Hadronic light-by-light contribution to muon $g - 2$

RBC/UKQCD update 2026

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Outline

1. RBC-UKQCD-23 calculation of HLbL
2. HLbL master formula
3. Improvement on direct, and π^0-exchange calculations
4. Results and summary

Results from previous work

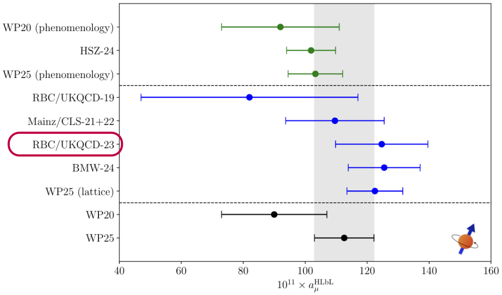
The HLbL contribution calculated with the 48I unitary mass ensemble in [Blum et al., 2023]:

$$a_\mu^{\mathrm{HLbL}} \times 10^{10} = 12.47\,(1.15)_{\mathrm{stat}}(0.95)_{\mathrm{syst}}\,[1.49].$$

Significant sources of systematic uncertainty come from a^2-correction and π^0-exchange contribution [Blum et al., 2023].

Previous work and improvements

48I light π^0-exchange	$2.00\ (0.11)_{\mathrm{stat}}(0.28)_{\mathrm{syst}}\ [0.30]$
This talk:	Improved long distance formula
48I light a^2-correction	$0.00\ (0.83)_{\mathrm{syst}}$
This talk:	New calculation on 64I



Summary of HLbL calculations.

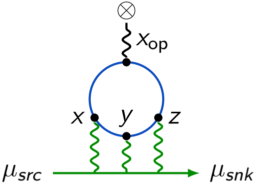
HLbL master formula

The master formula for the HLbL contribution to muon $g-2$ in position space is given by [Blum et al., 2017]:

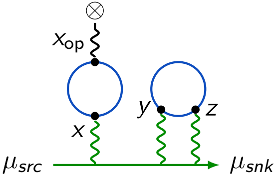
$$a_\mu^{\text{HLbL}} = \frac{2me^2}{3}\frac{1}{VT}\sum_{x_{\text{op}}}\sum_{x,y,z}\frac{1}{2}\epsilon_{ijk}\left(x_{\text{op}} - x_{\text{ref}}(x,y,z)\right)_j (6e^4)\,\mathcal{H}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z)\,\mathcal{M}_{i\rho\sigma\lambda}(x,y,z)$$

where $\mathcal{H}$ is the hadronic four-point function, $\mathcal{G}$ is the continuum QED weighting function evaluated in infinite volume, and $x_{\text{ref}}(x,z,y)$ is the moment-method reference position.

$$\mathcal{M}_{i\rho\sigma\lambda}(x,y,z) = \frac{1}{2}\operatorname{Tr}\left[\frac{1}{6}\,i^3\,\mathcal{G}_{\rho\sigma\lambda}(x,y,z)\,\Sigma_i\right] \qquad 6e^4\,\mathcal{H}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z) = \langle 0|\,T\{J_k(x_{\text{op}})J_\rho(x)J_\sigma(y)J_\lambda(z)\}\,|0\rangle$$



Connected diagram

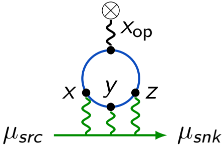


Leading order disconnected diagram

Long distance + short distance calculations

We define a cutoff function R_{max}:

$$R_{\text{max}} = \max\left(|x-y|,\ |x-z|,\ |y-z|\right).$$



The total can be written as the sum of all contributions from two regions:

$$a_\mu^{\text{HLbL}} = a_\mu\big(R_{\text{max}} < R_{\text{max}}^{\text{cut}}\big) + a_\mu\big(R_{\text{max}} > R_{\text{max}}^{\text{cut}}\big)$$

