Four Flavo(u)rs of MDWF

Lattice 2026
26 July - 1 August

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CoSMoN Collaboration
[cosmon-collaboration.github.io]

Why a new action?

- I have personally been interested in generating DWF for > 20 years [chiral symmetry]
  - "But they are expensive!" "And DWF don't have multi-grid!" …
- We have observed that calculations with MDWF in the valence sector require significantly less samples to achieve the same stochastic uncertainty [we believe this comes from a mix of χ-symmetry and smearing]
- DWF appear to have milder discretization effects [fewer and coarser $\{a\}$ to achieve same continuum?]
- DWF are significantly simpler to renormalize [less human time]
- DWF are more suited to heterogeneous architectures [more compute intense solves and greater percent wall clock time spent in highly optimized solve compared to the other pieces of measurement calcs]
- Generating physical pion mass ensembles is not an issue (aside from needing a machine larger than Perlmutter to fit within the queue)
- In aggregate, I suspect DWF are not more expensive than clover, especially when human time is included
- I was also interested in having a set of configurations to compute correlators involving baryons and scattering that were also free of the typical "politics" of use

What is our action?

- Lüscher Weisz gauge action $S_G = \frac{\beta}{2N_c} \sum \left(c_0 \square + c_1 \,\square\!\square \right)$ $\quad c_0 = 1 - 8c_1 \quad c_1 = -\frac{1}{12}$

- Möbius Domain Wall Fermions
  $b - c = 1$: rescaled Shamir operator
  $$D^M(M_5) = \frac{(b+c)D^W(M_5)}{2 + (b-c)D^W(M_5)} = (b+c)\frac{D^W(M_5)}{2 + D^W(M_5)} = \alpha D^{\text{Shamir}}(M_5)$$

- Dirac operator uses 8-steps of stout smeared gauge links with $\rho = 0.125$
  [Morningstar & Peardon, PRD 69 2004)]
  $$V_\mu = U_\mu^{(8)}, \qquad U_\mu^{(l+1)} = e^{i\rho Q_\mu^{(l)}} U_\mu^{(l)}, \qquad D^M[V_\mu](M_5)$$

- Most design decisions and all new software needed to generate our configurations are due to R. Jamie Hudspith — as well as all preliminary results shown here

Why 8 steps of stout smearing?

- Seems like a lot of smearing! 😱

- With MDWF/HISQ mixed-action calculations by CalLat with $t_{gf} = 1 \times a^2$ [Action paper - 1701.07559]
  $\pi^+ \leftarrow \pi^-$: 1805.02634
  g_A @ 0.74% uncertainty: 1805.12130, 2104.05226, 2503.09891
  F_K/F_π @ 0.37% uncertainty: 2005.04795
  w_0 scale @ 0.67% uncertainty: 2011.12166

  We observed improved chiral properties of $D^M[V_\mu]$
  Passed several high-precision tests
  No sign of over-smearing

- 8 steps of stout with $\rho = 0.125$ is similar to gradient-flow of length 1 in lattice units
  [see perturbative equivalence in Ammer & Durr, PRD 110 (2024) [2406.03493]]

- Clearly - we need to perform several high-precision tests of the action - see later

MDWF: $M_5, L_5, b(c), m_l, m_s, m_c$

- Parameters of the fermion action
- $M_5 = 1$ for all ensembles: as $a \to 0$, the $(-1)^t$ modes that appear in temporal dependence functions decouple — [Syritsyn & Negele LATTICE2007 [0710.0425]]
- tune b to minimize m_{res} and set L_5 such that $\sqrt{t_0} m_{res} \lesssim 0.0009$, or $m_{res} \lesssim 1.5$ MeV
- For each β, tune m_s such that connected η_s $8t_0 m_{\eta_s}^2 = 1.989(32)$ [Dowdall et al, 1110.6887]
- Set $am_c = 11.8 \times am_s$ from FLAG determination of ratio
- Scan m_l to build up ensembles with different m_π
- Most initial ensembles generated with $m_\pi L \simeq 3.5$

$\approx a$/fm	β	s.h	b	c	L_5^{*}	M_5	$L_{\rm traj}^{**}$	(am_s, am_c)	$10^5 am_{\rm res}$
0.12	4.008	a12	1.75	0.75	10	1.0	0.7	(0.0725,0.8555)	74(5)
0.10	4.068	a10	1.50	0.50	8	1.0	1.0	(0.0560,0.6608)	43(3)
0.08	4.160	a08	1.35	0.35	6	1.0	1.6	(0.0415,0.4897)*	22(3)
0.06	4.238	a06	1.20	0.20	4	1.0	2.6	(0.0305,0.3599)	39(1)
0.05	4.333	a05	1.16	0.16	4	1.0	4.0	(0.0230,0.2714)	23(0)

* Yes, we tried $L_5 = 2$ 😂

** I will return to this

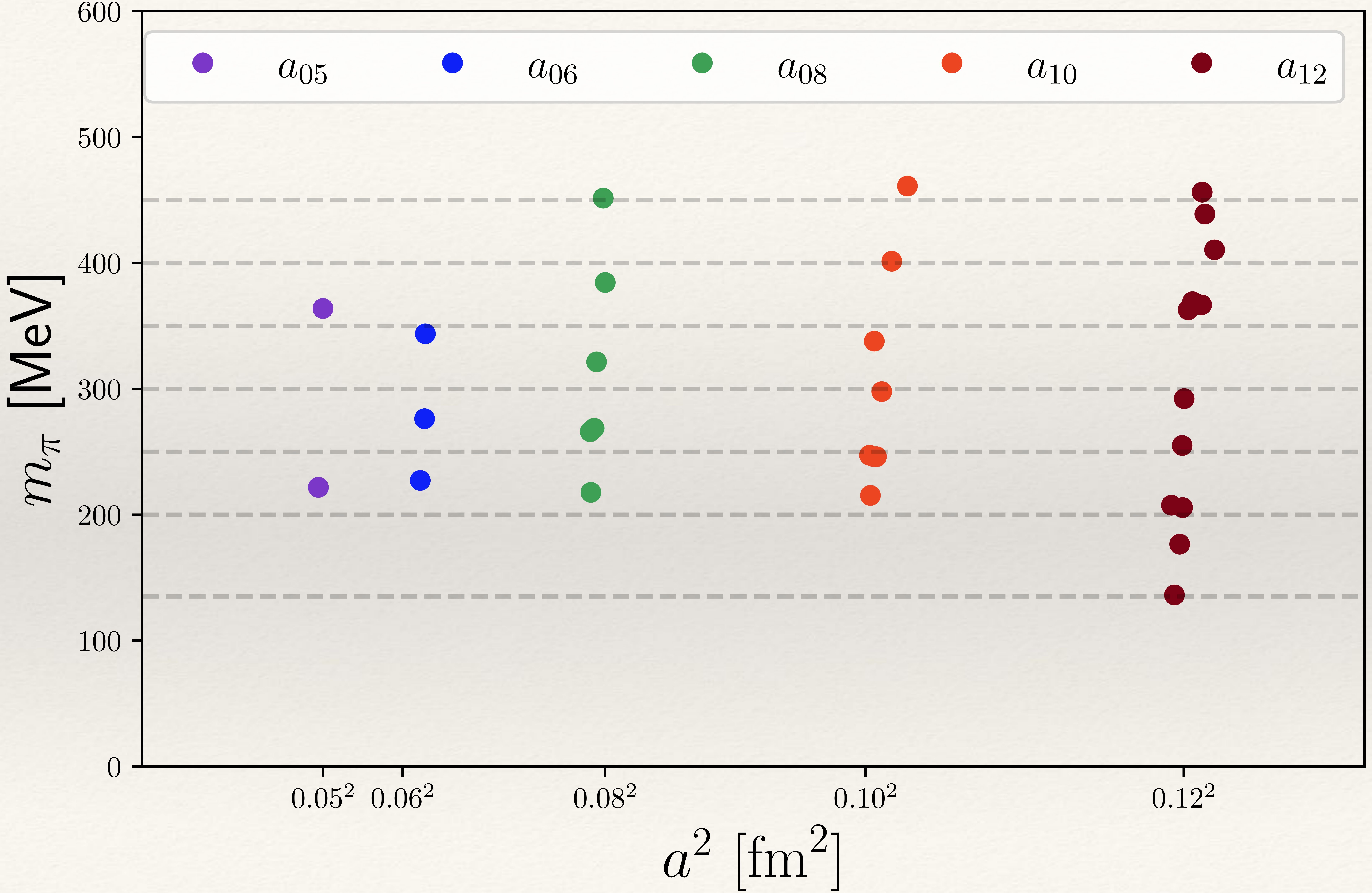
Landscape of current ensembles

- Ensembles from F_K/F_π [2605.06560]

TABLE III. Volume and light quark mass in lattice units for the ensembles used in this work. Additionally, the gradient flow scales (t_0, w_0), topological charge (Q) and their respective autocorrelation lengths are tabulated. The derived values of F_K/F_π, $\xi_\pi = m_\pi^2/(4\pi F_\pi)^2$, $\xi_K = m_K^2/(4\pi F_\pi)^2$, $m_\pi L$ and $\epsilon_a^2 = a^2/(4w_0^2)$ are listed as well as the number of independent configurations used $N_{\rm cfg}$ and the number of time-translated sloppy solves $N_{\rm src}$.

