Investigating the role of tetraquark operators in lattice QCD studies of the $a_0(980)$ and κ resonances: A cautionary tale

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This Talk

Presenting the findings of

"Investigating the role of tetraquark operators in lattice QCD studies of the $a_0(980)$ and κ resonances"
Hanlon et al., PRD 114, 014517 (2026)

Work done in collaboration with

Andrew D. Hanlon	Daniel Darvish	Sarah Skinner
Ruairí Brett	John Bulava	Jacob Fallica
Colin Morningstar	Fernando Romero-López	André Walker-Loud

Overview

- Earlier κ and $a_0(980)$ spectra were extracted without TQ interpolators [Wilson et al., PhysRevD.91.054008 (2015); Dudek et al., PhysRevD.93.094506 (2016)].
- Dedicated four-quark studies conflict: one found an extra light κ-like level, the other no extra $\kappa/a_0(980)$ level [Prelovsek et al., PhysRevD.82.094507 (2010); Alexandrou et al., JHEP04(2013)137 (2013)].
- Both omitted disconnected diagrams, which χPT shows are not negligible [Guo et al., PhysRevD.88.074506 (2013)].
- A later study included both, and found an extra $a_0(980)$ level [Alexandrou et al., PhysRevD.97.034506 (2018)].

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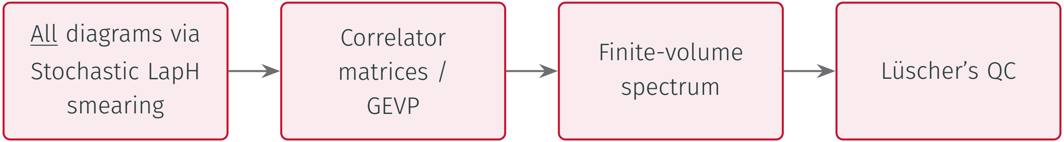

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<u>All</u> diagrams via Stochastic LapH smearing → Correlator matrices / GEVP → Finite-volume spectrum → Lüscher's QC

Do TQ operators alter the spectrum? If so, does the scattering description change?

Resonances of Interest

	$\kappa \equiv K_0^*(700)$	$a_0(980)$
Quantum numbers	$I(J^P) = \frac{1}{2}(0^+)$	$I^G(J^{PC}) = 1^-(0^{++})$
Strangeness	$S = 1$	$S = 0$
Dominant channels	$K\pi,\ K\eta$	$\pi\eta,\ K\overline{K},\ \pi\eta'$
Rest-frame irrep	A_{1g}	A_{1g}^-

Ensemble Info

Collaboration	Hadron Spectrum Collaboration
Fermion action	$N_f = 2+1$ anisotropic Wilson-clover
Lattice extent	$(L/a_s)^3 \times (T/a_t) = 32^3 \times 256,\ L = 3.74$ fm
Pion mass	$m_\pi \approx 230$ MeV
Anisotropy	$\xi = a_s/a_t = 3.451(11)$
Temporal spacing	$a_t = 0.033357(59)$ fm

N_{cf}	$a_t m_\pi$	$a_t m_K$	$a_t m_\eta$	$a_t m_{\eta'}$
412	0.03953(17)	0.08348(14)	0.0979(30)	0.165(12)

[Edwards et al., PhysRevD.78.054501 (2008); Lin et al., PhysRevD.79.034502 (2009)]

Extracting the Finite-Volume Spectrum

- We extract the finite-volume spectrum from correlation functions of a basis of interpolating operators.

$$\mathcal{C}_{ij}(t) = \langle 0|O_i(t+t_s)\overline{O}_j(t_s)|0\rangle .$$

- Isolate the contribution from the n-th state using GEVP, single pivot:

$$\lambda_n(t) \;\propto\; e^{-E_n t} \qquad \text{for large } t.$$

- For each momentum, solve a GEVP in the flavor basis

$$\tilde{\eta} = \overline{u}u + \overline{d}d, \qquad \tilde{\phi} = \overline{s}s, \qquad \text{ROT}\,0 \to \eta, \quad \text{ROT}\,1 \to \eta',$$

yielding variationally improved η and η' operators.

[Lüscher and Wolff, Nucl. Phys. B 339, 222 (1990)]

Single- and Two-Meson Operators

- Stout-smeared gauge links; LapH-smeared quark fields.
- Projected onto zero-momentum, A_{1g} and A_{1g}^{-} irreps.

Single-meson

κ	$a_0(980)$
K^{DDL2}	π^{SD2}
K^{TDO3}	π^{TDO3}
K^{TDU5}	

Two-meson

κ	$a_0(980)$
$K(0)^{SS0}\pi(0)^{SS0}$	$K(0)^{SS0}\overline{K}(0)^{SS0}$
$K(0)^{SS0}\tilde{\eta}(0)^{SS0}$	$K(1)^{SS1}\overline{K}(1)^{SS1}$
$K(0)^{SS0}\tilde{\phi}(0)^{SS0}$	$K(2)^{SS0}\overline{K}(2)^{SS0}$
$K(1)^{SS1}\pi(1)^{SS1}$	$K(2)^{SS1}\overline{K}(2)^{SS1}$
$K(1)^{SS1}\tilde{\eta}(1)^{SS1}$	$\tilde{\eta}(0)^{SS0}\pi(0)^{SS0}$
$K(1)^{SS1}\tilde{\phi}(1)^{SS1}$	$\tilde{\eta}(1)^{SS0}\pi(1)^{SS0}$
$K(2)^{SS0}\pi(2)^{SS0}$	$\tilde{\eta}(2)^{SS1}\pi(2)^{SS1}$
$K(3)^{SS0}\pi(3)^{SS0}$	$\tilde{\phi}(0)^{SS0}\pi(0)^{SS0}$
	$\tilde{\phi}(1)^{SS1}\pi(1)^{SS1}$
	$\tilde{\phi}(2)^{SS0}\pi(2)^{SS0}$

