Determination of $\alpha_s(m_Z)$ from gradient flow

Curtis Taylor Peterson*

Michigan State University, Postdoctoral Fellow

*on behalf of the Fermilab Lattice and MILC collaborations

UNIVERSITY OF MARYLAND

LATTICE 2026

MICHIGAN STATE

Alexei Bazavov[1], Akhil Chauhan[2], Mingwei Dai[2], Carleton DeTar[3], Steve Gottleib[4], Anna Hasenfratz[5], Leon Hostetler[4], Andreas Kronfeld[6], **Yash Mandlecha**[1], Ethan Neil[5], **Curtis Peterson**[1], James Simone[6]

[1]Michigan State University, [2]University of Illinois Urbana-Champaign, [3]University of Utah, [4]Indiana University, [5]University of Colorado, [6]Fermi National Accelerator Laboratory

recapitulation

$\alpha_s(m_Z)$ on and off the lattice

weak coupling analysis with HISQ

$\alpha_s(m_Z)$ from gradient flow β-function

recapitulation

$\alpha_s(m_Z)$ on and off the lattice

weak coupling analysis with HISQ

$\alpha_s(m_Z)$ from gradient flow β-function

recapitulation

$\alpha_s(m_Z)$ on and off the lattice

weak coupling analysis with HISQ

$\alpha_s(m_Z)$ from gradient flow β-function

recapitulation

$\alpha_{\mathrm{s}}(m_Z)$ on and off the lattice

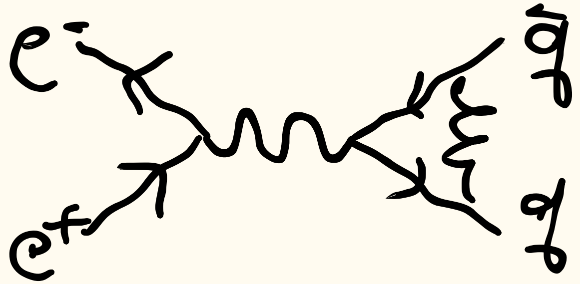
weak coupling analysis with HISQ

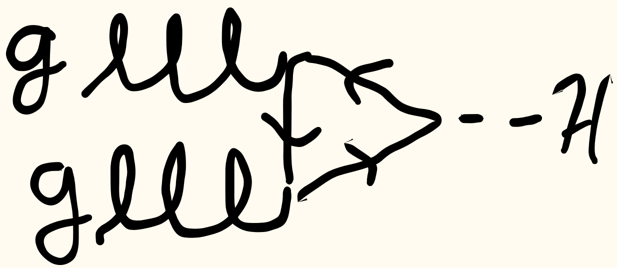
$\alpha_{\mathrm{s}}(m_Z)$ from gradient flow β-function

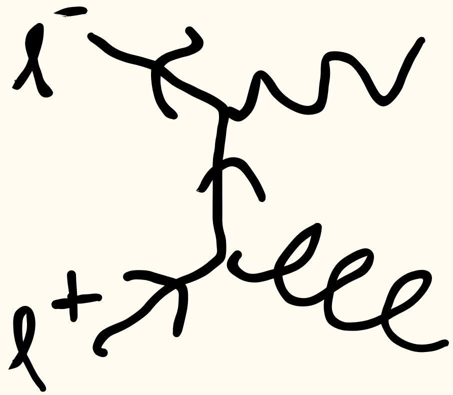
Motivation

[*Snowmass 2021 TF05* (2021)]
[*Snowmass 2022* (2021)]
[Dalla Brida, Mattia, *EPJA* **57**, 66 (2021)]

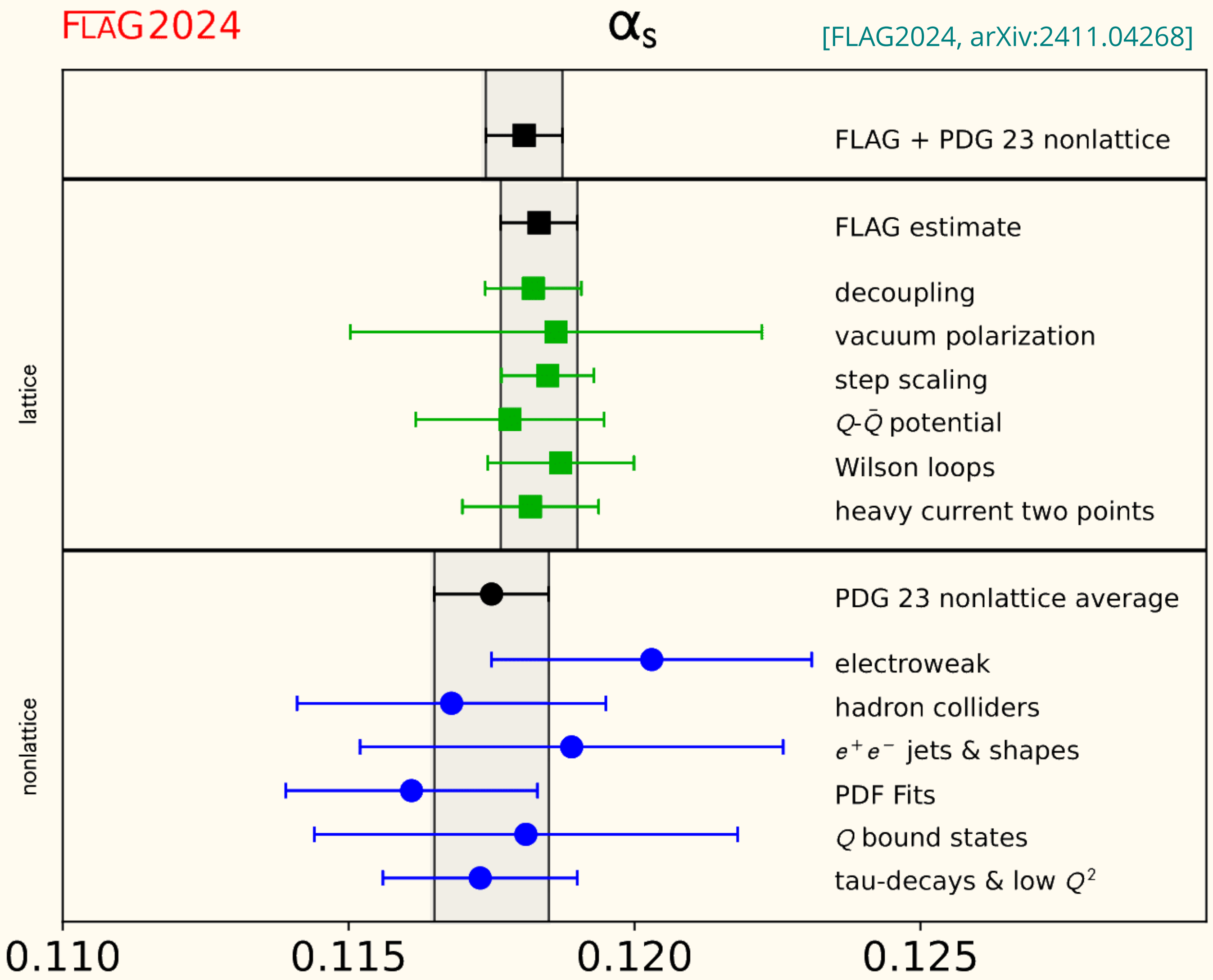
- Fundamental Standard Model parameter
  - pQCD processes ($\sigma, \tau \sim \alpha_s^n$)
  - t quark mass, width & Yukawa coupling
  - Contributes $2-4\%$ uncertainty to important Higgs production processes
  - Leading uncertainty in hadronic Z widths
- Future high-precision studies require well below 1% precision
  - Pheno. community aiming for $\sim 0.2\%$
  - Lattice community must keep up!