- The first term is the partial sum of the a_μ^{HLbL} summand over vertex coordinates within the range $0 \leq R_{\text{max}} < R_{\text{max}}^{\text{cut}}$; the second term is the reverse partial sum with respect to the same cutoff.
- $R_{\text{max}}^{\text{cut}}$ needs to be large enough so that $a_\mu\left(R_{\text{max}} > R_{\text{max}}^{\text{cut}}\right) \approx a_\mu^{\pi^0\text{-exchange}}$. Our previous work [Blum et al., 2023] used $R_{\text{max}}^{\text{cut}} = 4.0$ fm in the reported final result.

Long distance + short distance calculations

$$a_\mu^{\mathrm{HLbL}} = a_\mu(R_{\mathrm{max}} < R_{\mathrm{max}}^{\mathrm{cut}}) + a_\mu(R_{\mathrm{max}} > R_{\mathrm{max}}^{\mathrm{cut}})$$

Direct and π^0-exchange (indirect) calculations

$a_\mu(R_{\mathrm{max}} < R_{\mathrm{max}}^{\mathrm{cut}})$	$a_\mu(R_{\mathrm{max}} > R_{\mathrm{max}}^{\mathrm{cut}})$
• Direct calculation using two point source propagators ($\sim$ 2000 propagators per configuration). • Field-sparsening is employed to reduce contraction costs.	• Long distance region is dominated by the π^0-exchange contribution. • Indirect calculation using π^0 transition form factors (TFF).

Long-distance π^0 exchange contribution

In the long-distance region where $|z-y| \ll \min(|x-z|, |x-y|)$, the largest contribution to $\mathcal{H}_{k\rho\sigma\lambda}$ comes from the π^0 intermediate state:

$$\mathcal{H}_{k\rho\sigma\lambda}(x_{\mathrm{op}}, x, y, z) \approx \mathcal{H}^{\pi 0}_{k\rho\sigma\lambda}(x_{\mathrm{op}}, x, y, z),$$

$$(6e^4)\,\mathcal{H}^{\pi 0}_{k\rho\sigma\lambda}(x_{\mathrm{op}}, x, y, z) = \int \frac{d^3q}{(2\pi)^3}\,\frac{1}{2E_{\pi,\vec{q}}}\,\langle 0|T\{J_k(x_{\mathrm{op}})J_\rho(x)\}|\pi^0_{\vec{q}}\rangle\,\langle \pi^0_{\vec{q}}|T\{J_\sigma(y)J_\lambda(z)\}|0\rangle$$

The π^0 transition form factor is given by:

$$\widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, q) = \langle 0|T\{J_k(\tfrac{\tilde{x}}{2})J_\rho(-\tfrac{\tilde{x}}{2})\}|\pi^0_{\vec{q}}\rangle = \epsilon_{k\rho\alpha\beta}\,\tilde{x}_\alpha\, q_\beta\, \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}\left(\tilde{x}^2, q\cdot\tilde{x}\right)$$

Long-distance π^0 exchange contribution

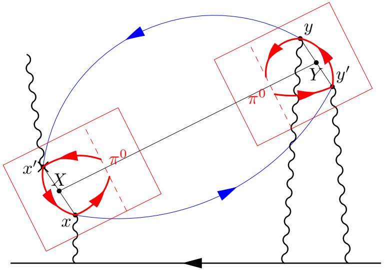
The π^0 transition form factor is given by:

$$\widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, q) = \langle 0| T\{J_k(\tfrac{\tilde{x}}{2}) J_\rho(-\tfrac{\tilde{x}}{2})\} |\pi^0_{\vec{q}}\rangle = \epsilon_{k\rho\alpha\beta}\, \tilde{x}_\alpha q_\beta\, \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}\left(\tilde{x}^2, q\cdot\tilde{x}\right)$$

$\mathcal{H}^{\pi_0}$ can then be expressed in terms of products of form factors and scalar pion propagator:

