Studying QED3 in radial quantization: Interacting system on coarse lattice

Nobuyuki Matsumoto (Boston U, RBRC)

BOSTON UNIVERSITY

In collab w/ P. Boyle (BNL), R. C. Brower (BU),
G. Fleming (FNAL), A. Katz (BU), NM, R. Misra (BU)

RIKEN BNL Research Center

Free limit: PRD [arXiv:2510.03085 [hep-lat]]

U.S. DEPARTMENT OF ENERGY
Office of Science
(Scidac5, USQCD)

Lattice 2026

- Lattice QCD has been successful in non-perturbative calculation of QFT

  Many branches in QFT have advanced meanwhile
  that will benefit from such first-principles calculation

  Example: Conformal field theories in higher dimensions

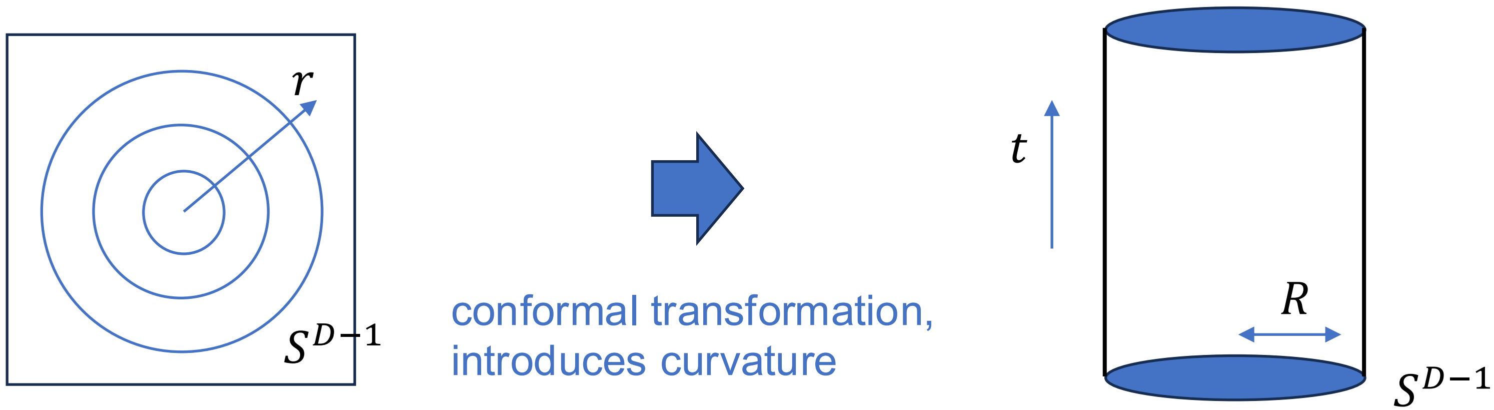
  Can be motivated by…

  - UV completion of the SM
  - Base ground for composite Higgs
  - Theoretical interests, e.g., for its structureless simplicity

- Ordinary IR regulator by the torus breaks scale invariance

- Good IR regulator: cylinder (radial picture; $S^{D-1} \times \mathbb{R}$)

  Scale transformation (aka dilatation) = translation in $\mathbb{R}$



- Quantum finite element (QFE) project: QFT on curved lattices

  Brower Fleming Neuberger Phys Lett ’13, Brower NM et al. PoS ’24

With some ambition of studying

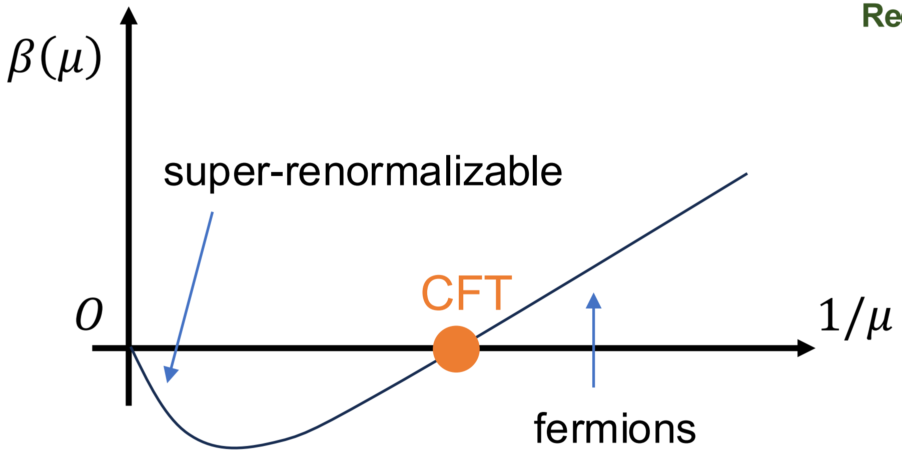
- Curvature-triggered phase transitions in QCD
- Strongly-interacting QFTs in astrophysical/cosmological backgrounds

$$S_{\text{cont}} = \int d^3x \sqrt{g(x)} \left[\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f \sigma^\mu (\partial_\mu + A_\mu + \omega_\mu) \psi_f \right]$$

- Conformal window for $N_f \gg 1$ (even)

Deser, Jackiw, Templeton Annals Phys 1982

Redlich PRL 1984, PRD 1984

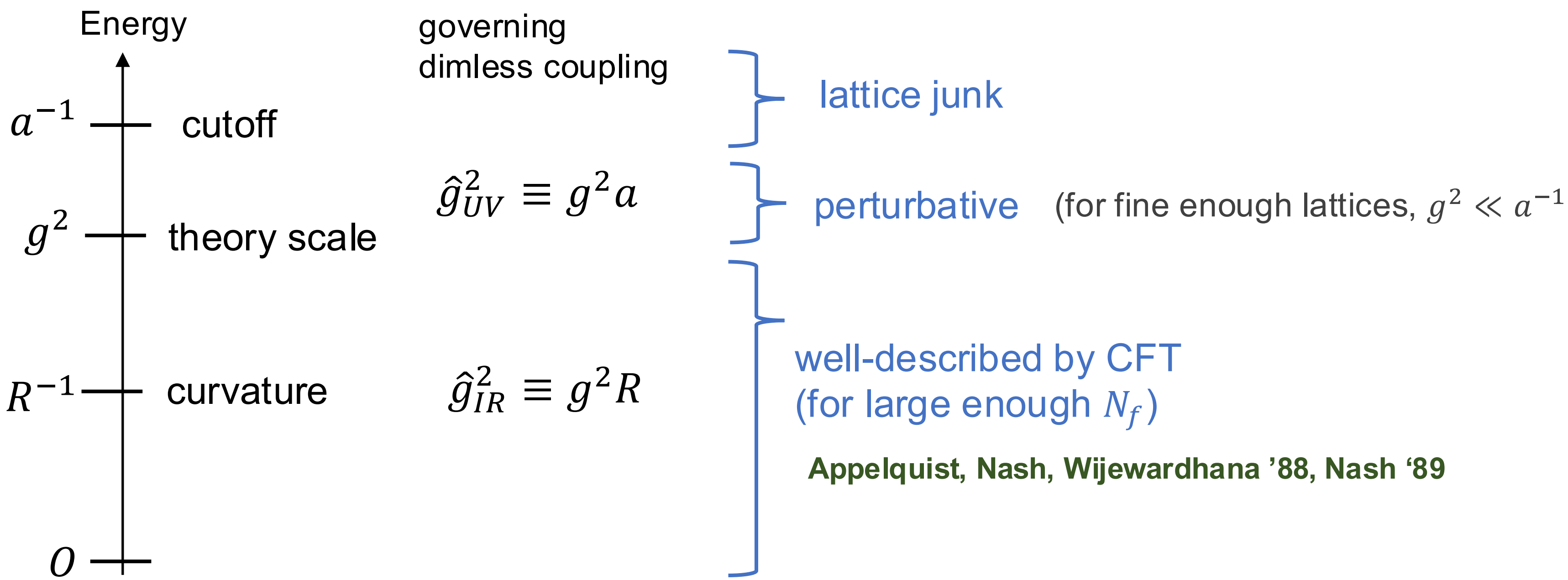


- Conformality breaking may be related to the SSB of $SU(N_f)$ flavor symmetry: $\langle \bar{\psi}_f \psi_f \rangle \neq 0$