[Morningstar and Peardon, PhysRevD.69.054501 (2004); Morningstar et al., PhysRevD.88.014511 (2013)]

Tetraquark Operators

- Schematically, a two-meson operator takes the form $M_1(\mathrm{p})M_2(-\mathrm{p})$.
- Our tetraquark annihilation operators are linear combinations of

$$\Phi^{ABCD\pm}_{\alpha\beta\mu\nu;ijkl}(t) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}(\delta_{ab}\delta_{cd} \pm \delta_{ad}\delta_{bc})$$
$$\times \overline{q}^{A}_{a\alpha i}(\mathbf{x},t)\; q^{B}_{b\beta j}(\mathbf{x},t)\; \overline{q}^{C}_{c\mu k}(\mathbf{x},t)\; q^{D}_{d\nu l}(\mathbf{x},t).$$

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  2. Flavor content

Isospin	Strangeness	Flavor content
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$\frac{1}{2}$	1	$\bar{s}u(\bar{u}u + \bar{d}d)$
$\frac{1}{2}$	1	$\bar{s}u(\bar{u}u - \bar{d}d) - \sqrt{2}\,\bar{s}d\bar{d}u$
1	0	$(\bar{u}u + \bar{d}d)\bar{d}u$
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  4. Spatial configuration

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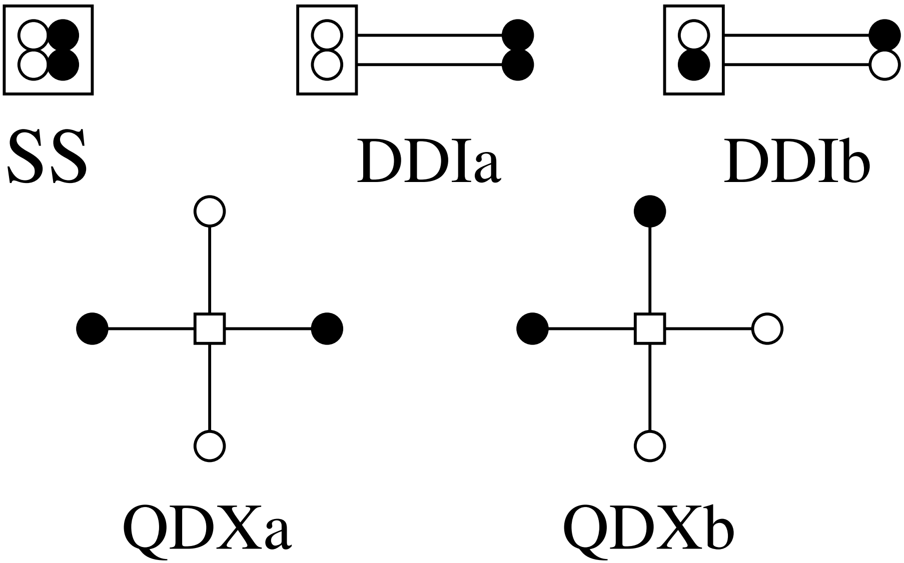
Tetraquark Spatial Configurations



Screening the TQ Operator Space

Hundreds of candidate T_i's to check

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Add one T_i to the meson basis

$$C^{(i)}(t) = \begin{pmatrix} C_{MM} & C_{MT_i} \\ C_{T_iM} & C_{T_iT_i} \end{pmatrix}$$

25 configurations

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usable effective energies

$\kappa: \; 60, \quad a_0: \; 32$

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Compare spectra

no TQ $\leftrightarrow +T_i$

select one per channel

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Hundreds of candidate T_i's to check → Add one T_i to the meson basis $C^{(i)}(t) = \begin{pmatrix} C_{MM} & C_{MT_i} \\ C_{T_iM} & C_{T_iT_i} \end{pmatrix}$ 25 configurations → GEVP usable effective energies κ : 60, a_0 : 32 → Compare spectra no TQ $\leftrightarrow$ $+T_i$ select one per channel

Which TQ structures change the low-lying spectrum?

Selected TQ Operator – κ

- $I = \frac{1}{2},\ S = 1,\ A_{1g}$
- Color-antisymmetric structure

$$T\left[\bar{s}u\bar{s}s\right]^{SS2(-)}$$

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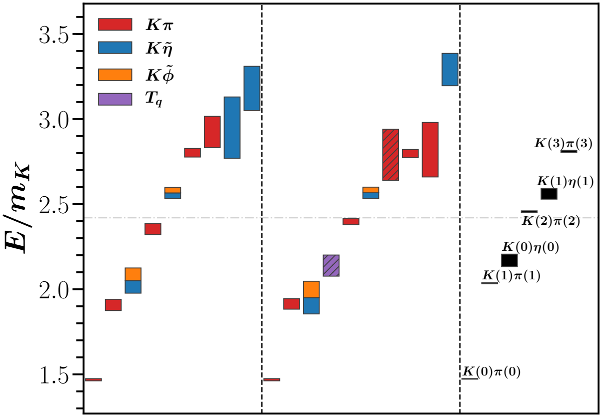