ensemble	$\frac{L}{a}^3 \times \frac{T}{a}$	am_l	$\sqrt{t_0}/a$	w_0/a	Q	$\tau_{\sqrt{t_0}}, \tau_{w_0}, \tau_Q$	F_K/F_π	ξ_π	ξ_K	$m_\pi L$	ϵ_a^2	$N_{\rm cfg} \times N_{\rm src}$
$\beta = 4.008,\ a \approx 0.12$ fm												
a12m456	$16^3 \times 32$	0.0300	1.1961(31)	1.4766(82)	0.3(2)	(16,16,16)	1.0800(20)	0.1262(19)	0.2106(28)	4.424(18)	0.1147(13)	173 × 32
a12m436	$16^3 \times 32$	0.0270	1.1946(20)	1.4766(82)	0.4(3)	(16,24,24)	1.0898(24)	0.1225(15)	0.2178(24)	4.255(16)	0.1141(15)	179 × 32
a12m410	$16^3 \times 32$	0.0235	1.1891(20)	1.4766(82)	-0.2(3)	(16,16,16)	1.1012(25)	0.1091(17)	0.2150(27)	3.979(18)	0.1162(11)	118 × 32
a12m375	$16^3 \times 32$	0.0195	1.2016(40)	1.5086(132)	0.2(3)	(24,24,16)	1.1109(33)	0.0918(17)	0.2070(30)	3.580(19)	0.1098(20)	106 × 32
a12m375M	$20^3 \times 48$	0.0195	1.1969(13)	1.5086(132)	0.2(3)	(24,24,16)	1.1061(17)	0.09031(66)	0.2048(12)	4.449(11)	0.11275(67)	204 × 48
a12m375L	$24^3 \times 48$	0.0195	1.2040(40)	1.5138(57)	0.4(5)	(16,16,8)	1.1007(20)	0.08770(76)	0.2003(16)	5.276(11)	0.10909(85)	95 × 32
a12m363	$16^3 \times 32$	0.0190	1.1964(13)	1.4943(39)	-0.1(2)	(8,8,12)	1.1198(32)	0.0959(17)	0.2221(30)	3.550(20)	0.11197(59)	235 × 32
a12m292	$20^3 \times 40$	0.0120	1.2064(7)	1.5258(27)	0.6(1)	(8,16,8)	1.1382(30)	0.06587(76)	0.2149(19)	3.541(12)	0.10738(39)	353 × 20
a12m250	$24^3 \times 48$	0.0090	1.2075(17)	1.5319(46)	-0.3(6)	(16,16,16)	1.1520(26)	0.05215(49)	0.2137(16)	3.7086(93)	0.10654(64)	116 × 48
a12m208	$28^3 \times 64$	0.0057	1.2138(6)	1.5528(28)	-0.6(3)	(12,24,8)	1.1743(28)	0.03752(30)	0.2204(14)	3.5223(73)	0.10368(38)	176 × 64
a12m208M	$32^3 \times 64$	0.0057	1.2073(6)	1.5639(20)	0.6(5)	(16,16,8)	1.1606(21)	0.03548(23)	0.2118(10)	3.9883(75)	0.10584(28)	154 × 64
a12m176	$32^3 \times 64$	0.0040	1.2090(8)	1.5460(22)	0.3(4)	(16,16,16)	1.1792(20)	0.02720(20)	0.2147(11)	3.4247(72)	0.10460(31)	157 × 64
a12m135	$48^3 \times 96$	0.0020	1.2120(3)	1.5514(11)	-0.1(10)	(12,24,12)	1.1894(36)	0.01733(11)	0.22251(99)	3.9631(74)	0.10386(21)	101 × 48
$\beta = 4.068,\ a \approx 0.10$ fm												
a10m460	$16^3 \times 32$	0.0220	1.4076(19)	1.7175(48)	-0.1(1)	(12,12,24)	1.0995(40)	0.1336(25)	0.2236(36)	3.745(22)	0.08475(47)	483 × 32
a10m401	$20^3 \times 40$	0.0185	1.4223(21)	1.7589(75)	-0.1(3)	(16,16,40)	1.0963(25)	0.1010(15)	0.1975(24)	4.074(18)	0.08081(75)	128 × 40
a10m340	$20^3 \times 40$	0.0135	1.4392(30)	1.8227(106)	-0.5(3)	(16,16,16)	1.1259(51)	0.0822(19)	0.2090(37)	3.430(21)	0.07525(90)	150 × 40
a10m300	$24^3 \times 48$	0.0100	1.4319(17)	1.7915(77)	-0.7(5)	(20,40,40)	1.1286(26)	0.06275(79)	0.1938(18)	3.628(14)	0.07789(70)	174 × 24
a10m247	$28^3 \times 64$	0.0067	1.4439(11)	1.8276(27)	-0.5(7)	(12,12,24)	1.1495(27)	0.04700(47)	0.1979(13)	3.514(12)	0.07484(22)	115 × 64
a10m247M	$32^3 \times 64$	0.0067	1.4370(12)	1.8128(38)	-0.5(3)	(16,16,8)	1.1464(31)	0.04659(46)	0.1988(15)	3.998(13)	0.07607(32)	115 × 32
a10m214	$32^3 \times 64$	0.0050	1.4430(11)	1.8266(30)	0.5(4)	(12,12,12)	1.1646(39)	0.03770(48)	0.2026(19)	3.497(12)	0.07493(25)	100 × 32
$\beta = 4.160,\ a \approx 0.08$ fm												
a08m430	$20^3 \times 40$	0.0170	1.8136(51)	2.2291(146)	0.4(2)	(20,20,30)	1.1242(58)	0.1378(41)	0.2396(62)	3.626(28)	0.05031(67)	233 × 20
a08m375	$24^3 \times 48$	0.0165	1.8097(46)	2.2108(118)	0.0(5)	(20,20,30)	1.1195(44)	0.0948(22)	0.2049(38)	3.705(21)	0.05115(58)	112 × 24
a08m323	$32^3 \times 64$	0.0090	1.8268(14)	2.2637(35)	-0.6(3)	(15,15,30)	1.1215(14)	0.06837(45)	0.18874(98)	4.1297(83)	0.04879(15)	307 × 64
a08m271	$32^3 \times 64$	0.0060	1.8318(24)	2.2767(55)	0.1(5)	(15,15,25)	1.1580(37)	0.05323(80)	0.2003(22)	3.453(13)	0.04823(23)	106 × 32
a08m271rs	$32^3 \times 64^*$	0.0060	1.8398(29)	2.3017(73)	0.6(7)	(20,20,40)	1.1538(32)	0.05315(65)	0.1972(18)	3.417(12)	0.04719(31)	101 × 64
a08m220	$40^3 \times 80$	0.0039	1.8383(12)	2.2304(28)	0.1(8)	(15,15,40)	1.1703(51)	0.03777(38)	0.2028(15)	3.4971(96)	0.04708(12)	147 × 20
$\beta = 4.238,\ a \approx 0.06$ fm												
a06m343	$32^3 \times 64$	0.0070	2.3142(49)	2.8498(164)	0.4(3)	(30,40,60)	1.1379(34)	0.0815(14)	0.2023(27)	3.491(15)	0.03078(38)	172 × 64
a06m275	$40^3 \times 80$	0.0045	2.3173(20)	2.8617(76)	1.5(5)	(10,20,30)	1.1453(32)	0.05389(67)	0.1879(18)	3.508(13)	0.03053(17)	149 × 40
$\beta = 4.333,\ a \approx 0.05$ fm												
a05m364	$40^3 \times 80$	0.0060	2.8964(55)	3.5177(132)	0.8(8)	(30,30,250)	1.1225(43)	0.0827(21)	0.1886(36)	3.675(26)	0.02020(15)	159 × 20
a05m218	$64^3 \times 128$	0.0021	2.9334(23)	3.6199(77)	-1.3(12)	(15,20,40)	1.1704(46)	0.03699(51)	0.1914(20)	3.583(15)	0.019078(88)	102 × 16

Landscape of current ensembles



HMC Integrator

- We use a 3-level integrator

$$\mathrm{Det}(M) = \mathrm{Sdet}\left(\frac{m_l}{h_1}\right) \mathrm{Sdet}\left(\frac{h_l}{h_2}\right) \cdots \mathrm{Sdet}\left(\frac{h_{n-1}}{h_n}\right) \mathrm{Sdet}\left(\frac{h_n}{m_{\mathrm{PV}}}\right) \mathrm{Sdet}_{\mathrm{EOFA}}\left(\frac{m_s}{m_c}\right) \mathrm{Sdet}\left(\frac{m_c}{m_{\mathrm{PV}}}\right) S_{\mathrm{Gauge}}$$

Level 1	Level 2	Level 3
OMF2	OMF2	OMF4
$O(10)$ steps	1 step	$1-2$ steps

- We tune the number and value of the Hasenbusch mass ratios to keep "Fdt" on each level $\lesssim 0.4$
  Therefore, at least $\mathrm{Sdet}\left(\frac{m_l}{h_1}\right)$ is on Level 1, with additional Hasenbush ratios added to Level 1 as needed to keep Fdt small
- Acceptance is kept $70\,\% \lesssim$ acceptance rate $\lesssim 80\,\%$
- For new physical pion mass ensembles with $a \simeq \{0.10,\ 0.08\}$ fm and $V = \{64^3 \times 128 \times 8,\ 80^3 \times 160 \times 6\}$, the Force-Gradient-Integrator for the light quarks has become cost effective

