Spectral reconstruction for reduced excited-state contamination in Euclidean correlators

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For the BaSc Collaboration

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Motivation

- Correlator analysis and extraction of energy levels hindered by excited-state contamination and signal degradation
- For multi-hadron systems we are interested in energy shifts, e.g. $\Delta E_{AB} = E_{AB} - E_A - E_B$

E.g. two-baryon systems

- $E_{AB} \approx O(\mathrm{GeV})$
- $\Delta E_{AB} \approx O(\mathrm{MeV})$

$\Longrightarrow$

Need enough statistical precision to resolve a **permille** correction, despite signal degradation and excited-state contamination

[*Phys.Rev.C* 113 (2026) 2, 024002]

Best options on the market (with related issues)

- Ratio fits: false plateaux (effective mass approaches asymptotic regime non-monotonically)
- Simultaneous fits: multi-exponential models, priors, often fully correlated fits not feasible

- **Is it possible to find a completely independent route to the extraction of energy shifts?**

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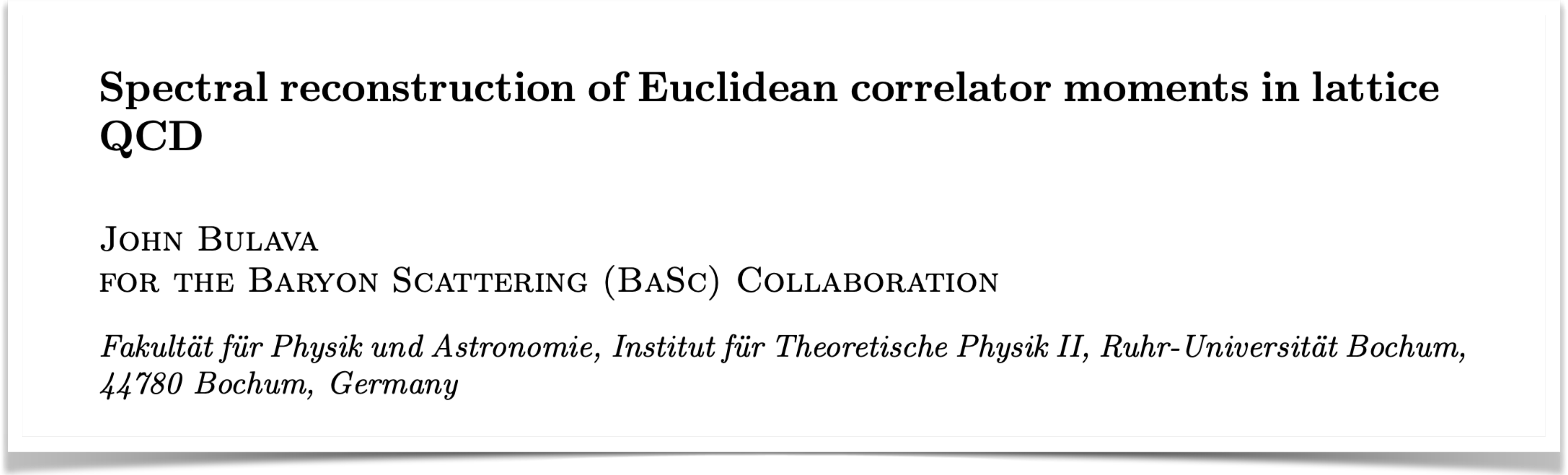
- Use spectral reconstruction as a <u>filter</u> of UV excitations to obtain a *filtered correlator* with reduced excited-state contamination

$$D(\alpha,\tau) = \int \mathrm{d}\omega\, \rho(\omega)\, e^{-\omega\tau} \kappa_\alpha(\omega)$$

$$\boxed{\rho(\omega) = \sum A_n \delta(\omega - \omega_n)}$$



Spectral reconstruction of Euclidean correlator moments in lattice QCD

John Bulava
for the Baryon Scattering (BaSc) Collaboration

Fakultät für Physik und Astronomie, Institut für Theoretische Physik II, Ruhr-Universität Bochum, 44780 Bochum, Germany

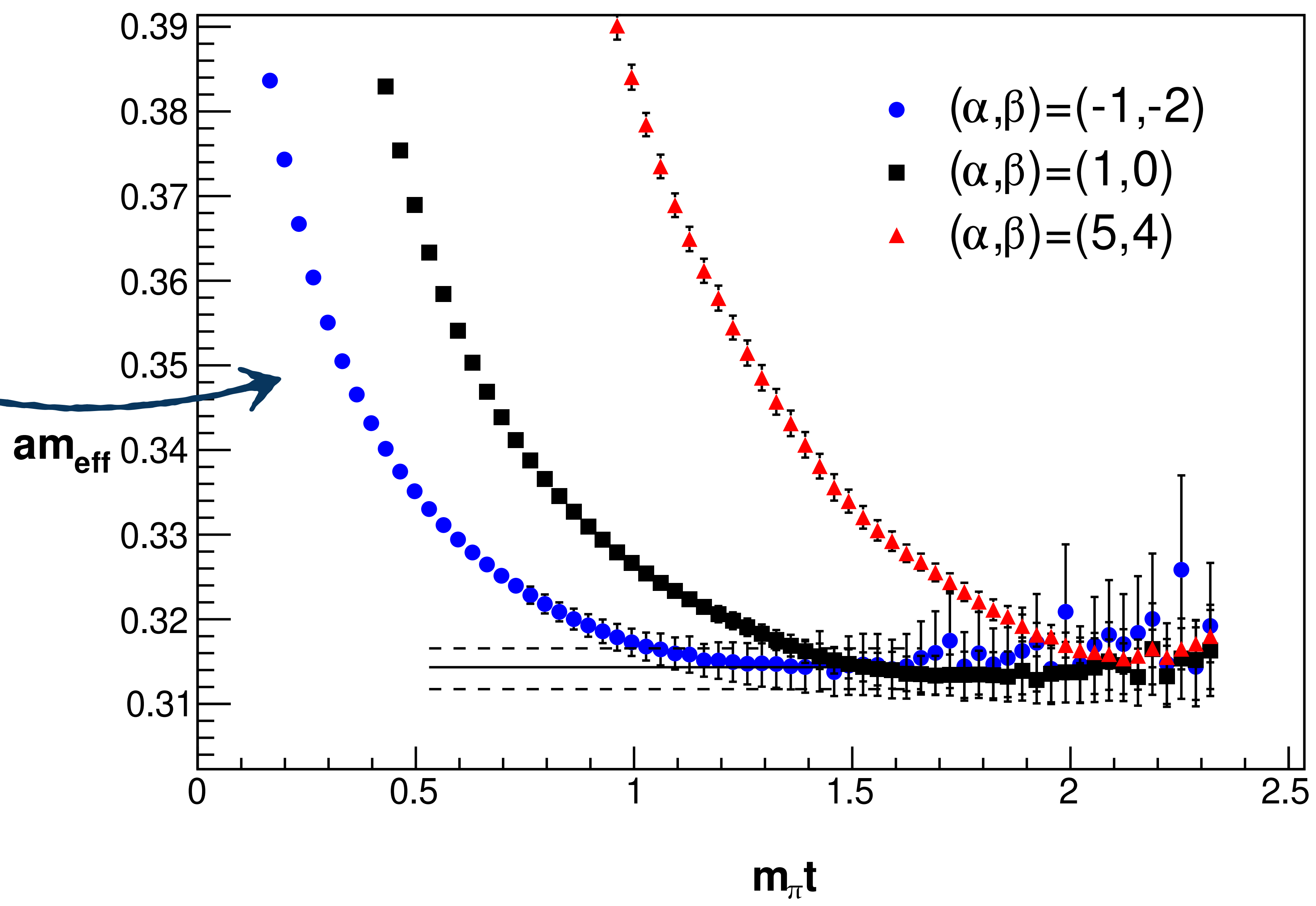
Reduced excited-state contamination is achieved if $\dfrac{\kappa_\alpha(\omega_0)}{\kappa_\alpha(\omega_1)} > 1$

Options:

$$\kappa_\alpha(\omega) = \omega^\alpha$$

$$\kappa_\alpha(\omega) = \frac{1}{1 + e^{\alpha(\omega - \bar{\omega})}}$$

[*Nuovo Cim.C* 47 (2024) 4, 199]



