Accessing (Disconnected) Nucleon Sigma Terms with Gradient Flow

Lattice 2026

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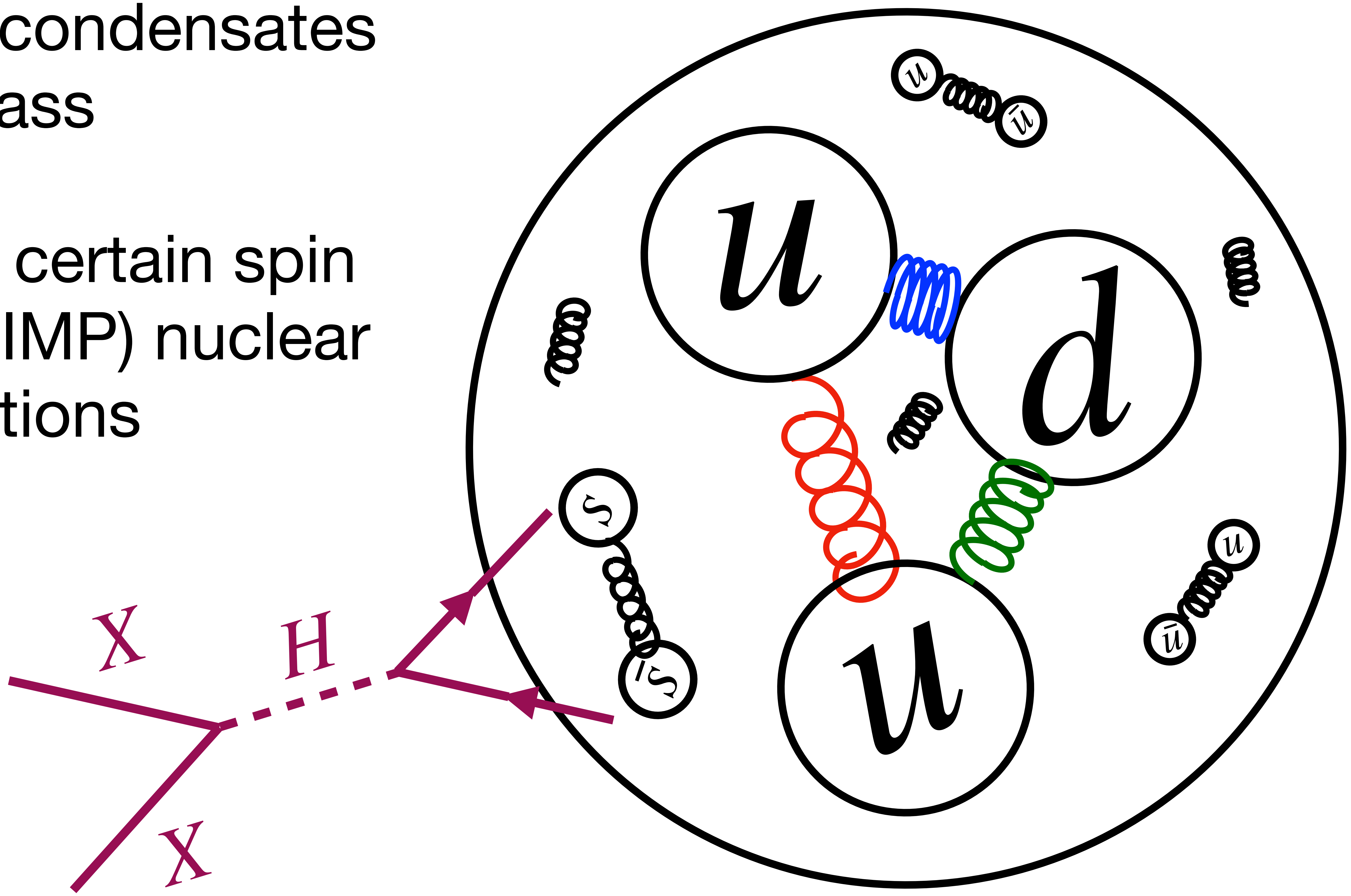
Nucleon sigma terms quantify the contribution of scalar quark condensates to the nucleon's mass

Critical input for calculating certain spin independant dark matter (WIMP) nuclear scattering cross sections

