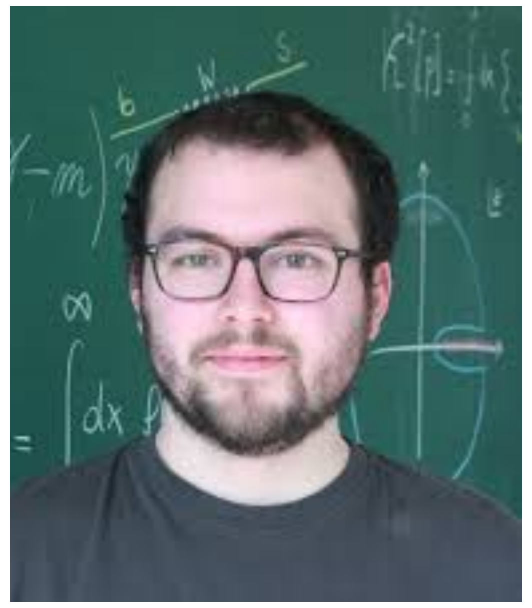
Update on hadronic *D* decays at the SU(3) flavour symmetric point

Maxwell T. Hansen

July 28th, 2026

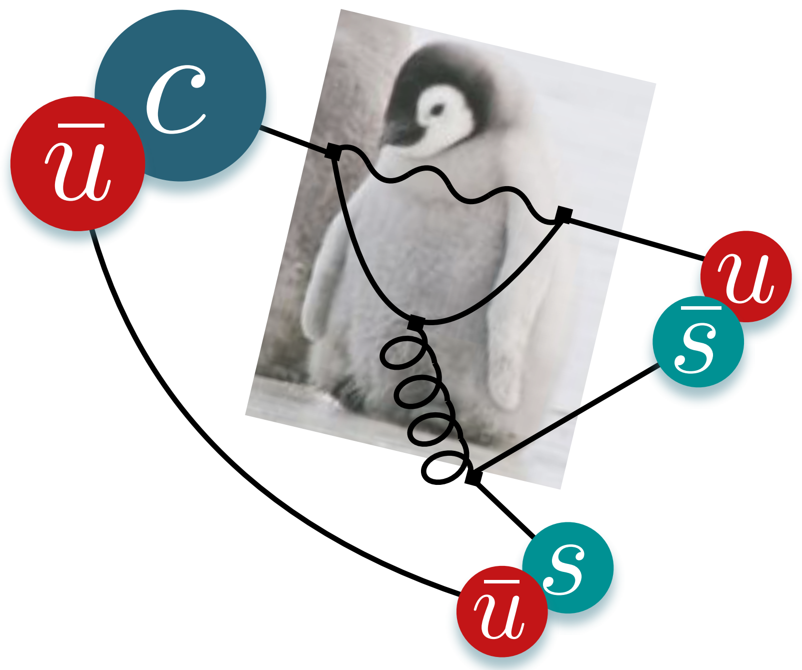


ongoing work with ***F Joswig***, F Erben, M Di Carlo, ***M Black***, M Hansen, N Pitanga Lachini, R Mukherjee, S Paul, A Portelli

THE UNIVERSITY *of* EDINBURGH

Motivation

- SM is well known to have CPV, but not enough for baryogenesis!



- 2019: LHCb observed CP violation in *hadronic charm decays* $D \to \pi\pi, K\overline{K}$

$$\Delta A_{\mathrm{CP}} = A_{\mathrm{CP}}(K^-K^+) - A_{\mathrm{CP}}(\pi^-\pi^+) \underset{\text{LHCb}}{=} (-15.4 \pm 2.9) \times 10^{-4}$$

New physics, or non-perturbative QCD?

• LHCb (PRL, 2019) •

- Lattice QCD can provide the needed SM amplitudes

- This work: pilot lattice calculation of $D \to K\pi$ at the $SU(3)_F$ point

Relevance of $SU(3)_F$ calculation

- $SU(3)_F$ / ***U-spin:*** organise abundant $D \to PP, PV$ data to "reduced" or "topological amplitudes"

Decay	$SU(3)_F$ Amplitude
$D^0 \to K^- \pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{5} \left(\sqrt{2} A_{27} + \sqrt{2} A_8 - \sqrt{5} C_8 \right)$
$D^0 \to \overline{K}^0 \pi^0$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{10} \left(3 A_{27} - 2 A_8 + \sqrt{10} C_8 \right)$
$D^0 \to \overline{K}^0 \eta$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(3\sqrt{2} A_{27} + \sqrt{2}(\sqrt{5}-2) A_8 - \sqrt{5}(\sqrt{5}-2) C_8 \right)$
$D^0 \to \overline{K}^0 \eta'$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{30\sqrt{3}} \left(3 A_{27} - 2(1+4\sqrt{5}) A_8 + \sqrt{10}(1+4\sqrt{5}) C_8 \right)$
$D^+ \to \overline{K}^0 \pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{\sqrt{2}} A_{27}$
$D_s^+ \to \overline{K}^0 K^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{5} \left(\sqrt{2} A_{27} + \sqrt{2} A_8 + \sqrt{5} C_8 \right)$
$D_s^+ \to \pi^+ \eta$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(6\sqrt{2} A_{27} - \sqrt{2}(4+\sqrt{5}) A_8 - \sqrt{5}(4+\sqrt{5}) C_8 \right)$
$D_s^+ \to \pi^+ \eta'$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{15\sqrt{3}} \left(3 A_{27} + 2(2\sqrt{5}-1) A_8 + \sqrt{10}(2\sqrt{5}-1) C_8 \right)$

• Bhattacharya et al. (2021) •

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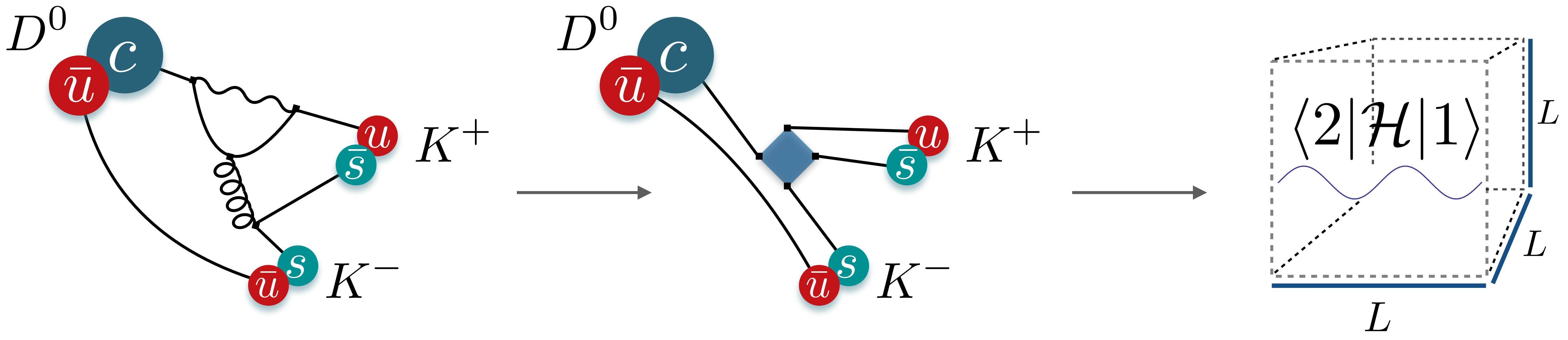
• Bhattacharya et al. (2021) •

- Fits to experiment require symmetry breaking, typically at the expected 20 - 30 % level

Lattice QCD can provide first direct determinations at $SU(3)_F$ while building technology/experience towards a calculation at the physical point

Basic approach

- Integrate out electroweak physics → basis of four-quark operators



- Finite-volume formalism required to relate these to amplitudes

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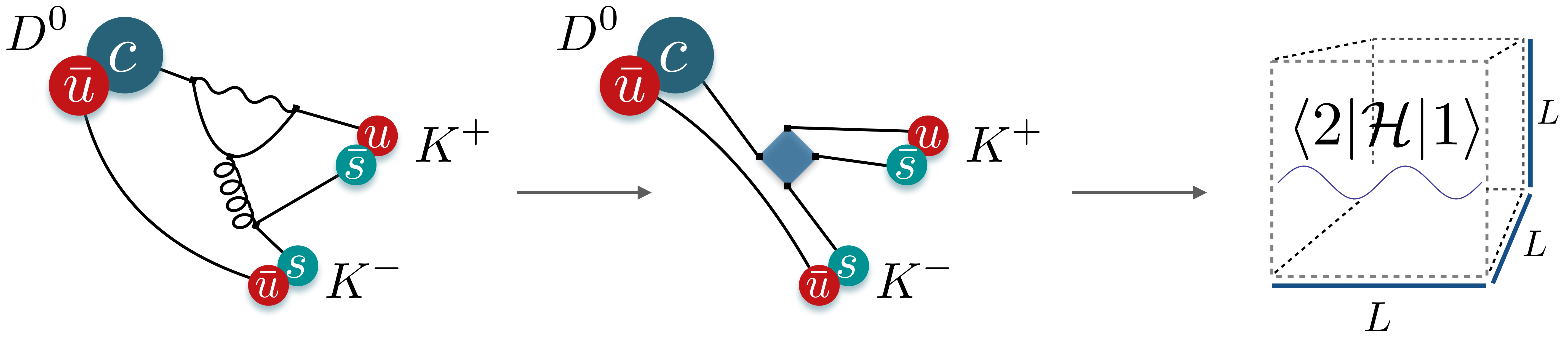

- Finite-volume formalism required to relate these to amplitudes
- Large body of related calculations:

K → ππ, ΔI = 3/2	RBC-UKQCD (Blum et al.) (2015)
K → ππ, ΔI = 1/2	RBC-UKQCD (Blum et al.) (2015, 2020)
π γ* → ππ, ρ → πγ*	Hadspec (Briceño, Dudek, Edwards, Shultz, Thomas, Wilson) (2015, 2016)
π γ → ππ, ρ → πγ	Alexandrou, Leskovec, Meinel, Negele, Paul, Petschlies, Pochinsky, Rendon, Syritsyn (2018)
K γ* → Kπ, K* → Kγ	Hadspec (Dudek, Edwards, Radhakrishnan) (2022)
B → ππ ℓν, B → ρ ℓν	Leskovec, Meinel, Petschlies, Negele, Paul, Pochinsky (2025)
D → Kπ ℓν, D → K* ℓν	HPQCD (Harrison et al.) (*ongoing*)
B → Kπ $\ell^+\ell^-$, B → K* $\ell^+\ell^-$	Erben, Black, Boyle, Di Carlo, Gülpers, Hansen, Lachini, Mukherjee, Portelli, Tsang (*ongoing*)

