Bridging Euclidean Methods for Parton Distributions with Machine Learning

Alex NieMiera
In Collaboration:
Brandon Kriesten, William Good, Tim Hobbs, and Huey-Wen Lin

Presentation Outline

I. Introduction to PDFs

II. PDFs from Lattice QCD

III. VAIM Framework

IV. Preliminary Results

V. Concluding Remarks

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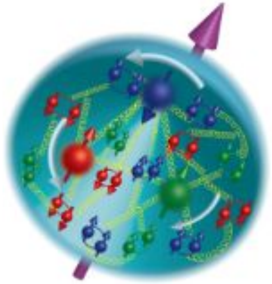
Aguilar et al., Eur. Phys. J. A 55, 190 (2019)
Arrington et al., J. Phys. G 48, 075106 (2021)
Achenbach at al., Nucl. Phys. A 1047, 122874 (2024)

Introduction to Pion Valence PDFs

Parton distribution functions (PDFs) $f_{i/H}(x, Q)$ describe the probability of finding a parton i in a hadron H, carrying a fraction x of the momentum at a given resolution scale Q.

What exactly is going on in here?



Credit: Argonne National Lab

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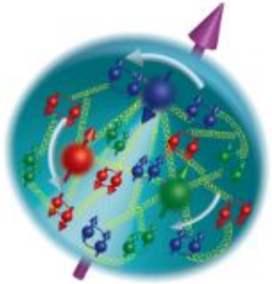

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In this work, we focus on the **pion valence PDF**…

- The pion's role as the Nambu–Goldstone boson of dynamical chiral symmetry breaking and the lightest QCD bound state, together with limited experimental constraints due to the absence of stable pion targets, makes it an ideal target for lattice QCD.

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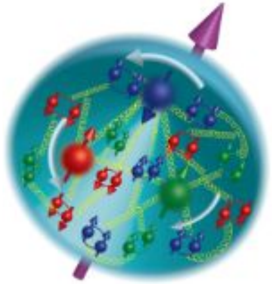

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Global Fits to Extract PDFs

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$$d\sigma_{eP} = \sum_i f_{i/P}(x,\mu) \otimes d\hat{\sigma}_{ei}(Q,\mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

something we can measure…

nonperturbative object we want!

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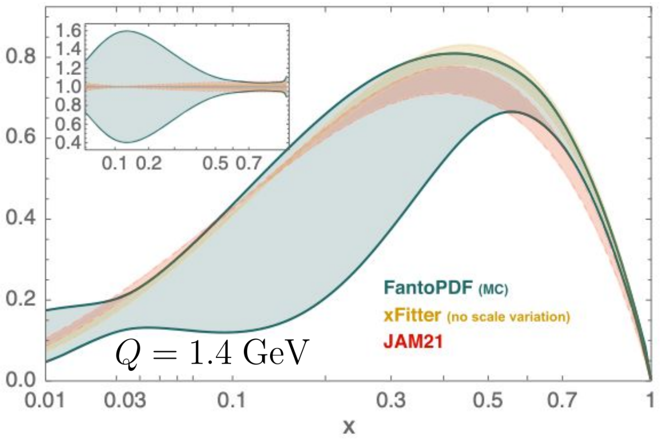

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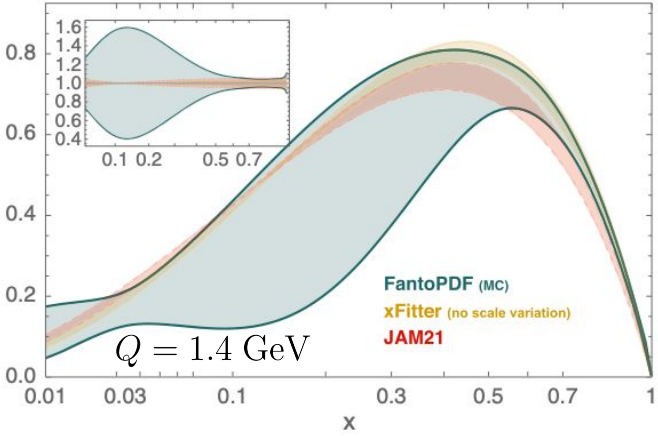

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recent global analyses have begun combining lattice QCD RpITDs and moments with experimental data to constrain kaon and pion PDFs…

Good et al., arXiv:2507.22730
Barry et al., arXiv:2510.11979

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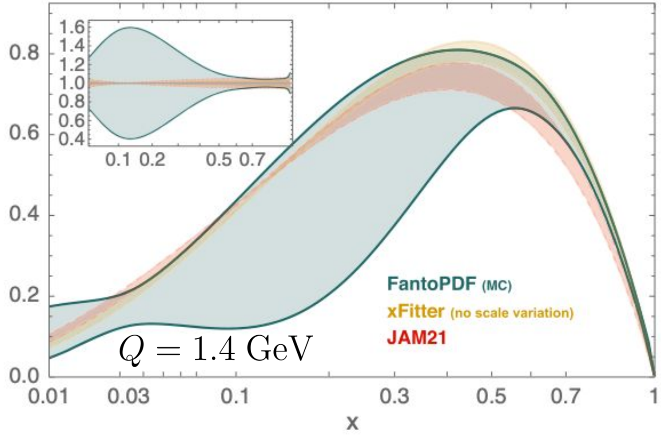

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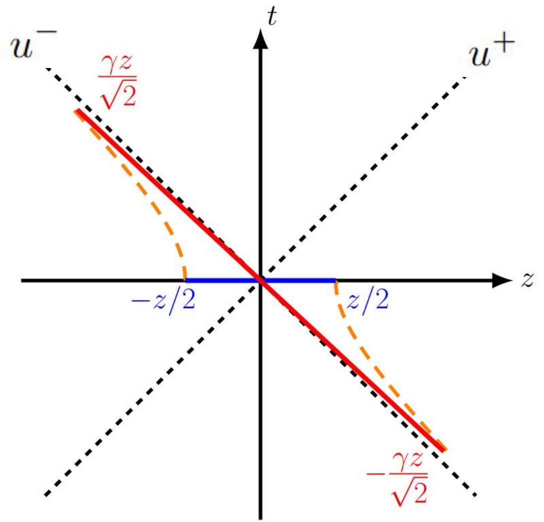
PDFs from Lattice QCD

$$u^{\pm} = \frac{u^0 \pm u^3}{\sqrt{2}}$$

It’s well known that we can formally define PDFs as the Fourier transform of light front correlators,

$$f(x) = \int dz^- e^{ixP^+z^-} \langle P|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^+ W\left(-\frac{z}{2}, +\frac{z}{2}\right)\psi\left(+\frac{z}{2}\right)|P\rangle$$



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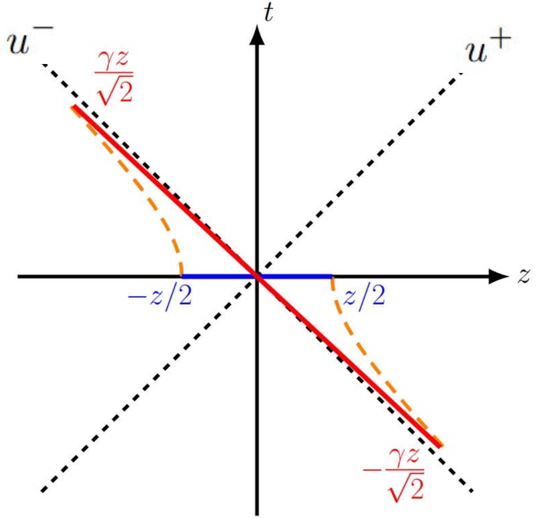

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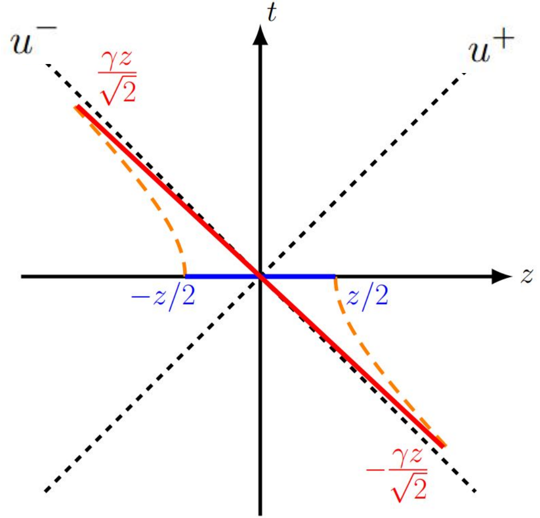