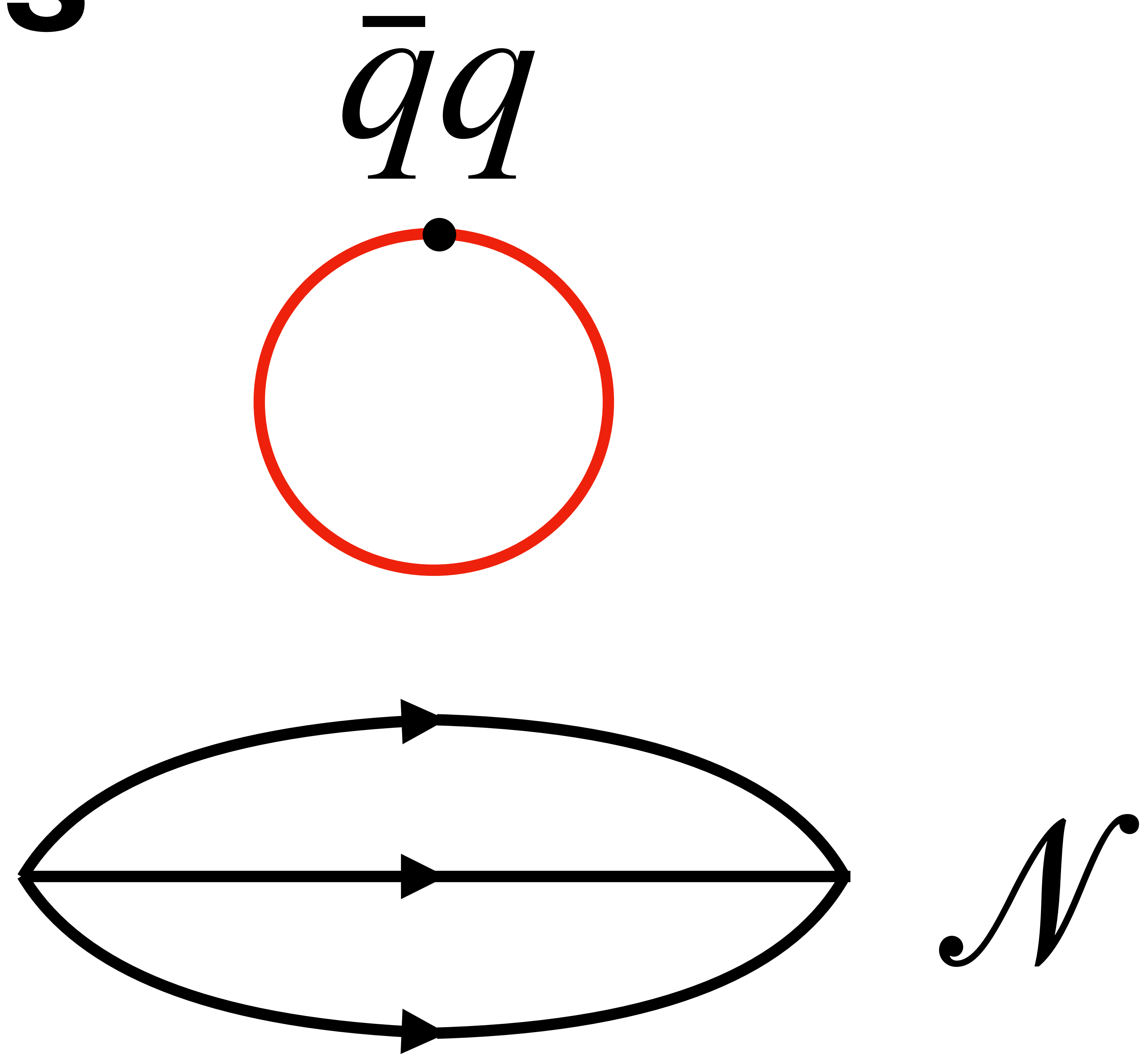
We will focus on strange and charm



Strange and Charm Content of the Nucleon: Purely Disconnected Diagrams

$$m_s \langle \mathcal{N} \,|\, \bar{s}s \,|\, \mathcal{N}^\dagger \rangle$$

$$m_c \langle \mathcal{N} \,|\, \bar{c}c \,|\, \mathcal{N}^\dagger \rangle$$



We want vacuum subtracted value

$$\langle \mathcal{N} \,|\, :\bar{q}q:\, |\, \mathcal{N}^\dagger \rangle = \langle \mathcal{N} \,|\, \bar{q}q \,|\, \mathcal{N}^\dagger \rangle - \langle \bar{q}q \rangle \langle \mathcal{N}\mathcal{N}^\dagger \rangle$$

Two large numbers;
Their difference is noisy

UV Divergent
UV artifacts may be present on the lattice

Status of strange and charm sigma terms

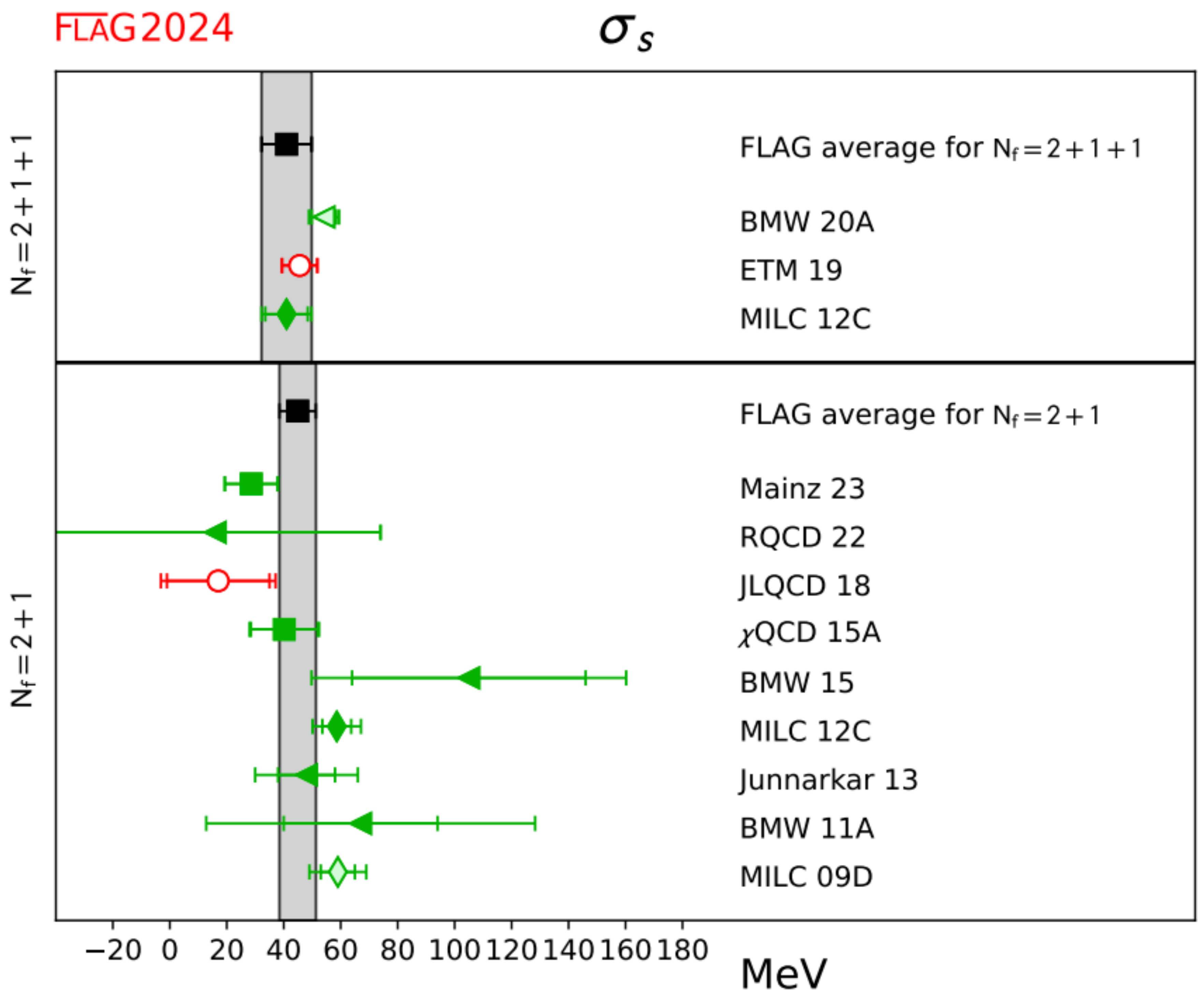


Figure 47: Lattice results and FLAG averages for σ_s for the $N_f = 2+1$, and $2+1+1$ flavour calculations. Determinations via the direct approach are indicated by squares and circles, the Feynman-Hellmann method by triangles and the hybrid approach by diamonds.

