Feynman-Hellman methods for four quark matrix elements

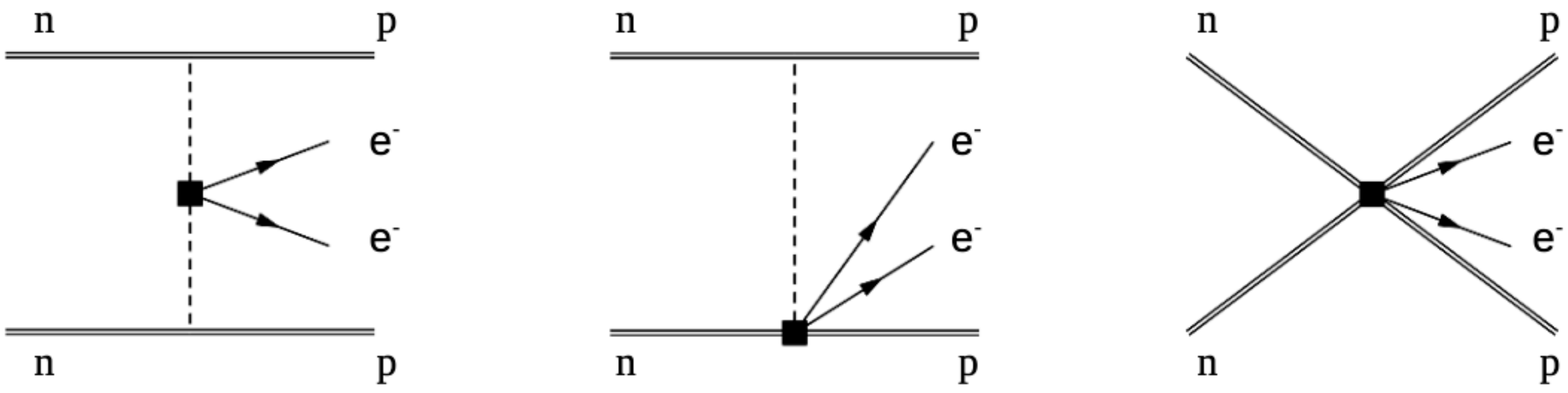
Toward finding QCD contributions to neutrinoless double beta decay

Presented by Charles Kacir at The 43rd International Symposium on Lattice Field Theory (July 29, 2026)
Collaborators: Henry Monge-Camacho (ORNL) and Amy Nicholson (UNC-CH)

Neutrinoless double beta decay ($0\nu\beta\beta$)

What is it?

- Process by which two neutrons are replaced by only two protons and two electrons
- Not yet conclusively observed

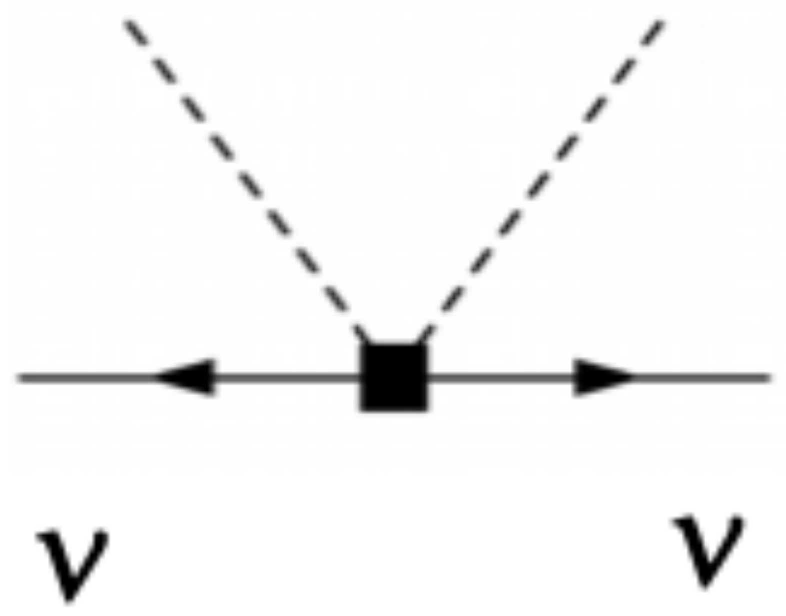


V. Cirigliano, W. Dekens, J. de Vries, M.L. Graesser, and E. Mereghetti (https://doi.org/10.1007/JHEP12(2018)097)

Neutrinoless double beta decay ($0\nu\beta\beta$)

Why do we expect to see it?

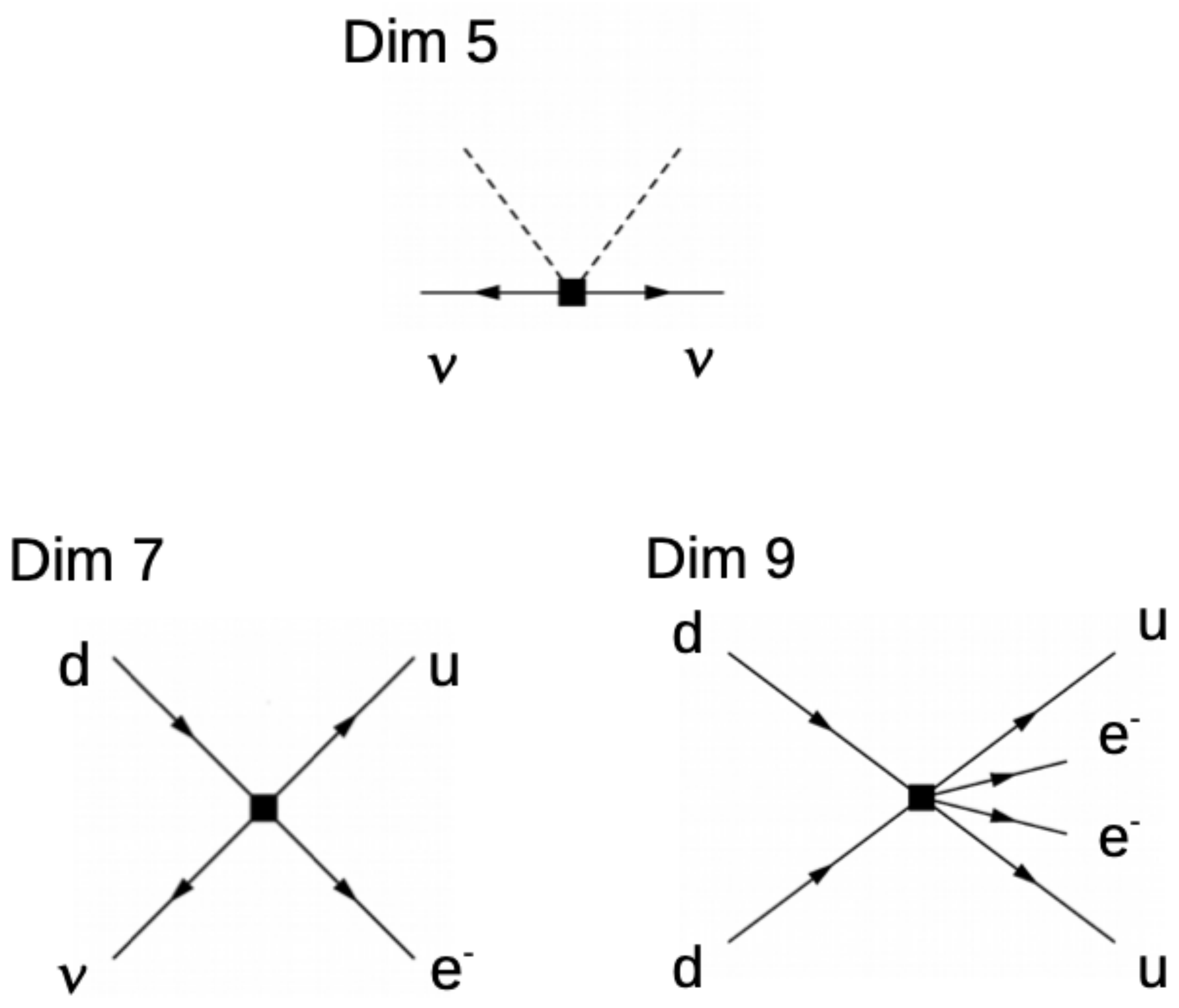
- Neutrinos have mass because they oscillate, but it isn't a simple Higgs mass because then beta decay wouldn't be maximally parity violating
- The popular seesaw mechanism hypothesizes that neutrinos are Majorana fermions, which clears up this confusion. It also naturally explains why the neutrino mass is so tiny.