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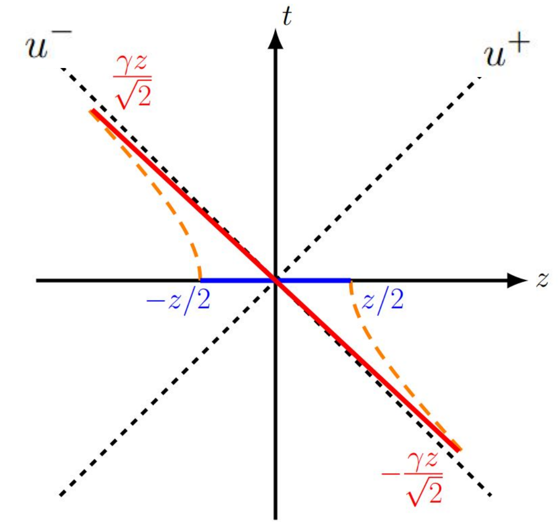

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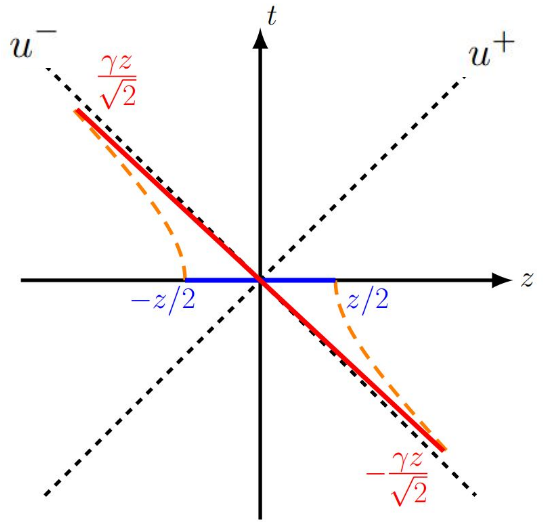

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- **Pseudo-PDF**, which uses the short-distance regime ($z^2\Lambda_{\mathrm{QCD}}^2 \ll 1$), where an operator product expansion allows for matching to PDFs.

(Like this slide? This was adapted from one of William Good's presentations!)

Procedural Overview: Quasi vs. Pseudo

PDFs are defined as the F.T. of light-front correlators, which can't be measured on a Euclidean lattice…

spatially separated matrix elements

$$\langle h(P_z)|O(z)|h(P_z)\rangle$$

from the lattice!

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$$\tilde{f}(x, P_z, \mu) = \int_{-1}^{+1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) f(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P_z)^2}\right)$$

Ji, PRL 110:262002 (2013)
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typically reliable in the range $x \in [0.2, 0.8]$…

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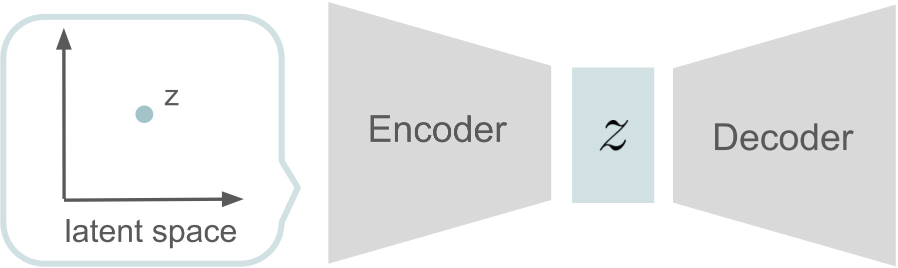

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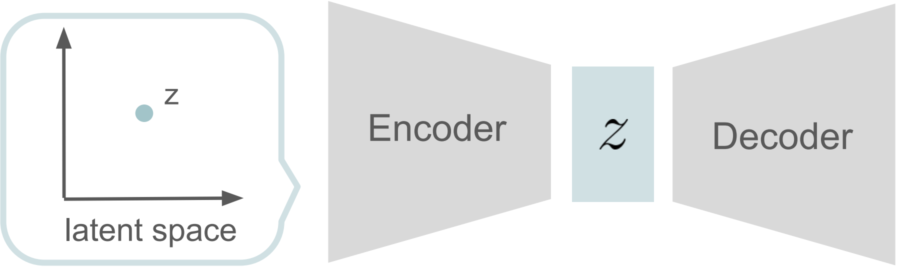

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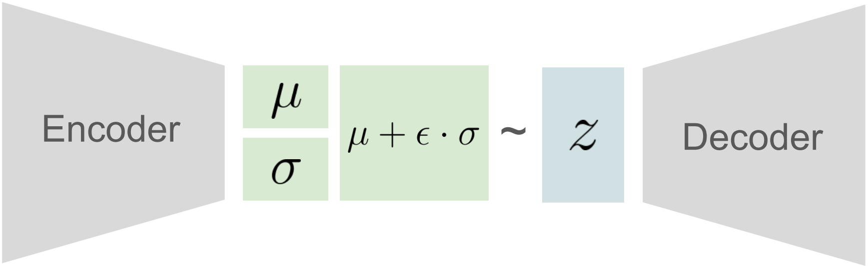
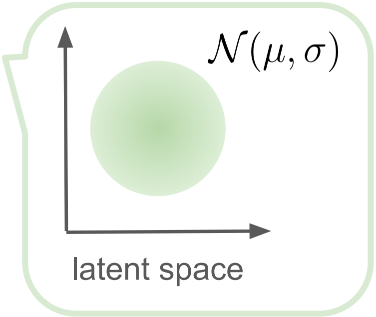
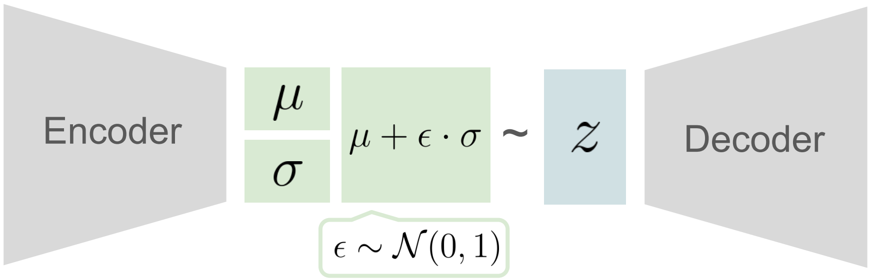


The **VAE** solves this by representing the input as a Gaussian probability…



Variational Autoencoders (VAEs)

An **autoencoder** takes an input, compresses it into a lower dimensional representation, and then decodes it back to the input space…



- The latent space is unstructured since each input maps to a single deterministic point, making sampling and interpolation unreliable!

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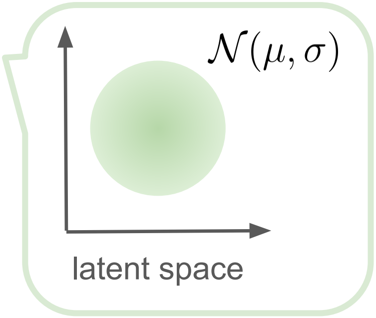
- Back propagation is achieved through a simple reparameterization in which one samples from a standard normal distribution, scales by the posterior variance, and shifts by the posterior mean…



Kingma et al., arXiv:1312.6114 (2013)

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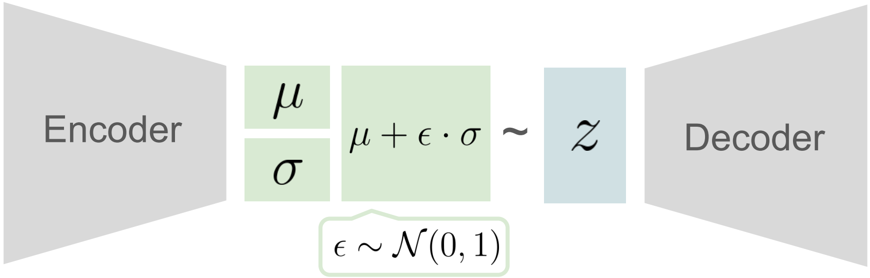

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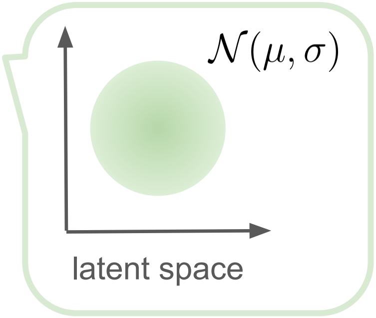
- This separates randomness from the learned parameters, making stochastic latent sampling differentiable and allowing for end-to-end training of the VAE using standard optimization techniques!



