Emergence of magnetic monopoles due to violation of the non-Abelian Bianchi identity

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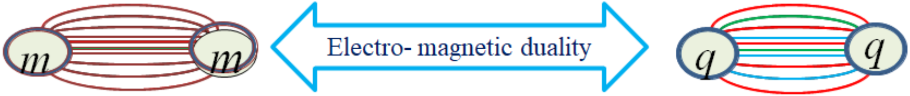
The 43rd International Symposium on Lattice Field Theory
the University of Maryland, College Park, USA,
From July 26 to August 1, 2026.

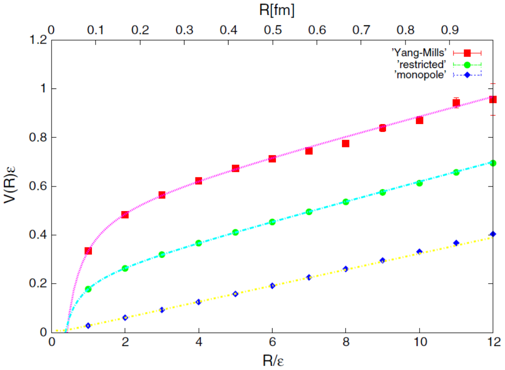
Introduction I

Dual superconductor picture is one of the most promising mechanism for quark confinement.
[Y.Nambu (1974). G.t Hooft, (1975). S.Mandelstam, (1976) A.M. Polyakov (1975)]
In this scenario, QCD vacuum is considered as a dual super conductor.

superconductor	dual superconductor
Condensation of electric charges	Condensation of magnetic monopoles
Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole	Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
Linear potential between monopoles	Linear potential between quarks



The perfect restricted-field dominance ("Abelian" dominance) and perfect magnetic-monopole dominance in the string tension, that is, the string tension from Yang-Mills field is completely reproduce by the restricted-field and the magnetic monopole.



$$V(r) = -\frac{C}{r} + \sigma r$$

$$F(r) = -\frac{d}{dr}V(r) = -\frac{C}{r^2} - \sigma$$

PRD 91, 034506 2015

Violation of the non-Abelian Bianchi identity and the emergence of magnetic monopoles (precieding works)

Bonati et.al. discussed the relation between the VNABI and magnetic monopoles, which are constructed from the 't Hooft tensor $F^a_{\mu\nu}$, where scalar fields, ϕ^a in the adjoint representation are expected:Phys. Rev. D **81** (2010) 085022]

$$\partial_\mu F^{a*}_{\mu\nu} = \mathrm{Tr}(\phi^a J_\nu) = \mathrm{Tr}(\phi^a D_\nu \mathscr{F}^*_{\mu\nu})$$

Suzuki et.al. studied a new scheme for color confinement under VNABI where the singularity/defect is assumed, i.e., $[\partial_\mu, \partial_\nu] \neq 0$ [Phys.Rev. D**97** (2018) 034501]:

$$D_\nu \mathscr{F}^*_{\mu\nu} = -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu\ ,\ [D_\rho\ ,\ D_\sigma]] = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho\ ,\ \partial_\sigma]A_\nu = \partial_\nu f^*_{\mu\nu}$$

Here, we study the pure Yang-Mills theory (without scalar fields), where NABI is satisfied within the classical field theory, and VNABI could be induced by singular/defect configurations in the quantum theory. (topological defect)

Gauge covariant decomposition I

[see for review: Phys. Rept. **579** (2015)]

Let us consider the gauge covariant decomposition via the Cho–Duan–Ge decomposition

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu} \in SU(2)\,,$$
$$U_{x,\mu} \to U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\hat{\mu}}\,,\; V_{x,\mu} \to U'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\hat{\mu}}\,,\; X_{x,\mu} \to X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x\,,$$

where $U_{x,\mu} = \exp(-ig\epsilon \mathscr{A}_\mu(x))$ represents Yang-Mills field, $V_{x,\mu} \simeq \exp(-ig\epsilon \mathscr{V}_\mu(x))$ the restricted field which plays a dominant role in the confinement, and $X_{x,\mu} \simeq \exp(-ig\epsilon \mathscr{X}_\mu(x)$ the remainder part. Those transform as gauge-covariant way under the gauge transformation $\Omega_x \in SU(2)$. The **decomposition** is define by so-called defining equation:

$$D_\mu[V]\mathbf{n}_x := V_{x,\mu}\boldsymbol{n}_{x+\hat{\mu}} - \boldsymbol{n}_x V_{x,\mu} = 0\,, \quad \mathrm{Tr}(X_{x,\mu}\boldsymbol{n}_x) = 0\,, \quad \boldsymbol{n}_x = \Theta_x H_3 \Theta_x^\dagger\,,$$

where we introduce the color-direction fields with $H_3 = \frac{\sigma^3}{2}$ and $\Theta_x \in SU(2)$.
For a given set of the gauge field configuration $\{U_{x,\mu}\}$ and the color-direction field configuration we obtain the decomposition:

$$V_{x,\mu} := \tilde{V}_{x,\mu} / \sqrt{\mathrm{tr}[\tilde{V}^\dagger_{x,\mu}\tilde{V}_{x,\mu}]/2}\,, \quad \tilde{V}_{x,\mu} := U_{x,\mu} + \mathbf{n}_x U_{x,\mu}\mathbf{n}_{x+\mu}\,, \quad X_{x,\mu} := U_{x,\mu}V^{-1}_{x,\mu}\,.$$

Gauge covariant decomposition II

Reduction condition

For a given gauge field configuration $\{U_{x,\mu}\}$, we determine the color-direction field configuration $\{\boldsymbol{n}_x\}$ (as the unique configuration up to the global color rotation) by minimizing the so-called *reduction condition*.

$$F_{\text{red}}[\{\mathbf{n}_x\}|\{U_{x,\mu}\}] := \sum_{x,\mu} \operatorname{tr}\left\{ \left(D_{x,\mu}[U]\mathbf{n}_x\right)^\dagger \left(D_{x,\mu}[U]\mathbf{n}_x\right) \right\} , \; D_{x,\mu}[U]\mathbf{n}_x := U_{x,\mu}\mathbf{n}_{x+\hat{\mu}} - \mathbf{n}_x U_{x,\mu}$$

Note that the reduction condition is **an equation of the motion** for the scalar field in the adjoint representation.

$$\boldsymbol{n}^* = \underset{\boldsymbol{n}}{\operatorname{argmin}} \, F_{\mathrm{red}}[\{\boldsymbol{n}\};\{U\}] , \quad \text{or} \quad \frac{\delta}{\delta n} F_{\mathrm{red}}[\{\boldsymbol{n}\};\{U\}] = 0$$

Magnetic monopoles I

Non-Abalian Stokes Theorem

[R. Matsudo and K.-I. Kondo,Phys. Rev. D **92** (2015) 125038]

Following the Non-Abelian Stokes theorem the Wilson loop operator in the representation R is given by

$$W_C[\mathscr{A}] = \int d\Omega \exp\left[\int_{\Sigma_C} \mathcal{F}\right] = \int d\Omega \exp\left\{\int \frac{1}{2}\left[(\omega_{\Sigma_C}, \mathcal{K}) + (N_{\Sigma_C}, \mathcal{J}_{\mathrm{e}})\right]\right\},$$

$$\mathcal{F} := 2\mathrm{tr}(\boldsymbol{n}\mathscr{F}_{\mu\nu}[\mathscr{V}]), \quad \mathcal{K} := {}^*d\mathcal{F}, \quad \mathcal{J}_{\mathrm{e}} := \delta\mathcal{F},$$

where $d\Omega$ is the product of the Haar measure over the surface Σ_C with boundary C, F^R represents the Abelian-like field strength, and $K^{(j)}$ represents the magnetic current (monopole). Λ_j denotes the j-th component of the highest-weight vector of the representation R, which corresponds to the color-direction field $\boldsymbol{n}^{(j)}$. $(\alpha, \beta) := \int_{\Sigma_C} \alpha \wedge {}^*\beta$ denotes the standard pairing of differential forms, and the one-form ω_{Σ_C} and N_{Σ_C} are the forms appearing in the Hodge decomposition