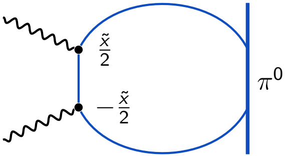
$$(6e^4)\, \mathcal{H}^{\pi_0}_{k\rho\sigma\lambda}(x', x, y', y) = \widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, -i\tfrac{\partial}{\partial X})\, \widetilde{\mathcal{F}}_{\sigma\lambda}(\tilde{y}, -i\tfrac{\partial}{\partial Y})\, D_{\pi_0}(|R_\pi|)$$

$$\boxed{\begin{aligned} &X = \tfrac{x'+x}{2}, \quad Y = \tfrac{y'+y}{2} \\ &R_\pi = X - Y \\ &\tilde{x} = x' - x, \quad \tilde{y} = y' - y \end{aligned}}$$

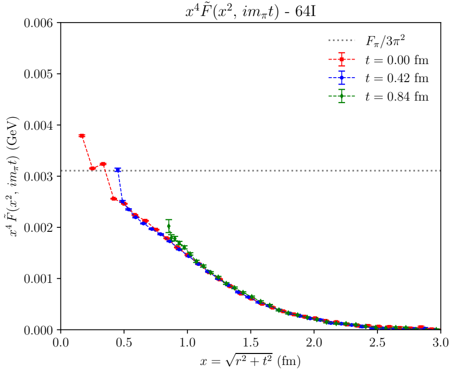


Long-distance π^0 exchange contribution

$$\widetilde{\mathcal{F}}_{k\rho}(\tilde{x}, q) = \langle 0 | T\{J_k(\tfrac{\tilde{x}}{2}) J_\rho(-\tfrac{\tilde{x}}{2})\} | \pi^0_{\vec{q}} \rangle = \epsilon_{k\rho\alpha\beta} \tilde{x}_\alpha q_\beta \, \tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(\tilde{x}^2, q \cdot \tilde{x})$$



$\pi^0 \to \gamma\gamma$ transition form factor



$\tilde{\mathcal{F}}_{\pi^0\gamma\gamma}(x^2, q \cdot x)$ has a weak dependence on $q \cdot x$.

Long-distance π^0 exchange contribution

$$(6e^4)\,\mathcal{H}^{\pi_0}_{\kappa\rho\sigma\lambda}(x',x,y',y) = \widetilde{\mathcal{F}}_{\kappa\rho}(\tilde{x}, -i\tfrac{\partial}{\partial X})\,\widetilde{\mathcal{F}}_{\sigma\lambda}(\tilde{y}, -i\tfrac{\partial}{\partial Y})\,D_{\pi_0}(|R_\pi|)$$

For leading order in $1/|R_\pi|$, the derivative with respect to the propagator can be written as:

$$-i\tfrac{\partial}{\partial X^\beta} D_\pi(|X-Y|) \approx \left(im_\pi \tfrac{(X-Y)_\beta}{|X-Y|}\right) D_{\pi_0}(|X-Y|).$$

Two formulations of $\mathcal{H}^{\pi^0}$

Previous work (2023): derivative approximation on the entire $\mathcal{H}^{\pi_0}$

$$(6e^4)\,\mathcal{H}^{\pi_0(RBC-UKQCD23)}_{\kappa\rho\sigma\lambda}(x',x,y',y) = \widetilde{\mathcal{F}}_{\kappa\rho}(\tilde{x}, im_\pi \hat{R}_\pi)\,\widetilde{\mathcal{F}}_{\sigma\lambda}(\tilde{y}, -im_\pi \hat{R}_\pi))\,D_{\pi_0}(|R_\pi|)$$