Y. Aoki et al. FLAG Review (2024)

FLAG average $\approx 20\,\%$ error

$$m_c\langle \mathcal{N} \,|\, : \bar{c}c : \,|\, \mathcal{N}^\dagger \rangle = 68.9(34.3)\ \text{MeV}$$

Data taken and processed from Freeman, Toussaint (2013), 10.1103/PhysRevD.88.054503

$$m_c\langle \mathcal{N} \,|\, : \bar{c}c : \,|\, \mathcal{N}^\dagger \rangle = 88.3(29.1)\ \text{MeV}$$

Data taken and processed from Gong M et al (2013) 10.1103/PhysRevD.88.054503

Nucleon		Individual p and n	
f^N_{ud}	0.0398(32)(44)	f^p_u	0.0142(12)(15)
f^N_s	0.0577(46)(33)	f^p_d	0.0242(22)(30)
f^N_c	0.0734(45)(55)	f^n_u	0.0117(11)(15)
f^N_b	0.0702(7)(9)	f^n_d	0.0294(22)(30)
f^N_t	0.0680(6)(7)		

Table taken from Borsanyi et al. (2020)

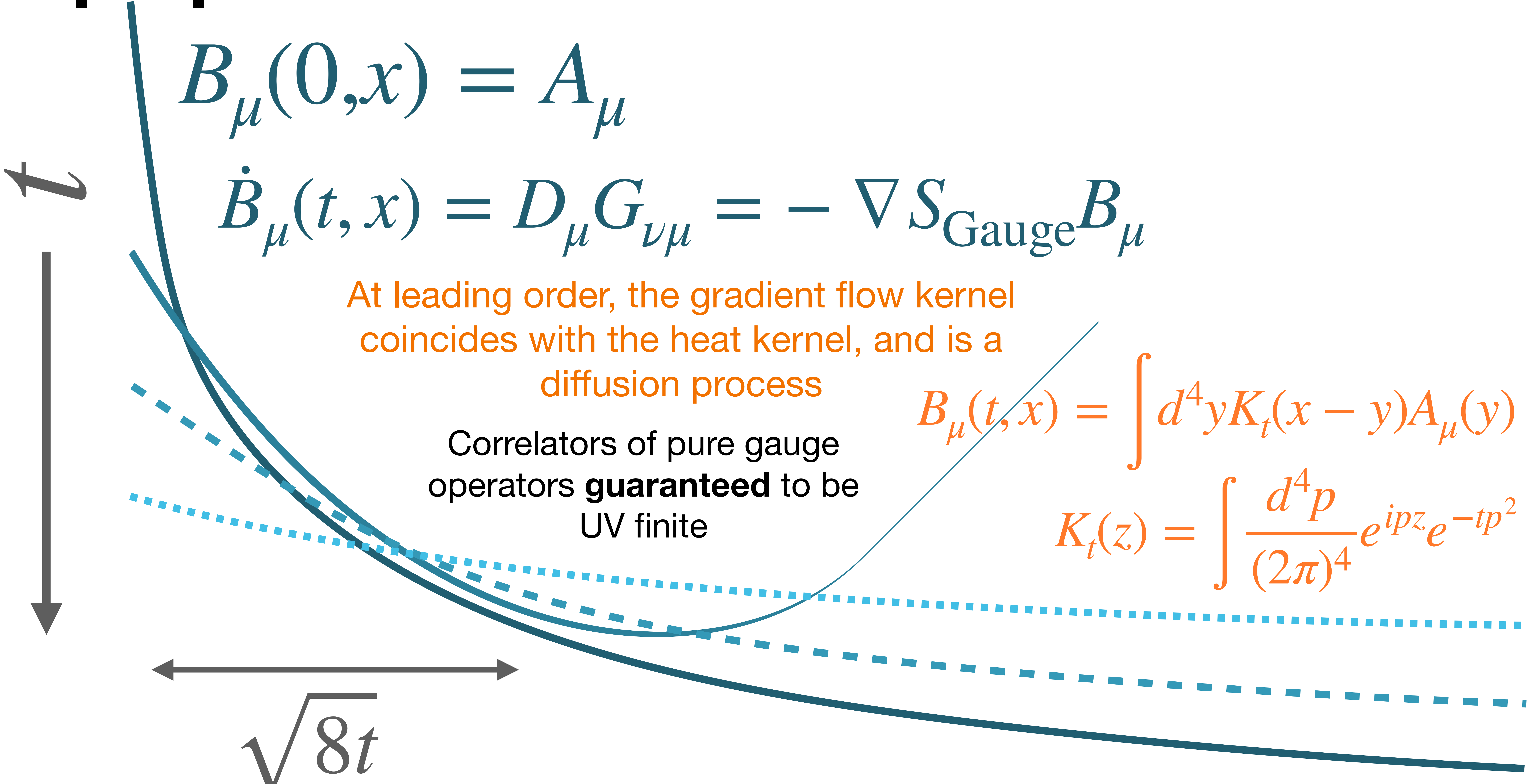
Can we find tools to improve systematics?

Luscher M, 10.1007/JHEP08(2010)071

Our proposal: Gradient Flow

$$B_\mu(0,x) = A_\mu$$

$$\dot{B}_\mu(t,x) = D_\mu G_{\nu\mu} = -\nabla S_{\text{Gauge}} B_\mu$$



t

At leading order, the gradient flow kernel coincides with the heat kernel, and is a diffusion process

$$B_\mu(t,x) = \int d^4y K_t(x-y) A_\mu(y)$$

$$K_t(z) = \int \frac{d^4p}{(2\pi)^4} e^{ipz} e^{-tp^2}$$

Correlators of pure gauge operators **guaranteed** to be UV finite

$$\sqrt{8t}$$

$$[t] = [a^2]$$

Can be easily implemented on the lattice numerically

Gradient Flow: Fermions

Fermions flow similarly to gauge fields

$$\partial_t \chi = \overrightarrow{\Delta} \chi(t,x) \qquad \Delta = D_\mu D_\mu$$

$$\partial_t \bar{\chi} = \bar{\chi}(t,x) \overleftarrow{\Delta} \qquad D_\mu = \partial_\mu + B_\mu$$