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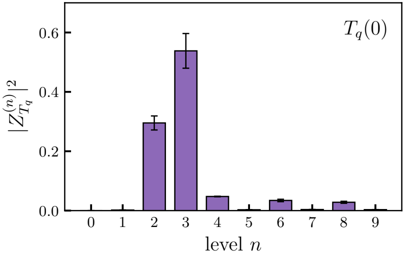

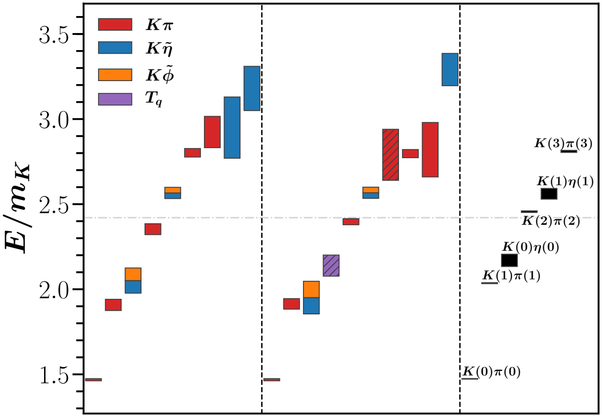

Selected TQ Operator – $a_0(980)$

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$$T\left[(\overline{u}u + \overline{d}d)\overline{d}u\right]^{SS\mathbf{2}(+)}$$

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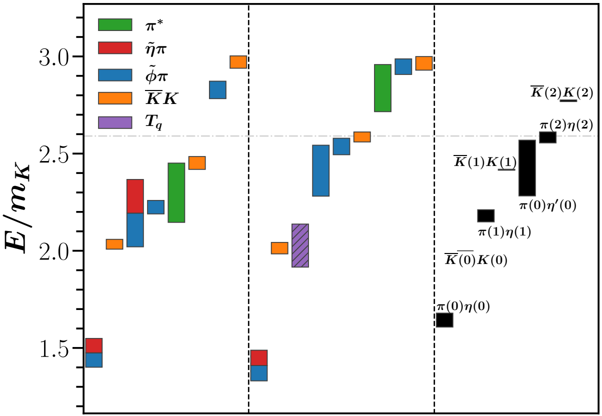

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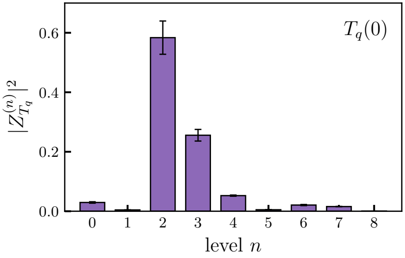
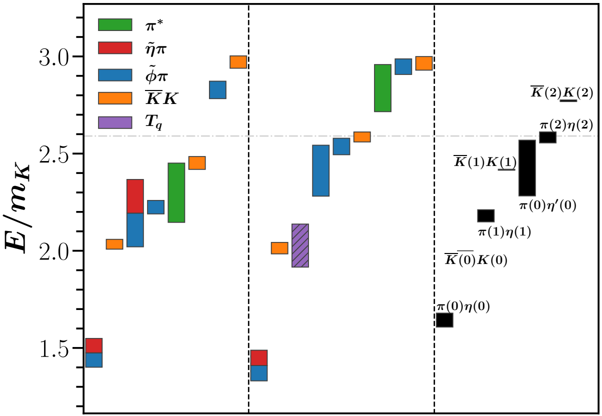




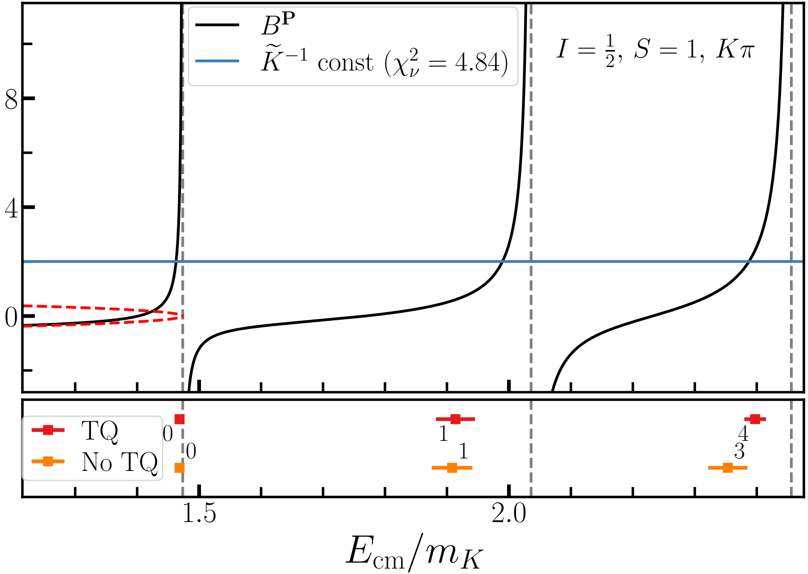
Lüscher QC – Does the IV scattering description change?