Q

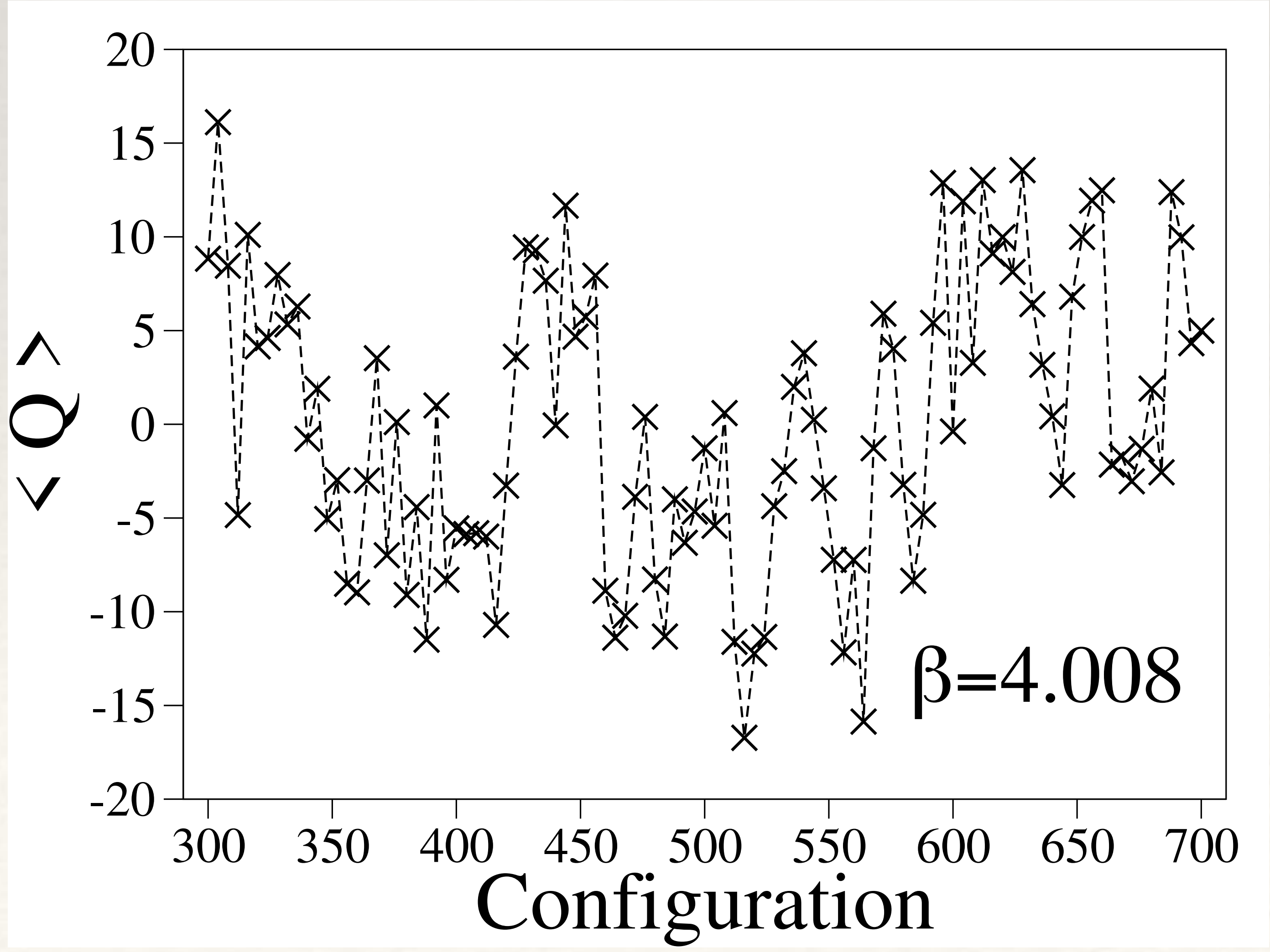
- Rather than keeping the MC integration time fixed as a function of β, the MD integration length is held fixed in physical units.

- We use t_0 as a proxy for this physical scale and on the 2nd coarsest, we have $L_{\rm traj}^{\beta=4.068} = 1$

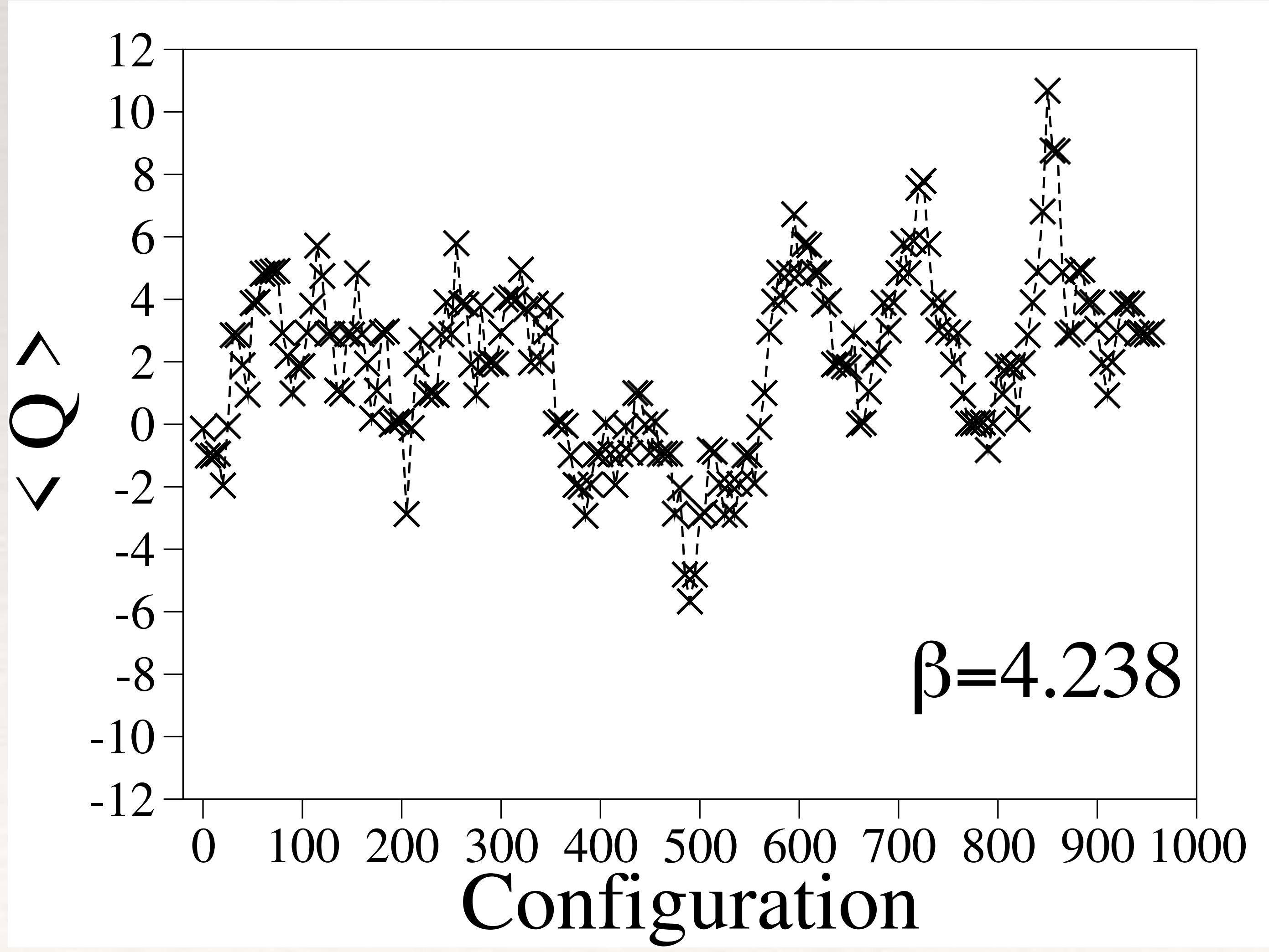
- This leads to setting $L_{\rm traj} = \{0.7,\ 1.0,\ 1.6,\ 2.6,\ 4.0\}$ on the $a \simeq \{0.12,\ 0.10,\ 0.08,\ 0.06,\ 0.05\}$ fm ensembles

- We find that Q keeps fluctuating down to the finest ensemble
  We speculate (but need to do a careful comparison) that the longer integration time improves the fluctuations

