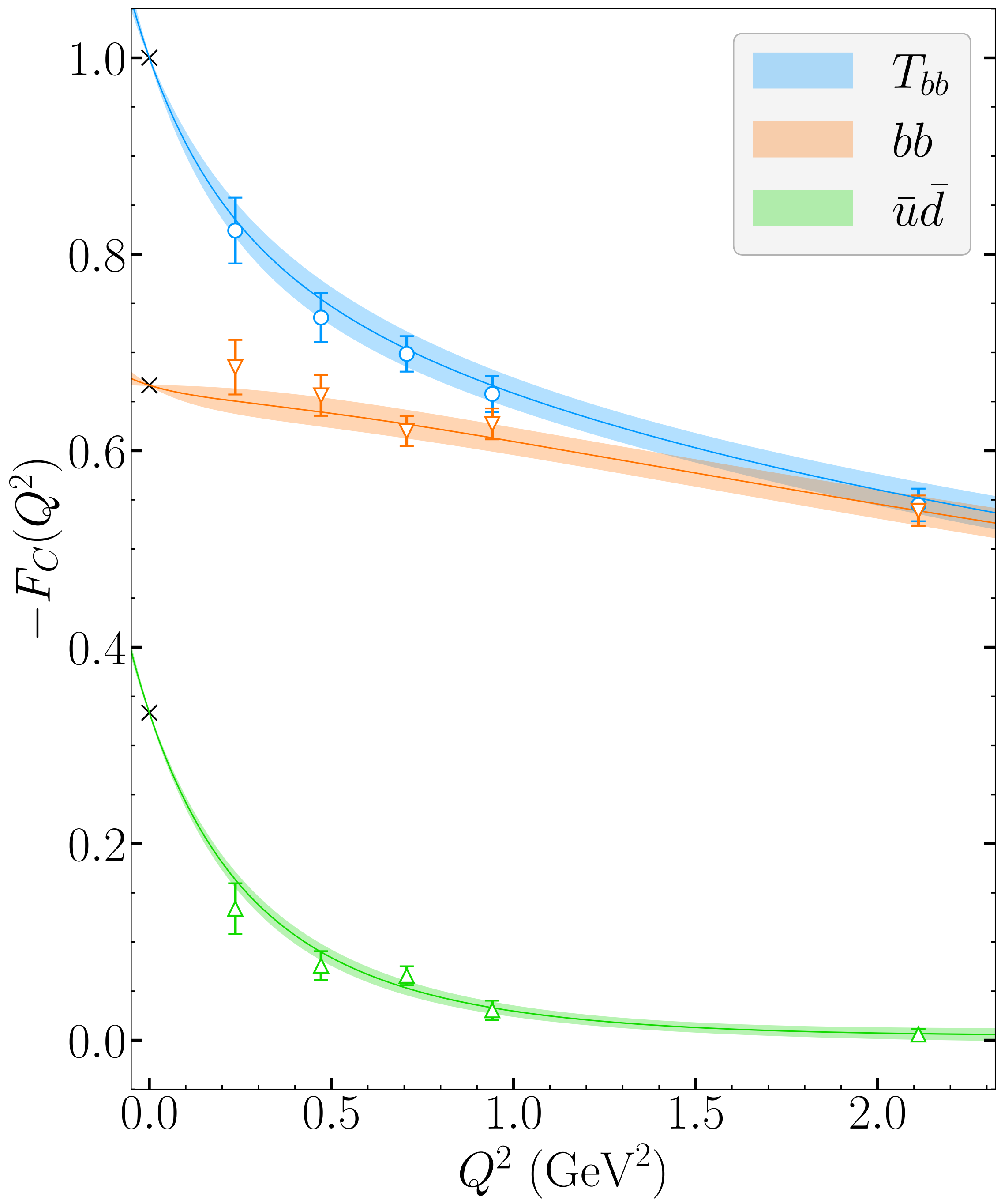
ExoHad
Exotic Hadrons Topical Collaboration

hadspec

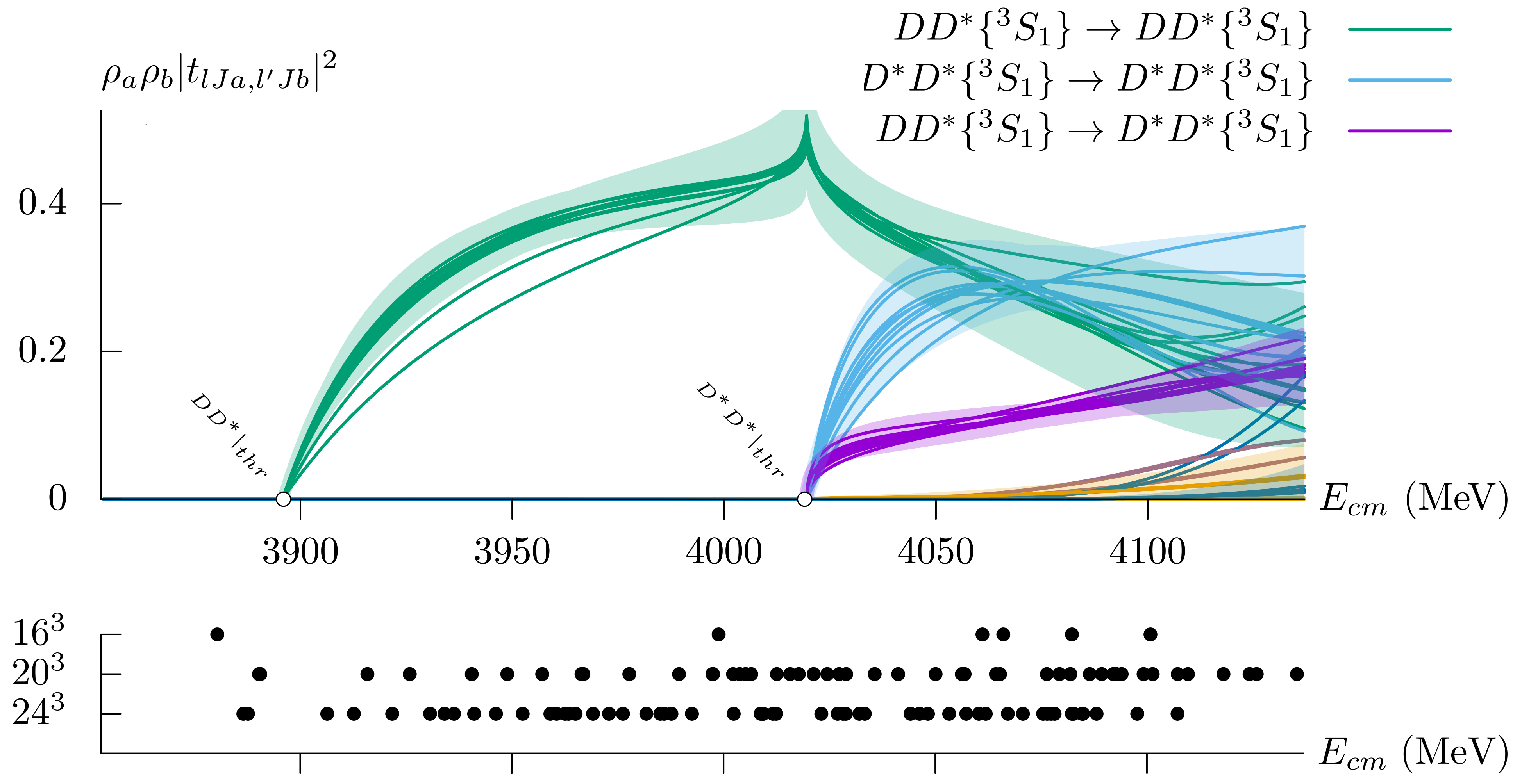
Structure of tetraquarks in LQCD

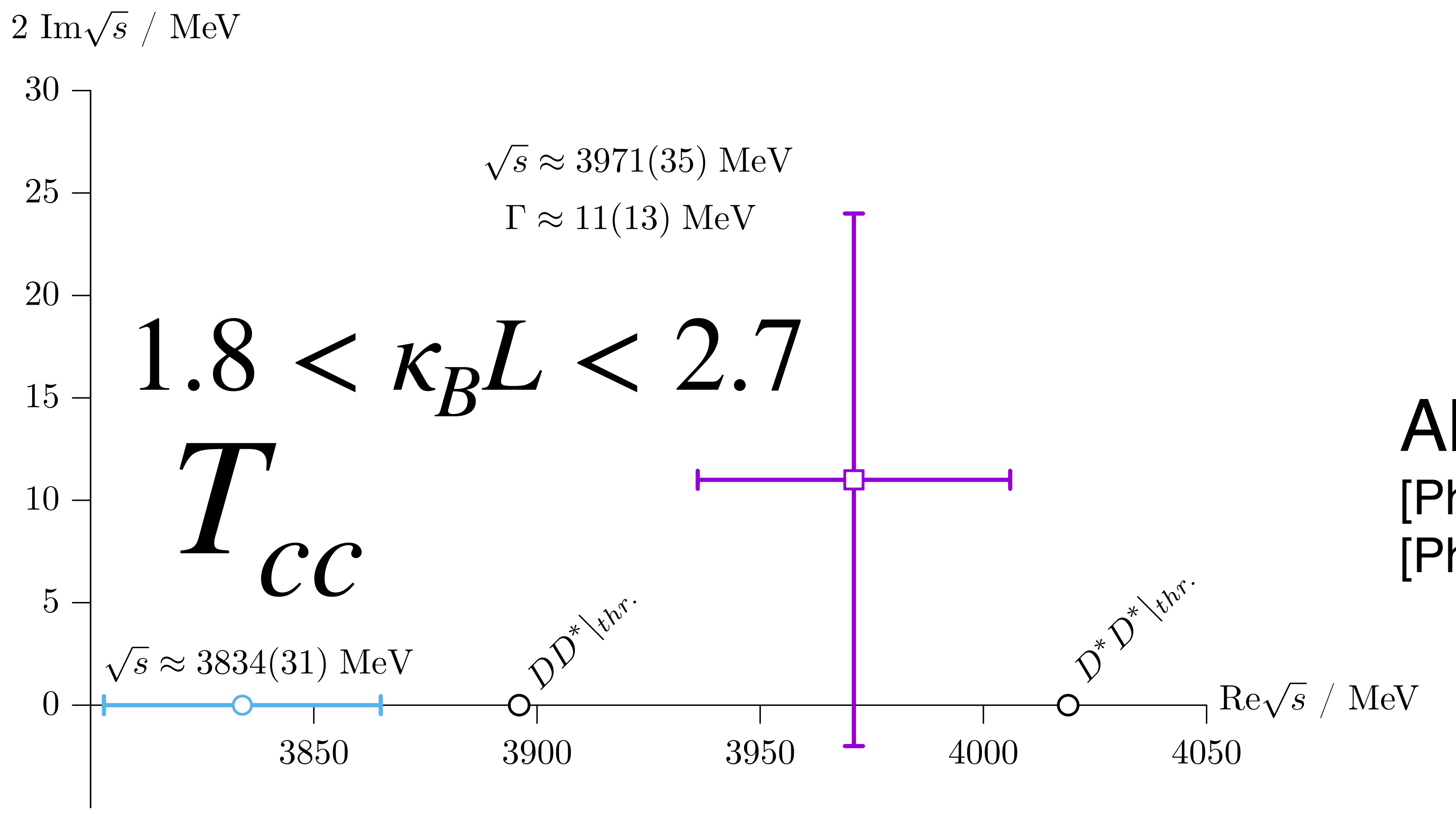
T_{bb} $\kappa_B L \approx 7.5 \gg 1$





[Phys. Rev. Lett. **136**, 161901]