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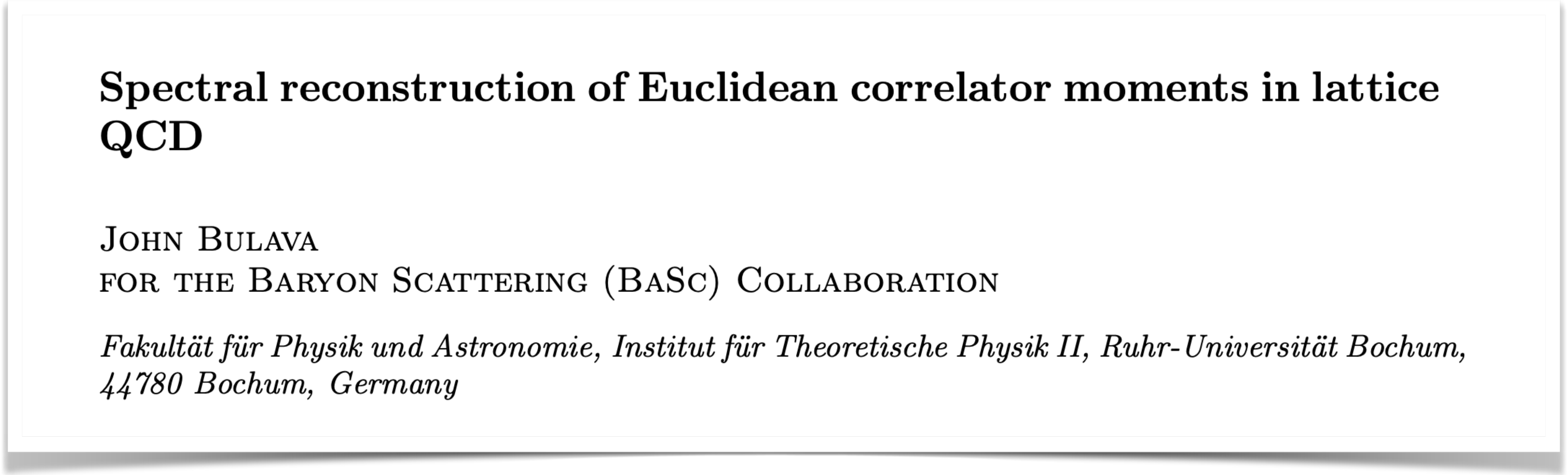
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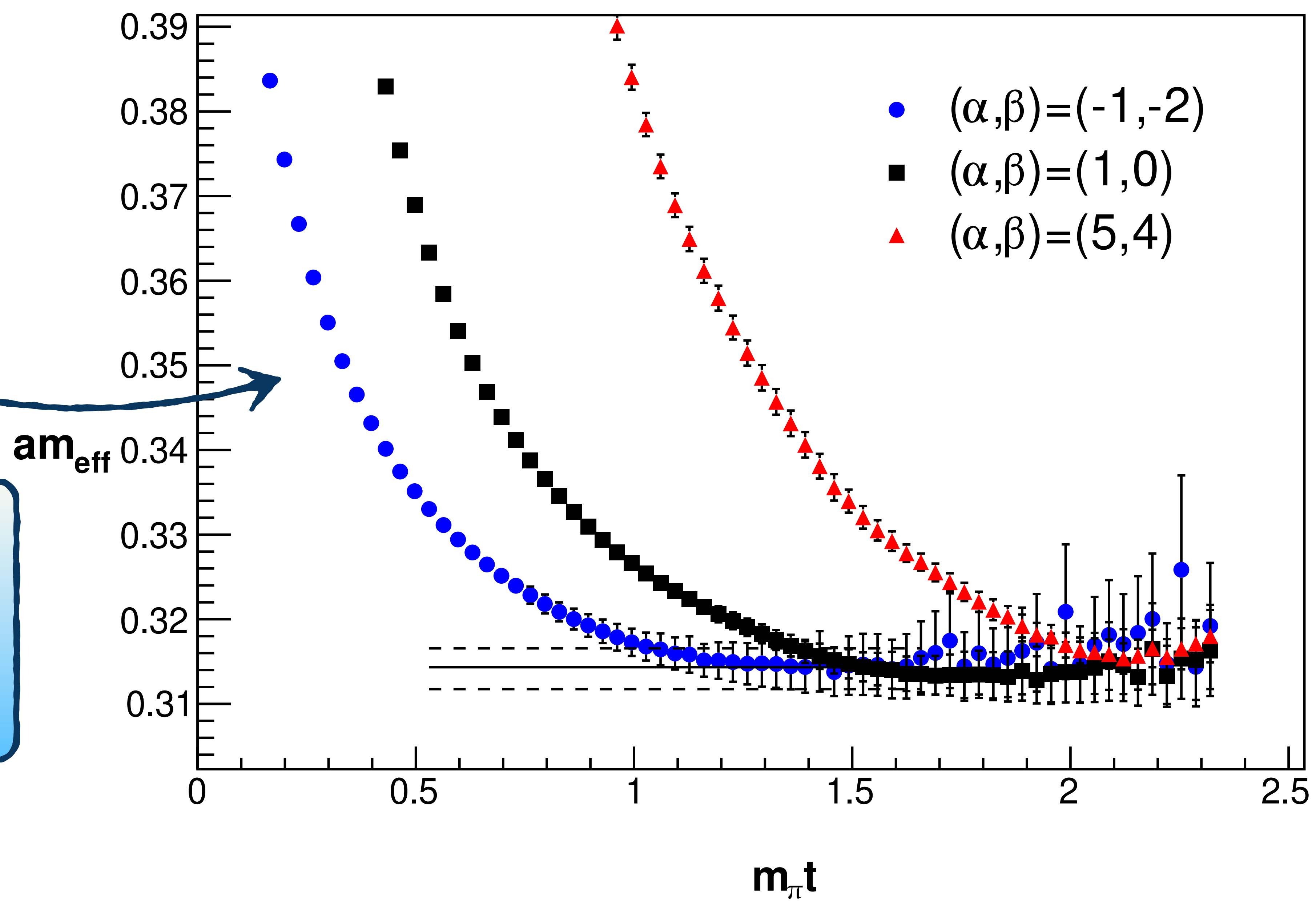
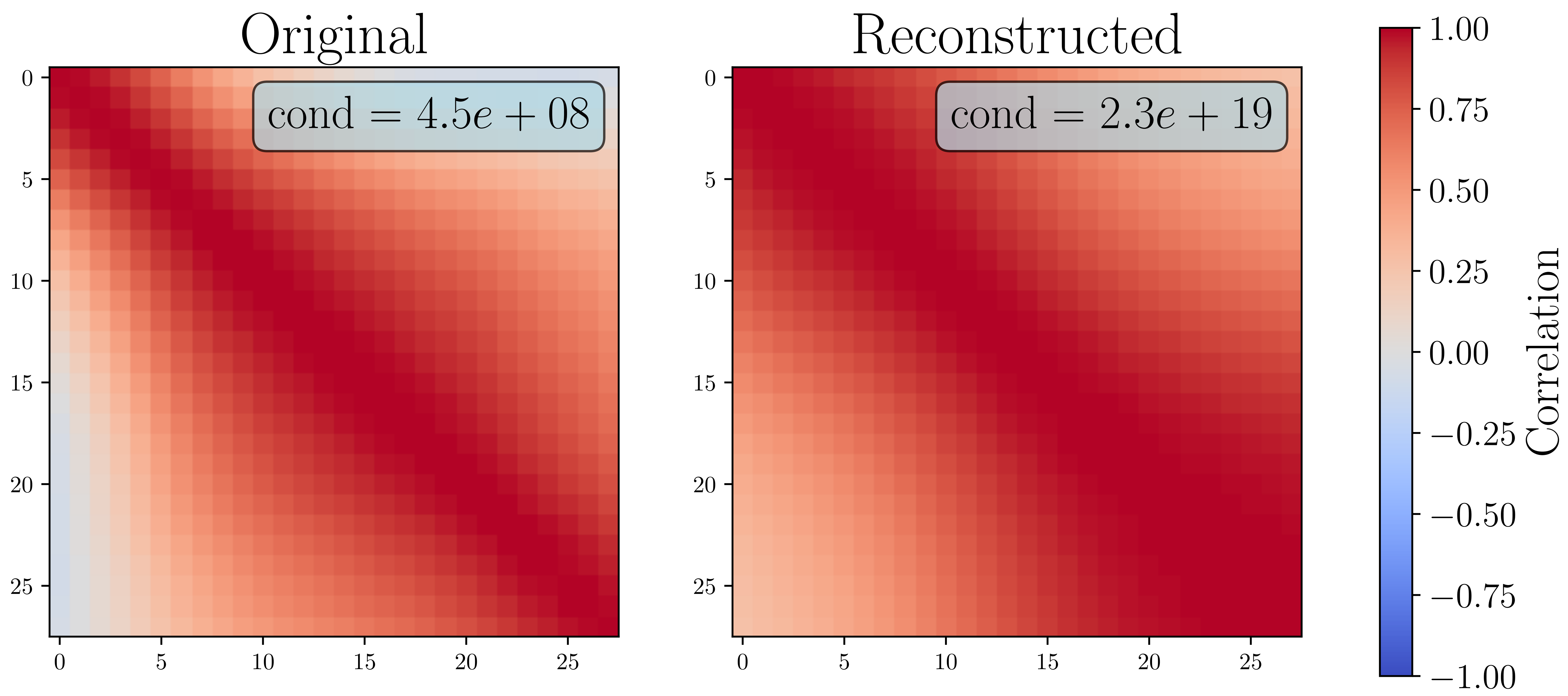


A possible idea: construct correlators with reduced excited-state contamination and fit them **simultaneously** to extract energy shifts

Filtered correlators

Simultaneous fit of reconstructed correlators could be <u>problematic</u> due to strong correlations coming from the reconstruction step



$$\kappa_\alpha(\omega) = \omega^\alpha$$

$$\alpha = -3$$

<u>We want to preserve correlations in the extraction of energy shifts</u>

Methodology

We need an alternative approach. Define the family of smearing kernels

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Leading correction:

$$\delta[\kappa_\epsilon] \approx \frac{A_1 K_\epsilon(\Delta)}{A_0 \kappa_\epsilon(0) + A_1 K'_\epsilon(\Delta)} \qquad \Delta \equiv \omega_1 - \omega_0$$