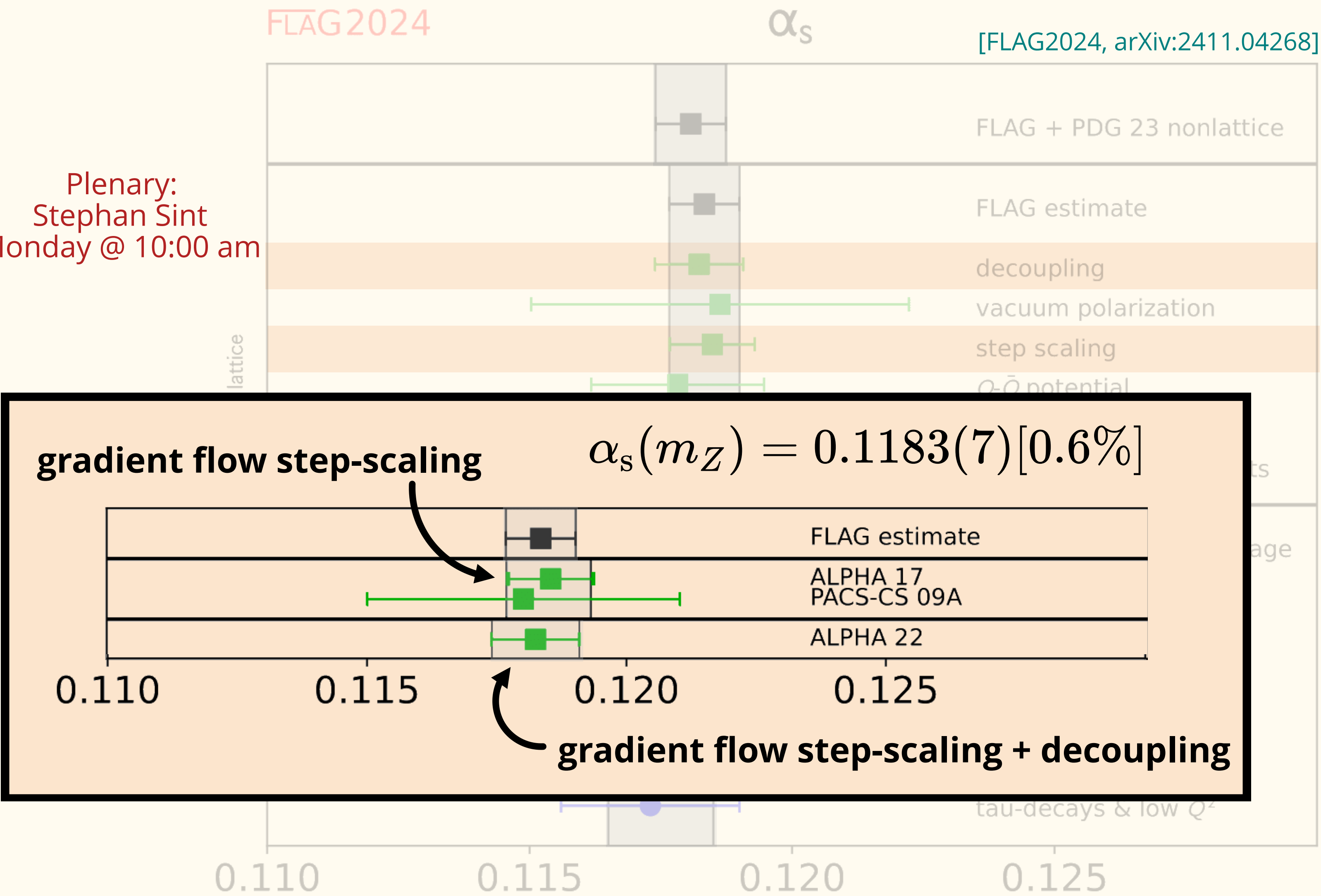
$\alpha_s(m_Z)$ from lattice gauge theory



$\alpha_s(m_Z)$ from lattice gauge theory

[FLAG2024, arXiv:2411.04268]

Plenary:
Stephan Sint
Monday @ 10:00 am



$\alpha_s(m_Z)$ and the Λ-parameter

- β-function describes running of $g_X^2(\mu)$:

$$\mu^2 \frac{\mathrm{d}g_X^2(\mu)}{\mathrm{d}\mu^2} \equiv \beta_X\left(g_X^2\right)$$

$X \equiv$ "some renormalization scheme"

- Elementary ODEs: equivalent approaches
  - Initial value problem (initial condition $g_X^2(m_Z)$)
  - Direct integration (integration constant Λ_X)

$$\frac{\Lambda_X^2}{\mu^2} \sim \exp\left[-\int^{g_X^2(\mu)} \mathrm{d}x \, \frac{1}{\beta_X(x)}\right]$$

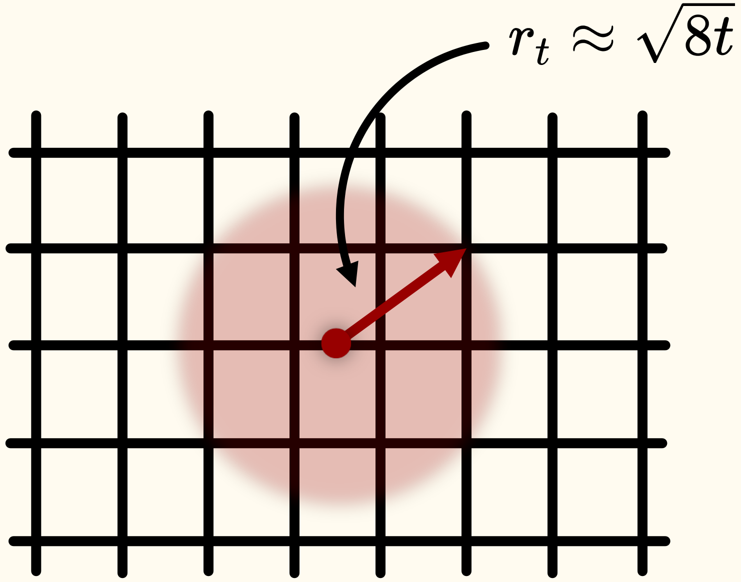
- Often easier to calculate Λ_X on lattice
  - Exact 1-loop relation between Λ_X and any other Λ_Y

Gradient flow β-function

- RG* β-functions calculable on the lattice!
- use continuous smearing; e.g., "gradient flow"

$$\frac{\mathrm{d}\mathcal{A}_\mu(x,t)}{\mathrm{d}t} = -g_0^2 \frac{\delta \mathcal{S}_{\mathrm{YM}}[\mathcal{A}_\mu]}{\delta \mathcal{A}_\mu(x,t)};$$
$$\mathcal{A}_\mu(x,0) = \mathcal{A}_\mu(x)$$

- Suppresses fluctuations above resolution scale $\mu \approx 1/r_t$
- *Describes* an RG transformation
  - Coarse-grain at level of expecation values