(Parity cannot be broken in vector-like theories **Vafa-Witten Phys Lett 1984**)

- Effective theory of a superconducting system **Dorey, Mavromators PLB 1990**

- The system is primarily a QFT on curved space



- CFT limit: $g^2 R \to \infty$ **e.g., Chester, Pufu '16**
  (strongly-interacting)

Lattice action

- Noncompact $U(1)$, Gaussian

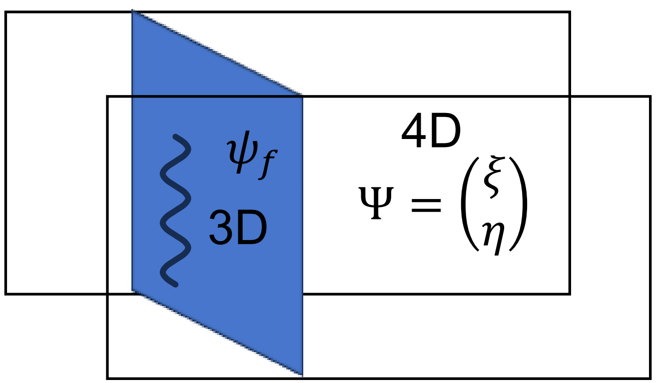
- Overlap fermions (Zolotarev): $N_f = 2, 4, 6$

  - HMC, Hasenbusch'ed
  - 15 poles (n=31) for accept/reject
  - 6 poles (n=11) for the force calculation

  - Exact flavor + parity symmetries on the lattice

Karthik, Narayanan PRD 2016ab

$$\mathcal{L}_{N_f} = \left(\xi^\dagger, \eta^\dagger\right) \gamma_4 \mathcal{D}_{\text{ov}} \begin{pmatrix} \xi \\ \eta \end{pmatrix} = \sum_f \bar{\psi}_f D_{\text{ov}} \psi_f$$



parity: $\xi \leftrightarrow \eta$
flavor basis: boundary modes ψ_f

Resources

USQCD Type B+C

Shared Computing Cluster

- Two chirality operators: γ_4, γ_5

- Two scalar operators:

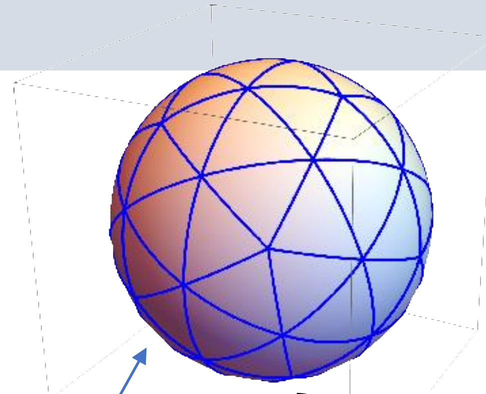
$$\sigma_{PS} \equiv \eta^\dagger \xi + \xi^\dagger \eta$$

(parity symmetric, flavor nonsinglet)

$$\sigma_{FS} \equiv \eta^\dagger \xi - \xi^\dagger \left(1 - D_{\text{ov}}^\dagger\right) \eta$$

(flavor symmetric, parity nonsinglet)

S^2 lattice and anisotropy



$L = 2$

$N_{\text{site}} = 10L^2 + 2$

- Refined icosahedron, projected onto S^2

  Refinement levels: $L = 1, 2, 4$

- Use the free-limit coupling, derived in the previous paper

A handwaving but plausible argument may support it (working hypothesis)

cf. Rohan's talk

- Spin connection fixed the curvature per plaquette:

$$\exp \oint dx^\mu \, \omega_\mu = \frac{1}{2} \int dx^{\mu\nu} R_{\mu\nu} = \frac{i\sigma_3}{2} A_\triangle$$

➡ S^2 geometry is protected

- Exactly conserved vector currents along the links, which do not renormalize

$$\sum_{y:\mathrm{nn}} J_{xy} = 0, \qquad J_{xy} = \kappa \bar{\Psi} K_{xy} \Psi$$

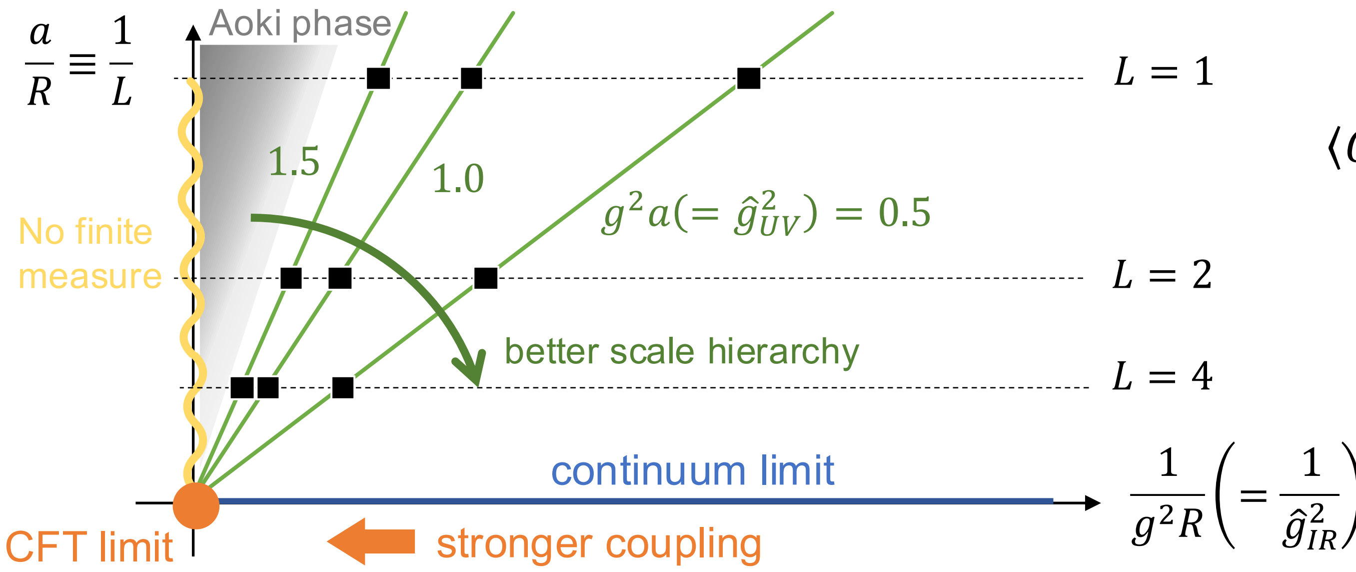
➡ fermion hopping κ is protected

- Theory is super-renormalizable

➡ gauge coupling β scales trivially

- Yet the space-time anisotropy can renormalize:

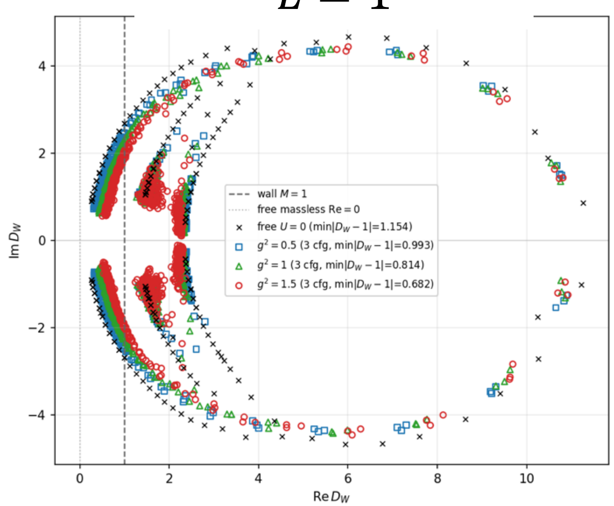
  ➡ Only the ratios of the energy levels (=operator dimensions Δ) are physical.