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Workflow

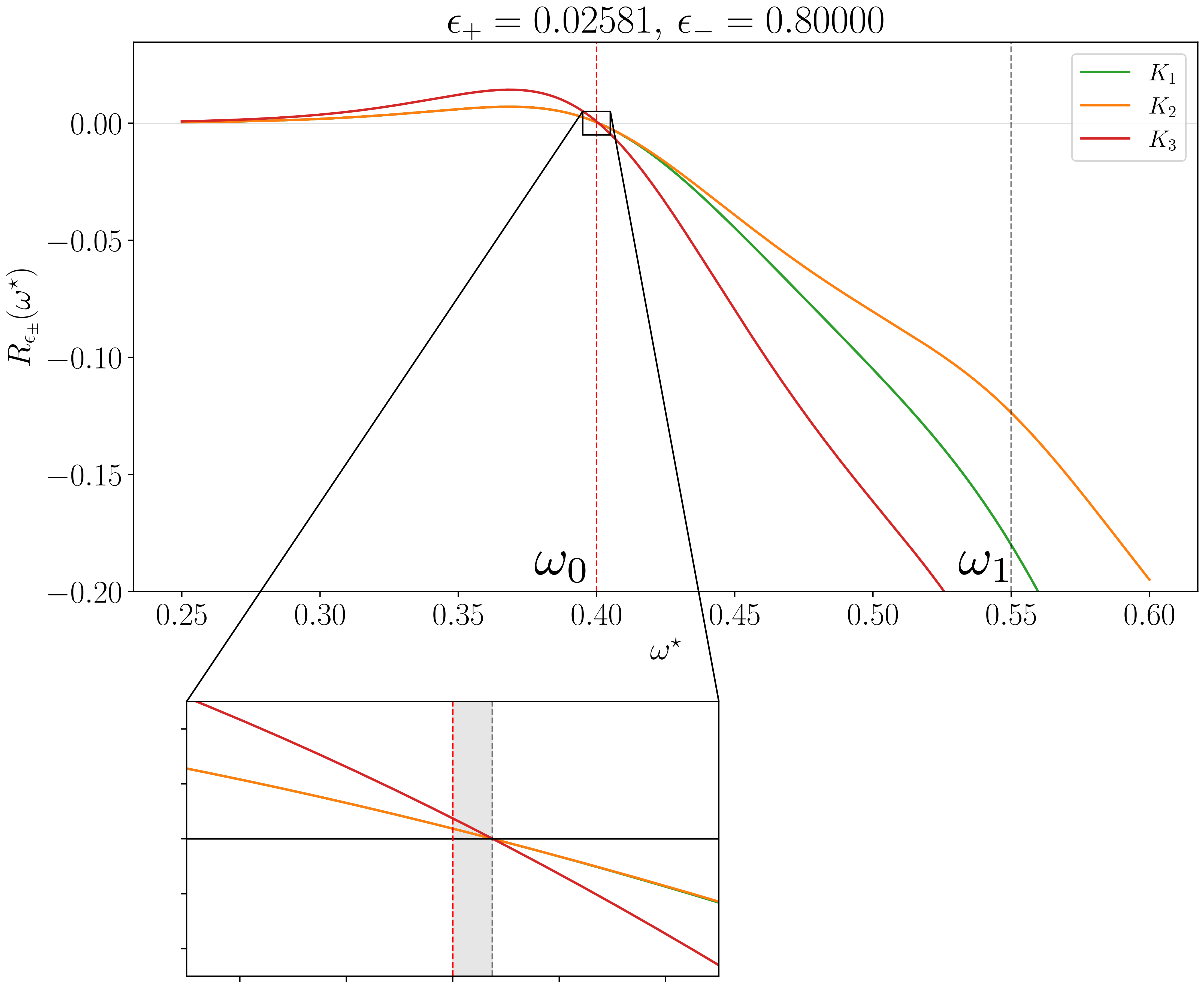
- Choose a suitable definition of $\kappa_\epsilon(\omega - \omega^\star)$ which guarantees $\delta[\kappa_\epsilon]$ to be small.
- Reconstruct R_ϵ and extract the estimator for ω_0 with a root-finding method: <u>direct output of the reconstruction</u>
- Explore different values of ϵ to ensure that the estimate is in the statistically dominated regime.

Smearing kernels

- $$\kappa_1(\omega - \omega^\star) = \frac{e^{-(\omega-\omega^\star)/\epsilon_-}}{1 + e^{(\omega-\omega^\star)/\epsilon_+}}$$

- $$\kappa_2(\omega - \omega^\star) = \frac{e^{-|\omega-\omega^\star|/\epsilon_-}}{1 + e^{(\omega-\omega^\star)/\epsilon_+}}$$

- $$\kappa_3(\omega - \omega^\star) = e^{-\alpha(\omega-\omega^\star)}\mathrm{sech}\left[\beta(\omega - \omega^\star)\right]$$



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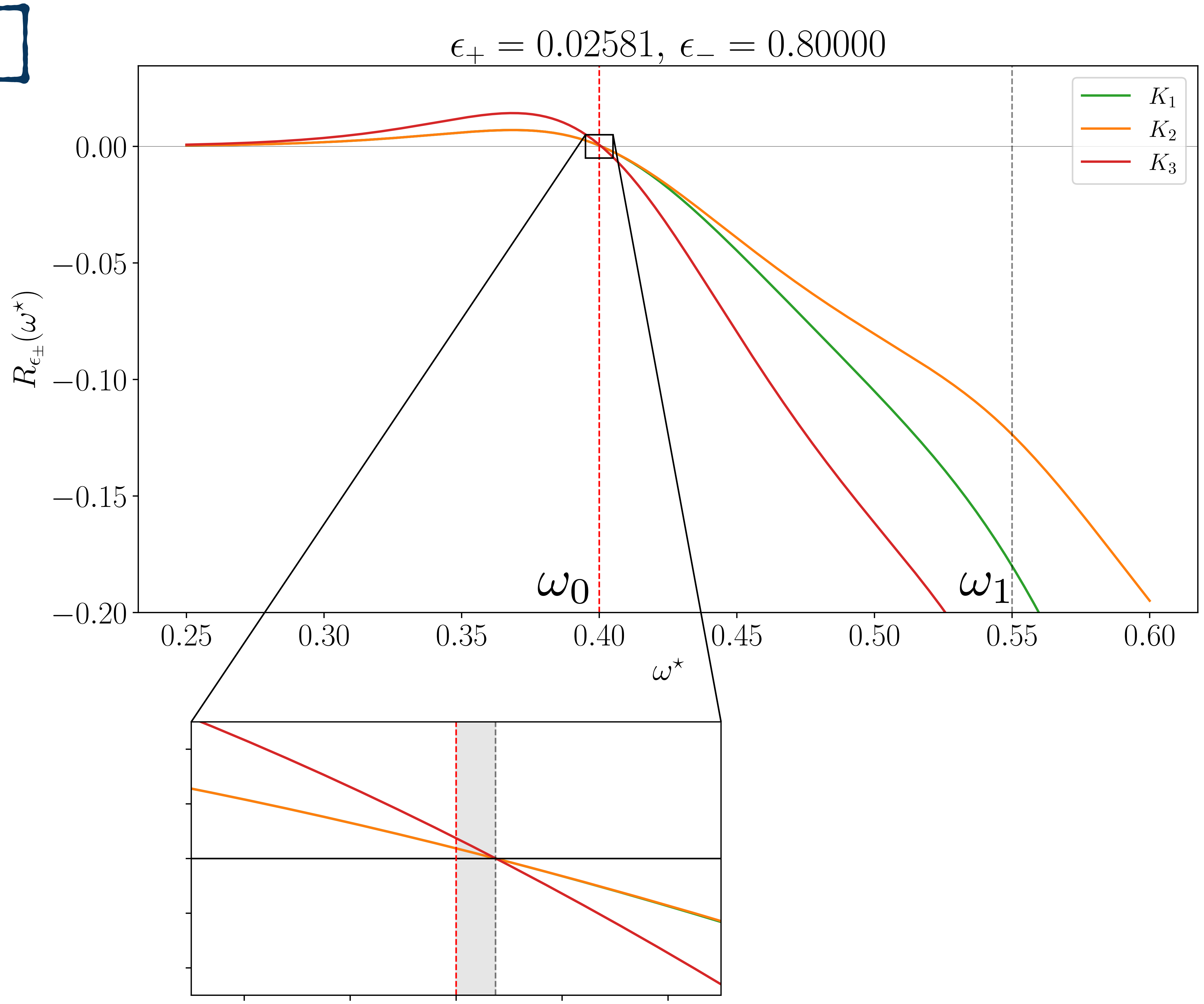

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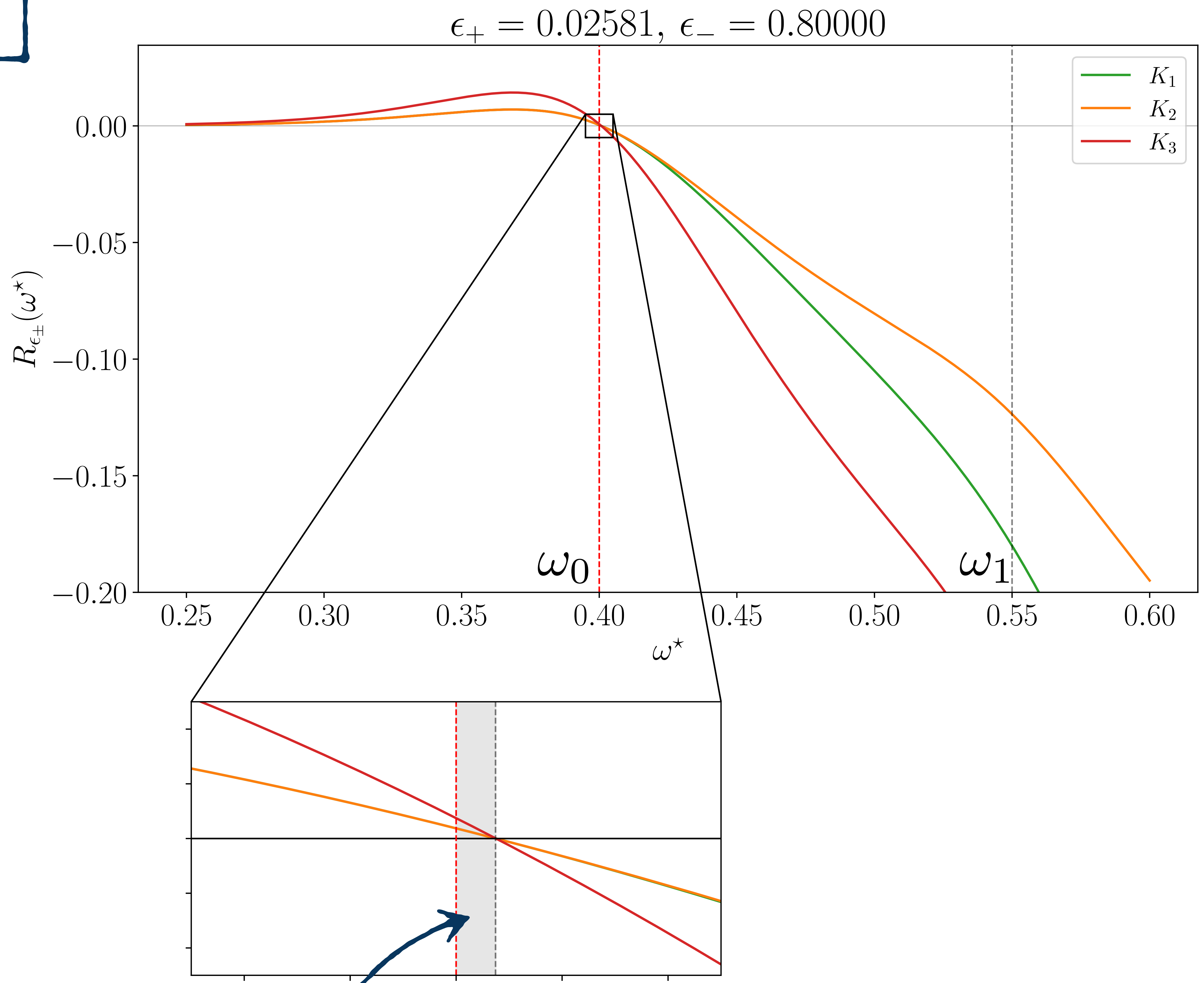
The leading correction to the root is the same if

$$\alpha = \frac{1}{2\epsilon_+} \qquad \beta = \frac{2\epsilon_+ + \epsilon_-}{2\epsilon_+\epsilon_-}$$

