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Symmetric mass generation in a three-dimensional fermion lattice model

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work with **S. Maiti & D. Banerjee**, and **S. Chandrasekharan**

[arXiv:2512.24836] and [arXiv:2602.18360]

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Outline

- Mass generation mechanisms
- 3D model and symmetries
- Fermion bag algorithm
- Phase diagram
- Summary and future directions

Maiti, Banerjee, Chandrasekharan, Marinkovic arXiv: 2512.24836
Maiti, Banerjee, Chandrasekharan, Marinkovic arXiv: 2602.18360


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Mass generation in QFTs

- Mechanisms of mass generation in lattice field theories alternate to spontaneous symmetry breaking [Catterall JHEP 01,121 (2015), Ayyar, Chandrasekharan, PhysRevD.91.065035 (2015)]

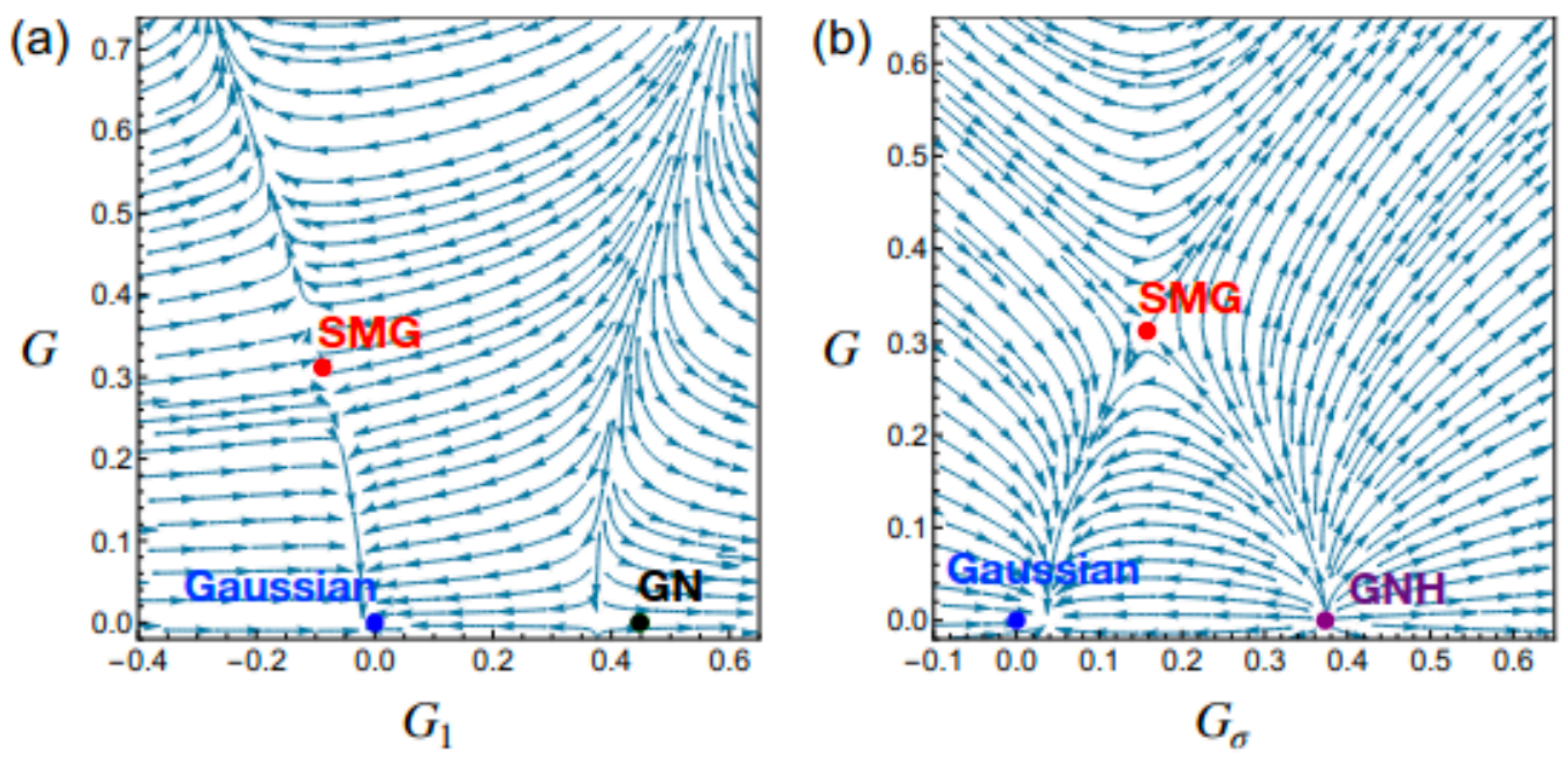
- Mass terms are often an emergent property
  - fermion bilinear condensates, spontaneously broken symmetry
  - exotic massive phases: topological origins, symmetric mass generation: four-fermion couplings

- Examples of mass generation:
  1. Explicit masses: $\mathscr{L} = \overline{\psi} i \partial\!\!\!/ \psi + m \overline{\psi} \psi$
  2. Dynamical spontaneous symmetry breaking: $\mathscr{L} = \overline{\psi} i \partial\!\!\!/ \psi + g(\overline{\psi}\psi)^2$, at couplings stronger than critical: $\langle \overline{\psi}\psi \rangle \neq 0$
  3. Topological origins [Callan-Harvey mechanism, 1984], …, [Talks by J. Dang and S. Onoda, Tuesday 4.50pm and 5.10pm, Theoretical Developments]

Symmetric mass generation

- Generate masses without topology or spontaneous symmetry breaking [Catterall JHEP 01,121 (2015), Ayyar, Chandrasekharan, PhysRevD.91.065035 (2015)]
- Strong interaction a prerequisite: typically multi-fermion interactions; symmetric gapped phase in the IR
- No t'Hooft anomalies $\longrightarrow$ relation to chiral gauge theories
- Studied in LFT from the 1980ies, but regarded as lattice artefacts [Hasenfratz, Neuhaus PRL 220, 435 (1989)]
- Recent work on pure fermionic theories, fermions and gauge theories, and anomaly cancelling mechnaisms [Hasenfratz, Xu, arXiv:2604.02424]
- New fixed point in the RG flow of fermionic theories

- Microscopic models where three phases could coexist:
    1. the massless fermion phase (MF)
    2. the traditional massive phase (SSB)
    3. the exotic massive phase (SMG)



Direct transition between MF and SMG phase [Martin, Grover, arXiv:2507.23032]

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Microscopic lattice model & symmetries: Gross-Neveu in 3D

- $N_f = 2$ massless staggered fermions ψ, χ, cubic lattice $n_0, n_1, n_2 = 0, \ldots, L-1$

- Two couplings, U_I and U_B such that both mechanisms of fermion mass generation are possible and can compete

$$S = \sum_{\langle x,y \rangle} \left(\bar{\psi}_x M_{x,y} \psi_y + \bar{\chi}_x M_{x,y} \chi_y \right) \underbrace{- U_{\mathrm{I}} \sum_x \left(\bar{\psi}_x \psi_x \bar{\chi}_x \chi_x \right)}_{S_I} - U_{\mathrm{B}} \sum_{\langle x,y \rangle} \left(\bar{\psi}_x \psi_x \bar{\psi}_y \psi_y + \bar{\chi}_x \chi_x \bar{\chi}_y \chi_y \right)$$

[Ayyar, Chandrasekharan, PRD 91.065035 (2015), PRD 93, 081701 (2016)]

- Rename $\psi \to u-$quark, $\chi \to d-$quark; η_{ij}: staggered phases

$$S = \sum_{\langle ij \rangle} \frac{\eta_{ij}}{2} \left(\bar{u}_i u_j - \bar{u}_j u_i + \bar{d}_i d_j - \bar{d}_j d_i \right) \underbrace{- U_I \sum_i \left(\bar{u}_i u_i \, \bar{d}_i d_i \right)}_{S_I} \underbrace{- U_B \sum_{\langle ij \rangle} \left(\bar{u}_i u_i \, \bar{u}_j u_j + \bar{d}_i d_i \, \bar{d}_j d_j \right)}_{S_B}$$

[addtional interaction, arXiv: 2512.24836, arXiv: 2602.18360]

Microscopic lattice model & symmetries: Gross-Neveu in 3D

$$S = \underbrace{\sum_{\langle ij \rangle} \frac{\eta_{ij}}{2} \left(\bar{u}_i u_j - \bar{u}_j u_i + \bar{d}_i d_j - \bar{d}_j d_i\right)}_{S_0} \underbrace{- U_I \sum_i \left(\bar{u}_i u_i \, \bar{d}_i d_i\right)}_{S_I} \underbrace{- U_B \sum_{\langle ij \rangle} \left(\bar{u}_i u_i \, \bar{u}_j u_j + \bar{d}_i d_i \, \bar{d}_j d_j\right)}_{S_B}$$

- S_0 invariant under SU(4)×U(1)

$$\begin{pmatrix} u_i \\ \bar{u}_i \\ d_i \\ \bar{d}_i \end{pmatrix} \longrightarrow V e^{i\theta} \begin{pmatrix} u_i \\ \bar{u}_i \\ d_i \\ \bar{d}_i \end{pmatrix} \qquad \begin{pmatrix} u_j \\ \bar{u}_j \\ d_j \\ \bar{d}_j \end{pmatrix} \longrightarrow V^* e^{-i\theta} \begin{pmatrix} u_j \\ \bar{u}_j \\ d_j \\ \bar{d}_j \end{pmatrix} \qquad V \in SU(4)$$

- S_I invariant under SU(4), but explicitly breaks the U(1) symmetry
- S_B invariant under SU(2) $\times \, \mathrm{U}(1)_{u-\mathrm{quark}}$ and SU(2) $\times \, \mathrm{U}(1)_{d-\mathrm{quark}}$

$$\begin{pmatrix} u_i \\ \bar{u}_i \end{pmatrix} \longrightarrow V_u e^{i\theta_u} \begin{pmatrix} u_i \\ \bar{u}_i \end{pmatrix} \qquad \begin{pmatrix} u_j \\ \bar{u}_j \end{pmatrix} \longrightarrow V_u^* e^{-i\theta_u} \begin{pmatrix} u_j \\ \bar{u}_j \end{pmatrix}$$