$1.8 < \kappa_B L < 2.7$

T_{cc}

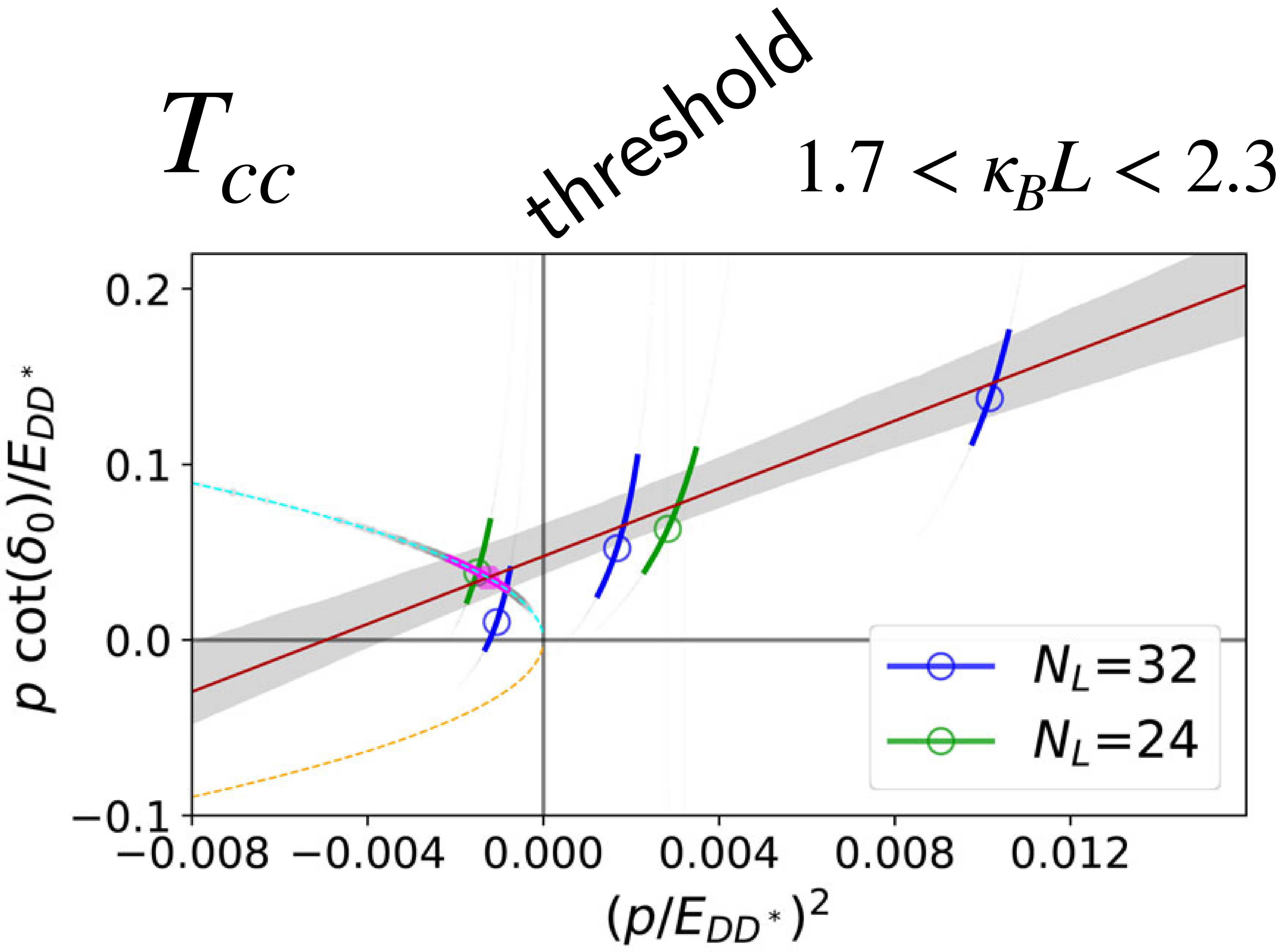
T_{bc}

Also found to be bound

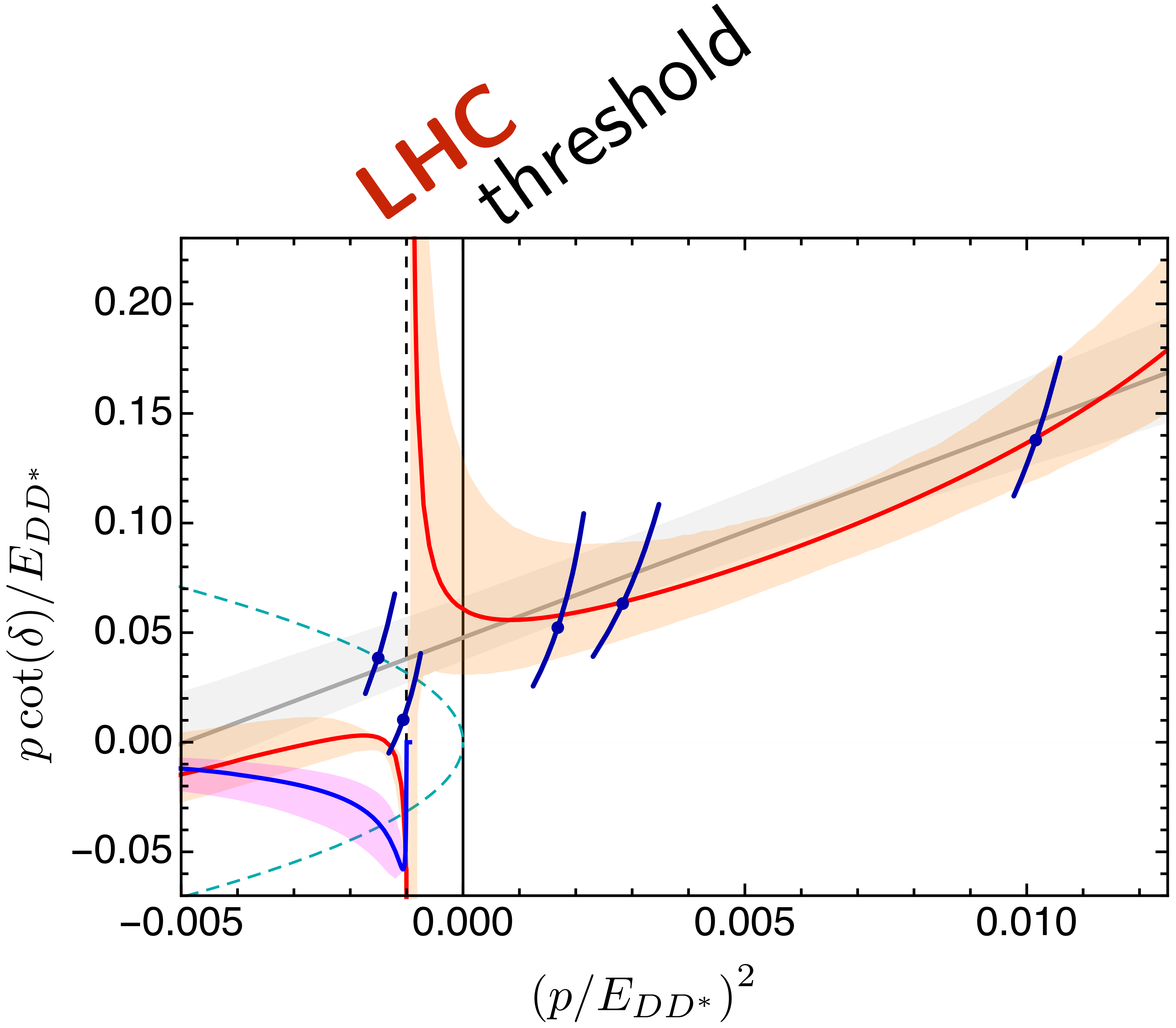
[Phys. Rev. Lett. **132**, 151902]
[Phys. Rev. Lett. **132**, 201902]

[Phys. Rev. D **111**, 034511]

Bound states and left hand cuts



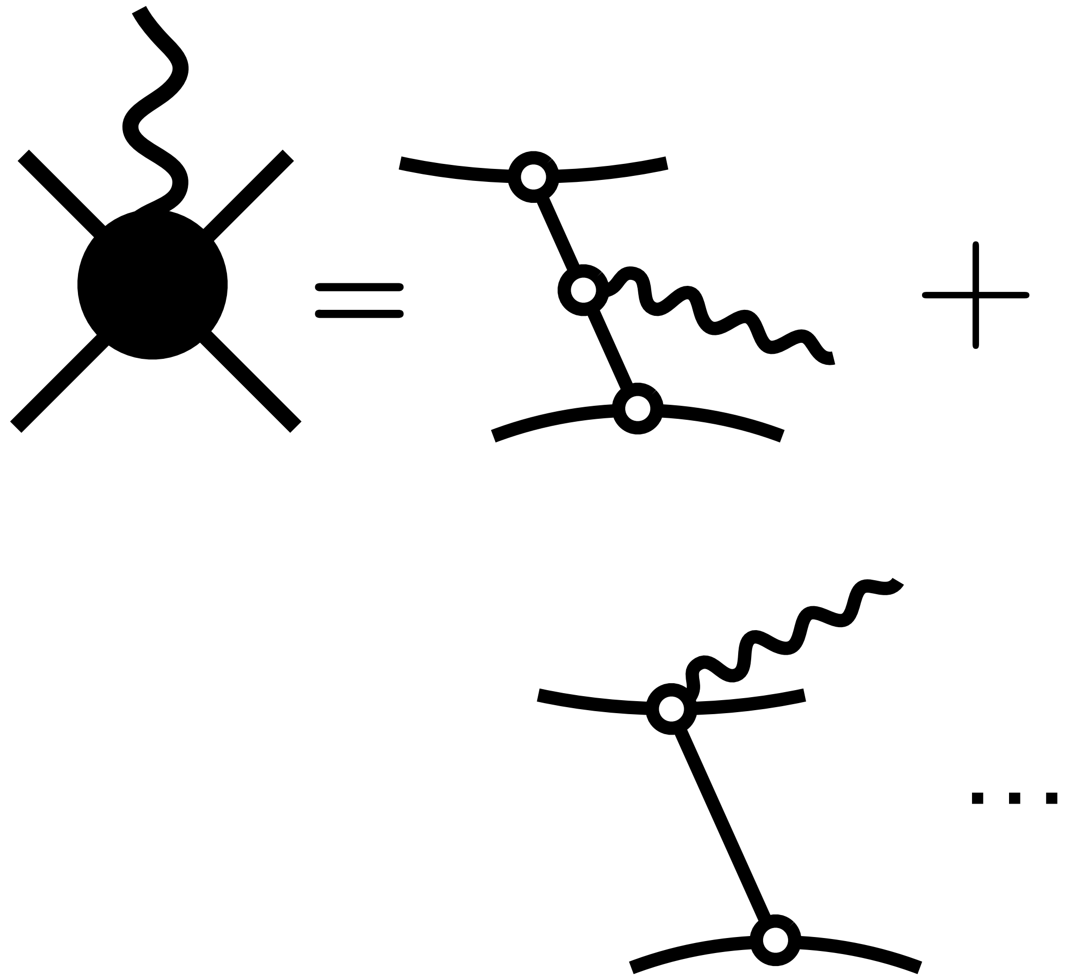
[Phys. Rev. Lett. **129**, 032002]



[Phys. Rev. Lett. **131**, 131903]