Control tail on the r.h.s

$$\delta[\kappa] \approx \frac{2A_1}{A_0}\Delta \exp\left[-\Delta\left(\frac{1}{\epsilon_+} + \frac{1}{\epsilon_-}\right)\right]$$



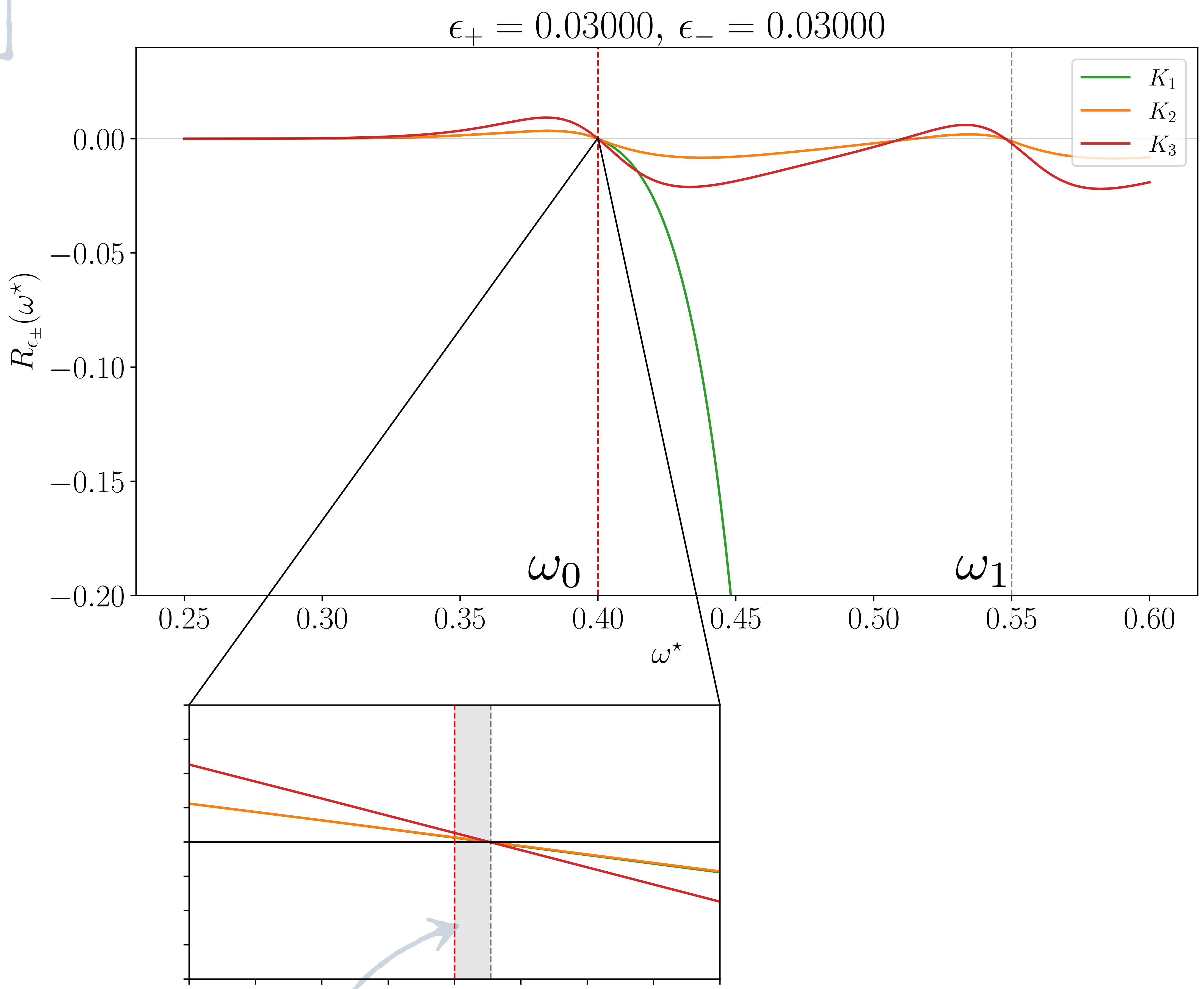
Smearing kernels

$$\kappa_1(\omega-\omega^\star)=\frac{e^{-(\omega-\omega^\star)/\epsilon_-}}{1+e^{(\omega-\omega^\star)/\epsilon_+}}$$

Control tail on the l.h.s

- Tune $\epsilon_\pm$ to reduce excited-state contamination
- Reconstruction with different kernels which provide the same excited-state contamination as a sanity check of the reconstruction.
- Using K_1 and K_2 the first excited-state is, in principle, accessible.

$$\delta[\kappa] \approx \frac{2A_1}{A_0}\Delta \exp\left[-\Delta\left(\frac{1}{\epsilon_+}+\frac{1}{\epsilon_-}\right)\right]$$



Setup of the reconstruction

- Consider an estimator for R_ϵ which is a linear combination of the input correlator

$$\hat{R}_{\epsilon_\pm}(\omega^\star) = \sum_{t_{\min}}^{t_{\max}} g_t(\omega^\star;\epsilon_\pm)C(t)$$

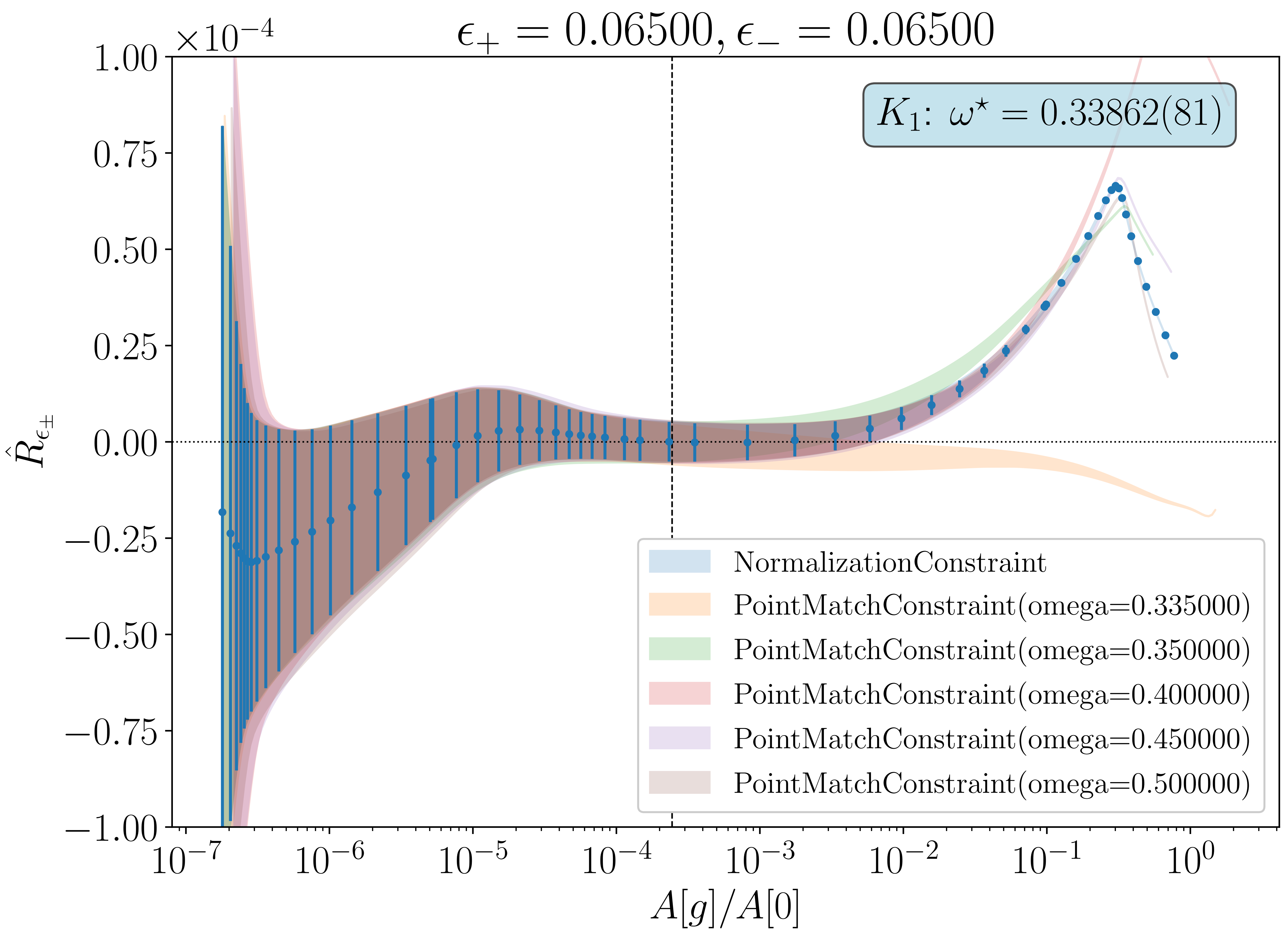
Coefficients $g_t(\omega^\star;\epsilon_\pm)$ chosen with Backus-Gilbert-like methods to balance systematic bias in the reconstruction and statistical error on $\hat{R}_{\epsilon_\pm}(\omega^\star)$.