$U_B = 0, \, U_I \neq 0$ $\;SU(4)$ symmetry intact

$U_B \neq 0, \, U_I \neq 0$ $\;SU(2) \times SU(2) \times \mathrm{U}(1)_\chi$

- In addition to the $U(1)$ symmetry of S_I for $\theta_u = \theta_d = \theta$, another independent symmetry of S_B: $\mathrm{U}(1)_\chi$ for $\chi = \theta_u = -\theta_d$
- Typically not sign-problem free

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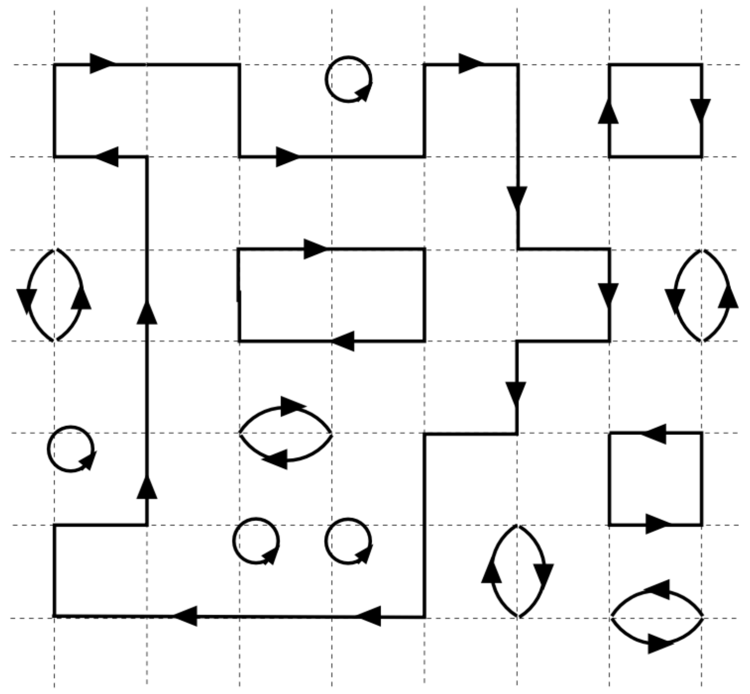
Worldline representation

- A fermionic model on a single layer

$$Z = \int [d\bar{\psi} d\psi] e^{\sum_{\langle ij \rangle} \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j} = \sum_{[C]} W(C)$$



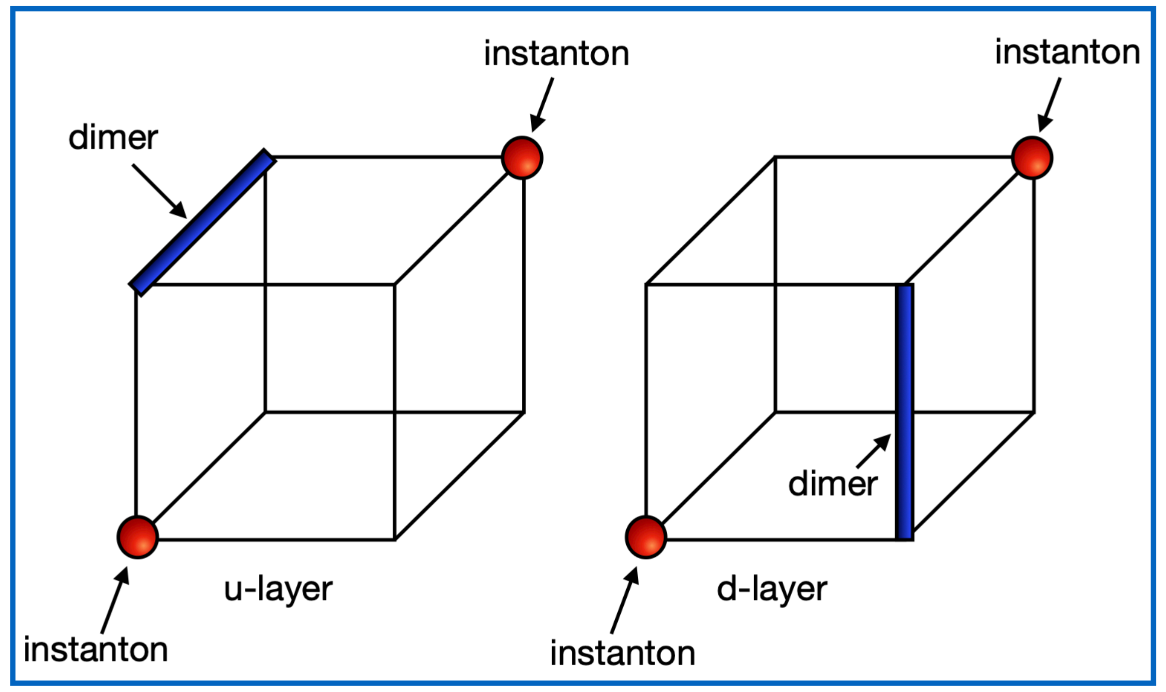
Example of C

- Our model: two "layers" of 3D cubic lattices: u-layer and d-layer

$$Z = \int [d\bar{\psi} d\psi]\ [d\bar{\chi} d\chi]\ e^{-\sum_{i,j} (\bar{\psi}_i M_{ij} \psi_j + \bar{\chi}_i M_{ij} \chi_j)} \times e^{U_I \sum_{i,j} \bar{\psi}_i \psi_i \bar{\chi}_i \chi_i} \times e^{U_B \sum_{i,j} (\bar{\psi}_i \psi_i \bar{\psi}_j \psi_j + \bar{\chi}_i \chi_i \bar{\chi}_j \chi_j)}$$

- Expand the interactions, e.g. $e^{U_B \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j} = 1 + \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j = \sum_{n_b^u = 0,1} (U_B \bar{\psi}_i \psi_i \bar{\psi}_j \psi_j)^{n_b}$

$$e^{U_I \bar{u}_i u_i \bar{d}_i d_i} = \sum_{n_i = 0,1} (U_I \bar{u}_i u_i \bar{d}_i d_i)^{n_i},$$
$$e^{U_B \bar{u}_i u_i \bar{u}_j u_j} = \sum_{n_b^u = 0,1} (U_B \bar{u}_i u_i \bar{u}_j u_j)^{n_b^u},$$
$$e^{U_B \bar{d}_i d_i \bar{d}_j d_j} = \sum_{n_b^d = 0,1} (U_B \bar{d}_i d_i \bar{d}_j d_j)^{n_b^d},$$



instanton-dimer configuraiton

Sign-problem free:

$$Z = \int [d\bar{\psi} d\psi] e^{-\sum_{x,\mu} \frac{\eta_{x,\mu}}{2} (\psi_x \psi_{x+\hat{\mu}} - \psi_{x+\hat{\mu}} \psi_x)} = \underbrace{\text{Det}\,(W_k[n])}_{\text{positive}}$$

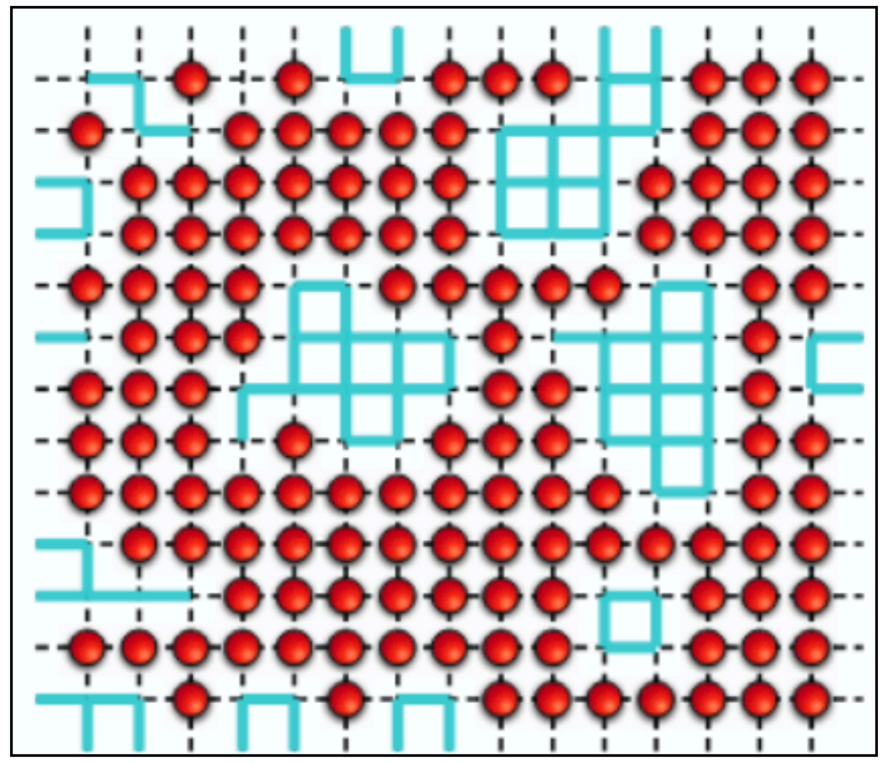
- $n_i = 0,1$ instanton at site i, $n_b^u, n_b^d = 0,1$ u, d - type dimer on a bond b

Monte Carlo algorithm: strong vs. weak coupling fermion bags

STRONG COUPLING FERMION BAGS:

$$Z = \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \det(W_u) \det(W_d)$$

- W_u and W_d: staggered fermion matrices restricted to the unoccupied sites
- Connected clusters of unoccupied sites
- Fermion bags: regions where fermions can freely propagate



$$\begin{array}{c} \\ bag1 \\ bag2 \\ bag3 \\ bag4 \end{array} \begin{array}{c} \begin{array}{cccc} bag1 & bag2 & bag3 & bag4 \end{array} \\ \left(\begin{array}{c|c|c|c} B_1 & 0 & 0 & 0 \\ \hline 0 & B_2 & 0 & 0 \\ \hline 0 & 0 & B_3 & 0 \\ \hline 0 & 0 & 0 & B_4 \end{array}\right) \end{array}$$

[Ayyar, Chandrasekharan, JHEP 10 (2016) 058]