Kriesten et al., PRD 109:014011 (2025)
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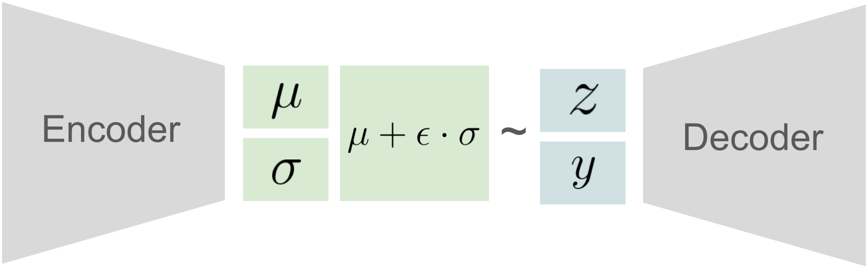
Variational Autoencoder Inverse Mapper (VAIM)

The **VAIM** extends the VAE by introducing an information pathway to incorporate known observables into the latent representation…



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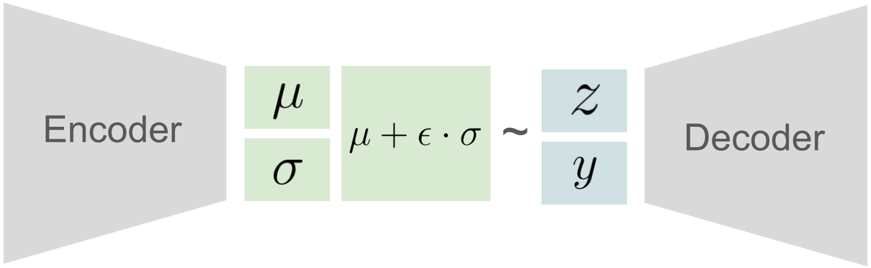

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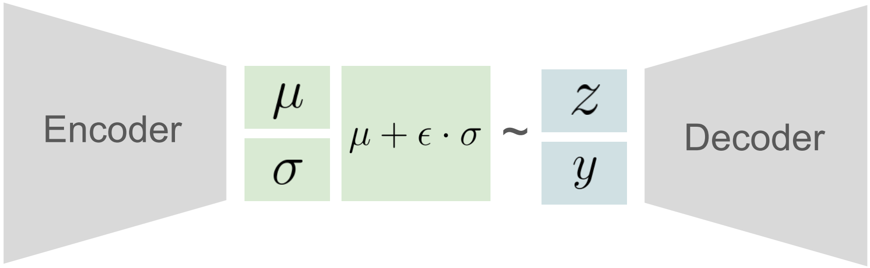

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Karpie, et al., JHEP 04:057 (2019)
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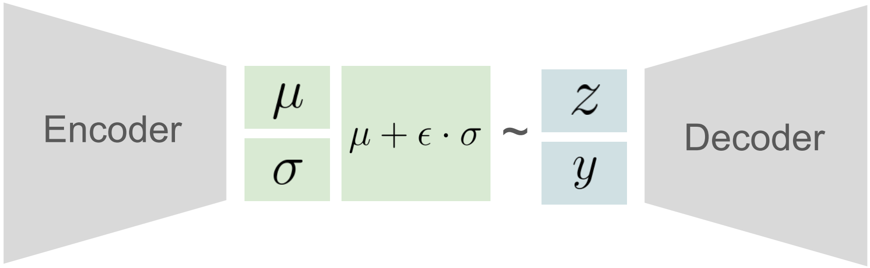

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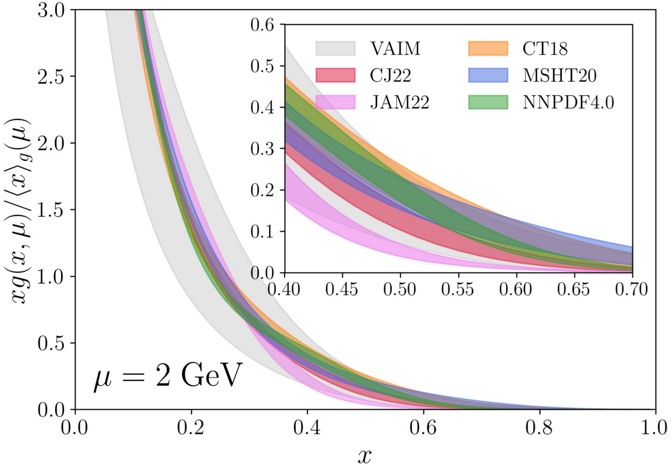

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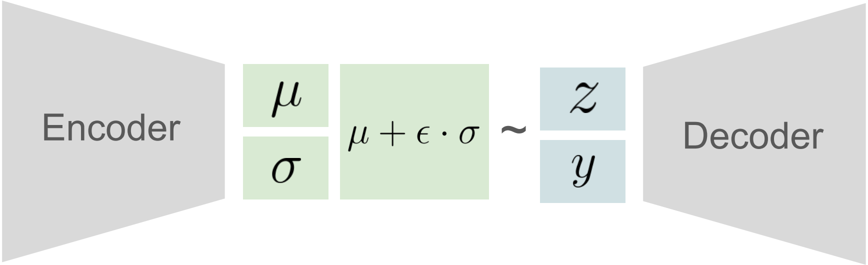

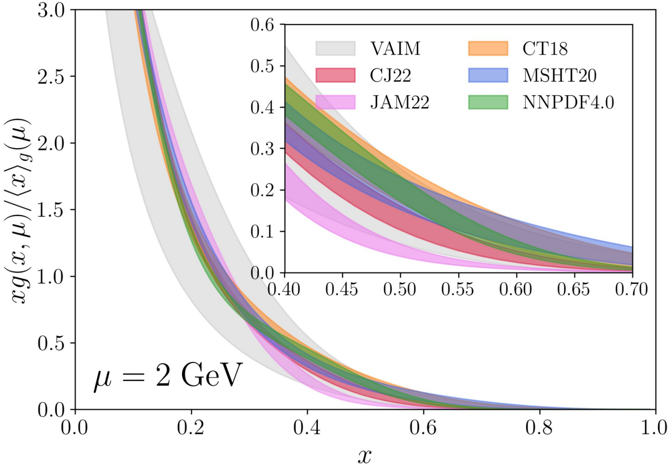

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- Here, we explore the complementarity of the pseudo- and quasi-PDF methodologies using pion valence lattice data.

Lattice Data Details

Thank you to Yong and Xiang for allowing us to use this data!
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For this work, we use lattices with $N_f = 2+1$ flavors of highly improved staggered quarks (HISQ) generated by the Hot QCD Collaboration with a lattice spacing of $a = 0.04\ \mathrm{fm}$ and a pion mass of $M_\pi = 300\ \mathrm{MeV}$.

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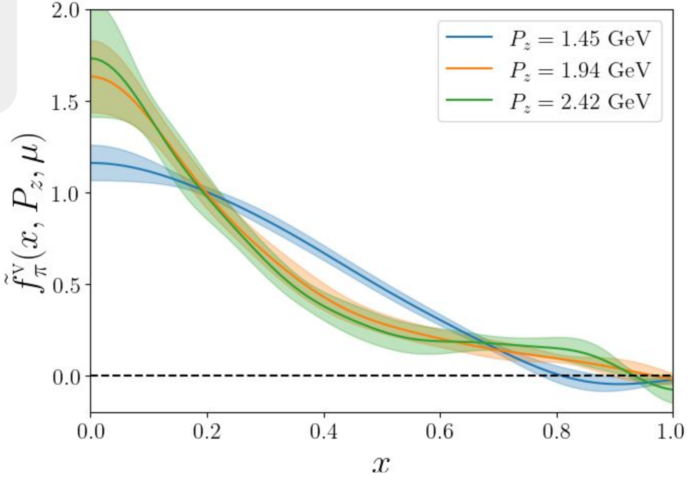

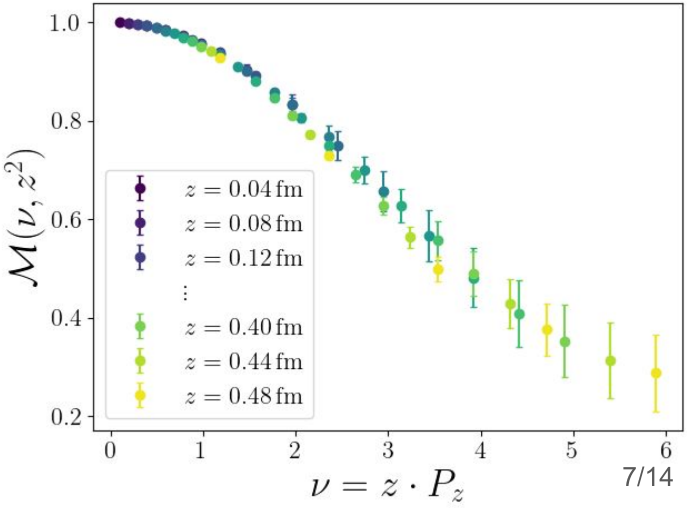

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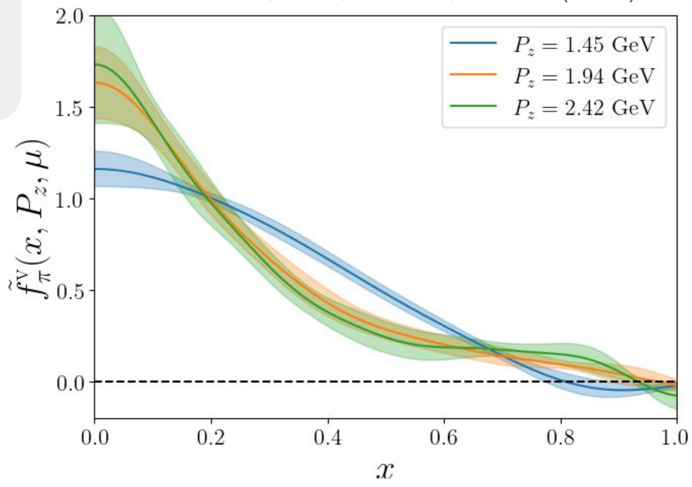