[*Geophys.J.Int.* 16 (1968) 2, 169-205]
[*Phys.Rev.D* 99 (2019) 9, 094508]

- Validate the strategy on a **single-nucleon** correlator produced by the BaSc Collaboration on the D251 CLS ensemble $(m_\pi \approx 280\,\mathrm{MeV})$. [See M. Salg's talk (Wed), M. Lazarow's talk (Fri) for more on this ensemble]
- Main goal here: extraction of the **ground-state energy**

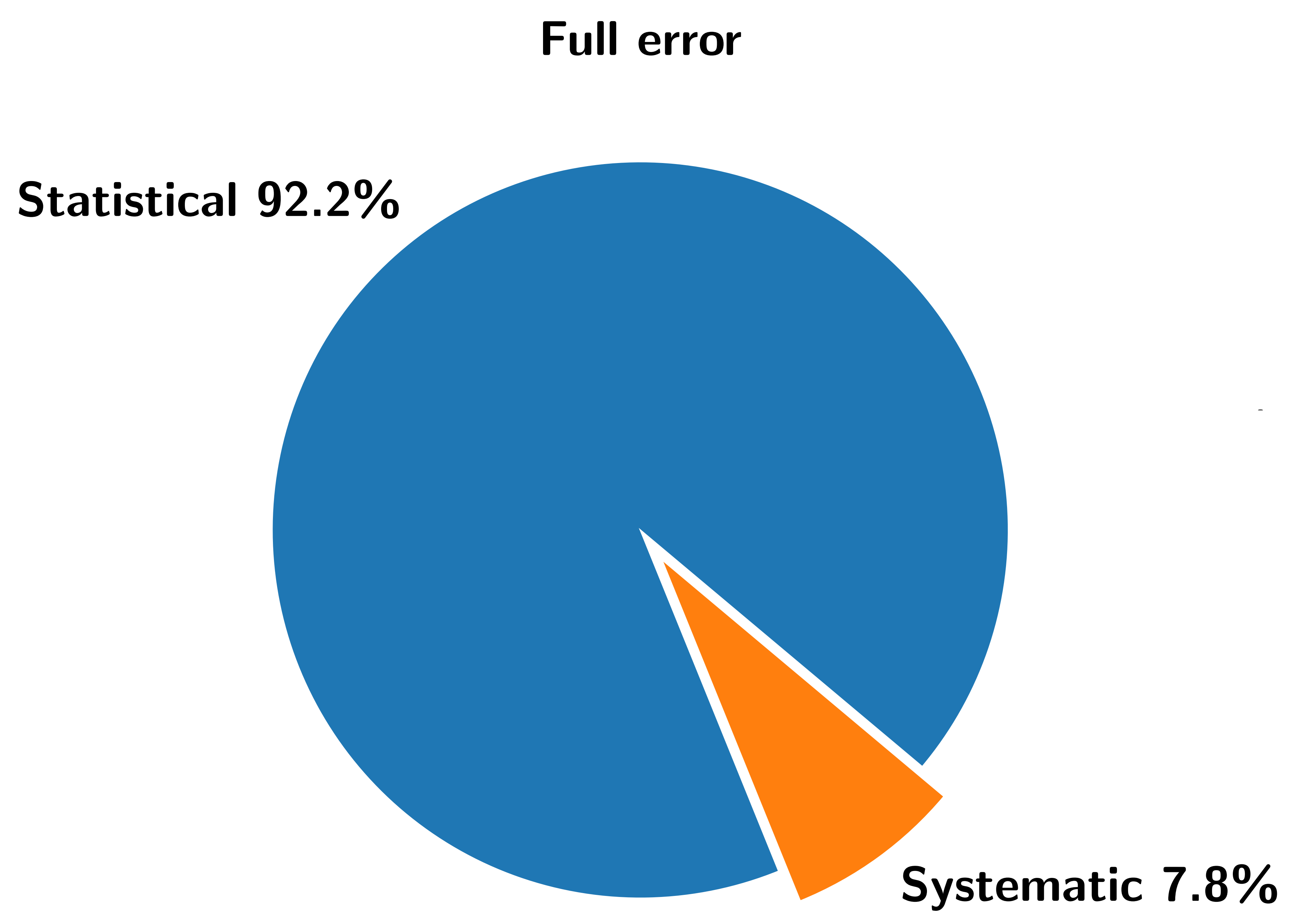
Tuning of the trade-off parameter

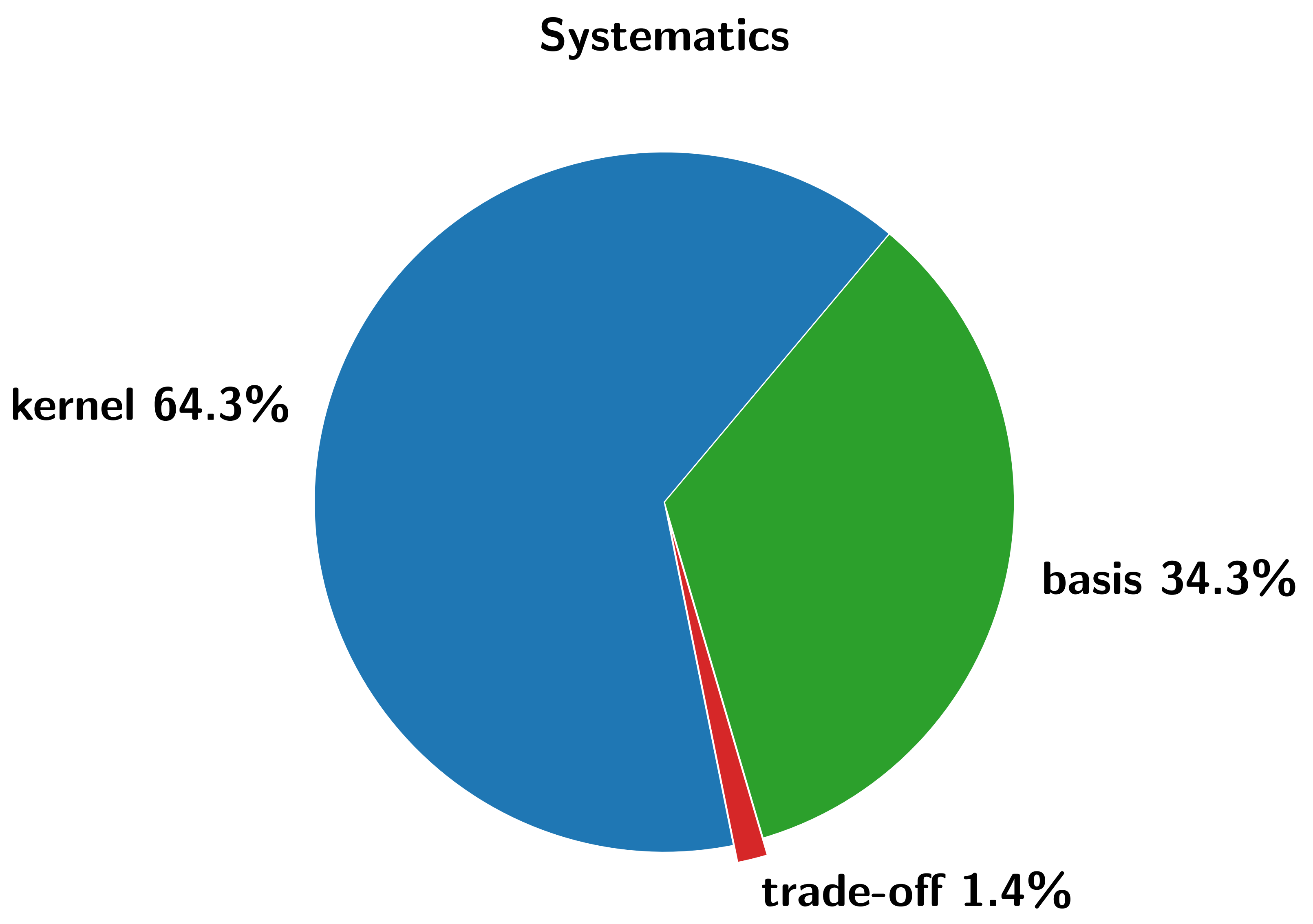
- Reconstruct $R_{\epsilon_\pm}$ with different linear constraints
- Constrain the target and the reconstructed kernel to be the same for energies:
    - On the r.h.s. tail (control over excited-state contamination)
    - On the l.h.s. tail (control over the position of the root)
- Choose λ_{opt} such that all the reconstructions agree within a given *tolerance* and they lie on a *plateau*