Improved (this work): derivative approximation only on $\widetilde{\mathcal{F}}_{\pi^0\gamma\gamma}(\tilde{x}^2, -i\frac{\partial}{\partial X})$

$$(6e^4)\,\mathcal{H}^{\pi_0(\mathrm{new})}_{\kappa\rho\sigma\lambda}(x',x,y',y) \approx \epsilon_{\kappa\rho\alpha\beta}\epsilon_{\sigma\lambda\gamma\zeta}\,\tilde{x}_\alpha\tilde{y}_\gamma\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{x}^2, im_\pi\hat{R}_\pi\cdot\tilde{x})\,\widetilde{\mathcal{F}}^E_{\pi^0\gamma\gamma}(\tilde{y}^2, -im_\pi\hat{R}_\pi\cdot\tilde{y})$$
$$\times\left(-i\tfrac{\partial}{\partial X^\beta}\right)\left(-i\tfrac{\partial}{\partial Y^\zeta}\right) D_{\pi^0}(|R_\pi|)$$

Calculation details

Using approximations for $\mathcal{H}^{\pi_0}$ described in the previous slide, we perform Monte Carlo importance sampling to evaluate:

$$a_\mu^{\pi_0\text{-exch}} = \frac{1}{VT} \sum_{x',x,y',y} I^{\pi^0\text{-exchange}}(x', x, y', y)$$

using the following **weighting function**:

$$w(x', x, y', y) = \frac{1}{|x'-x|^4 + \tilde{a}^2} \frac{1}{|y'-y|^4 + \tilde{b}^2} \frac{135\,\text{MeV}\ K_1(135\,\text{MeV}\ R_\pi)}{4\pi^2 R_\pi}$$

π^0 propagation distance: $R_\pi = \left|\frac{x'+x}{2} - \frac{y'+y}{2}\right|, \quad (\tilde{a} = \tilde{b} = 30.0)$

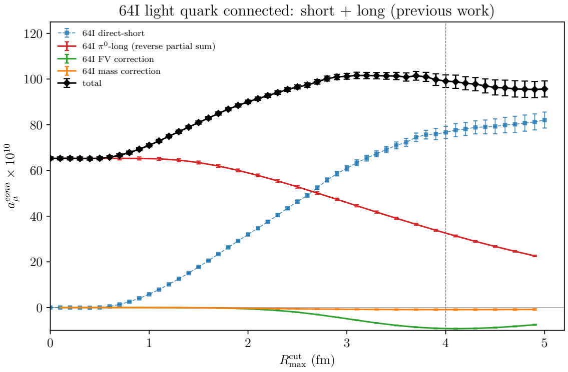
	48I	64I
m_π (MeV)	139	139
m_K (MeV)	499	508
a^{-1} (GeV)	1.730	2.359
a (fm)	0.114	0.084
L (fm)	5.47	5.35
# configs	113	119

Lattice parameters for 48I and 64I ensembles

64I Light quark connected - $\mathcal{H}^{\pi^0}$ (2023)

BLINDED, PRELIMINARY results

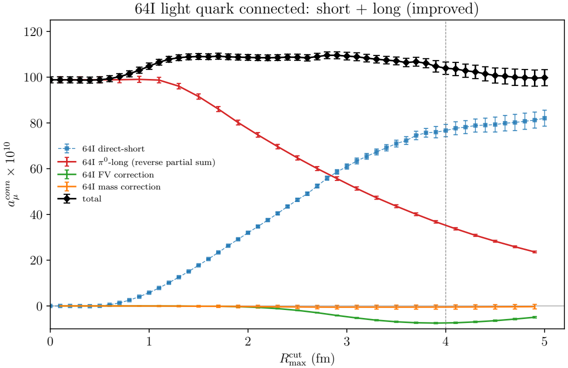
- Direct light quark contributions are evaluated with unitary $\pi^0 = 139$ MeV mass ensembles.
- Displayed curves are partial sums with respect to $R_{\text{max}}^{\text{cut}}$, except for 64I long distance, which is a reverse partial sum.
- In this plot, the long distance calculation is performed using RBC/UKQCD-23 formula for $\mathcal{H}^{\pi^0}$.