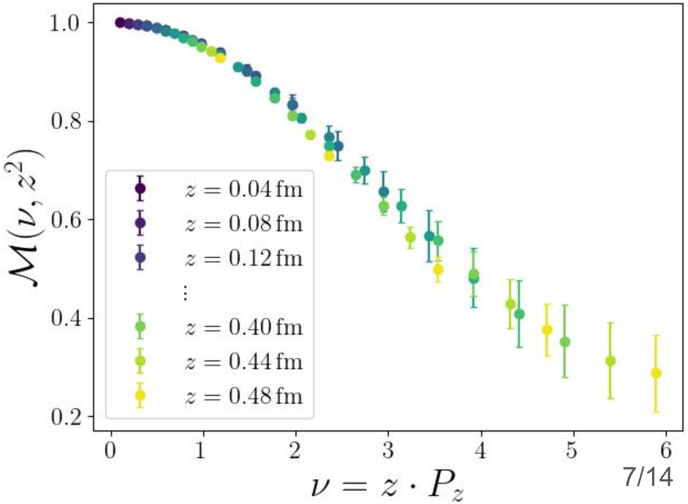

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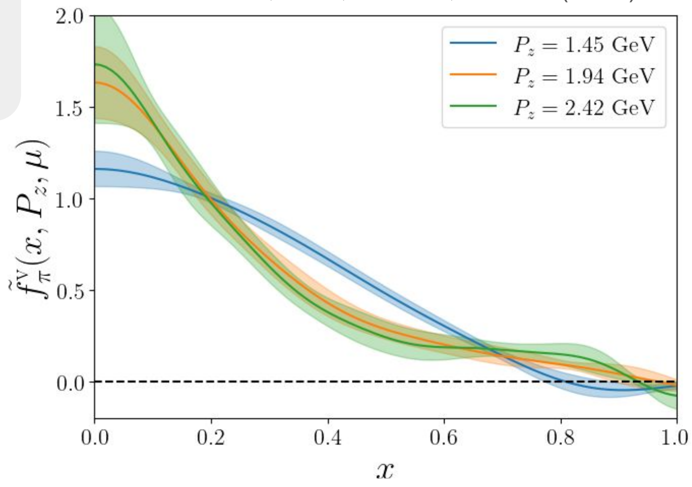

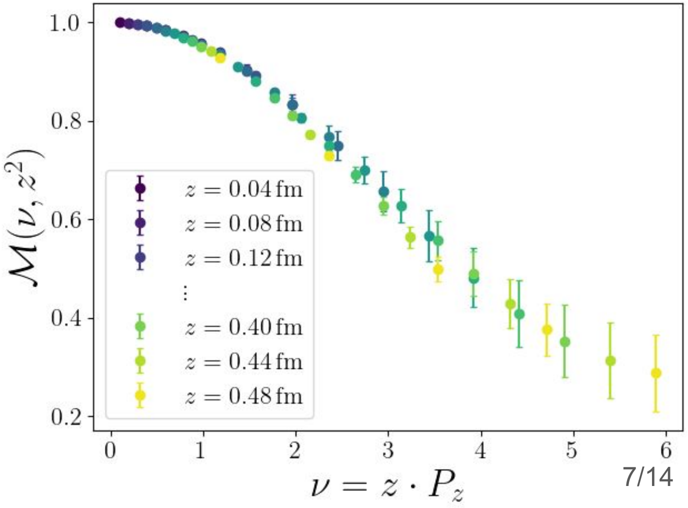

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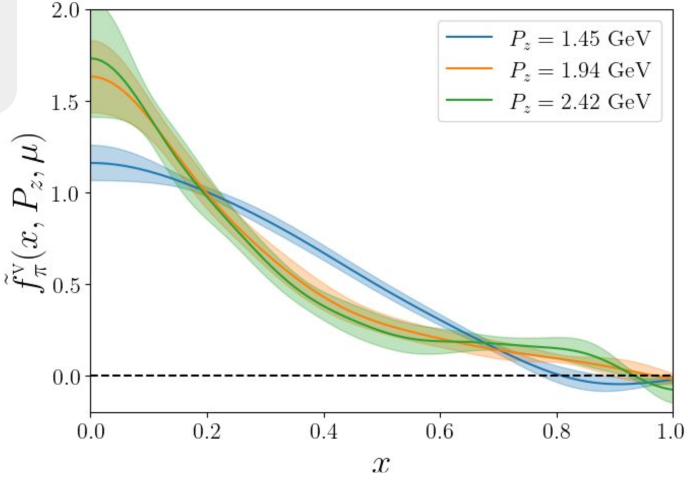
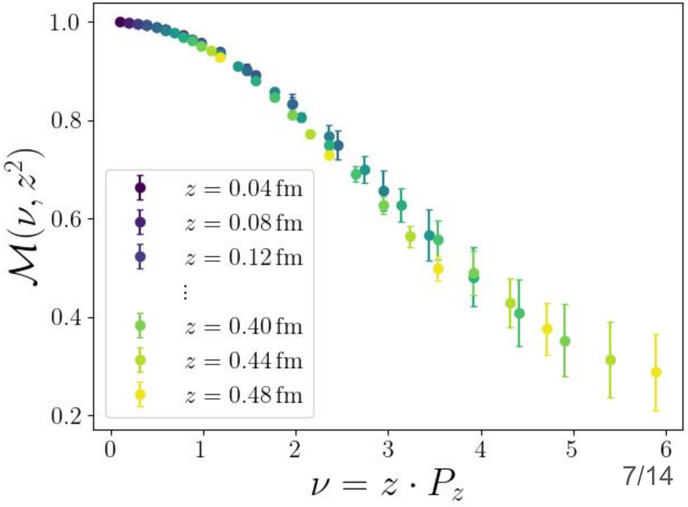
- For the pseudo-PDF method, we restrict ourselves to $z = [0.04, 0.48]$ fm. Radyushkin, PRD 96:034025 (2017)





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The parameters are determined through a random sampling of…

$\alpha \sim \mathcal{U}(-0.9, 1.0)$ $\beta \sim \mathcal{U}(0, 10)$ $\gamma, \delta \sim \mathcal{U}(-5.0, +5.0)$

To generate VAIM training data, we sample $\mathcal{O}(10^5)$ parameter sets $\{\alpha, \beta, \gamma, \delta\}$ from uniform distributions to create PDFs with a functional form $f_\pi^{\mathrm{v}}(x) = \mathcal{N} x^\alpha (1-x)^\beta (1 + \gamma\sqrt{x} + \delta x)$…

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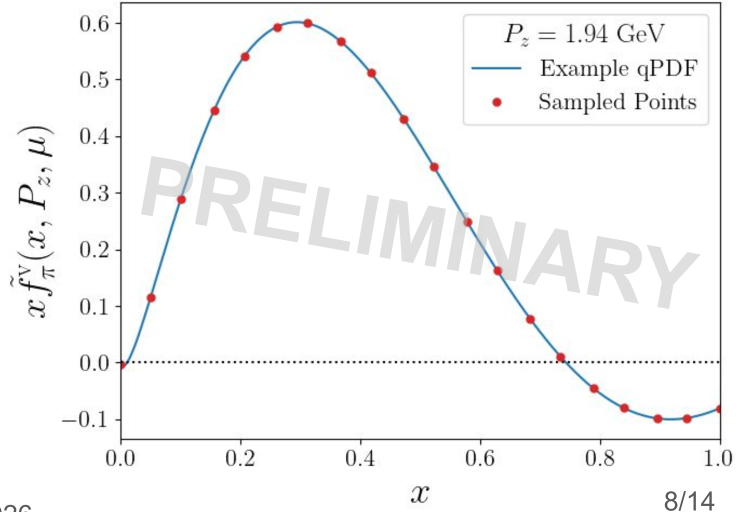

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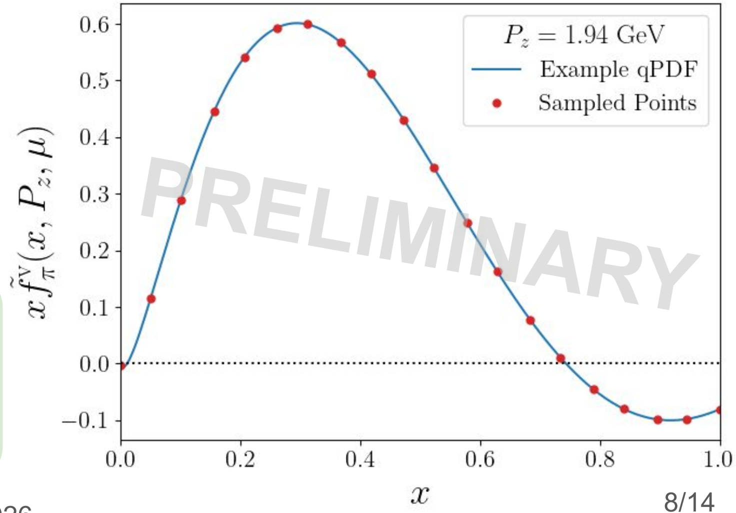

While we plan to explore additional sampling strategies, linear quasi-PDF sampling provides a natural starting point and is well motivated by ongoing efforts to directly extract x-space quasi-PDFs from lattice three-point correlators (**see Rui's talk, 07/28, 9:40 AM**).