Neutrinoless double beta decay (0νββ)

Why do we care about it?

- Requires lepton-number violation (LNV)
- Low energy discriminant between theories beyond the Standard Model (BSM)



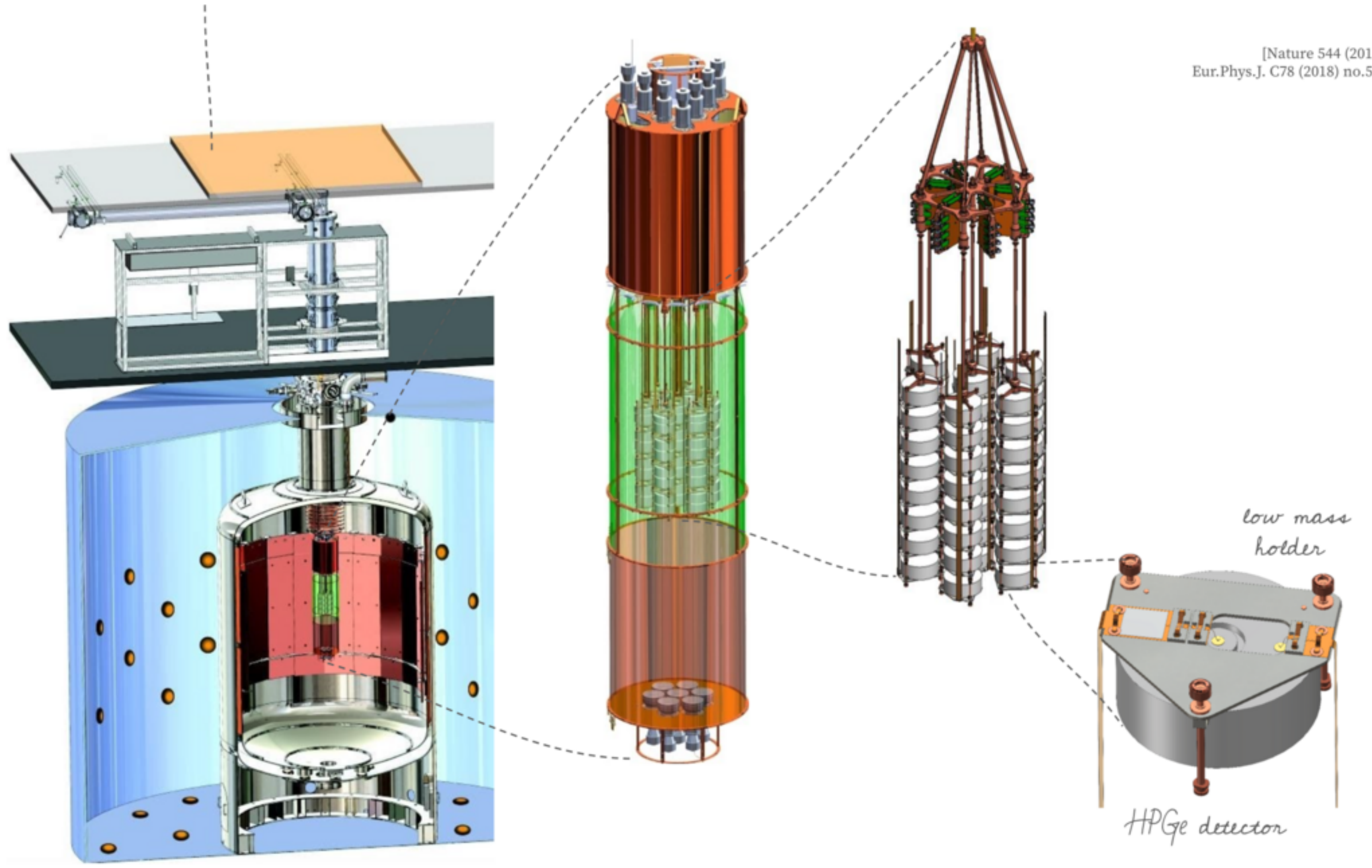
Representative lepton number violating operators in LEFT (from Cirigliano et al. submitted to Snowmass 2021)

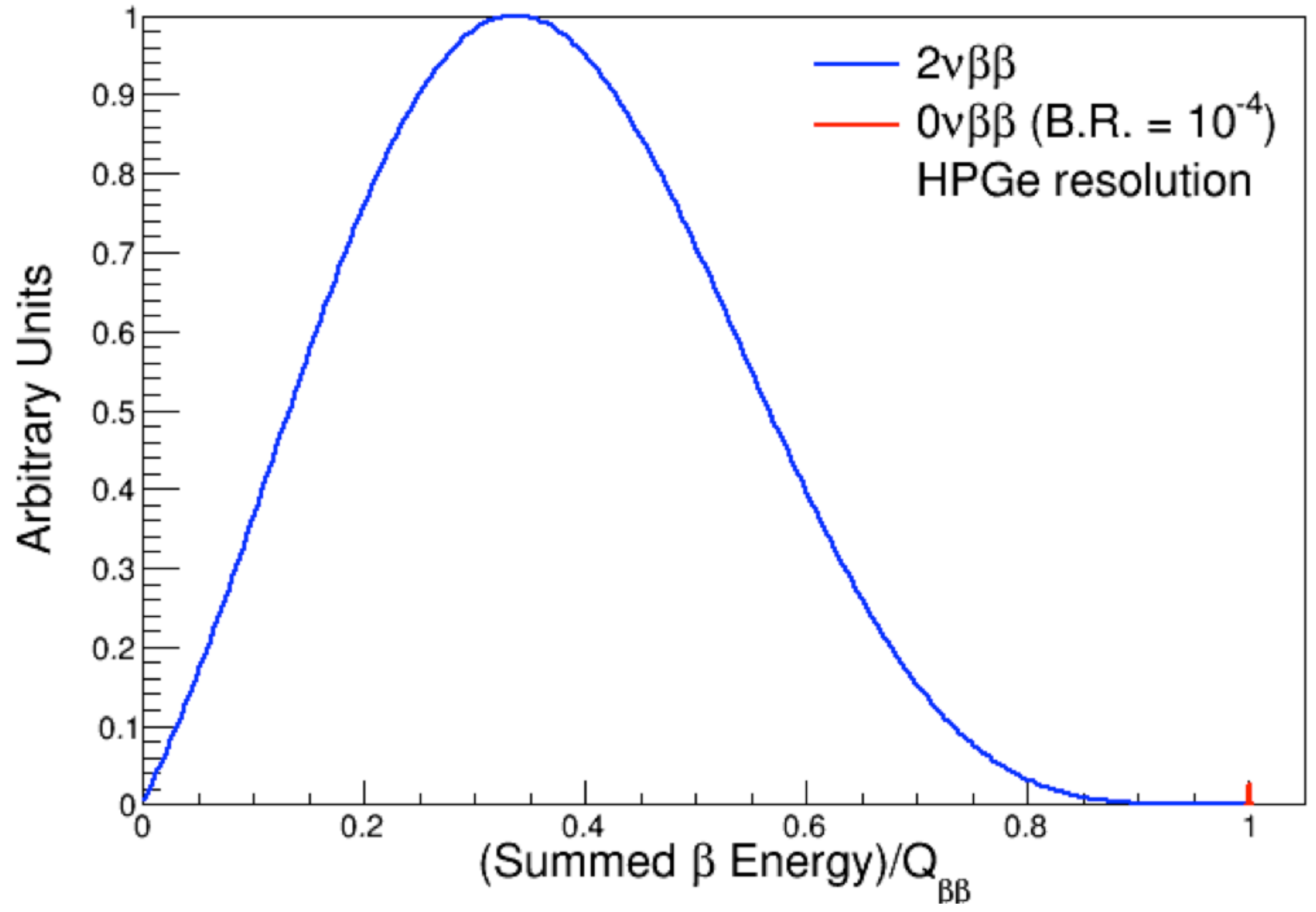
Neutrinoless double beta decay (0νββ)

Will we see it any time soon?