Light quark connected - $\mathcal{H}^{\pi^0}$(improved)

BLINDED, PRELIMINARY results

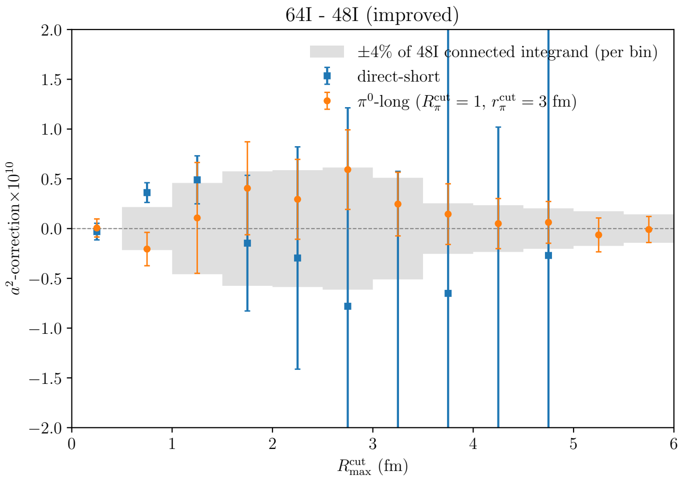
- Compared to $\mathcal{H}^{\pi^0}$(2023), the total plateaus at earlier at $R^{\text{cut}}_{\text{max}} \sim$ 1.5 fm.
- $\mathcal{H}^{\pi^0}$(improved) has less systematic uncertainty compared to $\mathcal{H}^{\pi^0}$(2023), due to the former using fewer derivative approximations.
- Earlier plateau allows us to consider lower R^{cut}_{max} cutoffs to reduce statistical uncertainty



Light quark connected: lattice-spacing effects

BLINDED, PRELIMINARY results

- The plot compares binned a^2-correction between direct and π^0-long distance calculations.
- Recall that we estimated $\pm$ 8% systematic error in our previous work. [Blum et al., 2023].
- 8% correction to 48I corresponds to 4% difference between 48I and 64I
- Shaded error-band is $\pm$ 4% of the 48I connected integrant.



Summary

Significant sources of systematic uncertainty come from a^2-correction and π^0-exchange contribution [Blum et al., 2023]

48I light π^0-exchange ($R_{\max} > 4\,\mathrm{fm}$)	$2.00\ (0.11)_{\text{stat}}(0.28)_{\text{syst}}\ [0.30]$
this talk:	Improved long distance formula ✓
48I light a^2-correction	$0.00\ (0.83)_{\text{syst}}$
this talk:	New calculation on 64I ✓

Future directions:

- Evaluate the charged pion contribution to HLbL to help improve the long distance calculation.
- Perform continuum limit extrapolation for the direct calculation; finalize estimates for π^0 mass and FV corrections; unblind the results.

Thank you!