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I. Introduction to PDFs

II. PDFs from Lattice QCD

III. VAIM Framework

IV. Preliminary Results

V. Concluding Remarks

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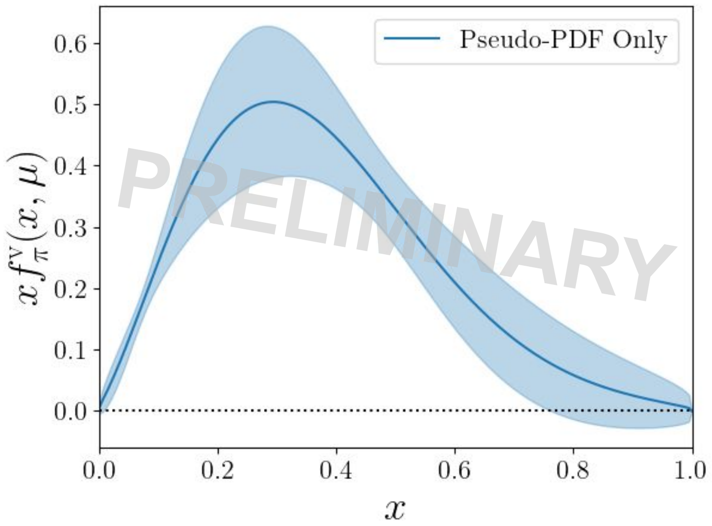

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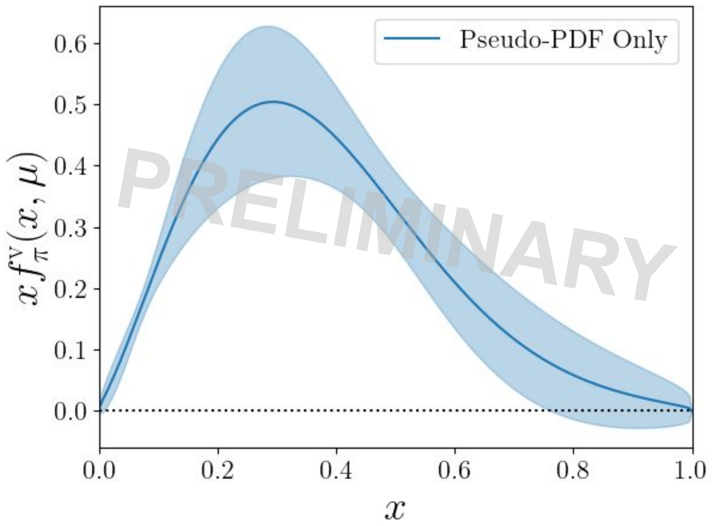

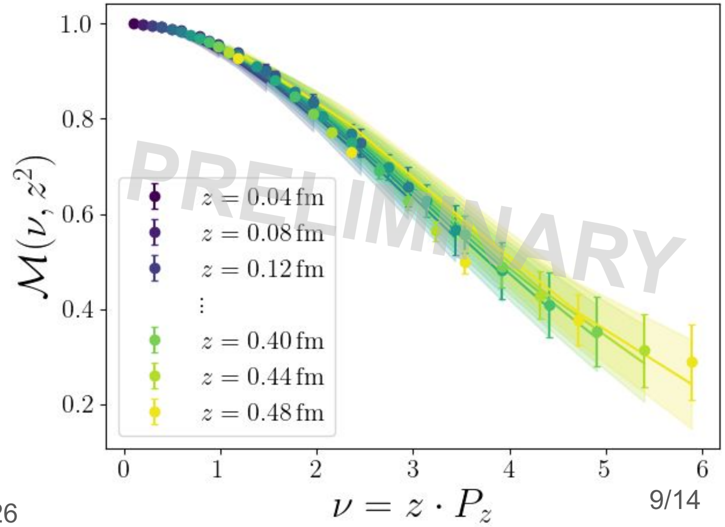

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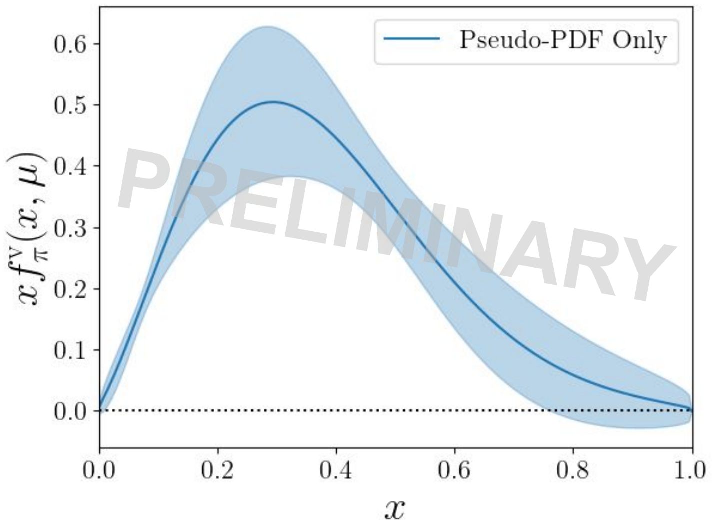

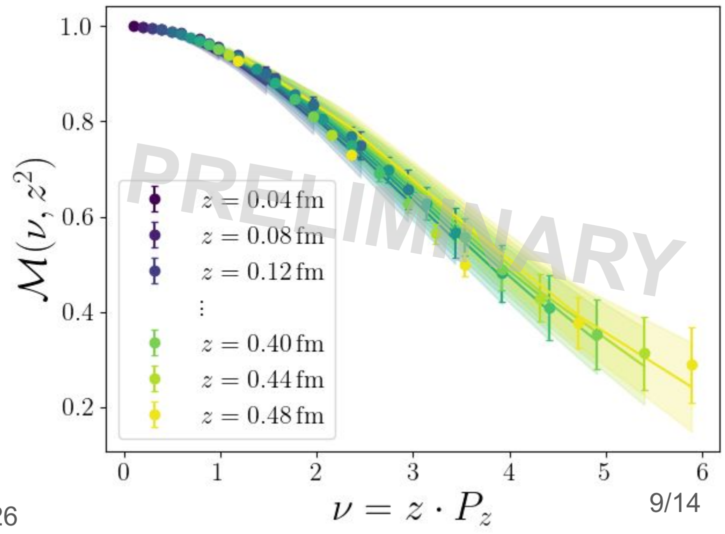

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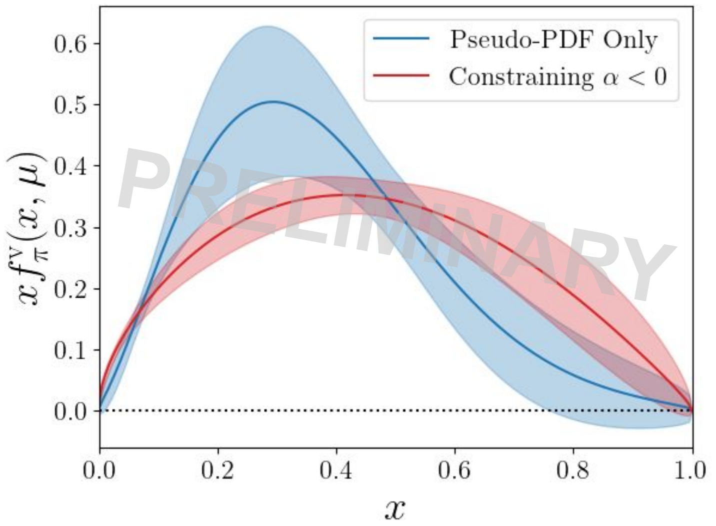
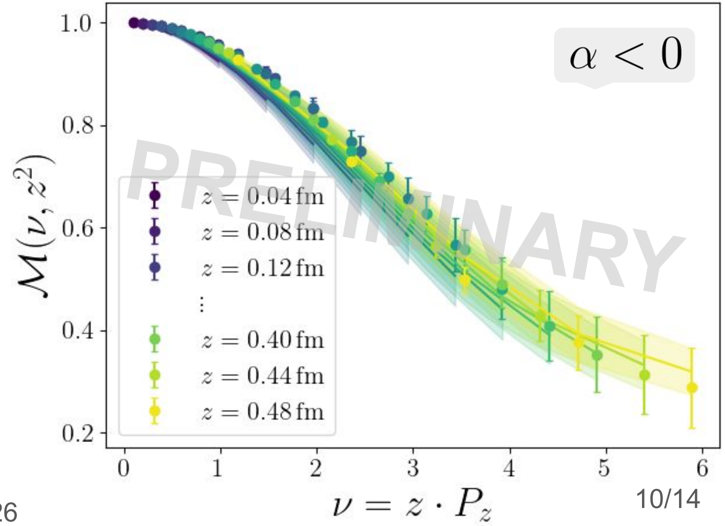
- Although VAIM constrains a solution, it is not unique, highlighting the ill-posedness of the pseudo-PDF method!
- Previous pion lattice studies constrain $\alpha < 0$, consistent with global fits favoring $\alpha \sim -0.5$ to -0.2, motivating a negative prior for the small-x behaviour.





