Analysis of isospin-zero two-pion scattering using rebased GEVP with physical quark masses

Clement Charles*, Christopher Kelly□ & Tianle Wang□

*University of Maryland, □Brookhaven National Lab

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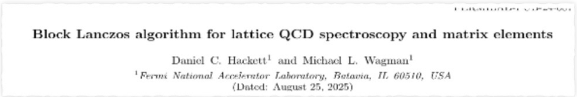
Outline

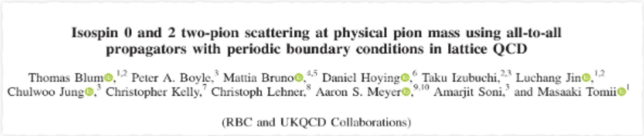
In recent years there has been much development in the field on spectral analysis for hadronic system to improve isolation of state energies.


Block Lanczos algorithm for lattice QCD spectroscopy and matrix elements

Daniel C. Hackett[1] and Michael L. Wagman[1]

[1] *Fermi National Accelerator Laboratory, Batavia, IL 60510, USA*

(Dated: August 25, 2025)

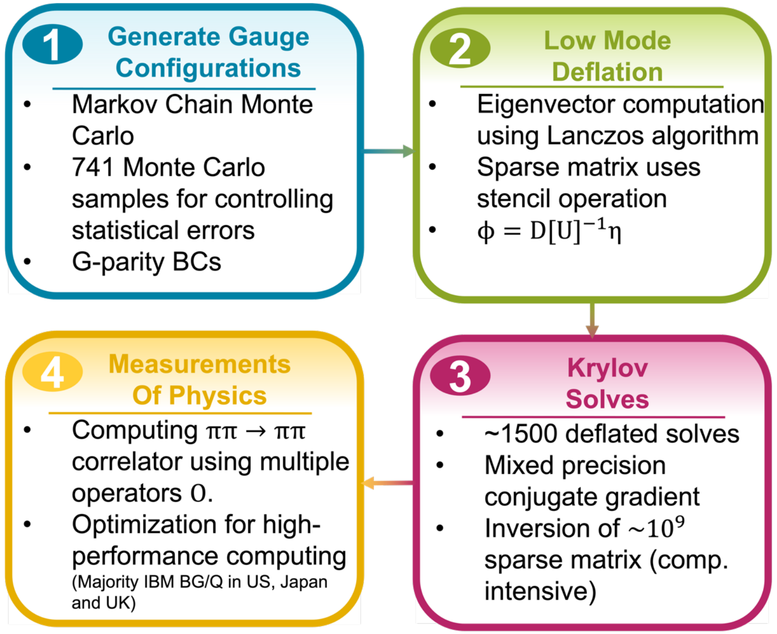

Isospin 0 and 2 two-pion scattering at physical pion mass using all-to-all propagators with periodic boundary conditions in lattice QCD

Thomas Blum,[1,2] Peter A. Boyle,[3] Mattia Bruno,[4,5] Daniel Hoying,[6] Taku Izubuchi,[2,3] Luchang Jin,[1,2] Chulwoo Jung,[3] Christopher Kelly,[7] Christoph Lehner,[8] Aaron S. Meyer,[9,10] Amarjit Soni,[3] and Masaaki Tomii[1]

(RBC and UKQCD Collaborations)

Introduced rebased technique

Reanalysis of 2020 data from RBC and UKQCD collaboration on 2 pion scattering at physical quark mass (I=0).



