

Investigating the role of tetraquark operators in lattice QCD studies of the $a_0(980)$ and κ resonances: A cautionary tale

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Presenting the findings of

**“Investigating the role of tetraquark operators in lattice
QCD studies of the $a_0(980)$ and κ resonances”
Hanlon et al., PRD 114, 014517 (2026)**

Work done in collaboration with

Andrew D. Hanlon

Daniel Darvish

Sarah Skinner

Ruairí Brett

John Bulava

Jacob Fallica

Colin Morningstar

Fernando Romero-López

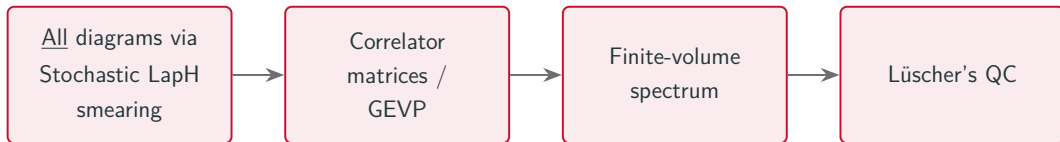
André Walker-Loud

Overview

- Earlier κ and $a_0(980)$ spectra were extracted without TQ interpolators [Wilson et al., PhysRevD.91.054008 (2015); Dudek et al., PhysRevD.93.094506 (2016)].
- Dedicated four-quark studies conflict: one found an extra light κ -like level, the other no extra $\kappa/a_0(980)$ level [Prelovsek et al., PhysRevD.82.094507 (2010); Alexandrou et al., JHEP04(2013)137 (2013)].
- Both omitted disconnected diagrams, which χ PT shows are not negligible [Guo et al., PhysRevD.88.074506 (2013)].
- A later study included both, and found an extra $a_0(980)$ level [Alexandrou et al., PhysRevD.97.034506 (2018)].

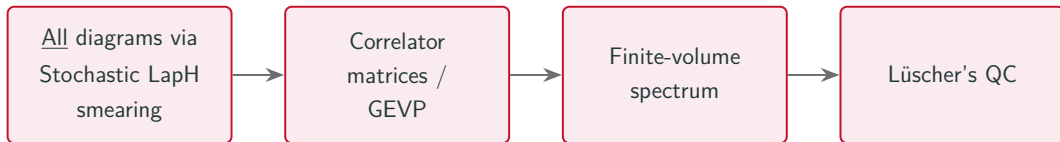
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Do TQ operators alter the spectrum? If so, does the scattering description change?

Resonances of Interest

	$\kappa \equiv K_0^*(700)$	$a_0(980)$
Quantum numbers	$I(J^P) = \frac{1}{2}(0^+)$	$I^G(J^{PC}) = 1^-(0^{++})$
Strangeness	$S = 1$	$S = 0$
Dominant channels	$K\pi, K\eta$	$\pi\eta, K\bar{K}, \pi\eta'$
Rest-frame irrep	A_{1g}	A_{1g}^-

Ensemble Info

Collaboration	Hadron Spectrum Collaboration			
Fermion action	$N_f = 2 + 1$ anisotropic Wilson-clover			
Lattice extent	$(L/a_s)^3 \times (T/a_t) = 32^3 \times 256, \ L = 3.74 \text{ fm}$			
Pion mass	$m_\pi \approx 230 \text{ MeV}$			
Anisotropy	$\xi = a_s/a_t = 3.451(11)$			
Temporal spacing	$a_t = 0.033357(59) \text{ fm}$			
N_{cf}	$a_t m_\pi$	$a_t m_K$	$a_t m_\eta$	$a_t m_{\eta'}$
412	0.03953(17)	0.08348(14)	0.0979(30)	0.165(12)

[Edwards et al., PhysRevD.78.054501 (2008); Lin et al., PhysRevD.79.034502 (2009)]

Extracting the Finite-Volume Spectrum

- We extract the finite-volume spectrum from correlation functions of a basis of interpolating operators.

$$\mathcal{C}_{ij}(t) = \langle 0 | O_i(t + t_s) \overline{O}_j(t_s) | 0 \rangle.$$

- Isolate the contribution from the n -th state using GEVP, single pivot:

$$\lambda_n(t) \propto e^{-E_n t} \quad \text{for large } t.$$

- For each momentum, solve a GEVP in the flavor basis

$$\tilde{\eta} = \bar{u}u + \bar{d}d, \quad \tilde{\phi} = \bar{s}s, \quad \text{ROT } 0 \rightarrow \eta, \quad \text{ROT } 1 \rightarrow \eta',$$

yielding variationally improved η and η' operators.