Roadmap of the calculation

Finite-volume matrix elements

$$M_{n,L} = \langle n, L|\mathcal{H}_W|D, L\rangle$$

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$$M_{n,L} = \langle n, L|\mathcal{H}_W|D, L\rangle = \sum_i C^{\overline{\mathrm{MS}}}_{\mathsf{wc},i} \lim_{a\to 0} Z_i^{\overline{\mathrm{MS}}} \lim_{\Delta t\to\infty} \langle 0|\Omega_n^\dagger(\Delta t/2)\mathcal{Q}_i(0)D(\Delta t/2)|0\rangle_L$$

Non-perturbative renormalization of four-quark operators

Control of discretization effects (enhanced by the charm mass)

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Infinite-volume decay amplitudes

$$C_{jk}(t) = \langle 0|\mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)|0\rangle_L$$

$$\mathcal{A}(D\to\alpha) = \mathcal{C}_L^{\mathrm{LL}}(\alpha|\alpha', \alpha'', \cdots)[M_{n_1,L_1}, M_{n_2,L_2}, \cdots]$$

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Lellouch-Lüscher framework ($\alpha, \alpha', \alpha''$ etc. represent open channels)

- Non-perturbative renormalization of four-quark operators
- Control of discretization effects (enhanced by the charm mass)
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- Extraction of the matrix element from three-point functions
- Formalism to relate finite-volume matrix elements to the amplitudes

Roadmap → Finite-volume formalism

Finite-volume matrix elements

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$SU(3)_F$ theory

- Project everything to an $SU(3)_F$ irreducible representation (irrep)

  incoming D, weak Hamiltonian ($\mathscr{H}_W$), outgoing two-hadron state ($\pi\pi$, $K\overline{K}$, $K\pi$, KK)

$SU(3)_F$ theory

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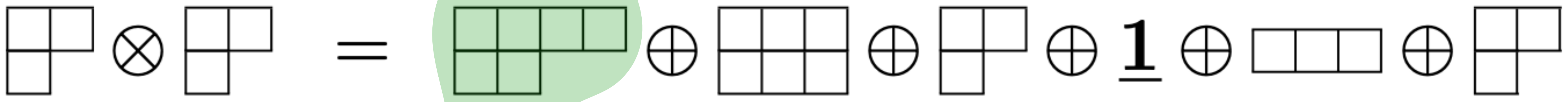
- Two-pseudoscalar state yields six irreducible representations (we focus on the **27**)



$$\mathbf{8} \otimes \mathbf{8} = \mathbf{27} \oplus \overline{\mathbf{10}} \oplus \mathbf{8} \oplus \mathbf{1} \oplus \mathbf{10} \oplus \mathbf{8}.$$

Contains
$I = 3/2\ K\pi$

$D^+ \to \overline{K}^0 \pi^+$	$\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* \ \frac{1}{\sqrt{2}} A_{27}$

• Bhattacharya et al. (2021) •

Simplifications for $SU(3)_F$

Finite-volume matrix elements (with irrep labels)

$$M_{n,L}^{I_D \otimes I_W \to I_n} = \langle n, I_n, L | \mathcal{H}_W^{I_W} | D, I_D, L \rangle$$

fewer operators
cheaper inversions
smaller physical volumes

Infinite-volume decay amplitudes

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Generic paradigm for many channels (only partially known)

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Generic paradigm for many channels (only partially known)

$$\mathcal{A}(D \to (\pi K)_{\mathbf{27}}) = \mathcal{C}_L^{\mathsf{LL}}\big((\pi K)_{\mathbf{27}} | (K\pi\pi\pi)_{\mathbf{27}}\big)[M_{n_1,L_1}, M_{n_2,L_2}, \cdots]$$

Reduce to one two- and one four-particle channel

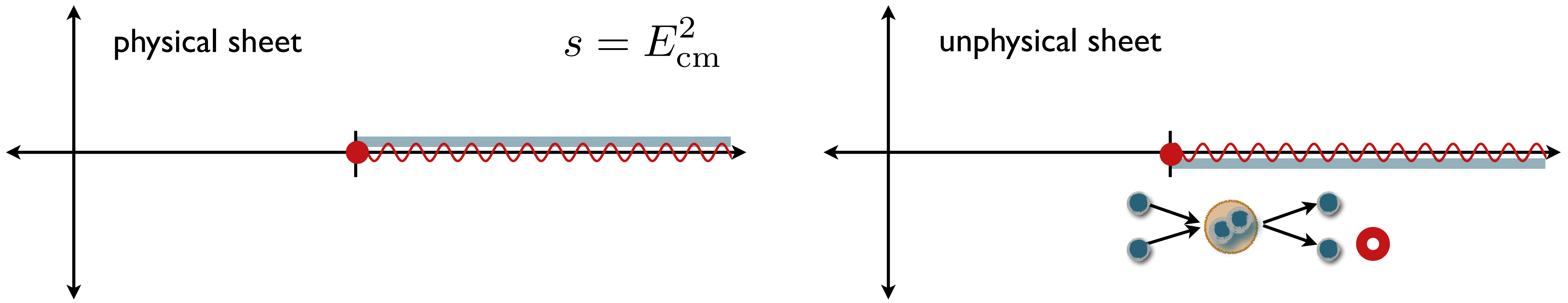
$$= \boxed{\mathcal{C}_{n,L,(\pi K)_{\mathbf{27}}}^{\mathsf{LL}} \times M_{n,L}}$$

Neglecting four particles yields standard Lellouch-Lüscher

More on finite-volume framework

- Applies for all energies $f(E) = \langle E, \pi K, \mathrm{out}|\mathcal{H}_W(0)|D\rangle$
- Interp-/extrapolate to $E = M_D$ for physical amplitude
- Use K-matrix framework to motivate fit forms