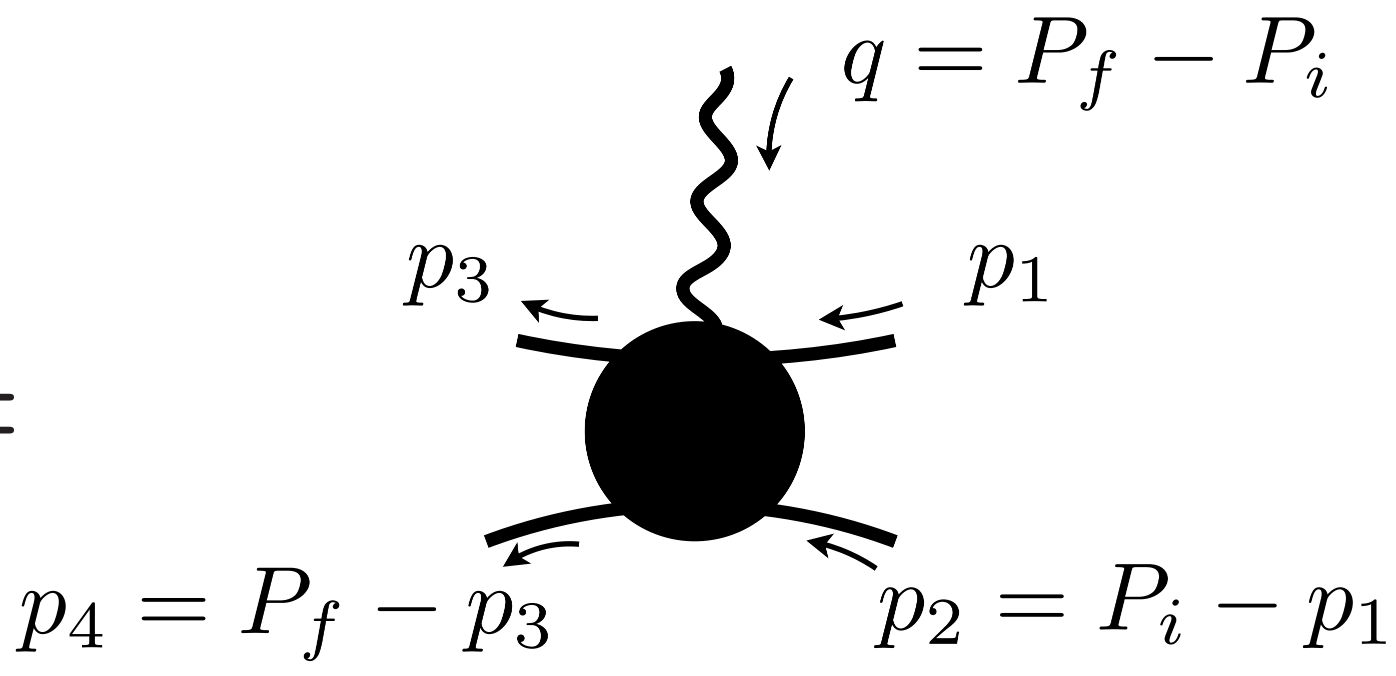
Analytic decomposition

arXiv:2607.24648

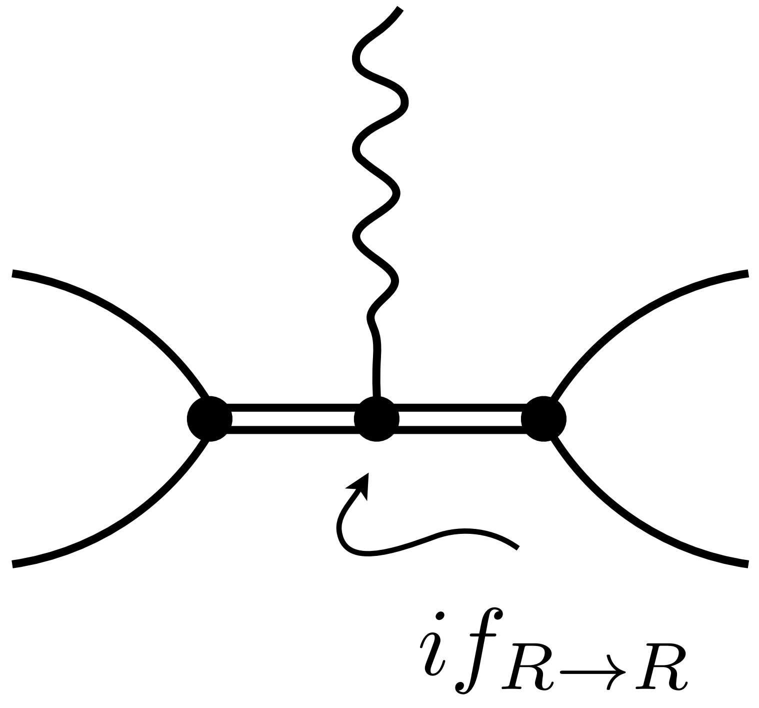


See also recent reanalysis of hadspec T_{cc} results arXiv:2607.19009

Two-hadron transitions $2+\mathcal{J}\to 2$

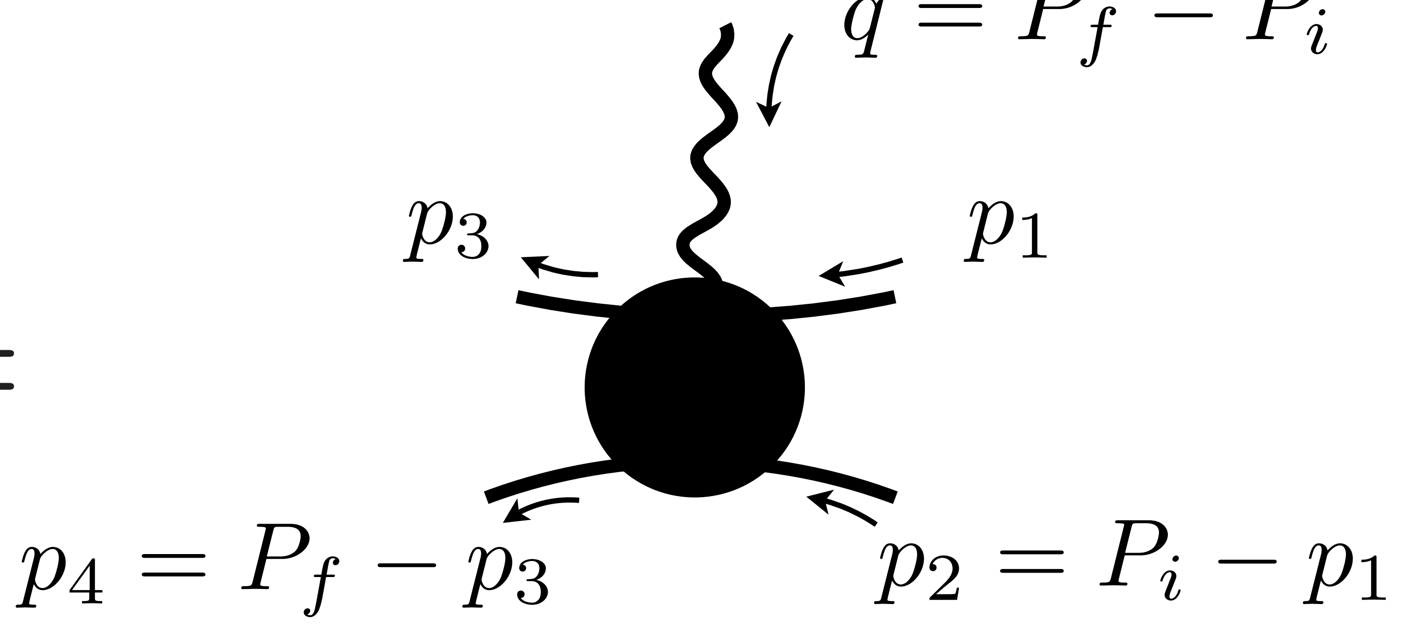
$\mathcal{W}^{\mu} =$ 

$P_f^2 \to s_B, \quad P_i^2 \to s_B$

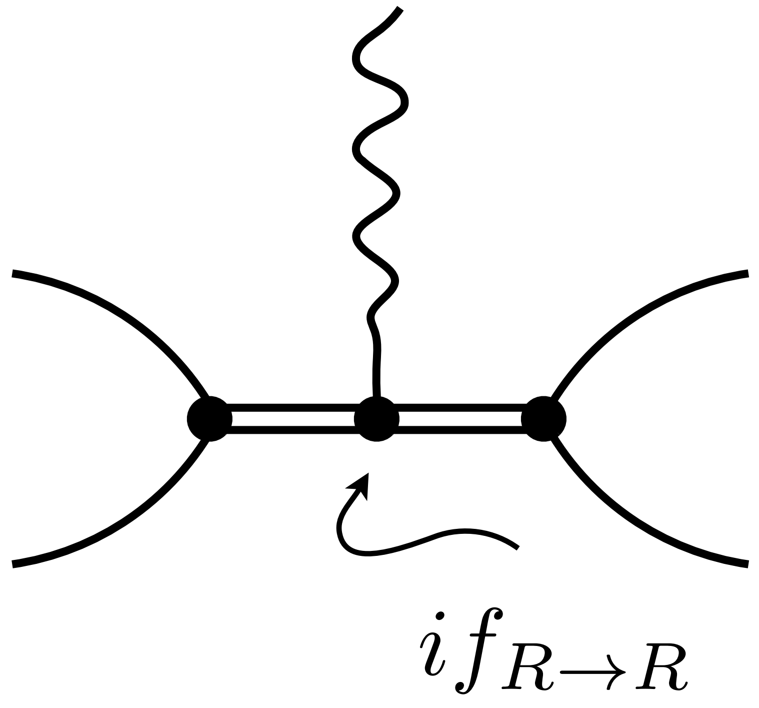


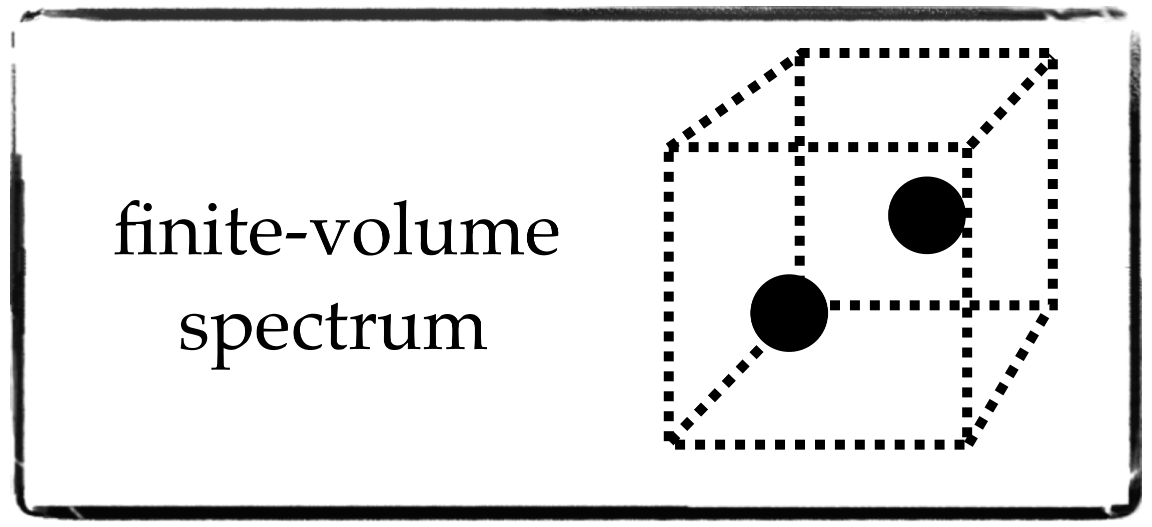
Two-hadron transitions $2 + \mathcal{J} \to 2$

$\mathcal{W}^{\mu} =$

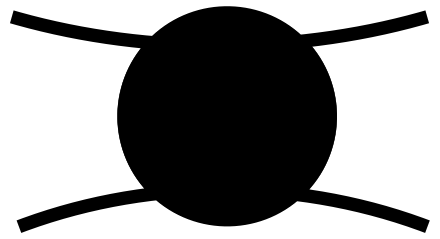


$P_f^2 \to s_B, \quad P_i^2 \to s_B$





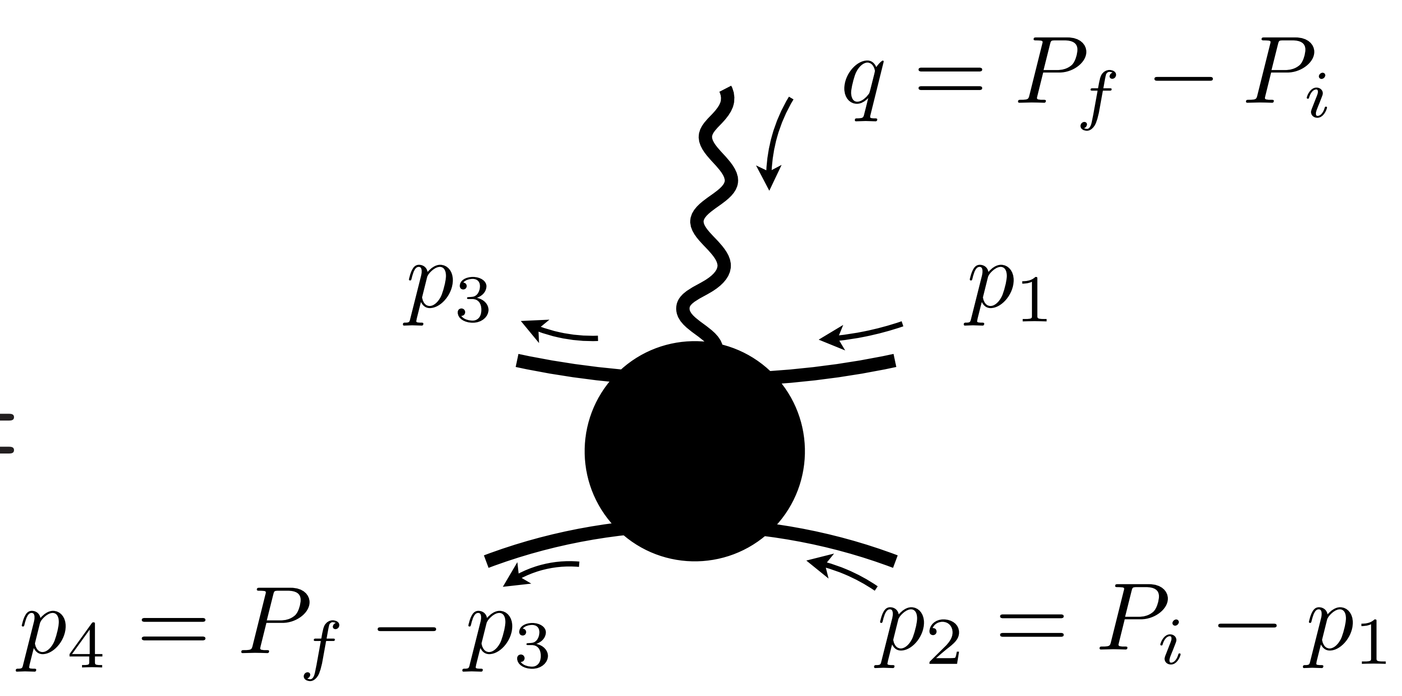
Lüscher QC



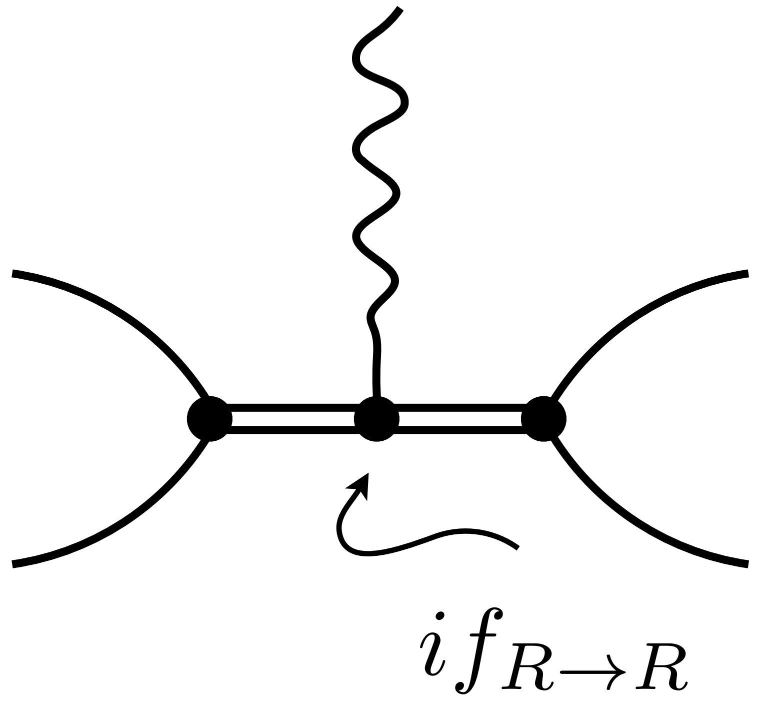
$\mathcal{M}$

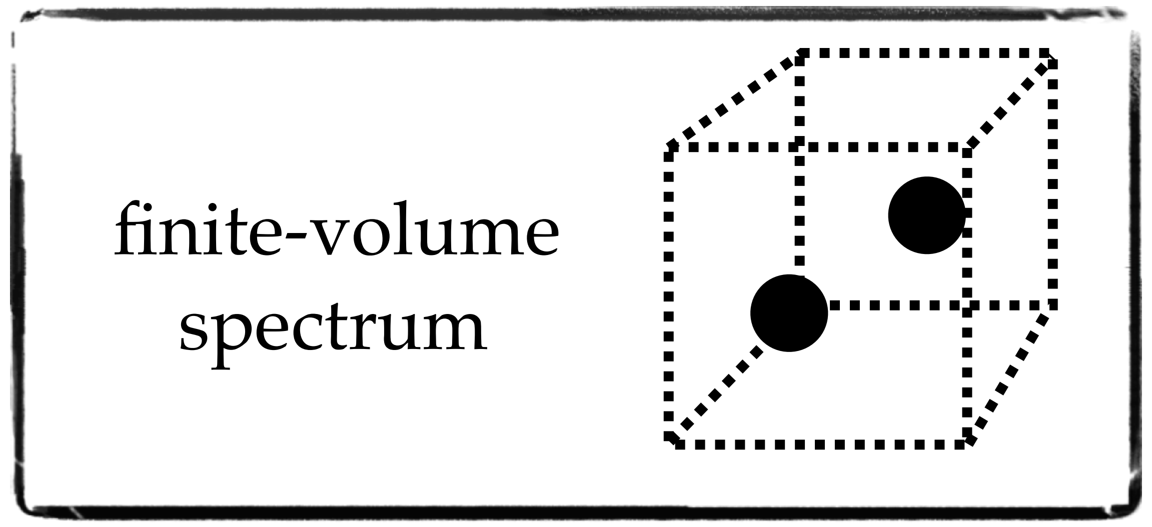
Two-hadron transitions $2 + \mathcal{J} \to 2$