Single- and Two-Meson Operators

- Stout-smeared gauge links;
LapH-smeared quark fields.
- Projected onto zero-momentum,
 A_{1g} and A_{1g}^- irreps.

Single-meson

κ	$a_0(980)$
K^{DDL2}	π^{SD2}
K^{TDO3}	π^{TDO3}
K^{TDU5}	

Two-meson

κ	$a_0(980)$
$K(0)^{SS0}\pi(0)^{SS0}$	$K(0)^{SS0}\bar{K}(0)^{SS0}$
$K(0)^{SS0}\tilde{\eta}(0)^{SS0}$	$K(1)^{SS1}\bar{K}(1)^{SS1}$
$K(0)^{SS0}\tilde{\phi}(0)^{SS0}$	$K(2)^{SS0}\bar{K}(2)^{SS0}$
$K(1)^{SS1}\pi(1)^{SS1}$	$K(2)^{SS1}\bar{K}(2)^{SS1}$
$K(1)^{SS1}\tilde{\eta}(1)^{SS1}$	$\tilde{\eta}(0)^{SS0}\pi(0)^{SS0}$
$K(1)^{SS1}\tilde{\phi}(1)^{SS1}$	$\tilde{\eta}(1)^{SS0}\pi(1)^{SS0}$
$K(2)^{SS0}\pi(2)^{SS0}$	$\tilde{\eta}(2)^{SS1}\pi(2)^{SS1}$
$K(3)^{SS0}\pi(3)^{SS0}$	$\tilde{\phi}(0)^{SS0}\pi(0)^{SS0}$
	$\tilde{\phi}(1)^{SS1}\pi(1)^{SS1}$
	$\tilde{\phi}(2)^{SS0}\pi(2)^{SS0}$

[Morningstar and Peardon, PhysRevD.69.054501 (2004); Morningstar et al., PhysRevD.88.014511 (2013)]

Tetraquark Operators

- Schematically, a two-meson operator takes the form $M_1(p)M_2(-p)$.
- Our tetraquark annihilation operators are linear combinations of

$$\begin{aligned}\Phi_{\alpha\beta\mu\nu;ijkl}^{ABCD\pm}(t) &= \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} (\delta_{ab}\delta_{cd} \pm \delta_{ad}\delta_{bc}) \\ &\times \bar{q}_{a\alpha i}^A(\mathbf{x}, t) q_{b\beta j}^B(\mathbf{x}, t) \bar{q}_{c\mu k}^C(\mathbf{x}, t) q_{d\nu l}^D(\mathbf{x}, t).\end{aligned}$$

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2. Flavor content

Isospin	Strangeness	Flavor content
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$\frac{1}{2}$	1	$\bar{s}u(\bar{u}u + \bar{d}d)$
$\frac{1}{2}$	1	$\bar{s}u(\bar{u}u - \bar{d}d) - \sqrt{2}\bar{s}d\bar{d}u$
1	0	$(\bar{u}u + \bar{d}d)\bar{d}u$
1	0	$\bar{s}s\bar{d}u$
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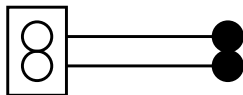
1. Color structure
2. Flavor content
3. Spin structure
4. Spatial configuration

Isospin	Strangeness	Flavor content
$\frac{1}{2}$	1	$\bar{s}u\bar{s}s$
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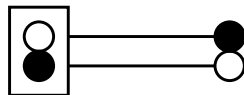
Tetraquark Spatial Configurations



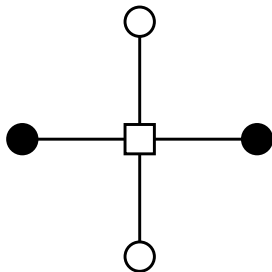
SS



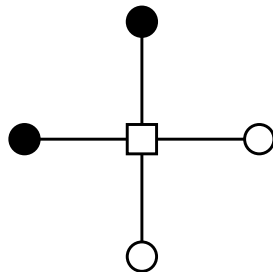
DDIa



DDIb



QDXa



QDXb

Screening the TQ Operator Space

Hundreds of candidate T_i 's to check

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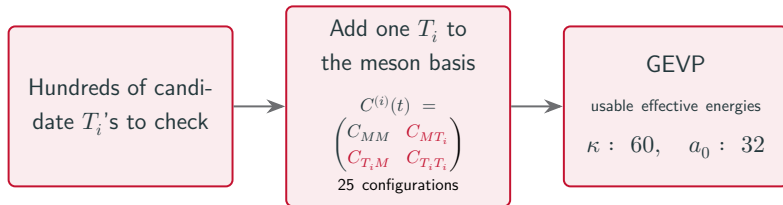