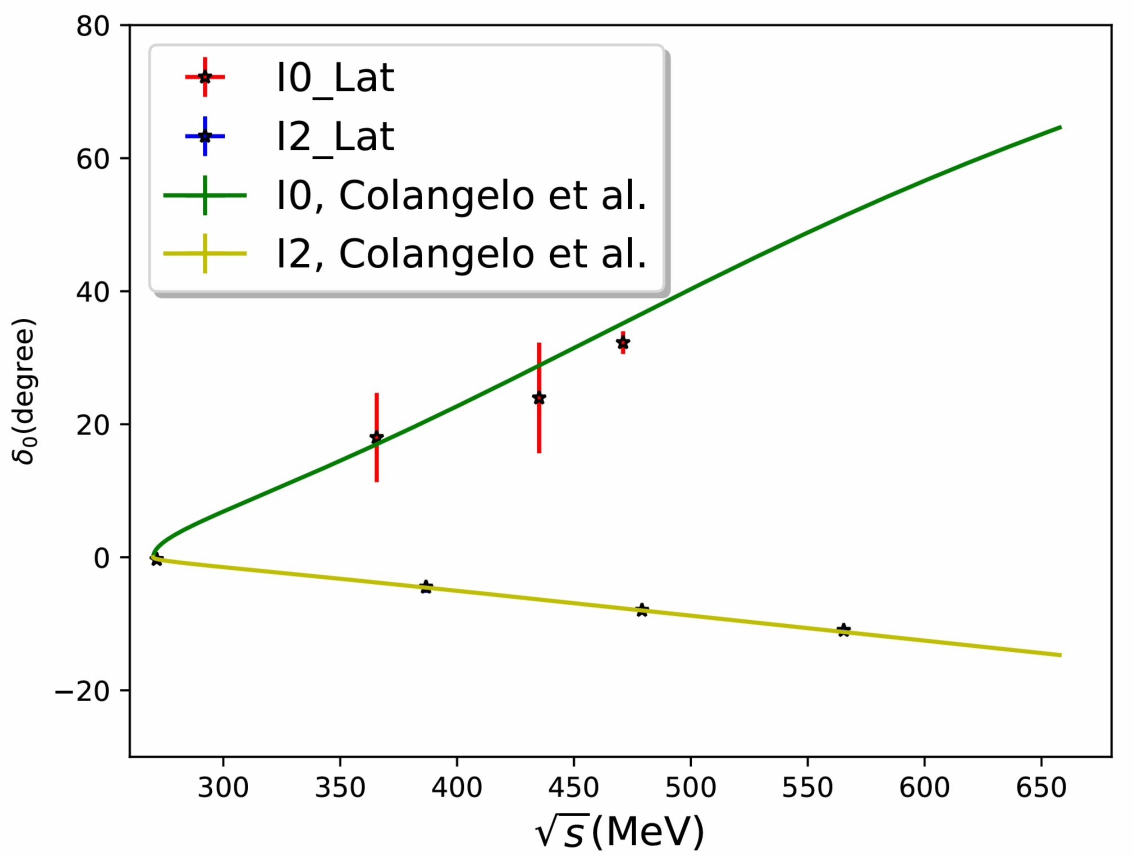
Why do we care about ππ → ππ

Important ingredient for process such as K → ππ decay (particularly isospin 0 and 2).

The K → ππ decay process itself is used to measure the **direct CP-violation** of the standard model.

$$\epsilon' = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}\, A_2}{\mathrm{Re}\, A_0} \left(\frac{\mathrm{Im}\, A_2}{\mathrm{Re}\, A_2} - \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \right)$$

Where A_2 and A_0 are the Kaon decay amplitudes for isospin 2 and 0 channel respectively, and δ_2 and δ_0 are the scattering phase shifts.



[arXiv:2103.15131]

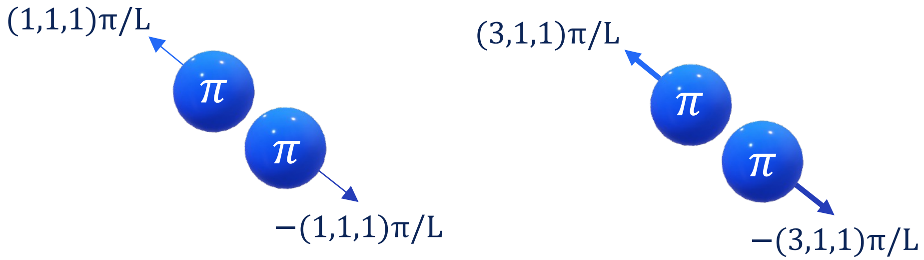
I=0 ππ → ππ

We focus here on the **isospin 0 channel** which is particularly challenging due to:

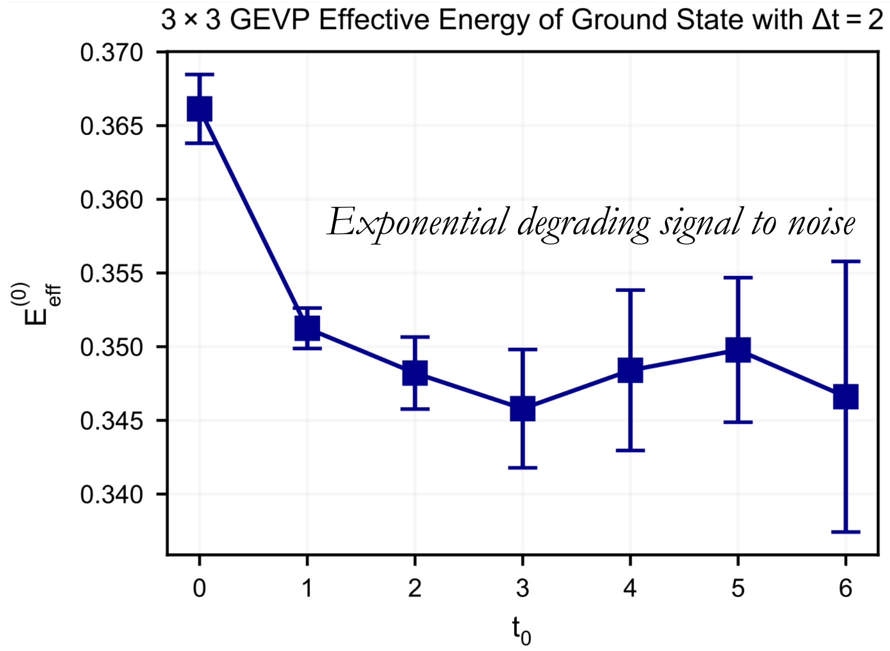
- The presence of disconnected diagrams
- Signal to noise issue for long times

This makes it a good testbed for comparing different multi-operator approaches. Here we have 3 operators:

$\pi\pi$-(1,1,1), $\pi\pi$ -(3,1,1) and σ (has same quantum numbers as two pions)



[C. Kelly Inspired]



We aim to solve the window problem that minimizes excited state contamination and also reduces statistical errors.

- Fast convergence (to plateau region)
- Slower growth in statistical errors

Generalized Eigenvalue Problem (GEVP)

Variational techniques are one possible way of determining the energy levels in lattice gauge theory. It utilizes the fact that Euclidean correlation functions are sums of exponential factors ($t \ll L_t/2$):

$$C_{ij}(t) = \sum_n A_{n,i} A^*_{n,j} e^{-E_n t}$$

Overlap of the i-th operator on the n-th state

One such variational technique is GEVP:

$$C(t) v_n(t, t_0) = \underbrace{\lambda_n(t, t_0)}_{\text{Eigenvalues}} C(t_0) \underbrace{v_n(t, t_0)}_{\text{Eigenvectors}}$$

The effectives 2-pions energies are related to the eigenvalues by:

$$E_n^{\text{eff}}(t) = \frac{1}{a} \log\left(\frac{\lambda_n(t, t_0)}{\lambda_n(t+a, t_0)}\right) = E_n + \underbrace{O\left(e^{-\Delta E_{N+1,n} t}\right)}_{\text{Vanishes as } t \to \infty}$$