$$\mathcal{A} = \frac{1}{1 - \mathcal{K}_2 \rho} \mathcal{H}$$



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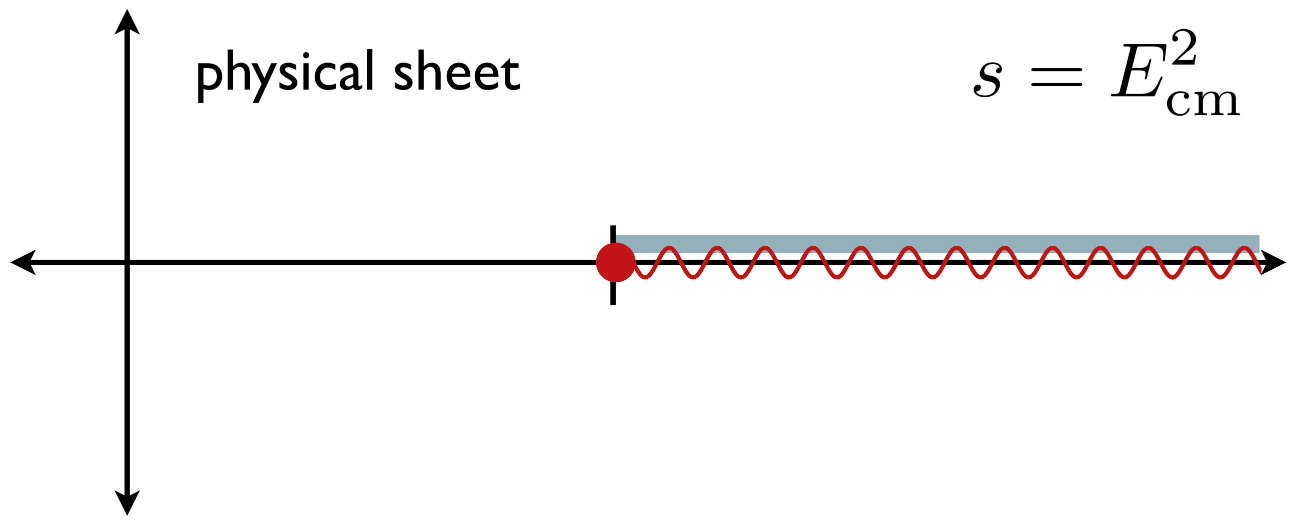
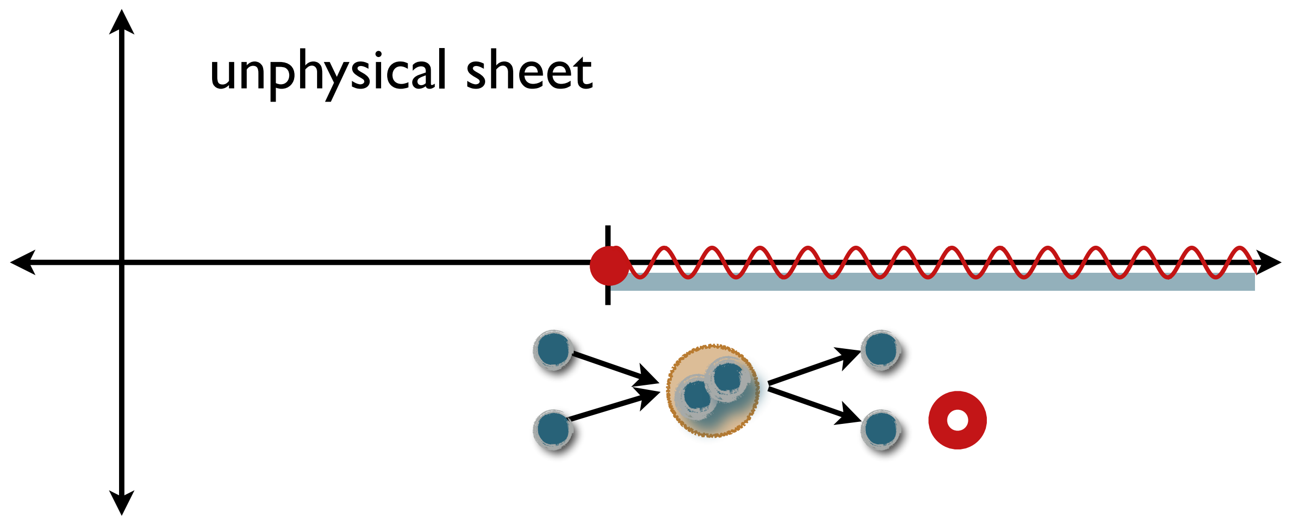
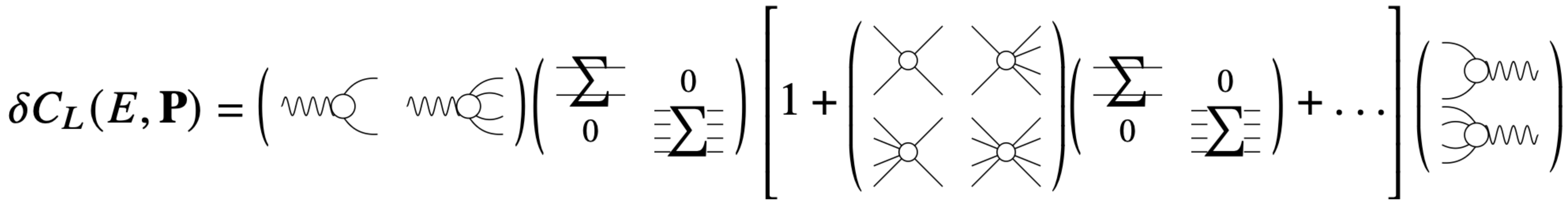
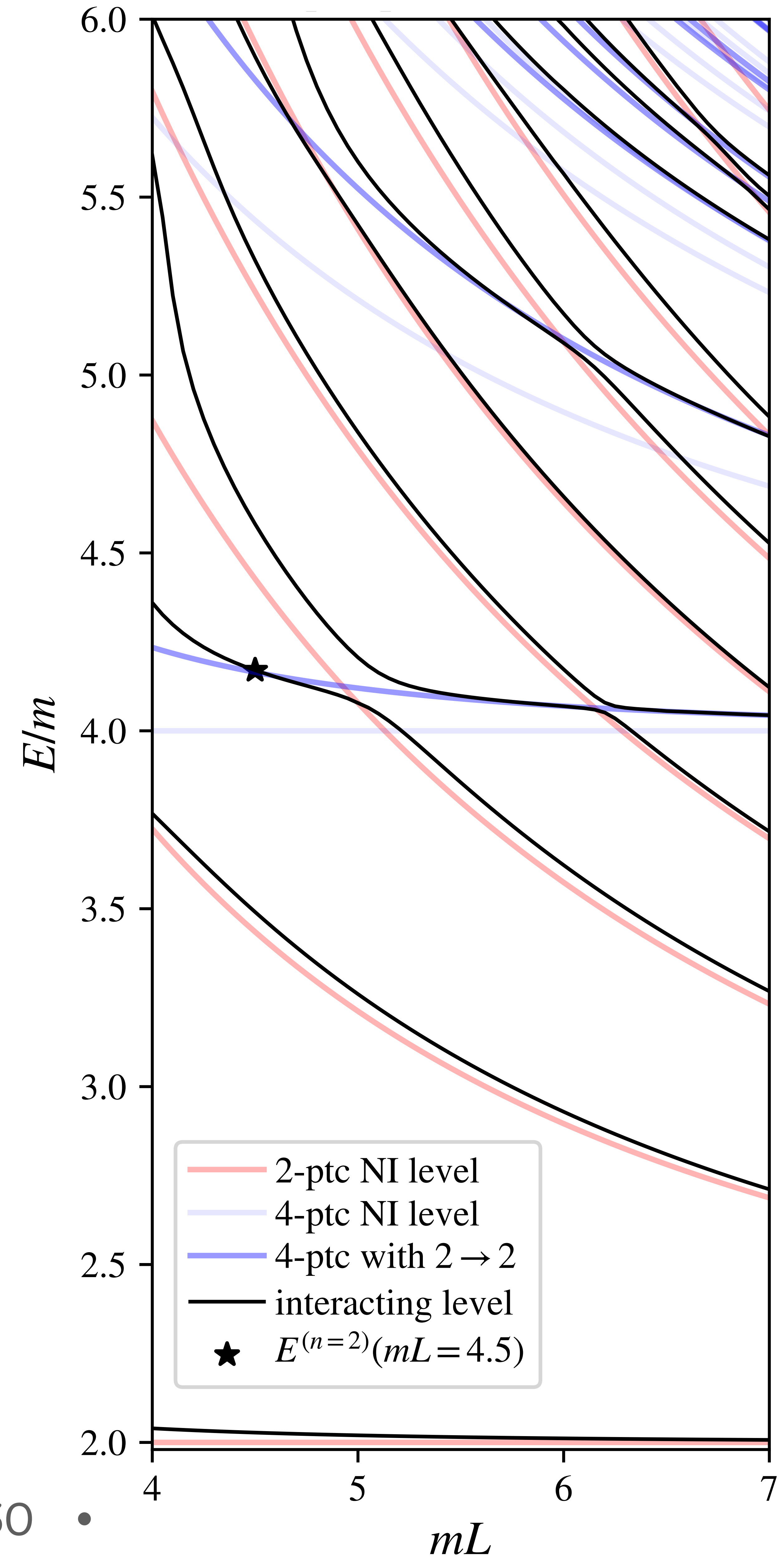




- Perturbative four-particle framework may also be useful





• MTH, Sharpe (2012) • Mukherjee, MTH (2026) & Wed 11:30 •

Roadmap → Renormalization

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- Non-perturbative renormalization of four-quark operators
- Control of discretization effects (enhanced by the charm mass)
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Non-perturbative renormalization

- Four-quark operators can be challenging with Wilson quarks

Power-divergent mixing	$\mathcal{O}_{\text{dim }6} + \frac{1}{a^n}\mathcal{O}_{\text{dim }(6-n)}$
Operator mixing	$\mathcal{O}_{\text{dim }6} + c\,\mathcal{O}_{\text{other dim }6}$
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- Discrete symmetries of $SU(4)_F$ theory highly constraining (even for Wilson quarks)
- Parity-negative part of $(V-A)^2$ operator does not mix

$$Q^{[\mathcal{S}=\pm]}_{[\mathcal{P}=-]} = O^{\pm}_{VA} + O^{\pm}_{AV}$$

$$O^{\pm}_{\Gamma_a\Gamma_b} = \frac{1}{2}\left[(\overline{\psi}_1\Gamma_a\psi_2)(\overline{\psi}_3\Gamma_b\psi_4) \pm (\overline{\psi}_1\Gamma_a\psi_4)(\overline{\psi}_3\Gamma_b\psi_2)\right]$$

- Donini et al. [hep-lat/9902030]

$$\begin{pmatrix} 1\pm \\ 2\pm \\ 3\pm \\ 4\pm \\ 5\pm \end{pmatrix}^{\text{RI-scheme}} = \begin{pmatrix} \square & & & & \\ & \square & \square & & \\ & \square & \square & & \\ & & & \square & \square \\ & & & \square & \square \end{pmatrix} \begin{pmatrix} 1\pm \\ 2\pm \\ 3\pm \\ 4\pm \\ 5\pm \end{pmatrix}^{\text{Wilson lattice}} .$$

- Apply RI-MOM and RI-SMOM with twisted momenta (*won't discuss more in this talk*)

Roadmap → Excited multi-hadron states & $a \to 0$

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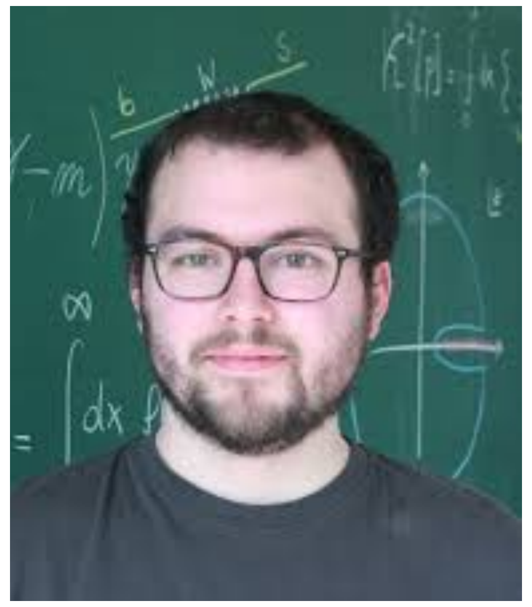

Computational set-up: *gauge-fields & software*

- Lattices generated by the OPEN LATtice initiative
    - Francis, Fritzsch, Karur, Kim, Pederiva, Pefkou, Rago, Shindler, Walker-Loud, Zafeiropoulos (2025)

- $N_f = 3$ Stabilised Wilson (exponentiated clover term) with $m_\pi = m_K = 410$ MeV, $m_D/m_\pi \approx 4.61$

Label	$T \times L^3/a^4$	a (fm)	$m_\pi L$	N_{cfg}
a12m400	96×24^3	0.12	5.988(28)	77
a09m400	96×32^3	0.094	6.201(19)	111
a06m400	96×48^3	0.064	6.383(14)	74

~7 % variation in physical box length

- Distillation framework is fully open source and based on

  Grid: data parallel C++ library (*github.com/paboyle/Grid*)

  Hadrons: Grid-based workflow management system (*github.com/aportelli/Hadrons*)

Variational method

$$C_{jk}(t) = \langle 0|\mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)|0\rangle_L \qquad C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0) \qquad \Omega_n^\dagger(t) = \sum_k v_k^{(n)}(t_d,t_0)O_k^\dagger(t)$$

- Exact distillation with 60 eigenvectors on the three ensembles
- $[K(\boldsymbol{p})\,\pi(\boldsymbol{P}-\boldsymbol{p})]_{\mathbf{27}}$ like operators up to $\boldsymbol{P}^2 = 4(2\pi/L)^2$ and up to 5 states
- Focus on trivial irrep in each frame (A_1^+ or A_1)