Add one T_i to the meson basis

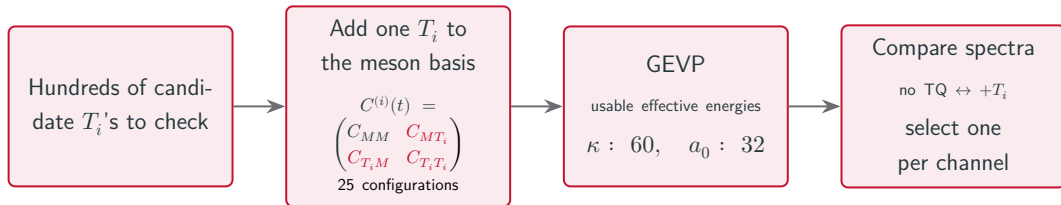
$$C^{(i)}(t) = \begin{pmatrix} C_{MM} & C_{MT_i} \\ C_{T_i M} & C_{T_i T_i} \end{pmatrix}$$

25 configurations

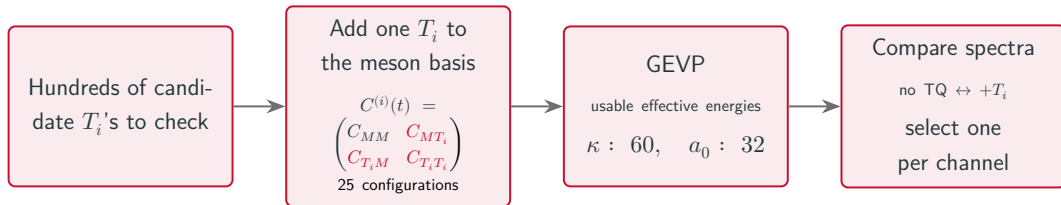
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Which TQ structures change the low-lying spectrum?

Selected TQ Operator – κ

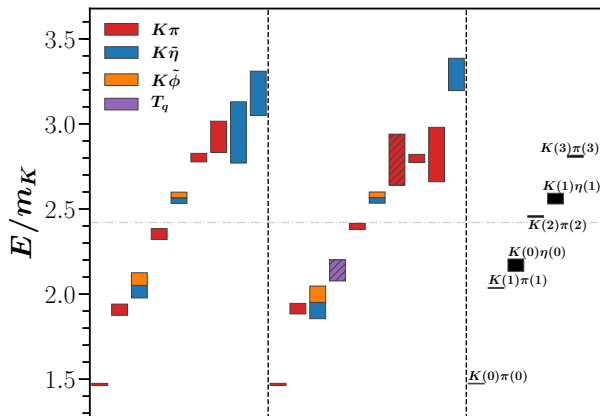
- $I = \frac{1}{2}, S = 1, A_{1g}$
- Color-antisymmetric structure

$$T \left[\bar{s} u \bar{s} s \right]^{SS2(-)}$$

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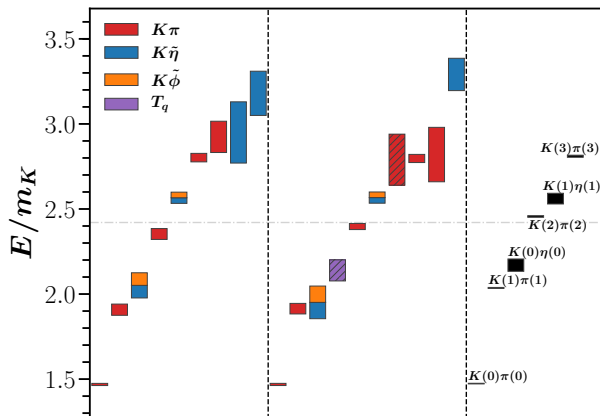
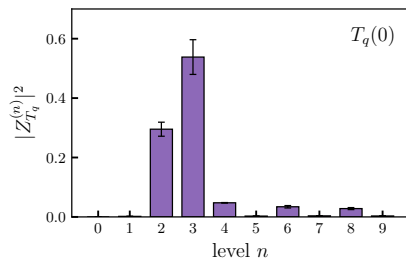
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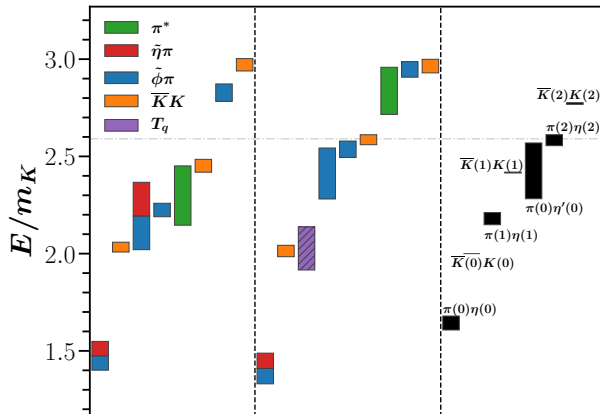
- $I = 1, S = 0, A_{1g}^-$
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$$T \left[(\bar{u}u + \bar{d}d) \bar{d}u \right]^{SS2(+)}$$