Not provably finite

$$\langle \chi(t,x)\bar{\chi}(s,y)\rangle = \int_p e^{ip(x-y)} \frac{e^{-(t+s)p^2}}{i\not{p} + m_0}$$

Fermions require renormalization, but this renormalization is purely multiplicative

$$\chi(t,x) = Z_\chi^{1/2} \chi(t,x) \quad \bar{\chi}(t,x) = Z_\chi^{1/2} \bar{\chi}(t,x)$$

Provided renormalization, GF UV regulates operators

Example: the most power divergent term $\langle \bar{q}q \rangle$ mixes with is $m_q I$

$$\frac{c}{a^2} m_q I \longrightarrow \frac{c}{t} m_q I$$

No power divergences when taking continuum limit at fixed flow time

Short Flow Time Expansion

SFTX is an Operator Product Expansion that separates short distance (perturbatively computable) terms from long distance (lattice) terms

(Getting back quantity free of flow time dependance)

$$\langle \bar{q}q \rangle(t) \sim c_0(t) m_q I + c_1(t) \mathrm{Tr}(M^2) m_q I + c_2(t)(MMM^\dagger)_{qq} + c_3(t) \langle \bar{q}q \rangle(0) + \mathcal{O}(t)$$

$\sim 1/t$

$\sim \log(t)$

We want this

Vacuum subtraction (normal ordering) isolates relevant operator $c_3(t)$

$$\langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(t) = \langle \mathcal{N} | \bar{q}q | \mathcal{N}^\dagger \rangle(t) - \langle \bar{q}q \rangle(t) \langle \mathcal{N} \mathcal{N}^\dagger \rangle$$

$$= c_3(t) \langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(0) + \mathcal{O}(t)$$

Shindler A, 10.48550/arXiv.1409.2735

Luscher M, 10.48550/arXiv.1308.5598

Luscher M, 10.48550/arXiv.1308.5598
Luscher M, 10.1007/JHEP04(2013)123
Shindler A, 10.48550/arXiv.1409.2735

How to determine $c_3(t)$?

$$\langle \mathcal{N} \,|\, : \bar{q}q : \,|\, \mathcal{N}^\dagger \rangle(t) \sim c_3(t) \langle \mathcal{N} \,|\, : \bar{q}q : \,|\, \mathcal{N}^\dagger \rangle(0) + \mathcal{O}(t)$$

Chiral Ward identities link the scalar condensate (Σ) to the pion correlator (G_π) in the $x_0 \to \infty$ limit

$$\Sigma = \Sigma_t \frac{G_\pi}{G_{\pi,t}}$$

$c_3(t)$ can be determined non-perturbatively from the ratio of pion correlators at 0 and non-zero flow time

$$c_3(t) = \frac{G_{\pi,t}}{G_\pi} + \mathcal{O}(t)$$

If fermions scheme only has multiplicative renormalization, nothing else needed to compute $c_3(t)$

We eventually want $m_q \langle \mathcal{N} \,|\, : \bar{q}q : \,|\, \mathcal{N}^\dagger \rangle$ (RGI), so no extra renormalization is needed

Finding a window where STFX is valid:

Task: scan $\langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(t) \dfrac{G_\pi}{G_{\pi,t}}$ in time

and **on dimensional grounds** identify window where $\langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(0) + \mathcal{O}(t)$ is "visible"

Very low t:
Discretization effects and divergences

High t:
Leave hadronic scale
Finite volume effects
Mixing with higher dimension operators
Mess up spectral decomposition

$a \longrightarrow \sqrt{8t}$

$\Lambda_{\mathrm{QCD}}^{-1}$ $\quad L_T$

$\dfrac{1}{m_q}$

Low t "window":

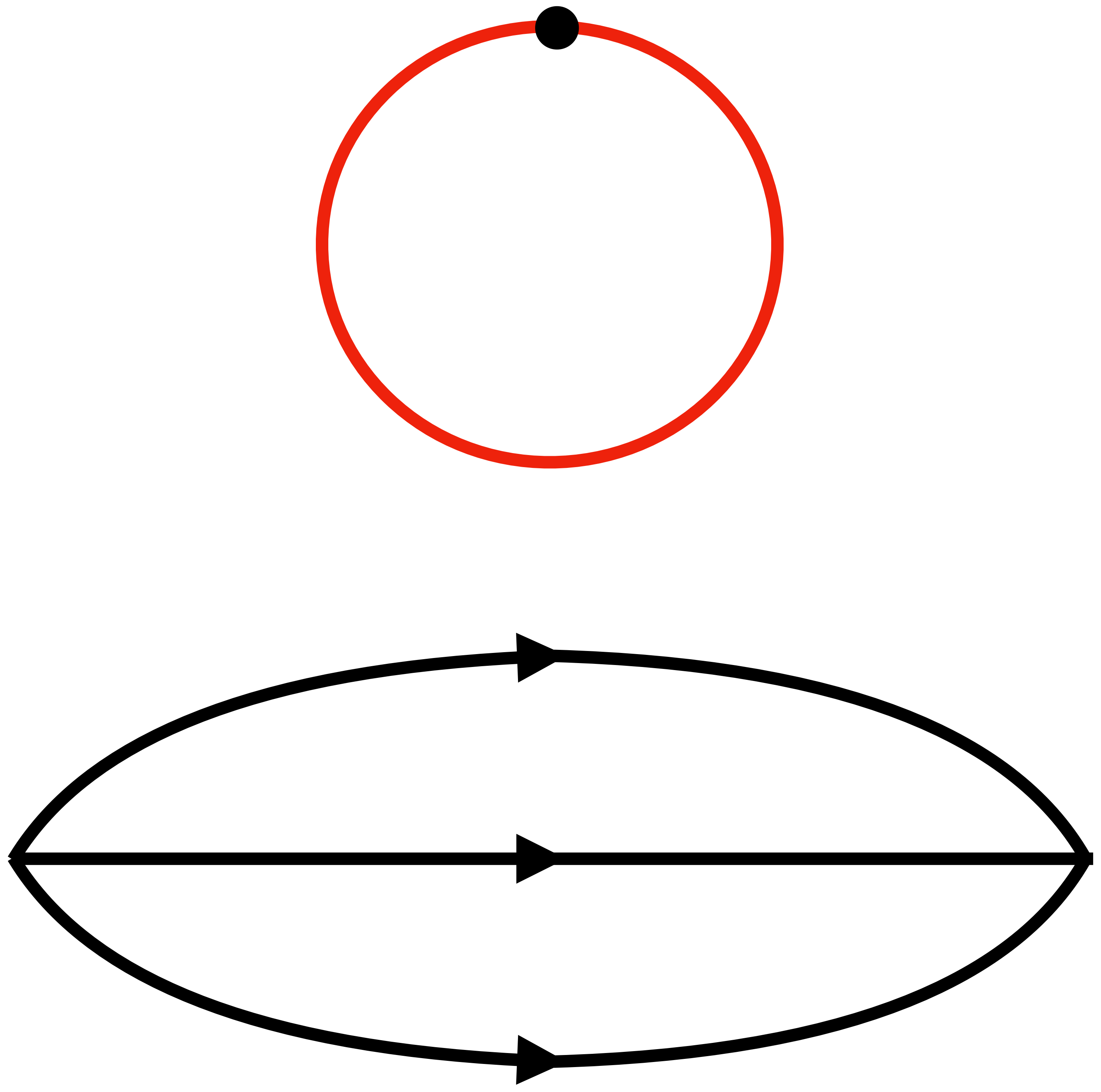
$\langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(t) \dfrac{G_\pi}{G_{\pi,t}}$