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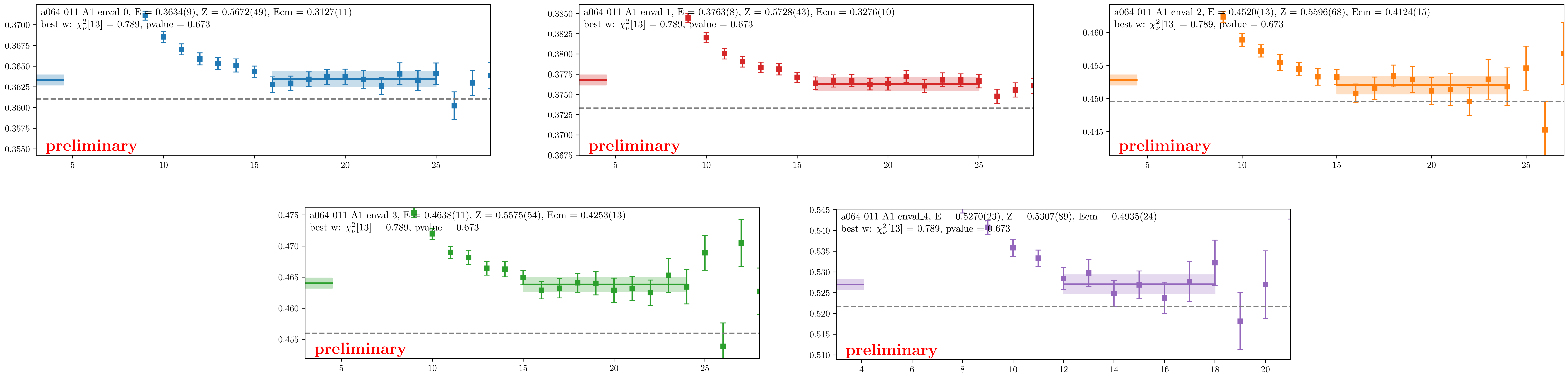
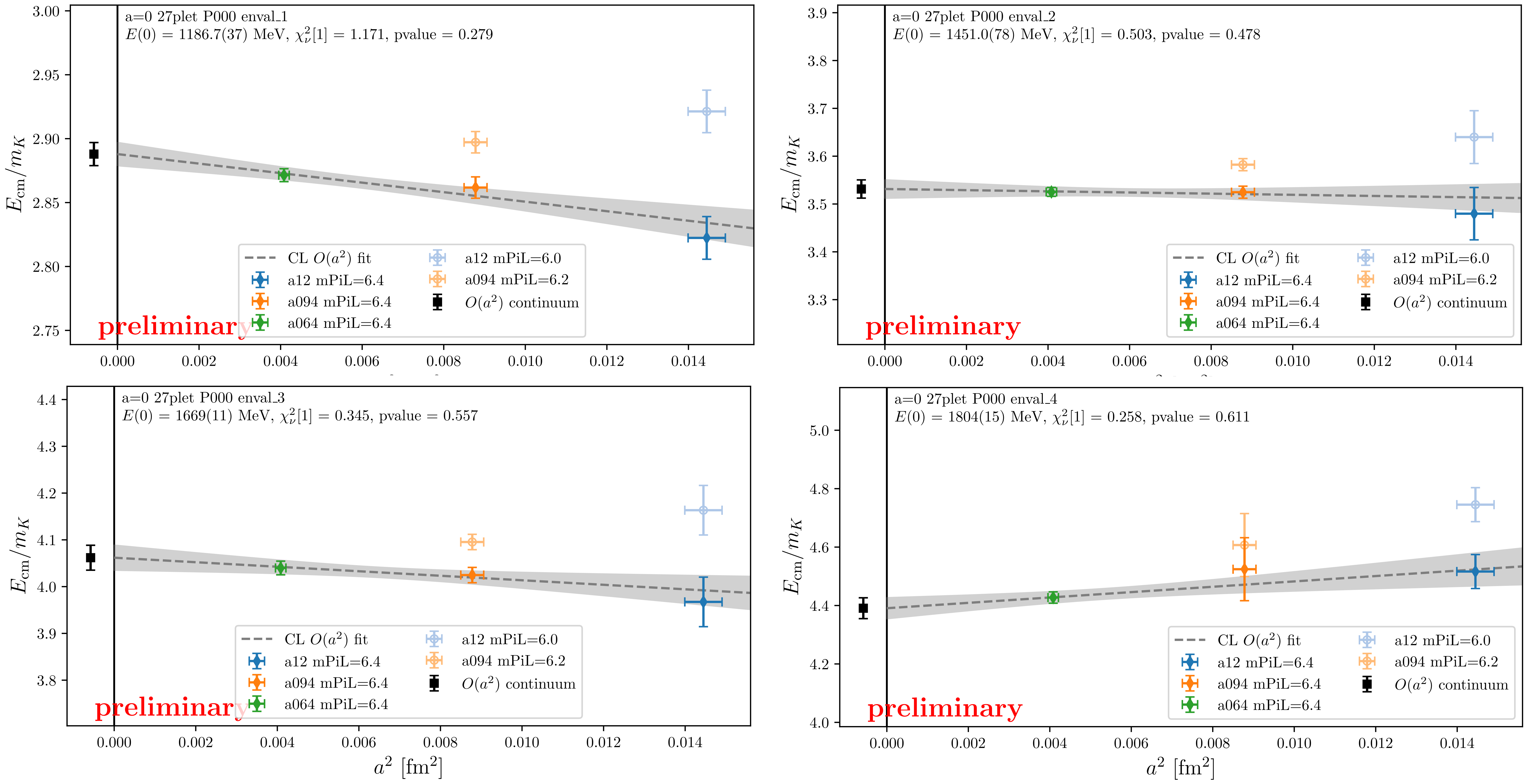
Example: [P] = [002]

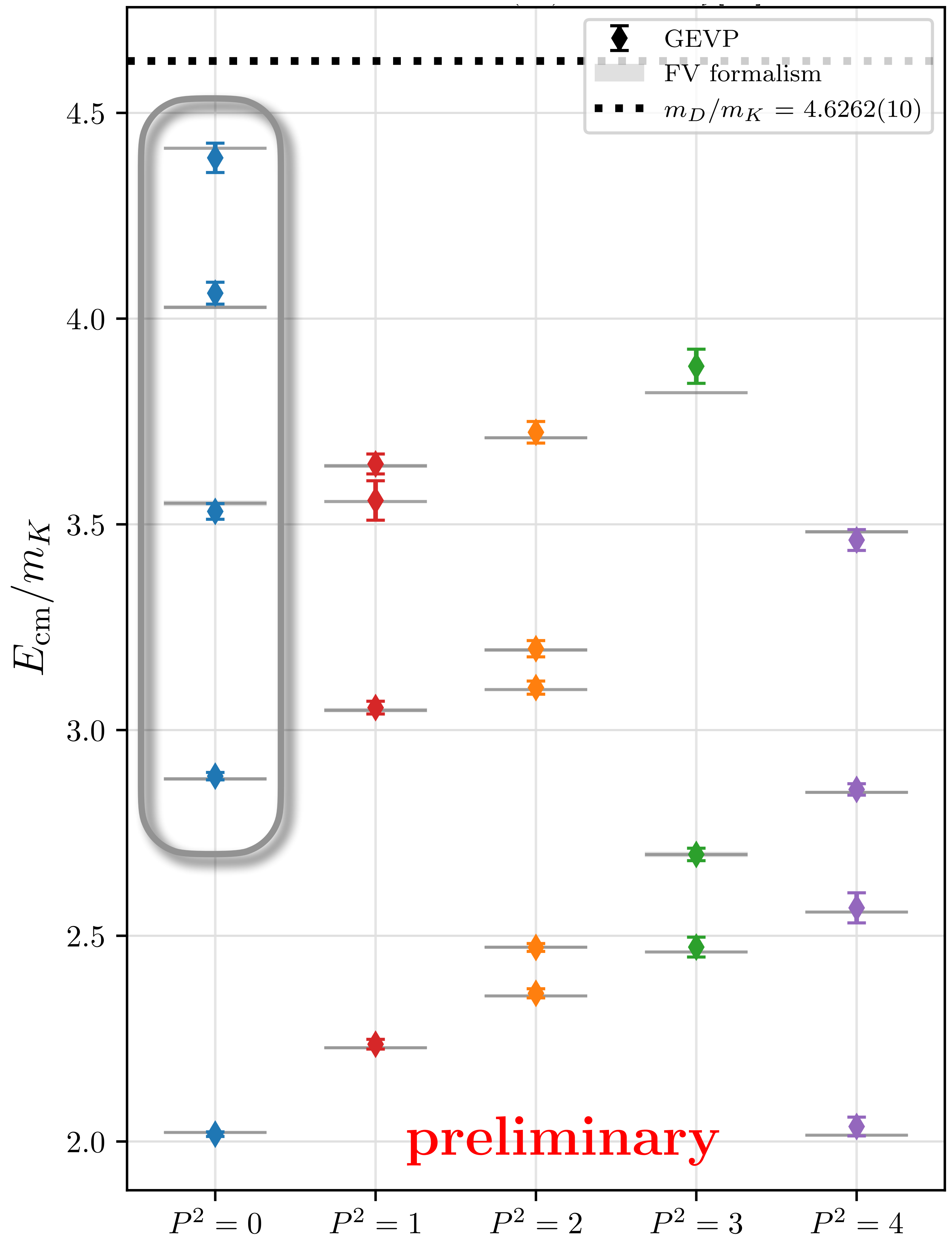


Spectrum + scattering-length-only fit

Continuum spectrum

- Physical volumes are reasonably well matched (~7% in L)
- Use per-ensemble scattering length to adjust to L
  → per-energy continuum limit at fixed physical volume