$\mathcal{W}^{\mu} =$

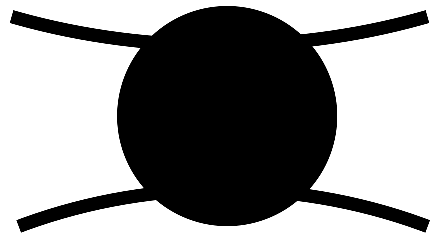


$P_f^2 \to s_B, \quad P_i^2 \to s_B$

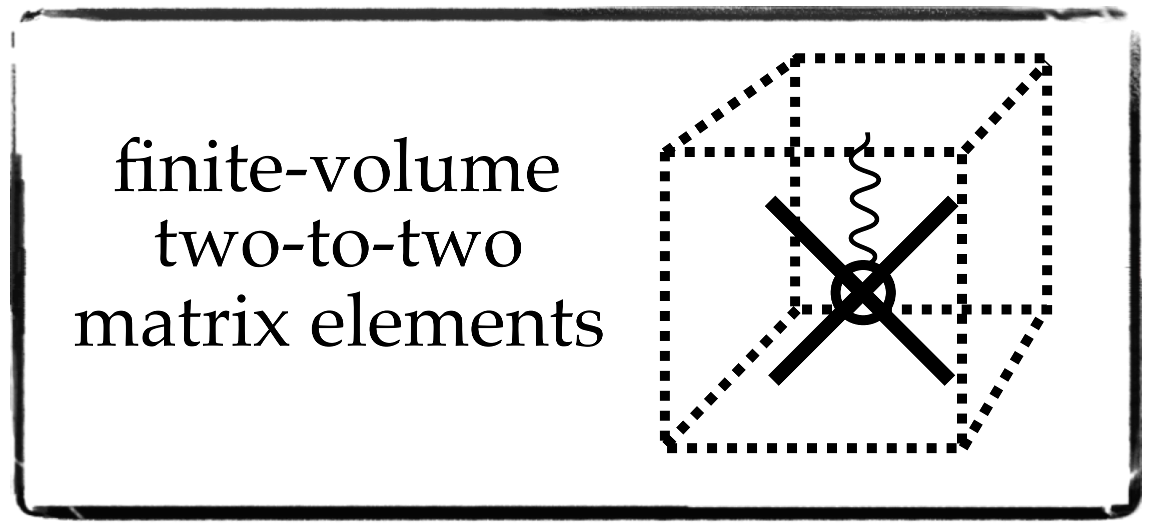




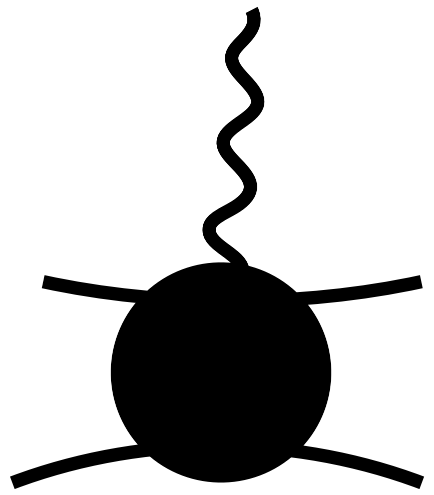
Lüscher QC



$\mathcal{M}$

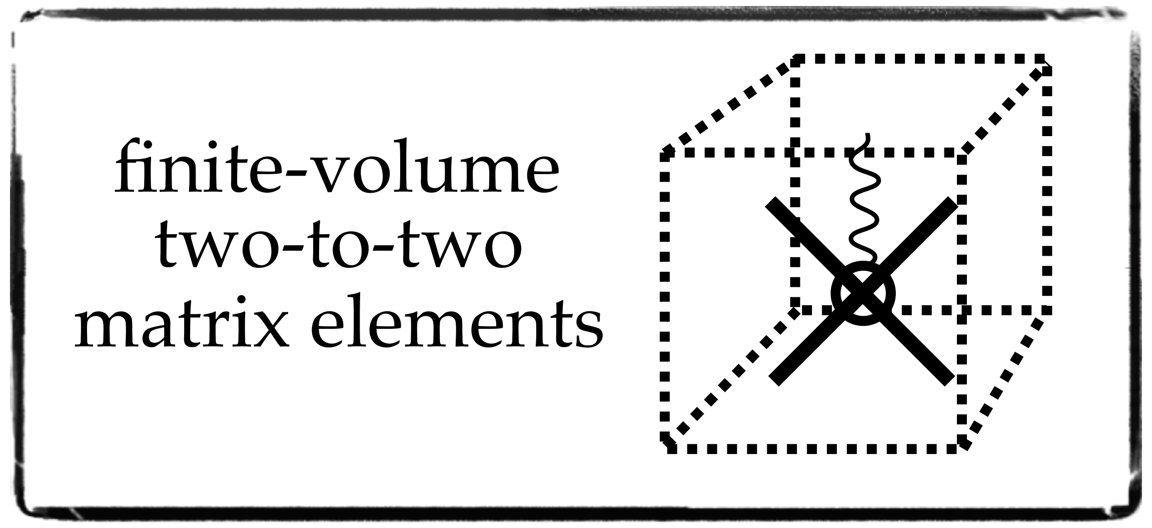


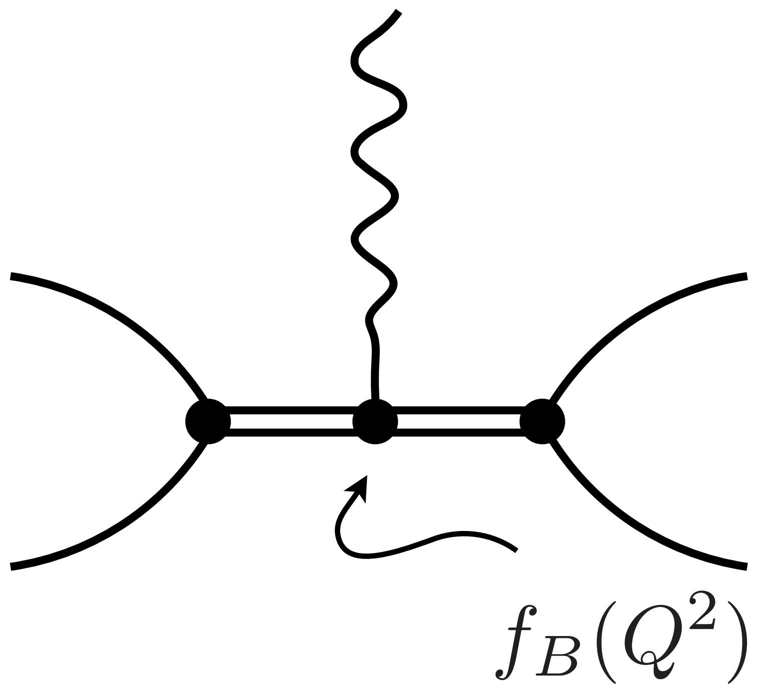
?

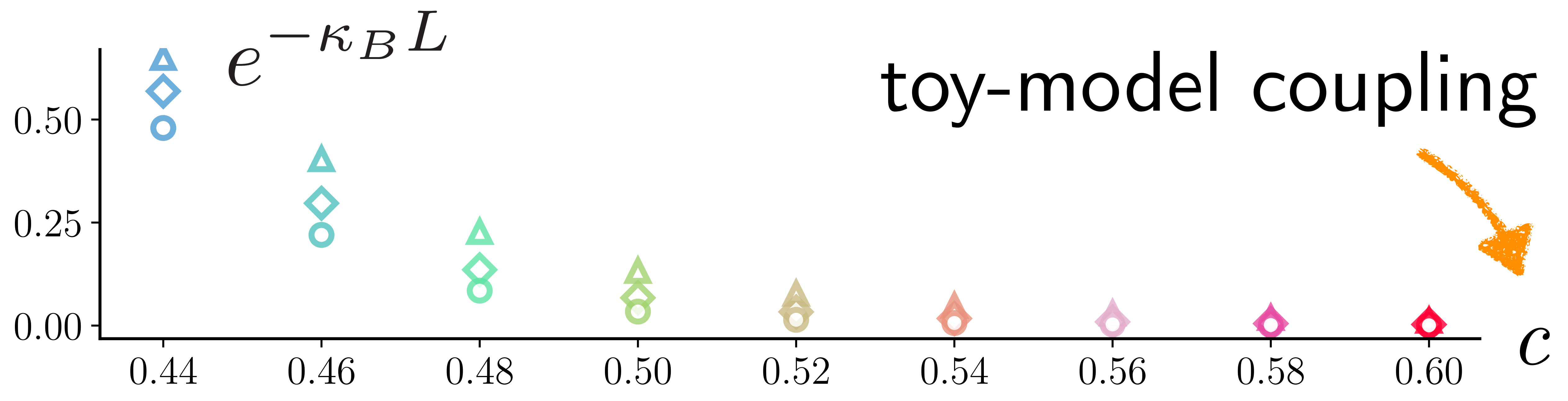


$\mathcal{W}^{\mu}$

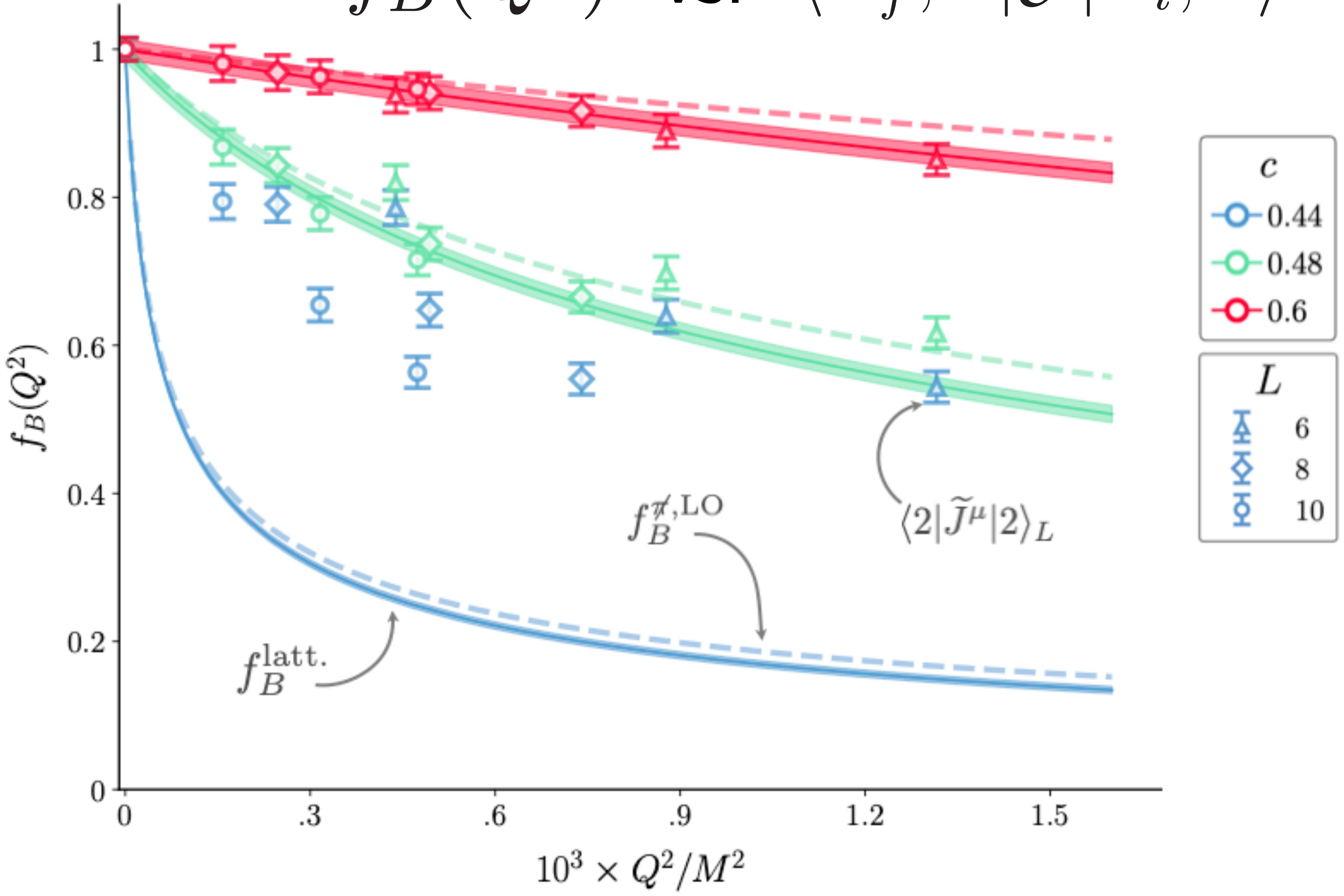
Bound state form factor



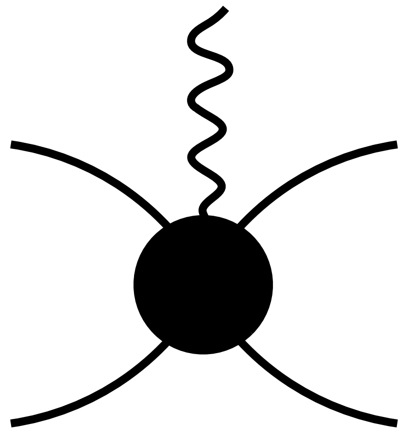




$f_B(Q^2)$ vs. $\langle E_f, L|\mathcal{J}|E_i, L\rangle$



- Matrix elements of two-body ground state
- The finite and infinite volume form factors match best for deepest bound state



Finite volume correction of $\mathcal{W}: 2 + \mathcal{J} \to 2$

1. Extract matrix elements from 3-point functions

$$\left\langle \mathcal{O}_{\pi\pi}(\Delta t)\mathcal{J}(t)\mathcal{O}^{\dagger}_{\pi\pi}(0)\right\rangle \sim \sum_{n,m} \langle E_n|\mathcal{J}(Q^2)|E_m\rangle \, e^{-E_n\Delta t-(E_m-E_n)t}$$

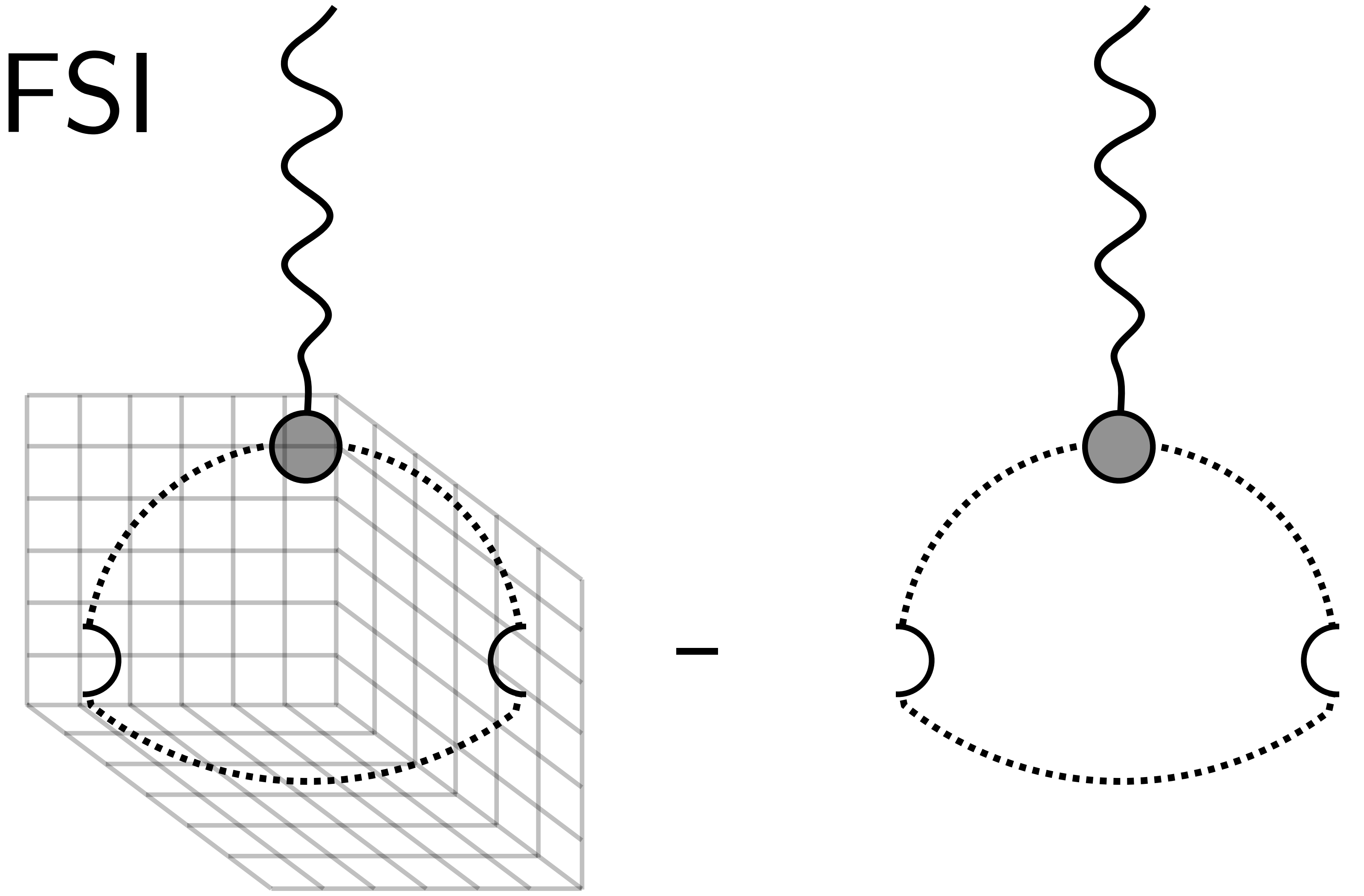
2. Correct for FV normalization/ add infinite volume ISI/FSI

$$\big|\langle E_n|\mathcal{J}(Q^2)|E_m\rangle\big|^2_L = \mathrm{tr}(\mathcal{R}_n\mathcal{W}_L(P_n,P_m)\mathcal{R}_m\mathcal{W}_L(P_m,P_n))$$

3. Correct for triangle function FV effects

$$\mathcal{W}_{\mathrm{df}} = \mathcal{W}_L - \mathcal{M}(E_n)(f\cdot G)\mathcal{M}(E_m)$$