Magnetic monopoles II

From the restricted field $V_{x,\mu}$, we define the field strength for the restricted field:

$$V_{x,\mu\nu} = V_{x,\mu} V_{x+\hat{\mu},\nu} V^{\dagger}_{x+\hat{\nu},\mu} V^{\dagger}_{x,\nu} = \exp\left(-ig\epsilon \mathscr{F}[\mathscr{V}]_{x,\mu\nu}\right) = \exp\left(iF_{x,\mu\nu}\boldsymbol{n}_x\right) \ ,$$

$$F_{x,\mu\nu} := 2g\epsilon \, \mathrm{Tr}(\mathscr{F}[\mathscr{V}]_{x,\mu\nu}\boldsymbol{n}_x)$$

$$= 2g\epsilon \left(\partial_\mu \mathrm{Tr}(\mathscr{V}_{x,\nu}\boldsymbol{n}) - \partial_\nu \mathrm{Tr}(\mathscr{V}_{x,\mu}\boldsymbol{n}_x) - ig^{-1}\mathrm{Tr}\left(\boldsymbol{n}_x[\partial_\mu \boldsymbol{n}_x, \partial_\nu \boldsymbol{n}_x]\right)\right) \ ,$$

where we have used $[V_{x,\mu\nu}, \boldsymbol{n}_x] = 0$ from defining equation $D_\mu[V]\mathbf{n}_x = 0$.
Note that $F_{x,\mu}$ is gauge invariant field strength, which we call the **" 't Hooft tensor"** associated to the color direction field.
We obtain the gauge-invariant magnetic monopole due to violation of Bianchi identity

$$\mathcal{K} := {}^{*}d\mathcal{F} \quad \Leftrightarrow \quad k_{x,\mu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu F_{x,\alpha\beta} \ ,$$

which satisfies the current conservation low $\partial_\nu k_{x,\mu} = 0$. Thus magnetic monopoles appear as closed loops.

Yang-Mills theory and Non-Abelian Bianchi Identity I

Let $\mathscr{A}_\mu(x) = \mathscr{A}_\mu^a(x)\,T^a \in su(2)$ be the $SU(2)$ Yang-Mills fields, and be $\mathcal{A}$ one-form variable: $\mathcal{A} = -ig\mathscr{A}_\mu^a(x)\,dx^\mu \otimes T^a$. The gauge field strength is given by

$$\mathcal{F}[\mathcal{A}] := d\mathcal{A} + \mathcal{A}\wedge\mathcal{A} = \frac{-ig}{2}\mathscr{F}^a_{\mu\nu}[\mathscr{A}](x)\,dx^\mu \wedge dx^\nu \otimes T^a\,,$$

where we have defined $\mathscr{F}_{\mu\nu}[\mathscr{A}] := \partial_\mu\mathscr{A}_\nu - \partial_\nu\mathscr{A}_\mu - ig[\mathscr{A}_\mu\,,\,\mathscr{A}_\nu] = \mathscr{F}^a_{\mu\nu}[\mathscr{A}]\,T^a$. The Lagrangian density is given by $L_{\mathrm{YM}} := -\frac{1}{2g^2}\mathrm{tr}^*\left(\mathcal{F}[\mathcal{A}]\wedge {}^*\mathcal{F}[\mathcal{A}]\right)$, where the symbol "$*$" denotes the Hodge star operator. Therefore, we obtain the equation of motion:

$$\begin{aligned}
\mathcal{D}[\mathcal{A}]^*\mathcal{F}[\mathcal{A}] &:= d\,{}^*\mathcal{F}[\mathcal{A}] + \mathcal{A}\wedge{}^*\mathcal{F}[\mathcal{A}] - {}^*\mathcal{F}[\mathcal{A}]\wedge\mathcal{A} = {}^*\mathcal{J}_e\,,\\
\mathcal{D}[\mathcal{A}]\mathcal{F}[\mathcal{A}] &:= d\mathcal{F}[\mathcal{A}] + \mathcal{A}\wedge\mathcal{F}[\mathcal{A}] - \mathcal{F}[\mathcal{A}]\wedge\mathcal{A} = {}^*\mathcal{J}_m = 0\,,
\end{aligned}$$

where $\mathcal{J}_e$ is a one-form electric current and $\mathcal{J}_m$ is a one-form magnetic current which is zero for the pure Yang-Mills theory.