- Multiple experimental searches are underway. I'm most familiar with LEGEND.
- Several unstable isotopes are energetically forbidden from single beta decay but not double beta decay, and some of them serve as very good electron detectors (EXO uses ^{176}Xe, LEGEND uses ^{76}Ge)
- Signal: seeing all decay energy in the two electrons from a decay

[Nature 544 (2017) 47, Eur.Phys.J. C78 (2018) no.5, 388]



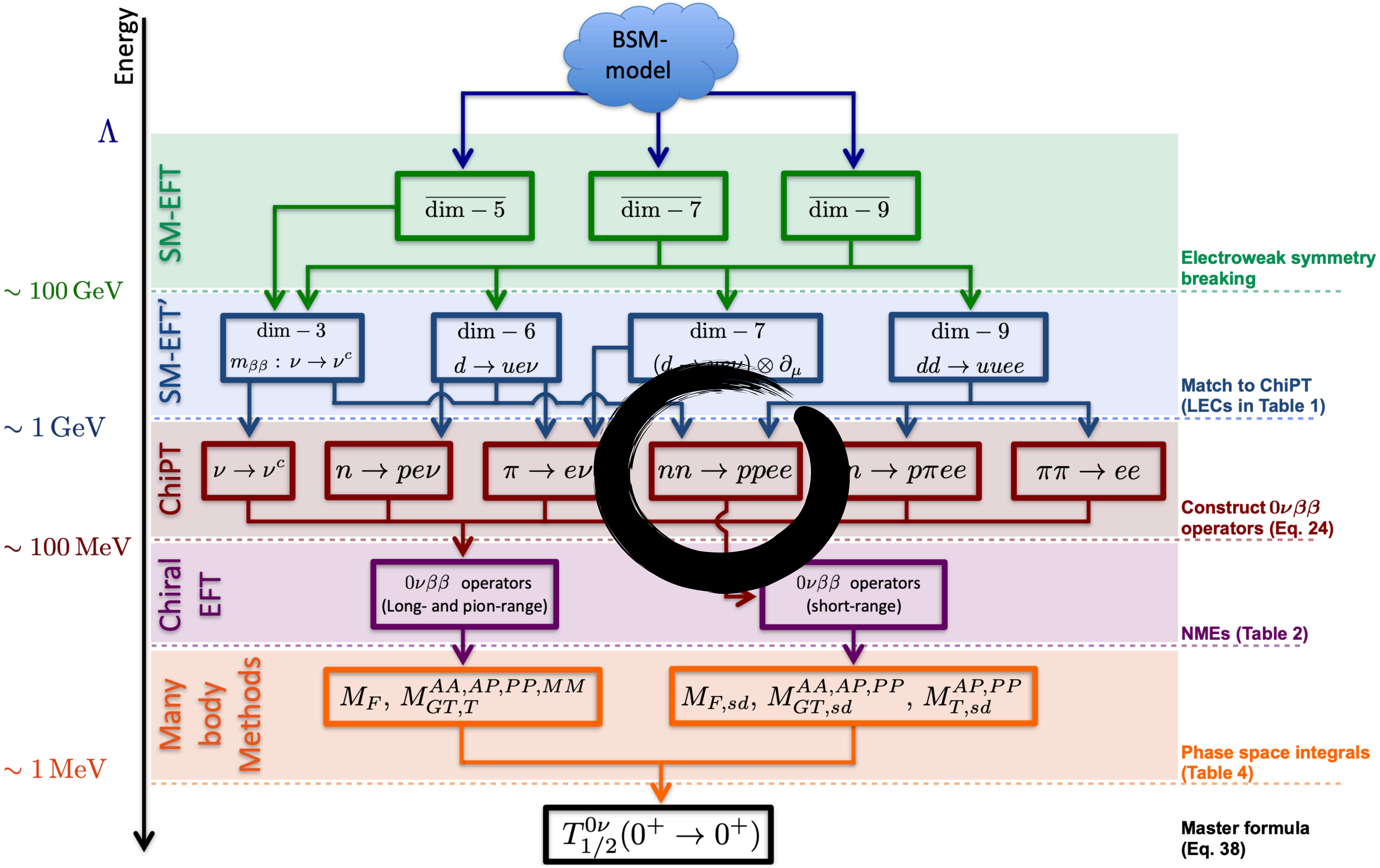


Credit: The MAJORANA DEMONSTRATOR
That red peak is too large for our universe.

Neutrinoless double beta decay (0νββ)

How are calculations done?

- Effective field theories generically extend the Standard Model
- Effective theories combine individual contributions from QCD, QED, BSM model, etc
- Circled interaction will be relevant to this work



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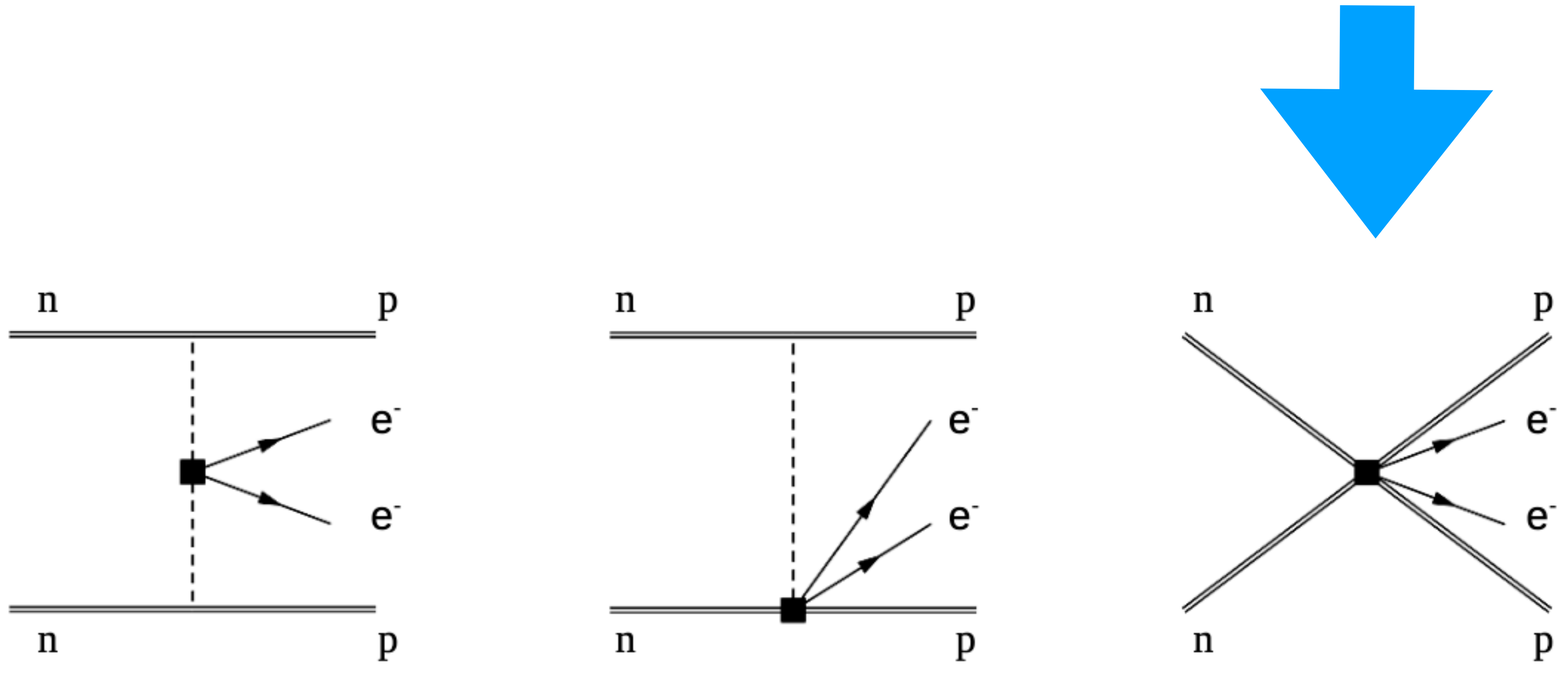
Neutrinoless double beta decay (0vββ)

Heavy physics contact interaction

- nn →ppee contact operator is a leading-order (in chiral EFT) short-range contribution as shown in 2018 paper shown below