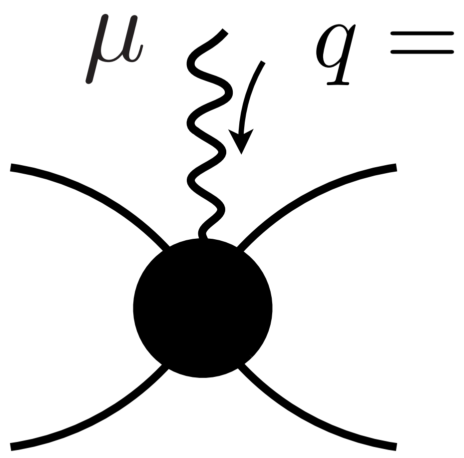
$f \cdot G \sim$



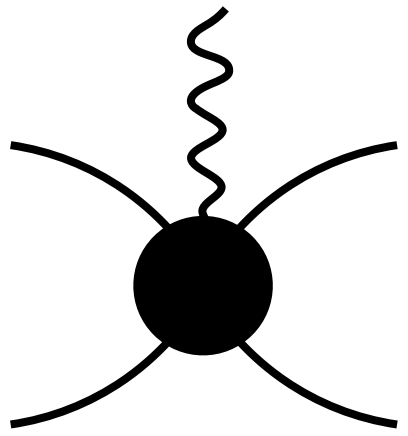
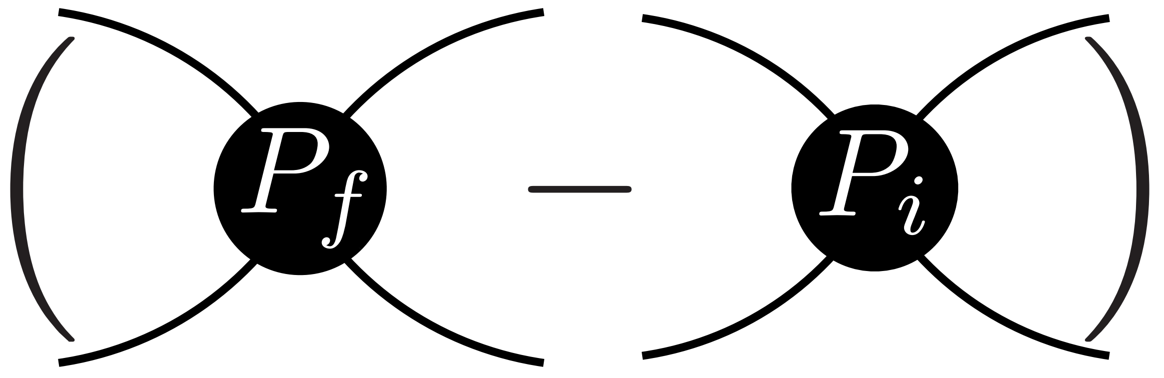
[Phys. Rev. D 94, 013008]
[Phys. Rev. D 100, 034511]

Constraints on the infinite volume transition

- Forward limit [Phys. Rev. D **100**, 114505]

Ward-Takashi Identity for conserved currents $q_\mu \times$  $= 0$

$+$

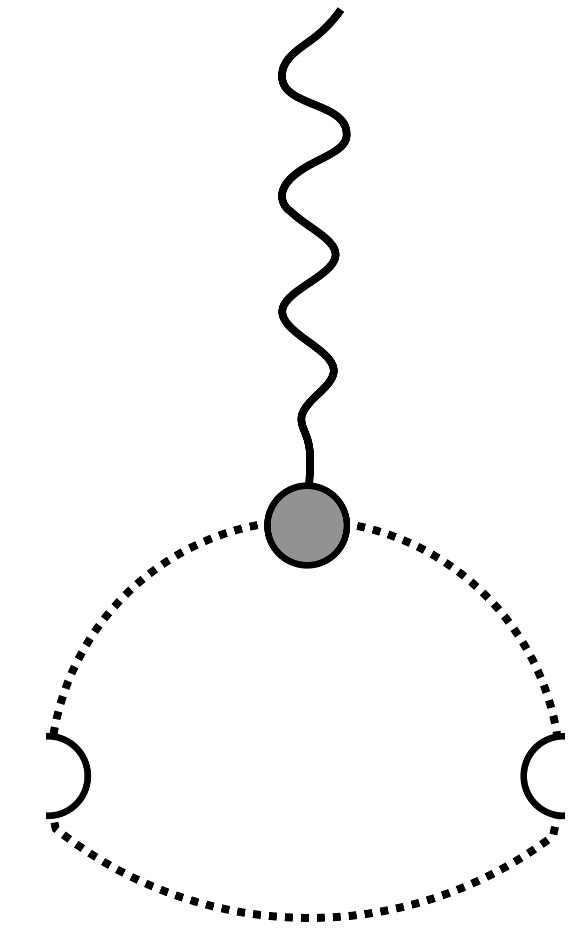
Low's Soft photon theorem  $\sim \frac{1}{q}$  $+\mathcal{W}^{\mu}_{\mathrm{df}}(P_f, P_i)$

$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P) = 2\mathrm{Q}_T P^{\mu}\frac{\partial}{\partial s}\mathcal{M}(s) = 2P^{\mu}\mathcal{W}_s(s)$$

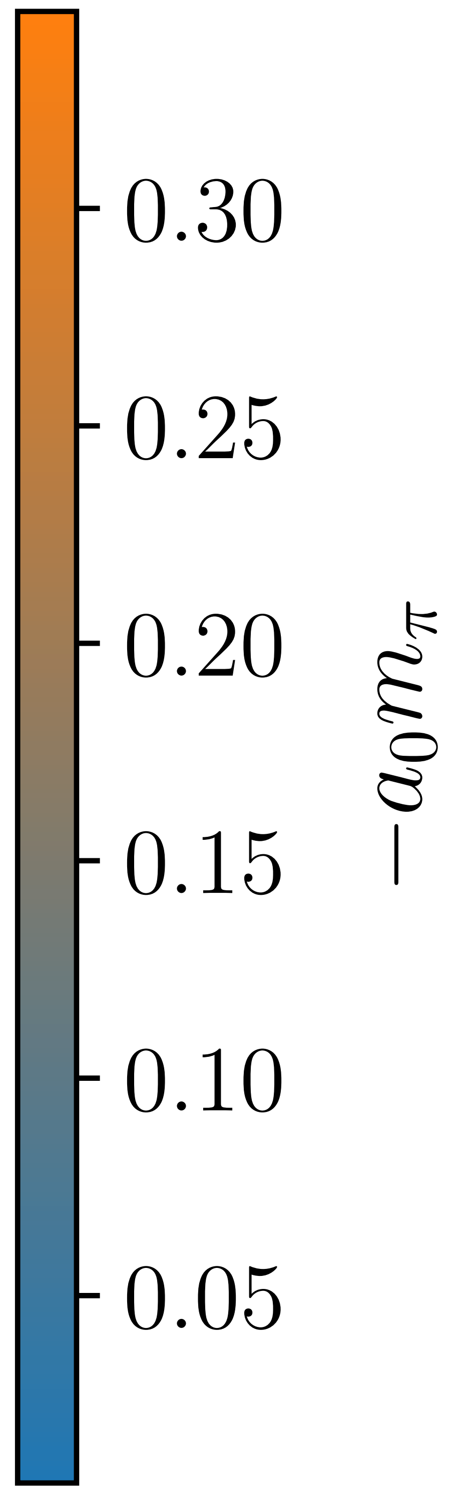
Total charge Lorentz decomposition

- Analytic decomposition [Phys. Rev. D **103**, 114512]

$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P) = \mathcal{M}(s)^2\left(\mathcal{A}^{\mu}(P,P) + \mathcal{G}^{\mu}(P,P)\right)$$

$\mathcal{G} \sim$ 

Matrix elements from FV formalism



$$p^\star \cot\delta = \frac{1}{a_0}$$

WT

$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P)$$

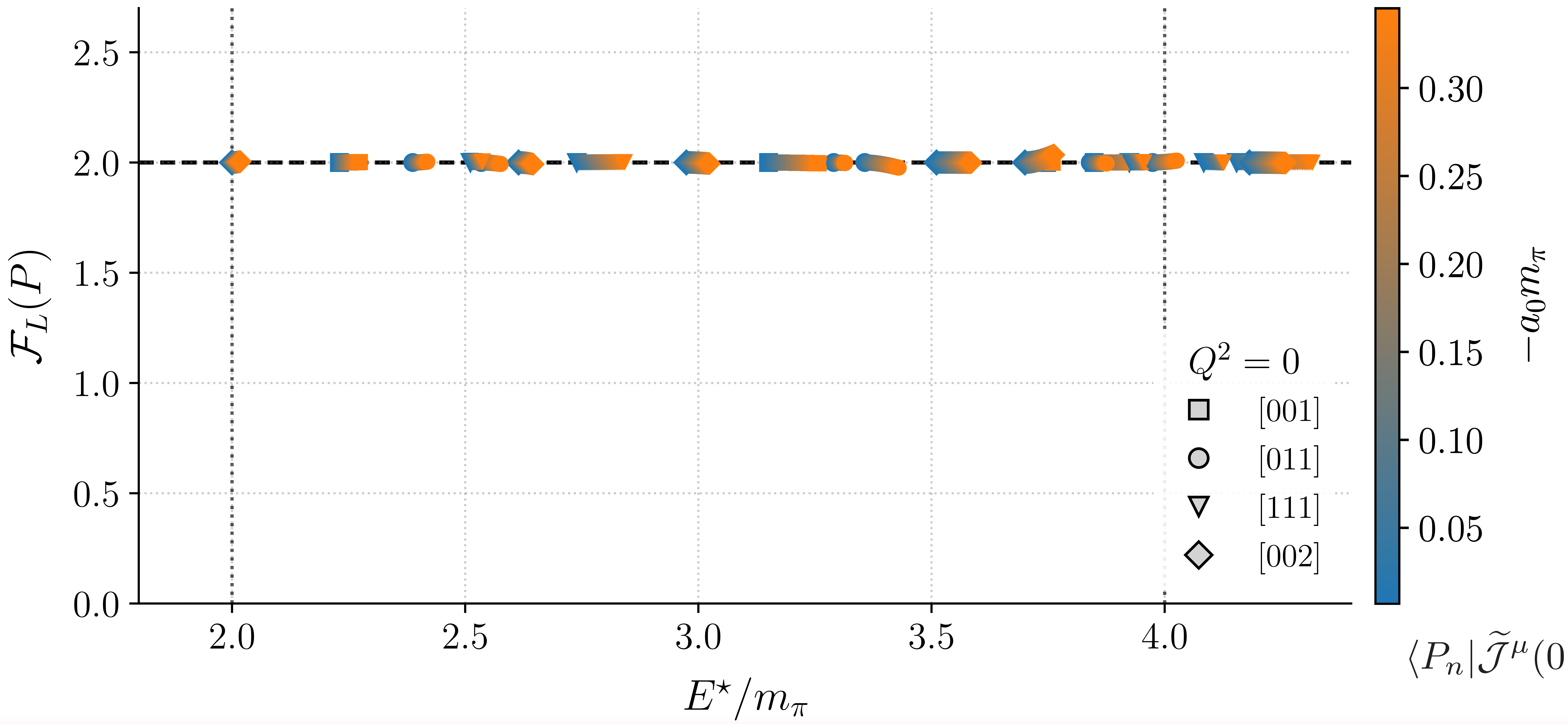
FV formalism

$$\langle P_n|\widetilde{\mathcal{J}}^{\mu}(0,\mathbf{q}=0)|P_n\rangle$$

$$\langle P_n|\widetilde{\mathcal{J}}^{\mu}(0,\mathbf{q}=0)|P_n\rangle = \frac{2P^\mu}{E_n}\frac{\mathcal{F}_L(P)}{2}$$

Matrix elements from FV formalism

Temporal component is just charge



$$p^\star \cot\delta = \frac{1}{a_0}$$

WT

$$\mathcal{W}^\mu_{\mathrm{df}}(P,P)$$

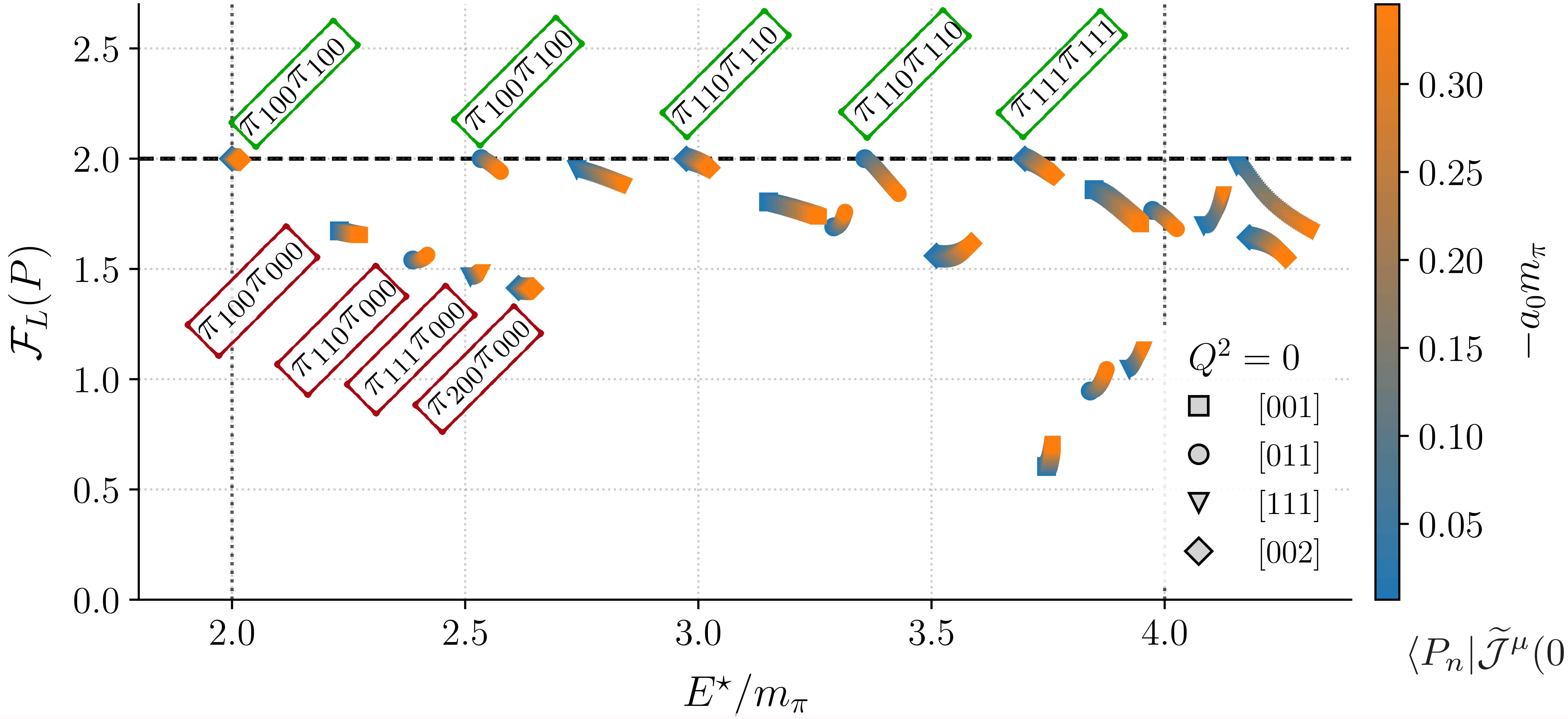
FV formalism

$$\langle P_n|\widetilde{\mathcal{J}}^\mu(0,\mathbf{q}=0)|P_n\rangle$$

$$\langle P_n|\widetilde{\mathcal{J}}^\mu(0,\mathbf{q}=0)|P_n\rangle = \frac{2P^\mu}{E_n}\frac{\mathcal{F}_L(P)}{2}$$