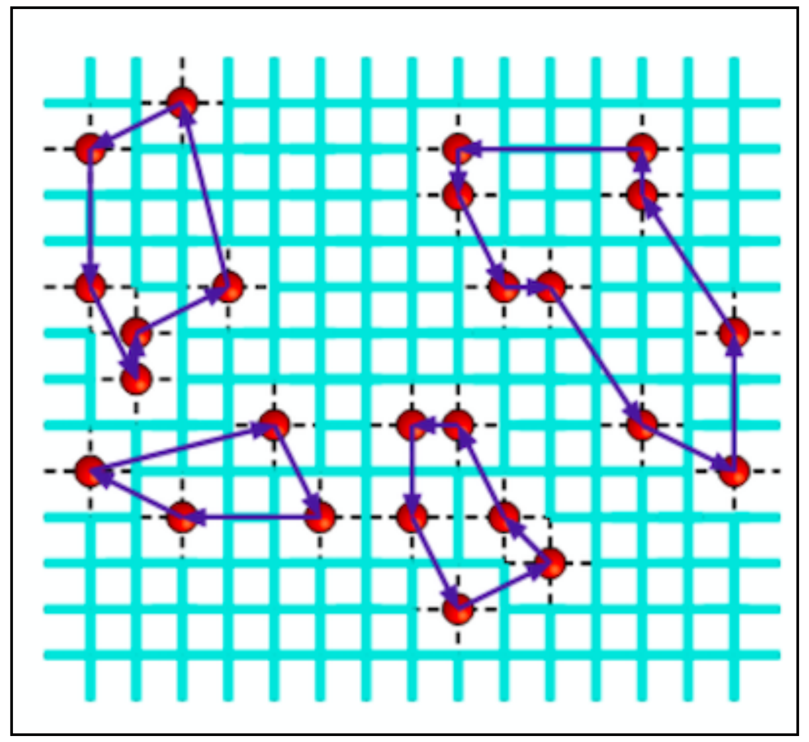
WEAK COUPLING FERMION BAGS:

$$Z = \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \{\bar{u}_{a_1} u_{a_1} \cdots \bar{u}_{a_l} u_{a_l}\} \{\bar{d}_{b_1} d_{b_1} \cdots \bar{d}_{b_k} d_{b_k}\}$$

- $\{a_1, \ldots, a_l\}$ and $\{b_1, \ldots, b_k\}$ sites at which interaction insertions occur in the u and d fermion sectors
- Operator insertions in an expansion about the free fermion

$$Z = (\det M)^2 \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u+N_d} \det G_u \det G_d$$

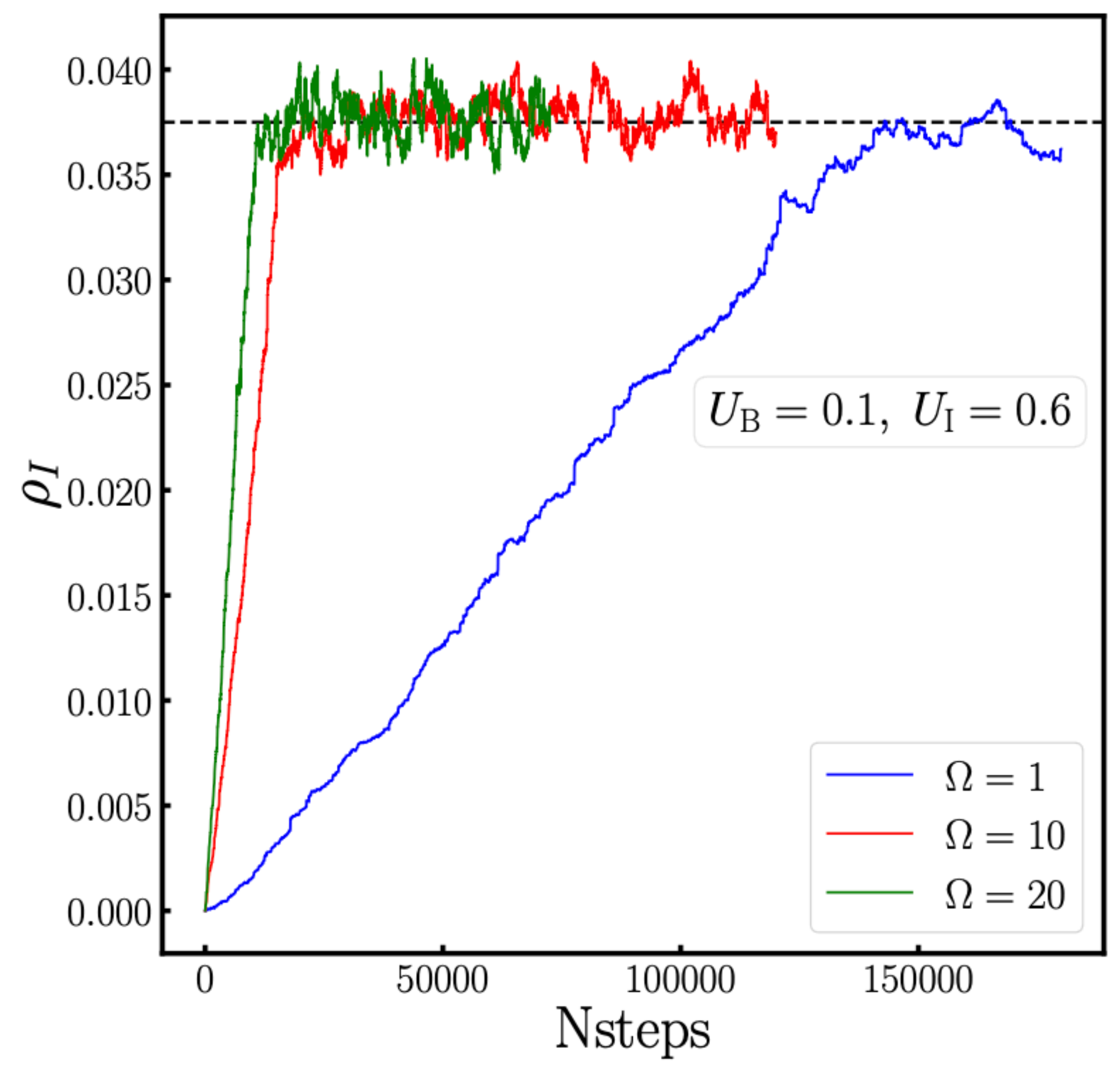
- G_u and G_d: staggered fermion matrices constructed from M^{-1}

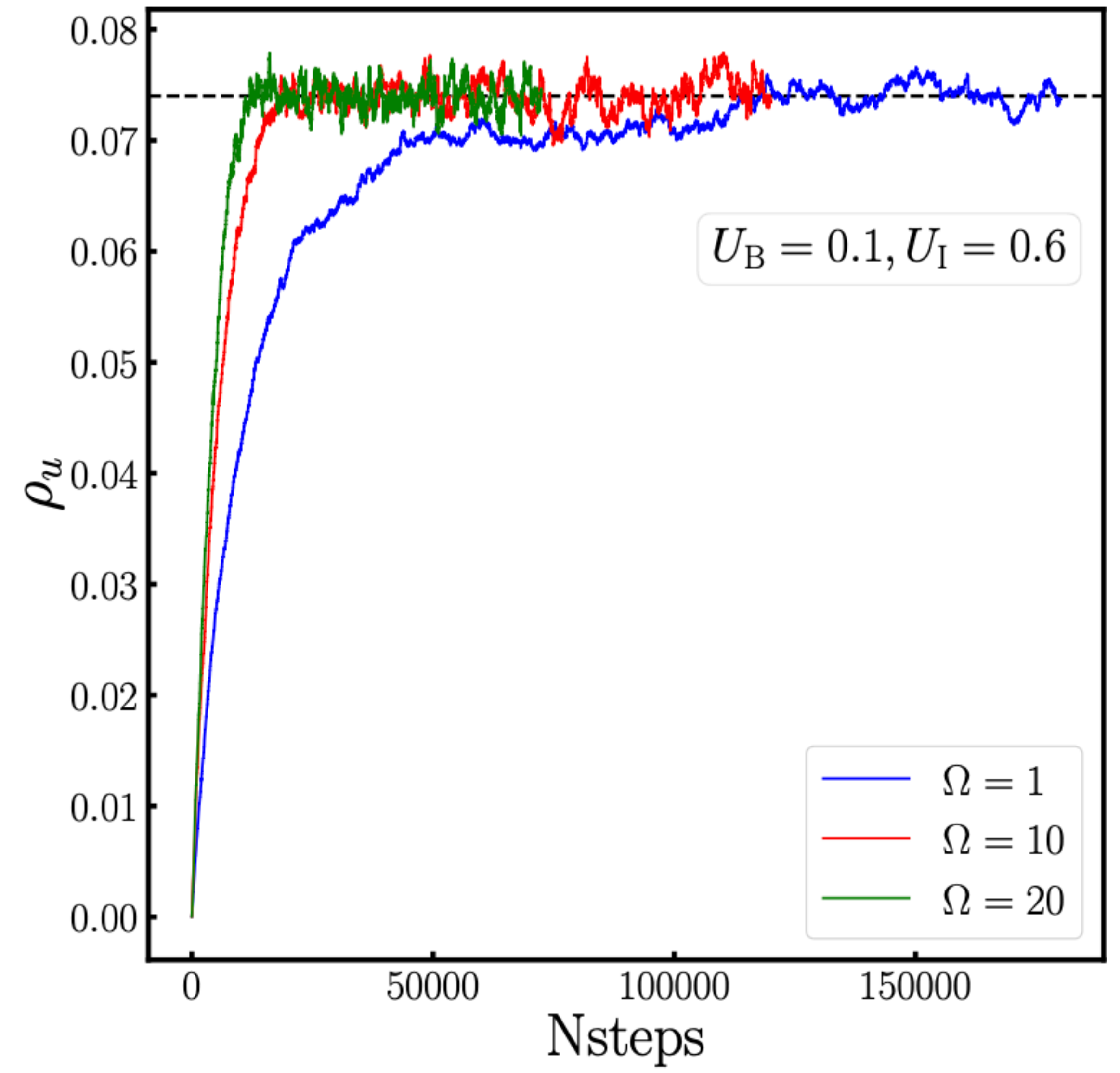


Fermion bags: connected clusters of interaction sites [Diagrammatic Determinant Monte Carlo]

Thermalization and autocorrelations

- Start from weak coupling $Z = (\det M)^2 \sum_{[b,i]} (U_I)^{N_i} (U_B)^{N_u + N_d} \det G_u \det G_d$
- Local update algorithm: add/remove bonds and two instantons at a time $\frac{\det G}{\det G_i} = \det F$
- Non-local updates: worm algorithm
- Algorithmic trick: reweighting factor Ω used to control autocorrelations and thermalization
    1. Ω = 1: 1.8×10^5 steps
    2. Ω = 10: 0.72×10^5 steps
    3. Ω = 20: 0.72×10^5 steps (mostly used)
- Statistics of $2.5 - 8 \times 10^6$ for GN transition, $L = 28\ldots56$
- Autocorrelations $\tau \approx 4000$ close to the phase transition
- Statistics of $2 - 4 \times 10^6$ for XY transition, $L = 12\ldots32$
- Autocorrelations $\tau \approx 1500$ close to the phase transition