Yang-Mills theory and Non-Abelian Bianchi Identity II

From the Jacobi identity for the covariant derivative acting on a Lie-algebra valued field Φ, $\mathscr{D}_\mu[\mathscr{A}]\Phi := \partial_\mu \Phi - ig[\mathscr{A}_\mu, \Phi]$, we obtain the non-Abelian Bianchi identity (NABI):

$$0 = ig^{-1} \sum_{\mu,\nu\,\lambda:\text{ cyclic}} \left[\mathscr{D}_\mu , [\mathscr{D}_\nu , \mathscr{D}_\lambda]\right] = \sum_{\mu,\nu\,\lambda:\text{cyclic}} \left[\mathscr{D}_\mu , \mathscr{F}_{\nu\lambda}\right] = \sum_{\mu,\nu\,\lambda:\text{cyclic}} \mathscr{D}_\mu \mathscr{F}_{\nu\lambda} ,$$

where we used

$$[\mathscr{D}_\nu , \mathscr{D}_\lambda] = -ig \left(\partial_\nu \mathscr{A}_\lambda - \partial_\lambda \mathscr{A}_\nu - ig[\mathscr{A}_\nu, \mathscr{A}_\lambda]\right) + [\partial_\nu , \partial_\lambda] = -ig\, \mathscr{F}_{\nu\lambda} ,$$

and assumed that there exists no singularity/defect for the classical field, i.e., $[\partial_\nu , \partial_\lambda] = 0$. We define the (Lie-algebra valued) magnetic current 1-form by

$$\mathcal{J}_{\rm m} := {}^*\mathcal{D}[\mathcal{A}]\mathcal{F}[\mathcal{A}] = \mathscr{J}_{m,\mu}\, dx^\mu, \quad \mathscr{J}^\mu_m := \frac{1}{2} \epsilon^{\mu\nu\alpha\beta}\, \mathscr{D}_\nu[\mathscr{A}]\, \mathscr{F}_{\alpha\beta}[\mathscr{A}] .$$

For smooth classical fields, NABI implies $\mathcal{J}_m = 0$. Note that $\mathcal{J}_m = 0$ is the equation of motion for the magnetic part.

Yang-Mills theory and Non-Abelian Bianchi Identity III

We also obtain a current conservation low :

$$\mathscr{D}_\mu \mathscr{J}_m^\mu = {}^*\mathcal{D}[\mathcal{A}]^*\mathcal{J}_m = {}^*\mathcal{D}[\mathcal{A}]\mathcal{D}[\mathcal{A}]\mathcal{F}[\mathcal{A}] = 0\,,$$

which is derived directly by calculating the covariant derivative.

Non-Abelian Bianchi identity for decomposed field

Let us consider a field decomposition $\mathcal{A} = \mathcal{V} + \mathcal{X}$ where we define $\mathcal{V} = -ig\mathscr{V}_\mu dx^\mu$, $\mathcal{X} = -ig\mathscr{X}_\mu dx^\mu$ with being $\mathscr{V}_\mu(x)$, $\mathscr{X}_\mu(x) \in su(2)$. The field strength is decomposed into

$$\mathcal{F}[\mathcal{A}] = \mathcal{F}[\mathcal{V}] + \widetilde{\mathcal{F}}[\mathcal{V}, \mathcal{X}] ,$$
$$\mathcal{F}[\mathcal{V}] := d\mathcal{V} + \mathcal{V} \wedge \mathcal{V} , \qquad \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] := d\mathcal{X} + \mathcal{V} \wedge \mathcal{X} + \mathcal{X} \wedge \mathcal{V} + \mathcal{X} \wedge \mathcal{X} .$$

Then, magnetic current $\mathcal{J}_{\mathrm{m}}$ is decomposed into

$$\mathcal{J}_{\mathrm{m}} = \mathcal{D}[\mathcal{A}]\mathcal{F}[\mathcal{A}] = \mathcal{D}[\mathcal{V} + \mathcal{X}] \left(\mathcal{F}[\mathcal{V}] + \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}]\right) = \mathcal{J}_{\mathrm{m}}^{V} + \mathcal{J}_{\mathrm{m}}^{x,v}$$
$$\mathcal{J}_{\mathrm{m}}^{V} := \mathcal{D}[\mathcal{V}]\mathcal{F}[\mathcal{V}]$$
$$\mathcal{J}_{\mathrm{m}}^{x,v} := \mathcal{D}[\mathcal{V}]\widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] + \mathcal{X} \wedge \mathcal{F}[\mathcal{V}] - \mathcal{F}[\mathcal{V}] \wedge \mathcal{X} + \mathcal{X} \wedge \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] - \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] \wedge \mathcal{X}.$$