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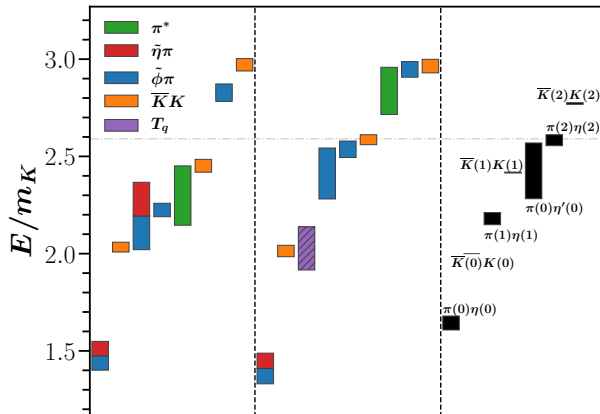
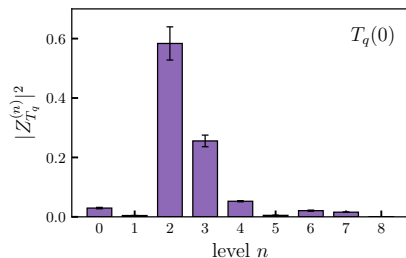
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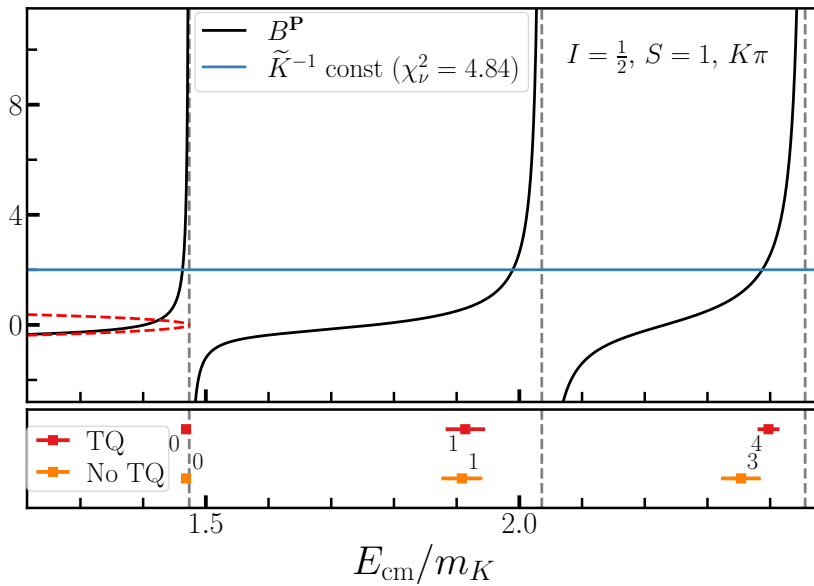


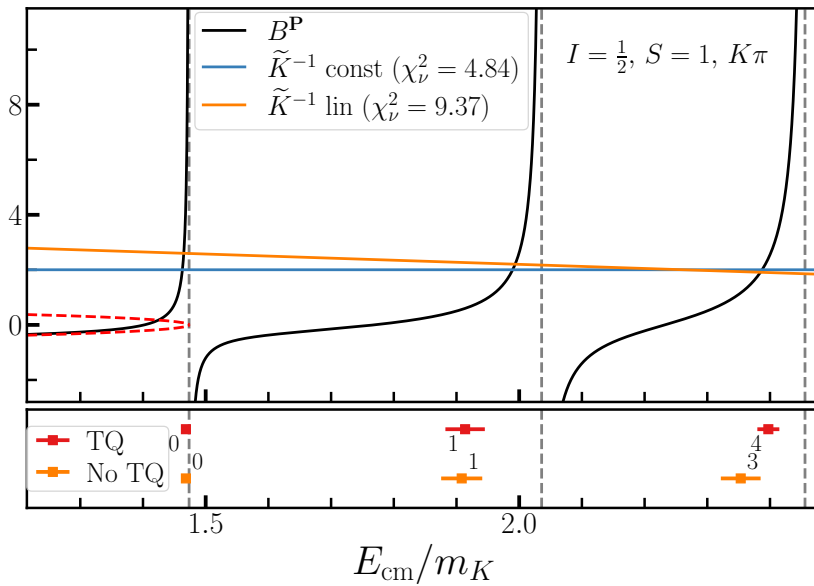
Lüscher QC – Does the IV scattering description change?