GEVP Methodology

Varying $\Delta t \equiv t - t_0$

Solve GEVP with suitable ordering of eigenvectors

Find t_{min} for each Δt

We include multiple time slices to improve error

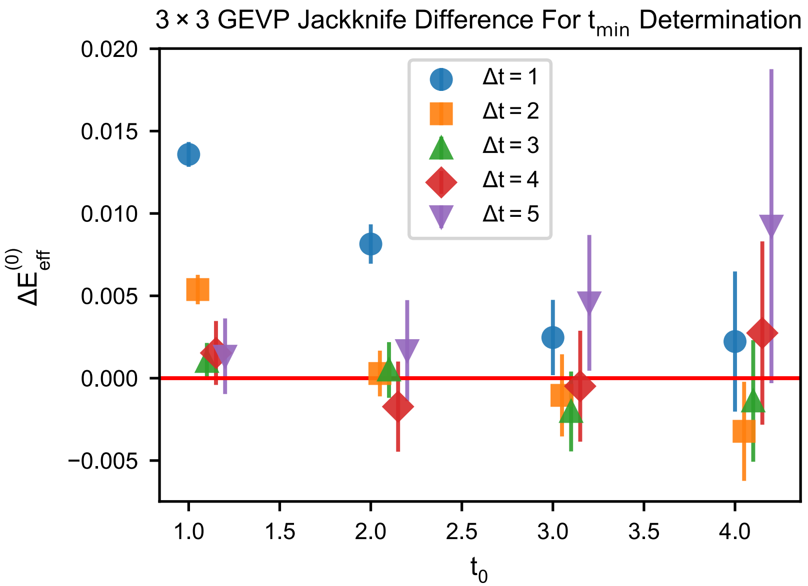
Fit over constant in range $t_{0_{\min}}$ $t_{0_{\max}}$

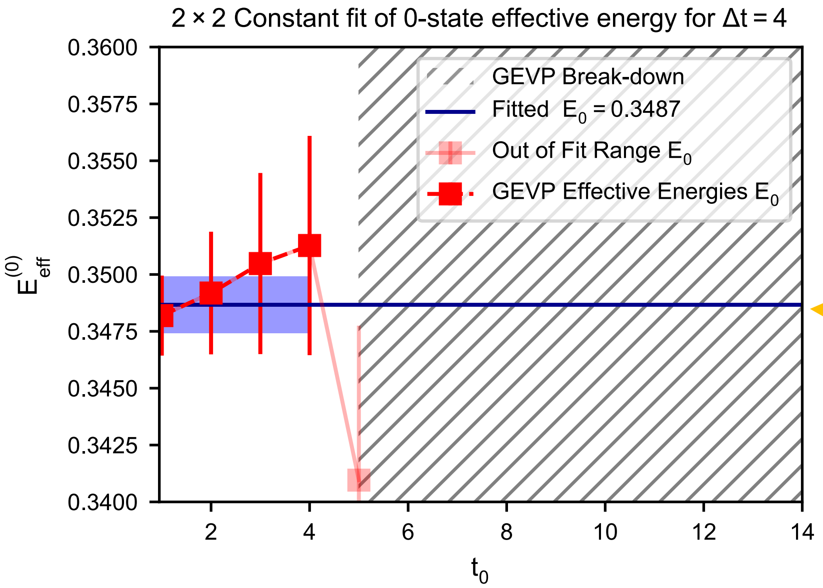
Optimize over lower statistical error and reasonable goodness of fit.