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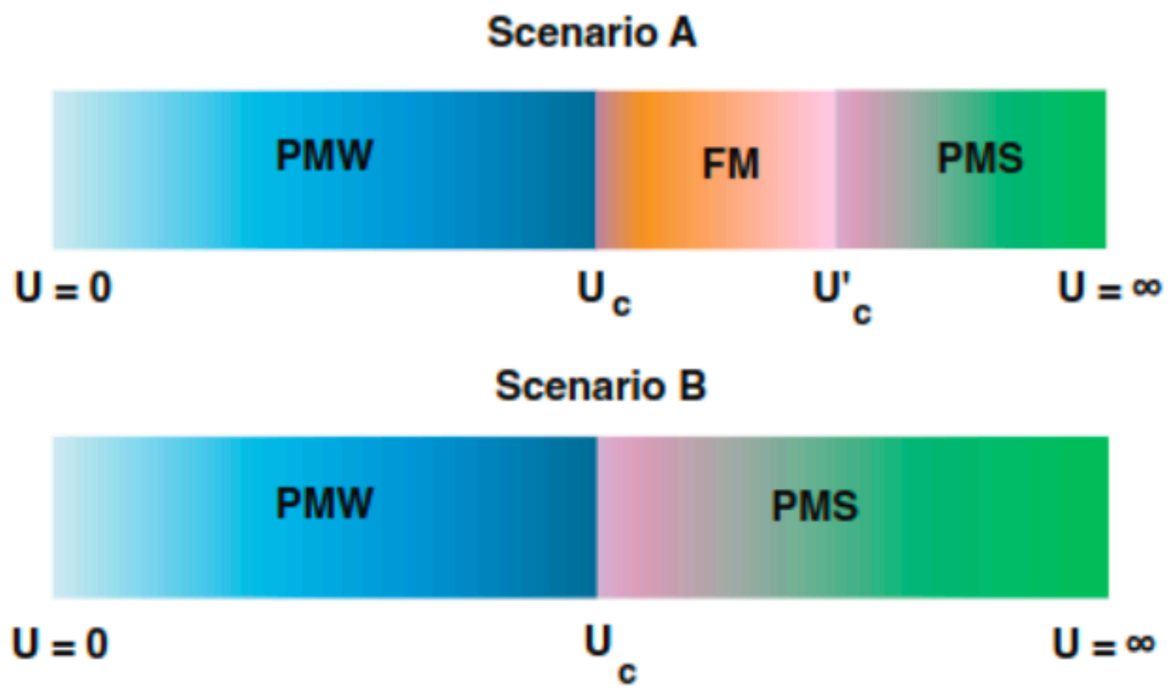
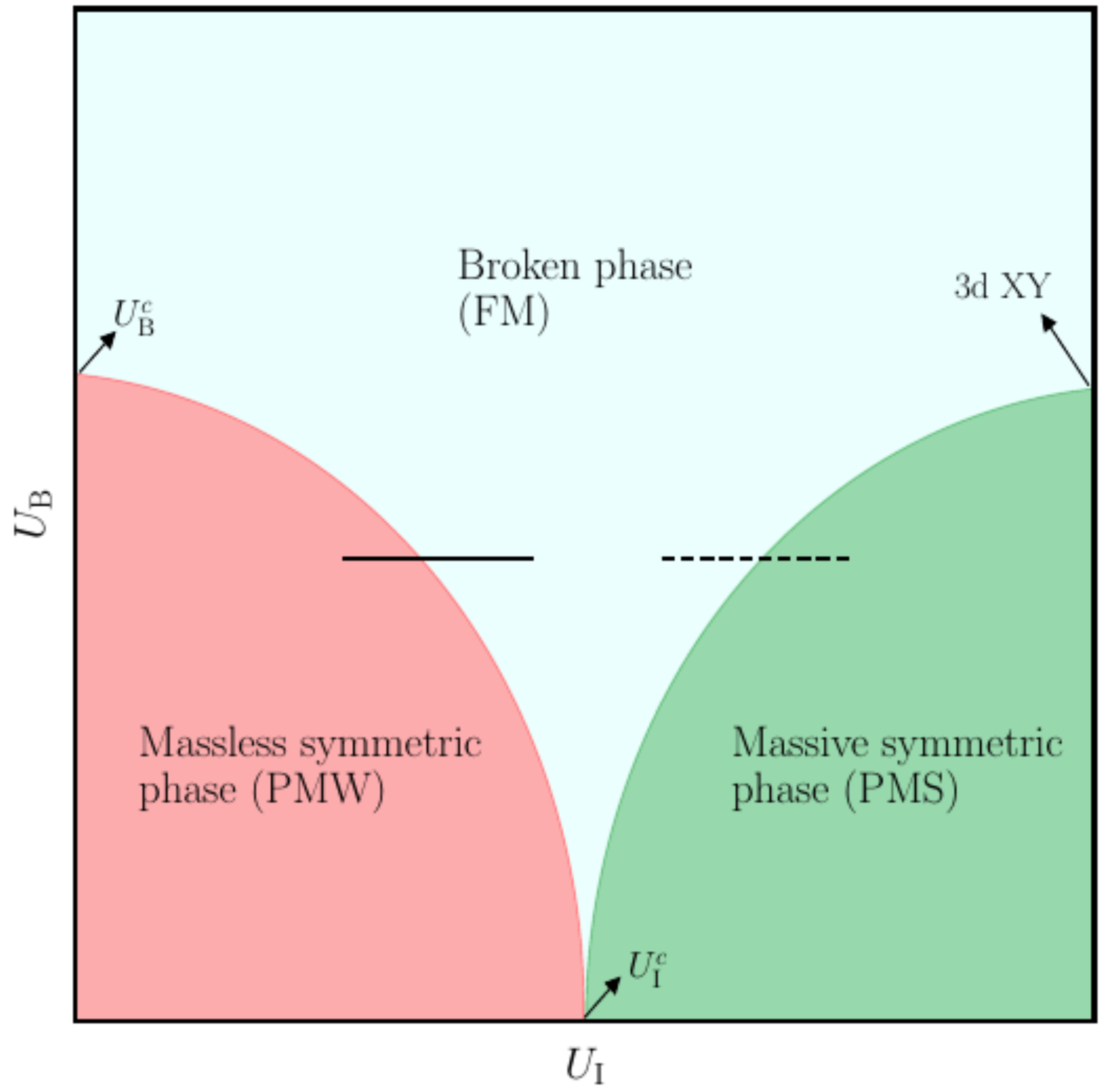
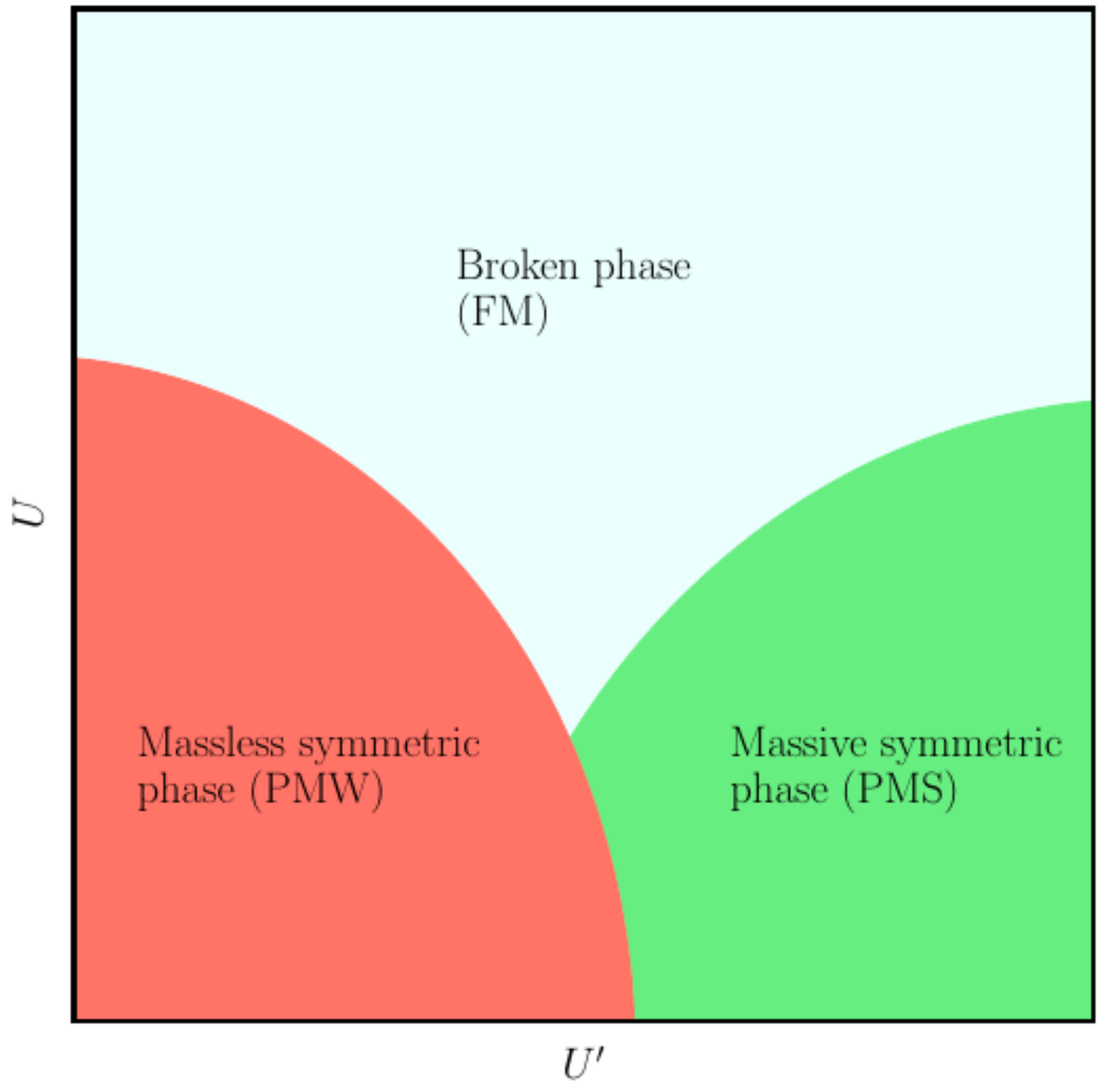
Postulated phase diagrams