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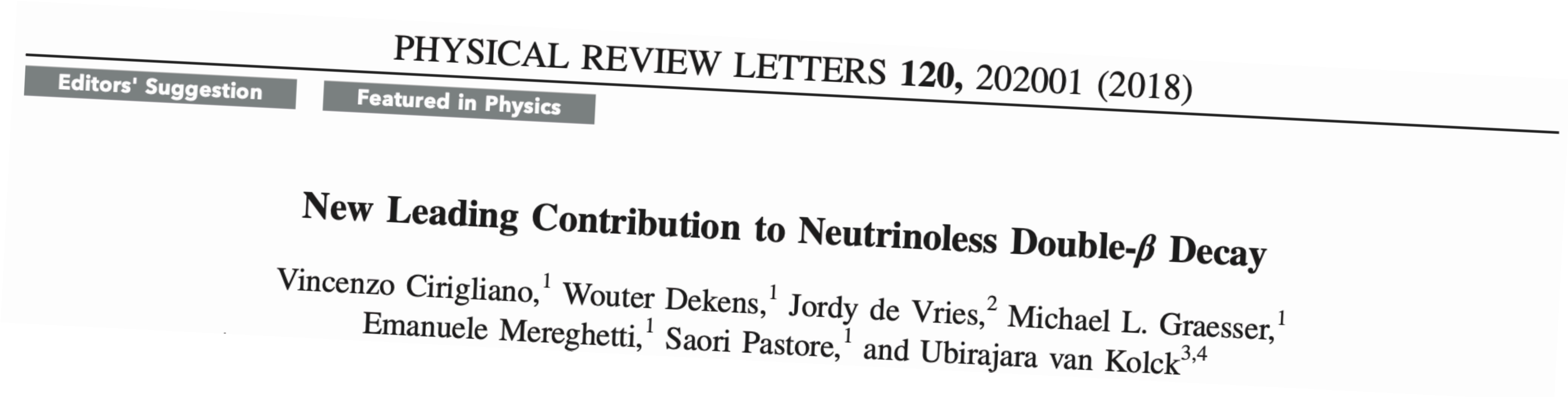


PHYSICAL REVIEW LETTERS **120**, 202001 (2018)

Editors' Suggestion Featured in Physics

New Leading Contribution to Neutrinoless Double-β Decay

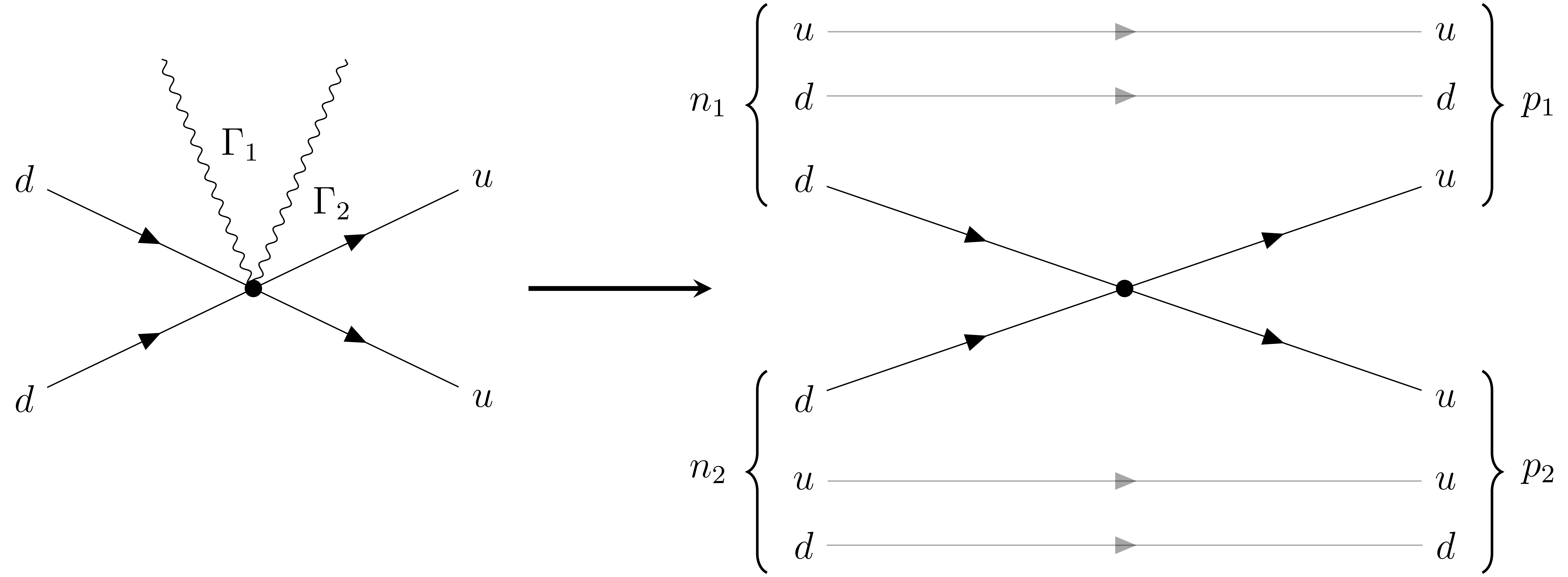
Vincenzo Cirigliano,[1] Wouter Dekens,[1] Jordy de Vries,[2] Michael L. Graesser,[1] Emanuele Mereghetti,[1] Saori Pastore,[1] and Ubirajara van Kolck[3,4]

"In this Letter, we demonstrate that Weinberg's scheme for 0vββ assumed in Ref. [26] breaks down and any consistent power counting requires a leading-order short-range ΔL=2 operator, whose effect is missing in all current calculations."

Four quark matrix elements for 0νββ

Heavy physics contact interaction

- At QCD level, these are four quark operators
- Expect extreme signal-to-noise issues for nucleon calculation



Two weak currents inserted at a single intermediate spacetime point.
The block can be contracted with observer quarks to make nuclear contact interaction

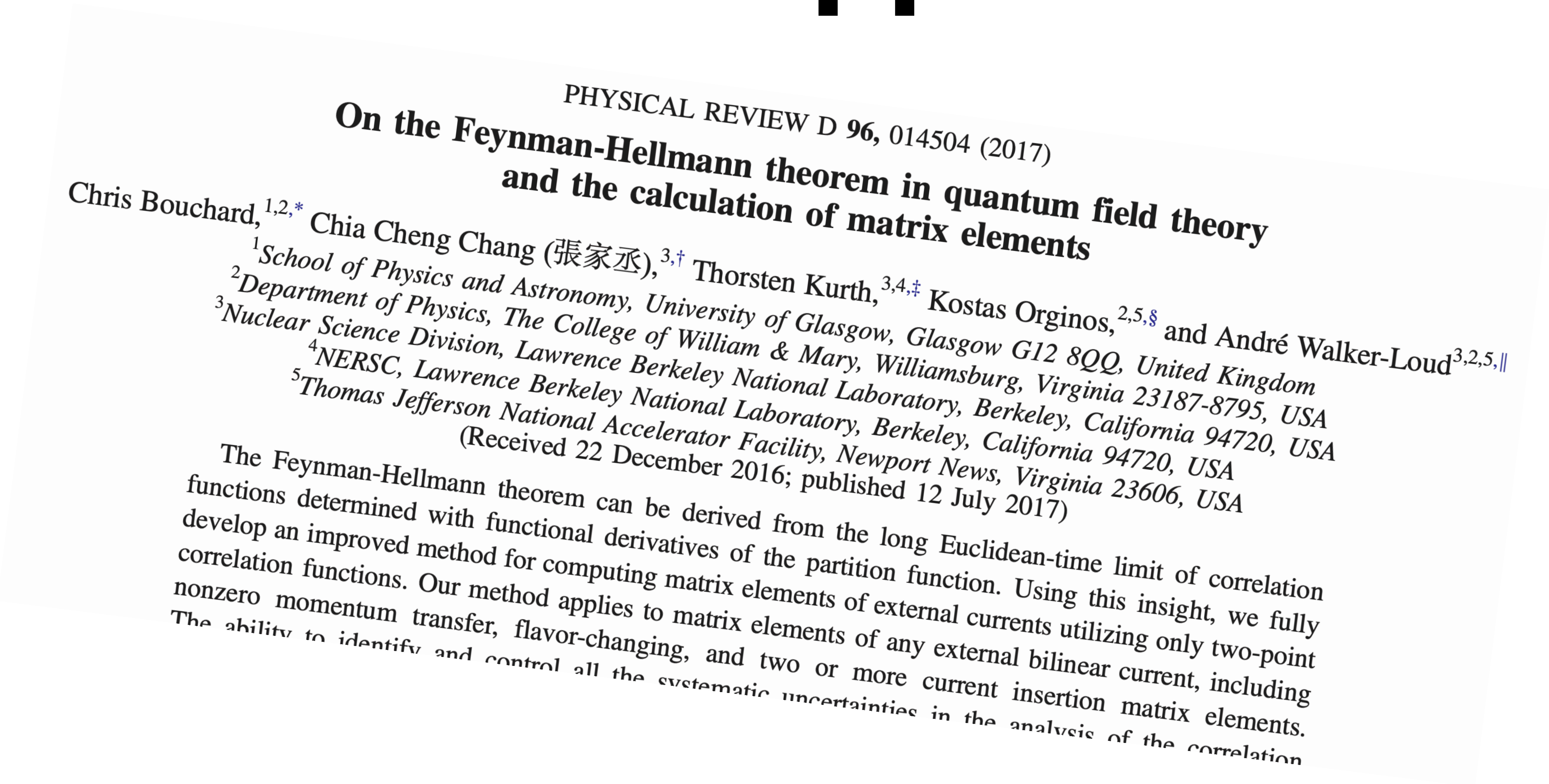
Four quark matrix elements for 0νββ

Proposal: FH method

- Feynman-Hellmann (FH) methods can boost the signal of the current matrix element.
- Expect $\mathcal{O}$(t) stochastic enhancement over sequential propagator

$$\frac{\partial E_n}{\partial \lambda} = \langle n | \frac{\partial H}{\partial \lambda} | n \rangle \;\rightarrow\; \left.\frac{\partial m_{eff}(\lambda)}{\partial \lambda}\right|_{\lambda=0} = \frac{\langle 0|J|0\rangle}{2E_0}$$