Continuum spectrum

- Physical volumes are reasonably well matched (~7% in L)
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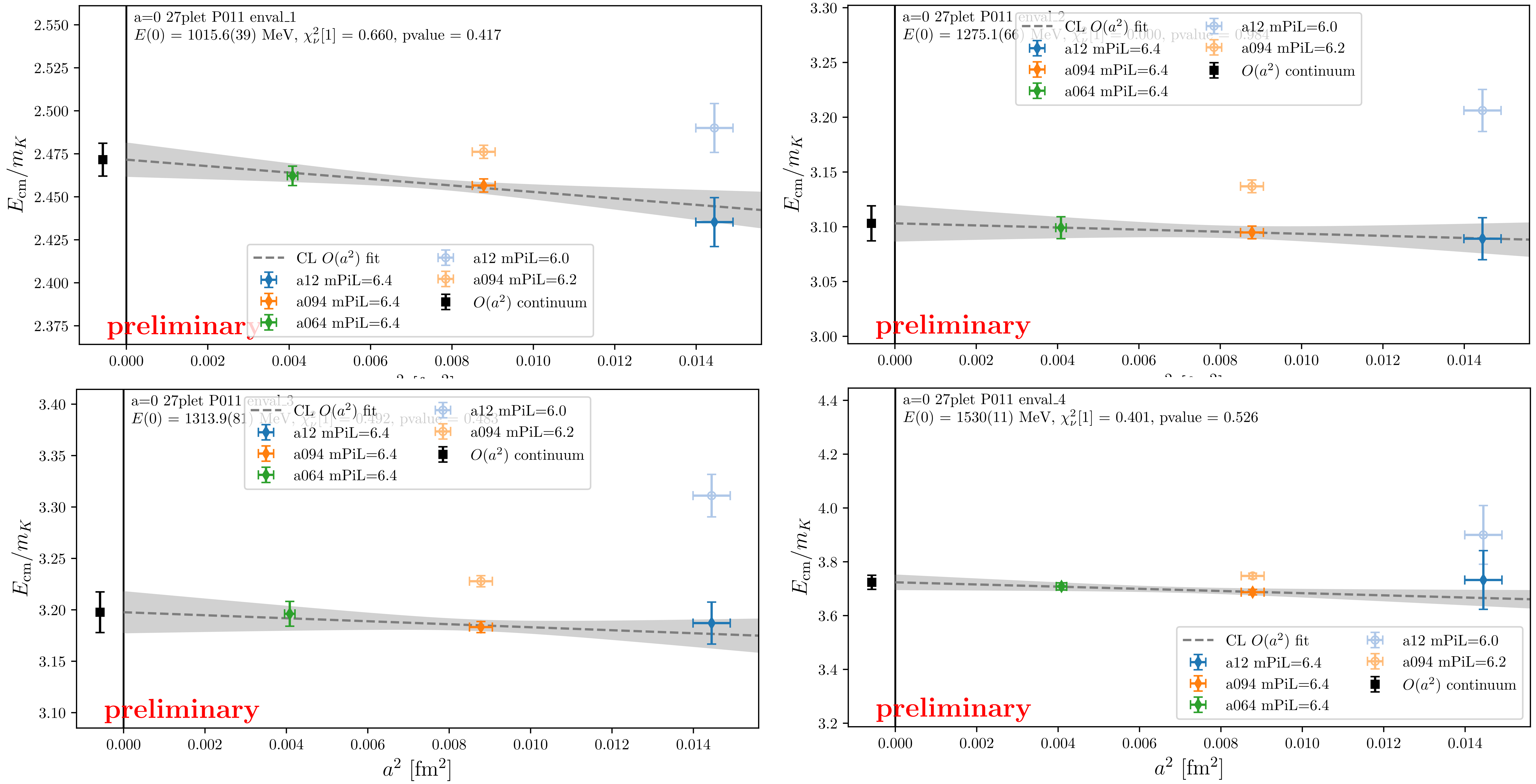
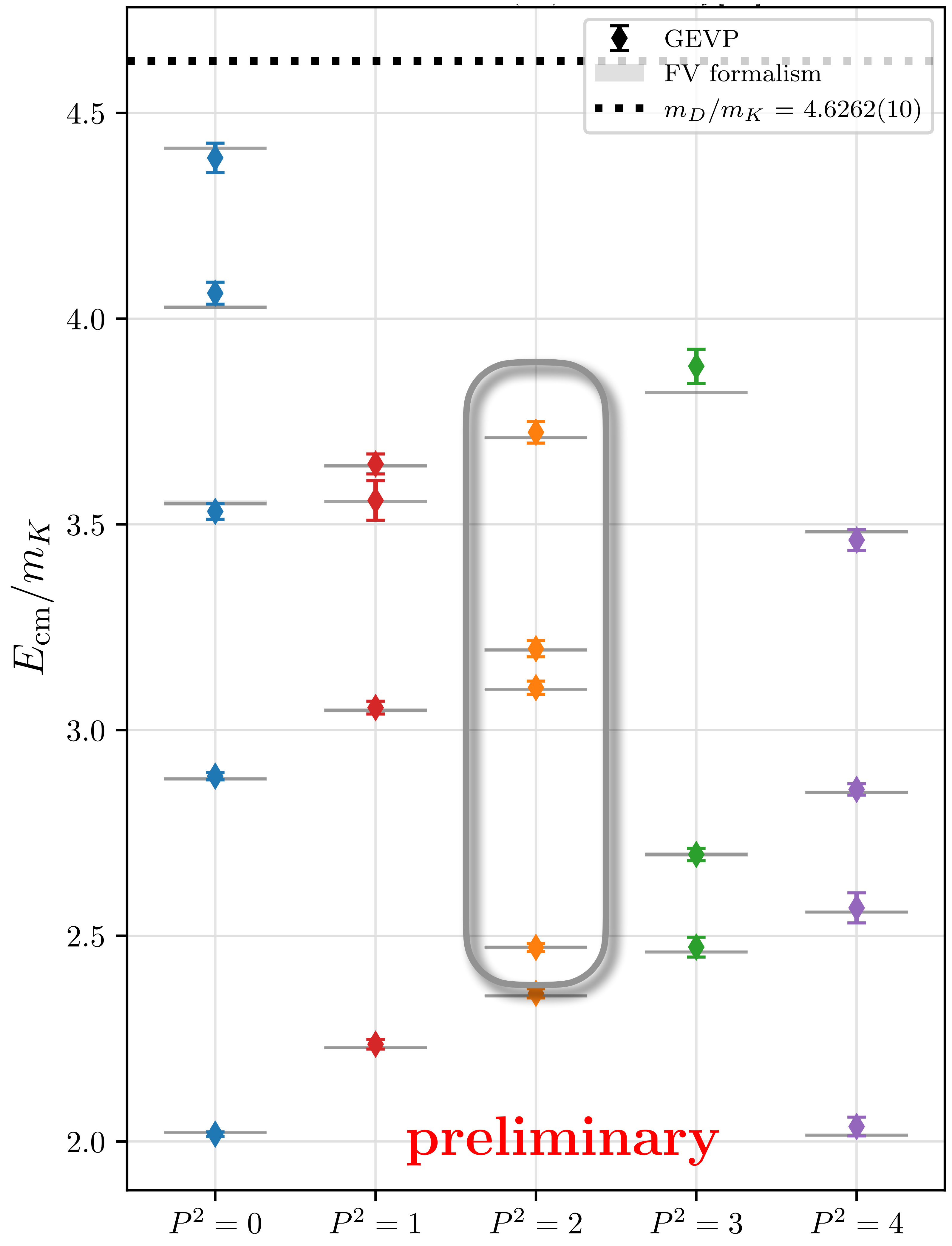
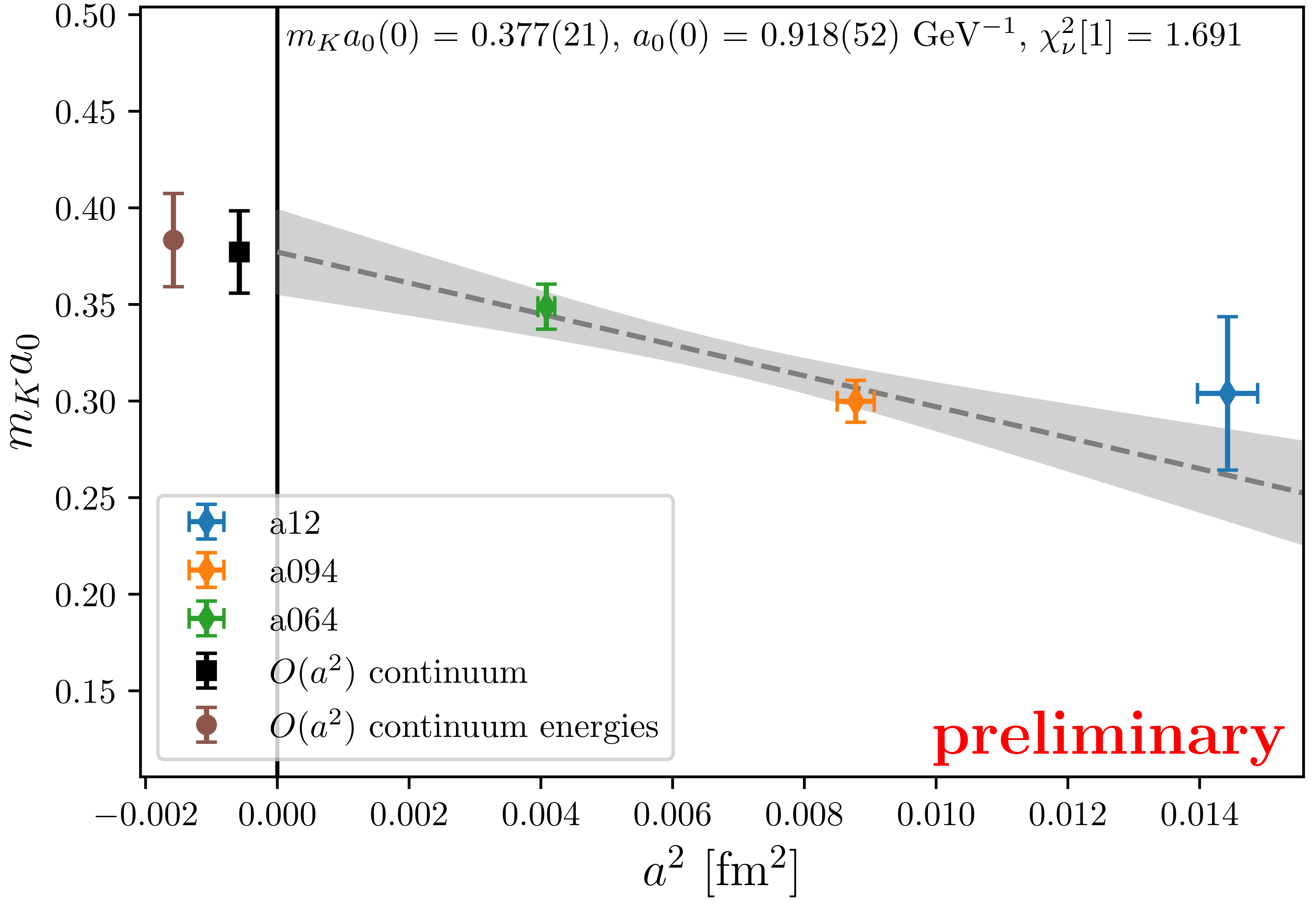