Matrix elements from FV formalism

Spatial component is current: non-trivial



Non-interacting limit

$$\langle \mathbf{P}, A_1 | \widetilde{\mathcal{J}}^i(0, \mathbf{q}=0) | \mathbf{P}, A_1 \rangle = \frac{1}{\nu} \sum_{\substack{R \in LG(\mathbf{P}) \\ \mathbf{p}_1 + \mathbf{p}_2 = \mathbf{P}}} \left(\frac{R\mathbf{p}_1}{\omega_{\pi 1}} + \frac{R\mathbf{p}_2}{\omega_{\pi 2}} \right)^i$$

Equal energy/momentum

$$= \frac{2\mathbf{P}^i}{E}$$

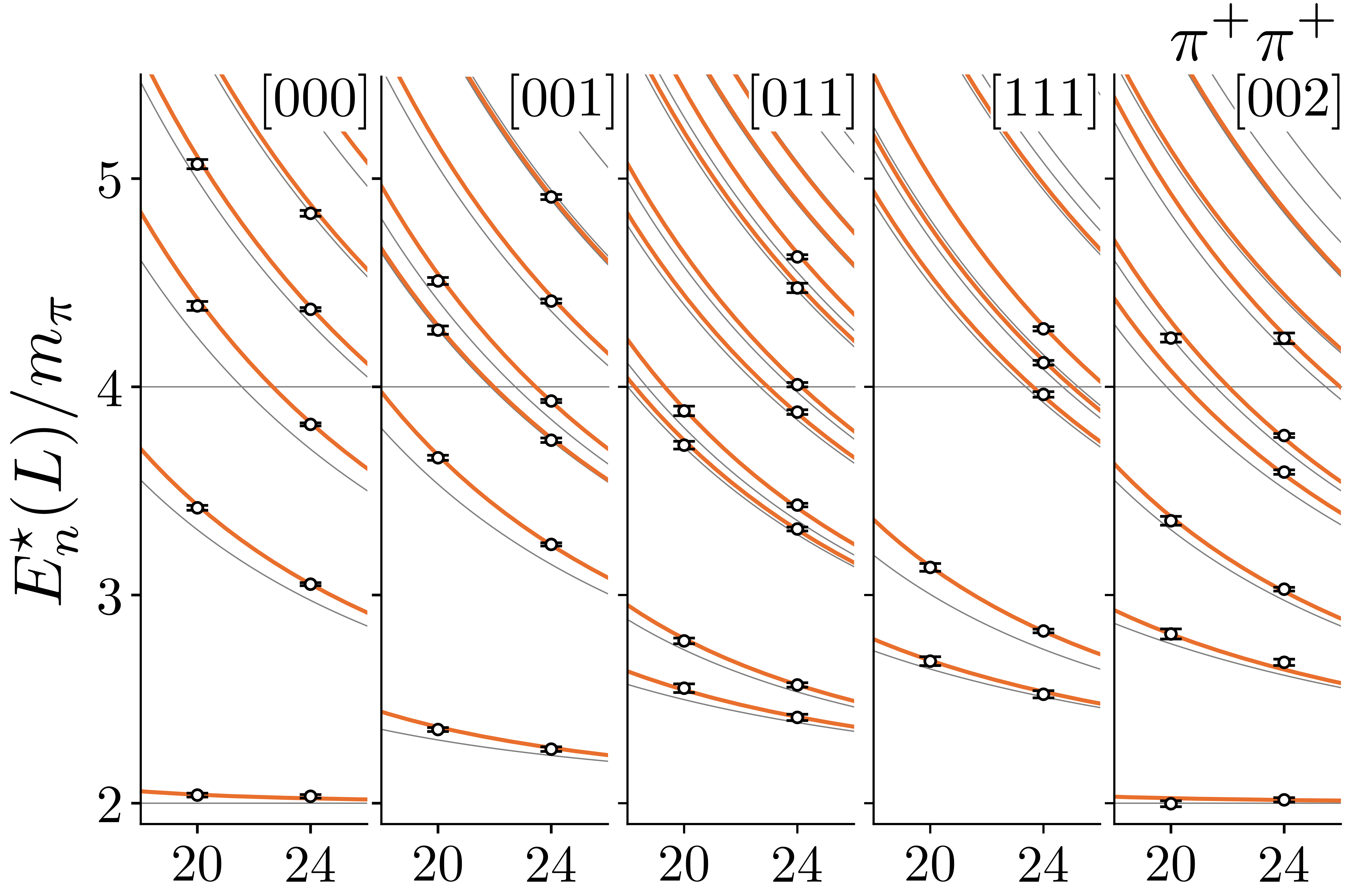
One vanishing momentum

$$= \frac{\omega_{\pi 1} + m_\pi}{2\omega_{\pi 1}} \frac{2\mathbf{P}^i}{E}$$

$$\langle P_n | \widetilde{\mathcal{J}}^\mu(0, \mathbf{q}=0) | P_n \rangle = \frac{2P^\mu}{E_n} \frac{\mathcal{F}_L(P)}{2}$$

Lattice details

m_π	L/a_s	$m_\pi L$	a_s/a_t	a_t
396 MeV	24	5.71	3.444	1.7×10^{-4} MeV^{-1}



[Phys. Rev. Lett. **126**, 012001]

GEVP with 2π ops

- Spectrum $E_n^\star$
- Optimized ops Ω_n

Single pion ops with distillation and multi-derivative construction à la hadspec.

Matrix element extraction

Average 3pt over:

- Momentum direction P
- Current irrep row
- Source-sink reversal

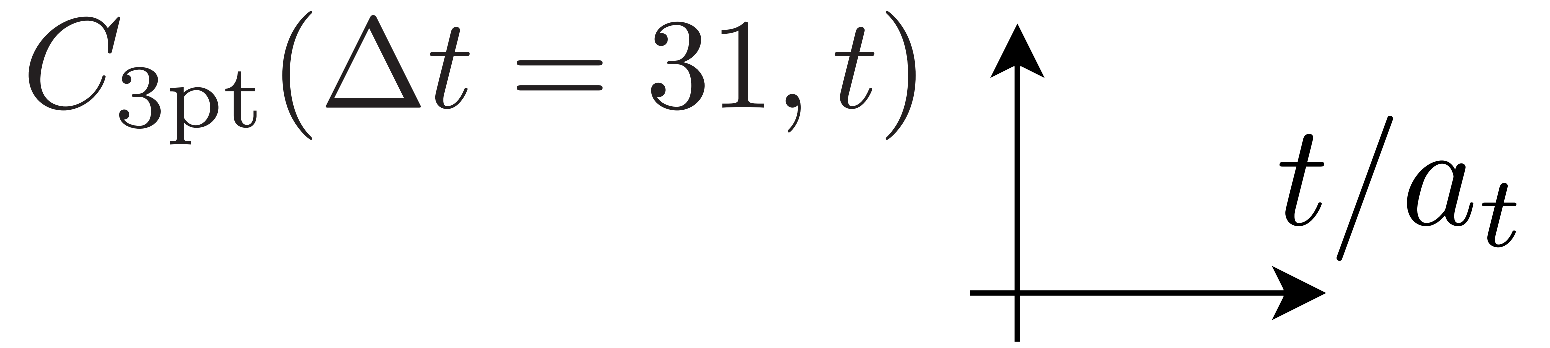
$$C_{3\text{pt}}(\Delta t, t) = \langle \Omega_{\mathfrak{n}}(\Delta t)\, \mathcal{J}(t)\, \Omega^\dagger_{\mathfrak{n}}(0) \rangle$$
$$= \sum_{n,m} e^{-(E_m - E_n)t} Z_{\mathfrak{n}}^{(n)} Z_{\mathfrak{n}}^{(m)*} e^{-E_n \Delta t} \mathfrak{M}_{n,m}$$

Two-step fitting scheme:

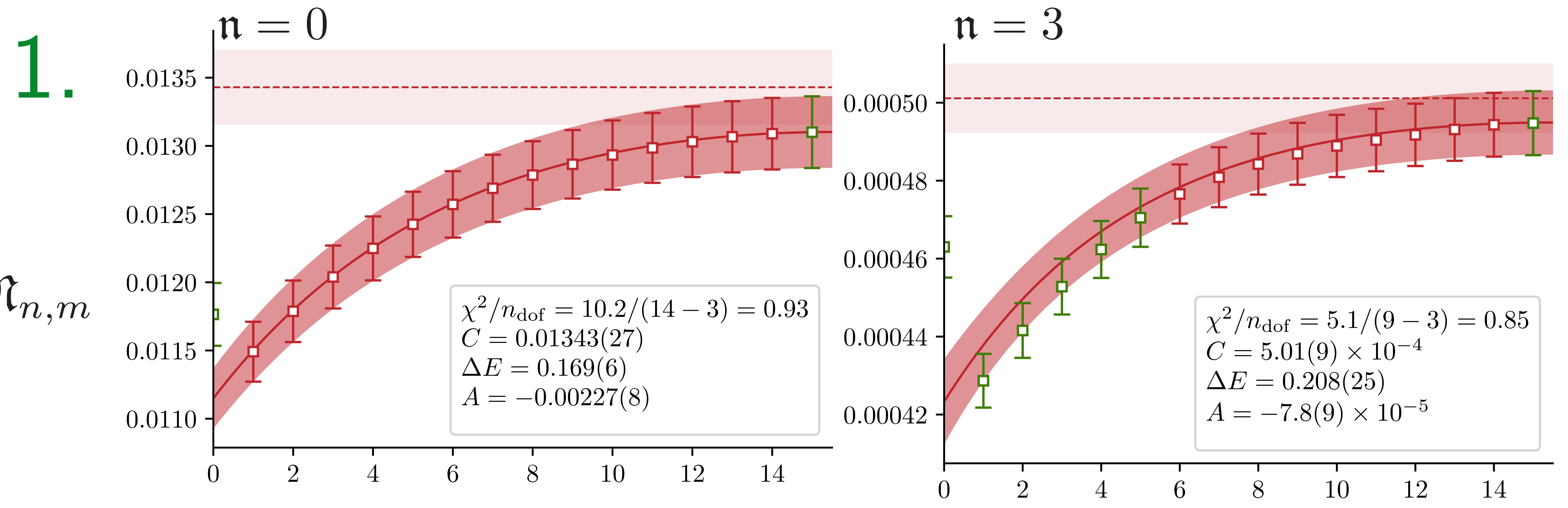
1. Fit data with fixed Δt

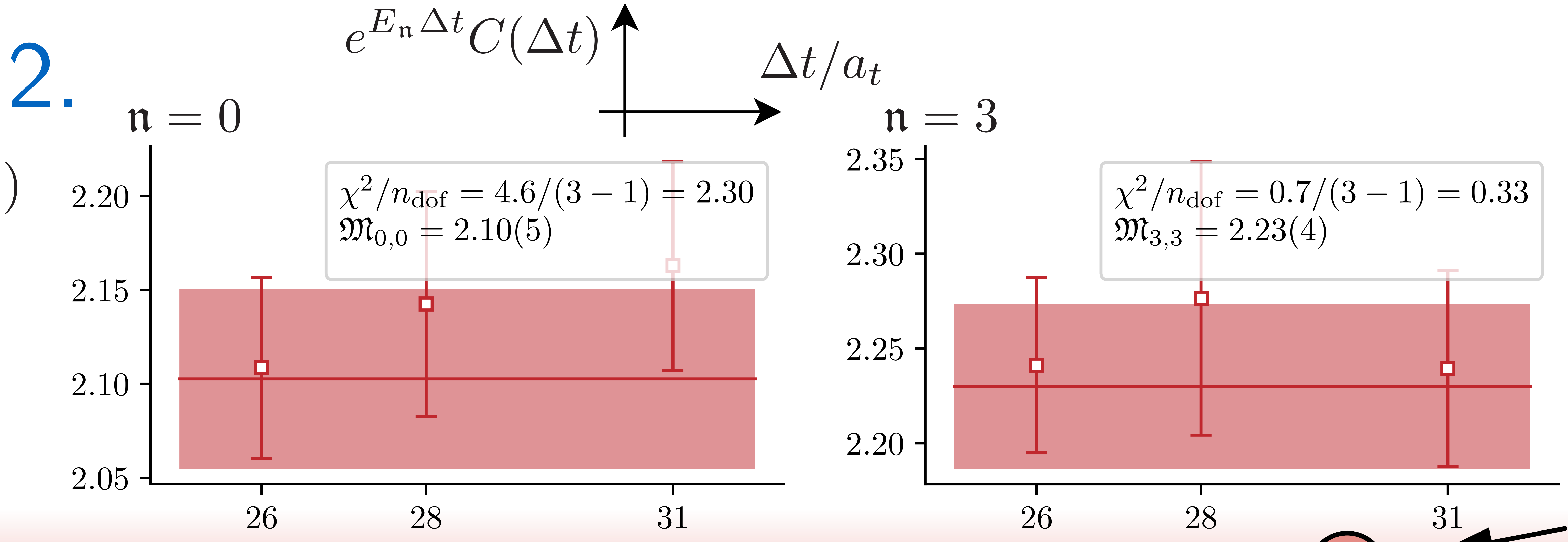
$$C_{3\text{pt}}(\Delta t, t) = C(\Delta t) + A(e^{-\Delta E\, t} + e^{-\Delta E(\Delta t - t)})$$