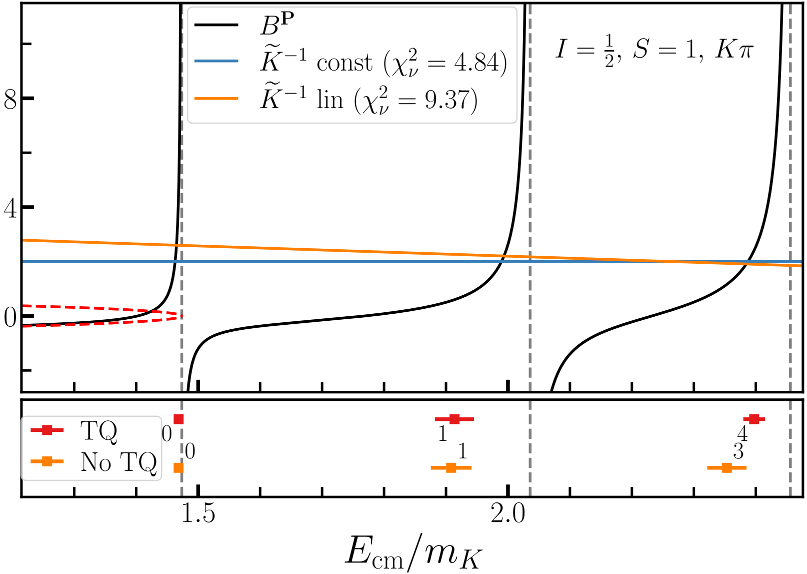
$$\det\left[\widetilde{K}^{-1}(E_{\mathsf{cm}}) - B^{(\mathrm{P},\Lambda)}(E_{\mathsf{cm}})\right] = 0$$

- Truncate to S-wave, consider levels below 4-particle threshold.
- Experiment and Z-factors indicate weak interchannel mixing [Navas et al., PhysRevD.110.030001 (2024)].
- Only one $\pi\eta'$ level in A_{1g}^{-} below 4-particle threshold – exclude.

$$\begin{aligned}\det\left(\widetilde{K}^{-1} - B\right) &= \det\left(\begin{bmatrix}\widetilde{K}_{11}^{-1} - B_{11} & 0 \\ 0 & \widetilde{K}_{22}^{-1} - B_{22}\end{bmatrix}\right) \\ &= \det\left(\widetilde{K}_{11}^{-1} - B_{11}\right)\det\left(\widetilde{K}_{22}^{-1} - B_{22}\right).\end{aligned}$$

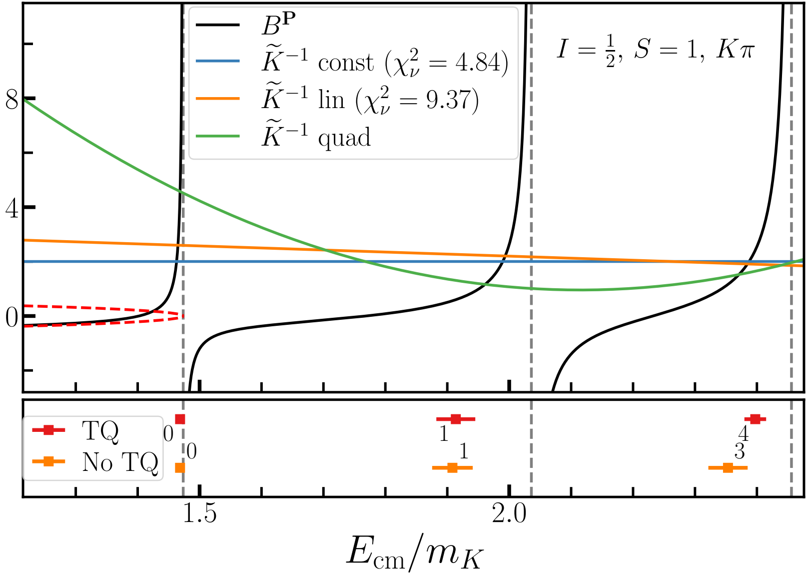
Lüscher QC – A_{1g}, $K\pi$





$B^{\mathbf{P}}$
$\widetilde{K}^{-1}$ const ($\chi^2_\nu = 4.84$)
$\widetilde{K}^{-1}$ lin ($\chi^2_\nu = 9.37$)
$I = \frac{1}{2},\ S = 1,\ K\pi$
8
4
0
TQ
No TQ
0
0
1
1
4
3
1.5
2.0
E_{cm}/m_K

Lüscher QC – A_{1g}, $K\pi$



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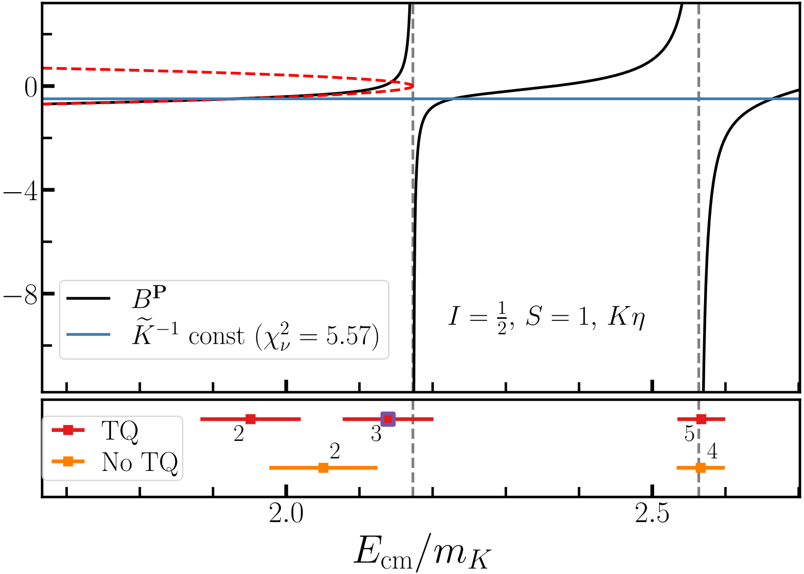