Continuum scattering length

- Compare a_0 from $E_n(L, a \to 0)$ vs $\lim_{a \to 0} a_0(a)$
- To do: Consider other K-matrix ansätze

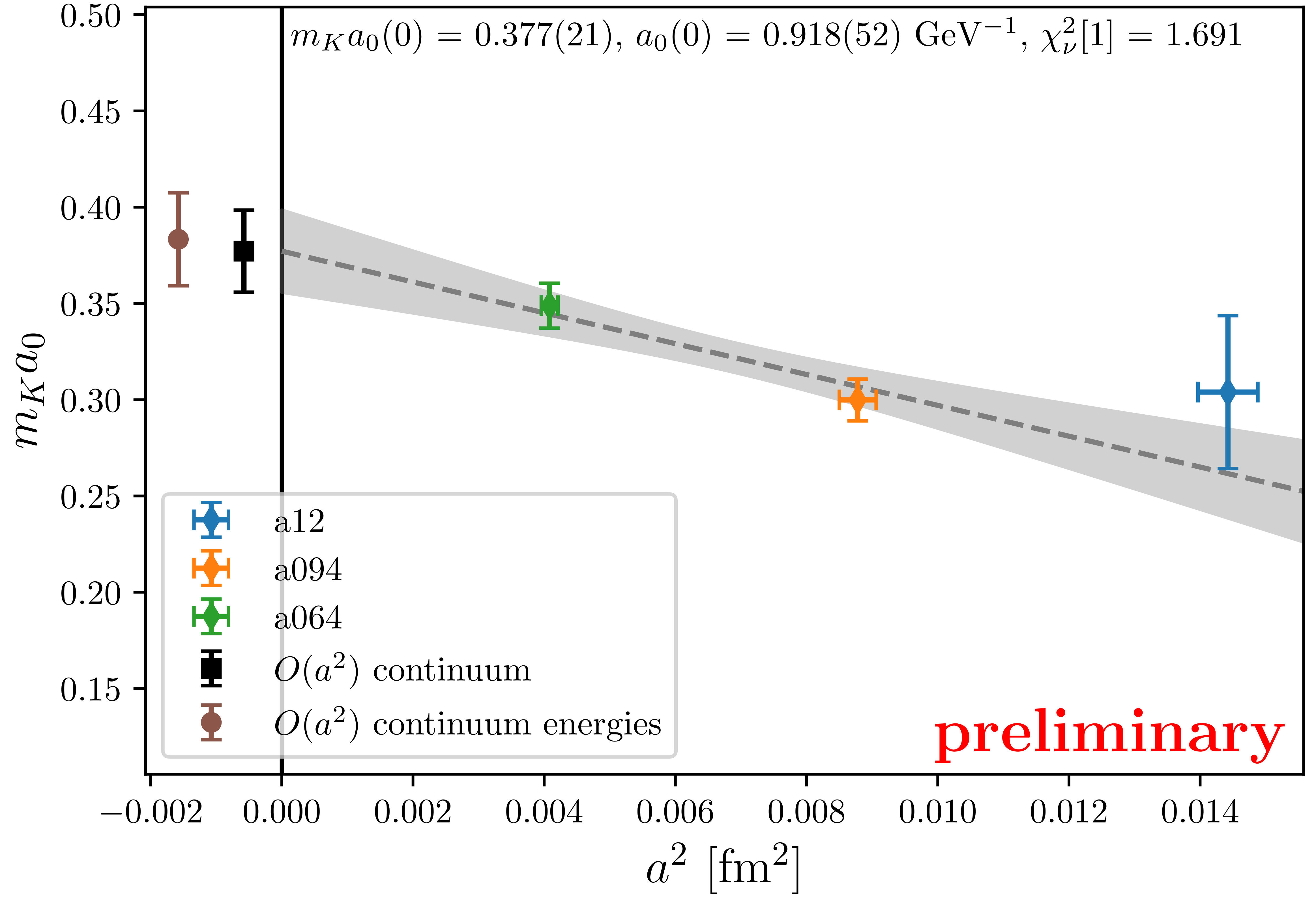


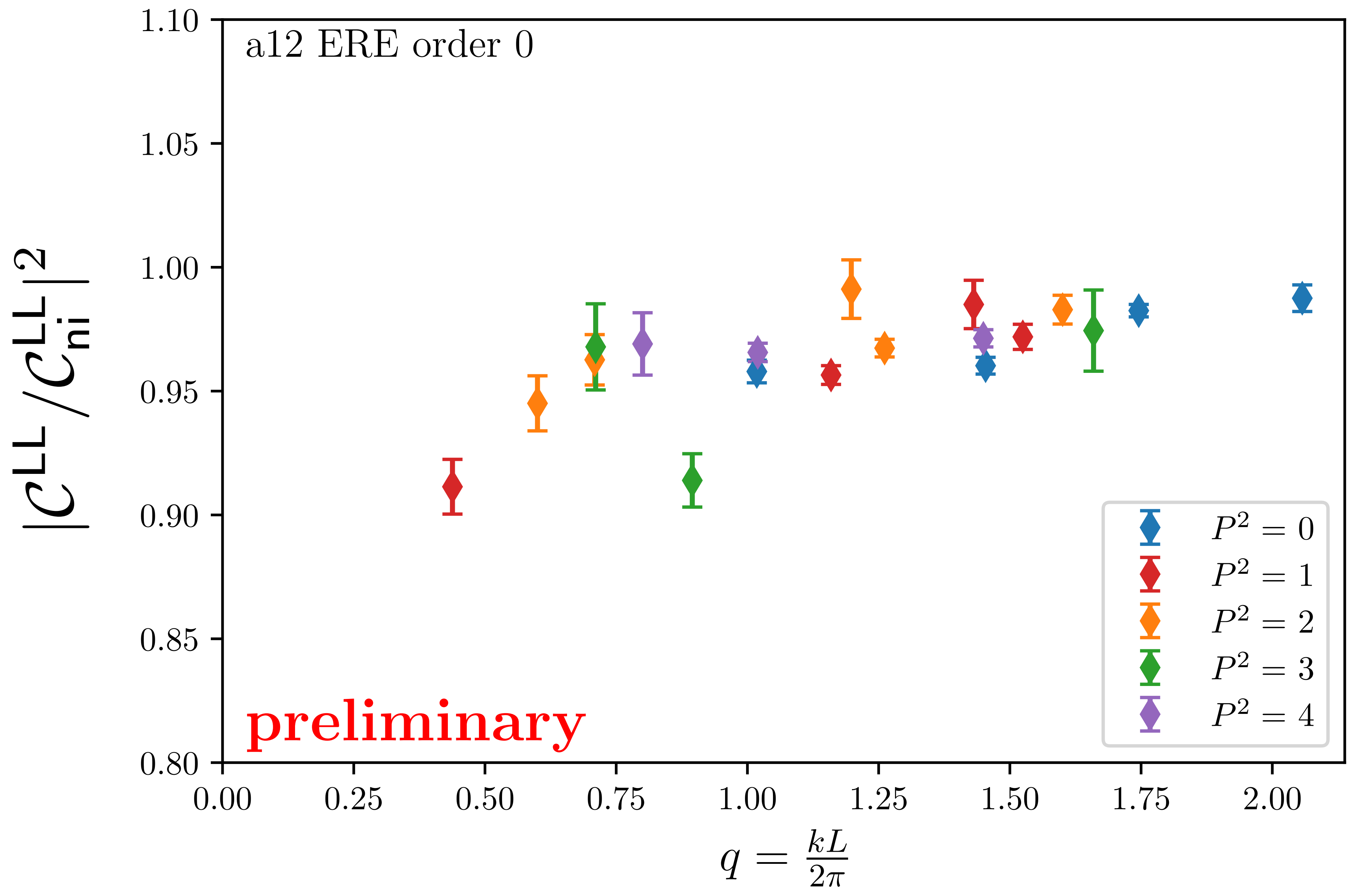
Continuum scattering length

- Compare a_0 from $E_n(L, a \to 0)$ vs $\lim_{a\to 0} a_0(a)$
- To do: Consider other K-matrix ansätze
- Results will give:

  Confidence in operator construction

  Scattering information for finite-volume formalism





$$\left|\mathcal{C}^{\mathrm{LL}}\right|^2 = 8\pi \left(q \frac{\partial \phi}{\partial q} + k \frac{\partial \delta_0}{\partial k} \right)_{k=k_n} \frac{E_n^2 m_D}{k_n^3}$$

Roadmap → Three-point functions

Finite-volume matrix elements *(suppressing irrep labels)*

$$M_{n,L} = \langle n, L|\mathcal{H}_W|D, L\rangle = \sum_i C^{\overline{\mathrm{MS}}}_{\mathsf{wc},i} \lim_{a\to 0} Z_i^{\overline{\mathrm{MS}}} \lim_{\Delta t\to\infty} \langle 0|\Omega_n^\dagger(\Delta t/2)\mathcal{Q}_i(0)D(\Delta t/2)|0\rangle_L$$

Infinite-volume decay amplitudes

$$C_{jk}(t) = \langle 0|\mathcal{O}_j^\dagger(t)\mathcal{O}_k(0)|0\rangle_L$$

$$\mathcal{A}(D \to (\pi K)_{\mathbf{27}}) = \mathcal{C}^{\mathsf{LL}}_{n,L,(\pi K)_{\mathbf{27}}} \times M_{n,L}$$