2. Use spectrum to remove lead source-sink separation dependence



$P = [100]$





Spectrum and Lellouch-Luscher factor

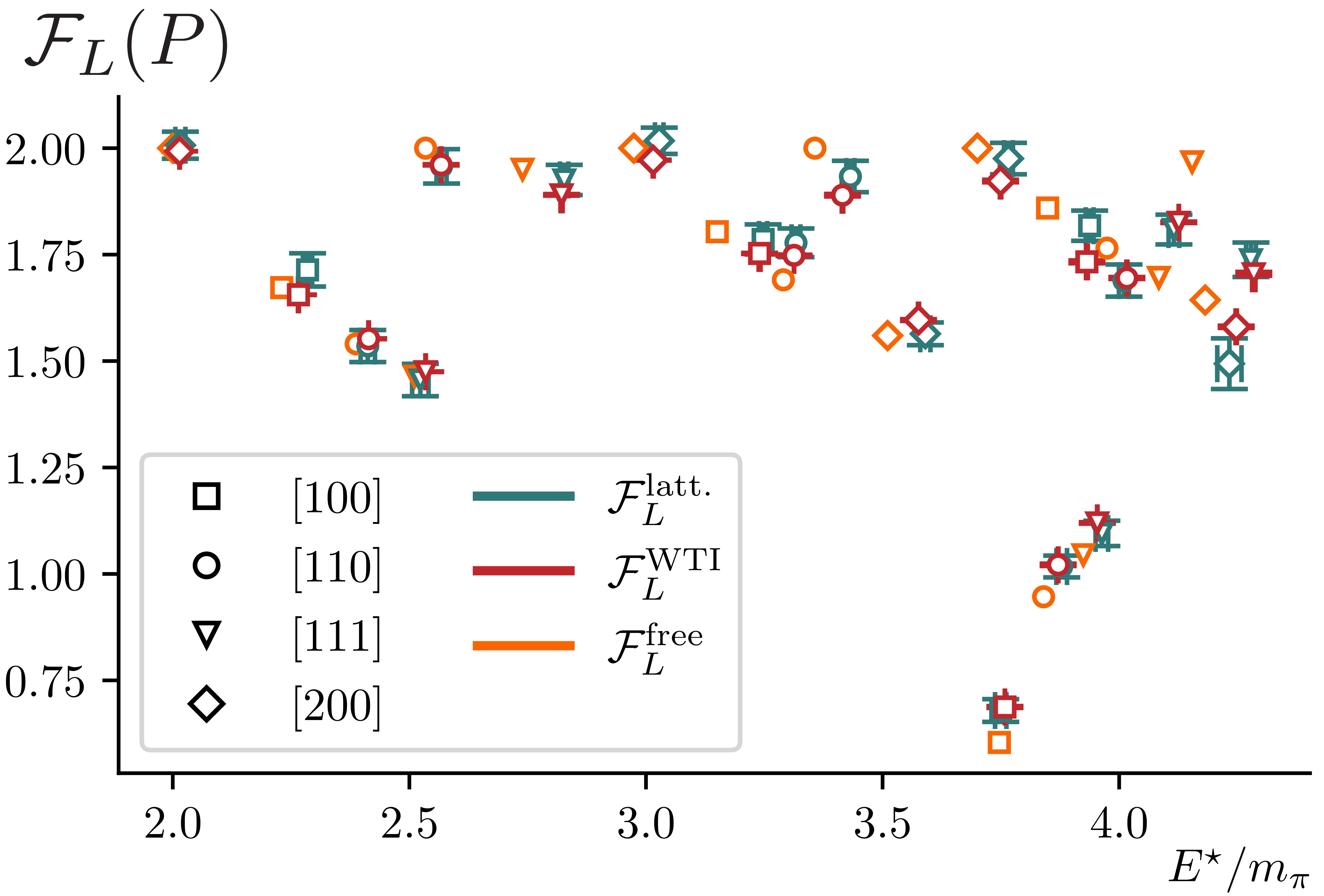
S-wave

LL factor $\widetilde{\mathcal{R}}^{1/2} = \left(-2E_n^\star \lim_{E\to E_n^\star} \frac{E - E_n^\star}{\mathcal{M}^{-1}(E) + F^{\mathbf{P}}(E,L)}\right)^{1/2}$

Lüscher QC $\mathcal{M}^{-1}(E) + F^{\mathbf{P}}(E,L) = 0$

$$p^\star \cot\delta = \frac{1}{a}$$

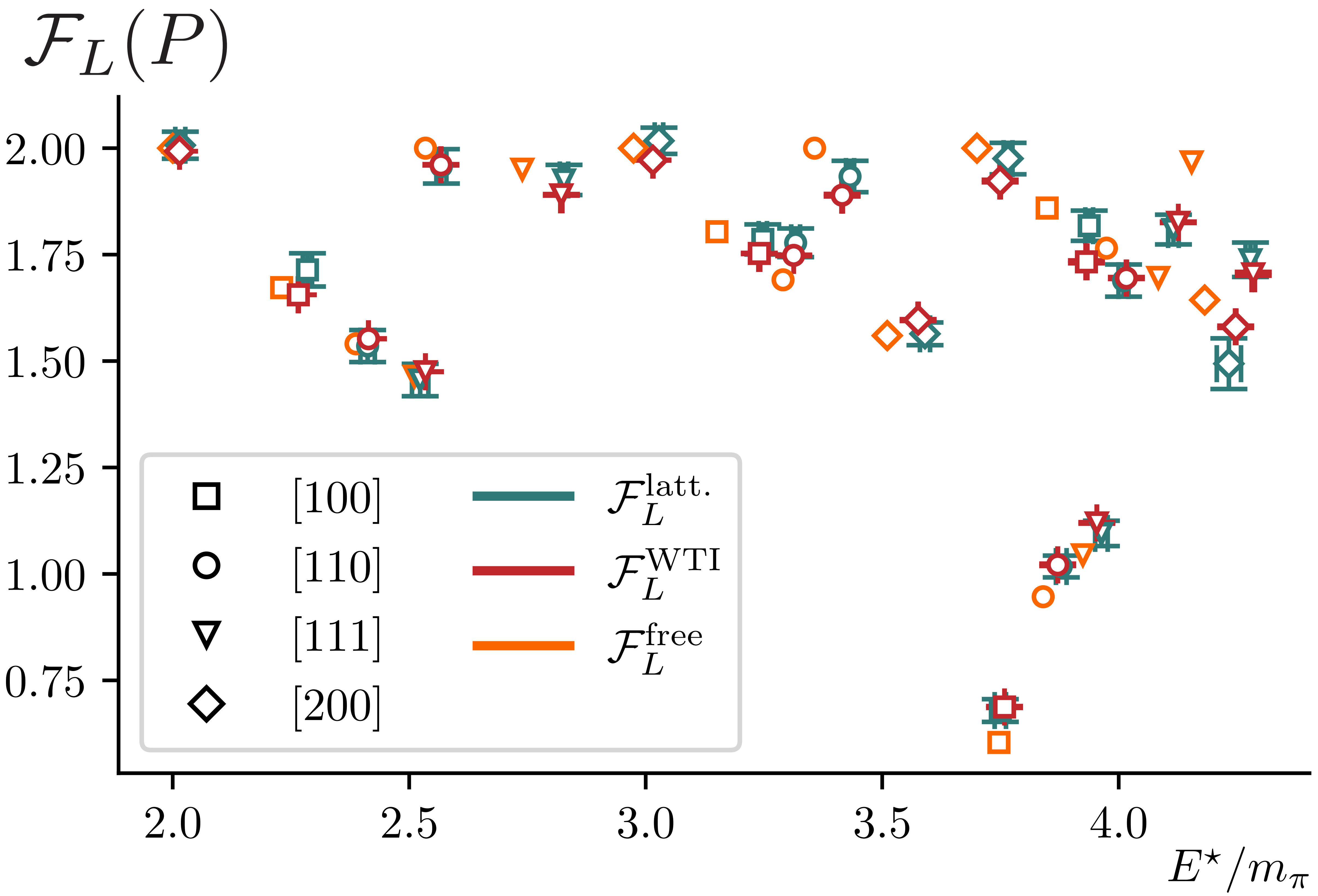
LQCD vs Ward-Takahashi



$$\chi^2 = (\mathcal{F}_L^{\mathrm{latt}} - \mathcal{F}_L^{\mathrm{WTI}}) \cdot \mathrm{Cov}_{\mathrm{latt}}^{-1} \cdot (\mathcal{F}_L^{\mathrm{latt}} - \mathcal{F}_L^{\mathrm{WTI}}) = 44.6$$

20 matrix elements (no fit)

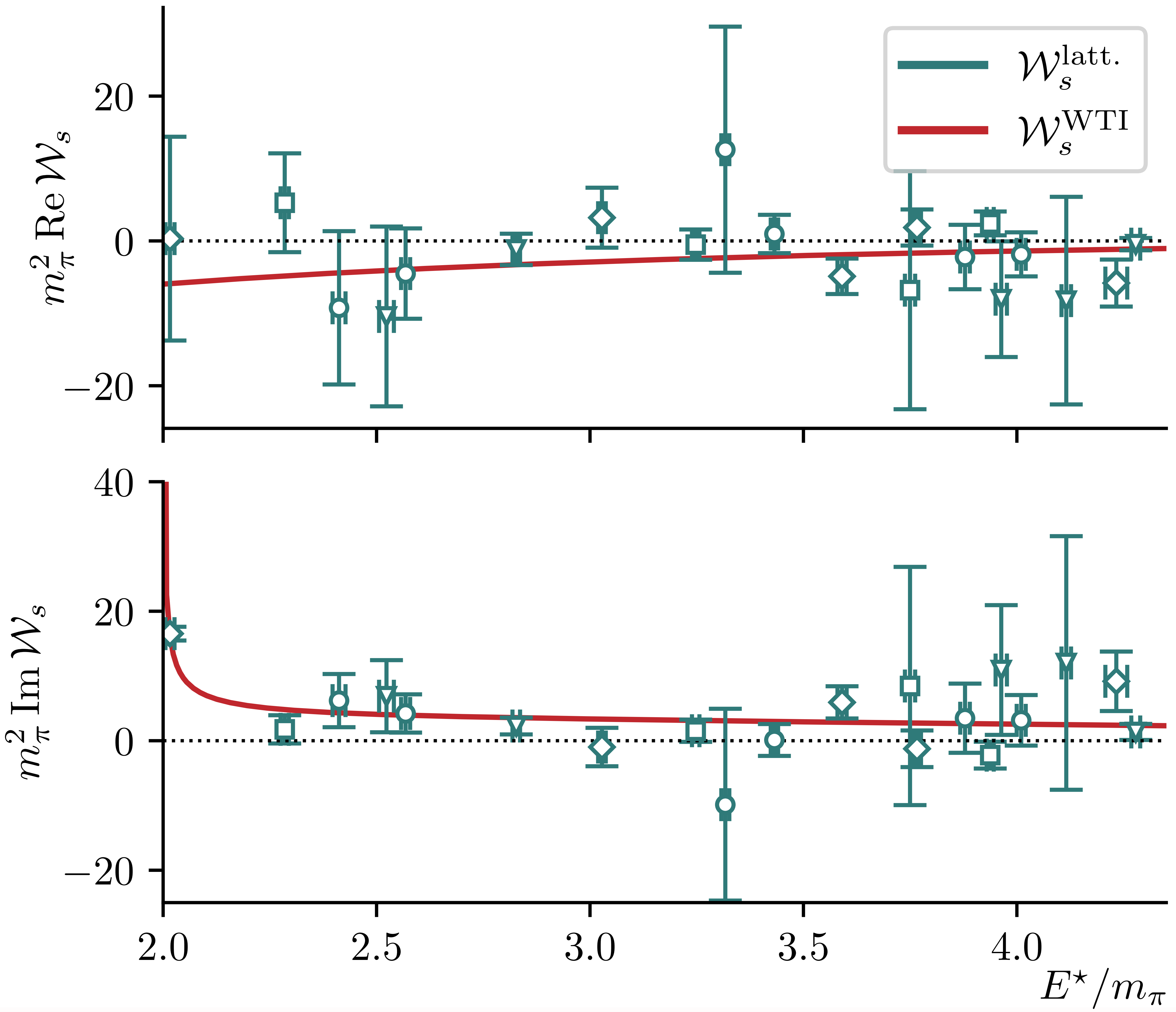
LQCD vs Ward-Takahashi



$$\chi^2 = (\mathcal{F}_L^{\rm latt} - \mathcal{F}_L^{\rm WTI}) \cdot {\rm Cov}_{\rm latt}^{-1} \cdot (\mathcal{F}_L^{\rm latt} - \mathcal{F}_L^{\rm WTI}) = 44.6$$

20 matrix elements (no fit)

$$\mathcal{W}^\mu_{\rm df}(P,P) = 2P^\mu \mathcal{W}_s(s)$$



Summary and outlook

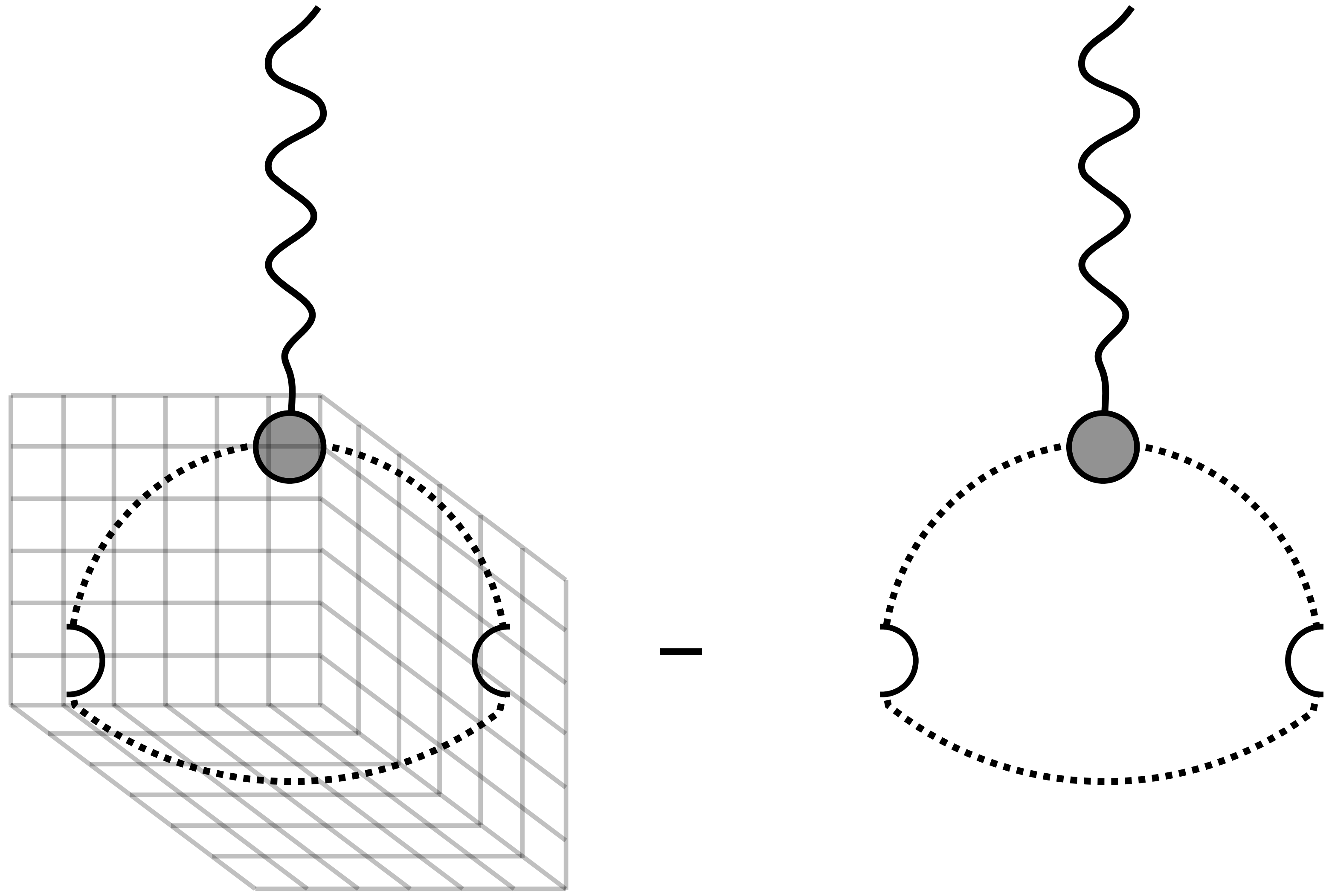
- **Two-pion transition calculation** mediated by a photon in the forward limit.
- **GEVP and operator construction** techniques allow us to extract the matrix elements across the entire elastic range.
- **Finite volume correction** of matrix elements appears consistent with the Ward-Takahashi constraint on the amplitude.
- **Structure of states near two-particle thresholds** can be accessed with these kind of amplitudes, as shown in the lattice toy-model.
- **Future work:** inclusion of LHC will be needed for tetraquarks, but this technique is immediately applicable to many other channels.

Thanks for your attention! Questions?