Fit effective energy over t_0 for fixed Δt

Consistency Checks

GEVP Methodology – Fit Range and Results



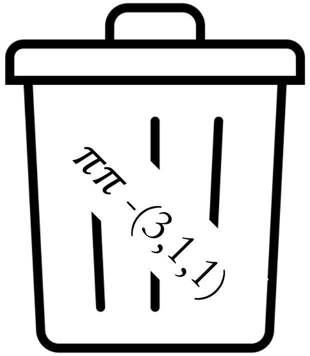


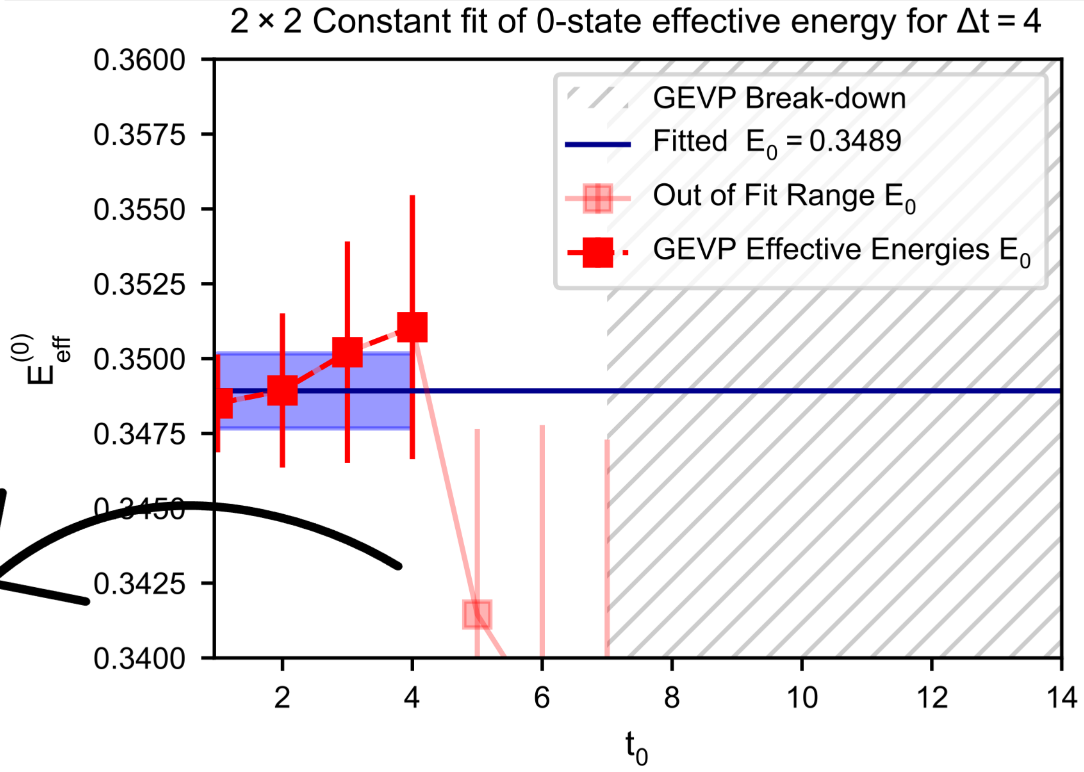
Δt	Fit Range	GS energy	Est. P-Value
1	4-5	0.3476(21)	0.3603
2	3-5	0.3479(18)	0.6087
3	2-5	0.3481(13)	0.8075
4	1-4	0.3489(12)	0.9067
5	1-3	0.3497(22)	0.8949

Fit range determined qualitatively from jackknife difference.

2×2 GEVP

What if we throw out $\pi\pi$ -(3,1,1) operator. This allows us to determine the effectiveness of rebasing (which also reduces the correlator matrix size).





Marginal effect of the stability of GEVP for the ground state. **Increases** region before GEVP break-down.