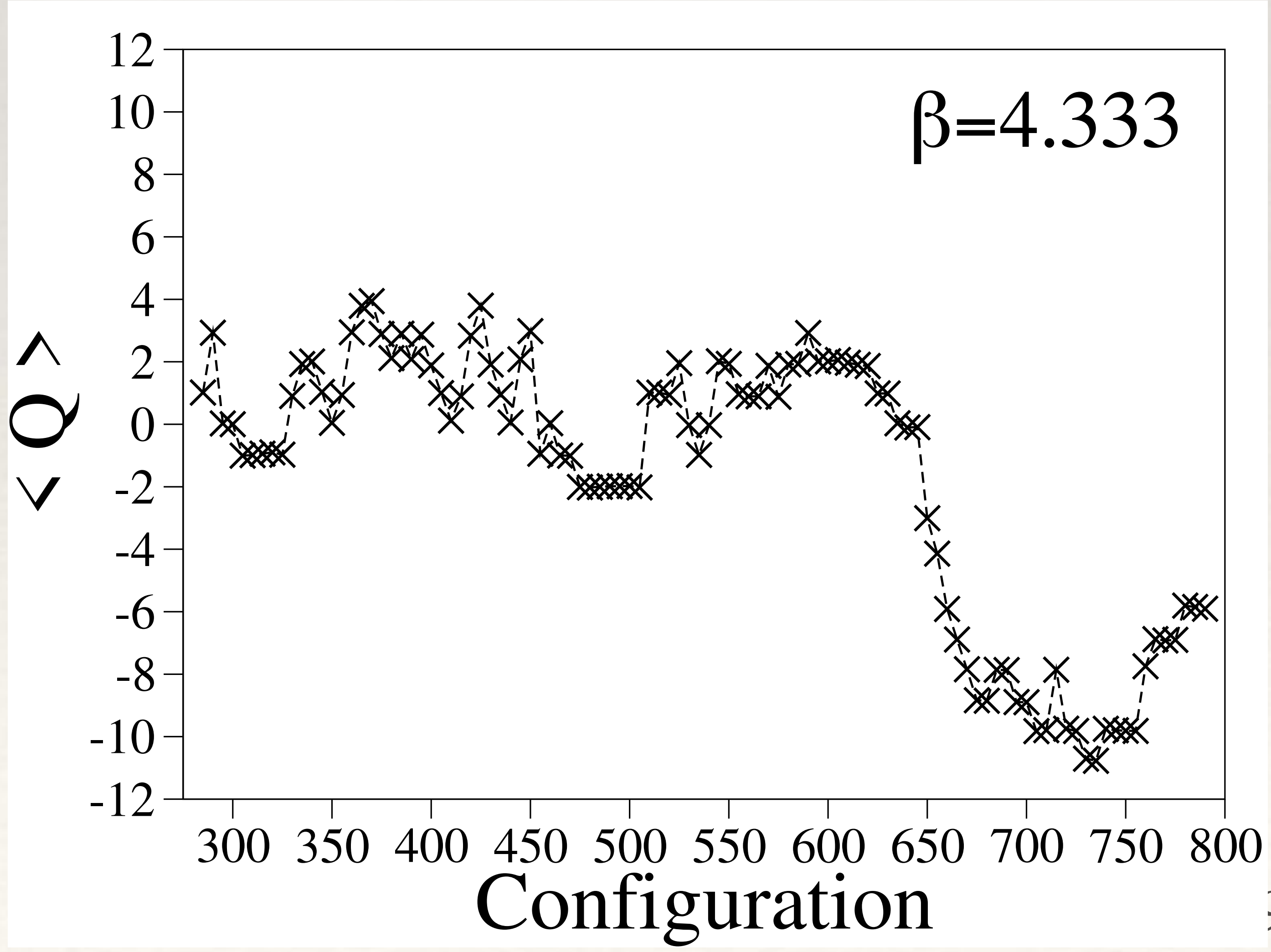
$a \simeq 0.12$ fm



$a \simeq 0.06$ fm



$a \simeq 0.05$ fm



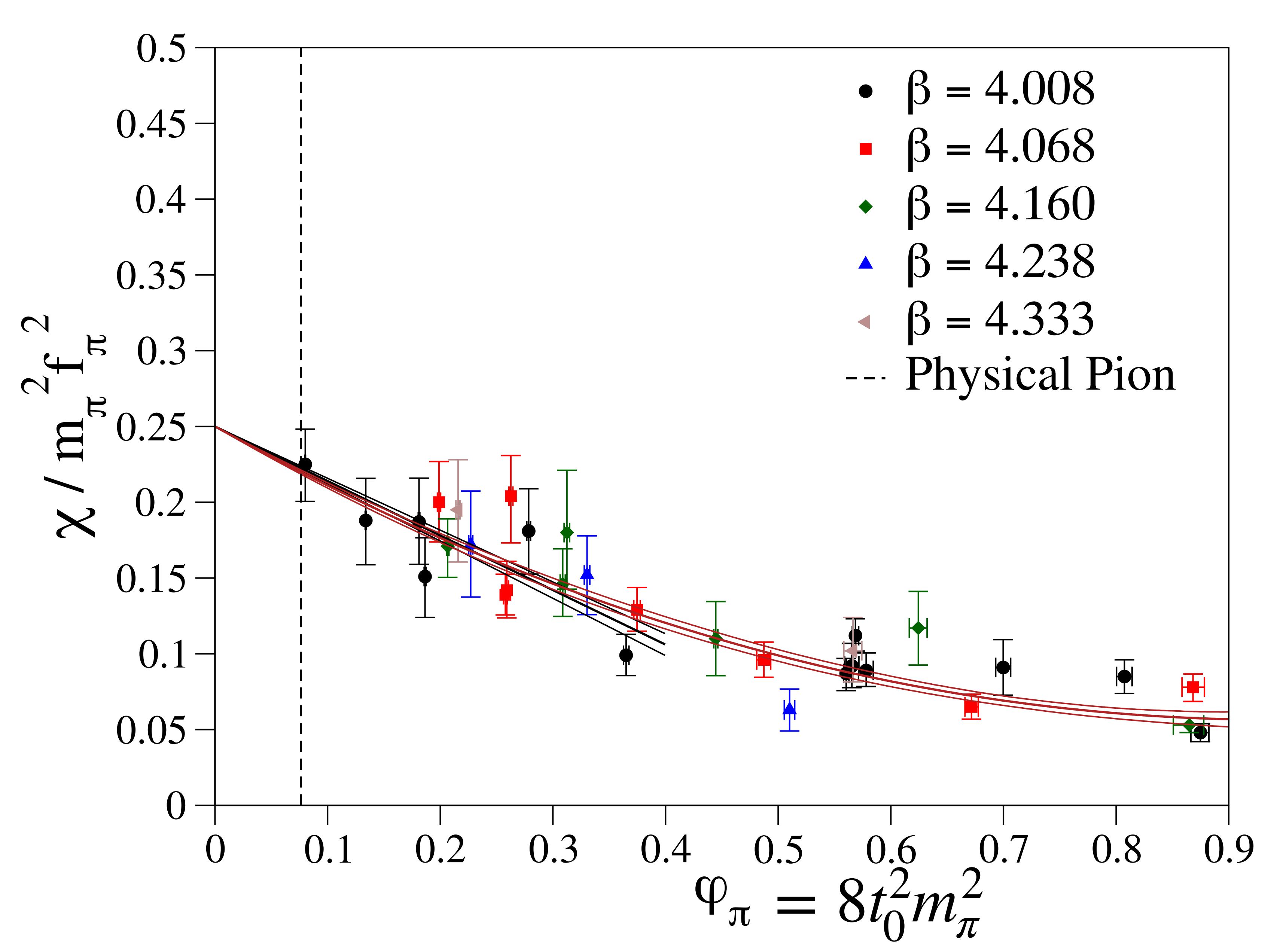
Topological Susceptibility: Preliminary

- Topological susceptibility is computed with the "slab" method [Bietenholz, de Forcrand, Gerber 1509.06433] $\chi(\tau) = C + \chi V \frac{\tau}{T} + \mathcal{O}(e^{-m_\eta L})$

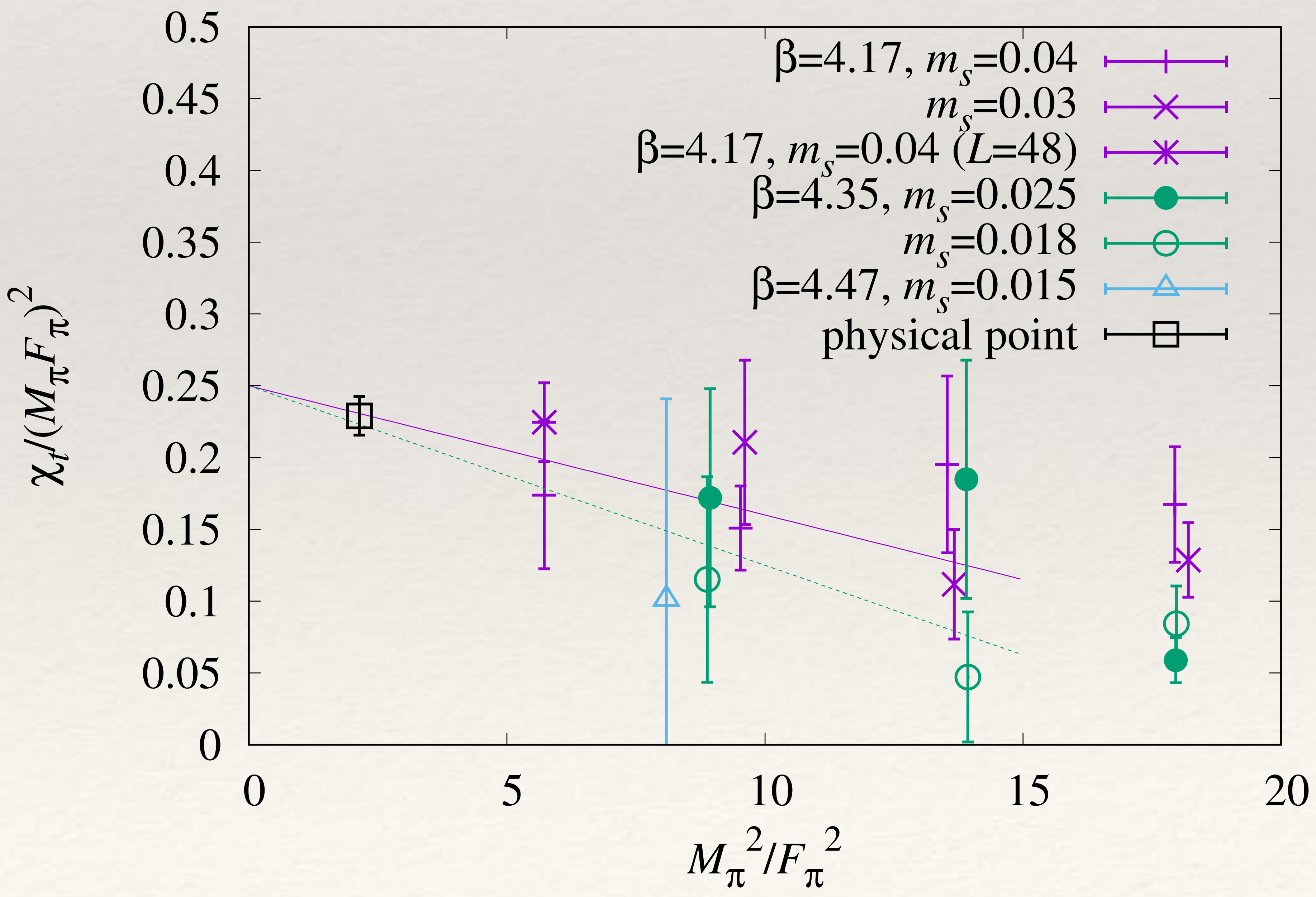
- 30 iterations of HYP smearing with $\alpha_1 = 0.75$, $\alpha_2 = 0.5$, $\alpha_3 = 0.25$ are used