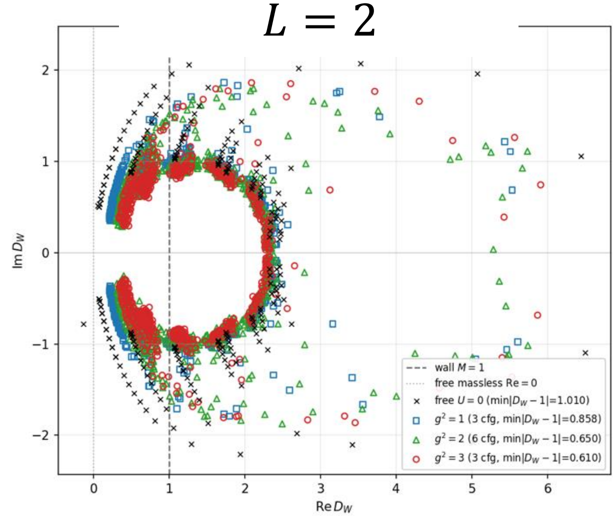
$$\langle O \rangle_{\mathrm{CFT}} = \langle O \rangle_{\mathrm{lat}} + c_{a^2} \left(\frac{a}{R} \right)^2 + \frac{c_{g^2}}{g^2 R} + \cdots$$

<u>Wilson spectrum</u> (around the domain-wall height, by Arnoldi; fixed a_t)

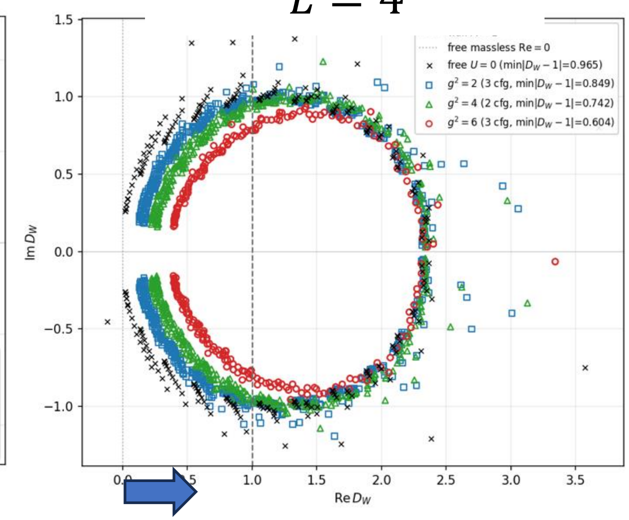
$L = 1$



$L = 2$



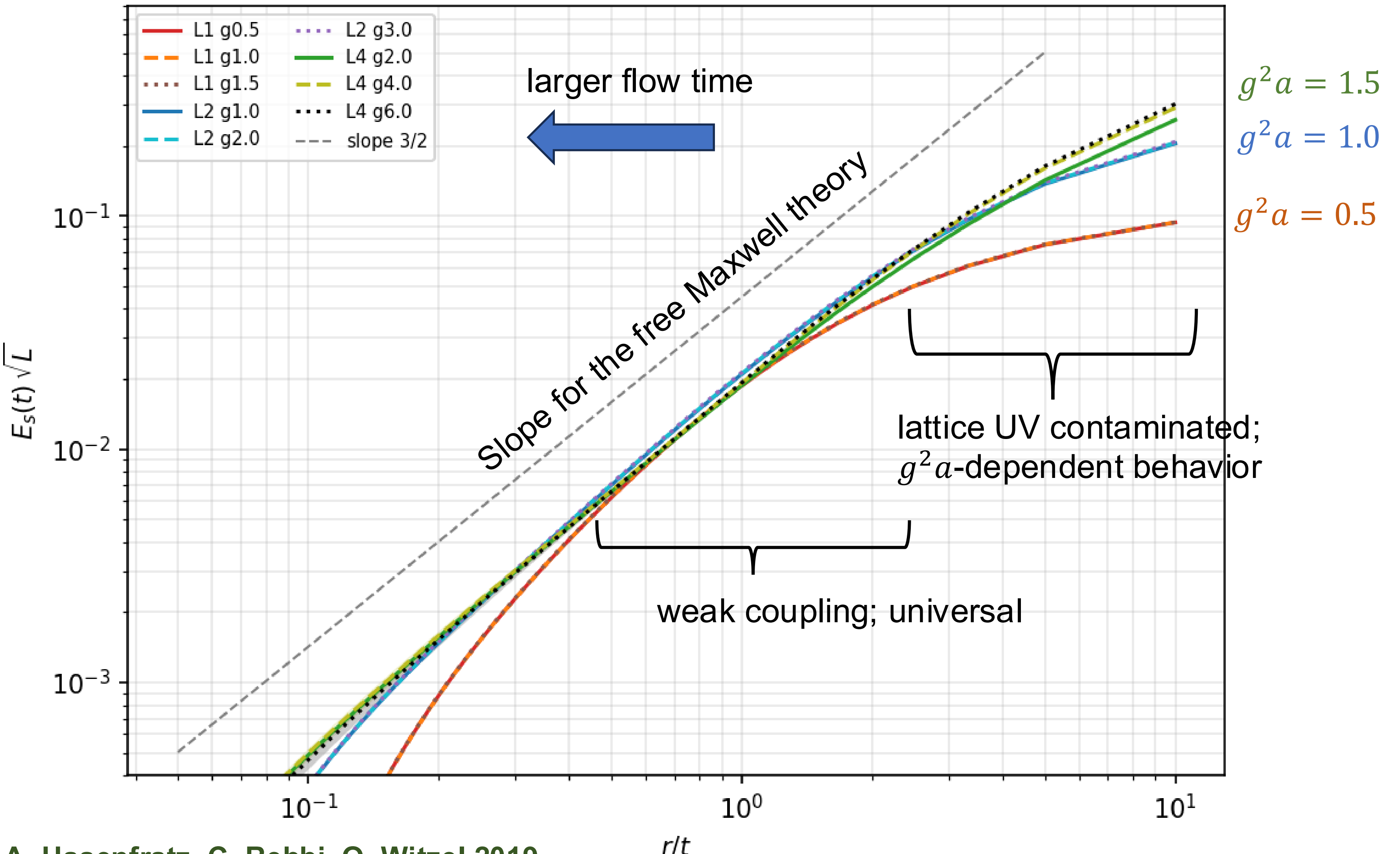
$L = 4$



additive mass shift

Scale scan with the gradient flow

- Scaling curve for the smeared energy $E(t)$ from the gradient flow: $\dot{A}_\mu(t) = -\frac{\delta S}{\delta A_\mu(x)}$