The name is an analogy to the Feynman Hellmann theorem


PHYSICAL REVIEW D **96,** 014504 (2017)

On the Feynman-Hellmann theorem in quantum field theory and the calculation of matrix elements

Chris Bouchard,[1,2,*] Chia Cheng Chang (張家丞),[3,†] Thorsten Kurth,[3,4,‡] Kostas Orginos,[2,5,§] and André Walker-Loud[3,2,5,∥]

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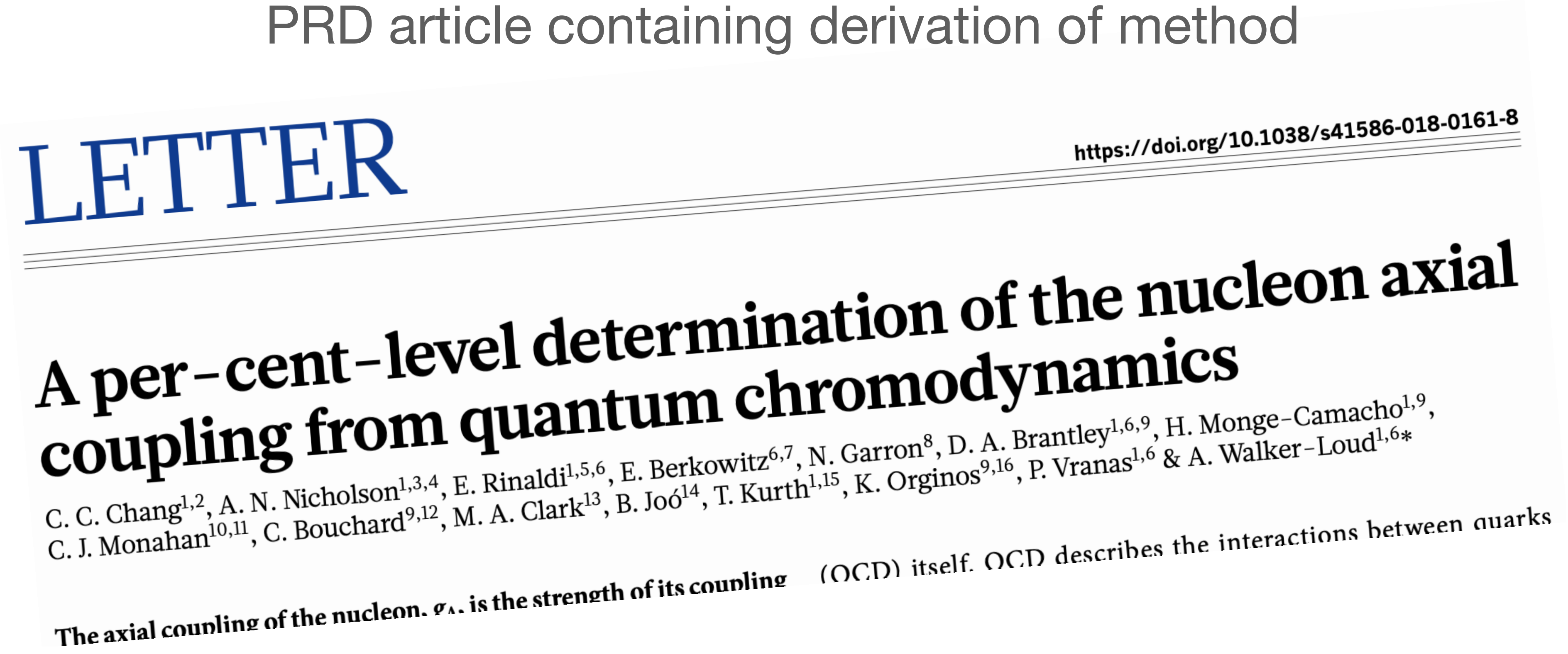
[2]Department of Physics, The College of William & Mary, Williamsburg, Virginia 23187-8795, USA
[3]Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[4]NERSC, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
[5]Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA

(Received 22 December 2016; published 12 July 2017)

The Feynman-Hellmann theorem can be derived from the long Euclidean-time limit of correlation functions determined with functional derivatives of the partition function. Using this insight, we fully develop an improved method for computing matrix elements of external currents utilizing only two-point correlation functions. Our method applies to matrix elements of any external bilinear current, including nonzero momentum transfer, flavor-changing, and two or more current insertion matrix elements. The ability to identify and control all the systematic uncertainties in the analysis of the correlation

PRD article containing derivation of method


LETTER

https://doi.org/10.1038/s41586-018-0161-8

A per-cent-level determination of the nucleon axial coupling from quantum chromodynamics

C. C. Chang[1,2], A. N. Nicholson[1,3,4], E. Rinaldi[1,5,6], E. Berkowitz[6,7], N. Garron[8], D. A. Brantley[1,6,9], H. Monge-Camacho[1,9], C. J. Monahan[10,11], C. Bouchard[9,12], M. A. Clark[13], B. Joó[14], T. Kurth[1,15], K. Orginos[9,16], P. Vranas[1,6] & A. Walker-Loud[1,6*]

The axial coupling of the nucleon, g_A, is the strength of its coupling … (QCD) itself. QCD describes the interactions between quarks

2018 Nature Letter demonstrating method

Four quark matrix elements for 0νββ

The challenge

- The "FH propagator" contains an integral over space and time, which translates to a sum over the entire volume of the lattice

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)\tau} - \frac{\partial_\lambda C_\lambda(t)}{C(t)\tau}$$

- Analysis is easier because the current insertion is now contained in a two-point function, but you have an expensive sum of a lattice volume worth of three-point functions

$$\partial_\lambda C_\lambda(t) = \int dt' \left\langle T\left(\underset{\text{Quark}}{\underset{\uparrow}{\mathcal{O}(t)}} \int d^3x\, \underset{\text{Current}}{\underset{\uparrow}{j(t',\vec{x})}}\underset{\text{Quark}}{\underset{\uparrow}{\mathcal{O}(0)}}\right)\right\rangle$$

$$F^i(z,y,x)_{\substack{\beta\alpha\\ ba}} = P^u(z,y)_{\substack{\beta\beta'\\ bc}}\Gamma^i_{\beta'\alpha'}P^d(y,x)_{\substack{\alpha'\alpha\\ ca}}$$

Each current insertion has two quark propagators

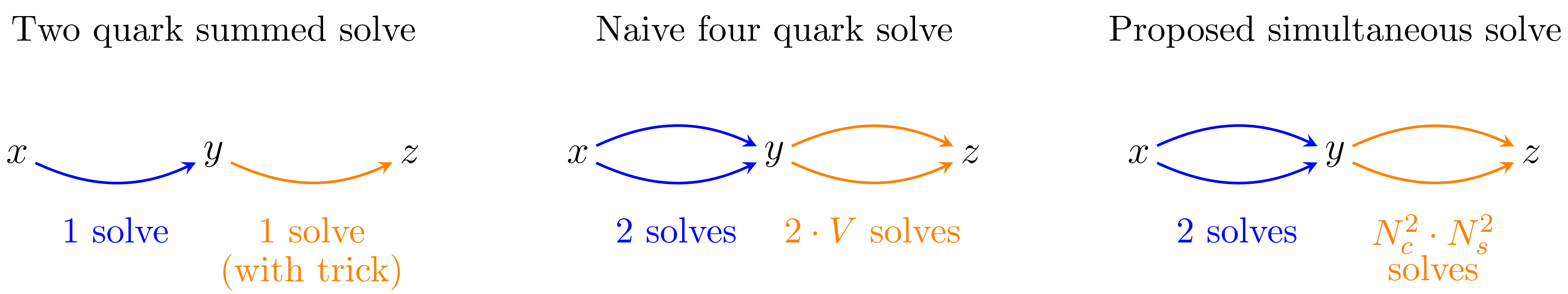
$$B_\Sigma(z,x_0)_{\substack{\beta\alpha\\ \nu\mu\\ ba\\ dc}} = \sum_{y'} F^2(z,y',x_0)_{\substack{\nu\mu\\ dc}}F^1(z,y',x_0)_{\substack{\beta\alpha\\ ba}}$$

Each F block depends on same y'. Sum doesn't cleanly factor.