- $$\chi = \frac{m_\pi^2 F_\pi^2}{4}\left(1 + 2l' \frac{m_\pi^2}{F_\pi^2} + \mathcal{O}(m_\pi^4/F_\pi^4)\right)$$

$\chi = 0.223(4)_{\text{stat}}(4)_{\phi_\pi}$

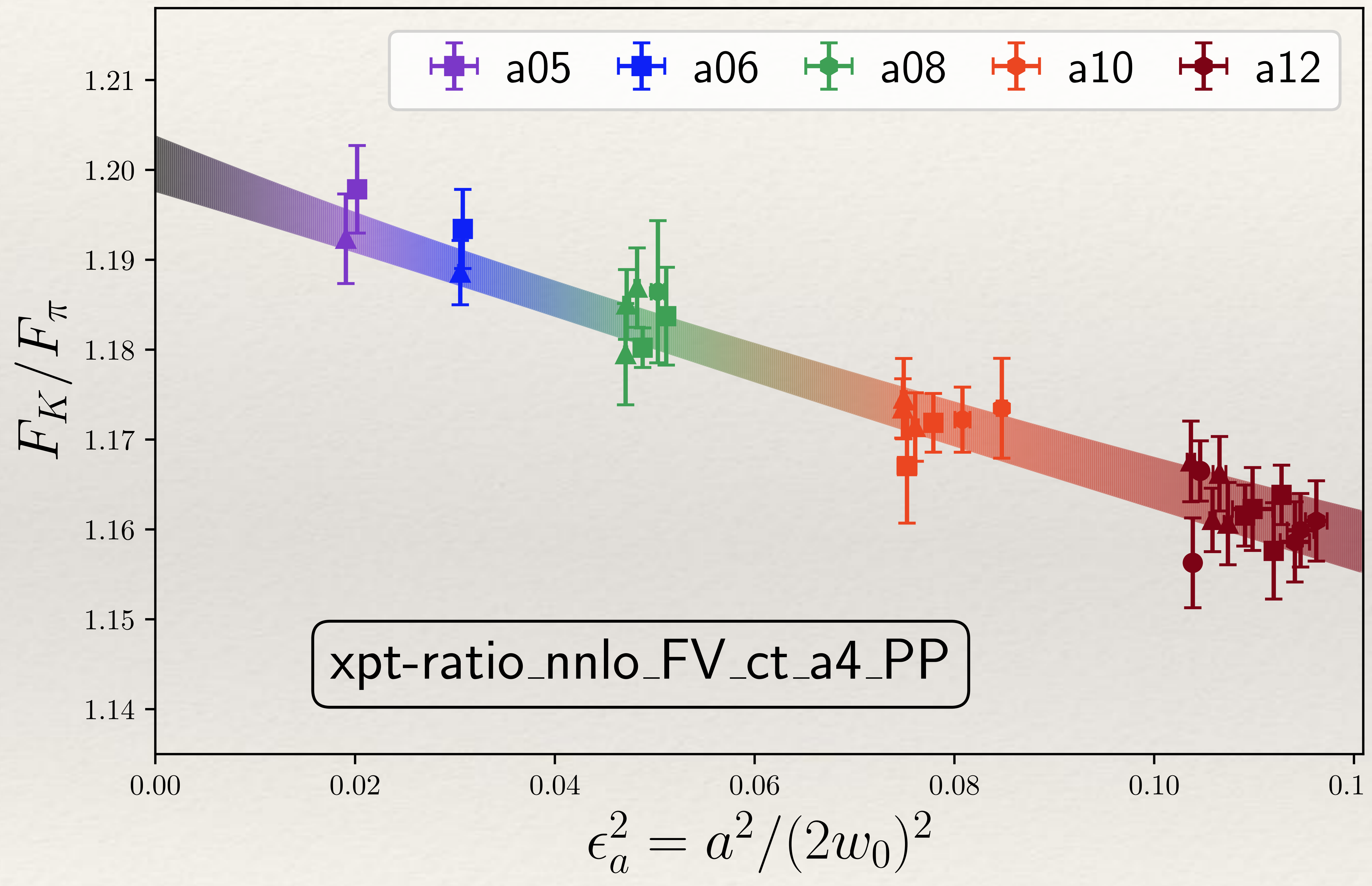


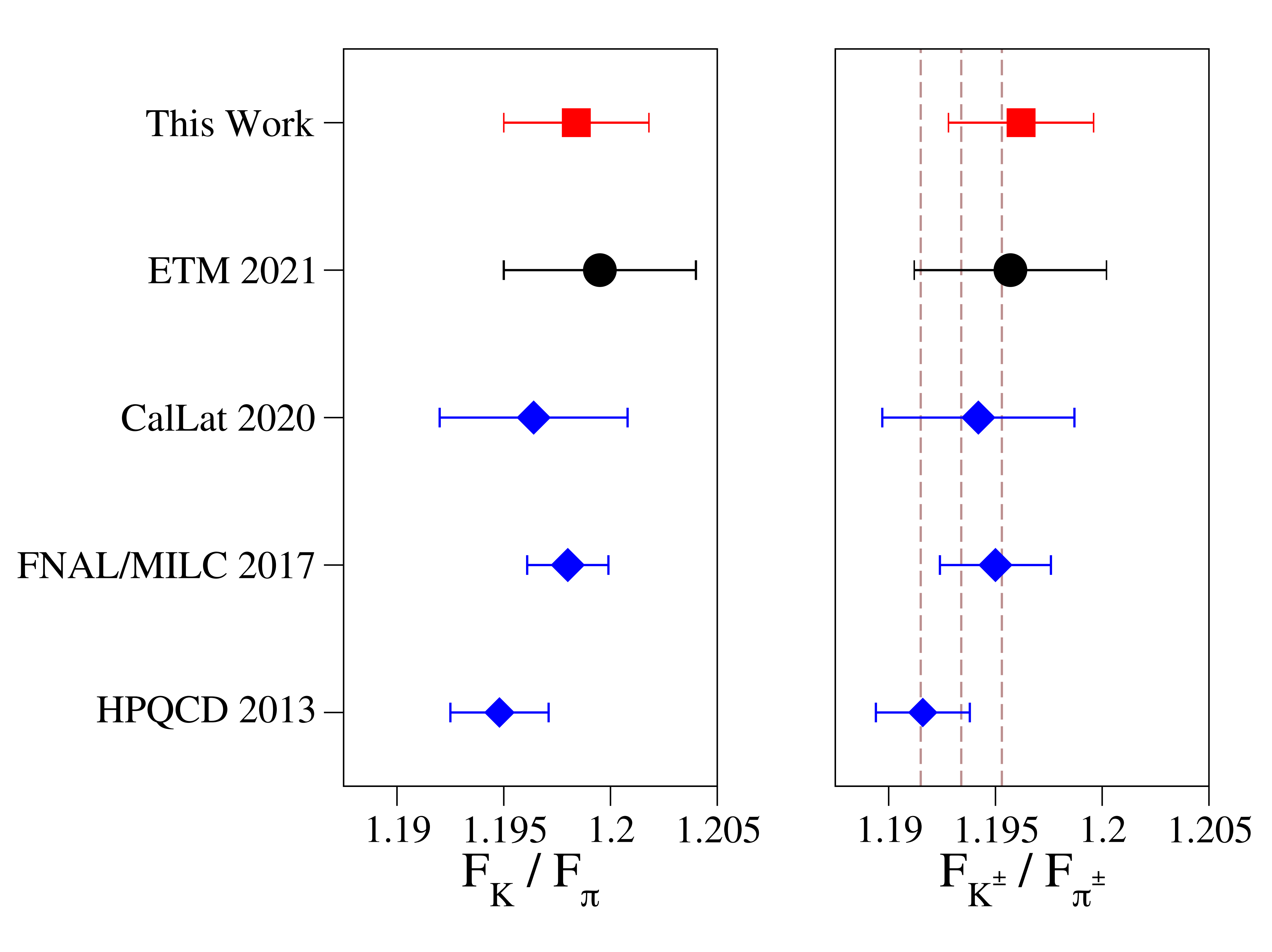
Aoki et al, JLQCD, PTEP 4 (2018) [1705.10906]