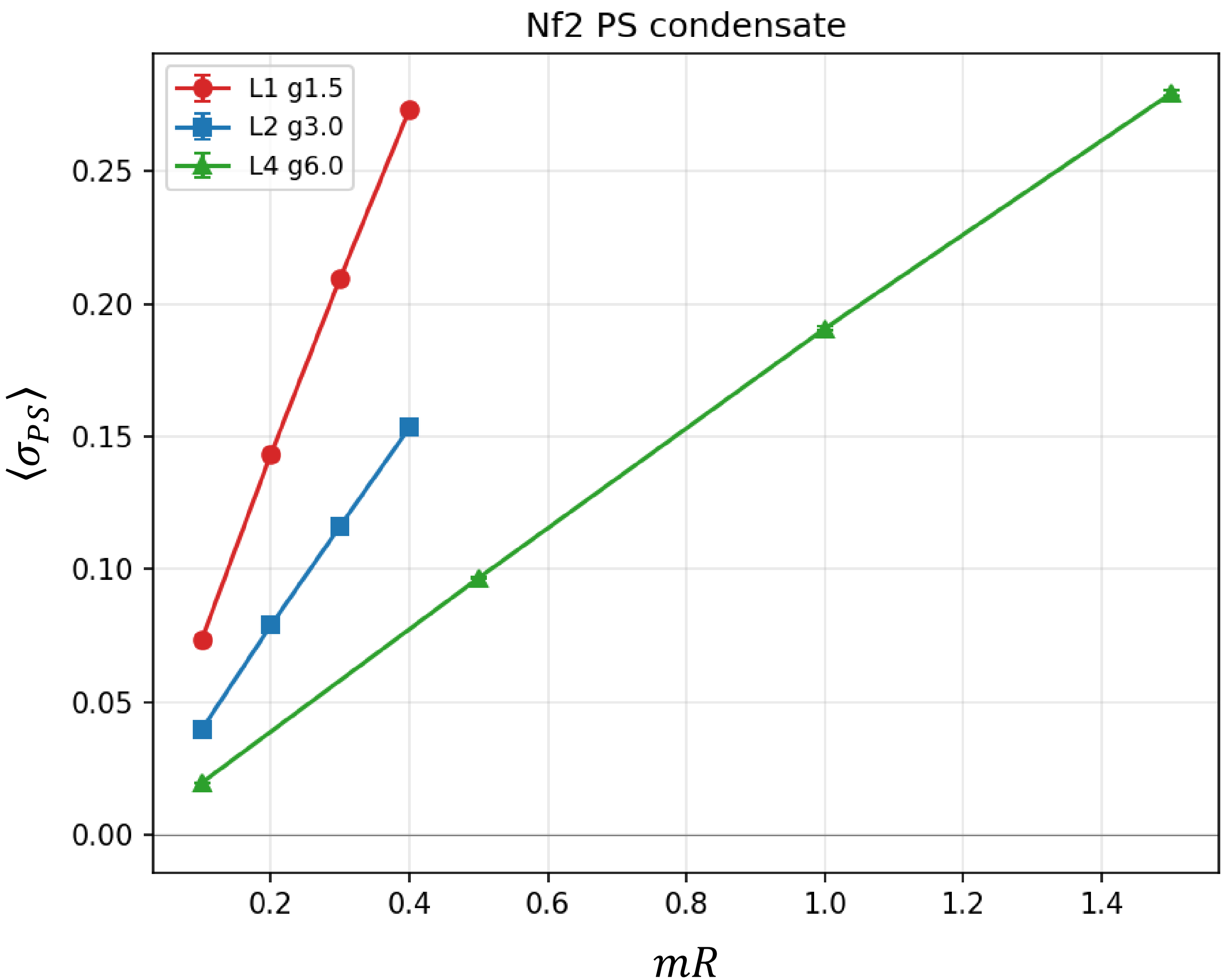


cf. A. Hasenfratz, C. Rebbi, O. Witzel 2019,
A. Hasenfratz, C. Peterson 2024
L. G. Robert V. Harlander, R. H. Mason 2026

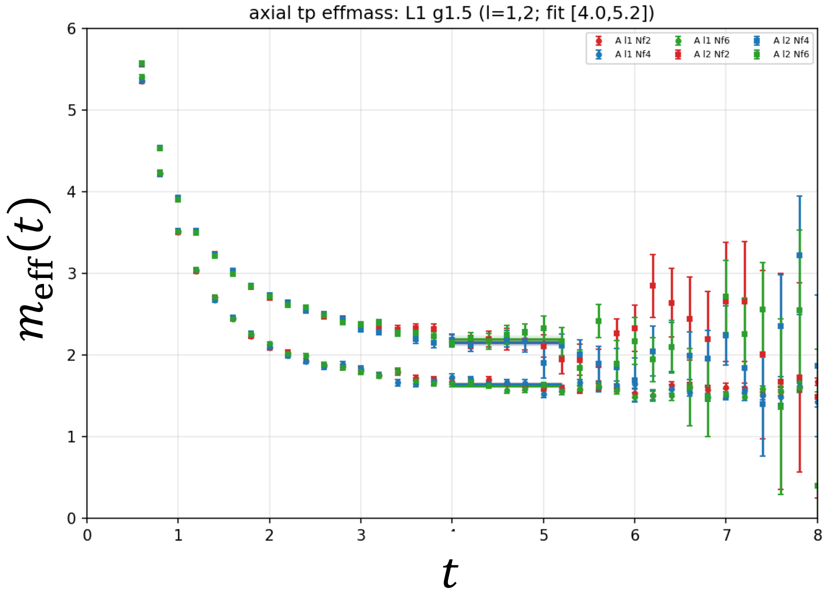
Conformal window study: Condensate

- Soft mass term with σ_{PS} (break the flavor symmetry)
- No sign of SSB even with $N_f = 2$

  - cross-checked with the overlap spectrum
  - consistent with the result of **Karthik, Narayanan PRD 2016ab**



Cross-check 1: Conserved current (pseudovector)



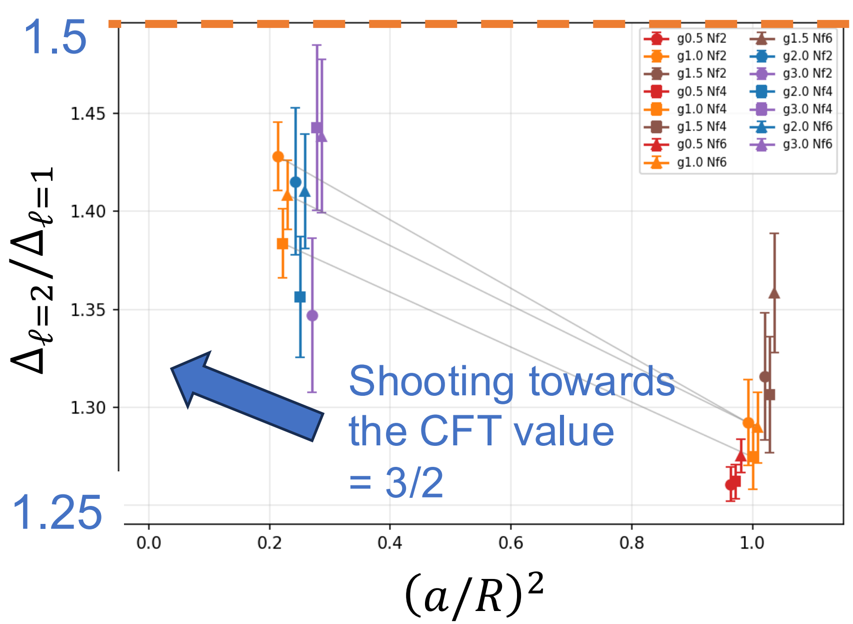
- CFT formula in the flat space:

$$\langle J_\mu(x) J_\nu(y) \rangle = \frac{I_{\mu\nu}(x-y)}{(x-y)^{2(D-1)}}$$

$$\left(I_{\mu\nu}(x) \equiv \delta_{\mu\nu} - \frac{2x_\mu x_\nu}{x^2} \right)$$