*renormalization group

[Luescher, M., *JHEP* 08 (2010) 71]
[Carosso, A., Hasenfratz, A., Neil, E. *PRL* 121, 201601]

Gradient flow β-function

- RG* β-functions calculable on the lattice!
- use continuous smearing; e.g., "gradient flow"

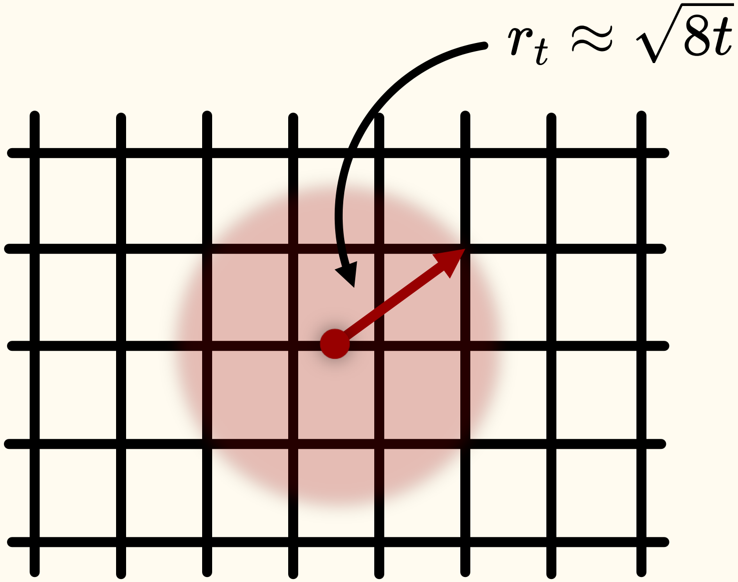
$$\frac{\mathrm{d}\mathcal{A}_\mu(x,t)}{\mathrm{d}t} = -g_0^2 \frac{\delta \mathcal{S}_{\mathrm{YM}}[\mathcal{A}_\mu]}{\delta \mathcal{A}_\mu(x,t)};$$

$$\mathcal{A}_\mu(x,0) = \mathcal{A}_\mu(x)$$

$$E(t) \equiv \mathrm{Tr}_c\left[\mathcal{F}_{\mu\nu}(x,t)\mathcal{F}^{\mu\nu}(x,t)\right]$$

$$g^2_{\mathrm{GF}}(t) \propto t^2 \langle E(t) \rangle$$

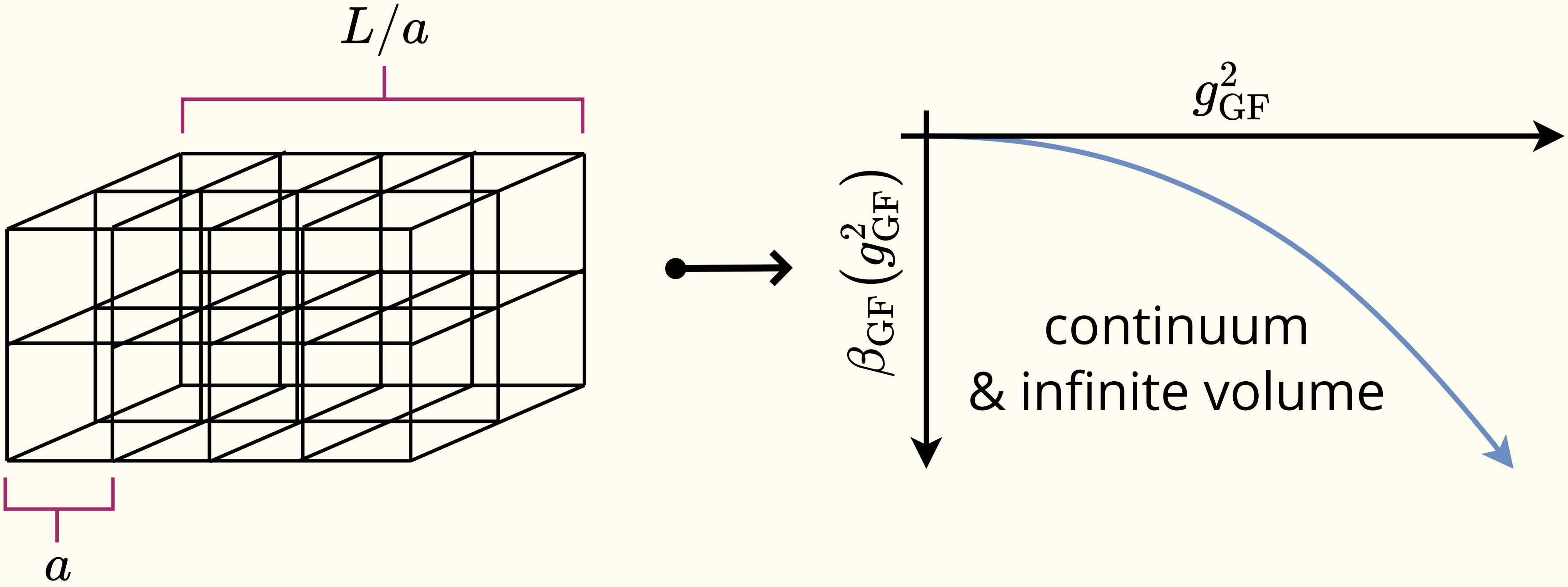
$$\beta_{\mathrm{GF}}(g^2_{\mathrm{GF}}) \equiv -t\frac{\mathrm{d}}{\mathrm{d}t} g^2_{\mathrm{GF}}(t)$$



*renormalization group

[Luescher, M., *JHEP* 08 (2010) 71]
[Fodor, Z., Holland, K., Kuti, J., Nogradi, D., Wong, C.H., EPJ Web Conf. 175 (2018) 08027]
[Hasenfratz, A., Witzel, O., Phys. Rev. D 101 (2020) 3,034514]

Gradient flow β-function from the lattice



1. finite-volume coupling & β-function
2. infinite volume ($a/L \to 0$) limit
3. continuum ($a^2/t \to 0$) limit

*a.k.a. infinite-volume β-function

[Fodor, Z., Holland, K., Kuti, J., Nogradi, D., Wong, C.H., EPJ Web Conf. 175 (2018) 08027]
[Hasenfratz, A., Witzel, O., Phys. Rev. D 101 (2020) 3,034514]

From $\beta_{\mathrm{GF}}(g^2_{\mathrm{GF}}) \to \alpha_{\mathrm{s}}(m_Z)$

- schematically ($\mu^2 \propto 1/8t$):

requires matching

$$8t_0\Lambda^2_{\mathrm{GF}} \sim \exp\left[-\int^{g^2_{\mathrm{GF}}(t_0)} \frac{\mathrm{d}x}{\beta_{\mathrm{GF}}(x)}\right]$$

- reuse (must know t_0 in fm in chiral limit):

$$\frac{m_Z^2}{\Lambda^2_{\overline{\mathrm{MS}}}} \sim \exp\left[\int^{g^2_{\overline{\mathrm{MS}}}(m_Z)} \frac{\mathrm{d}x}{\beta_{\overline{\mathrm{MS}}}(x)}\right]$$

5-loop β-function

- $\alpha_{\mathrm{s}}(m_Z) = 0.08286(28)$ [0.3%] ($N_{\mathrm{f}} = 0$)