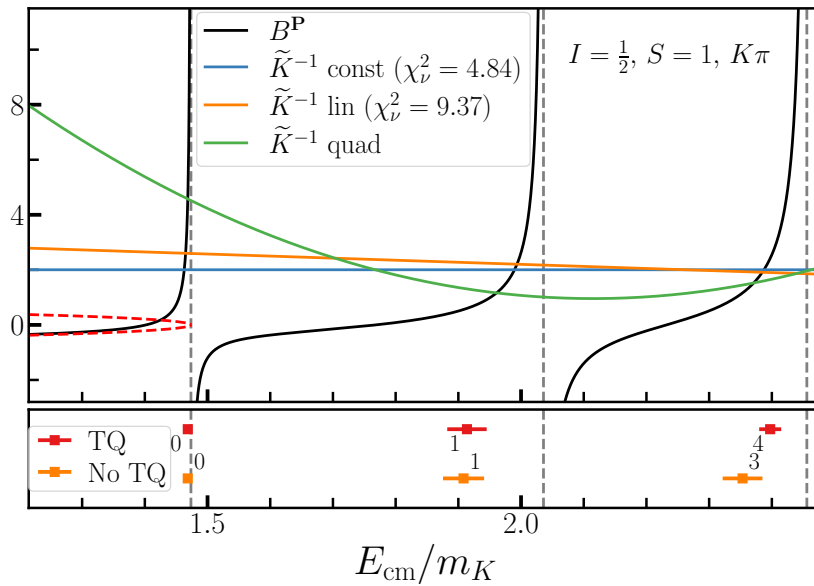
$$\det \left[\widetilde{K}^{-1}(E_{\text{cm}}) - B^{(\text{P},\Lambda)}(E_{\text{cm}}) \right] = 0$$

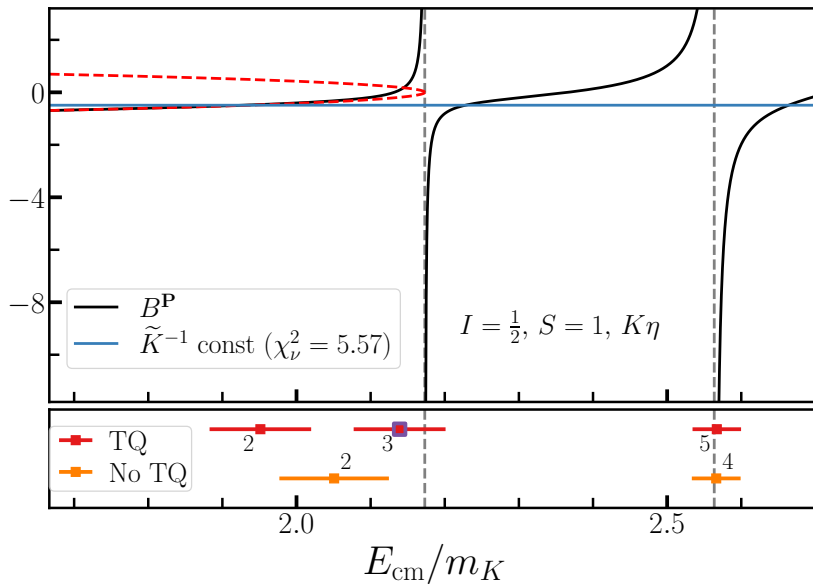
- Truncate to S -wave, consider levels below 4-particle threshold.
- Experiment and Z -factors indicate weak interchannel mixing [Navas et al., PhysRevD.110.030001 (2024)].
- Only one $\pi\eta'$ level in A_{1g}^- below 4-particle threshold – exclude.