Back up slides

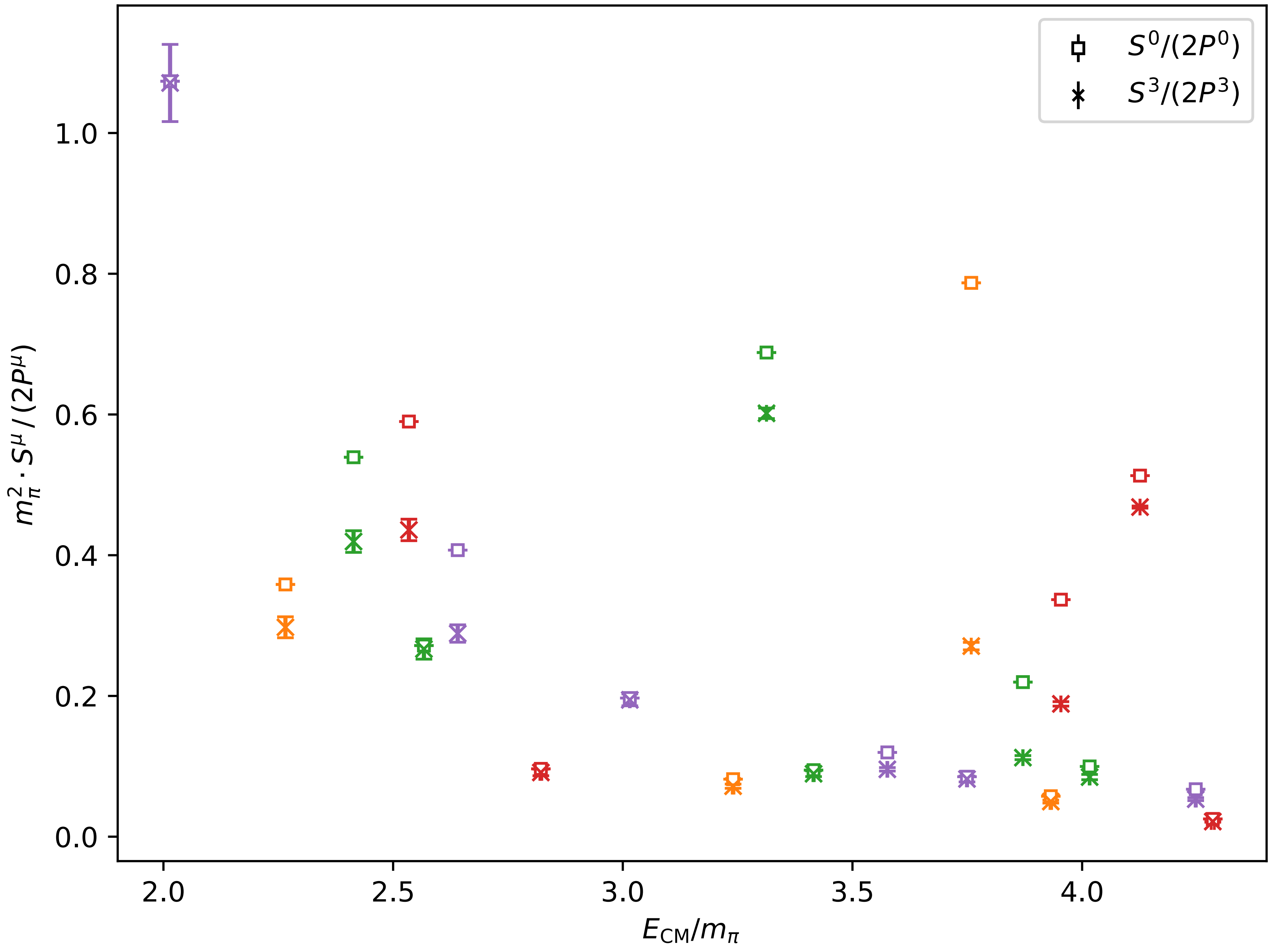
G-function

$f \cdot G \sim$

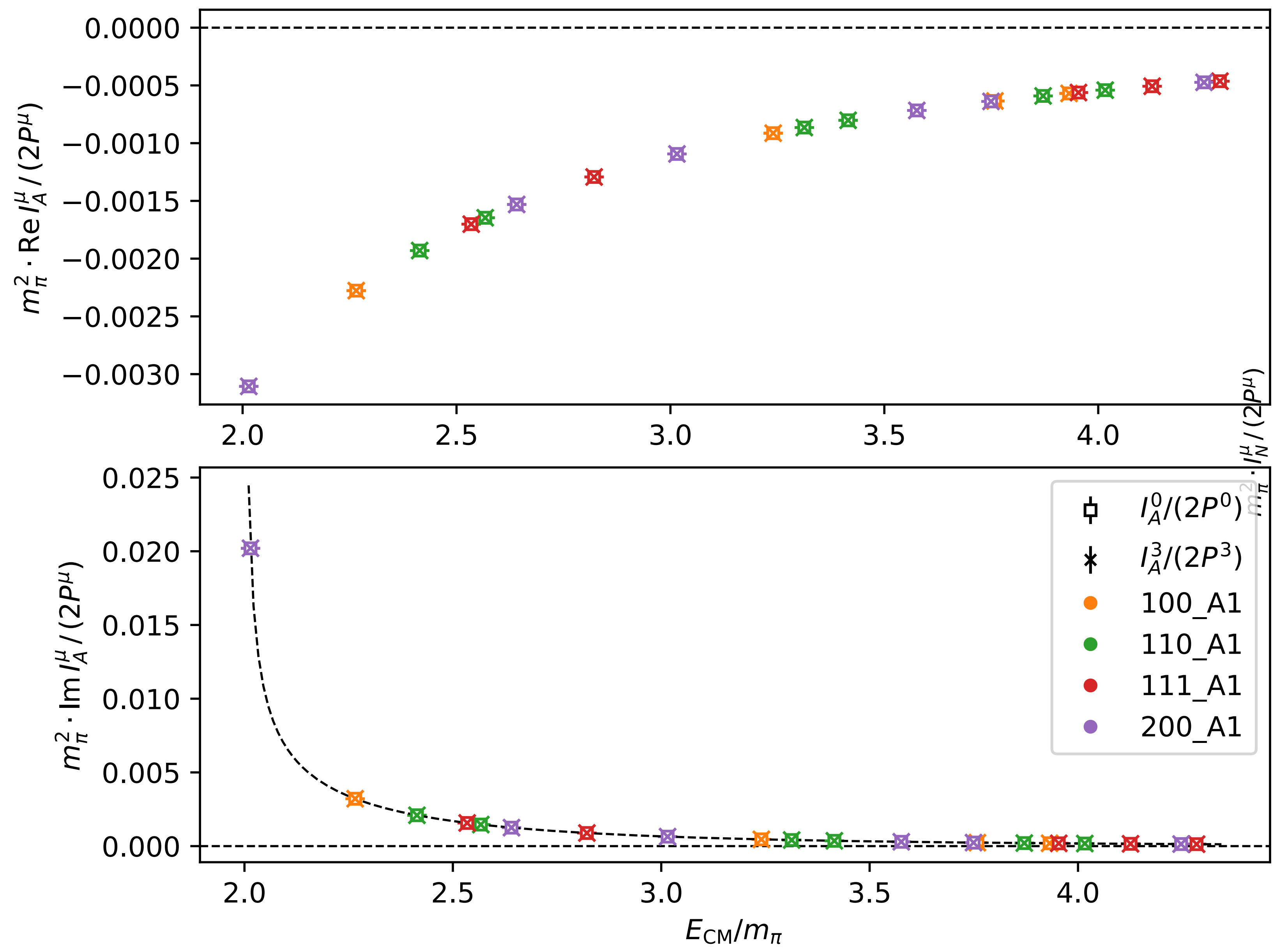


The only difference in the current zero and spatial components is the additive Lorentz non-covariant triangle loop sum

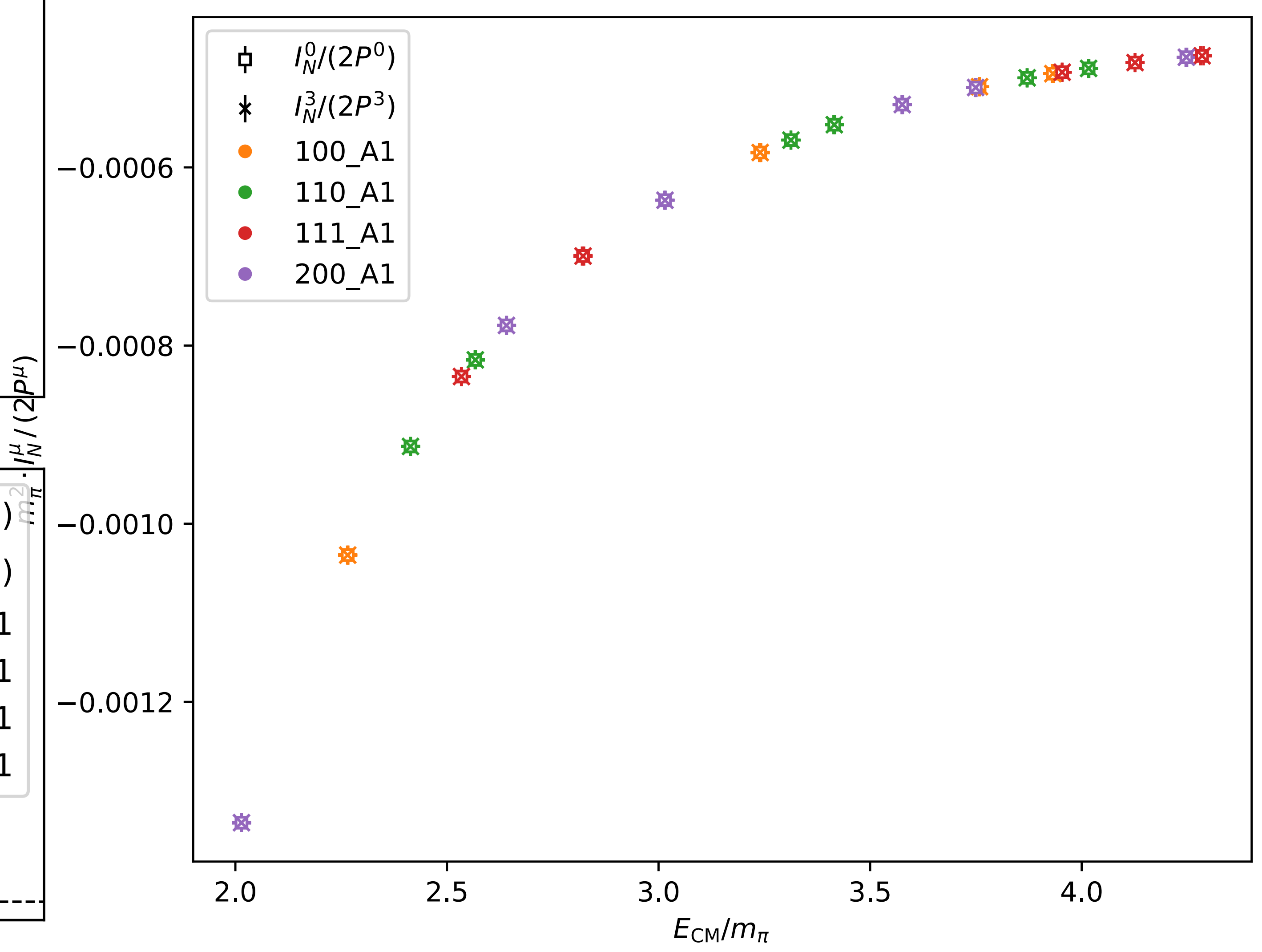
Sum



Triangle loop integral

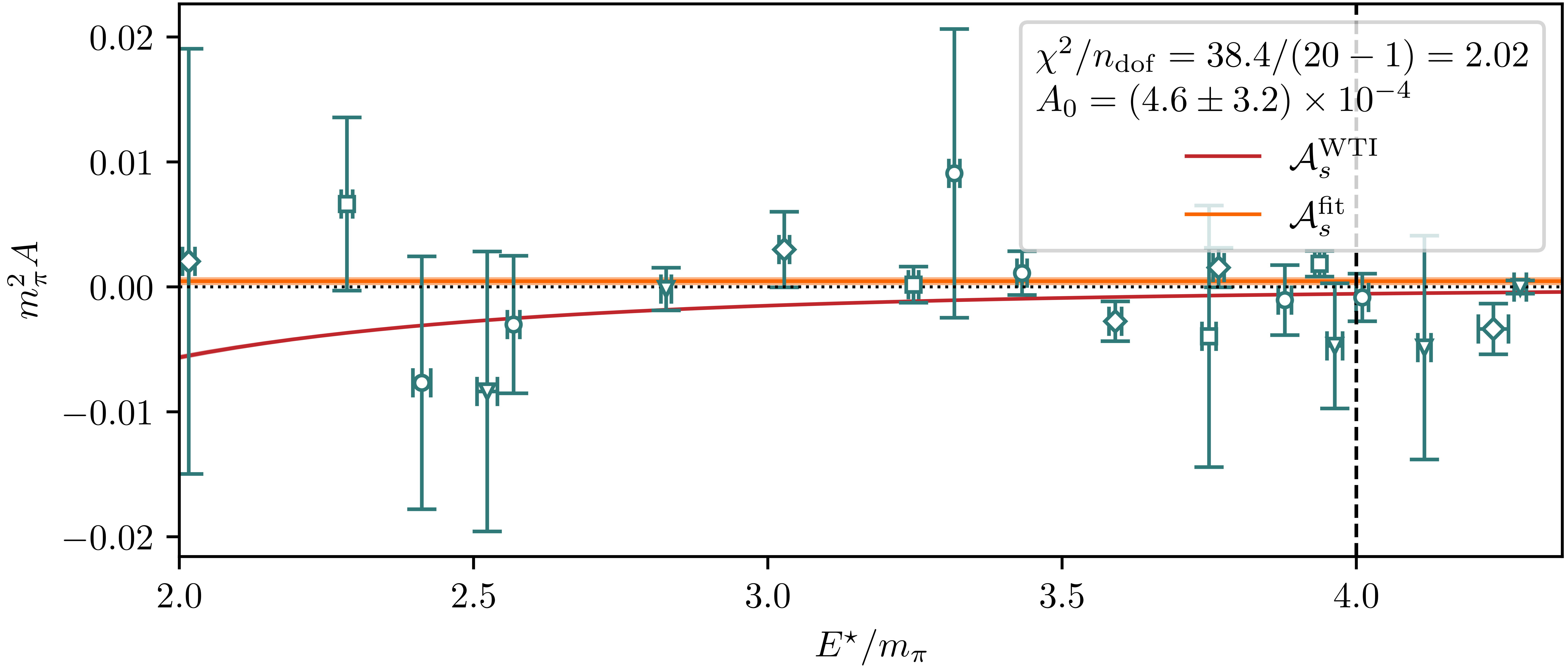


Scheme dependent smooth piece



Smooth function decomposition

$$\mathcal{W}^{\mu}_{\mathrm{df}}(P,P) = \mathcal{M}(s)^2 \left(\mathcal{A}^{\mu}(P,P) + \mathcal{G}^{\mu}(P,P)\right)$$



Similar to the K-matrix in scattering, the A function is suitable to parametrize and fit to lattice data.