- Direct transition from massive fermion (MF/PMW) to massive symmetric phase (SMG/PMS)? [Ayyar, Chandrasekharan, PhysRevD.91.065035 (2015)]





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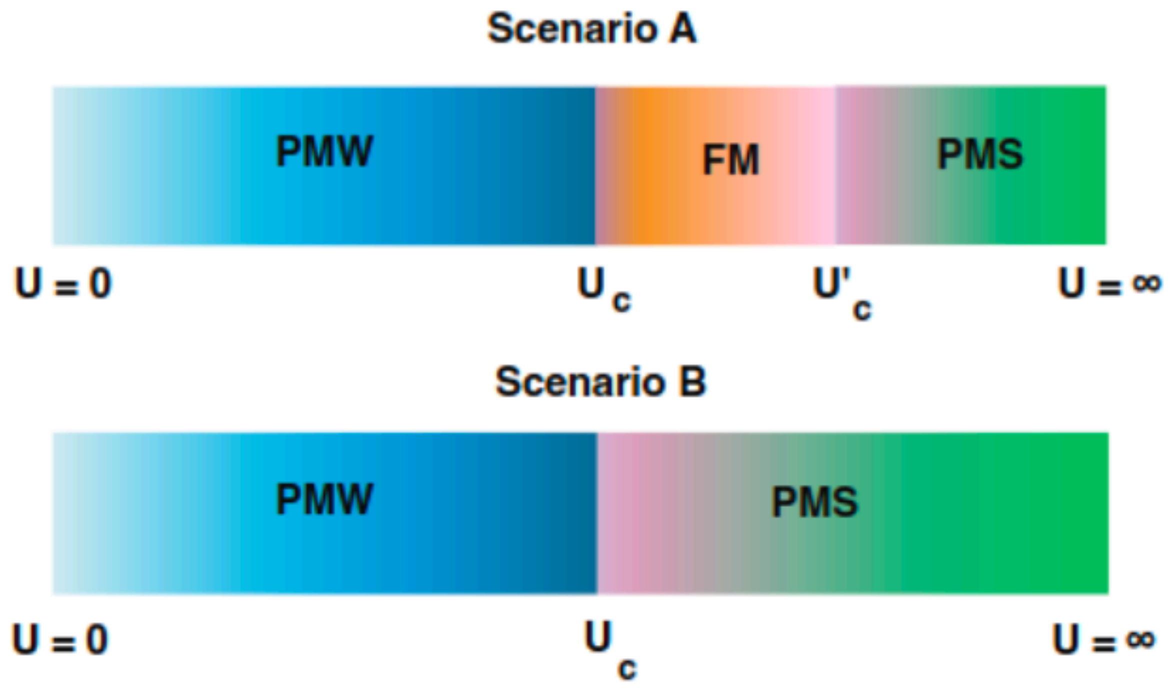

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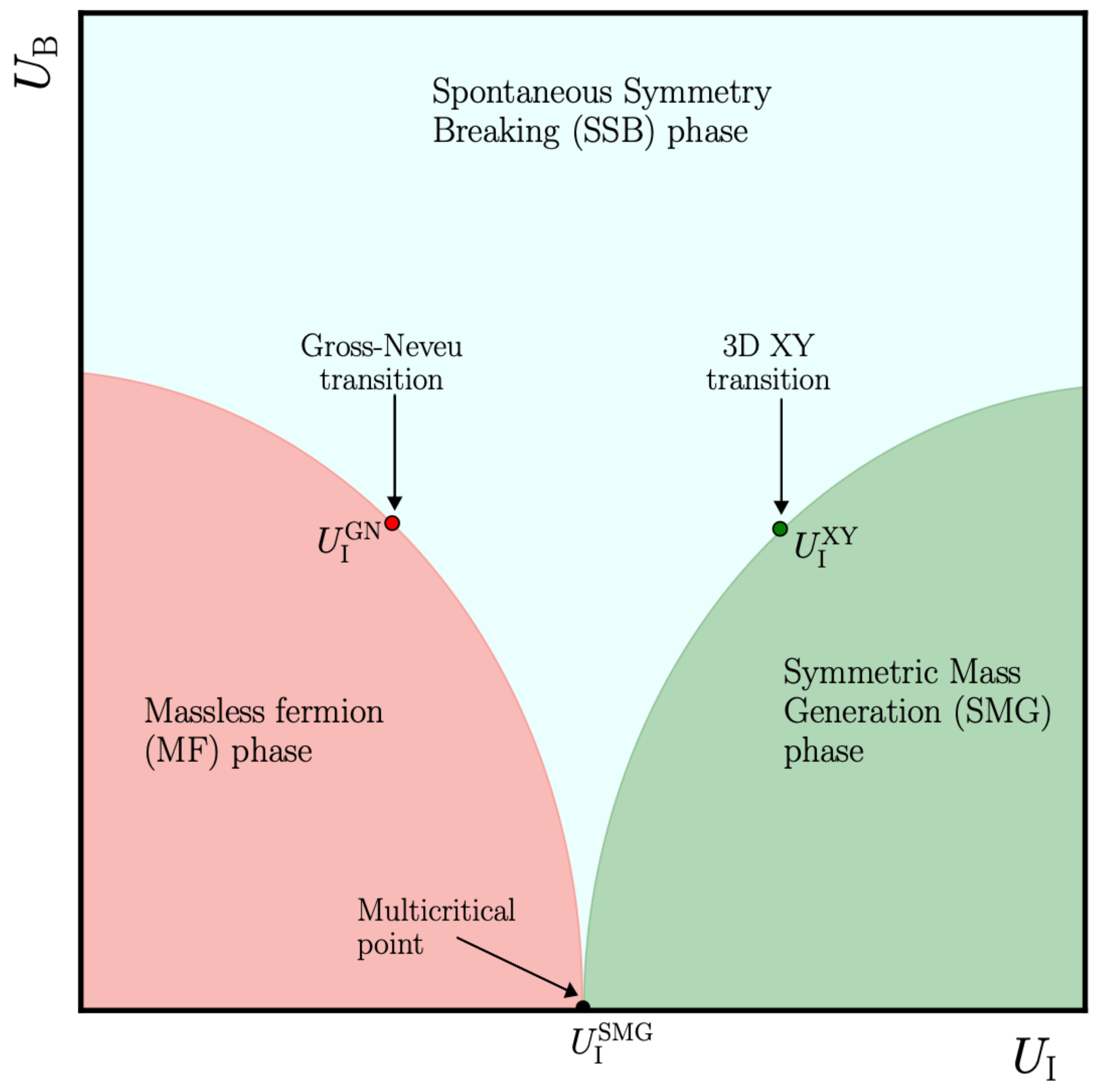
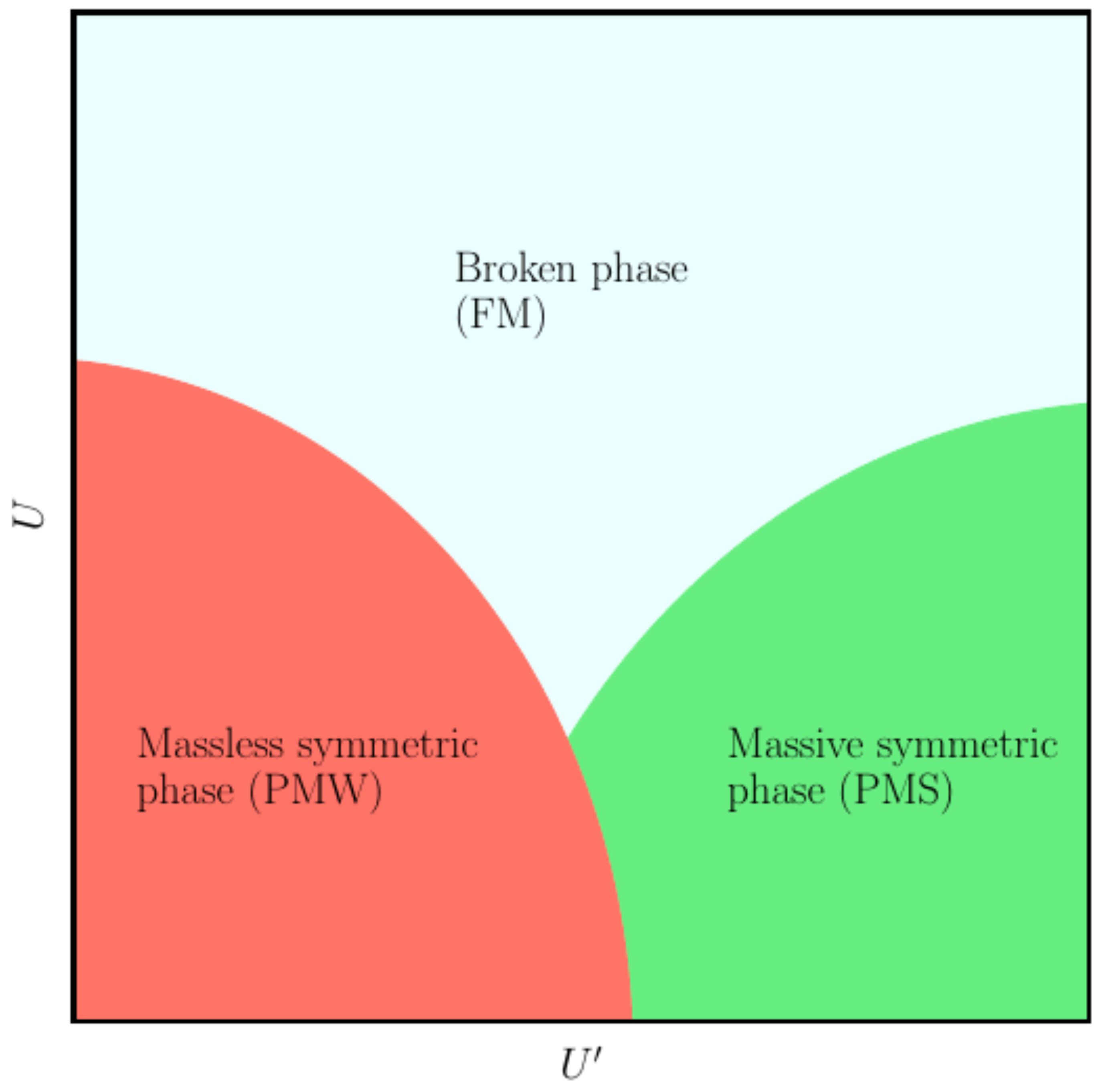
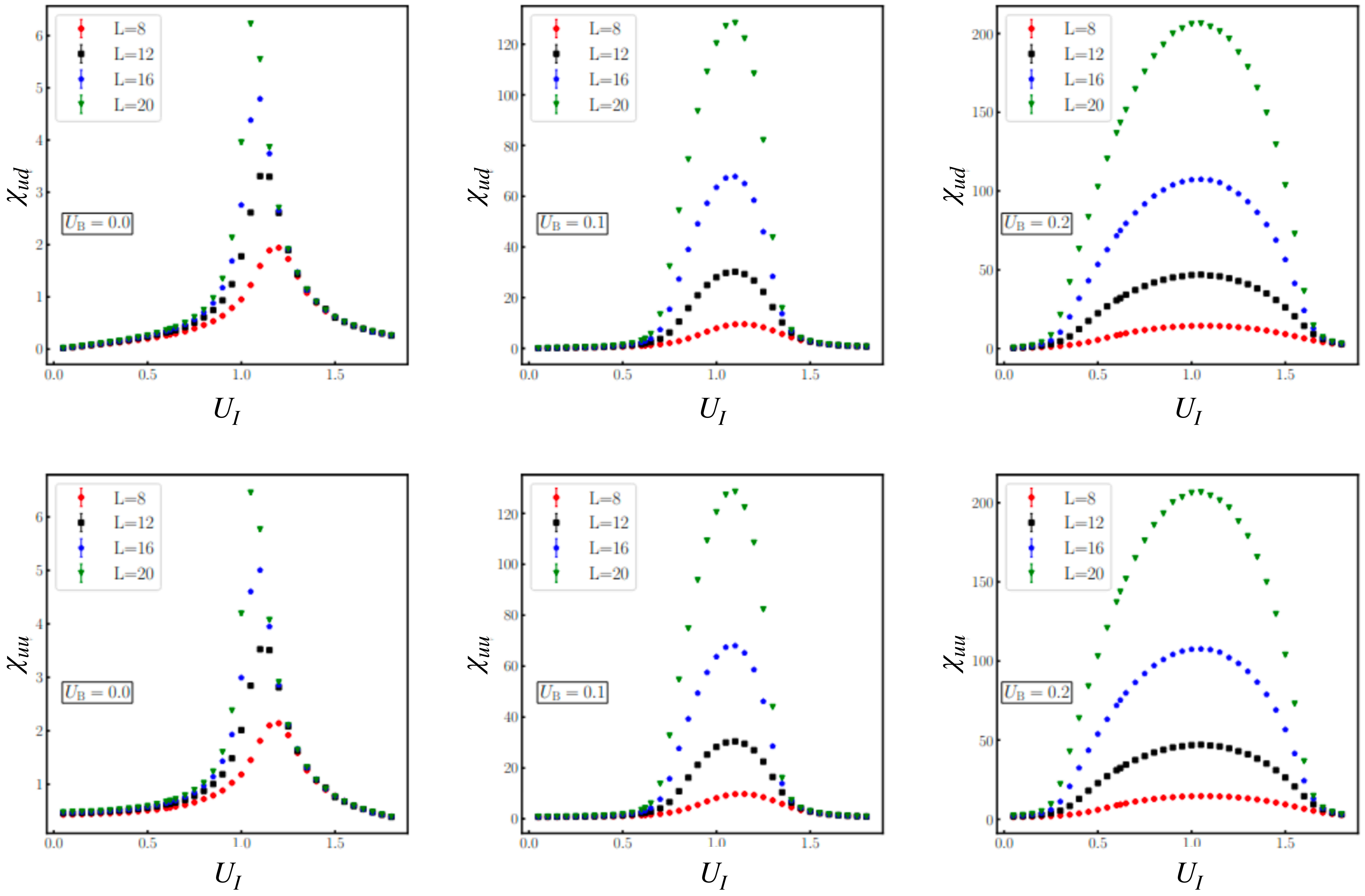