Error budget at fixed ϵ

$$\boxed{\epsilon \equiv (1/\epsilon_- + 1/\epsilon_+)^{-1} = 0.065}$$





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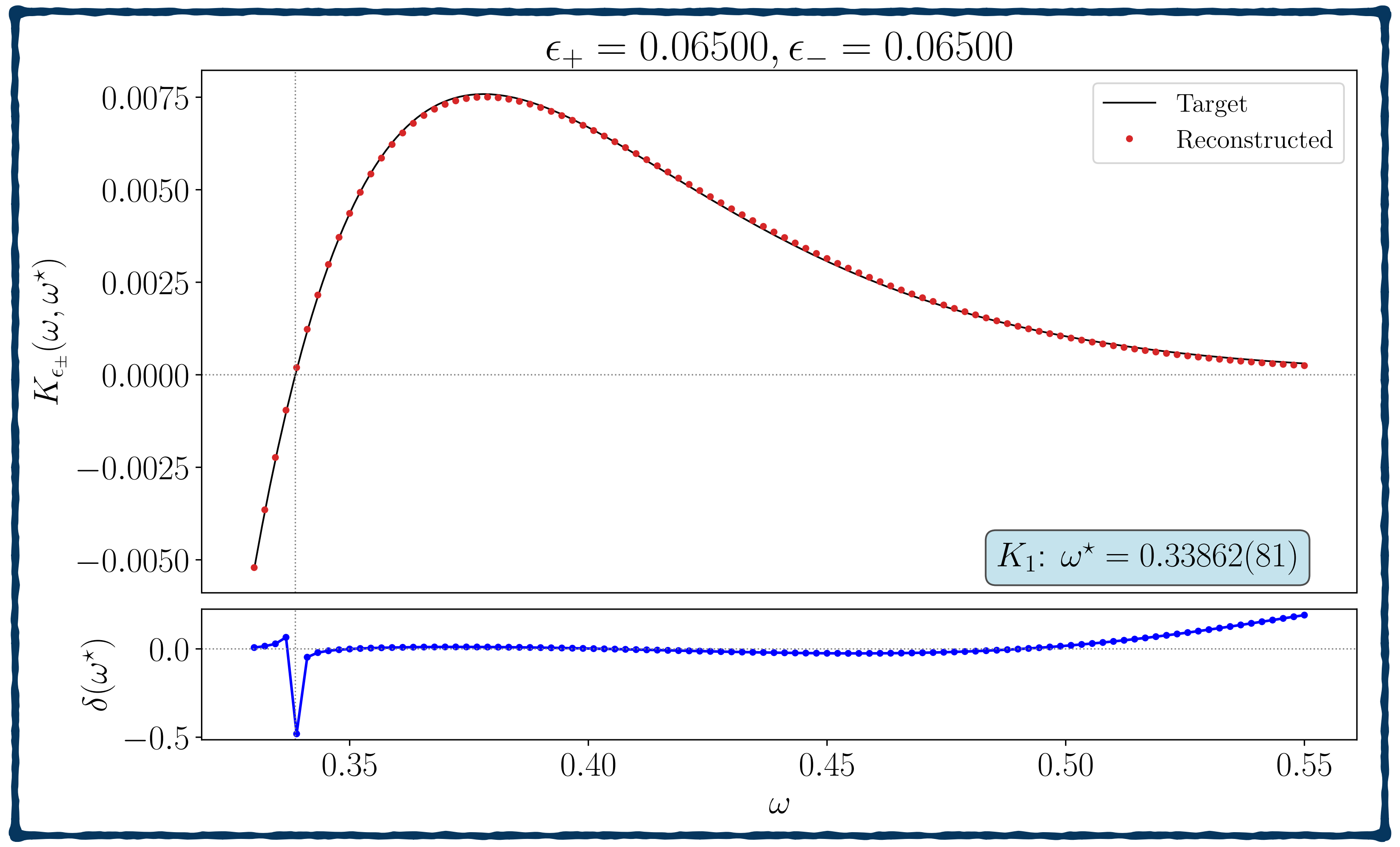
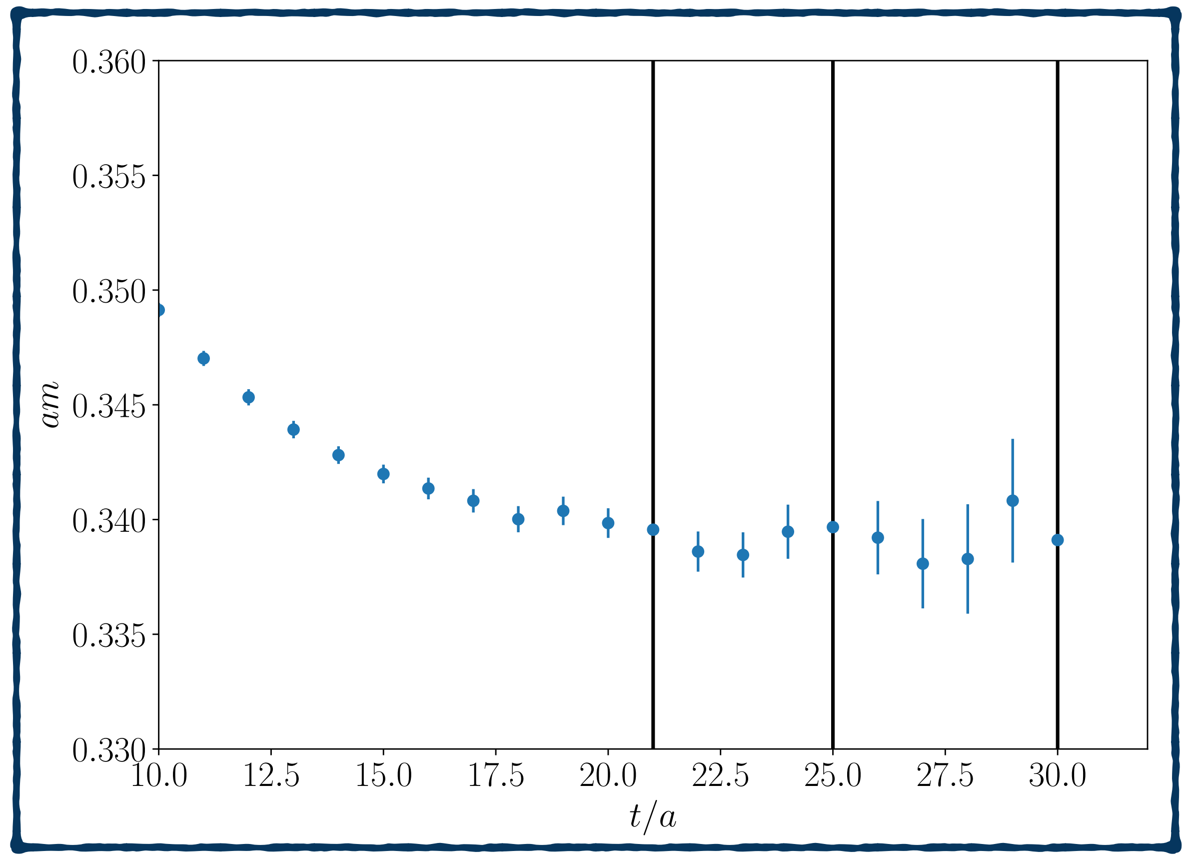
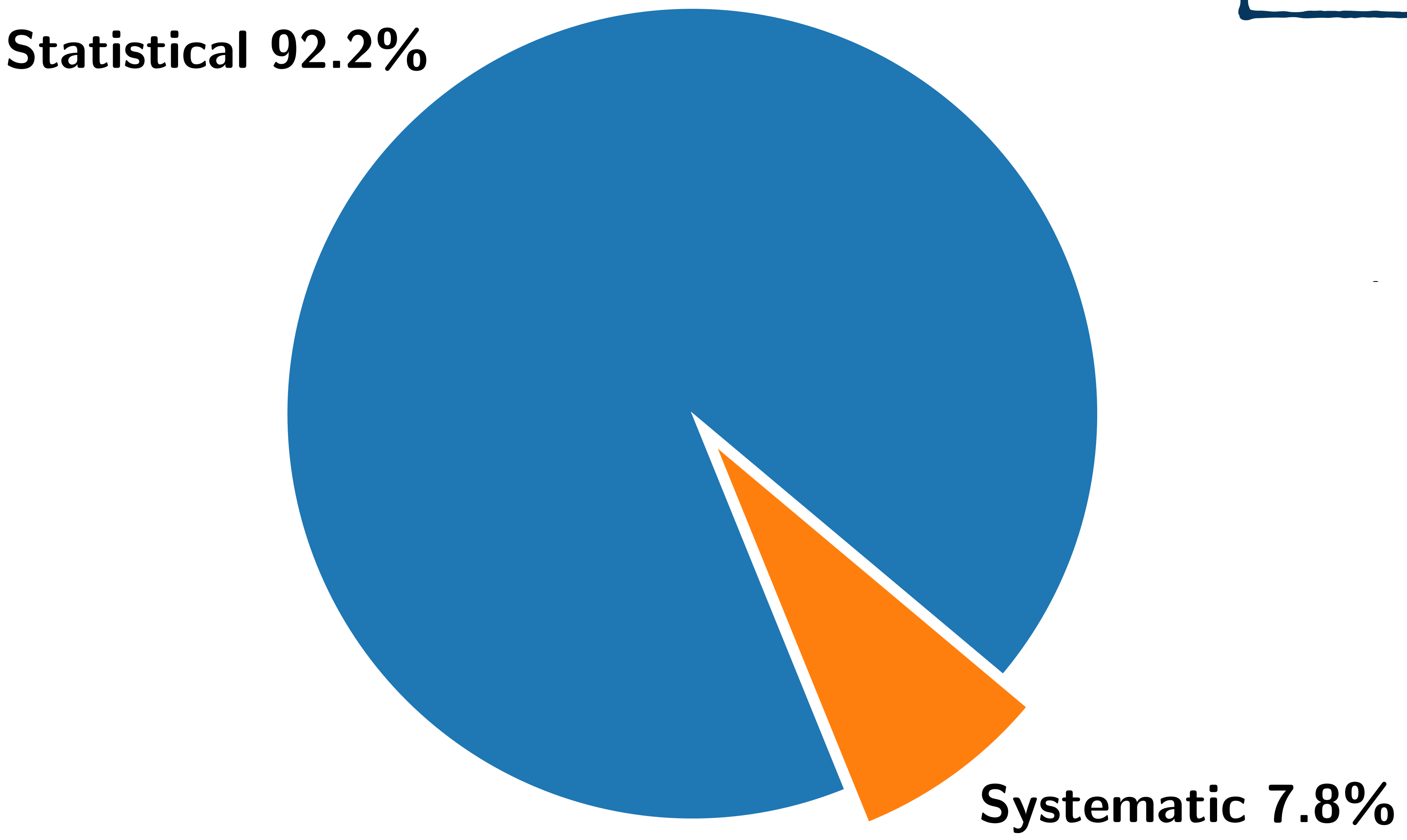
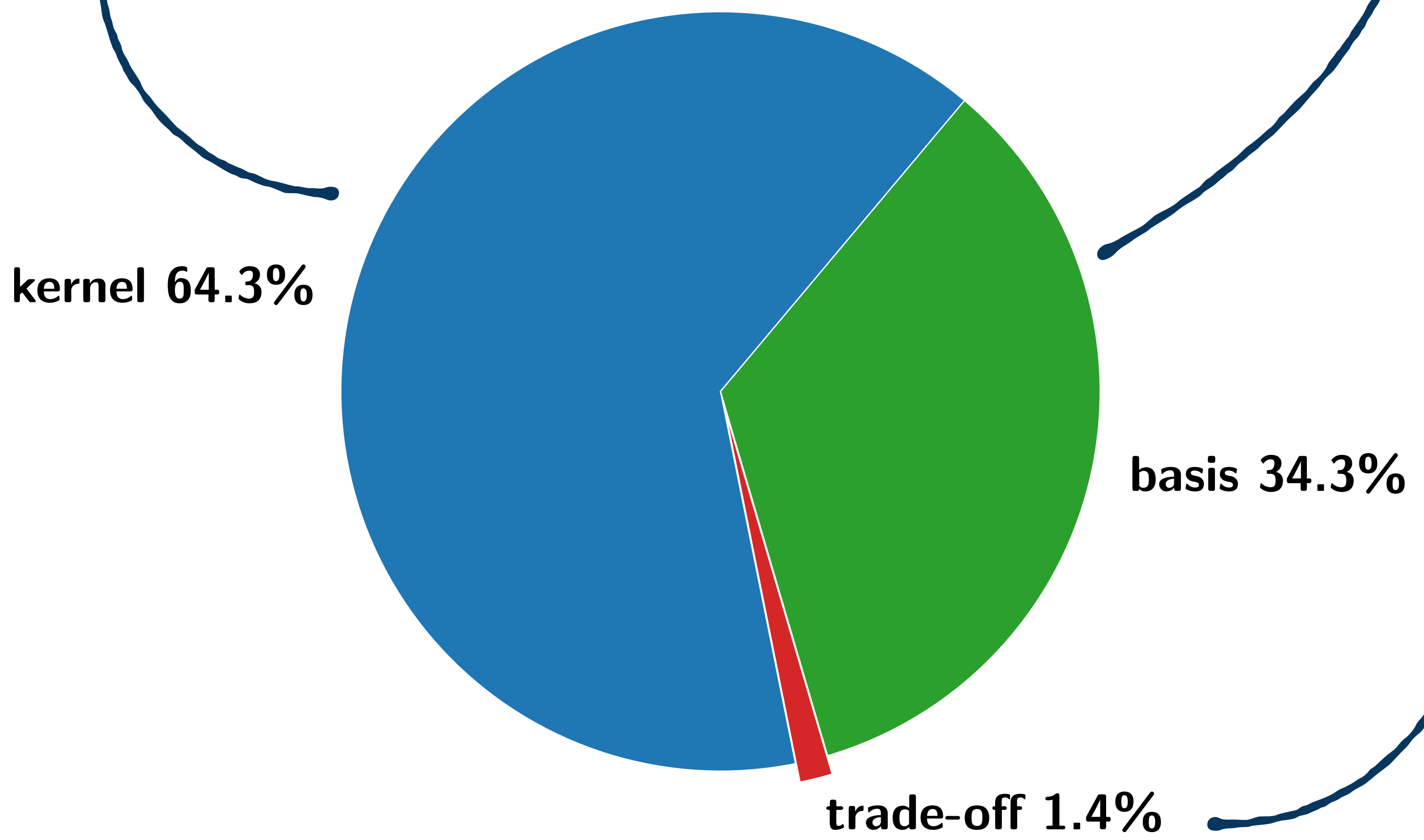