When we substitute the field strength of the classical solution $\mathcal{F}[\mathcal{V}]$ and $\widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}]$ into the above equation we obtain the Bianchi Identy for the decomposed fields by direct calculation:

$$\mathcal{J}_{\mathrm{m}}^{V} = \mathcal{D}[\mathcal{V}]\mathcal{F}[\mathcal{V}] = 0 ,$$
$$\mathcal{J}_{\mathrm{m}}^{x,v} = \mathcal{D}[\mathcal{V}]\widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] + \mathcal{X} \wedge \mathcal{F}[\mathcal{V}] - \mathcal{F}[\mathcal{V}] \wedge \mathcal{X} + \mathcal{X} \wedge \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] - \widetilde{\mathcal{F}}[\mathcal{V} , \mathcal{X}] \wedge \mathcal{X} = 0 .$$

Violation of Bianchi identity and magnetic monopole I

We here focus on the magnetic current $\mathcal{J}_{\rm m}^{V} = \mathcal{D}[\mathcal{V}]\mathcal{F}[\mathcal{V}]$ for the restricted field, . For the maximal option, we have field strength for the restricted field $\mathcal{A}$ as

$$\mathcal{F}[\mathcal{V}] = \mathcal{F}\,\boldsymbol{n}(x)\,, \qquad \mathcal{F} := \frac{1}{2}F_{\mu\nu}dx^{\mu}\wedge dx^{\nu}\,,\; F_{\mu\nu} := (\boldsymbol{n}\cdot\mathscr{F}_{\mu\nu}) = 2\mathrm{tr}(\boldsymbol{n}\mathscr{F}_{\mu\nu})\,.$$

Therefore, we obtain

$$^{*}\mathcal{J}_{\rm m} = \mathcal{D}[\mathcal{V}]\mathcal{F}[\mathcal{V}] = \{\boldsymbol{n}\,d\mathcal{F} + \mathcal{F}\,\mathcal{D}[\mathcal{V}]\boldsymbol{n}\} = \boldsymbol{n}\,d\mathcal{F}$$

where we have used the defining equation $\mathcal{D}[\mathcal{V}]\boldsymbol{n} = 0\,.$ On the other hand, by direct calculation we can show

$$\mathrm{Tr}\left\{\boldsymbol{n}\left(\mathcal{D}[\mathcal{V}]\widetilde{\mathcal{F}}[\mathcal{V},\mathcal{X}] + \mathcal{X}\wedge\mathcal{F}[\mathcal{V}] - \mathcal{F}[\mathcal{V}]\wedge\mathcal{X} + \mathcal{X}\wedge\widetilde{\mathcal{F}}[\mathcal{V},\mathcal{X}] - \widetilde{\mathcal{F}}[\mathcal{V},\mathcal{X}]\wedge\mathcal{X}\right)\right\} = 0$$

Therefore we obtain the magnetic current

$$\mathcal{K} = \mathrm{Tr}\left(\mathcal{J}_{\rm m}\boldsymbol{n}\right) = d\mathcal{F}\,.$$

Numerical result

(*) Numerical simulations are performed on a 16^4 lattice using the Wilson action at $\beta = 2.4$.

(*) For Yang-Mills field $U_{x,\mu}$ and CDG decomposition $U_{x,\mu} = X_{x,\mu} V_{x,\mu}$, we obtain field strength by using the plaquette as

$$U_{x,\alpha\beta} := U_{x,\alpha} U_{x+\bar{\alpha},\beta} U^{\dagger}_{x+\bar{\beta},\alpha} U^{\dagger}_{x,\beta} = \exp(-ig\epsilon \mathscr{F}_{x,\alpha\beta}[\mathscr{A}])$$

$$V_{x,\alpha\beta} := V_{x,\alpha} V_{x+\bar{\alpha},\beta} V^{\dagger}_{x+\bar{\beta},\alpha} V^{\dagger}_{x,\beta} = \exp(-ig\epsilon \mathscr{G}_{x,\alpha\beta}[\mathscr{V}])$$

(*) Several types of operators were employed to measure the magnetic current (magnetic monopole) associated with the violation of the non-Abelian Bianchi identity, as will be discussed later. $\mathcal{O} \in \{\tilde{J}, J, \tilde{K}, K\}$

(*) On the lattice, it is possible to construct a gauge-invariant magnetic current. However, such a current does not, in general, satisfy an exact conservation law.

(*) To investigate the differences among the operators, histograms of the magnetic current magnitude were constructed.