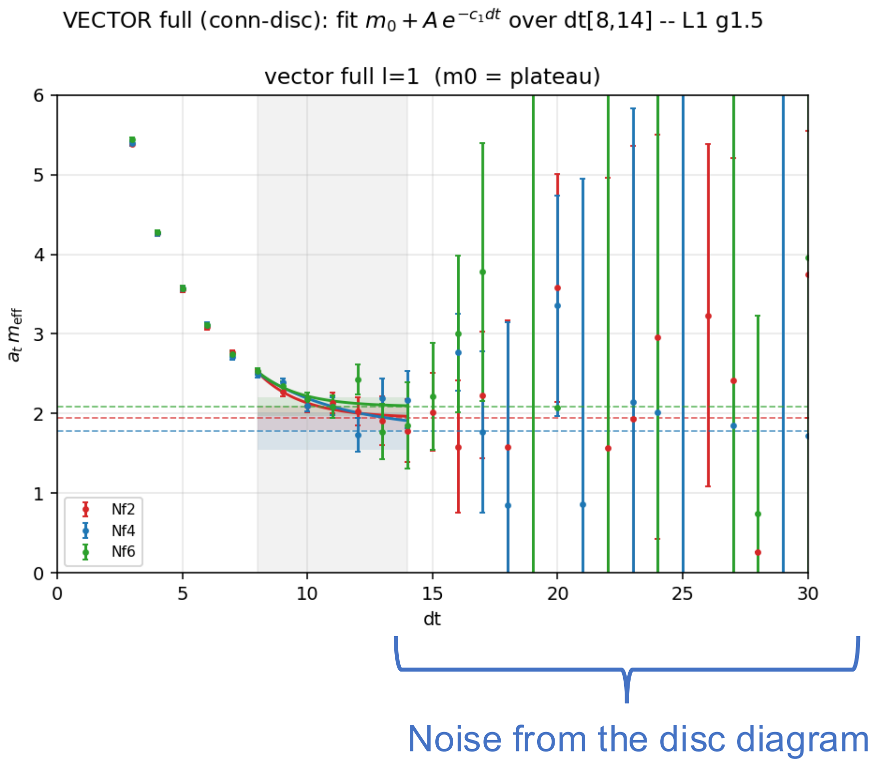
- $\Delta_J = D - 1 = 2$

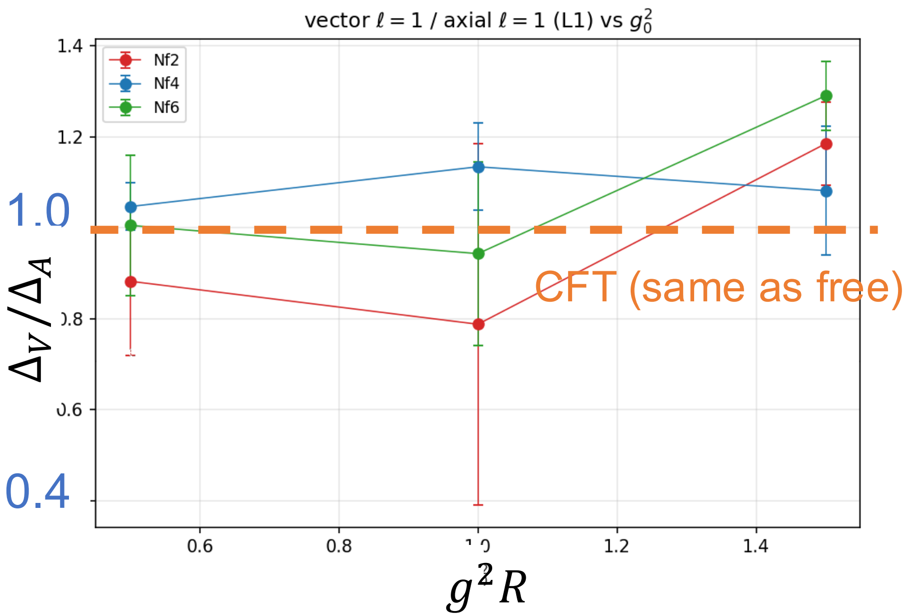
- $\Delta_{\ell=2}/\Delta_{\ell=1}$ = first descendant/primary



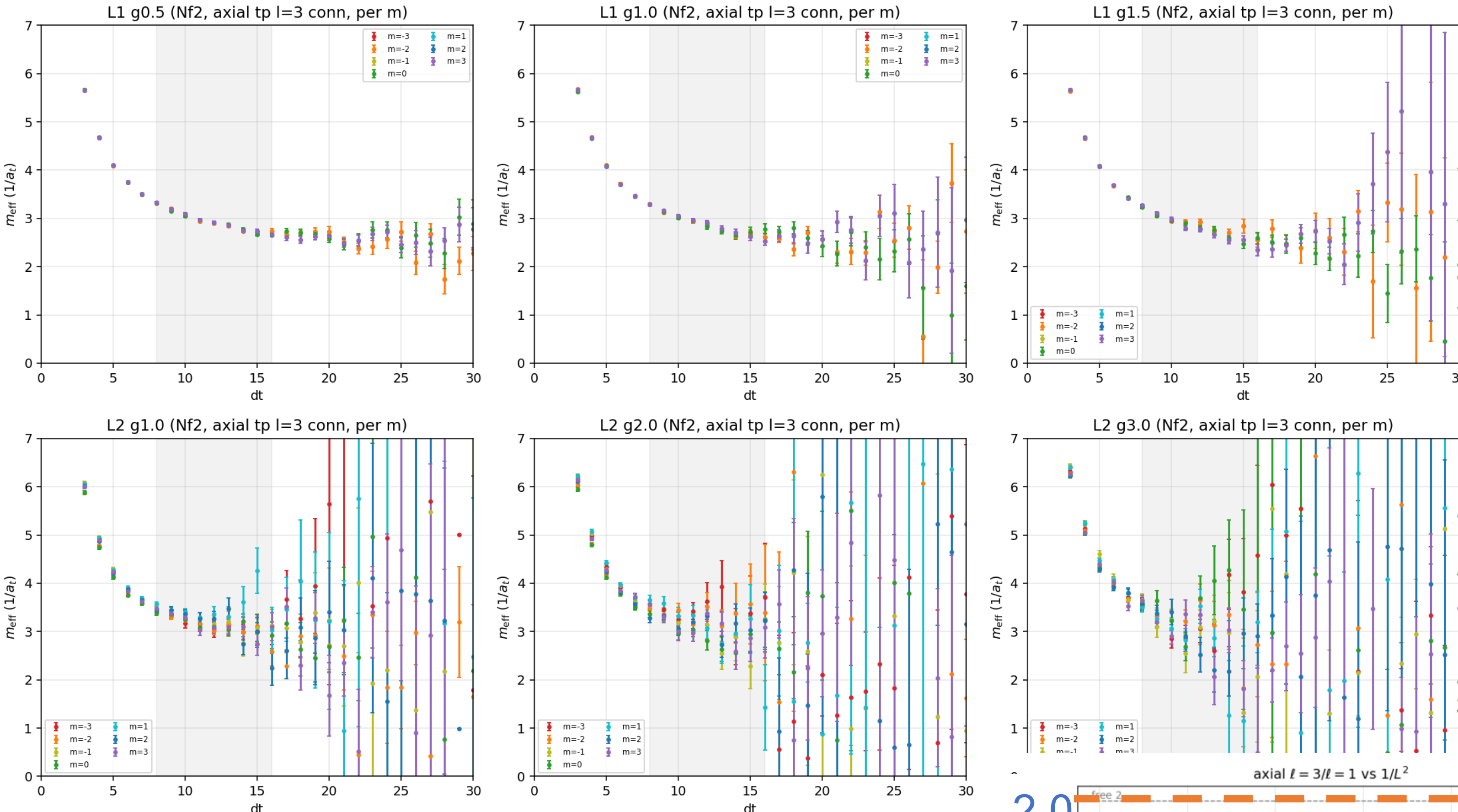
CFT (same as free; no renormalization)