Aicher et al., PRL 105:252003 (2010)
Wijesooriya et al., PRC 72:065203 (2005)
Hecht et al., PRC 63:025213 (2001)
Glück et al., EPJC 10:313 (1999)
Joó et al., PRD 100:114512 (2019)
Sufian et al., PRD 99:074507 (2019)

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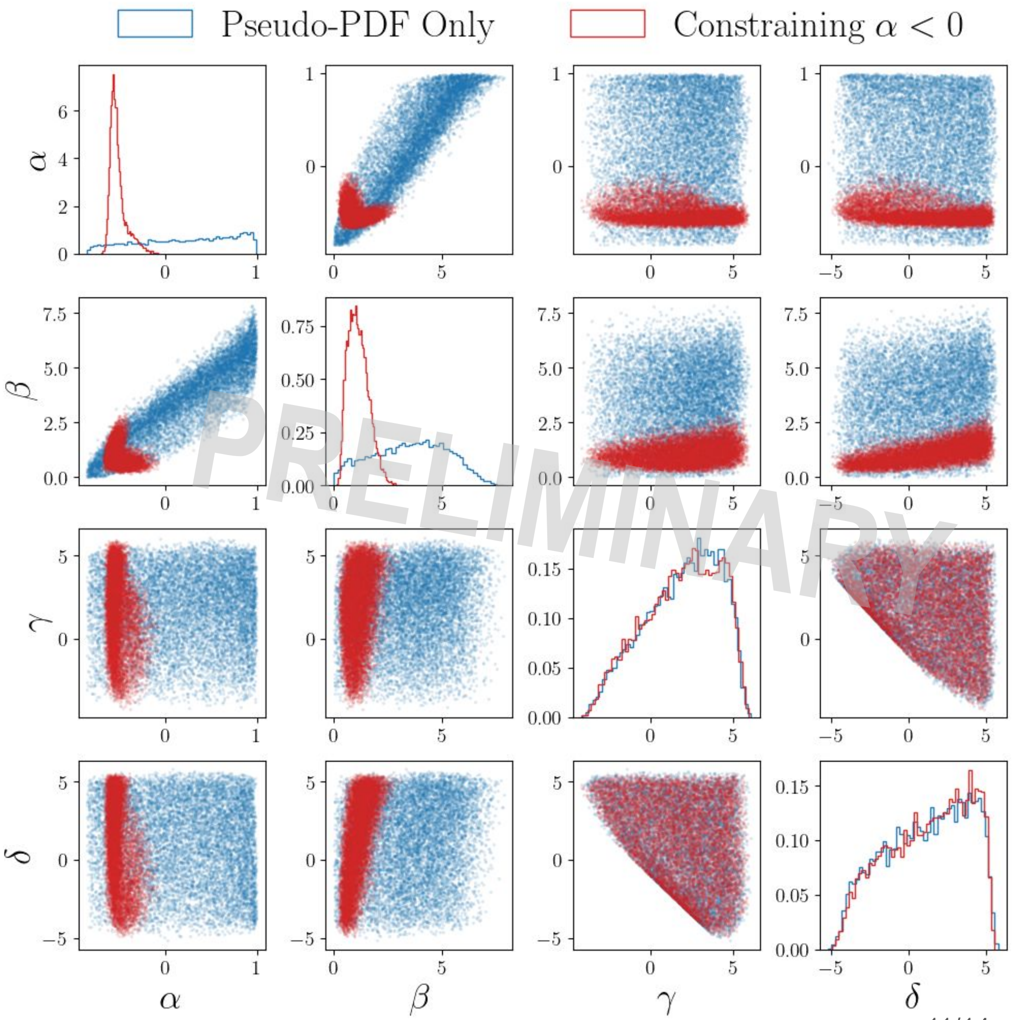

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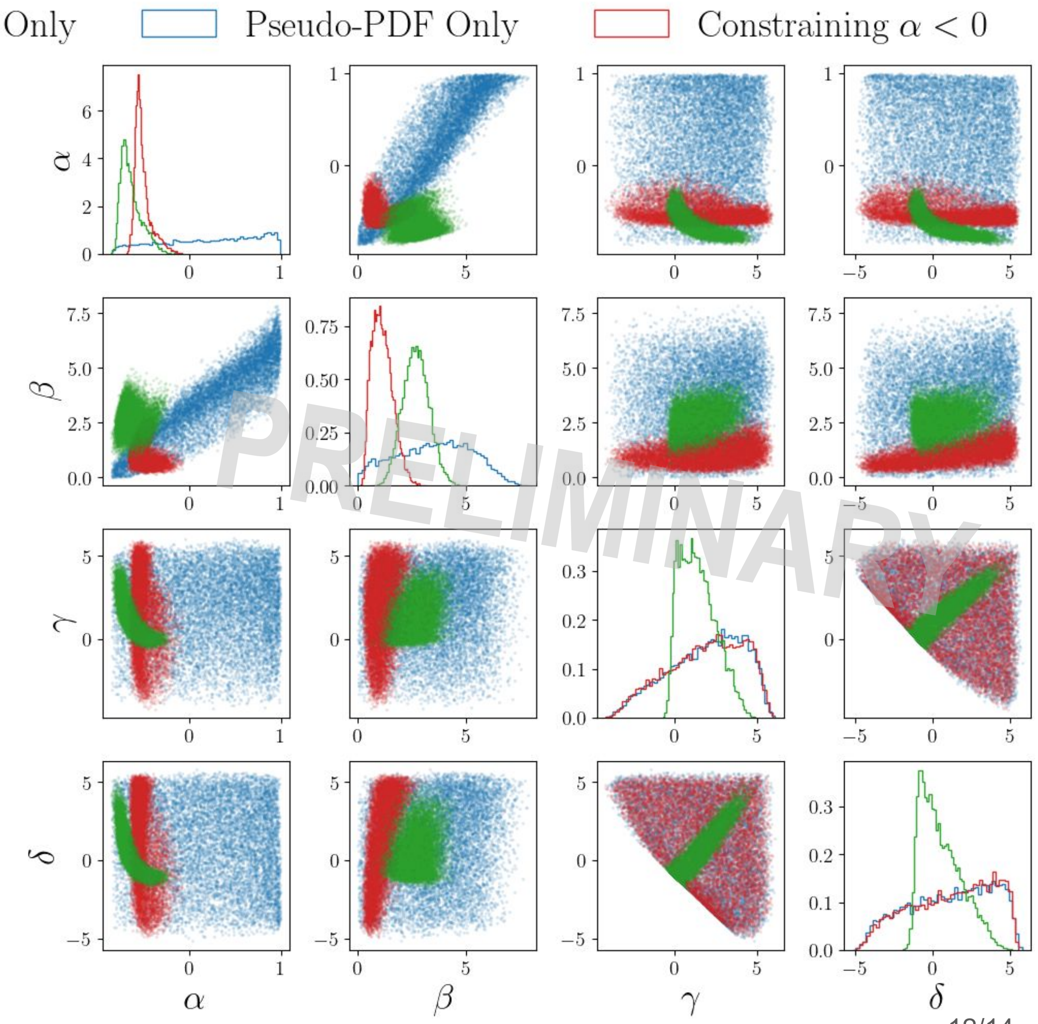