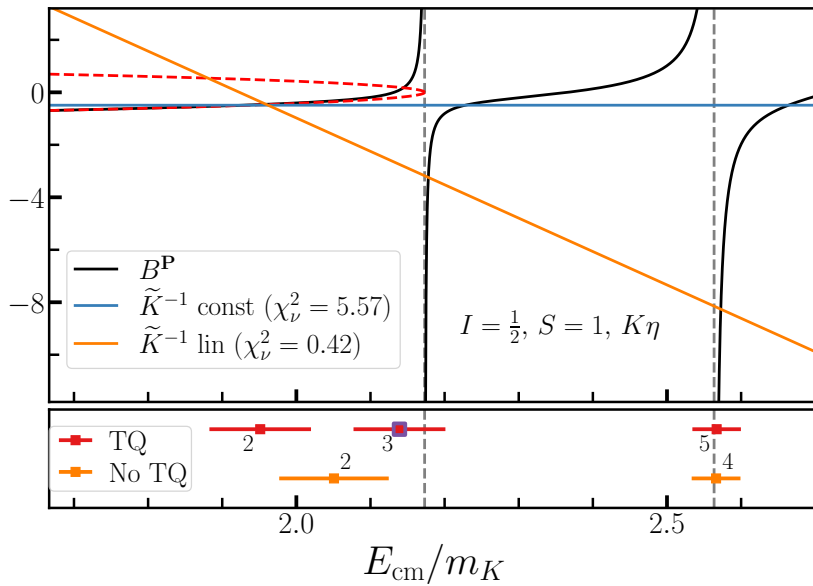
$$\begin{aligned} \det(\widetilde{K}^{-1} - B) &= \det \left(\begin{bmatrix} \widetilde{K}_{11}^{-1} - B_{11} & 0 \\ 0 & \widetilde{K}_{22}^{-1} - B_{22} \end{bmatrix} \right) \\ &= \det(\widetilde{K}_{11}^{-1} - B_{11}) \det(\widetilde{K}_{22}^{-1} - B_{22}). \end{aligned}$$