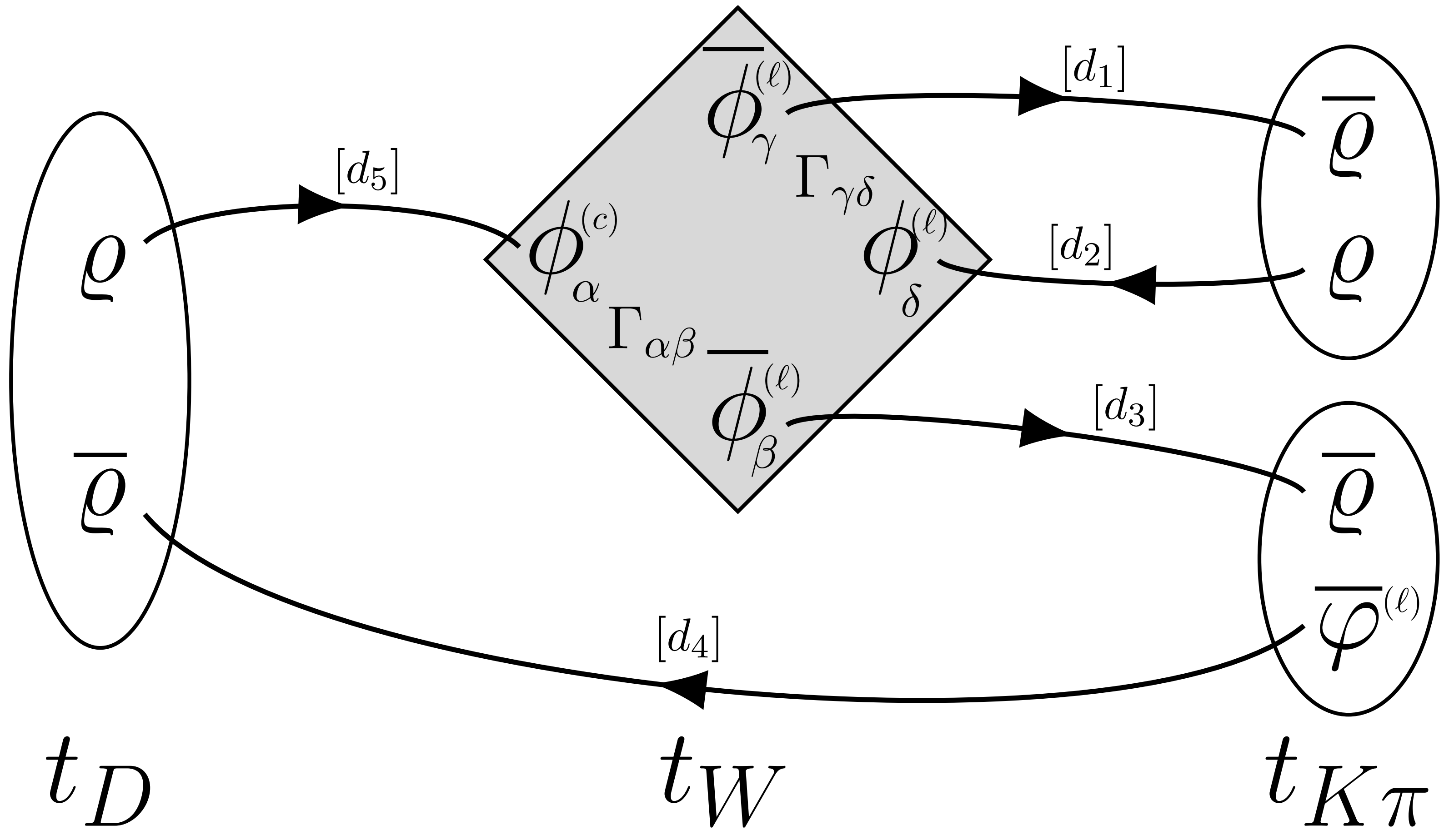
$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$$

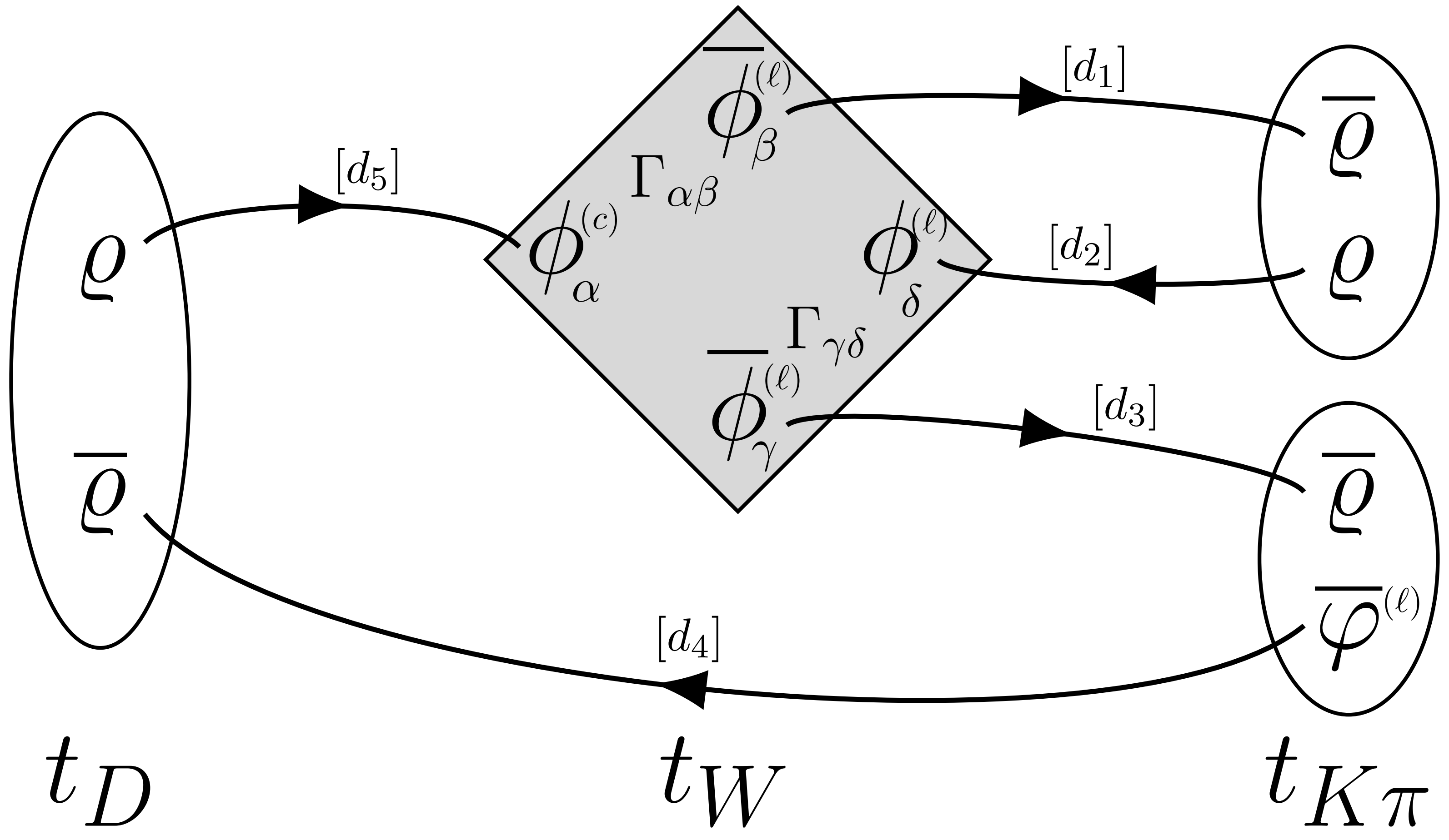
$$\Omega_n^\dagger(t) = \sum_k v_k^{(n)}(t_d,t_0)O_k^\dagger(t)$$

- Non-perturbative renormalization of four-quark operators
- Control of discretization effects (enhanced by the charm mass)
- Reliable creation of excited multi-hadron final states
- Extraction of the matrix element from three-point functions
- Formalism to relate finite-volume matrix elements to the amplitudes

Three-point function set-up

- Introduce unsmeared/generalized perambulators so the local four-quark operator insertion retains full position-space information at t_H
- Decay contractions are organized into four contributions: T_{Q_i} and C_{Q_i} with $i = 1,2$
- All four can be factorized into product of two traces over distillation indices

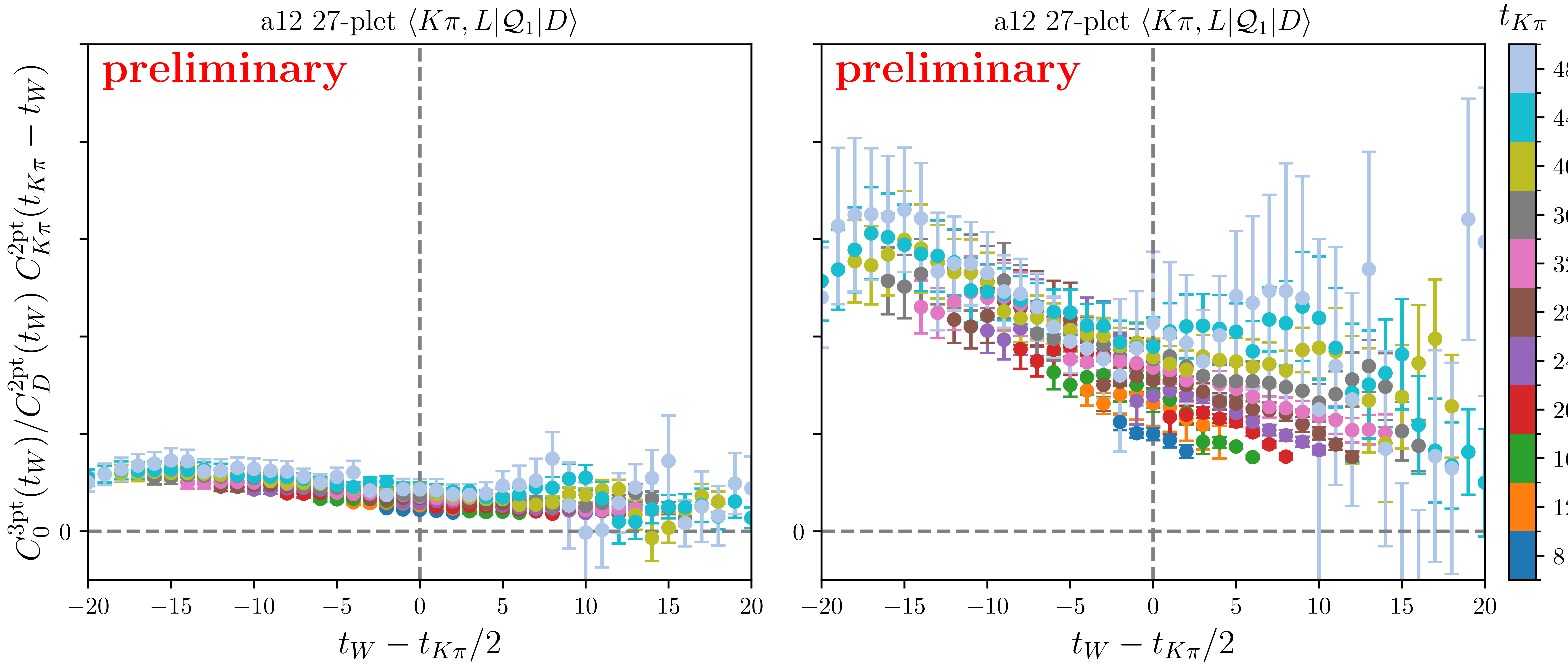




First preliminary results (*coarsest ensemble*)