*$\Lambda_{\overline{\mathrm{MS}}} =$ "Λ_{QCD}"

*$\langle t_0^2 E(t_0)\rangle \equiv 0.3$

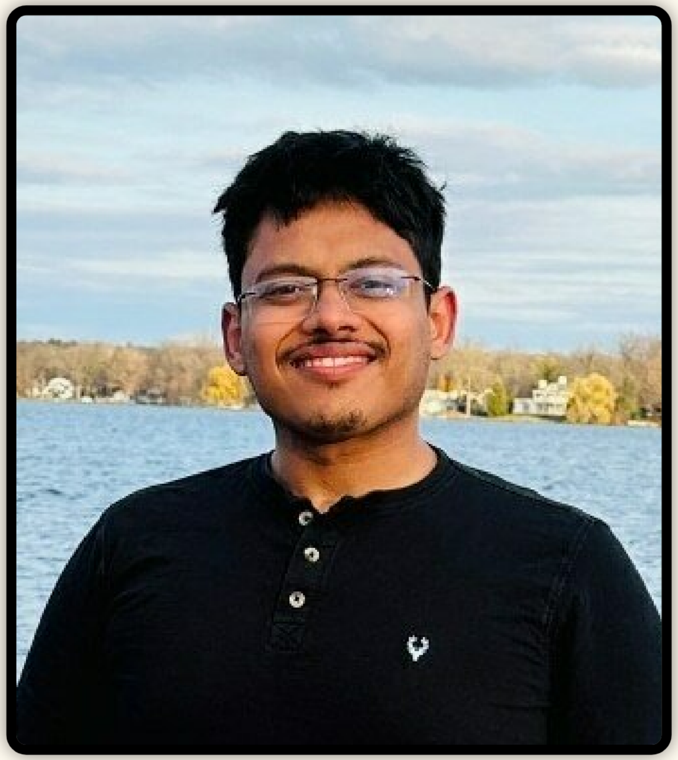
[Hasenfratz, A., Peterson, C.T., van Sickle, J., Witzel, O. *PRD* 108, 014502 (2023)]

$\alpha_s(m_Z)$ from HISQ

- $\alpha_s(m_Z)$ $(N_f = 5)$
  - target precision $\lesssim 0.3\%$
  - $N_f = 4$ HISQ action (no rooting)
  - $N_f = 4 \to 5$: perturbative decoupling
- Weak coupling
  - $m_f = 0$ (massless RG scheme)
  - $(L/a)^3 \times (2L/a)$ with $L/a = 32, 40, 48$
  - Eight $8.50 \leq \beta_b \leq 20.0$
- Strong coupling
  - $m_f \neq 0$ (must extrapolate $m_f \to 0$)
  - in the works (not discussed here)

Weak coupling analysis

Yash Mandlecha (graduate student)



Michigan State University

Weak coupling analysis

corrects for gauge zero-modes and tree-level discretization effects

- Renormalized coupling:

$$g^2_{\mathrm{GF}}(t; L, \beta_{\mathrm{b}}) \propto \frac{\langle t^2 E(t) \rangle_{\beta_{\mathrm{b}}}}{1 + \delta(t, L)}$$

- β-function:

$$\beta_{\mathrm{GF}}(t; L, \beta_{\mathrm{b}}) \equiv -t \frac{\mathrm{d}g^2_{\mathrm{GF}}(t; L, \beta_{\mathrm{b}})}{\mathrm{d}t}$$

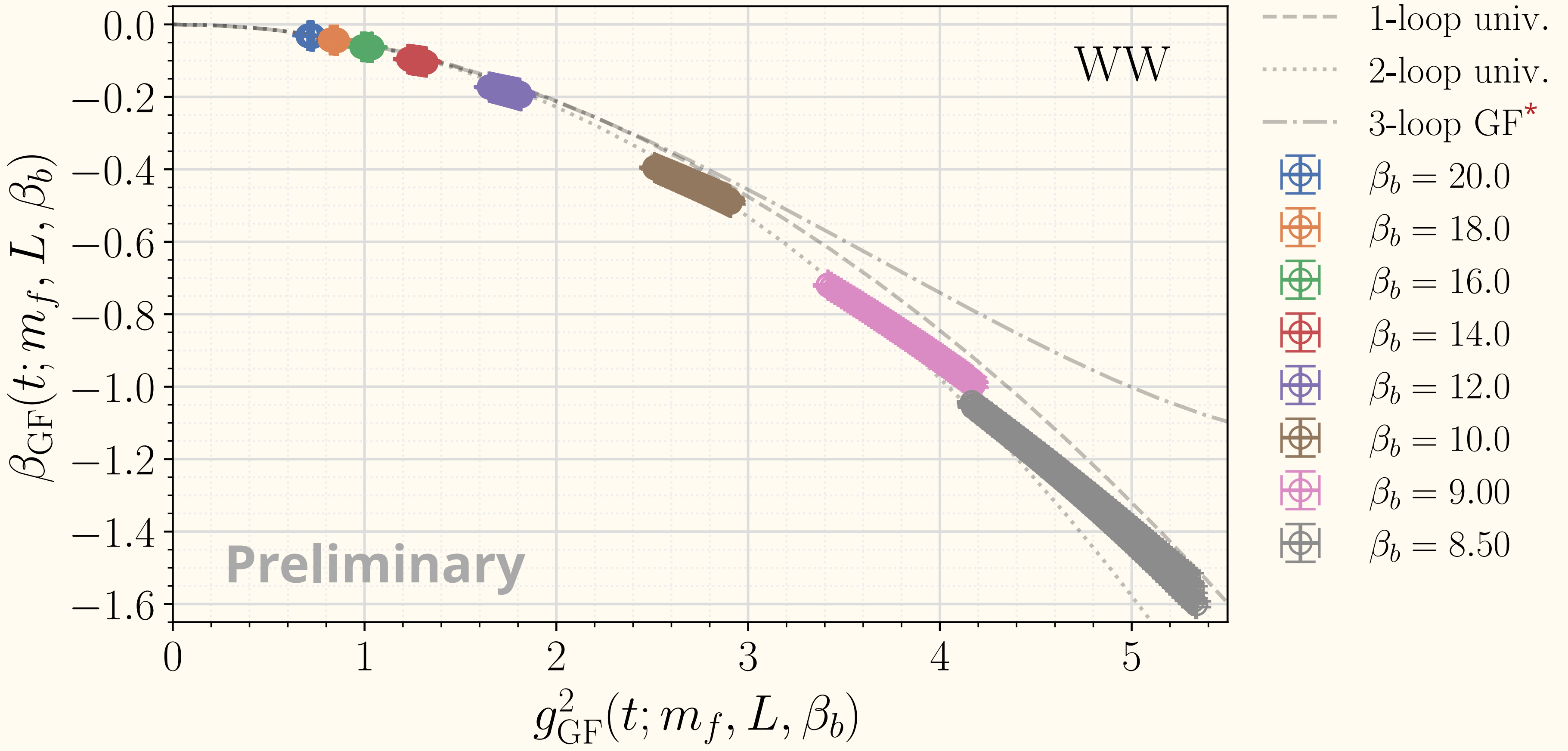
estimate from 5-point stencil

- Wilson flow + Wilson (WW), Symanzik (WS), and clover (WC) $E(t)$ discretization