Four quark matrix elements for 0νββ

Method: summed sequential block solver

- The summed sequential propagator is very expensive to build with standard one-to-all propagators. We want to build a custom solver which performs a partial inversion of a combination of two Dirac operators at once. Adapting existing tools to do that turned out to be nontrivial.
- We need to develop entirely new machinery for the FH four quark operator



Four quark matrix elements for 0νββ

The code: building on USQCD's Chroma library

- Chroma with QUDA allows for scalable GPU-accelerated quark propagator solvers, we are building on these libraries.

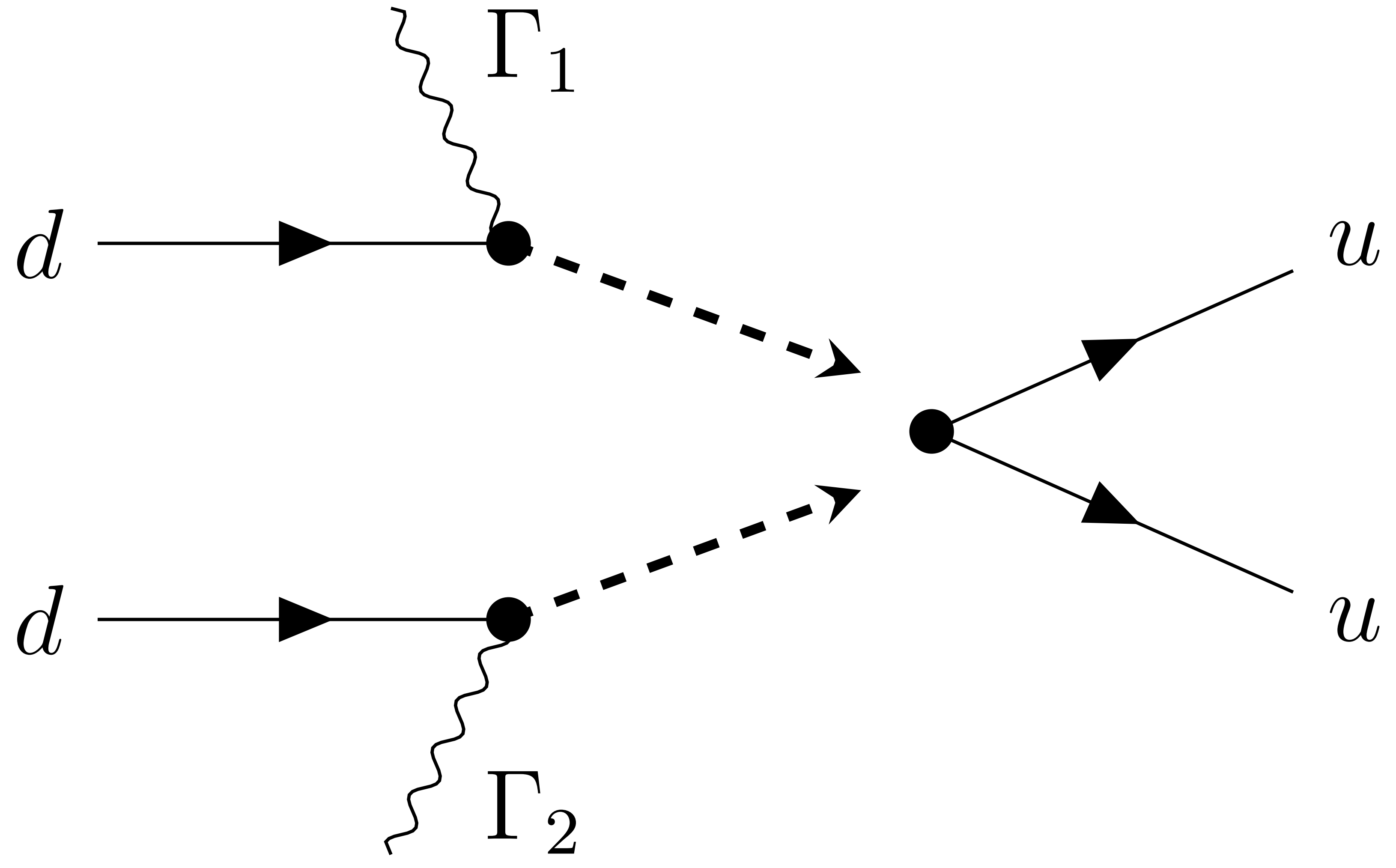
- Chroma: R. G. Edwards (LHPC Collaboration), B. Joó (UKQCD Collaboration), "*The Chroma Software System for Lattice QCD*", arXiv:hep-lat/0409003, Proceedings of the 22nd International Symposium for Lattice Field Theory (Lattice2004), Nucl. Phys B1 40 (Proc. Suppl) p832, 2005. (https://jeffersonlab.github.io/chroma/)

- QUDA: M. A. Clark, R. Babich, K. Barros, R. C. Brower, C. Rebbi, Solving Lattice QCD systems of equations using mixed precision solvers on GPUs arXiv:0911.3191v1 [hep-lat] (https://lattice.github.io/quda/)

Four quark matrix elements for 0νββ

Block solver structural highlights

- Our solver will effectively apply two copies of the Dirac operator at once, enabling two sequential solves summed over every intermediate lattice point
- Sparseness of Dirac operator makes two applications at once tractable
- Each term of sum contains two propagators at the same intermediate point, so we can share Dirac operator elements between the two applications.