Cross-check 2: Conserved current (vector)





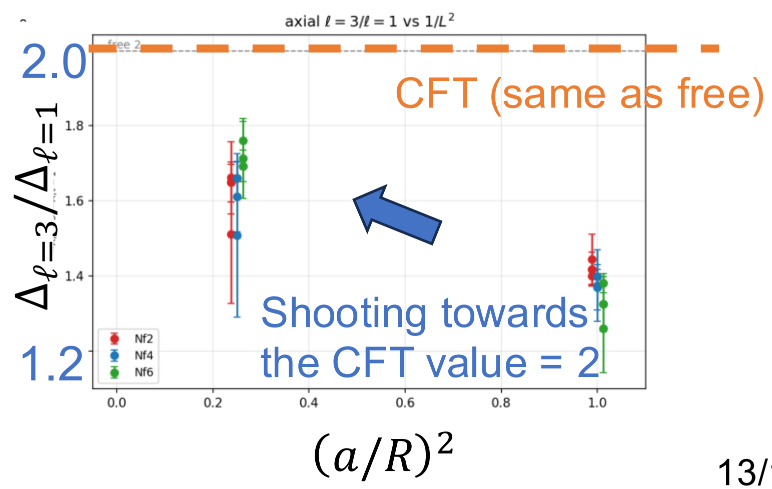
More check 1: Spherical symmetry breaking—$\ell = 3$ *icosahedral symmetry protects $\ell = 1, 2$



Visible but small (~3%) systematic spread in $L = 2$;
→ strength of the spherical breaking terms