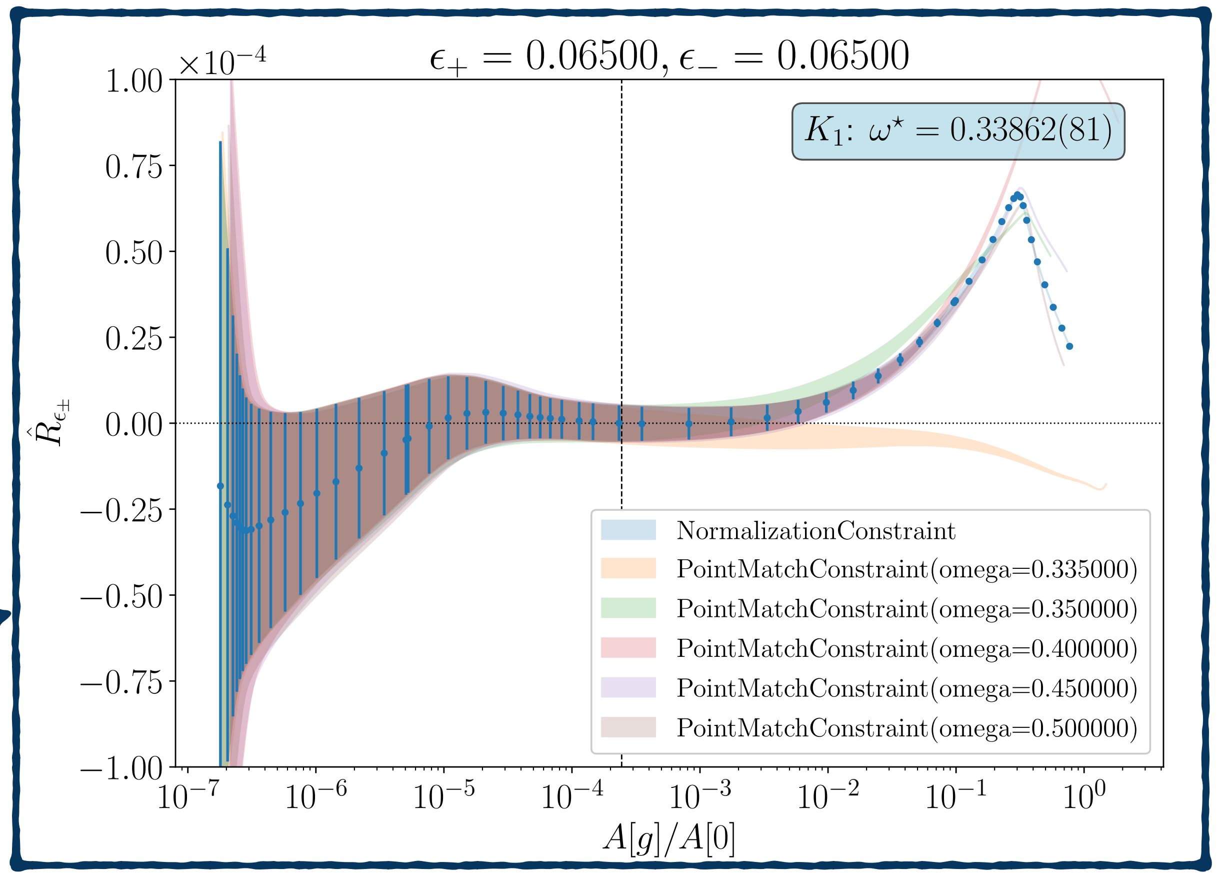


Full error



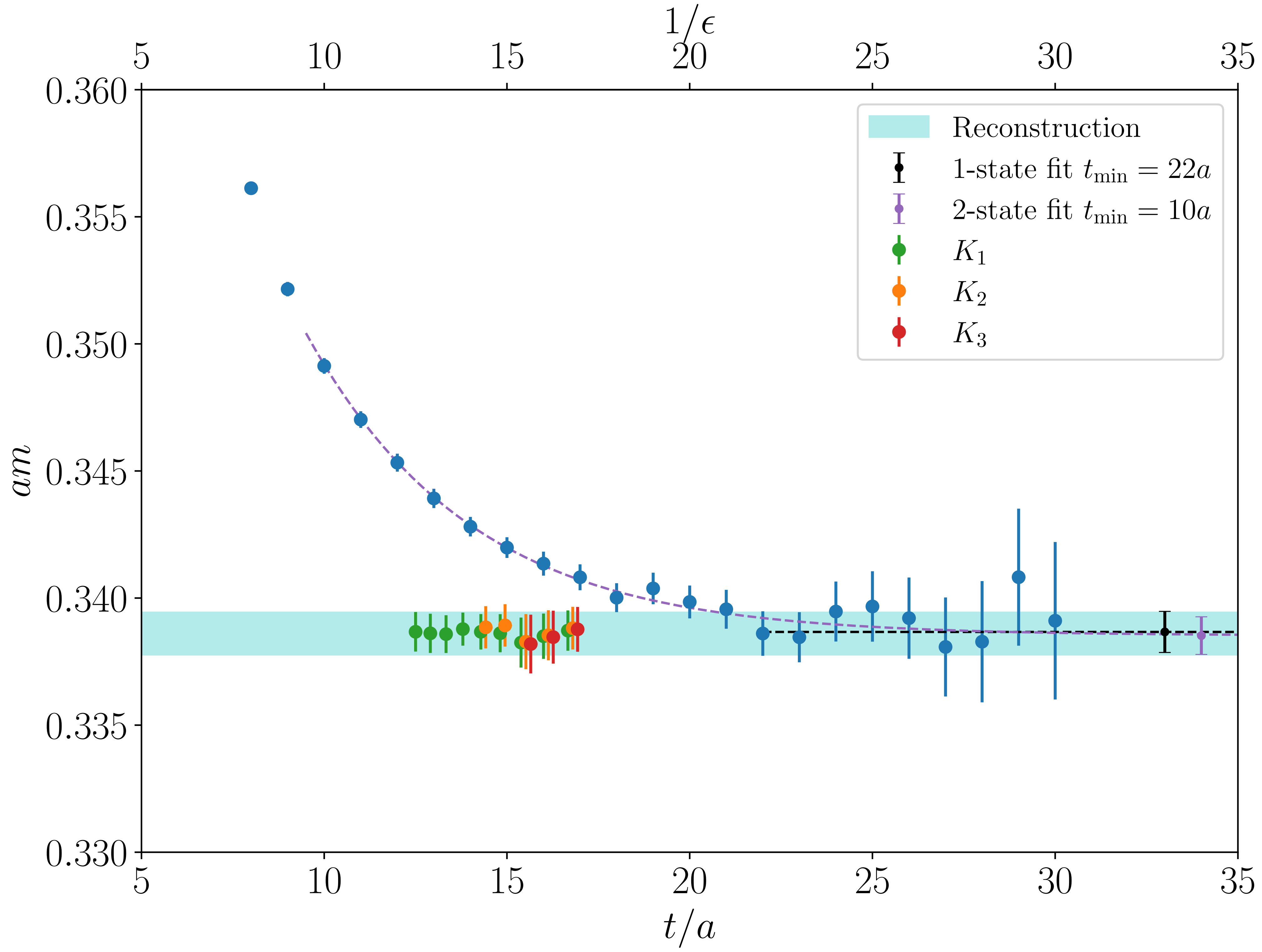
Systematics





Results for single-nucleon correlator

- Reconstruction performed for $\epsilon \in [0.06, 0.08]$
- ϵ-dependence is negligible within the statistical precision
- All the reconstructions reproduce results from standard one- and two-exponential fits to the correlator with a similar statistical error



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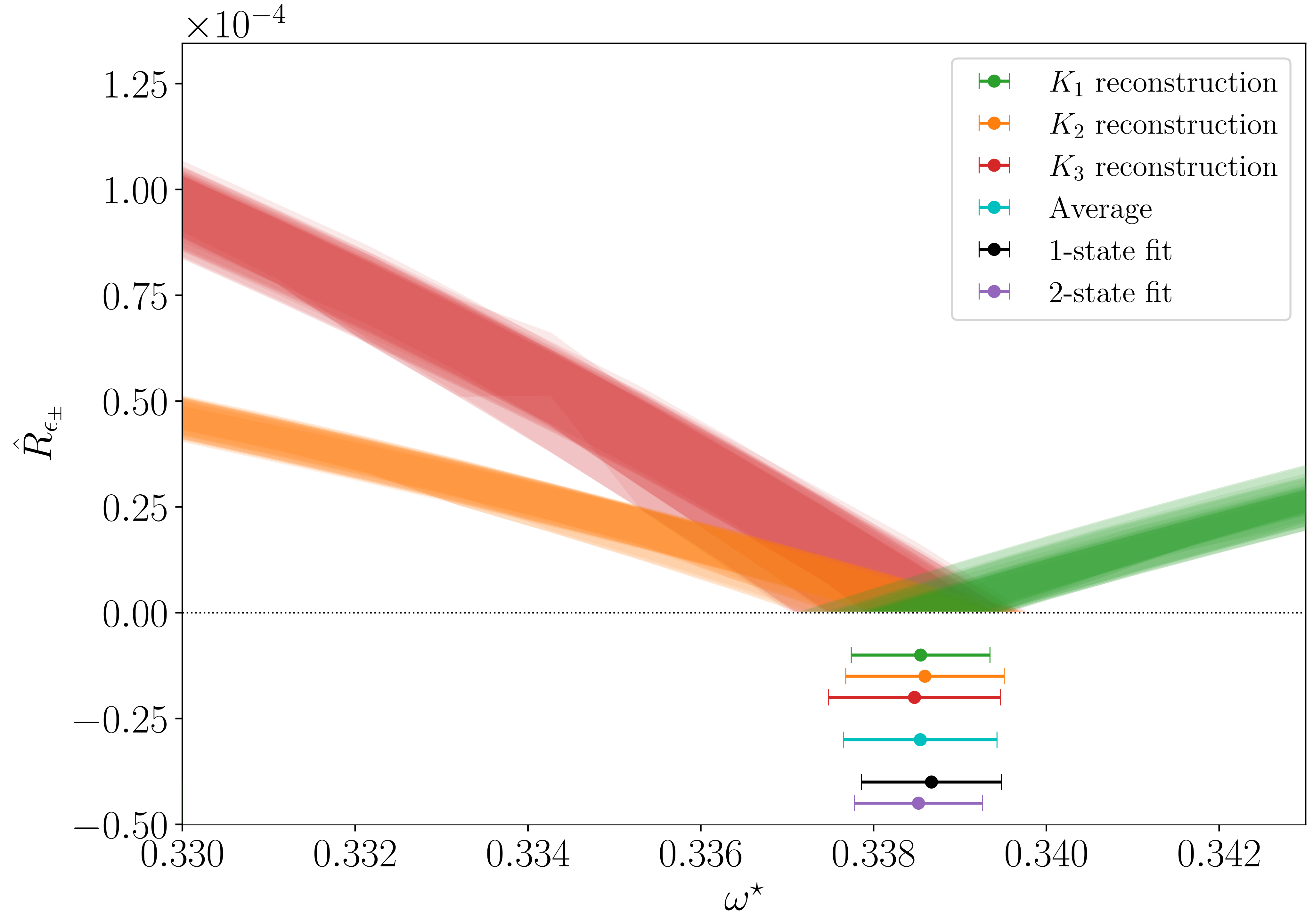


Future plans

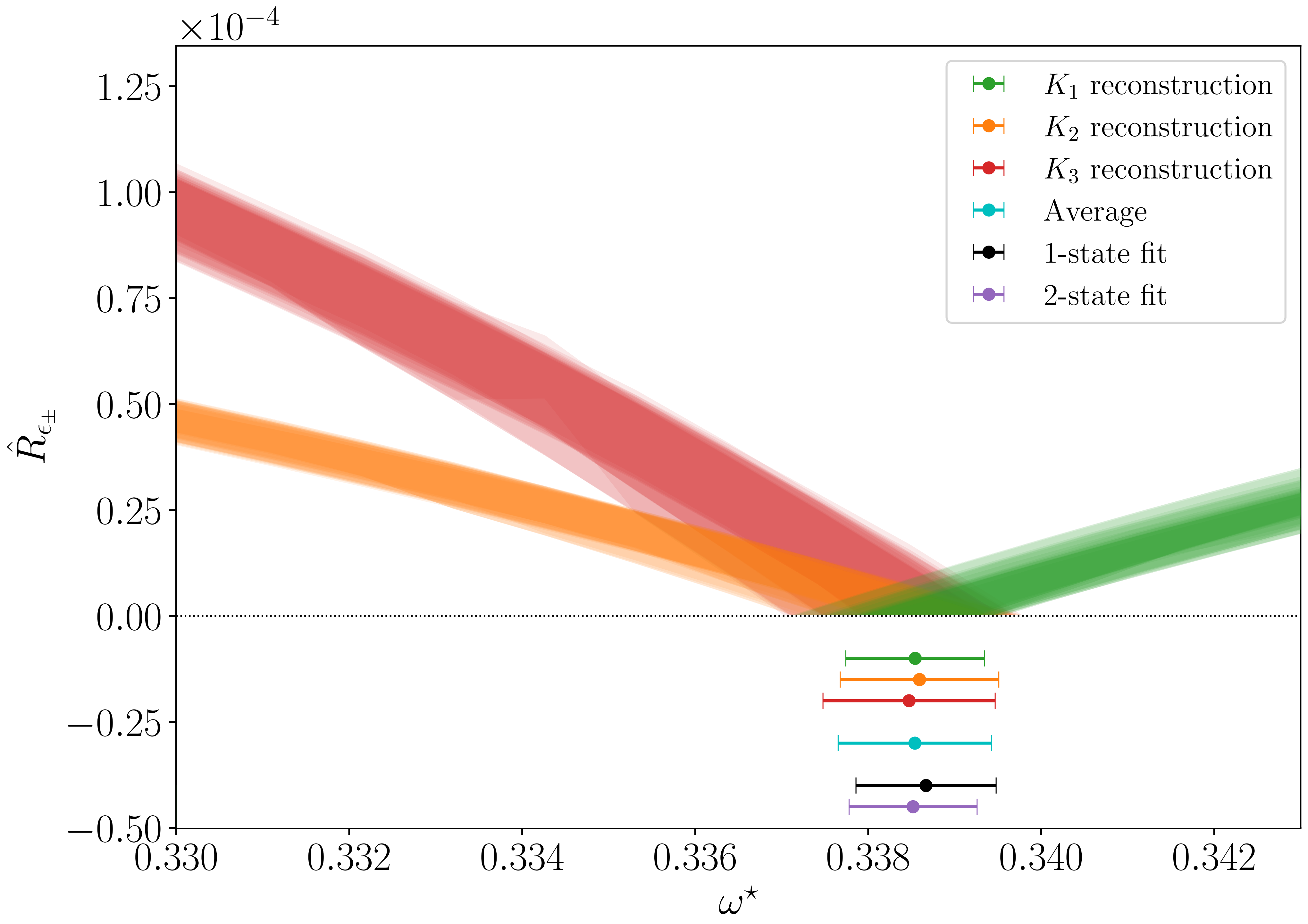
Extend this strategy to the extraction of energy shifts from multi-hadron correlators

Workflow

- Simultaneous reconstruction from single- and multi-hadron correlators
- Simultaneous root-finding on all the reconstructed $R_{\epsilon_\pm}$ to constrain the energy shift
- Leading correction expected to be smaller compared to a single correlator if shifts due to interactions are small (e.g. two nucleon systems)

Summary

- Spectral reconstruction to exploit <u>all the information</u> contained in the short-time region of a correlator
- Estimator of the ground-state energy as a direct <u>output of the reconstruction step</u>
- Provided that the extracted root saturates to the ground-state energy (no ϵ-dependence), the procedure does not rely on any <u>parameterization</u> of the correlator or of the reconstructed observable



➡ Move to the energy shifts now!

Thank you!

Reconstructions