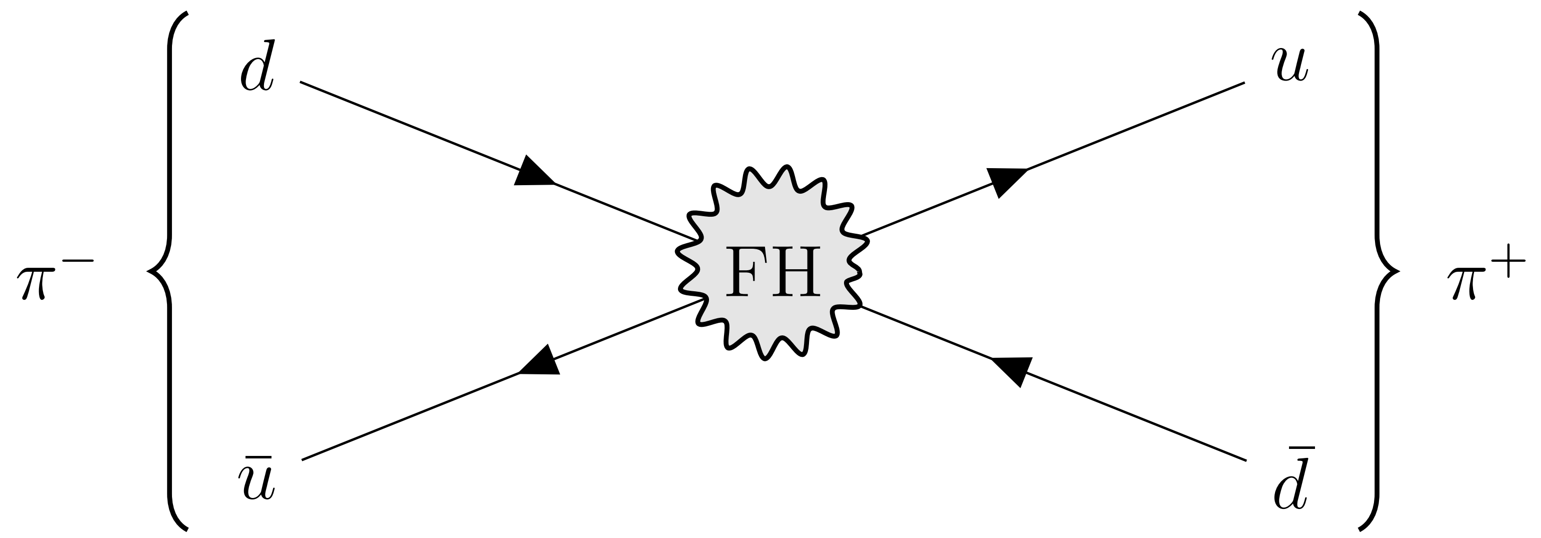


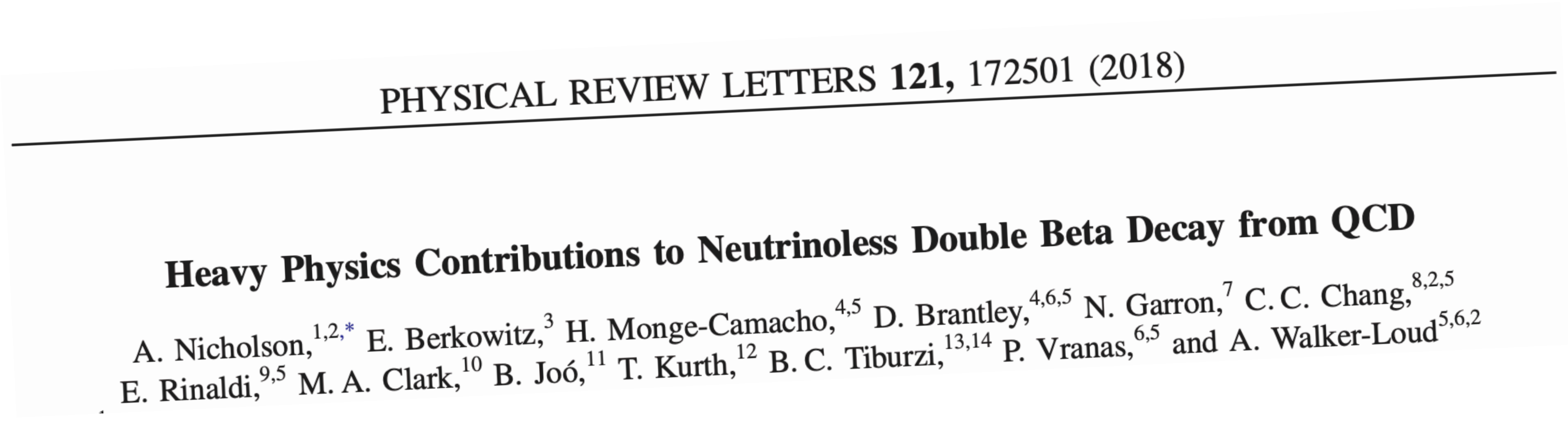
On left we have one-to-all solves for the incoming lines which are contracted with current insertions.
Those are combined into a source for our proposed simultaneous solve of the outgoing lines.

Four quark matrix elements for 0νββ

Testing on the pion

- Start by testing those tools against previously calculated $\pi^- \to \pi^+$ matrix element
  - This is a long-range heavy physics contribution to 0νββ
- Good calculations have already been performed for this matrix element

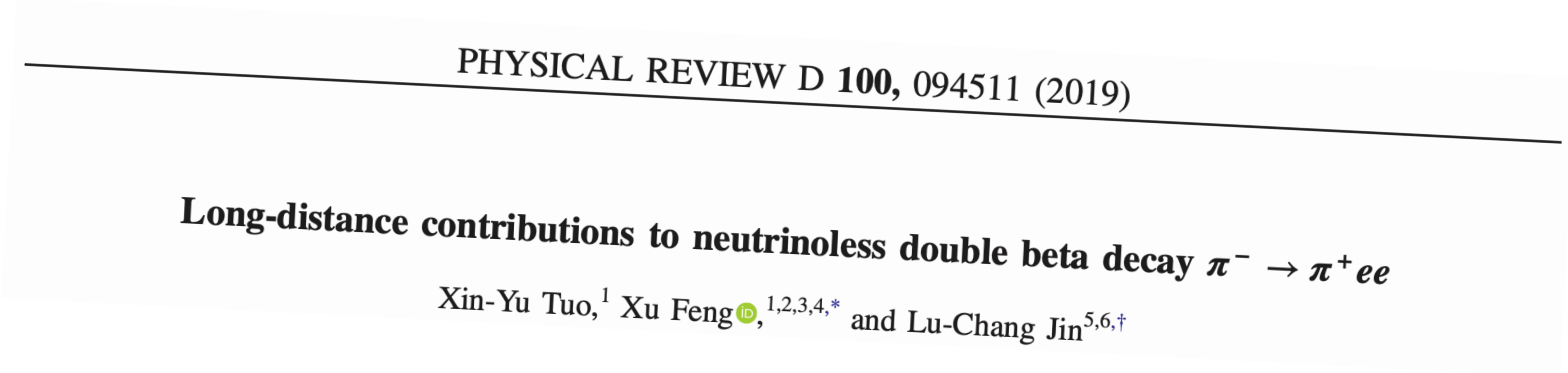




PHYSICAL REVIEW LETTERS **121**, 172501 (2018)

Heavy Physics Contributions to Neutrinoless Double Beta Decay from QCD

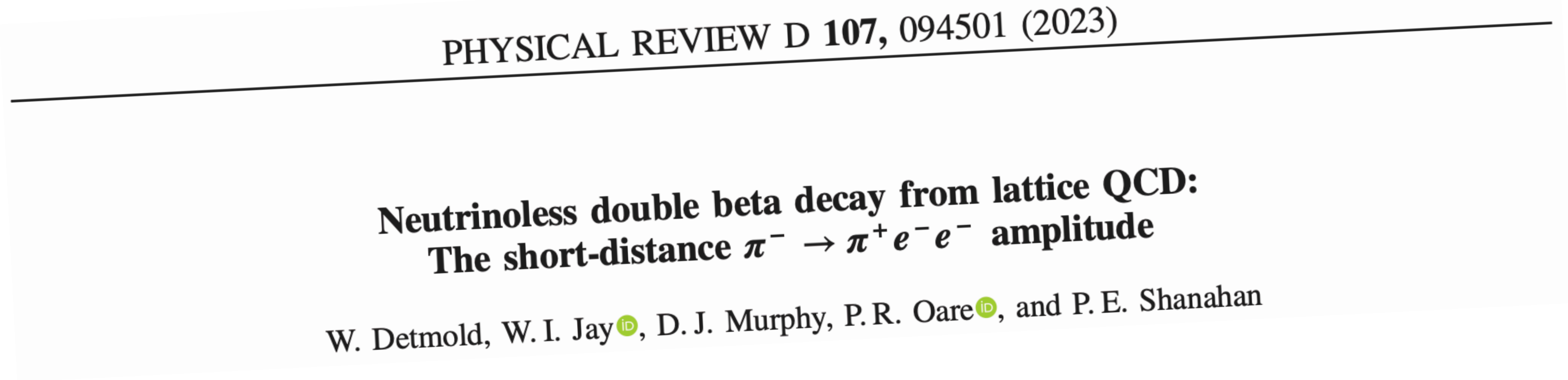
A. Nicholson,[1,2,*] E. Berkowitz,[3] H. Monge-Camacho,[4,5] D. Brantley,[4,6,5] N. Garron,[7] C. C. Chang,[8,2,5] E. Rinaldi,[9,5] M. A. Clark,[10] B. Joó,[11] T. Kurth,[12] B. C. Tiburzi,[13,14] P. Vranas,[6,5] and A. Walker-Loud[5,6,2]



PHYSICAL REVIEW D **100**, 094511 (2019)

Long-distance contributions to neutrinoless double beta decay $\pi^- \to \pi^+ ee$

Xin-Yu Tuo,[1] Xu Feng,[1,2,3,4,*] and Lu-Chang Jin[5,6,†]



PHYSICAL REVIEW D **107**, 094501 (2023)

Neutrinoless double beta decay from lattice QCD: The short-distance $\pi^- \to \pi^+ e^- e^-$ amplitude