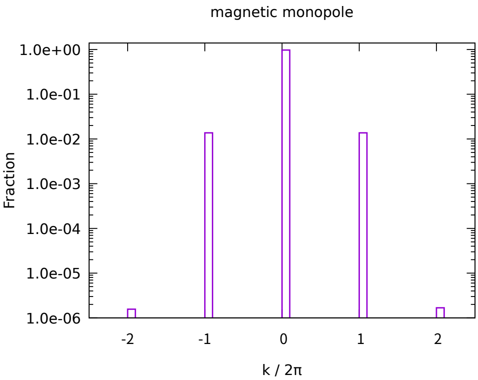
(*) The magnetic current density was measured using each operator:

$$\rho = \frac{1}{N_{site}} \sum_{x} \sqrt{\sum_{\mu,A} \mathcal{O}^{A}_{x,\mu} \mathcal{O}^{A}_{x,\mu}}, \quad \mathcal{O} \in \{\tilde{J}, J, \tilde{K}, K\}$$

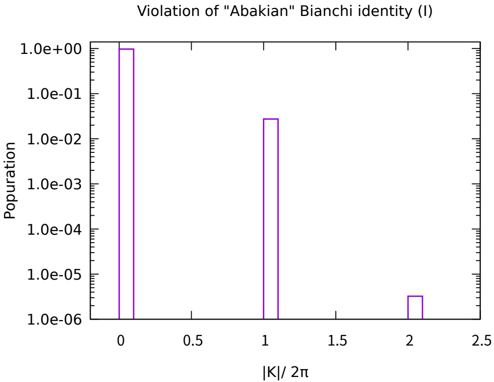
magnetic monopole [Preliminary]

$$\Theta^{\pm}_{x,\alpha\beta} := \arg \mathrm{Tr}\left((\mathbf{1} \pm \boldsymbol{n})\, V_{x,\mu\nu}\right)$$
$$k^{\pm}_{x,\mu} := \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(\Theta^{\pm}_{x+\bar{\nu},\alpha\beta} - \Theta^{\pm}_{x,\alpha\beta}\right)$$

$$\mathscr{G}_{x,\alpha\beta} := \mathrm{Log}(V_{x,\alpha\beta})\,, \quad \mathscr{G}_{x+\hat{\nu},\alpha\beta} := \mathrm{Log}(V_{x+\hat{\nu},\alpha\beta})$$
$$K_{x,\mu} := \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(V_{x,\nu}\mathscr{G}_{x+\bar{\nu},\alpha\beta}V_{x,\nu} - \mathscr{G}_{x,\alpha\beta}\right)$$



$$\rho = 0.5864133 \pm 0.00338$$



$$\rho = 0.5864133 \pm 0.00338$$

Magnetic current and violation of NABI [Preliminary]

$$\tilde{\mathscr{G}}_{x,\alpha\beta} := \frac{i}{2g\epsilon}\left(V_{x,\alpha\beta} - V^{\dagger}_{x,\alpha\beta}\right)$$

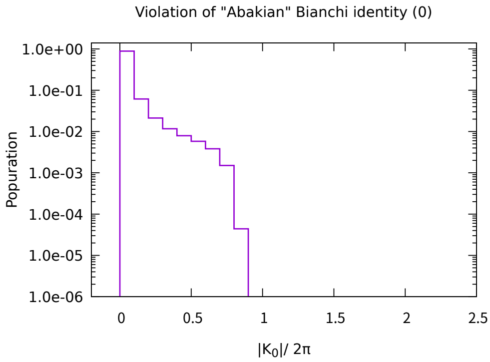
$$\tilde{\mathscr{G}}_{x+\hat{\nu},\alpha\beta} := \frac{i}{2g\epsilon}\left(V_{x+\hat{\nu},\alpha\beta} - V^{\dagger}_{x+\hat{\nu},\alpha\beta}\right),$$

$$\tilde{K}^{(0)}_{x,\mu} := \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(U_{x,\nu}\tilde{\mathscr{G}}_{x+\bar{\nu},\alpha\beta}U_{x,\nu} - \tilde{\mathscr{G}}_{x,\alpha\beta}\right)$$