varies like low order polynomial

Can extrapolate to $t \to 0$

Shindler A, 10.48550/arXiv.1409.2735

Gradient Flow of Disconnected loops



$$\xi_k(t;s,w) = \sum_v K(t;v;s,w)^\dagger \eta_k(v)$$

Forwards flow

Backwards flow

$$\langle S_t^{rr}(x)\rangle = \langle \xi_k(t;0,v)^\dagger S(v,w)_{rr} \xi_k(t;0,w)\rangle$$

Backwards flow time evolution is exponentially unstable when calculated numerically

From Backwards Flow to Adjoint Flow

Workaround

Suppose we create a stochastic vector in fermion space and **assign** it time $-t$

What if this fermion field was the result of backwards flow from time 0 to $-t$? Then what is the value of the field at time 0?

This is *adjoint flow*, and is well defined

Adjoint flow

Starting Normal Fermion

$$\chi_0 \xrightarrow{\text{Normal flow}} \chi_n$$

U_n Starting gauge field U_0 U_n

$$\xi_{-n} \xrightarrow{\text{Adjoint flow}} \xi_0$$

Starting Adjoint Fermion

Luscher M, 10.1007/JHEP04(2013)123

Adjoint flow

$$R_t^{\epsilon\dagger}(U_t)\xi_{t+\epsilon}^{\epsilon} = \xi_t$$

Adjoint flow step $R_t^{\epsilon\dagger}$ transposes operators of normal flow

$$U_n \quad U_{n-1} \quad U_{n-2} \quad \cdots\cdots \quad U_0$$

Starting gauge field

Starting Adjoint Fermion (e.g. noise η)

$$\xi_{-n} \quad \xi_{1-n} \quad \xi_{2-n} \quad \cdots\cdots \quad \xi_0$$

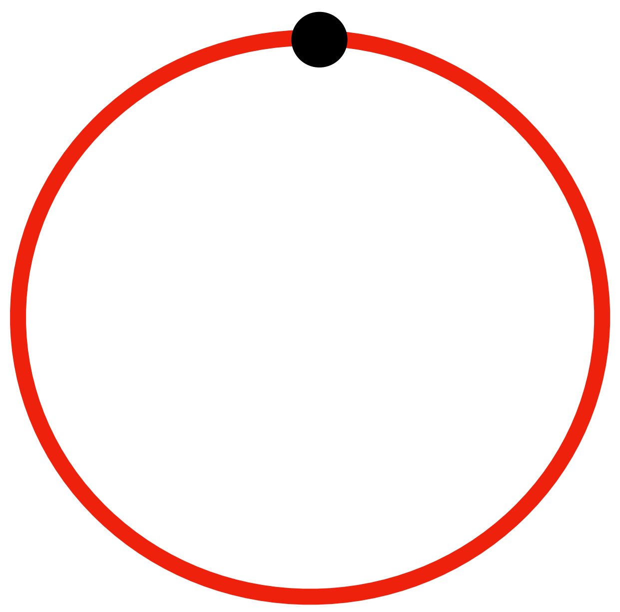
"Safe" algorithm:
Only U_0 is saved, and computation scales as $\mathcal{O}(n^2)$

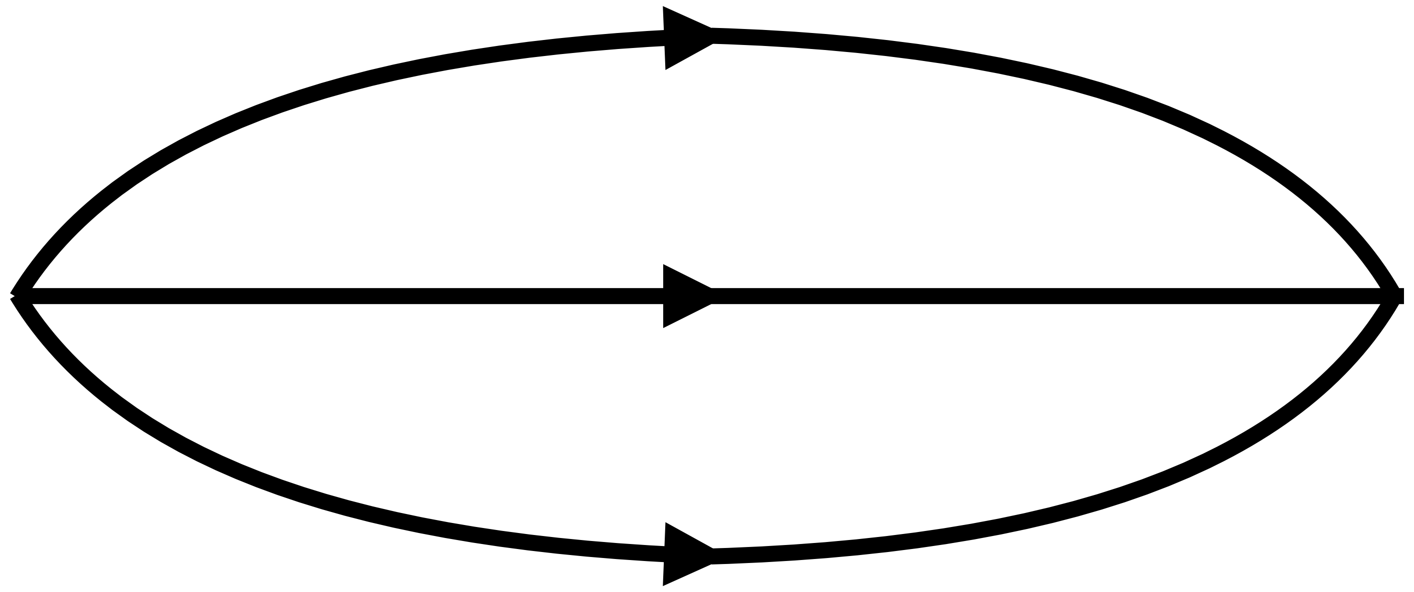
"Hierarchical" algorithm:
Reserve memory for s gauge fields to be saved. Then divide total n flow steps into n_b blocks to be hierarchically flowed. Choices of s, n_b can achieve $\mathcal{O}(n)$ performance.
This algorithm is crucial when running production data.