W. Detmold, W. I. Jay, D. J. Murphy, P. R. Oare, and P. E. Shanahan

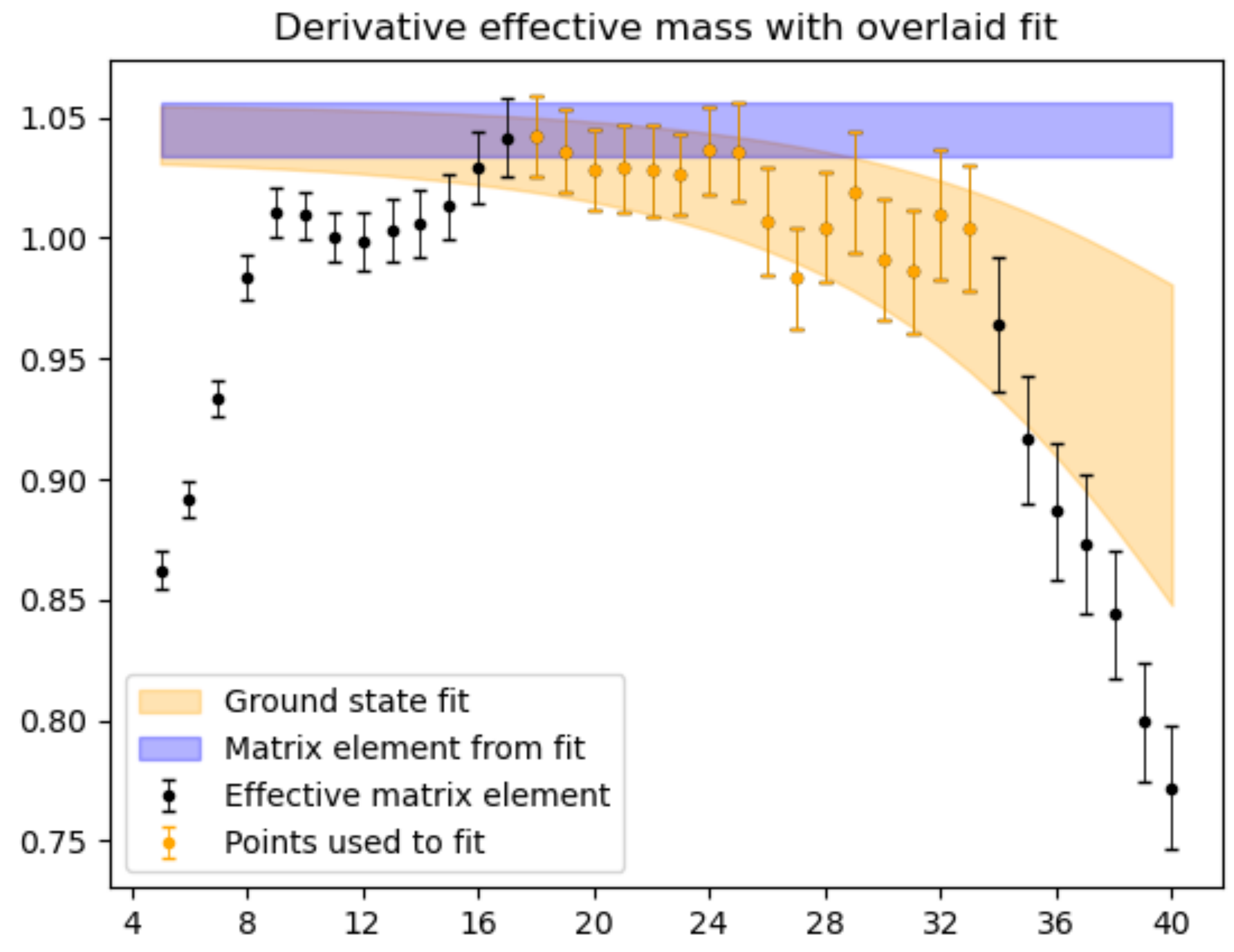
Four quark matrix elements for 0νββ

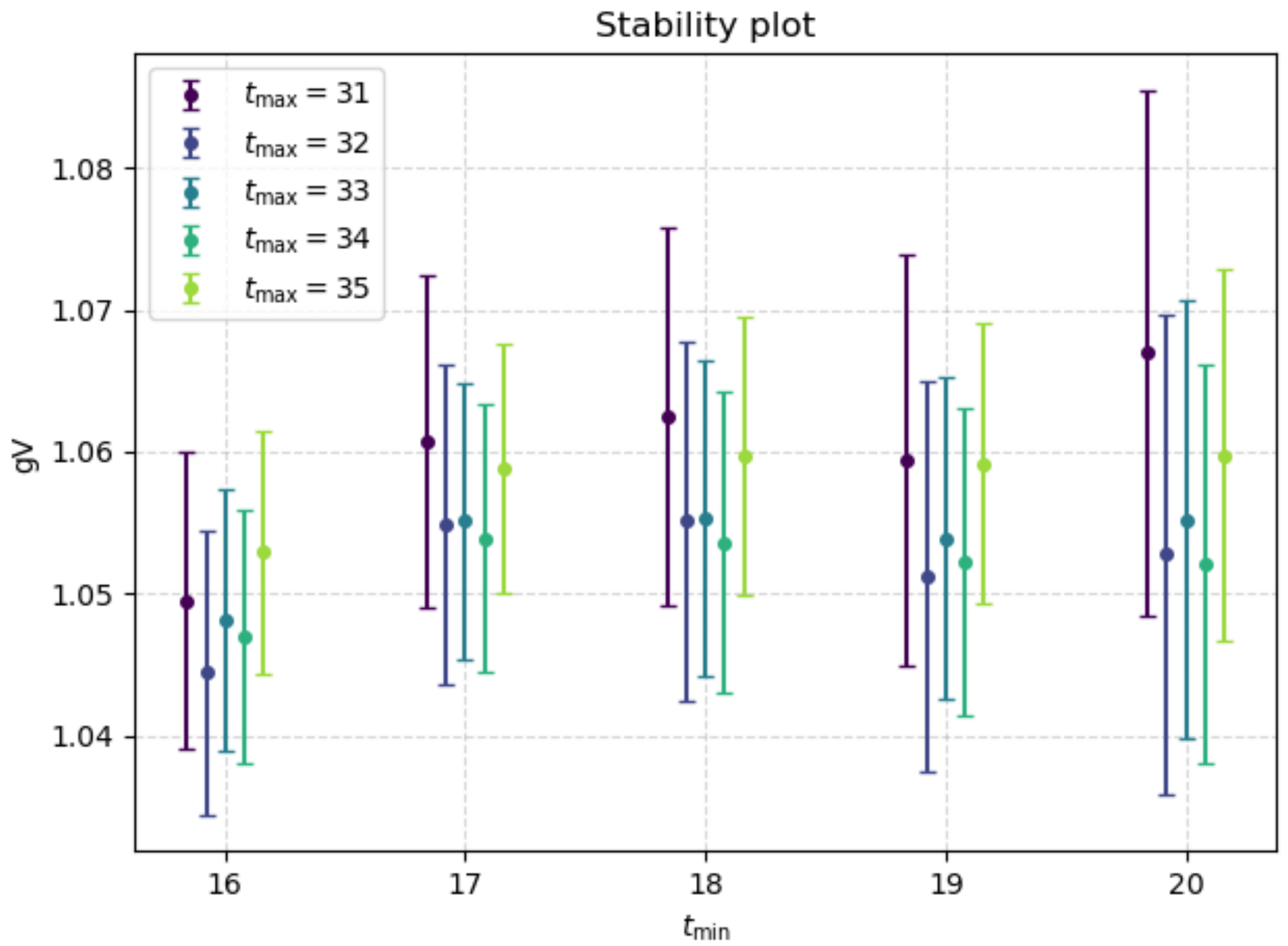
Testing the pion: analysis pipeline

- We derived derivative effective mass formula at long time and $\lambda \to 0$

- We used weak vector current FH propagators from CalLat to test our analytical derivation on the simpler $\pi^+ \to \pi^0$ current insertion which has the same FH structure.

$$\frac{N(t+\tau)+N(t-\tau)-2\cosh(E_0\tau)N(t)}{2\tau\sinh(E_0\tau)C(t)} = \frac{\mathcal{J}_{00}}{2E_0} + \mathcal{J}_{\Omega\Omega} + \frac{c'}{C(t)}$$





Four quark matrix elements for $0\nu\beta\beta$

Solver work in progress

- Double Dirac operator in Chroma is constructed and tested against independent Python implementation. Integrating it into a solver requires further development.
    - The four quark block has the shape of a Propagator with an extra set of spin and color indices. Chroma does not deal with this natively.
- We have constructed brute-force measurements in Chroma for use on small lattices to make sure that contraction code is functioning correctly.
- Brute force data generation and validation is in progress.

Four quark matrix elements for 0νββ

Summary

- Four quark matrix elements are necessary to account for short range nucleon contact contributions to neutrinoless double beta decay, allowing discrimination between BSM theories
- We will apply Feynman-Hellman methods for an $\mathcal{O}$(t) stochastic enhancement of the matrix element in this noisy two-nucleon calculation
- We have begun to write a solver to efficiently generate the summed sequential quark block necessary for FH with a simultaneous double Dirac operator
- We have constructed a pipeline to calculate the matrix element for the $\pi^- \to \pi^+$ interaction in order to validate this new machinery