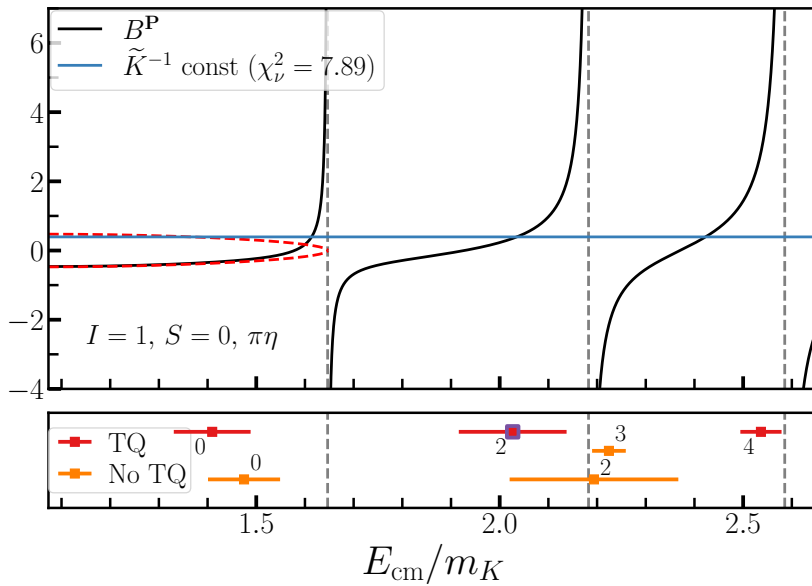




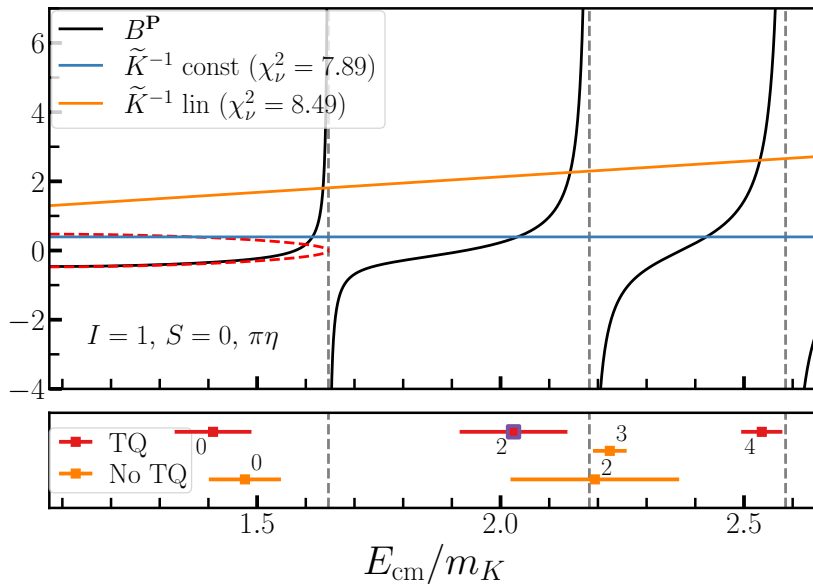




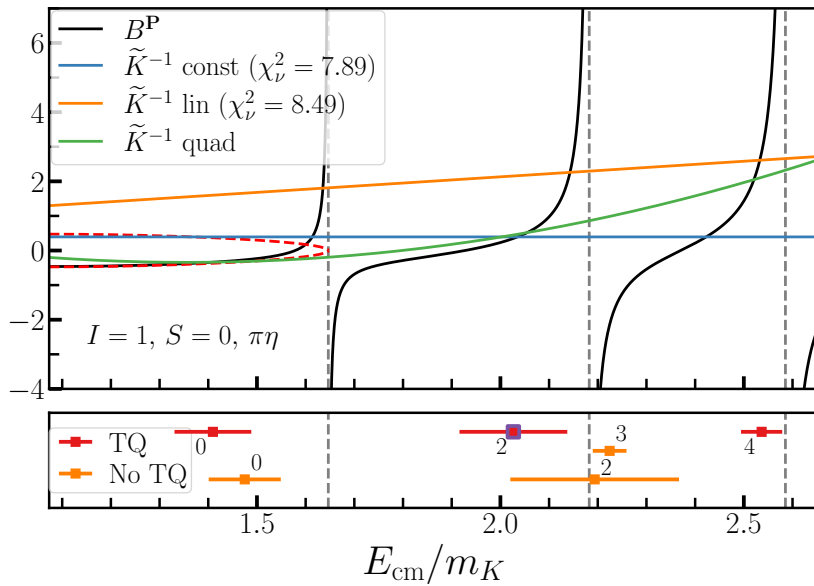
Lüscher QC – A_{1g}^- , $\pi\eta$

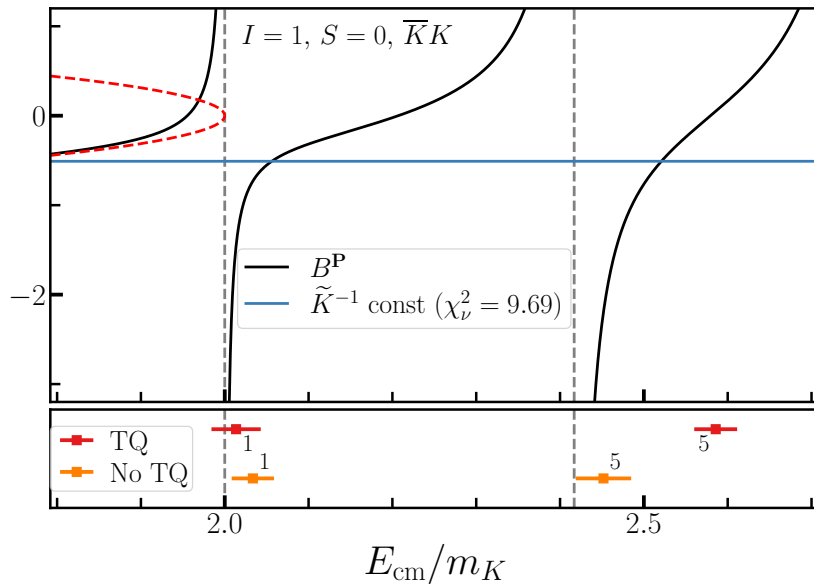


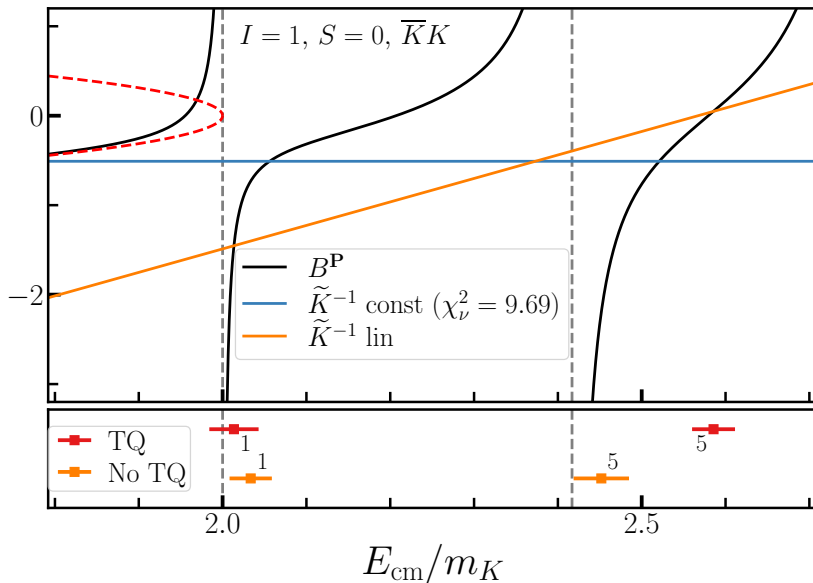
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$K\pi$ energies are only slightly affected.