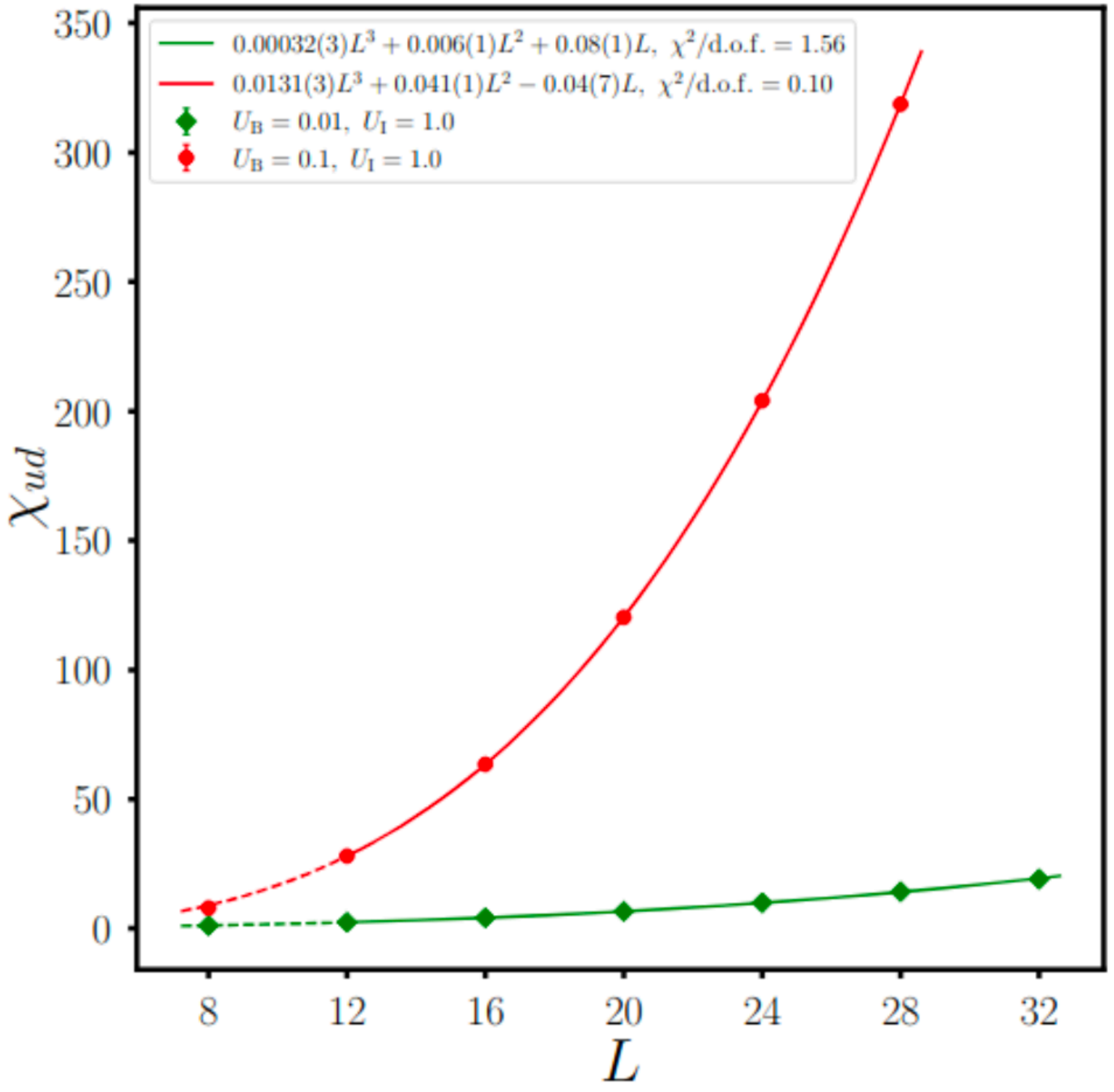
- We find the diagram on the left corresponds to the studied system with two four-fermion couplings U_I and U_B

Susceptibilities and scaling

- Bilinear condensates $\langle \bar{u}u \rangle$ and $\langle \bar{d}d \rangle$ spontaneously break $\mathrm{U}(1)_\chi$ (reminder: $\chi = \theta_u = -\theta_d$)
- Evidence from susceptibilities involving fermion bilinear fields $\chi_{uu} = \dfrac{1}{2L^3} \sum_{i,j} \langle \bar{u}_i u_i \bar{u}_j u_j \rangle$, $\chi_{ud} = \dfrac{1}{2L^3} \sum_{i,j} \langle \bar{u}_i u_i \bar{d}_j d_j \rangle$