$$\left\langle (K_\mathbf{0}\pi_\mathbf{0})_\mathbf{27}(t_{K\pi})\ \mathcal{Q}(t_W)\ D_\mathbf{0}(0) \right\rangle$$

$$Q_1^q = \bar{q}^\alpha\gamma_\mu(1-\gamma_5)c^\beta \otimes \bar{u}^\beta\gamma^\mu(1-\gamma_5)q^\alpha \qquad Q_2^q = \bar{q}^\alpha\gamma_\mu(1-\gamma_5)c^\alpha \otimes \bar{u}^\beta\gamma^\mu(1-\gamma_5)q^\beta$$



Roadmap → Status

Finite-volume matrix elements

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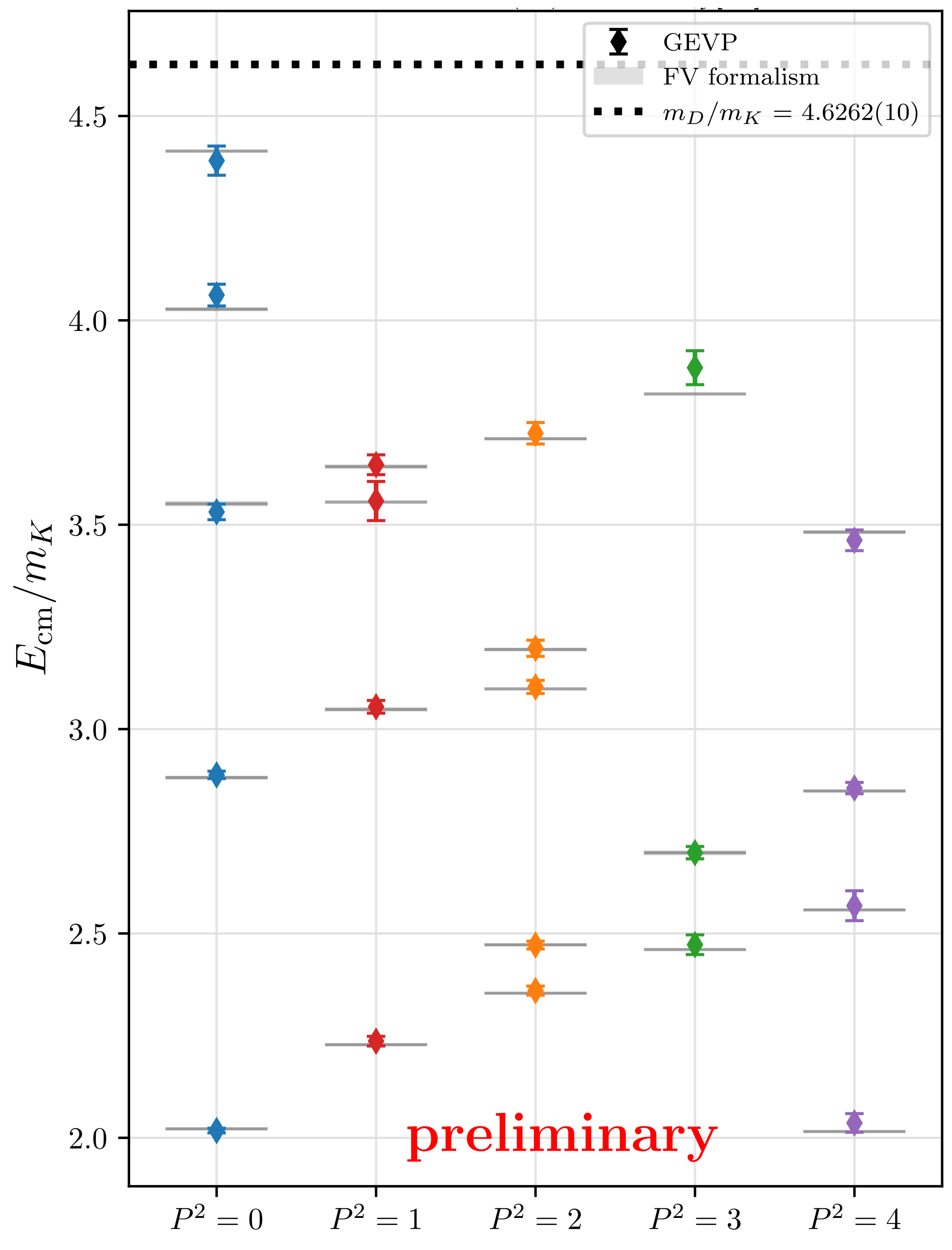
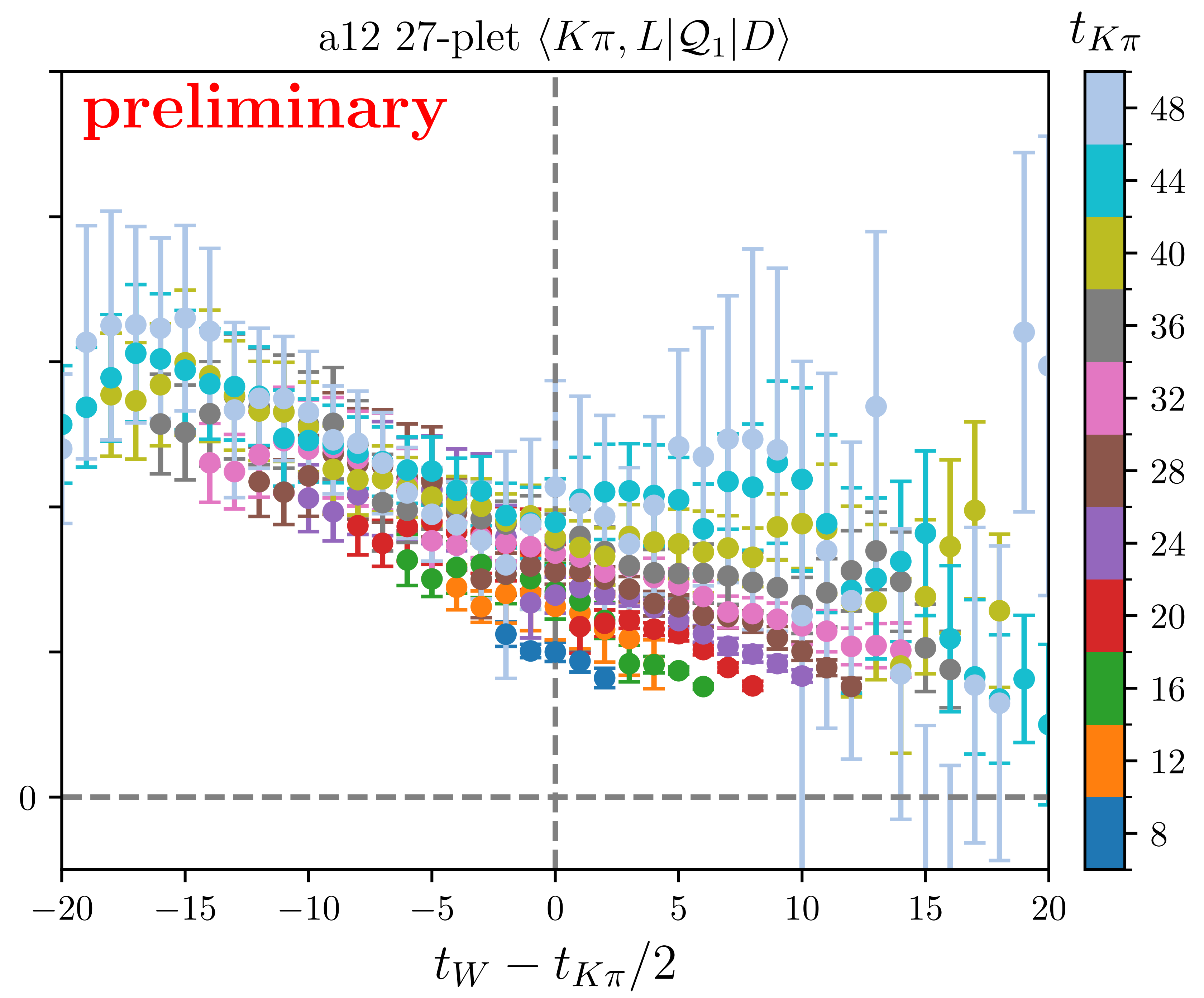
Non-perturbative renormalization of four-quark operators	●●●●◐
Control of discretization effects	●●●●○
Reliable creation of excited multi-hadron final states	●●●●○
Extraction of the matrix element from three-point functions	●●●●○
Formalism to relate finite-volume matrix elements to the amplitudes	●●●●●

Conclusions and Outlook

- Pilot study of $D \to (K\pi)_{27}$ at the $SU(3)_F$ point is reaching a mature stage
- Future calculations:
  - Remaining $SU(3)_F$ final states
  - Amplitudes with operator mixing *(switch action or apply gradient flow methods?)*
  - Move towards physical masses
- Possible technical improvements:
  - Improved operator construction
  - Sparse grid or ***blending*** for $\mathscr{H}_W$

• Zhi-Cheng Hu et al. (2025) •





Thanks for listening!

UK Research and Innovation

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