Anticipated, scaling toward the cont important

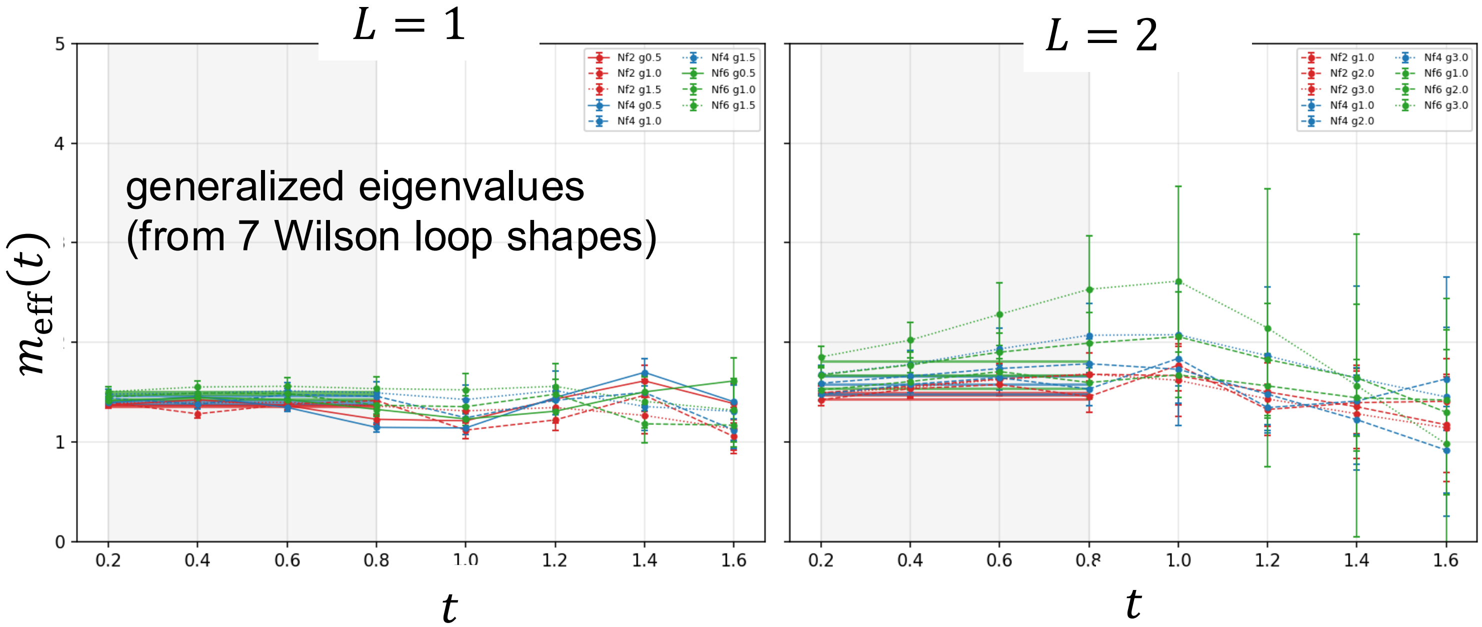
The average seems to scale correctly

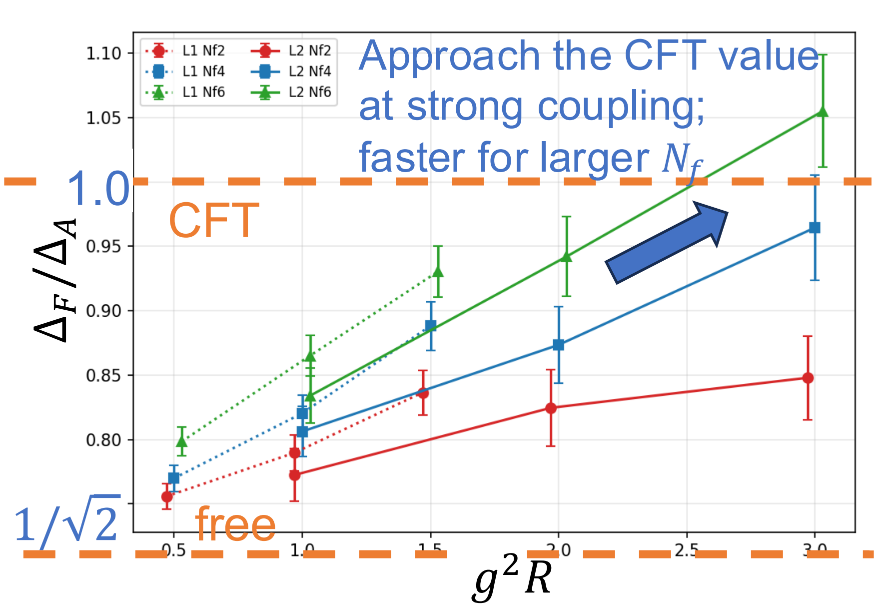


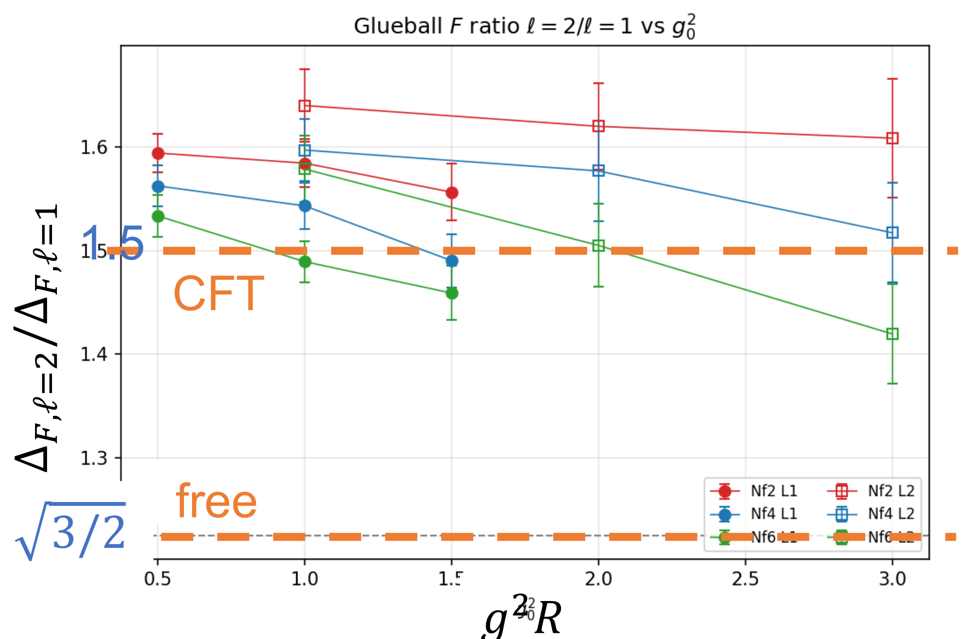
More check 2: Fermionic vs gluonic sector