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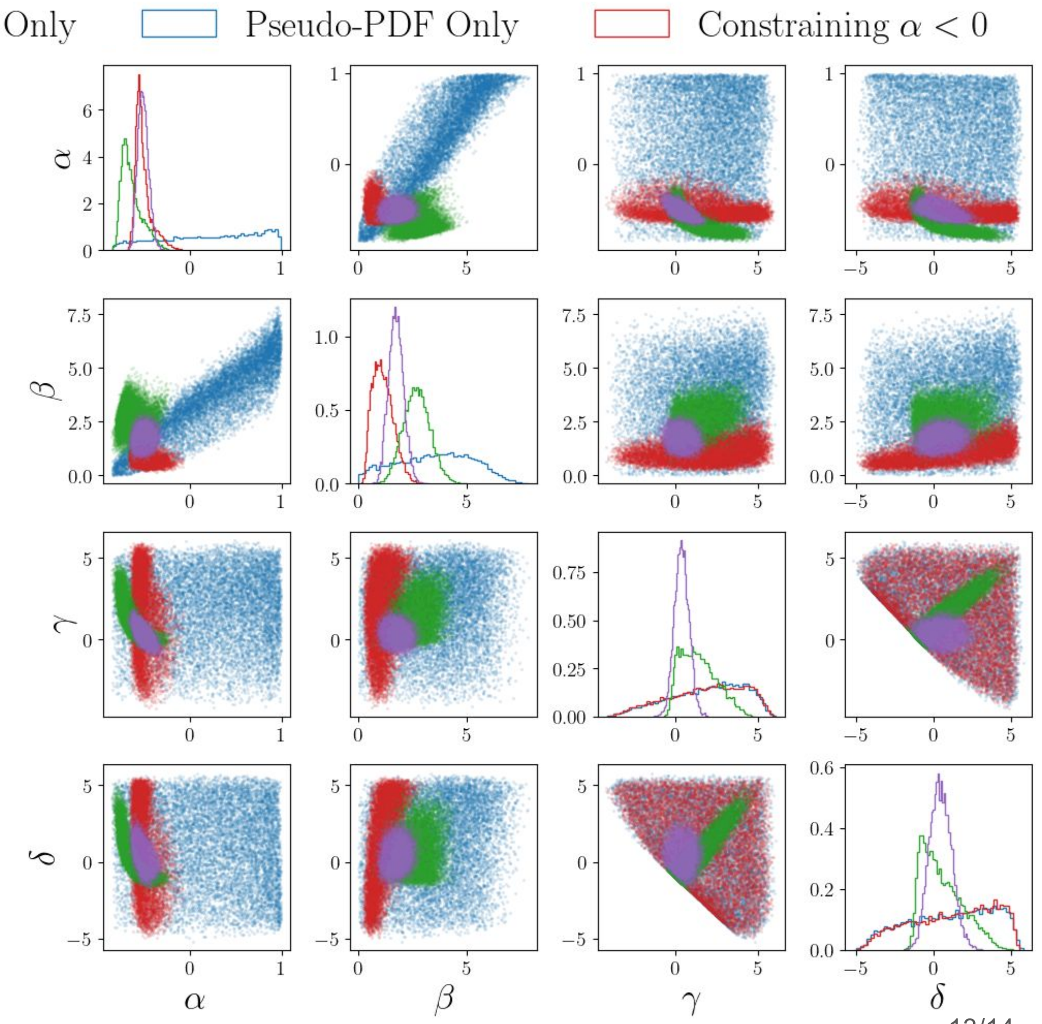
- Taking the unconstrained pseudo-PDF information and allowing the model to see this alongside of the quasi-PDF information during training, we see that the additional information replaces the need for a constraint on $\alpha < 0$



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Outlook & Concluding Remarks

- We have made preliminary progress toward developing a unified VAIM framework capable of incorporating information from both the quasi- and pseudo-PDF methodologies.
- Future work will extend this framework by including higher-twist corrections and systematic uncertainties from both approaches, allowing the model to learn these additional contributions directly.
- We also plan to investigate the dependence of the extracted PDFs on the underlying parameterization.

Thank You!

Backup Slides

Good et al., arXiv:2507.22730
Barry et al., PRD 105.114051 (2022)
Karpie et al., JHEP 11:024 (2021)

Accounting for Higher-Twist Effects

To account for systematic effects in the **pseudo-PDF** training data, we follow the standard phenomenological treatment, which we have adopted in our previous work,

$$\mathcal{M}(\nu, z^2) = \int_0^1 dx\ f_\pi^{\mathrm{v}}(x) R_{qq}(x\nu, z^2) + z^2 B_1(\nu) + \frac{1}{z} P_1(\nu)$$

Where the terms $\{B_1, P_1\}(\nu) = \sum_{n=1}^{N} \{b_n, p_n\} \sigma_{0,n}(\nu)$ respectively account for higher-twist and discretization effects.

- Thus, we sample parameters $\{a, b, b_n, p_n\}$, which are passed to the VAIM model during training in addition to the PDF parameters.
- The model learns the relationship between these systematic variations and their impact on the resulting RpITDs.

the weighted cosine transforms of Jacobi polynomials, shifted to the domain $0 \leq x \leq 1$,

$$\sigma_{0,n}(\nu) = \int_0^1 dx\ \cos(x\nu) x^a (1-x)^b J_n^{(a,b)}(x)$$

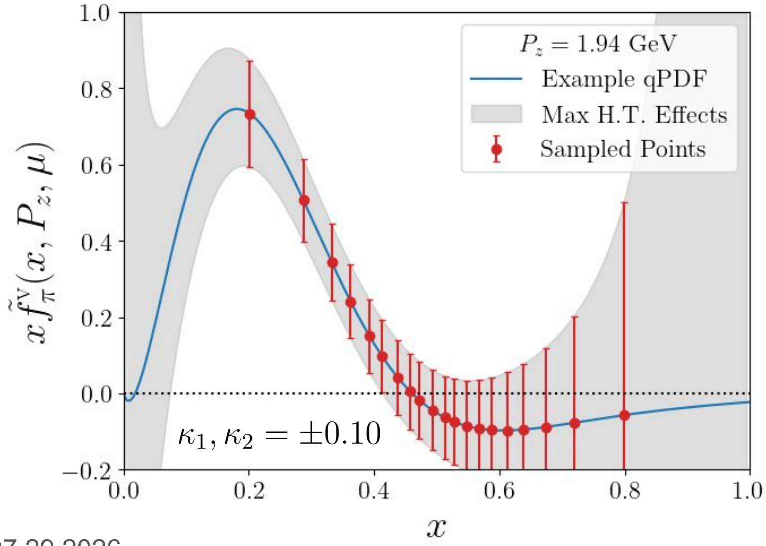
which ensures $\sigma_{0,n}(0) = 0$, as required for proper RpITD normalization…

Accounting for Higher-Twist Effects

To account for systematic effects in the **quasi-PDF** training data, we use a similar treatment in which higher-twist contributions are included as

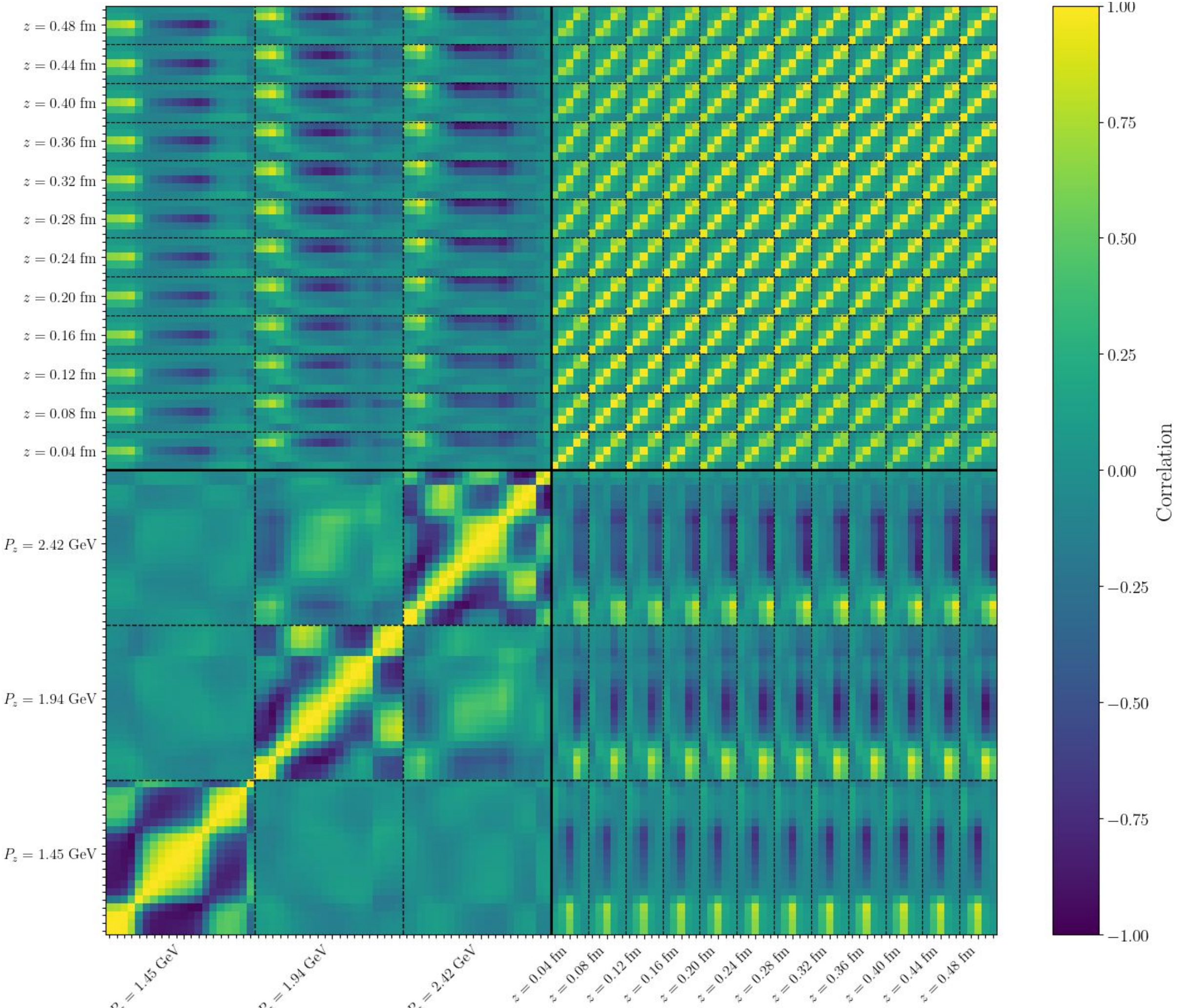
$$\tilde{f}_\pi^{\mathrm{v}}(x, P_z, \mu) = \int_{-1}^{+1} \frac{dy}{|y|} C_{\gamma_t}^{\mathrm{hyb-ratio}}\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) f_\pi^{\mathrm{v}}(y, \mu) + \kappa_1 \left(\frac{\Lambda_{\mathrm{QCD}}}{xP_z}\right)^2 + \kappa_2 \left(\frac{\Lambda_{\mathrm{QCD}}}{(1-x)P_z}\right)^2$$