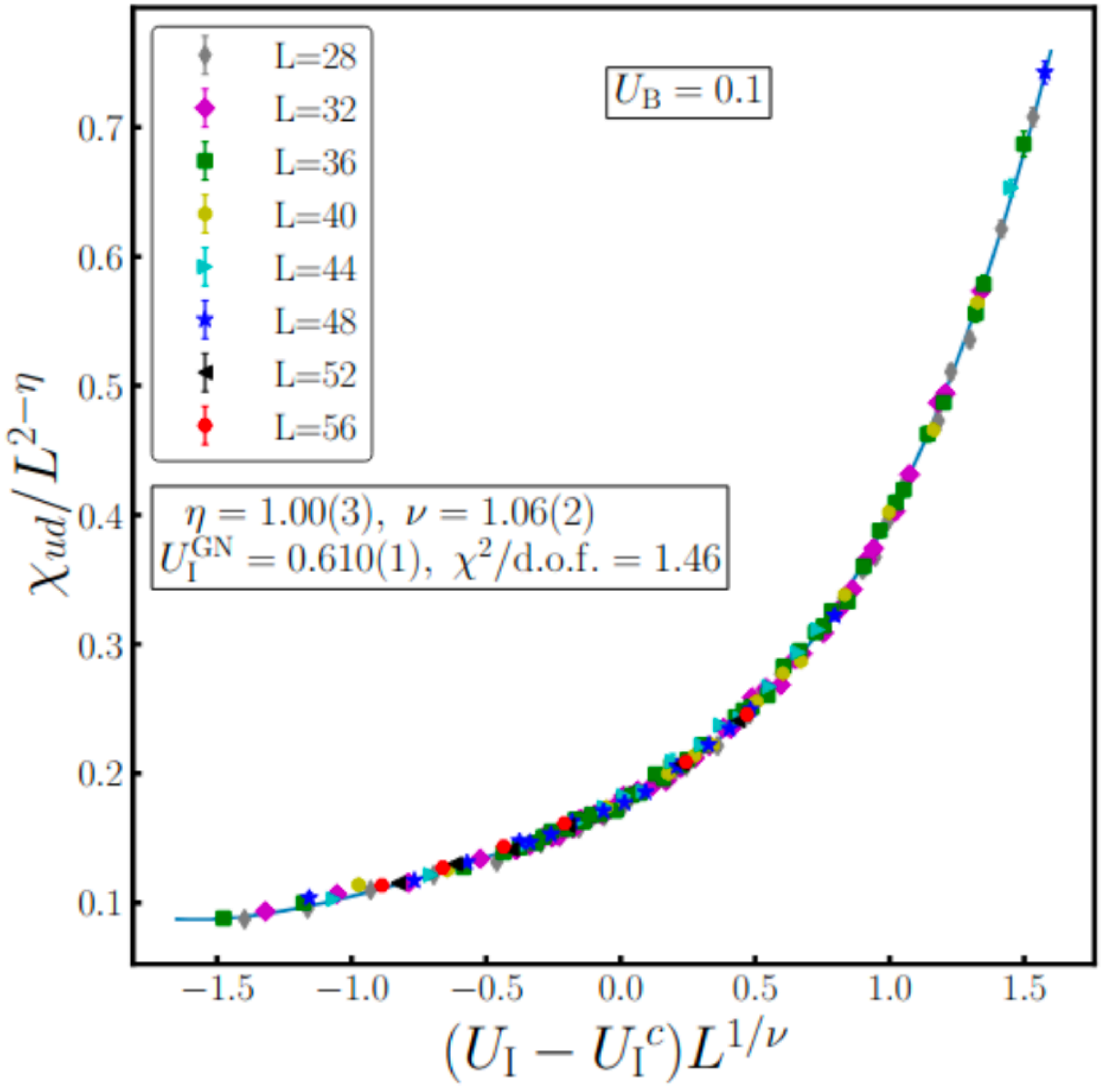


Multicritical point at $U_B = 0$



- Even $U_B = 0.01$is sufficient to cause SSB at $U_I(c)$ detected by χPT inspired fits
- SSB breaks the $U_\chi(1)$ subgroup in $SU(4) = SU(2) \times SU(2) \times U_\chi(1)$

Massless fermion to SSB phase transition



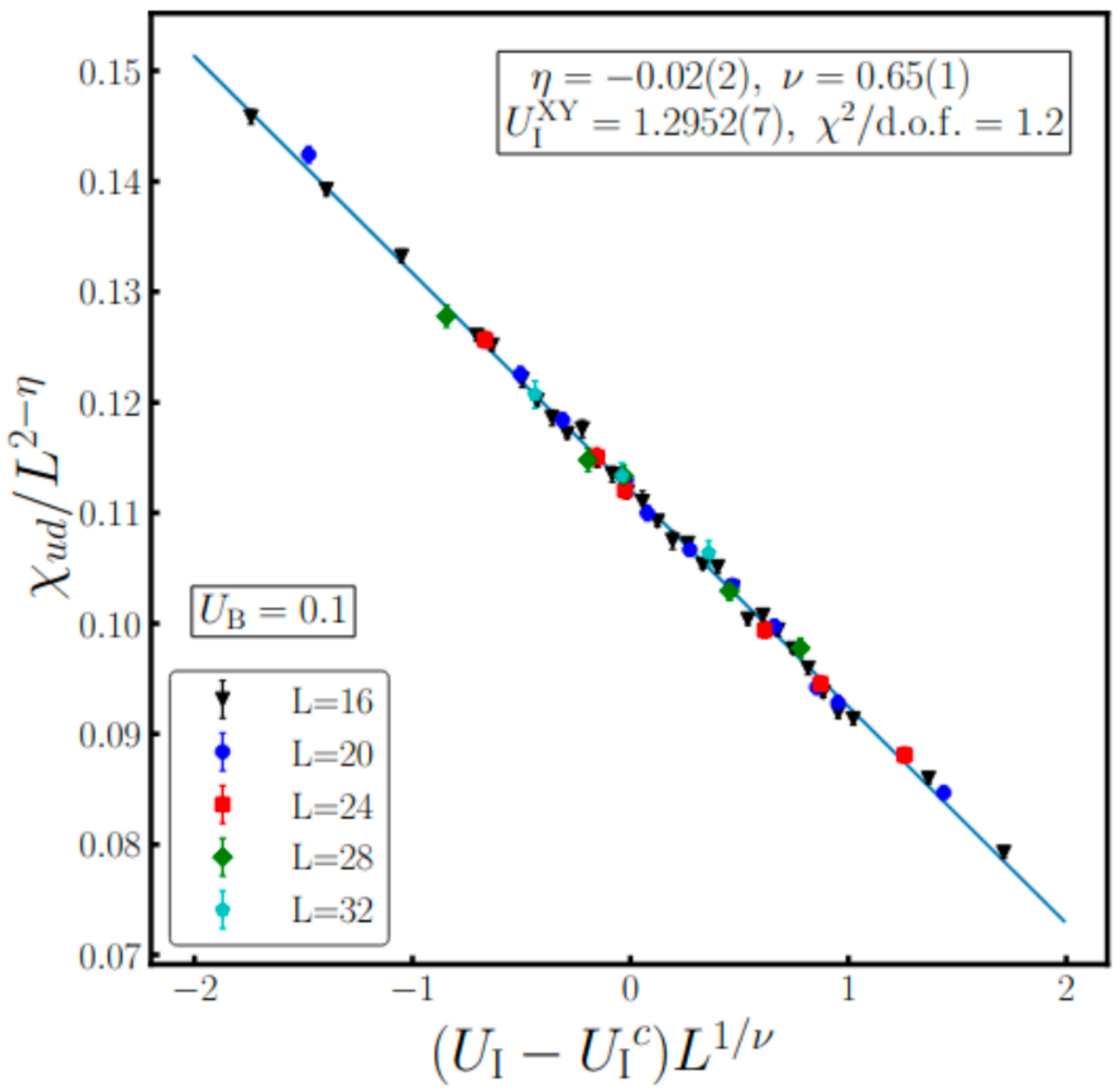
ν	η	Citation
1.06(2)	1.00(3)	This work
1.13	0.93(09)	MC study (bilayer graphene) [44]
1.06(9)	0.99(5)	ε-expansion (4-loop) [45]
1.092(6)	0.926(13)	ε-expansion (interpolation) [46]

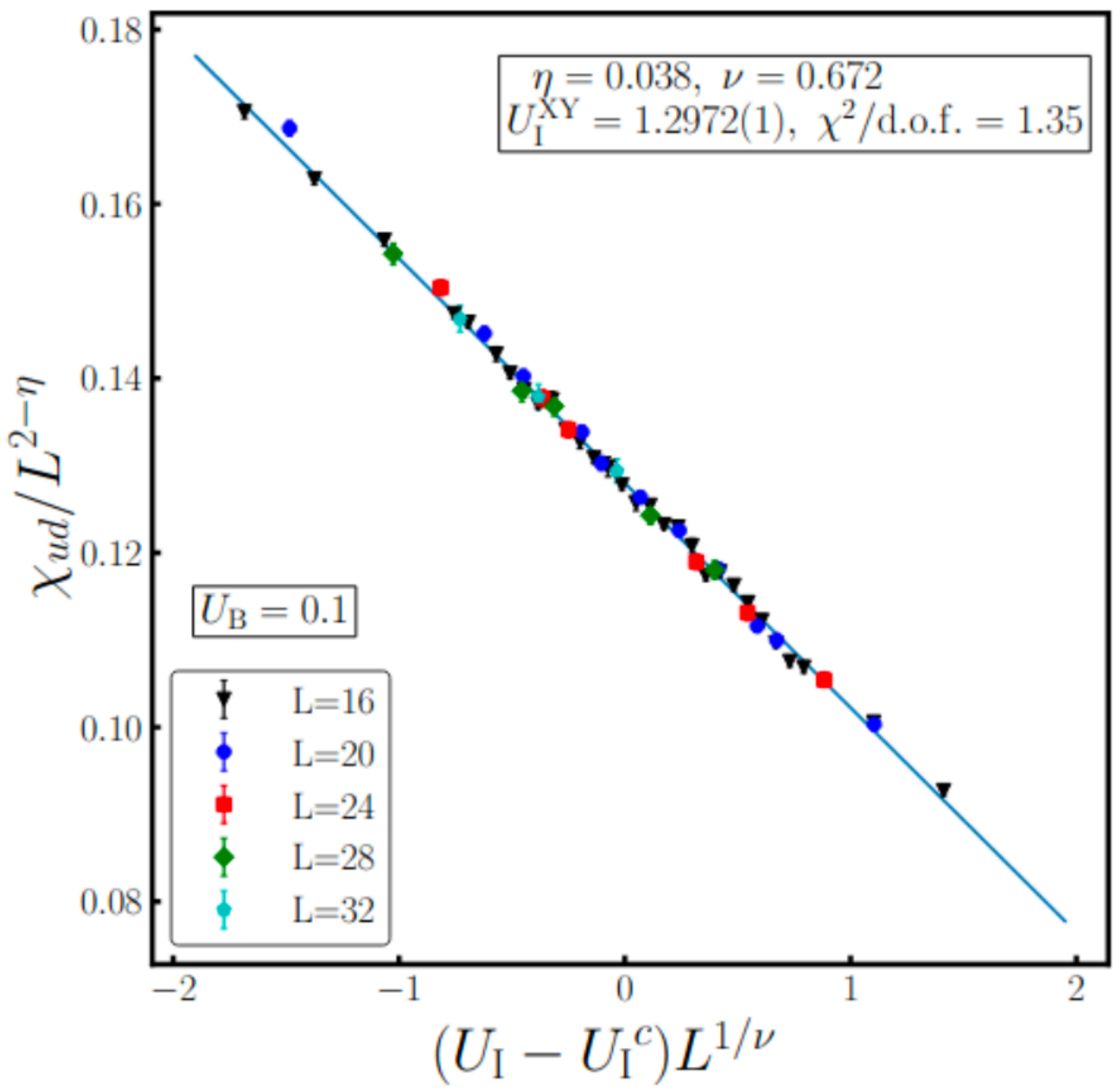
TABLE II. Critical exponents reported in various studies for the Gross–Neveu phase transition in three Euclidean dimensions with four flavors of four-component Dirac fermions. At this transition, massless fermions acquire a mass through the formation of a fermion–bilinear condensate that spontaneously breaks a $U(1)$ symmetry of the theory. The results from earlier studies are summarized in Table I of Ref. [46].