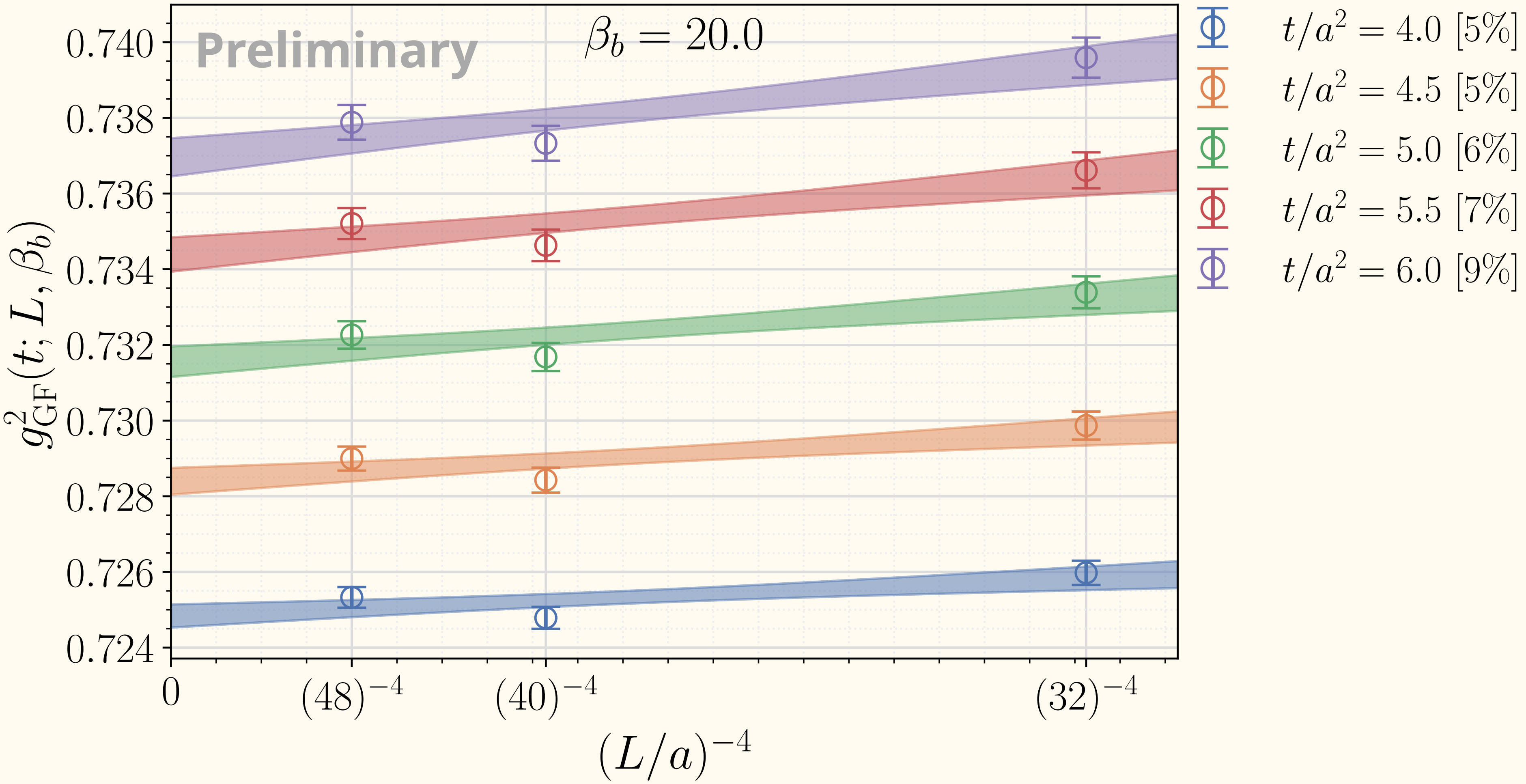
[Fodor, Z., Holland, K., Kuti, J., Mondal, S., Nogradi, D., JHEP 09 (2014) 018]

Unextrapolated dataset

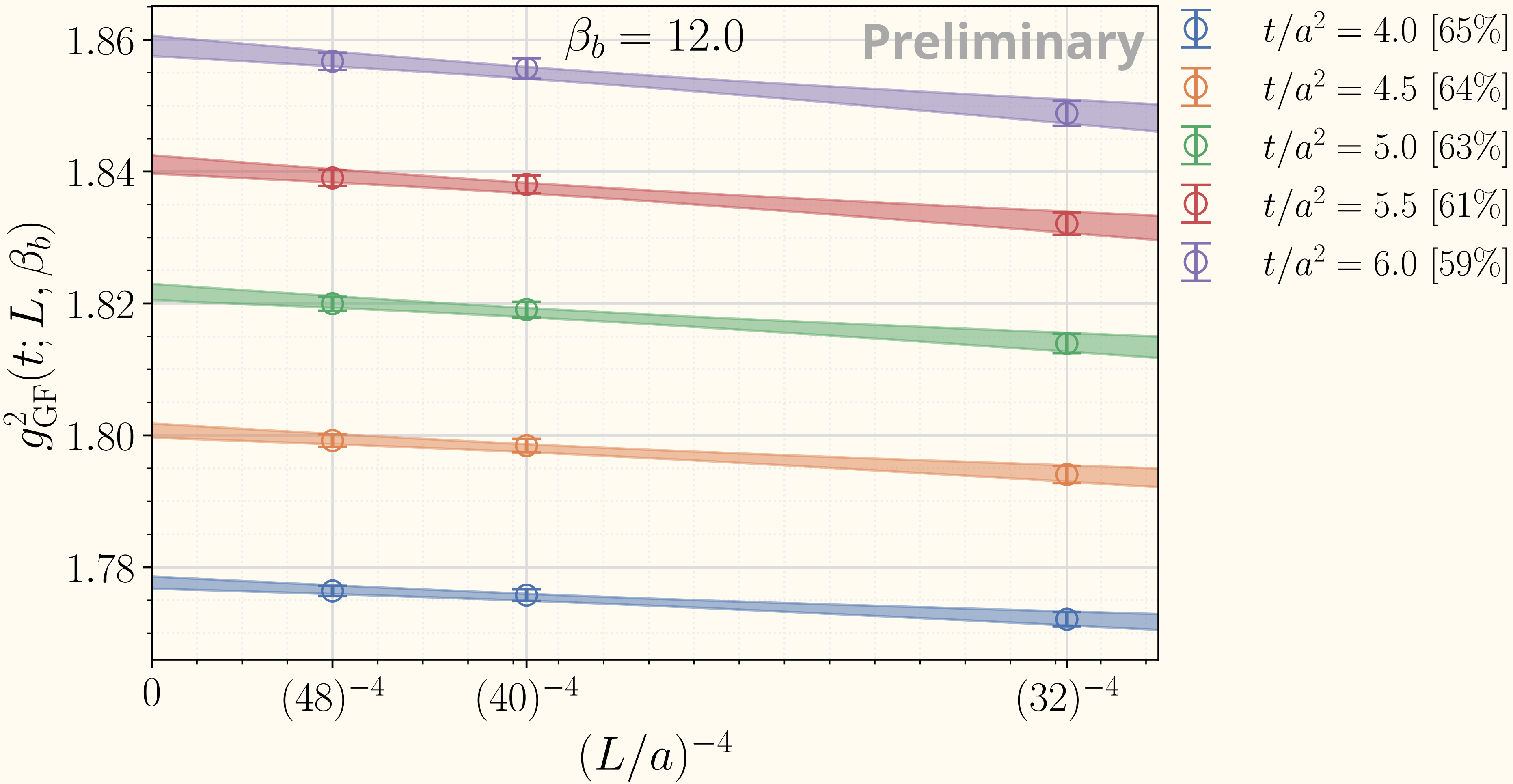
*[Artz, Harlander, Lange, Neumann, Prausa JHEP 06 (2019) 121]