quda

We now maintain both safe and hierarchical algorithms in QUDA

Test Ensemble





Gauge fields generated with HISQ action,

$N_f = 2 + 1 + 1$

Flowed HISQ fermions for disco loop

MDWF nucleon 2-pt correlators

16 sources each for positive and negative parity

$$a = 0.12 \text{ fm}$$

$$M_\pi = 310 \text{ MeV}$$

$$V = 24^3 \times 64$$

1053 gauge configs

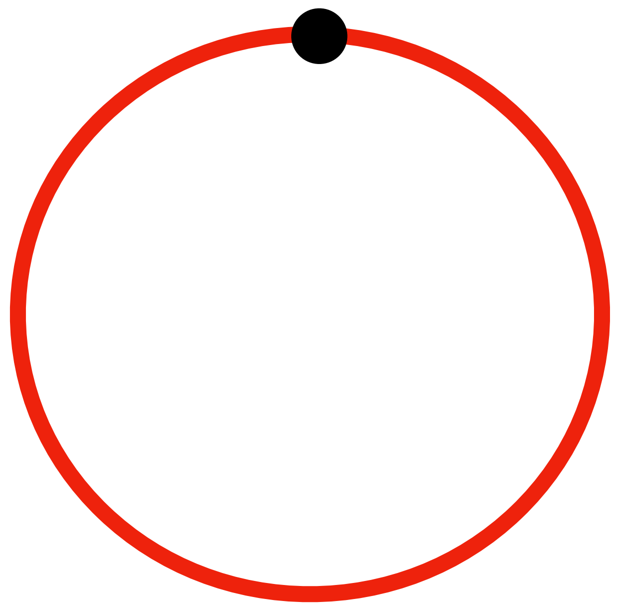
$$t_0 = 1.7 \text{ fm}^2$$

$$am_c = 0.6382$$

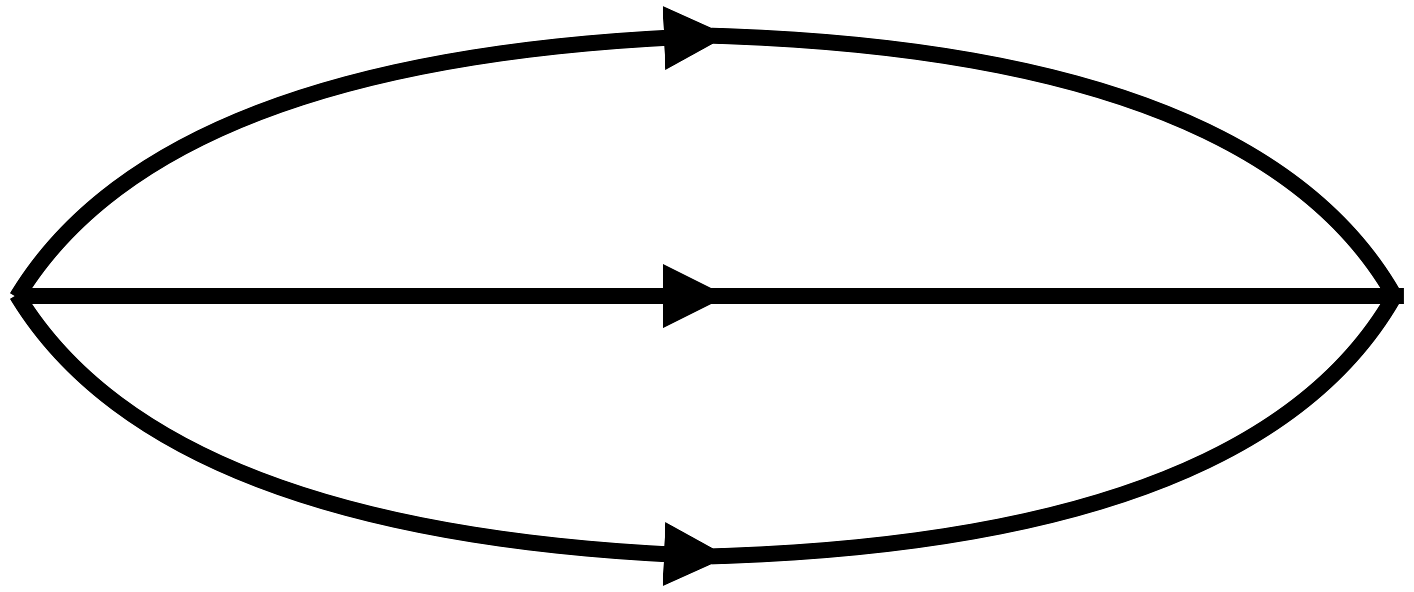
$$am_s = 0.052520$$

$$am_l = 0.011$$

Extracting Ground State



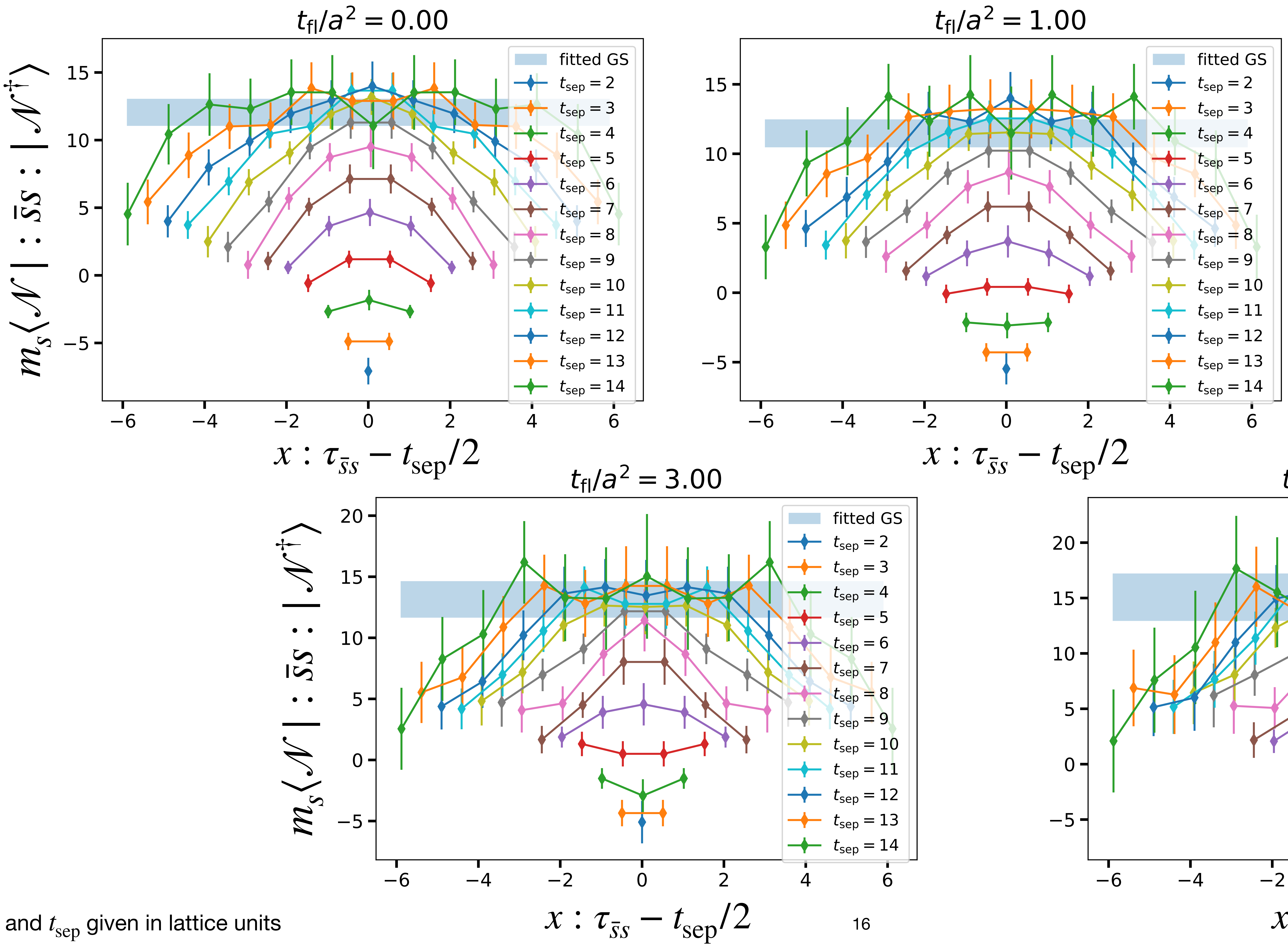
$$C_3(t_{\text{fl}}; t_{\text{sep}}, \tau) = \sum_{\mathbf{x},\mathbf{y}} \langle \mathcal{N}(t_{\text{sep}}, \mathbf{y}) \,|\, :\bar{q}q(t_{\text{fl}}, \tau):\, |\, \mathcal{N}^\dagger(0,\mathbf{x}) \rangle$$



$$C_2(t_{\text{sep}}) = \sum_{\mathbf{x},\mathbf{y}} \langle \mathcal{N}(t_{\text{sep}}, \mathbf{y}) \mathcal{N}^\dagger(0,\mathbf{x}) \rangle$$

Identify plateau with ground state

$$R_{\bar{q}q}(t_{\text{fl}}; t_{\text{sep}}, \tau) = \frac{C_3(t_{\text{fl}}; t_{\text{sep}}, \tau)}{C_2(t_{\text{sep}})} \xrightarrow[t_{\text{sep}}-\tau\to\infty]{t_{\text{sep}}\to\infty} \langle \mathcal{N}_0 \,|\, \bar{q}q \,|\, \mathcal{N}_0^\dagger \rangle$$