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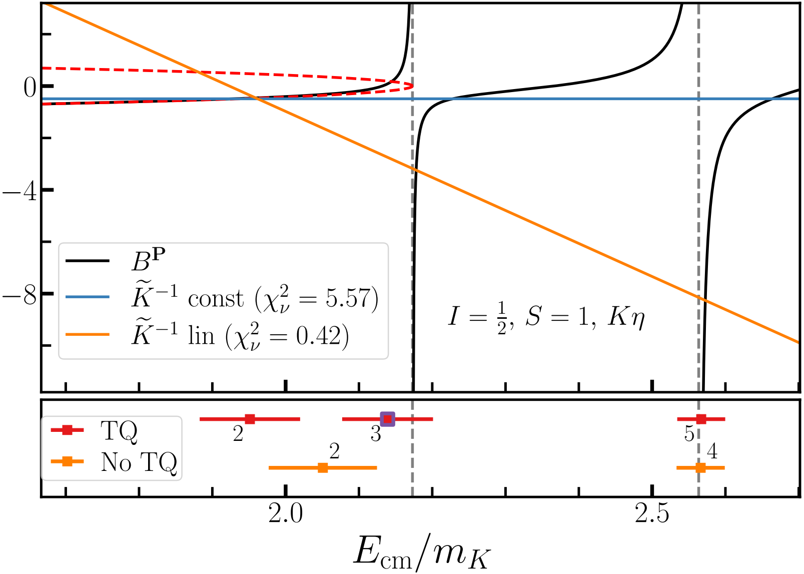
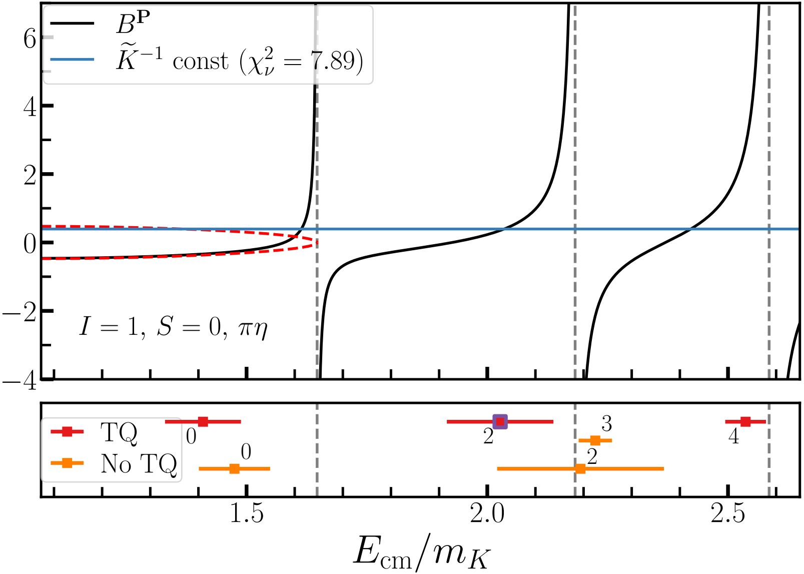
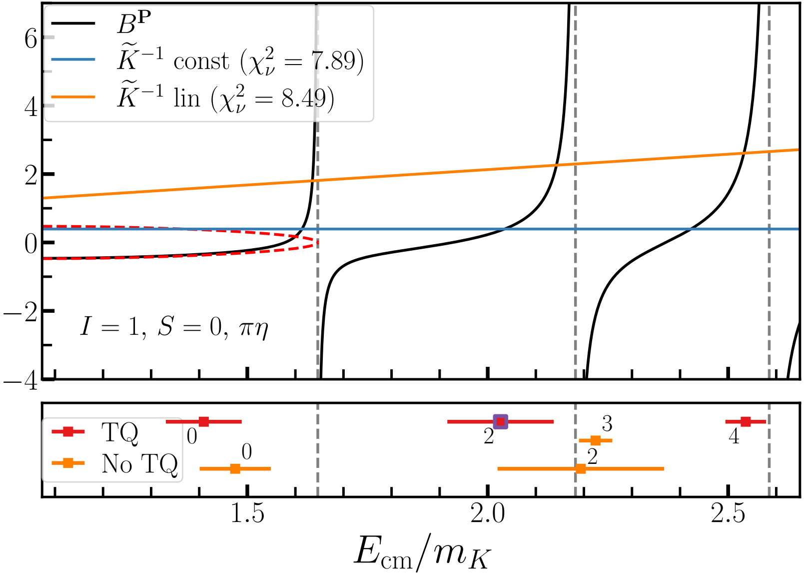