Implication: κ studies appear unaffected;
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OUTLOOK
resonances

Higher statistics ■ moving frames ■ multiple ensembles ■ other light

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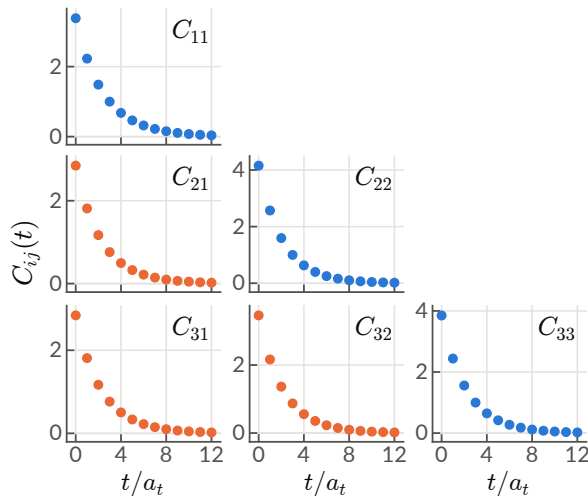
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Apparent plateaux are insufficient without overlap onto all relevant states.

Thank you for your attention

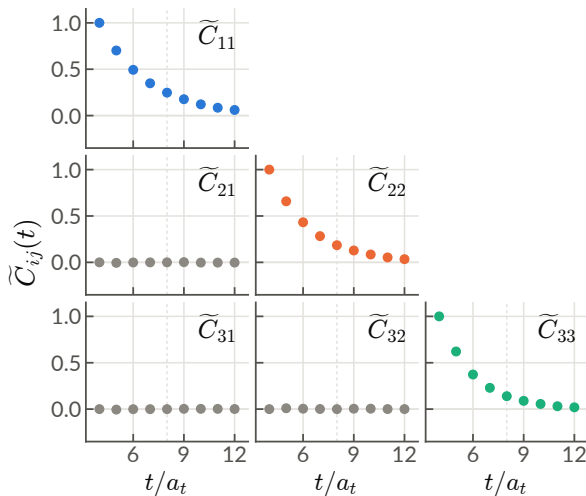
Single Pivot Method – Toy Example



$$C_{ij}(t) = \frac{\mathcal{C}_{ij}(t)}{\sqrt{\mathcal{C}_{ii}(t_N) \mathcal{C}_{jj}(t_N)}}$$

$$C_{ij}(t) = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)*} e^{-E_n t}$$

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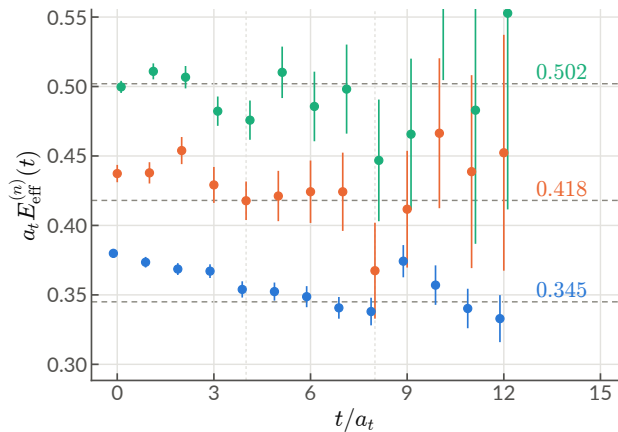
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$$C(t_0)^{-1/2} C(t_D) C(t_0)^{-1/2} U = U D$$

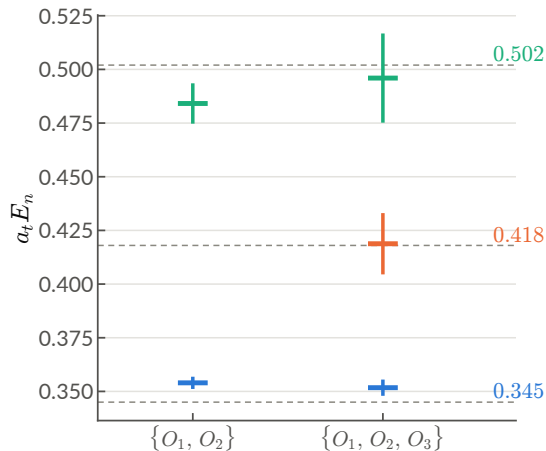
$$\tilde{C}(t) = U^\dagger C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} U$$

Effective Energies



$$a_t E_{\text{eff}}^{(n)}(t) = \ln \left[\frac{\tilde{C}_{nn}(t)}{\tilde{C}_{nn}(t+1)} \right]$$

The Cost of a Missing Operator



$$Z_i^{(n)} = \langle 0 | O_i | n \rangle$$

$$Z_i^{(n)} = \begin{pmatrix} 1.00 & 0.35 & 0.42 \\ 0.38 & 0.42 & 0.92 \\ 0.41 & 0.88 & 0.55 \end{pmatrix}$$

Lots of options! Compare TQ spectrum to NO TQ