Rebased GEVP

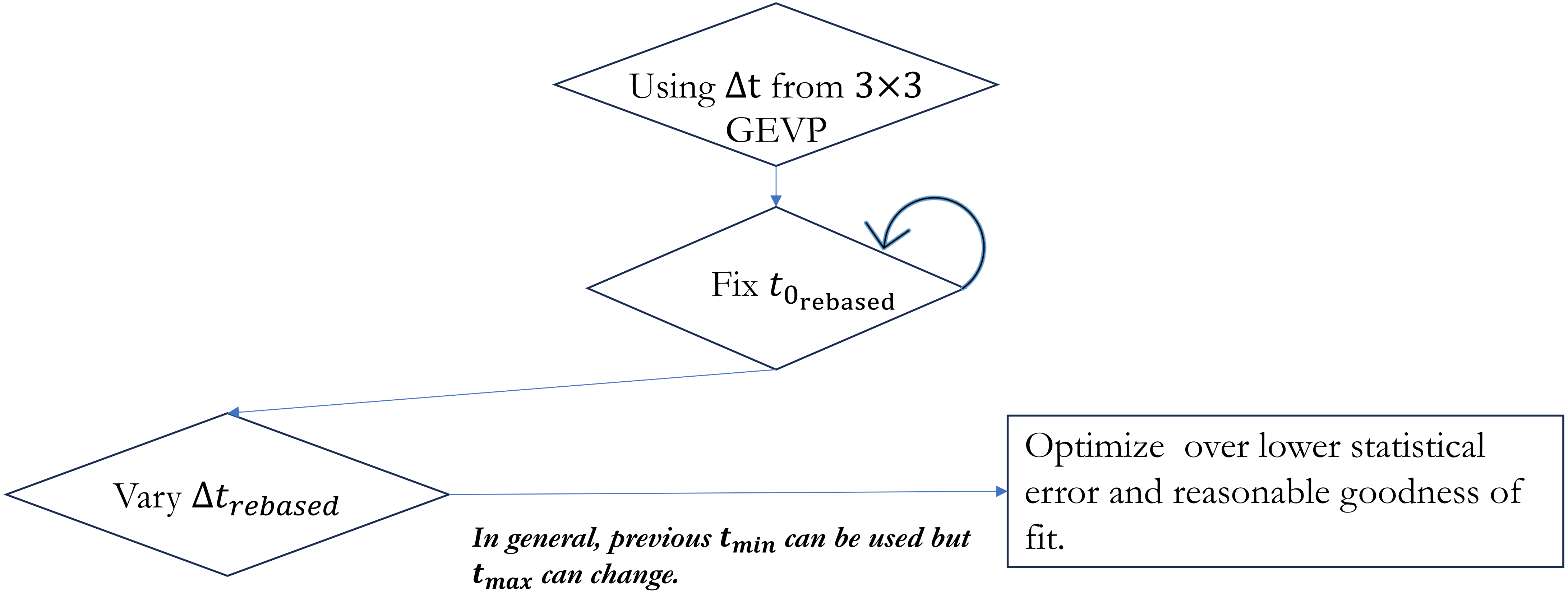
Rebased GEVP means projecting the original correlator matrix onto a smaller subspace using previously determined eigenvectors, so the remaining states can be isolated more cleanly with reduced excited-state contamination.

[PhysRevD.107.094512]

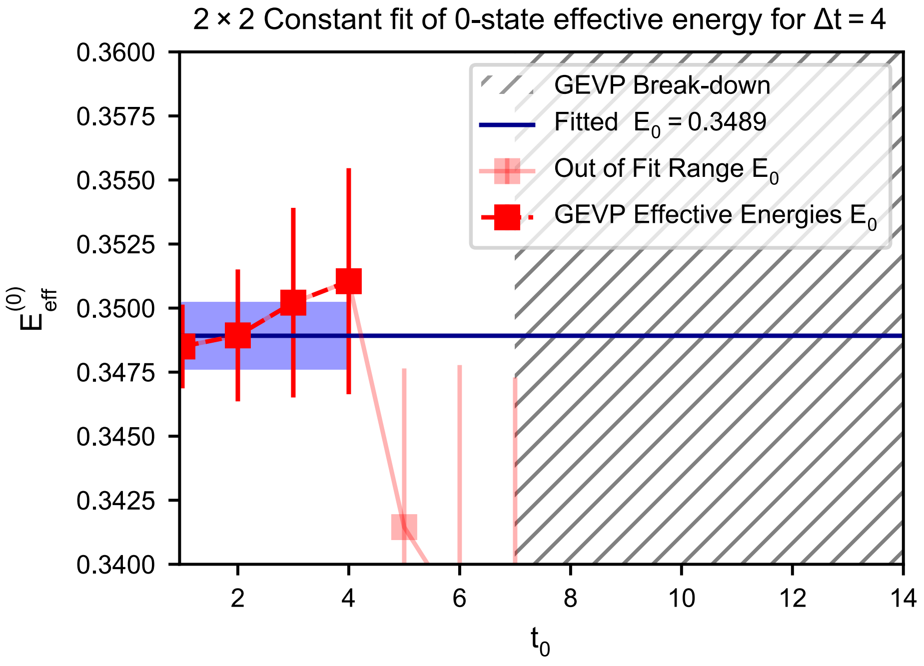
$$\boxed{V^\dagger}\ \boxed{C(t)}\ \boxed{V} = \boxed{\tilde{C}}$$