$$J^{\mu}_{\mathrm{top}} \equiv \epsilon^{\mu\nu\rho} F_{\nu\rho}$$

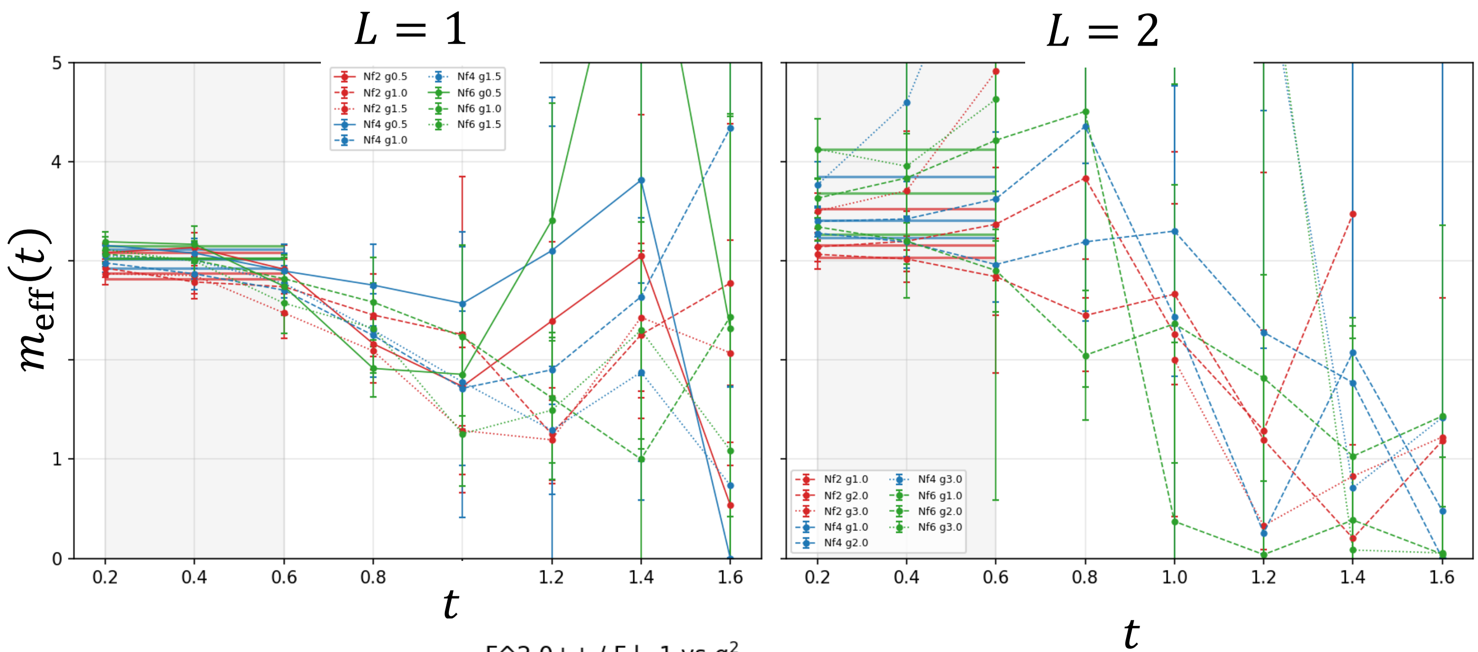
topologically conserved

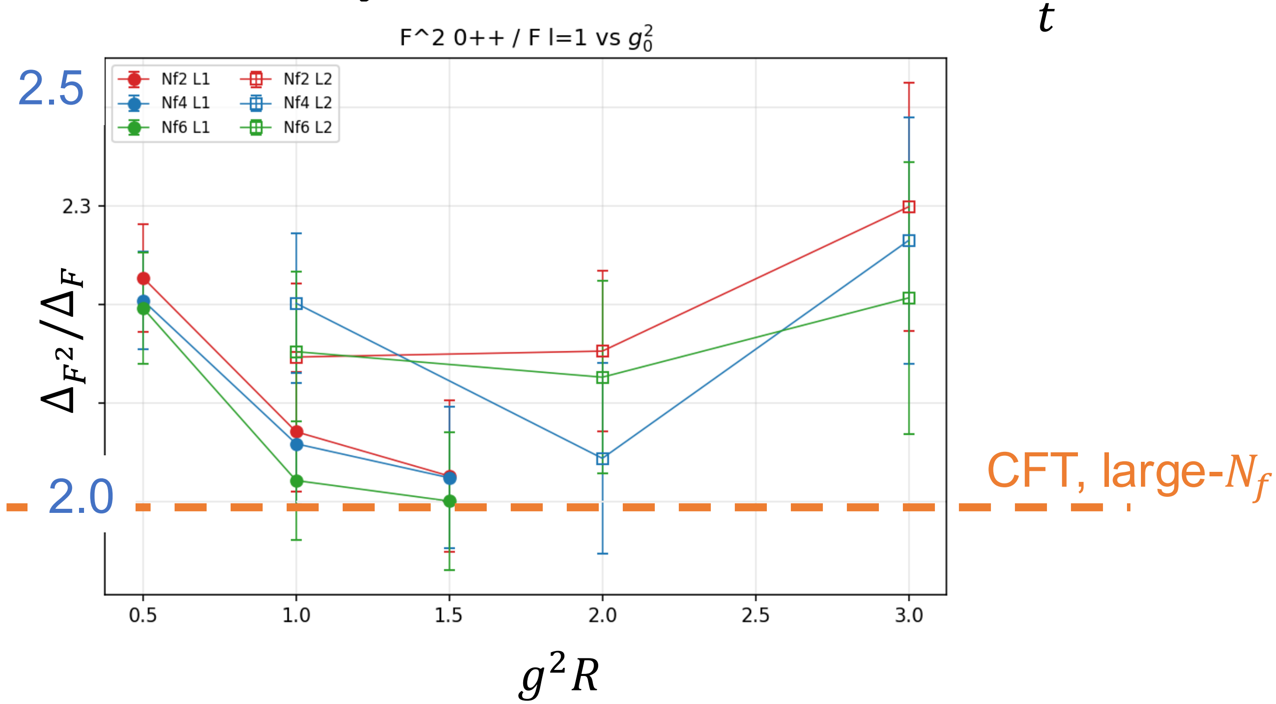






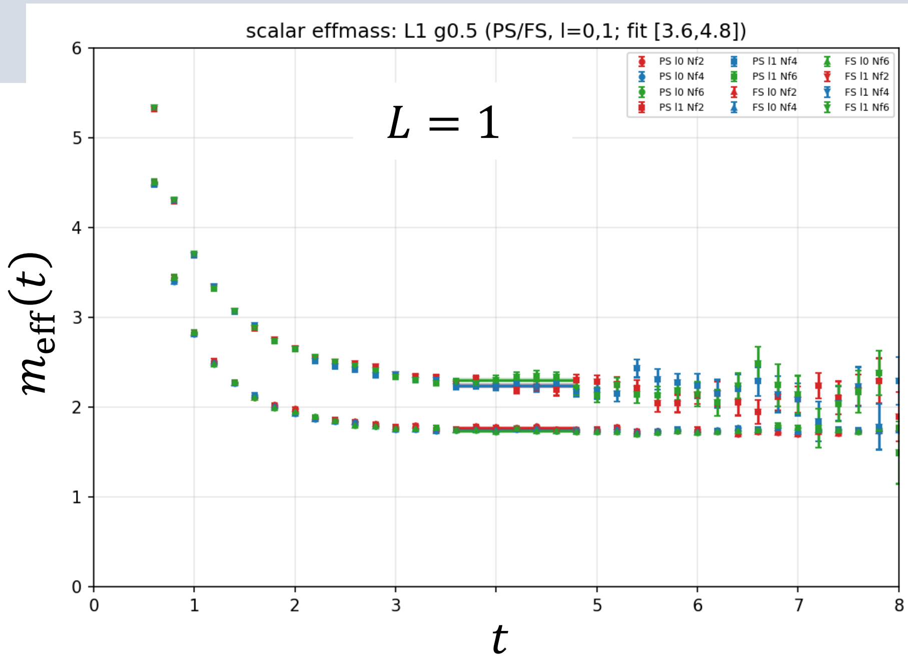
Prediction 1: F^2 — the relevant term in the action

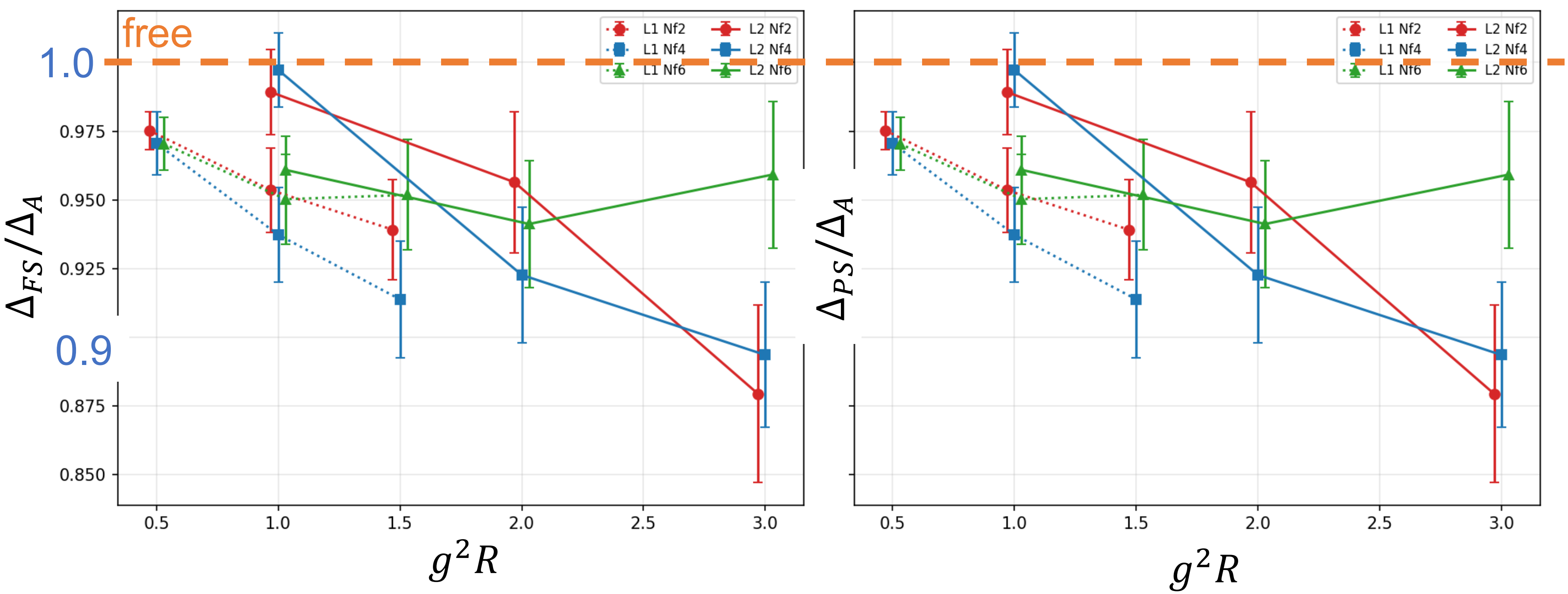




Prediction 2: Scalar correlators

- Nontrivial lattice prediction; can be compared to the bootstrap
- σ_{PS} & σ_{FS} give identical spectra





Summary

- Performed an analysis of QED3 in $S^2 \times \mathbb{R}$ on coarse lattices
- Conformal window prediction: $N_f \geq 2$
- Consistent results with the analytic CFT predictions for the conserved currents
- Nontrivial predictions deliverable from lattice to the CFT community

Discussion

- Increasing statitstics; elaborate GEVP methods; **Quite a few talks in this conference**
  $(a/R)^2, g^2R$ global fit towards the CFT limit, finite a_t/R correction

- Order of limits is important; Aoki phase can have SSB

  Compactness of U(1) can be relevant (∃finite measure with $g^2 a \to \infty$)

  - Monopole proliferation w/ compact U(1)? **Polyakov 1977 cf. Karthik, Narayanan PRD 2019**
  - Nontrivial scaling limit w/ $g^2 a = \text{const}$?
  - Can it induce zero modes/topological excitations though $\pi_1(S^2) = 0$? **Banks, Casher '80 Marinari, Parisi, Rebbi '81 Leutwyler, Smilga '92**

- Finer lattice: large scale code w/ domain-wall fermions **Peter, NM++ work in progress**

- *If* the non-renormalization hypothesis is correct:
  QFTs on curved spacetime; chiral fermions with curved manifolds?

Hersh Singh's Plenary; Aoki Fukaya 2023, Kaplan Sen 2024, Golterman Shamir 2024, Clancy Kaplan 2025, Yamamoto Kan Fukaya 2025

Thanks!