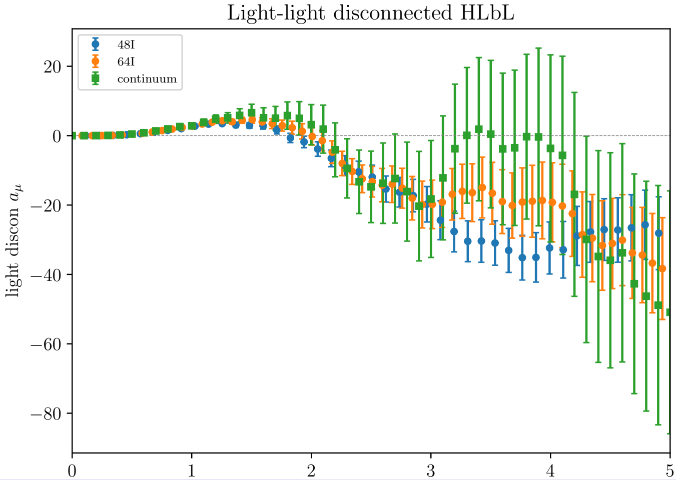
References I

- Bijnens, Johan and Johan Relefors [2016]. "Pion light-by-light contributions to the muon $g-2$". In: *JHEP* 09, p. 113. DOI: 10.1007/JHEP09(2016)113. arXiv: 1608.01454 [hep-ph].
- Blum, Thomas et al. [Aug. 2017]. "Using infinite-volume, continuum QED and lattice QCD for the hadronic light-by-light contribution to the muon anomalous magnetic moment". In: *Phys. Rev. D* 96 [3], p. 034515. DOI: 10.1103/PhysRevD.96.034515. URL: https://link.aps.org/doi/10.1103/PhysRevD.96.034515.
- Blum, Thomas et al. [2023]. "Hadronic light-by-light contribution to the muon anomaly from lattice QCD with infinite volume QED at physical pion mass". In: *Phys. Rev. D* 111.1, p. 014501. DOI: 10.1103/PhysRevD.111.014501. arXiv: 2304.04423 [hep-lat].

Light quark disconnected (unitary $\pi^0 = 139$ MeV mass ensembles)

BLINDED, PRELIMINARY results

- Large statistical error in the non-negligible long-distance tail of the light quark disconnected diagram.
- Instead of adding the noisy disconnected contribution at long-distance $R_{\text{max}}^{\text{cut}}$ we use the hybrid approach.



Results: hybrid approach

BLINDED, PRELIMINARY results

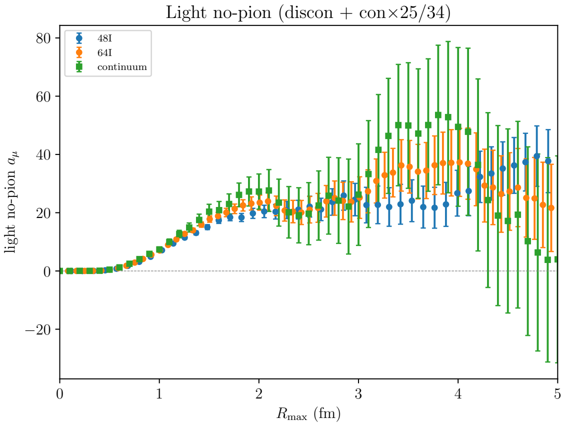
- Long-distance discon/con ratio [Bijnens and Relefors, 2016]:

$$\lim_{R\to\infty} \frac{a_\mu^{\text{discon}}(R_{\text{max}} > R)}{a_\mu^{\text{con}}(R_{\text{max}} > R)} = -\frac{1}{2} \cdot \frac{(e_u^2 + e_d^2)^2}{e_u^4 + e_d^4} = -\frac{25}{34}$$

- Hybrid approach:

$$a_\mu^{\text{no-pion}} = a_\mu^{\text{discon}} + \frac{25}{34}\, a_\mu^{\text{con}},$$
$$a_\mu^{\text{total}} = a_\mu^{\text{no-pion}} + \frac{9}{34}\, a_\mu^{\text{con}}.$$

- We can truncate $a_\mu^{\text{no-pion}}$ at an earlier plateau value, $\sim$ 2.5 fm, and add it to less noisy connected contributions at larger $R_{\text{max}}^{\text{cut}}$