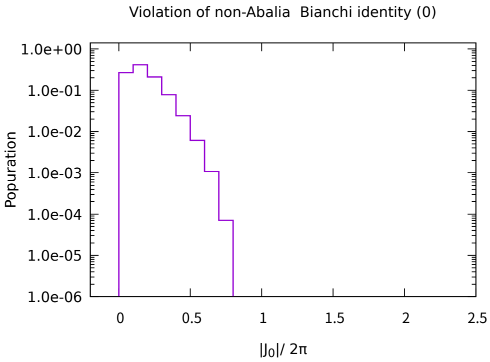
$$\tilde{\mathscr{F}}_{x,\alpha\beta} := \frac{i}{2g\epsilon}\left(U_{x,\alpha\beta} - U^{\dagger}_{x,\alpha\beta}\right)$$

$$\tilde{\mathscr{F}}_{x+\hat{\nu},\alpha\beta} := \frac{i}{2g\epsilon}\left(U_{x+\hat{\nu},\alpha\beta} - U^{\dagger}_{x+\hat{\nu},\alpha\beta}\right)$$

$$\tilde{J}^{(0)}_{x,\mu} := \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(U_{x,\nu}\tilde{\mathscr{F}}_{x+\bar{\nu},\alpha\beta}U_{x,\nu} - \tilde{\mathscr{F}}_{x,\alpha\beta}\right)$$



$$\rho = 0.7931049 \pm 0.00204$$

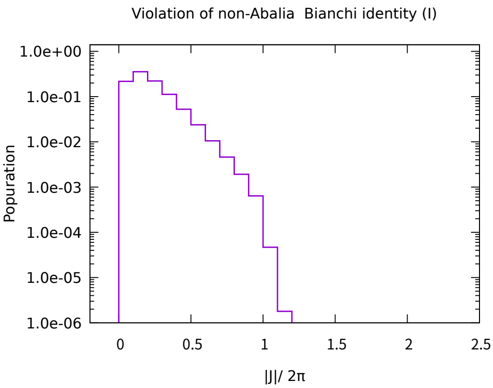


$$\rho = 2.345133 \pm 0.000698$$

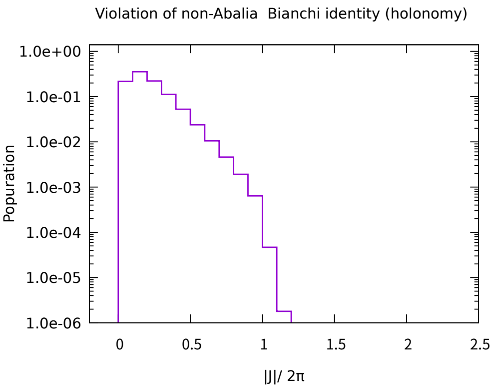
Magnetic current and vioration of NABI [Preliminary]

$$\mathscr{F}_{x,\alpha\beta} := \frac{i}{2g\epsilon}\mathrm{Log}(U_{x,\alpha\beta})$$
$$\mathscr{F}_{x+\hat{\nu},\alpha\beta} := \frac{i}{2g\epsilon}\mathrm{Log}(U_{x+\hat{\nu},\alpha\beta})$$
$$J^{(I)}_{x,\mu} := \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\left(U_{x,\nu}\mathscr{F}_{x+\bar{\nu},\alpha\beta}\,U^{\dagger}_{x,\nu} - \mathscr{F}_{x,\alpha\beta}\right)$$

$$M_{x,\nu\alpha\beta} := \mathrm{Log}\left(U_{x,\nu}U_{x+\nu,\alpha\beta}U^{\dagger}_{x,\mu}U^{\dagger}_{x,\alpha\beta}\right)$$
$$\simeq -ig\epsilon^{3}\mathcal{D}_{\nu}[\mathscr{A}]\mathscr{F}_{x,\alpha\beta}[\mathscr{A}] + o(\epsilon^{4})$$
$$J^{hc}_{x,\mu} := \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}M_{x,\nu\alpha\beta}$$



$$\rho = 2.912078 \pm 0.00113$$



$$\rho = 2,272125 \pm 0.000610$$

Summary and Outlook

Summary:

* We have drived the direct relation between the violation of the non-Abelian Bianchi identity (NABI) and the emergence of magnetic monopole through the non-Abelian Stokes theorem.
* As a consequence, emergence of magnetic monopoles relates to violation of the NABIs.
* We constructed lattice operators representing the violation of the non-Abelian Bianchi identity and measured the associated magnetic monopoles in SU(2) pure Yang-Mills theory.
* We performed the first direct measurement of the relationship between the violation of the non-Abelian Bianchi identity and the emergence of magnetic monopoles.

Outlook:

* We plan to perform more detailed measurements by applying the Gradient Flow to remove high-energy (ultraviolet) fluctuations on the lattice.