F_K/F_π as a precision test

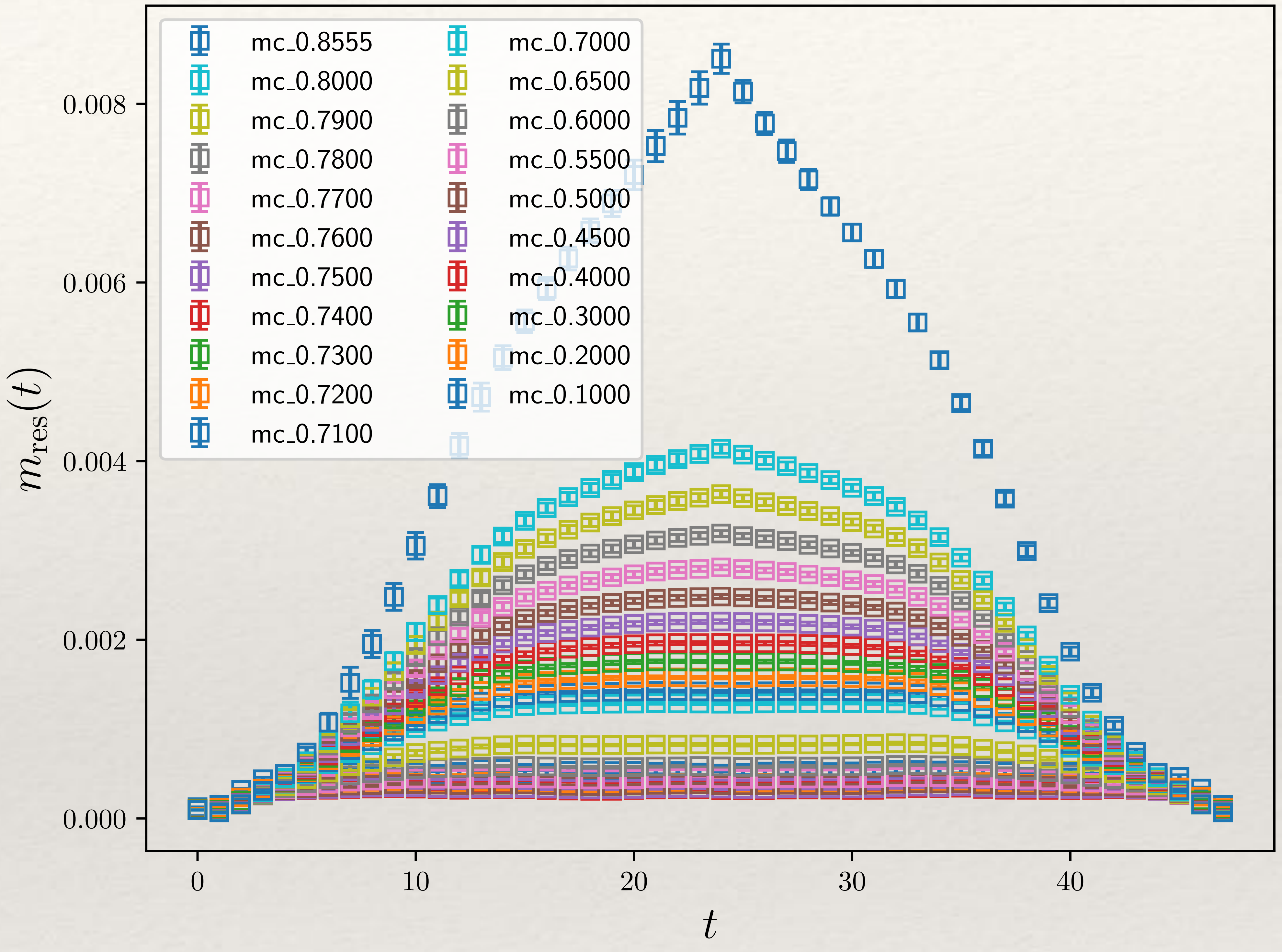
- We computed F_K/F_π as a high-precision test - [2605.06560] — see Zack Hall's talk after this one





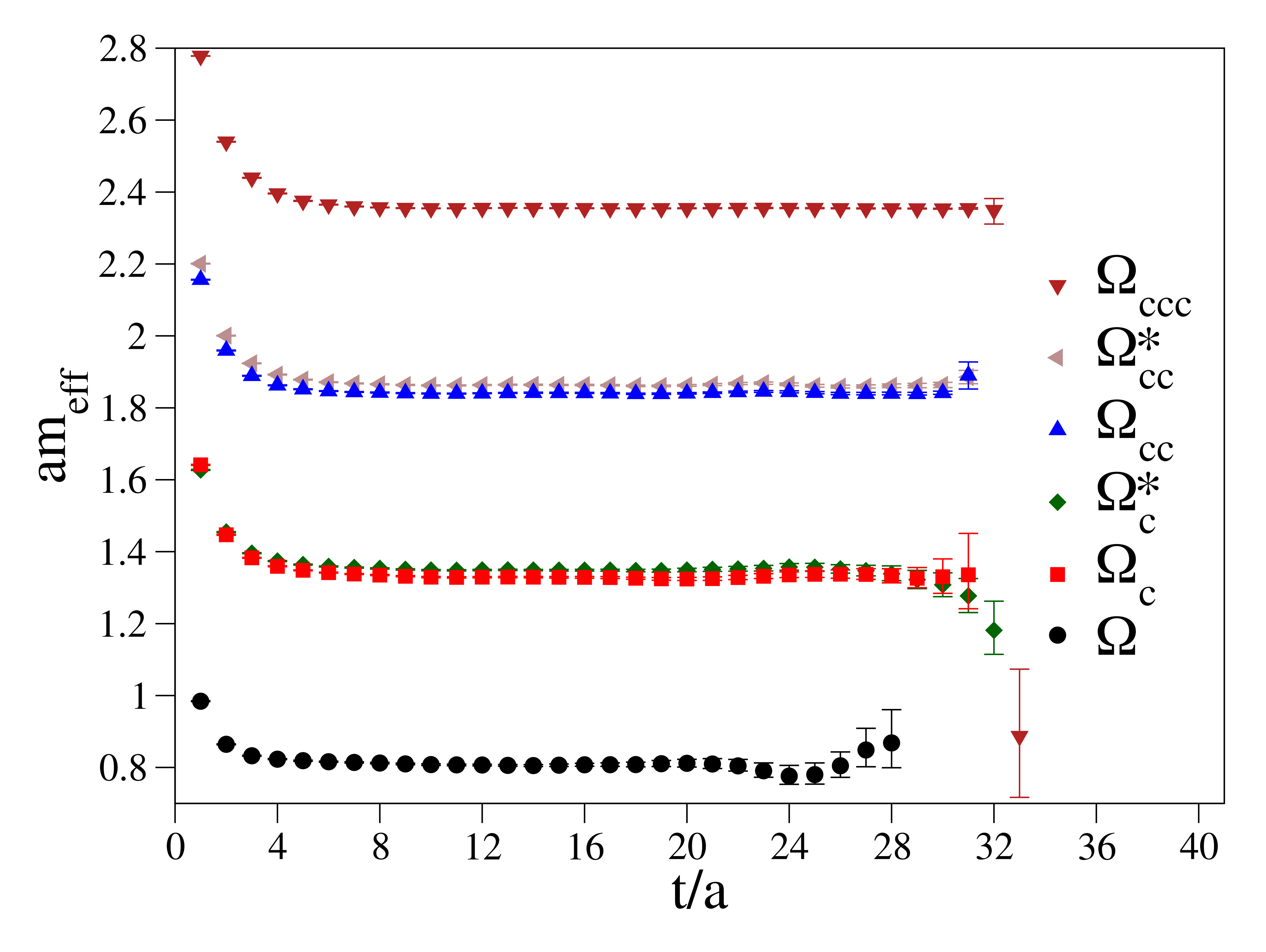
Charmed Quarks

- On the coarsest ensemble, we observe some issues with the charmed fermion
  - m_{res} correlator does not plateau
  - η_{cc} correlators with A_4 and P sinks do not exhibit same effective mass ground state
  - F_{D_s}/F_D not obviously wrong on coarsest lattice spacing
  - We are investigating if we can extrapolate quantities such as m_{res} and the η_c mass using partially quenched valence charm mass from lighter up to sea value $am_c = 0.8555$



Ω_c, Ω_{cc}: Preliminary

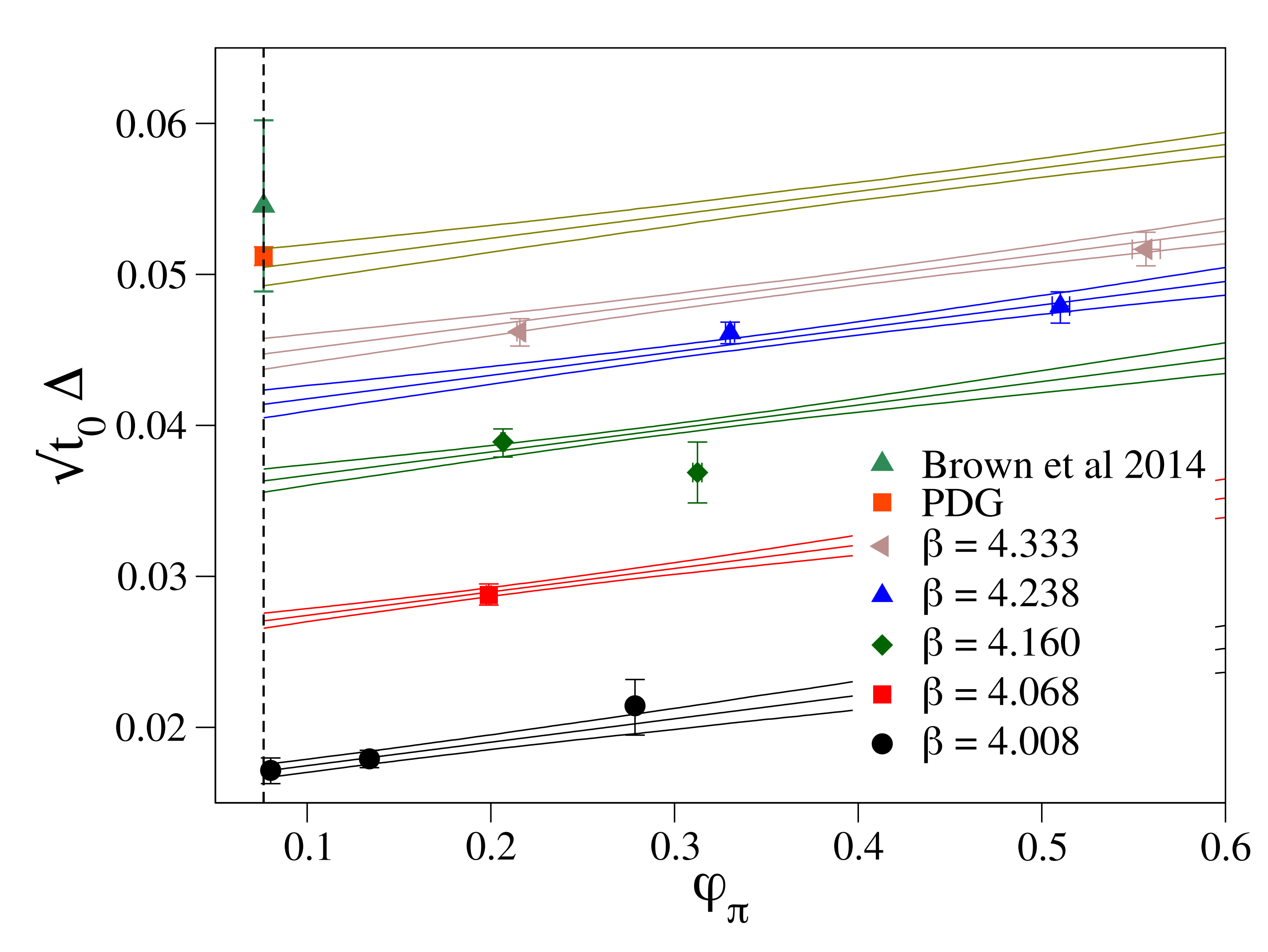
- In contrast, the $\Omega_c = (s^T C\gamma_i s)c$ and $\Omega_{cc} = (c^T C\gamma_i c)s$ correlators show no issues
- We explore the ratio of $J = 3/2$ to $J = 1/2$ correlators to fit the mass splitting
  $$\frac{C^{3/2|+}(t)}{C^{1/2|+}(t)} \approx Ae^{-\Delta t}$$
- LHCb (Beauty 2026) have preliminary $J = 1/2$ Ω_{cc} mass (3725.9(1.5) MeV) and $J = 3/2$ unknown
- The Ω_c splitting is known from Belle 2009 and BaBar 2006: $\Delta = 70.7(9)$ MeV
- Right: sample effective mass plots from Coulomb gauge fixed wall sources with Gaussian profile for single to triply charmed Ω