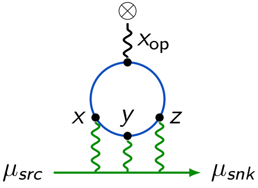


HLbL master formula

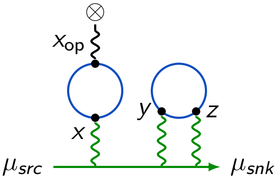
The master formula for the HLbL contribution to muon $g-2$ in position space is given by:

$$a_\mu^{\text{HLbL}} = \frac{2me^2}{3}\frac{1}{VT}\sum_{x_{\text{op}}}\sum_{x,y,z}\tfrac{1}{2}\,\epsilon_{ijk}\left(x_{\text{op}} - x_{\text{ref}}(x,z,y)\right)_j (6e^4)\,\mathcal{H}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z)\,\mathcal{M}_{i\rho\sigma\lambda}(x,y,z)$$

- Quark-gluon interactions to all orders are included in the calculation, but not shown in the diagrams.
- $\mathcal{H} = \mathcal{H}^{\text{con}} + \mathcal{H}^{\text{discon}} + \mathcal{H}^{\text{sub-leading-discon}} + \mathcal{H}^{\text{charm}}$
- The sub-leading disconnected diagrams vanish in the SU(3) limit.



Connected diagram



Leading order disconnected diagram

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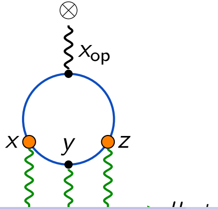
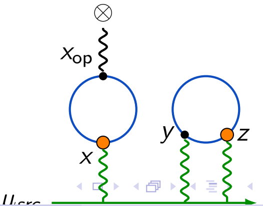
$$(6e^4)\,\mathcal{H}^{\text{con-no-perm}}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z) = -6\Big\langle \mathrm{Re}\sum_{q=u,d,s} e_q^4\,\mathrm{Tr}\big[\gamma_k S_q(x_{\text{op}},x)\gamma_\rho S_q(x,z)\gamma_\lambda S_q(z,y)\gamma_\sigma S_q(y,x_{\text{op}})\big]\Big\rangle$$

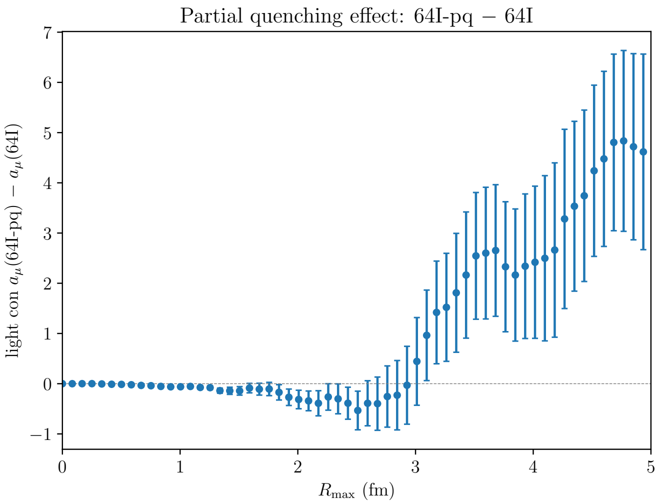
$$(6e^4)\,\mathcal{H}^{\text{discon-no-perm}}_{k\rho\sigma\lambda}(x_{\text{op}},x,y,z) = 3\Big\langle \Pi_{k\rho}(x_{\text{op}},x)\big(\Pi_{\lambda\sigma}(z,y) - \langle\Pi_{\lambda\sigma}(z,y)\rangle\big)\Big\rangle$$

$$\Pi_{\rho\sigma}(x,y) \equiv \sum_q e_q^2\,\mathrm{Tr}\big[\gamma_\rho S_q(x,y)\gamma_\sigma S_q(y,x)\big]$$