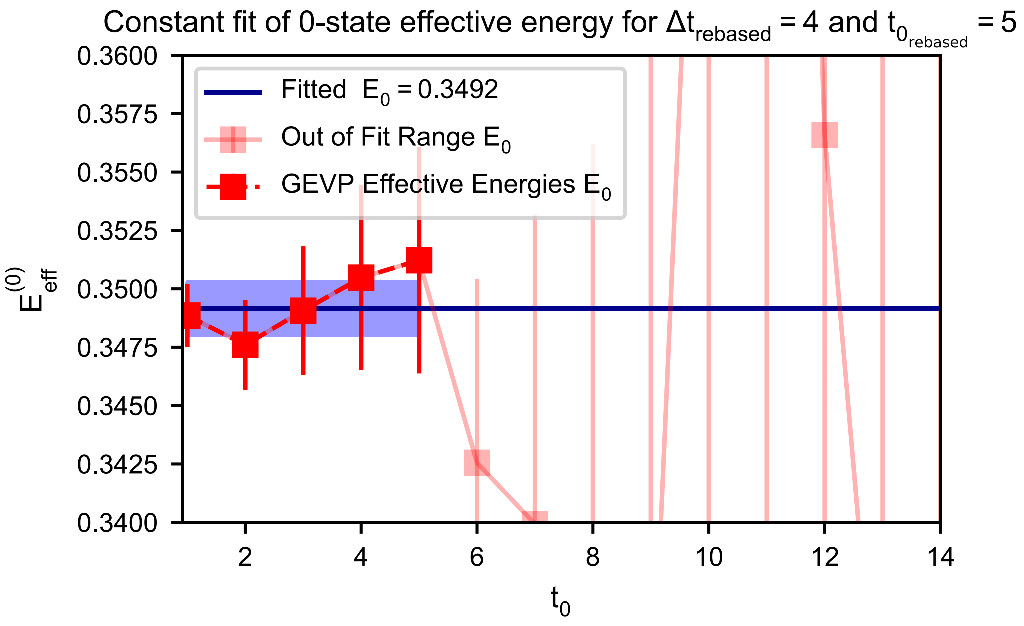
We perform rebasing using **central values eigenvectors** for all jackknife samples such that it maintains configuration independence for the new operator basis.