- Exponents extracted by fitting χ_{uu} and χ_{dd} using the finite size scaling relation $\frac{\chi}{L^{2-\eta}} = f((U_I - U_I^C)\, L^{1/\nu})$ at the critical point
- GN universality: critical exponents checked: expectation to get $N_f = 4$ four component Dirac fermions and mean-field

SSB to SMG phase transition: 3d XY universality

- Low-energy modes in the SSB phase are Goldstone bosons associated with $U_\chi(1)$, while fermionic excitations are massive
- Long-distance critical behaviour: 3D XY universality
- Not a priori obvious if all fermionic efects are irrelevant at this phase transition
- Bosonic limit: (easy) $U_B \sim U_I \gg t$, and define $\lambda = \dfrac{U_I}{U_B}$





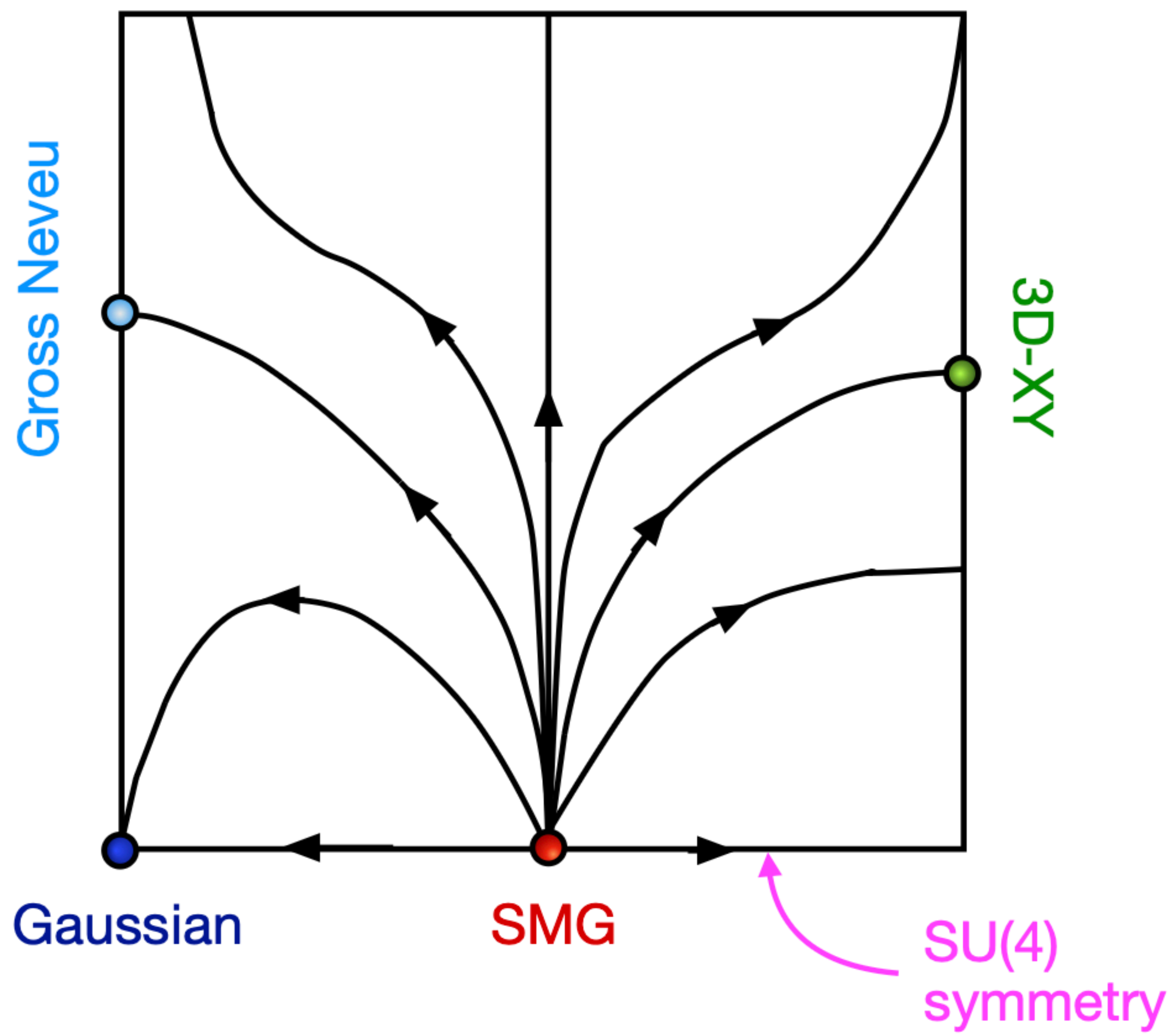
- Two different analysis performed: fitting critical exponents (left), while fitting the critical exponents to the 3D XY values (right)

RG flow diagram

- Conjectured RG flows in 3D for fermionic GN-Yukawa models near the SMG fixed point (FP)

    1. Gaussian FP: massless fermions
    2. Gross-Neveu FP: interacting massless fermions and bosons
    3. XY FP: interacting massless bosons
    4. SMG FP: massless fermions

- No parity anomaly for $N_f = 4$ four component Dirac fermion
- Anomalies due to other global symmetries are also expected to cancel

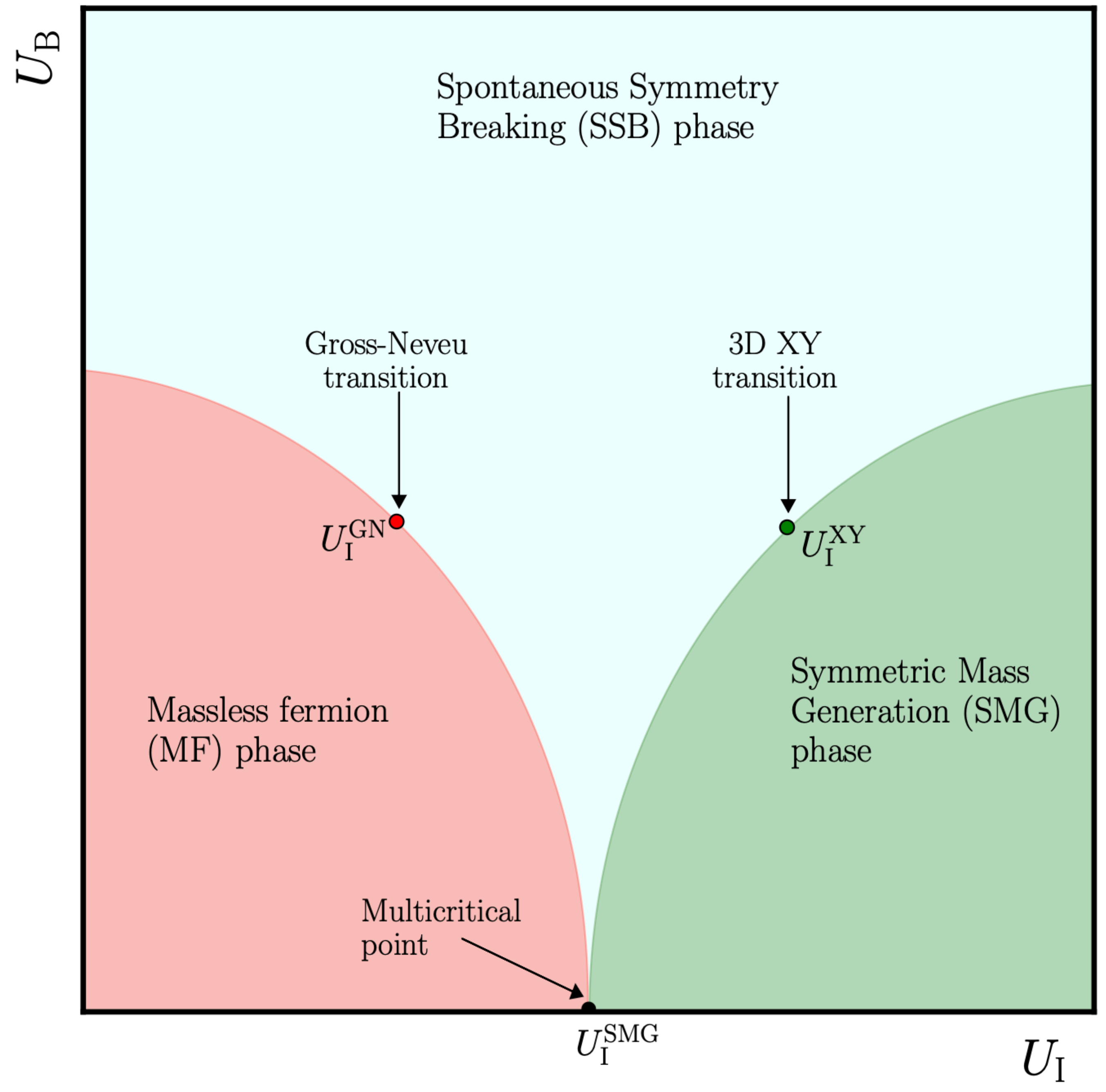


Summary & Outlook

Maiti, Banerjee, Chandrasekharan, Marinkovic arXiv: 2512.24836
Maiti, Banerjee, Chandrasekharan, Marinkovic arXiv: 2602.18360

- Two massless fermions + two four-fermion couplings U_B and U_I
- The three phases
    1. the massless fermion phase (MF)
    2. the traditional massive phase (SSB)
    3. the exotic massive phase (SMG)

  coexist at a multicritical SU(4) symmetric point at $U_I^{\rm SMG}$



- Strong coupling fermion bag at small U_B to further explore fermionic effects
- Spectroscopy in the SMG phase
- Hamiltonian formulation for real time dynamics
- Does a similar multicritical point exist in 4D, e.g. with Yukawa interactions? [Butt, Catterall, Schaich, PhysRevD.98.114514 (2018)]
  [Talk by G. Hartshaw, Thursday, 4.10pm, Theoretical Developments]
  [Plenary talk by S. Catterall, Saturday, 9am]