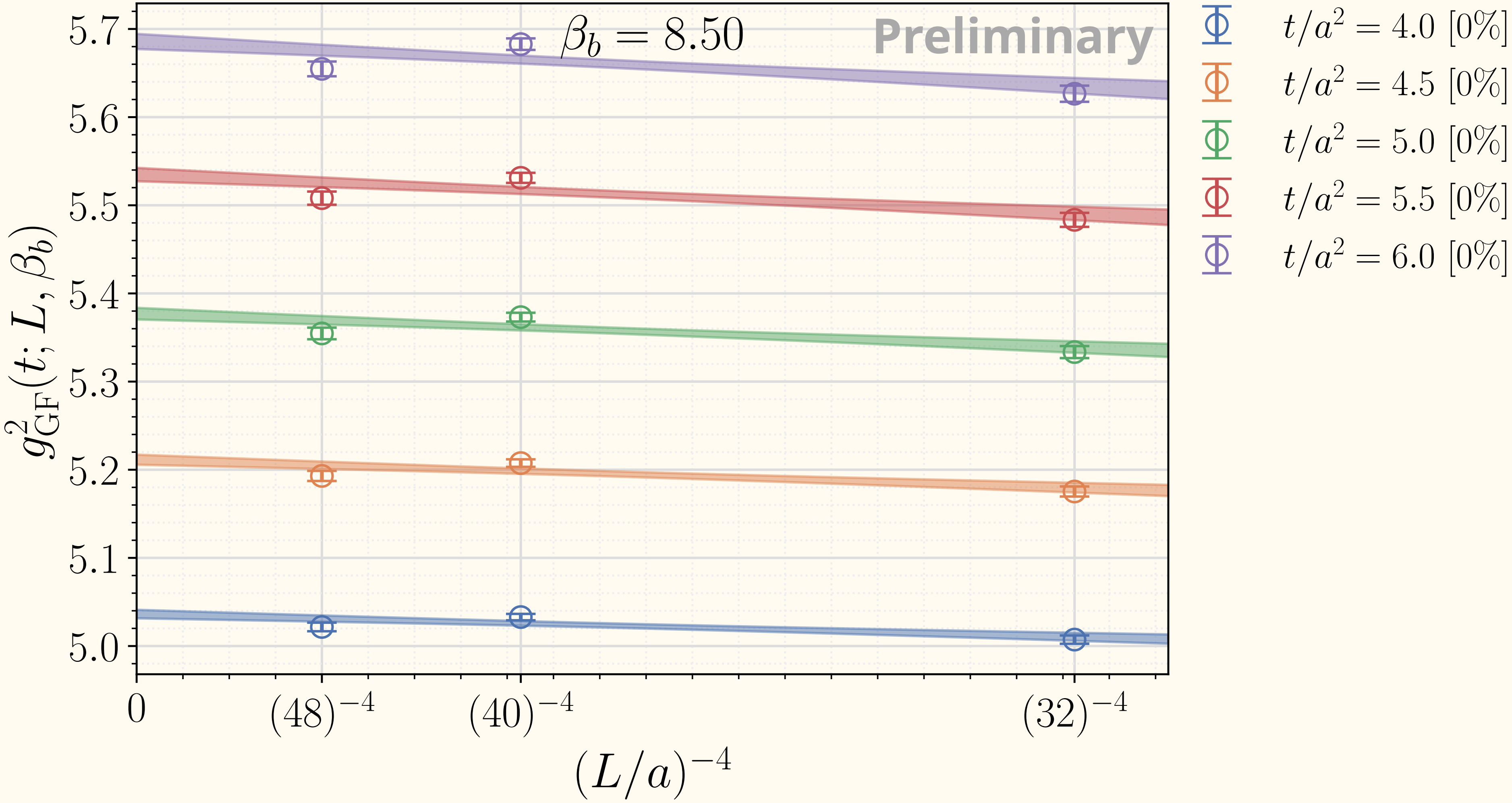
Infinite volume extrapolation



Infinite volume extrapolation

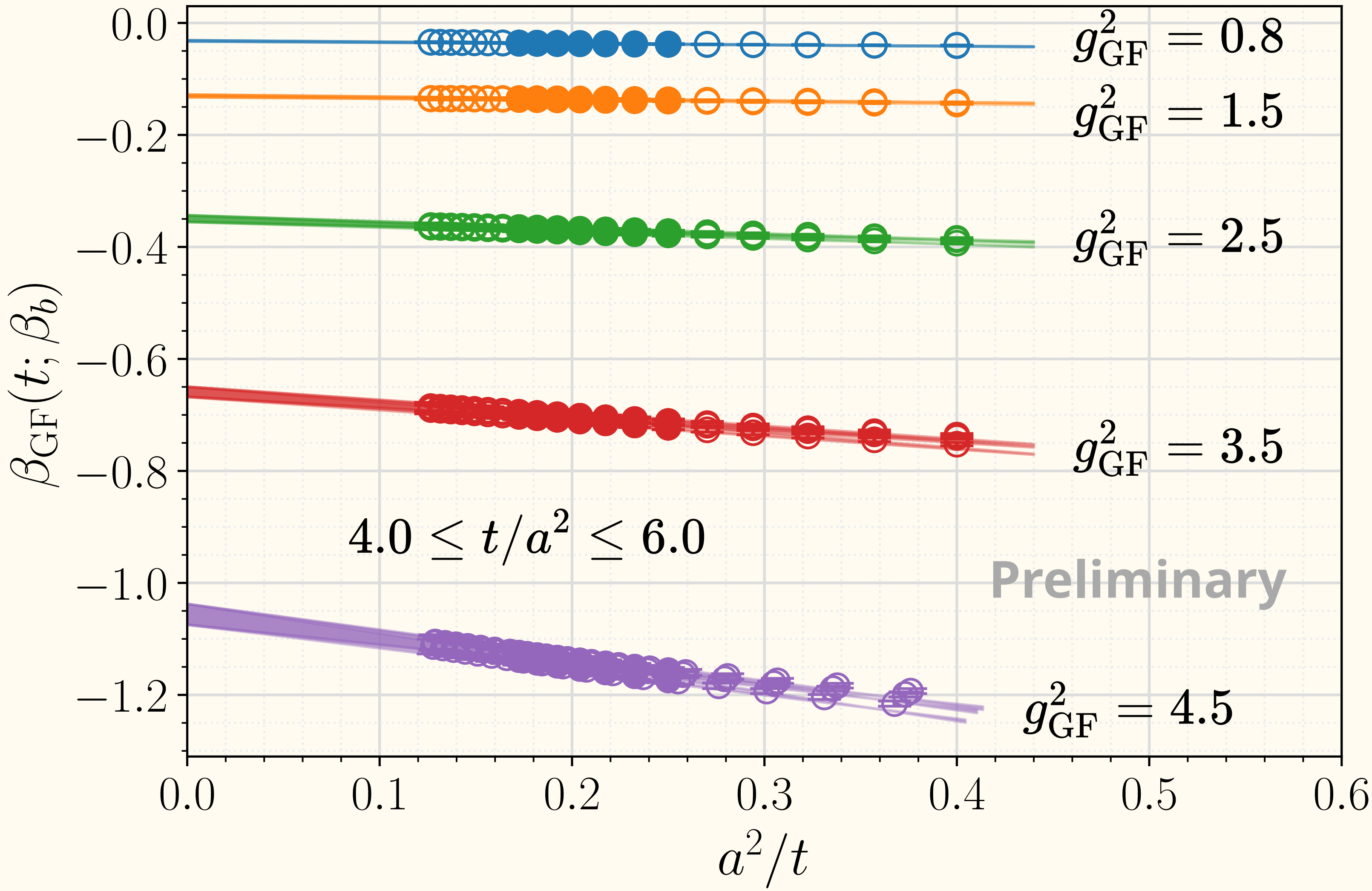


Infinite volume extrapolation



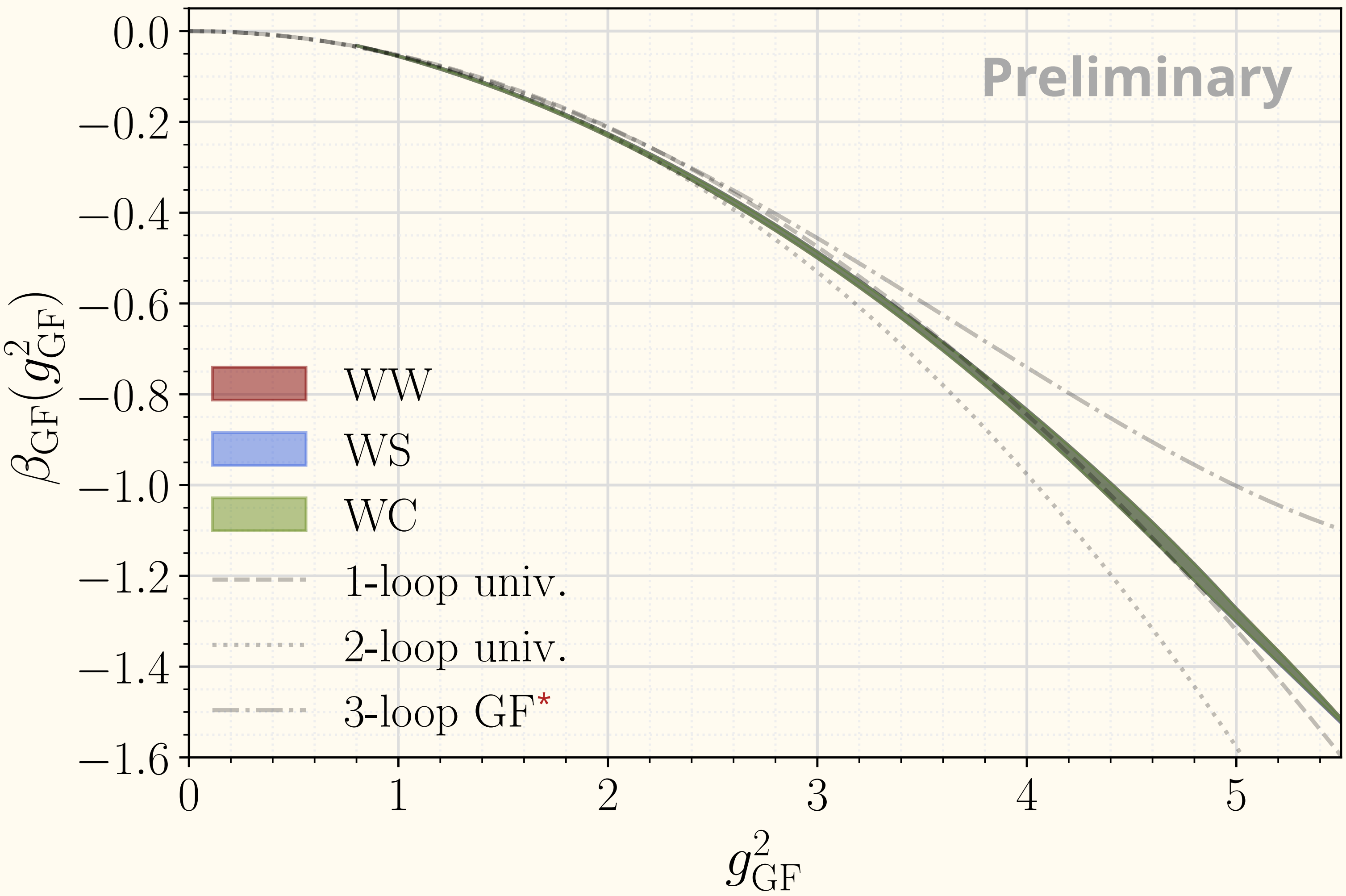
MICHIGAN STATE

Continuum extrapolation



Continuum β-function

*[Artz, Harlander, Lange, Neumann, Prausa JHEP 06 (2019) 121]



Recapitulation

- Targeting precise determination of $\alpha_\mathrm{s}(m_Z)$ from gradient flow from HISQ
  - continuous β-function method
  - direct integration of continuum β-function
- Weak coupling
  - add $L/a = 64$ volume
  - increase statistics
  - fill in β_b
- Strong coupling
  - in production (not discussed)

U.S. DEPARTMENT OF ENERGY

USQCD US Lattice Quantum Chromodynamics

Quantum EXpressions (QEX)