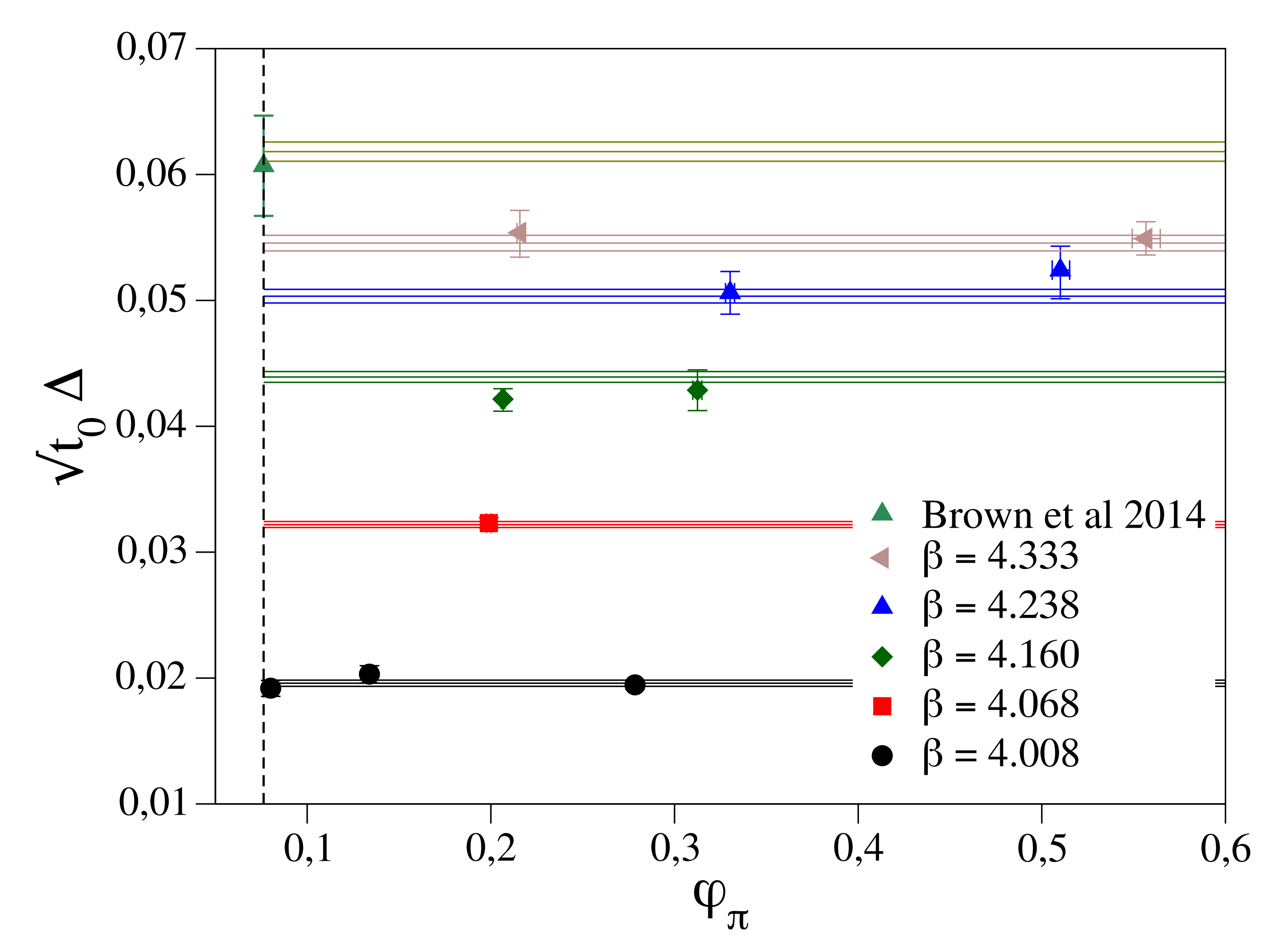
Ω_c, Ω_{cc}: Preliminary

- Simple continuum + chiral fit to mass splittings

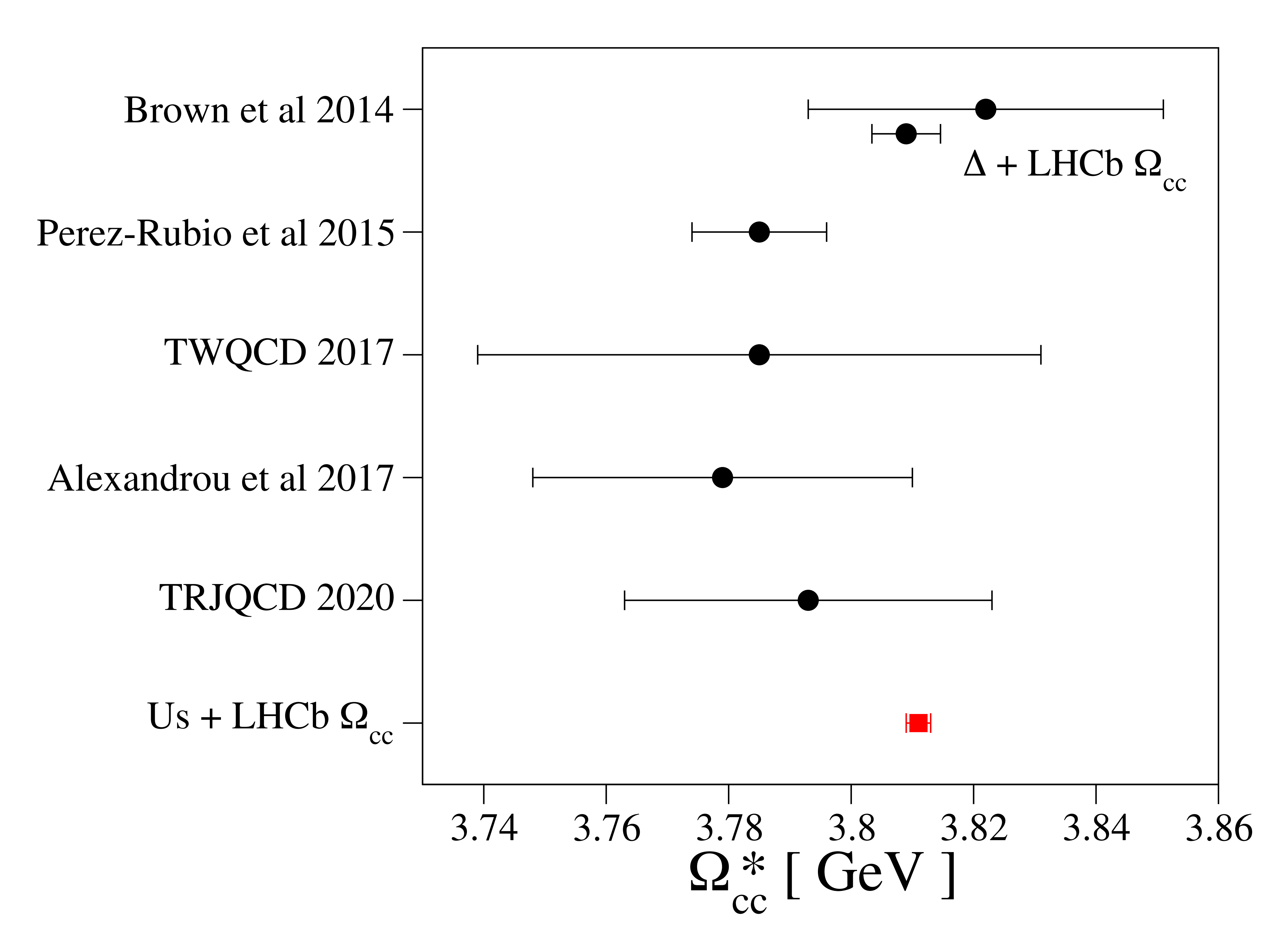
$$\sqrt{t_0}\Delta(\phi_\pi^2, a^2/t_0) = \sqrt{t_0^{\mathrm{Cont}}}\Delta(\phi_\pi^{\mathrm{Cont}},0)\left(1 + A(\phi_\pi - \phi_\pi^{\mathrm{Cont}}) + Ba^2/t_0\right)$$