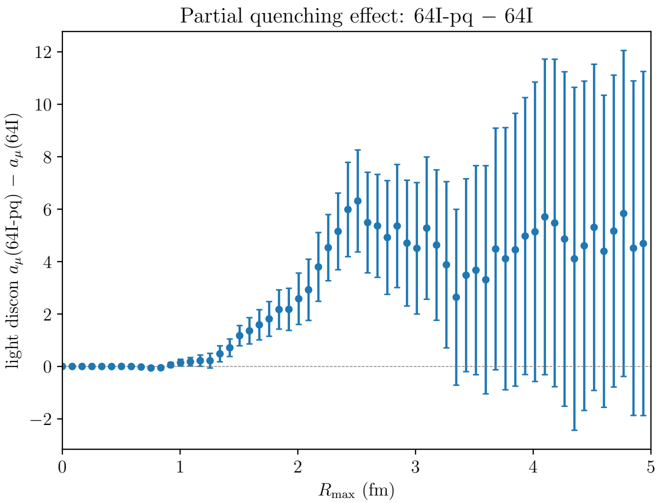




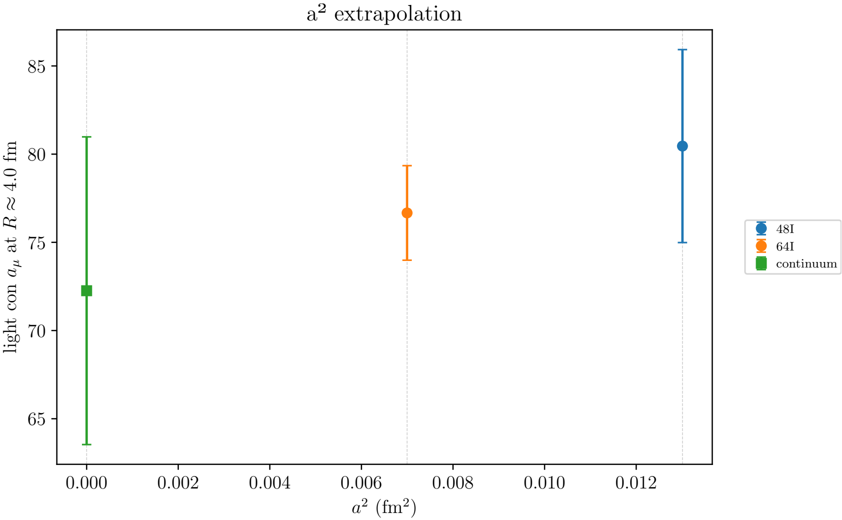
Light quark connected: π^0 mass correction



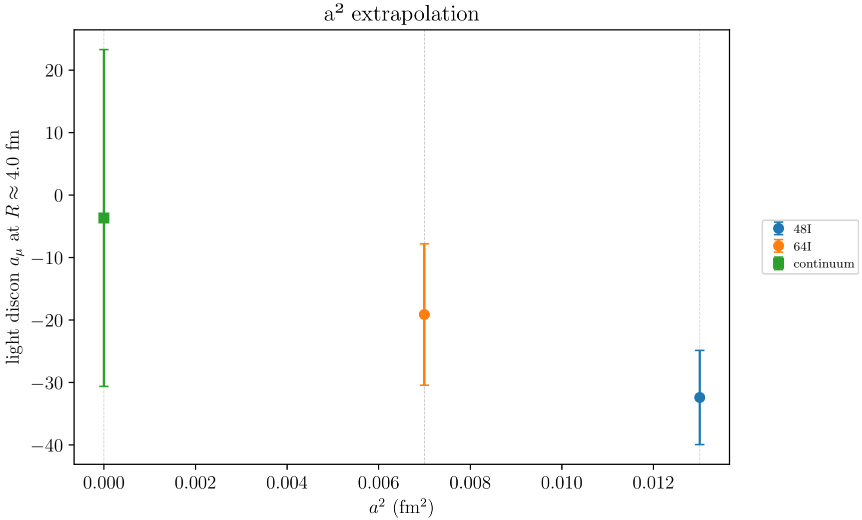
Light quark disconnected: π^0 mass correction



Light quark connected: continuum limit



Light quark disconnected: continuum limit



Strange quark connected: (partially quenched $\pi^0 = 135$ MeV mass ensembles)

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