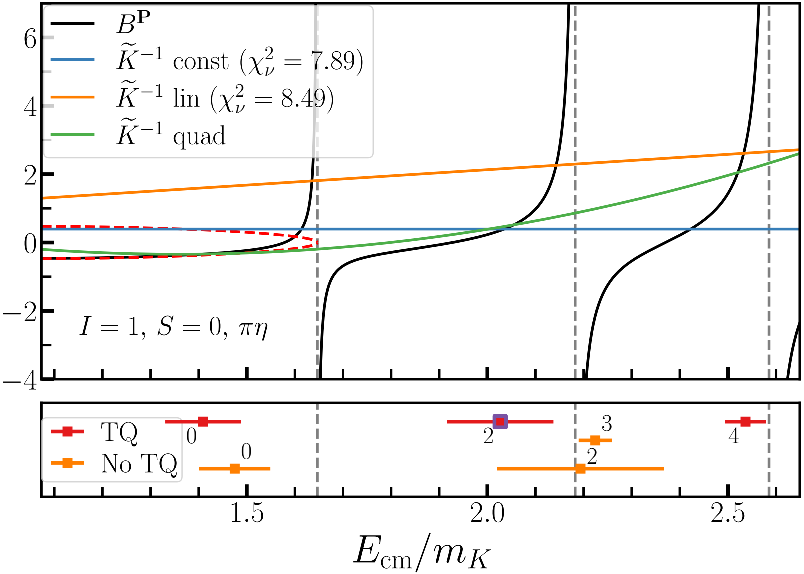
Lüscher QC – A_{1g}^{-}, $\pi\eta$



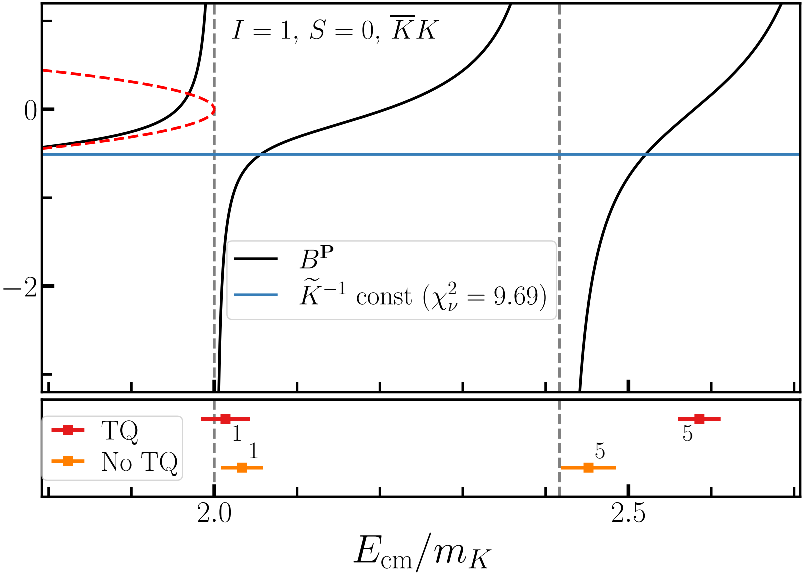
Lüscher QC – A_{1g}^{-}, $\pi\eta$



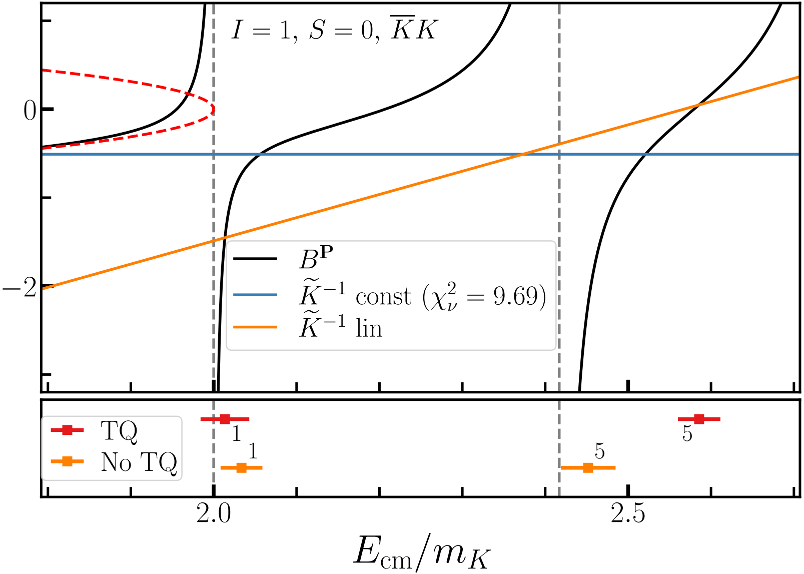
Lüscher QC – A_{1g}^{-}, $\pi\eta$



Lüscher QC – A_{1g}^{-}, $\overline{K}K$



Lüscher QC – A_{1g}^{-}, $\overline{K}K$



κ

Without TQ: one $K\eta$ level is missed.

$K\pi$ energies are only slightly affected.

Implication: κ studies appear unaffected;
TQ is likely important for $K_0^*(1430)$.

Conclusions and Outlook

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Inferred $\pi\eta$ and $\overline{K}K$ descriptions differ dramatically.

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OUTLOOK Higher statistics · moving frames · multiple ensembles · other light resonances

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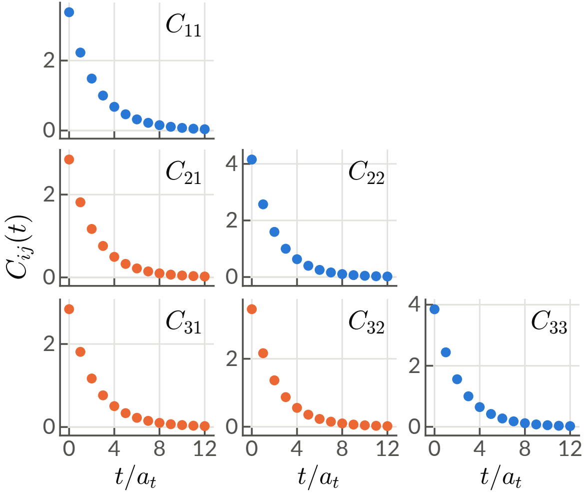
Apparent plateaux are insufficient without overlap onto all relevant states.