Δ_{Ω_c}



$\Delta_{\Omega_{cc}}$



$$\Delta_{\Omega_{cc}} = 85.4(1.2)\ \mathrm{MeV}$$

$$M_{\Omega_{cc}^*} = 3811(2)\ \mathrm{MeV}$$

Future studies of the action

- Spectroscopy study using $\pi\pi$ systems is underway
  eg. "perambulators" computed by T. Donovan/A. Hanlon
  $I = 0$ $\pi\pi$ correlators computed by M. Salg
  a12m375 ensemble
  - Note: $I = 0$ may require use of tetra-quark operators to get the correct spectrum — see Hanlon et al, PRD 114 (2026) [2603.19192]
- From our h-dibaryon continuum study with many actions we do not expect any surprises, but good to check

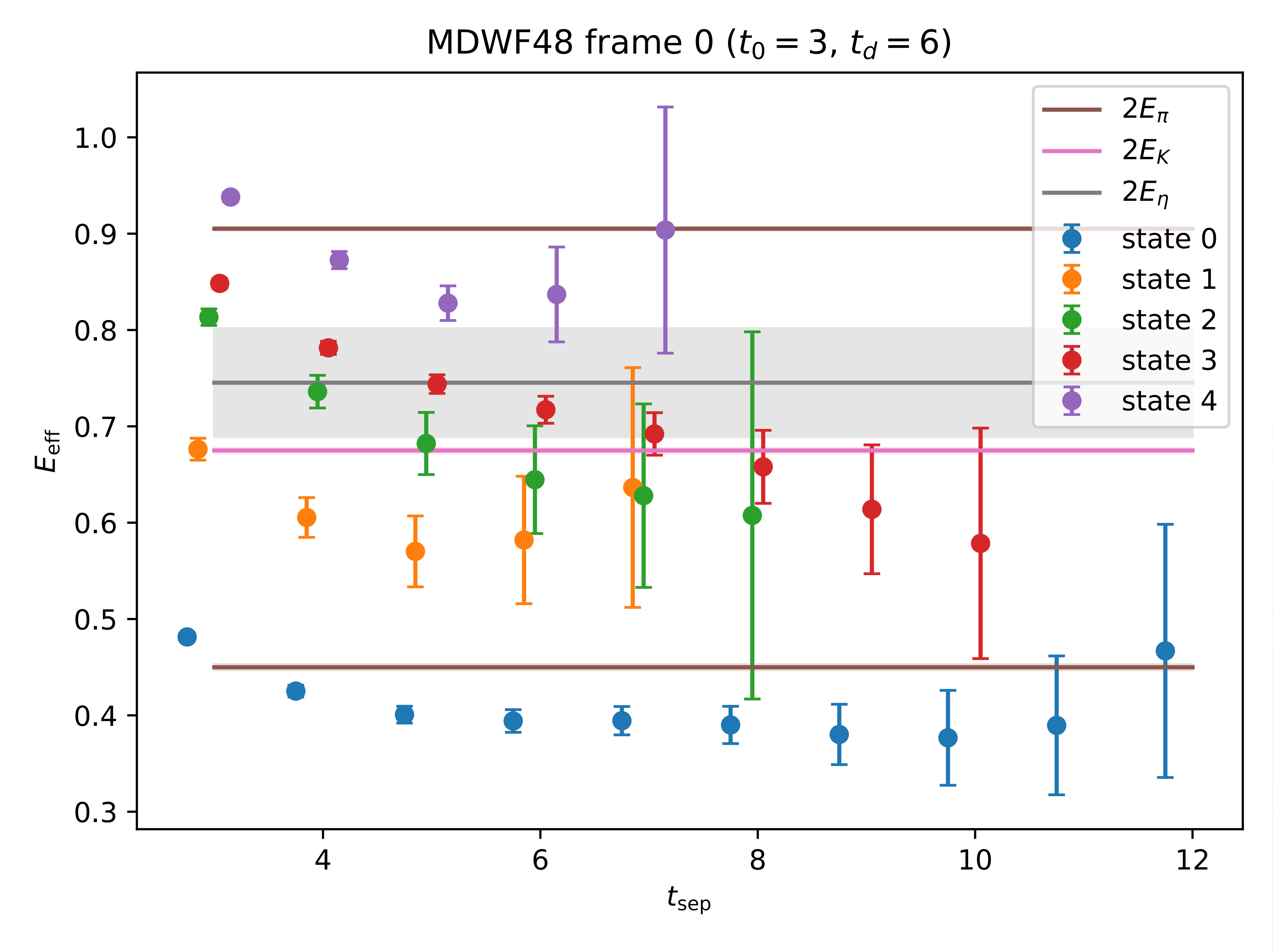
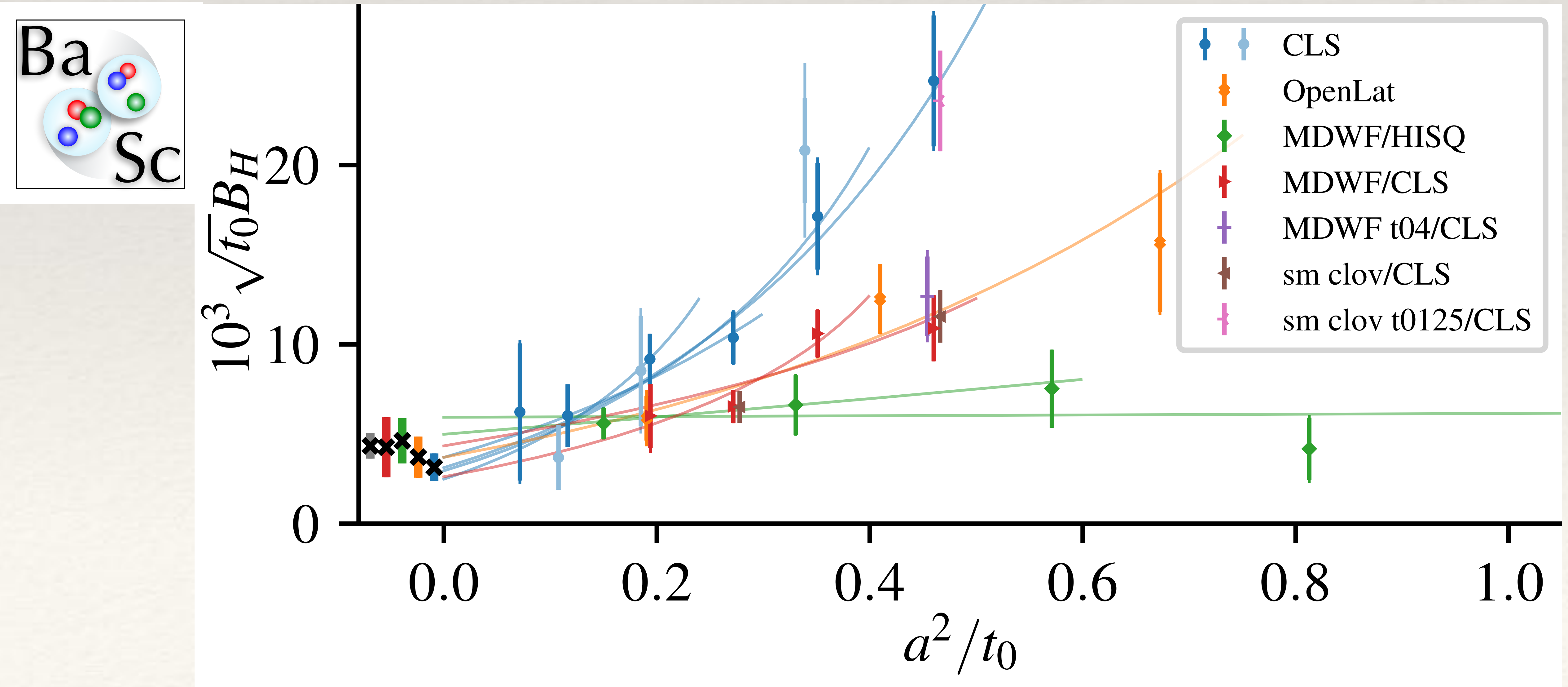


figure: M. Salg



to be arXiv'd

figure: J. Green

Future studies of the action

- An “action paper” is in preparation that will contain the details presented here as well as more
- High-precision calculation of F_{D_s}/F_D is underway
- We have started a pilot study of $K \to \pi \ell \nu$ on a constant line of physics @ $m_\pi \approx 375$ MeV
- We have started generating two more physical pion mass ensembles
  and several larger volume ensembles at lighter pion mass
- We are generating SU(3)-flavor symmetric ensembles to cary out various studies
  - t_0/t_1
  - NPR
  - …
- Assuming no “gotchas”, we hope to shift our full physics program to these ensembles

Data availability

- All configurations from our first paper — 2605.06560 — are available via Globus from the Globus "File Manager", search for the "NERSC Gauge Connection" (which is unfortunately down at the moment from the NERSC upgrade)

F_K/F_π as a precision test of a new four flavor Domain Wall fermion action

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- We intend to keep dumping configs in this public domain as we use them in publications — feel encouraged to check them out!

- Configurations generated with a fork of Grid [Boyle, Yamaguchi, Cossu, Portelli, 1512.03487]
  https://github.com/RJHudspith/Grid
  that supports efficient force computations from stout smearing
  as well as different integrators on different levels

- Perambulators computed with quda_laph — https://github.com/cosmon-collaboration/quda_laph

- Pion/Kaon/Omega etc. 2pt functions computed with Wit [R.J. Hudspith, J. Green, S. Paul]

Conclusions & Outlook

- We have generated a promising new 2+1+1 MDWF action

- I didn't emphasize, but, the 30 ensembles from our first paper were generated with a modest allocation on NERSC in ~1.2 years — signaling a relatively inexpensive set of MDWF ensembles

- Our first high-precision scrutniy is available on the arXiv: F_K/F_π with more to come, eg. F_{D_s}/F_D

- An action paper is being prepared

- The configurations are in the public domain

- If you have any questions, let us know!

Thank You!