- Similarly, we sample parameters $\{\kappa_1, \kappa_2\}$, which are passed to the VAIM model during training in addition to the PDF parameters…
- To prevent the small- and large-x samples from blowing up, we adopt a more sophisticated scheme to sample the x-dependence of the quasi-PDF.
- The beauty of allowing VAIM to learn systematics this way lies in it's extendability!



Data Correlations

The RpITDs and quasi PDFs originate from the same bare lattice matrix elements, which are very correlated!



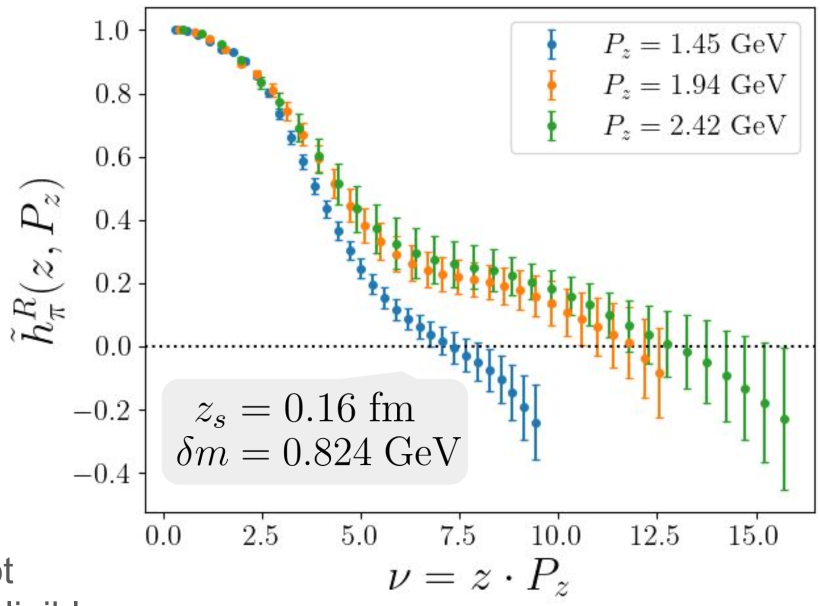
Hybrid Ratio Renormalization

We renormalize the quasi-PDF matrix elements using hybrid ratio renormalization to remove the linear divergences arising from the Wilson line self-energy at large separations…

$$h^R(z,P_z) = \begin{cases} \frac{h^B(0,0)}{h^B(0,P_z)} \frac{h^B(z,P_z)}{h^B(z,0)} & z \le z_s \\ \frac{h^B(0,0)}{h^B(0,P_z)} \frac{h^B(z,P_z)}{h^B(z_s,0)} e^{(\delta m + m_0)(z - z_s)} & z > z_s \end{cases}$$

- Here, z_s denotes the distance scale, which should not exceed 0.3 fm and below which the divergence is negligible.

Gao, et al., PRL 128 14:142003 (2022)



The term $\delta m + m_0$ is fit by matching to the appropriate Wilson coefficient $C_0^{\mathrm{NLO}}(z,\mu)$, with the following ansatz:

$$(\delta m + m_0)z + I_0 \approx \ln\left[\frac{C_0^{\mathrm{NLO}}(z,\mu)}{h^B(z,0)}\right]$$

Quasi-PDF Sampling

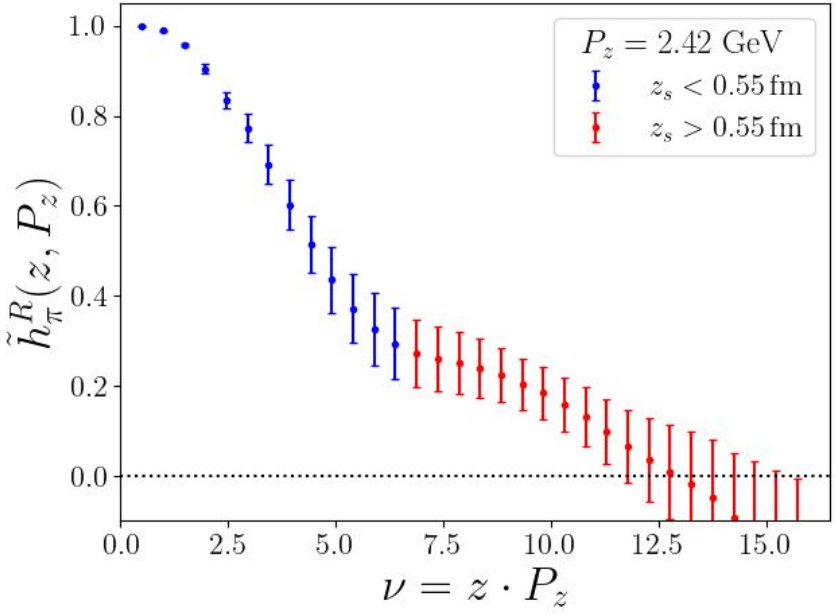
Ji et al., arXiv:2601.12189
Gao et al., PRL 128:142003 (2022)

In the quasi-PDF method, we take one value of momentum P_z and perform the large distance extrapolation using $z_s > 0.55 - 0.60$ fm.

- The NLA ansatz for this extrapolation is:

$$\tilde{h}^{\mathrm{NLO}}(z, P_z) = \left[A_2 + \frac{A_2'}{|z|} + 2A_1 \cos(\phi - zP_z) + \frac{2A_1' \cos(\phi' - zP_z)}{|z|} \right] e^{-\Lambda|z|}$$

which has 7 fitting parameters…



To ensure we're not over-sampling the quasi-PDFs in comparison to the pseudo-PDF data, we take the number of samples to be:

~13 data points (small-ν) + 7 data points (extrapolation ansatz) = ~20 total data points!

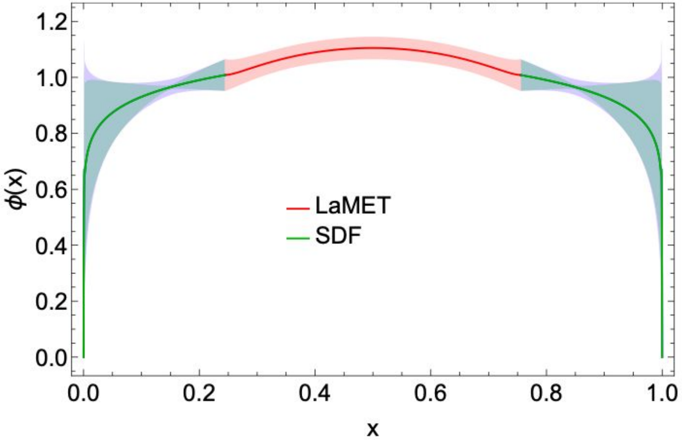
A Brief Literature Review

Building on these efforts, we use ML to investigate the interplay between lattice systematics and higher-twist effects in PDF reconstruction, with the goal of developing reliable strategies to work towards a lattice global fit.

While many groups have focused on developing individual methodologies, recent efforts have begun exploring strategies to combine these approaches…

Holligan et al. combines large momentum effective theory (LaMET) with short distance factorization (SDF) constraints to control the coordinate-space matrix element across short and long distances, enabling a precision pion DA determination!

Holligan et al, NPB 993:116282 (2023)



Chu et al. explores a unified neural-network framework for reconstructing PDFs from lattice observables, incorporating flexible PDF parametrizations and systematic variations within the extraction.

Chu et al, arXiv:2509.15931