Thank you for your attention

Single Pivot Method – Toy Example



$$C_{ij}(t) = \frac{\mathcal{C}_{ij}(t)}{\sqrt{\mathcal{C}_{ii}(t_N)\,\mathcal{C}_{jj}(t_N)}}$$

$$C_{ij}(t) = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)*} e^{-E_n t}$$

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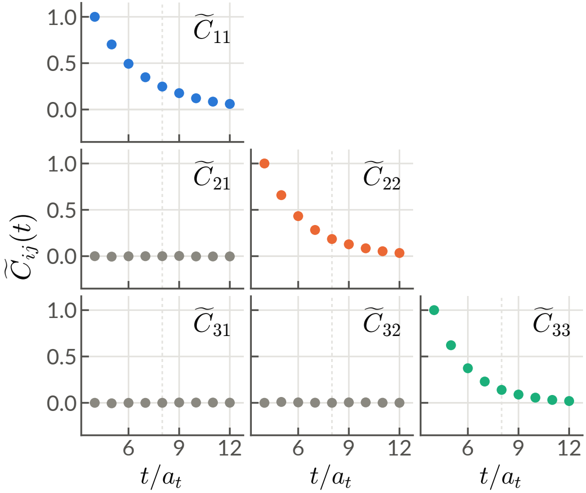

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$$C(t_0)^{-1/2}\, C(t_D)\, C(t_0)^{-1/2}\, U = U D$$

$$\widetilde{C}(t) = U^{\dagger}\, C(t_0)^{-1/2}\, C(t)\, C(t_0)^{-1/2}\, U$$

Single- and Two-Meson Operators

Single-meson

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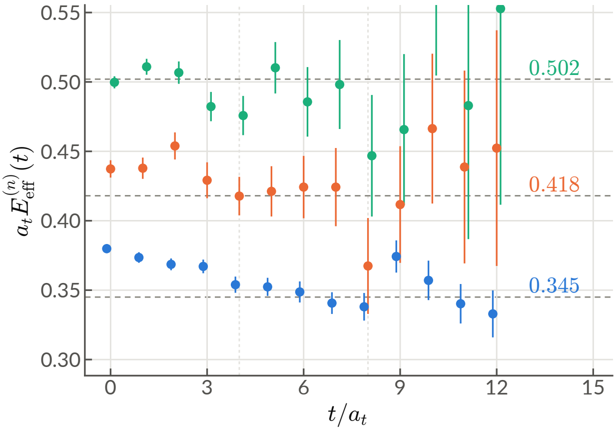
$K^{DDL2}_{A_{1g}}$	$\pi^{SD2}_{A^-_{1g}}$
$K^{TDO3}_{A_{1g}}$	$\pi^{TDO3}_{A^-_{1g}}$
$K^{TDU5}_{A_{1g}}$	

Two-meson

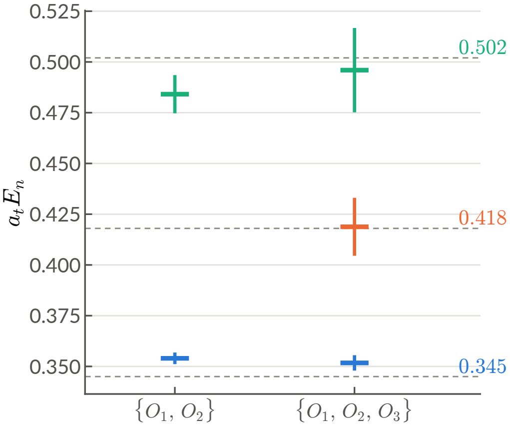
κ	$a_0(980)$
$K(0)^{SS0}_{A_{1u}}\,\pi(0)^{SS0}_{A^-_{1u}}$	$K(0)^{SS0}_{A_{1u}}\,\overline{K}(0)^{SS0}_{A_{1u}}$
$K(0)^{SS0}_{A_{1u}}\,\tilde{\eta}(0)^{SS0}_{A^+_{1u}}$	$K(1)^{SS1}_{A_2}\,\overline{K}(1)^{SS1}_{A_2}$
$K(0)^{SS0}_{A_{1u}}\,\tilde{\phi}(0)^{SS0}_{A^+_{1u}}$	$K(2)^{SS0}_{A_2}\,\overline{K}(2)^{SS0}_{A_2}$
$K(1)^{SS1}_{A_2}\,\pi(1)^{SS1}_{A^-_2}$	$K(2)^{SS1}_{A_2}\,\overline{K}(2)^{SS1}_{A_2}$
$K(1)^{SS1}_{A_2}\,\tilde{\eta}(1)^{SS1}_{A^+_2}$	$\tilde{\eta}(0)^{SS0}_{A^+_{1u}}\,\pi(0)^{SS0}_{A^-_{1u}}$
$K(1)^{SS1}_{A_2}\,\tilde{\phi}(1)^{SS1}_{A^+_2}$	$\tilde{\eta}(1)^{SS0}_{A^+_2}\,\pi(1)^{SS0}_{A^-_2}$
$K(2)^{SS0}_{A_2}\,\pi(2)^{SS0}_{A^-_2}$	$\tilde{\eta}(2)^{SS1}_{A^+_2}\,\pi(2)^{SS1}_{A^-_2}$
$K(3)^{SS0}_{A_2}\,\pi(3)^{SS0}_{A^-_2}$	$\tilde{\phi}(0)^{SS0}_{A^+_{1u}}\,\pi(0)^{SS0}_{A^-_{1u}}$
	$\tilde{\phi}(1)^{SS1}_{A^+_2}\,\pi(1)^{SS1}_{A^-_2}$
	$\tilde{\phi}(2)^{SS0}_{A^+_2}\,\pi(2)^{SS0}_{A^-_2}$