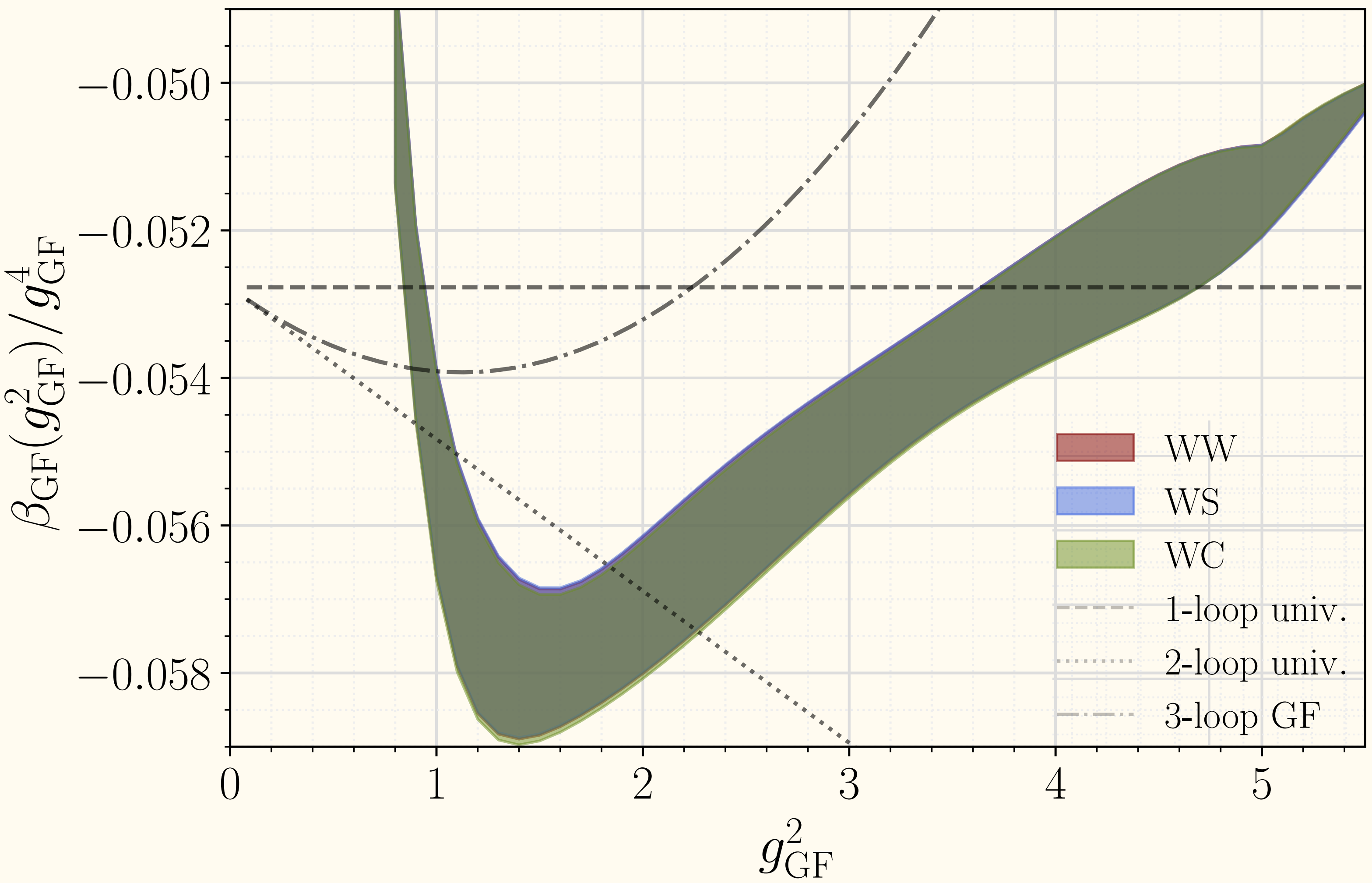
James Osborn (ANL)

Xiao-Yong Jin (ANL)

Plenary:
Xiao-Yong Jin
Monday @ 12:00 pm

Questions

The perturbative regime

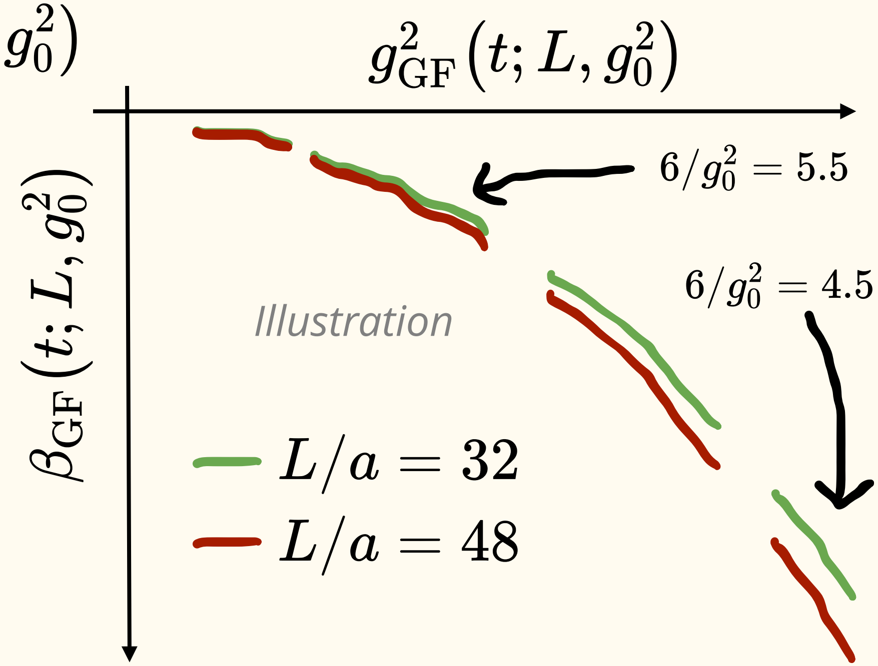


Continuous β-function method

1. Finite-volume coupling & β-function

$$g^2_{\mathrm{GF}}(t; L, g_0^2) \propto \frac{1}{1+\delta(t,L)} \langle t^2 E(t;L)\rangle_{g_0^2}$$

$$\implies \beta_{\mathrm{GF}}(t; L, g_0^2) = -t\frac{\mathrm{d}}{\mathrm{d}t} g^2_{\mathrm{GF}}(t; L, g_0^2)$$



Continuous β-function method

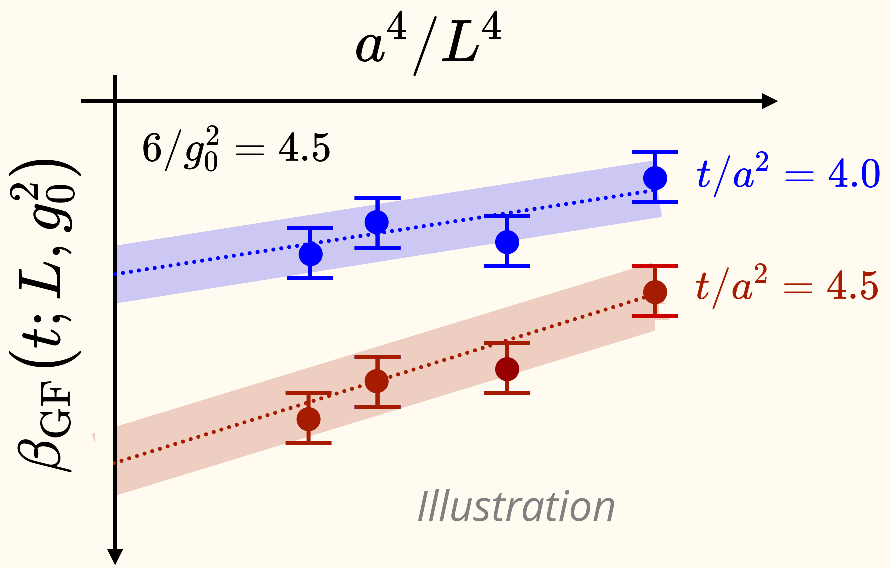
1. Finite-volume coupling & β-function
2. Infinite volume ($a/L \to 0$) limit

Extrapolate...

$$g^2_{\rm GF}(t; L, g_0^2) \overset{a/L\to 0}{\longrightarrow} g^2_{\rm GF}(t; g_0^2)$$

$$\beta_{\rm GF}(t; L, g_0^2) \overset{a/L\to 0}{\longrightarrow} \beta_{\rm GF}(t; g_0^2)$$

(at each fixed g_0^2 & t/a^2)



Continuous β-function method (CBFM)

1. Finite-volume coupling & β-function
2. Infinite volume ($a/L \to 0$) limit
3. Continuum ($a^2/t \to 0$) limit

Extrapolate...

$$\beta_{\mathrm{GF}}\left(t; g_0^2\right) \xrightarrow{a^2/t \to 0} \beta_{\mathrm{GF}}\left(g_{\mathrm{GF}}^2\right)$$

(each fixed g_{GF}^2)