Method - Rebasing

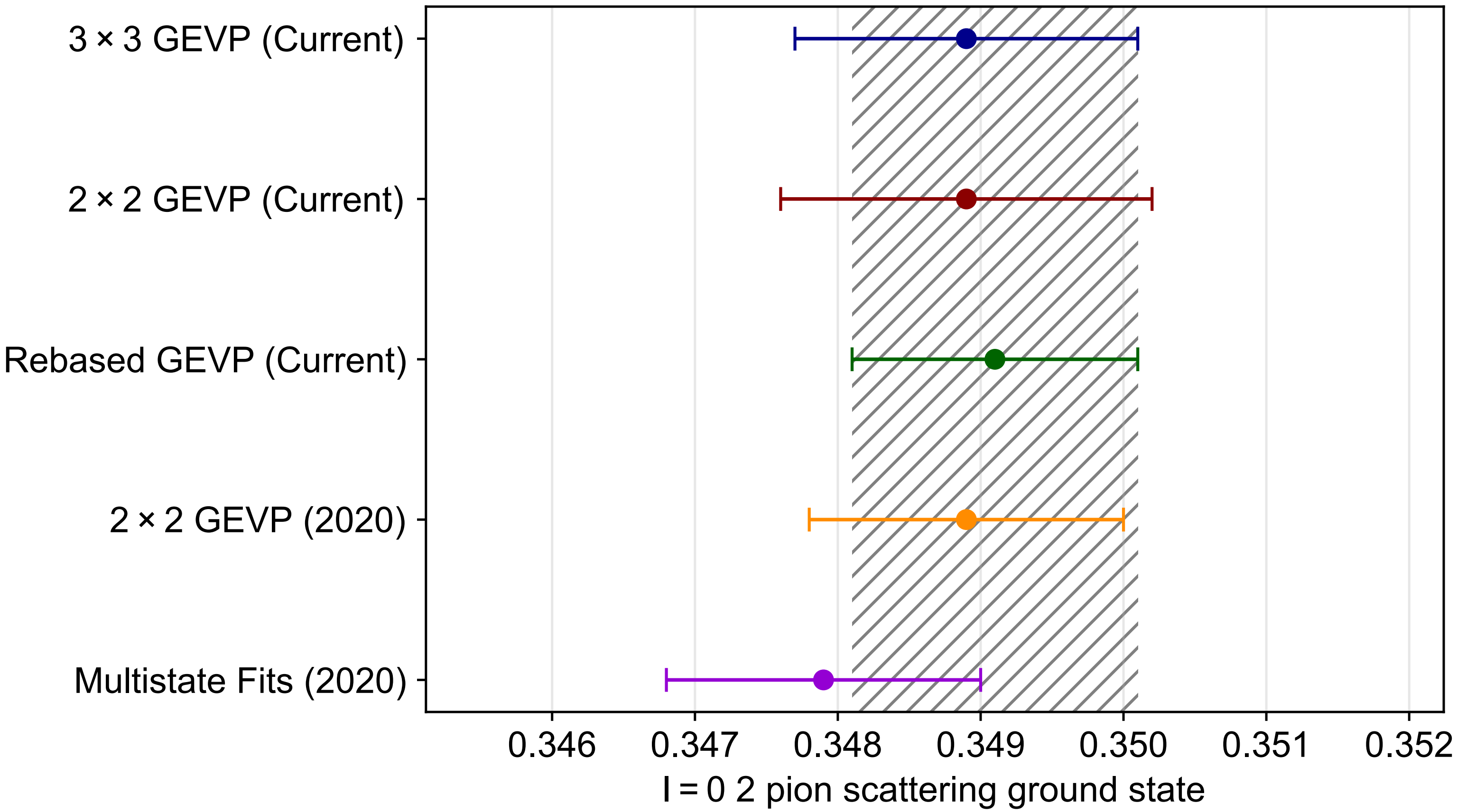


Results – Rebasing





Results – Summary



Comprehensive Python Package

This work was produced using a data analysis package (PySALaC) which is based on the C/C++ SARLaC package. **https://github.com/giltirn/PySARLaC**

Using this one can simply call a function to perform GEVP/Rebased GEVP:

```
run_GEVP(E_n = [0], rebasing = False, input_data = raw5, 2)
```

Rebased:
```
run_GEVP(E_n = [0,1], rebasing = True, input_data = raw5, 2, t0_rebased = 1, dt_rebased = 1)
```

Next Steps

In the next steps of this analysis, we plan to:

- Using Block Lanczos Algorithm on this dataset as studies have shown that there can be faster convergence than GEVP making comparisons and implementing various modifications to further improve on the existing algorithm.

$$C(t) \rightarrow T_{ij}$$

[arXiv:2412.04444]

- We also plan to factor in other sources of systematic error in our calculation like the 2020 analysis.

- Using test-data we plan to further study the effect of rebasing to determine when such techniques are most suitable.

- Future developments may also include developing agentic workflows for tuning hyperparameters.

Conclusions

- Rebased GEVP has small but a noticeable effect on reducing the statistical errors.
- Rebased GEVP was found to extend the region before GEVP fails substantially (more than just removing the $\pi\pi$ -(3,1,1) operator).
- Increased accessible fit range (window).

Thank You!

Acknowledgements

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