Effective Energies



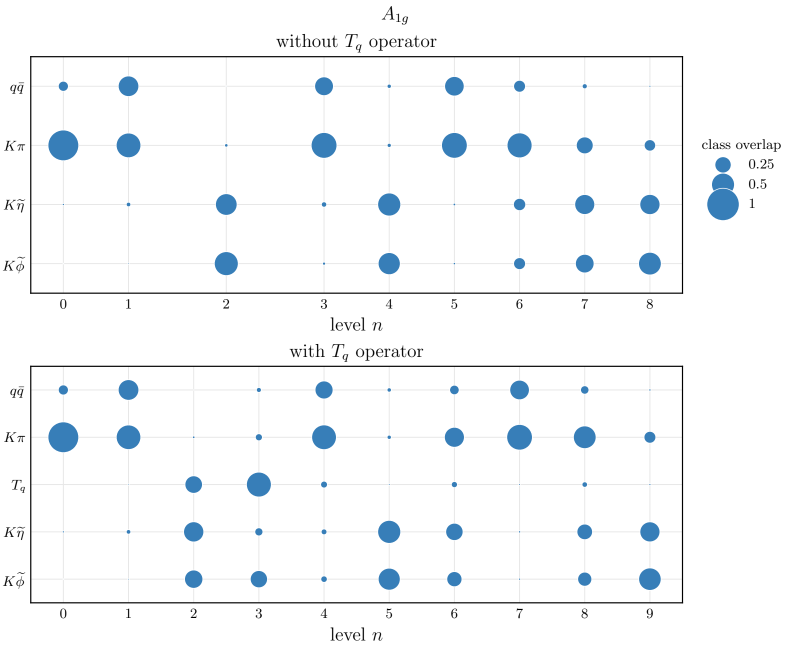
$$a_t E^{(n)}_{\mathsf{eff}}(t) = \ln \left[\frac{\widetilde{C}_{nn}(t)}{\widetilde{C}_{nn}(t+1)} \right]$$

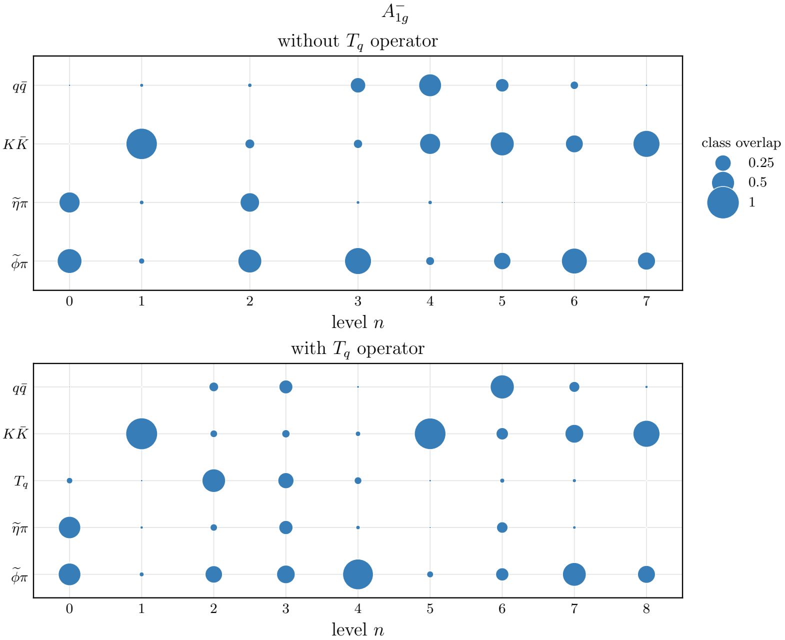
The Cost of a Missing Operator



$$Z_i^{(n)} = \langle 0 | O_i | n \rangle$$

$$Z_i^{(n)} = \begin{pmatrix} 1.00 & 0.35 & 0.42 \\ 0.38 & 0.42 & 0.92 \\ 0.41 & 0.88 & 0.55 \end{pmatrix}$$


A_{1g}
without T_q operator
$q\bar{q}$
$K\pi$
$K\widetilde{\eta}$
$K\widetilde{\phi}$
0 1 2 3 4 5 6 7 8
level n
class overlap
0.25
0.5
1
with T_q operator
$q\bar{q}$
$K\pi$
T_q
$K\widetilde{\eta}$
$K\widetilde{\phi}$
0 1 2 3 4 5 6 7 8 9
level n


A_{1g}^{-}
without T_q operator
$q\bar{q}$
$K\bar{K}$
$\widetilde{\eta}\pi$
$\widetilde{\phi}\pi$
0 1 2 3 4 5 6 7
level n
class overlap
0.25
0.5
1
with T_q operator
$q\bar{q}$
$K\bar{K}$
T_q
$\widetilde{\eta}\pi$
$\widetilde{\phi}\pi$
0 1 2 3 4 5 6 7 8
level n

Lots of options! Compare TQ spectrum to NO TQ