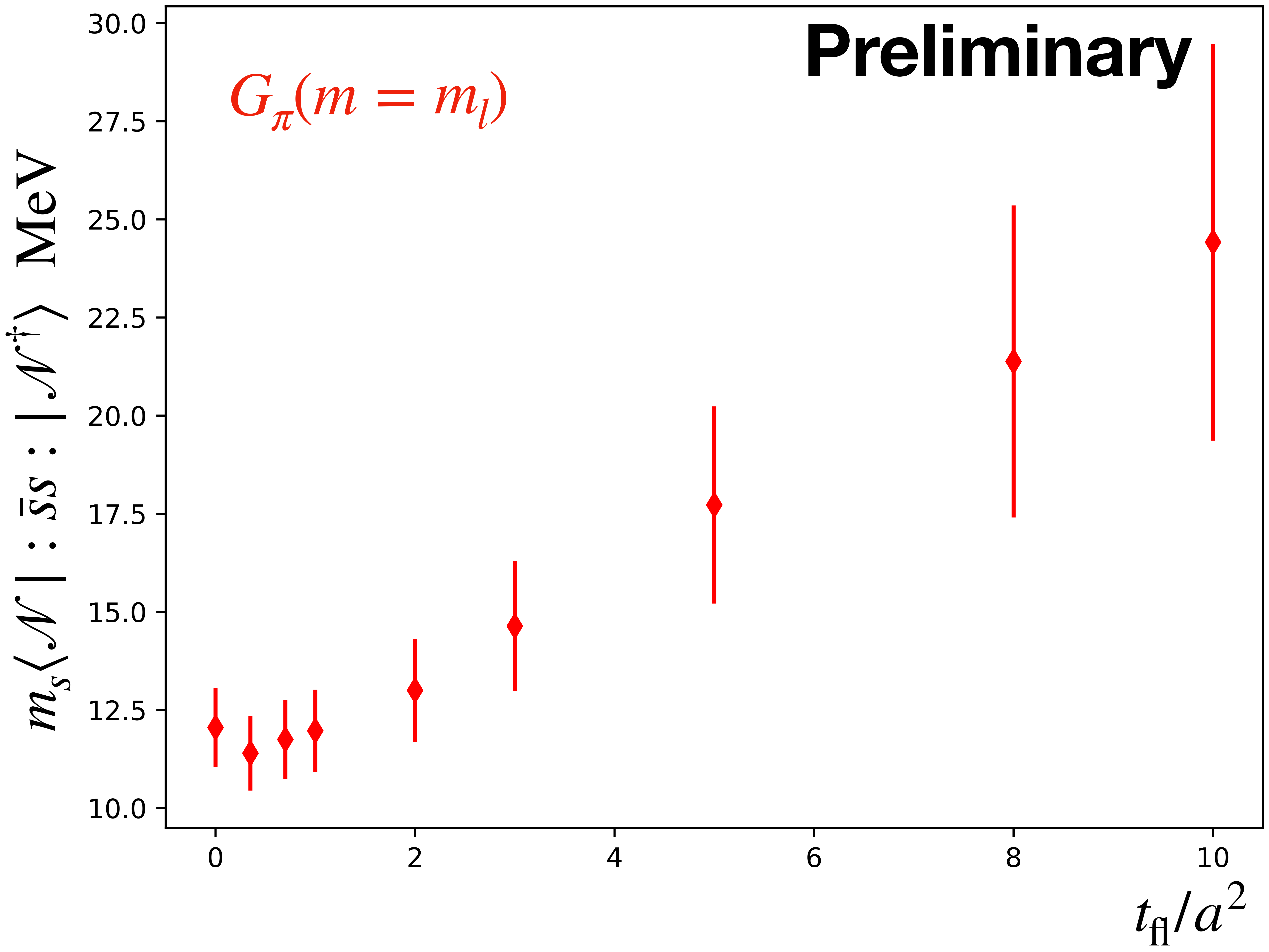
$R_{\bar{s}s}$ plots

$G_\pi(m = m_l)$

Preliminary

τ and t_{sep} given in lattice units

For strange, SFTX window seemingly easy to spot



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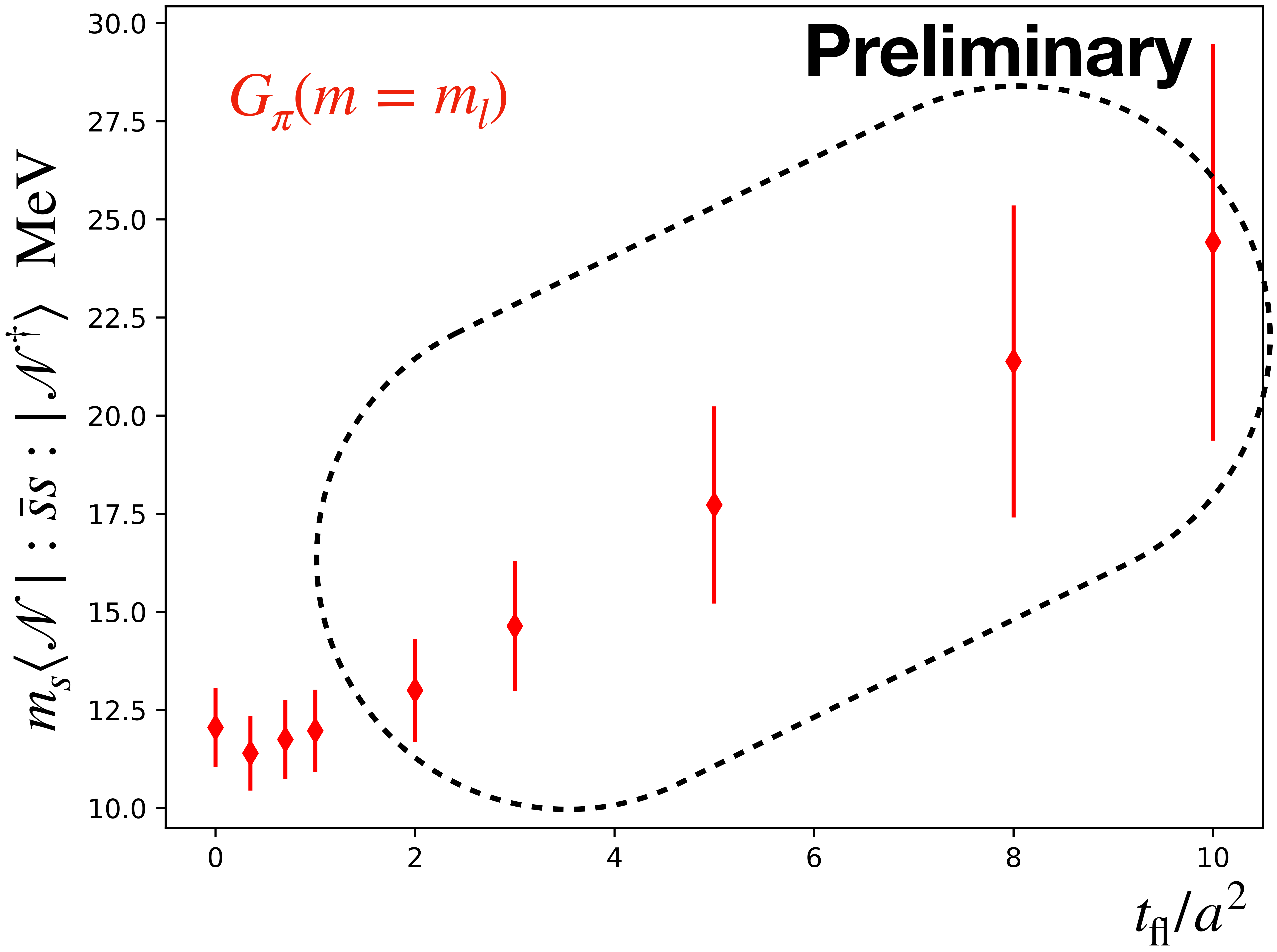
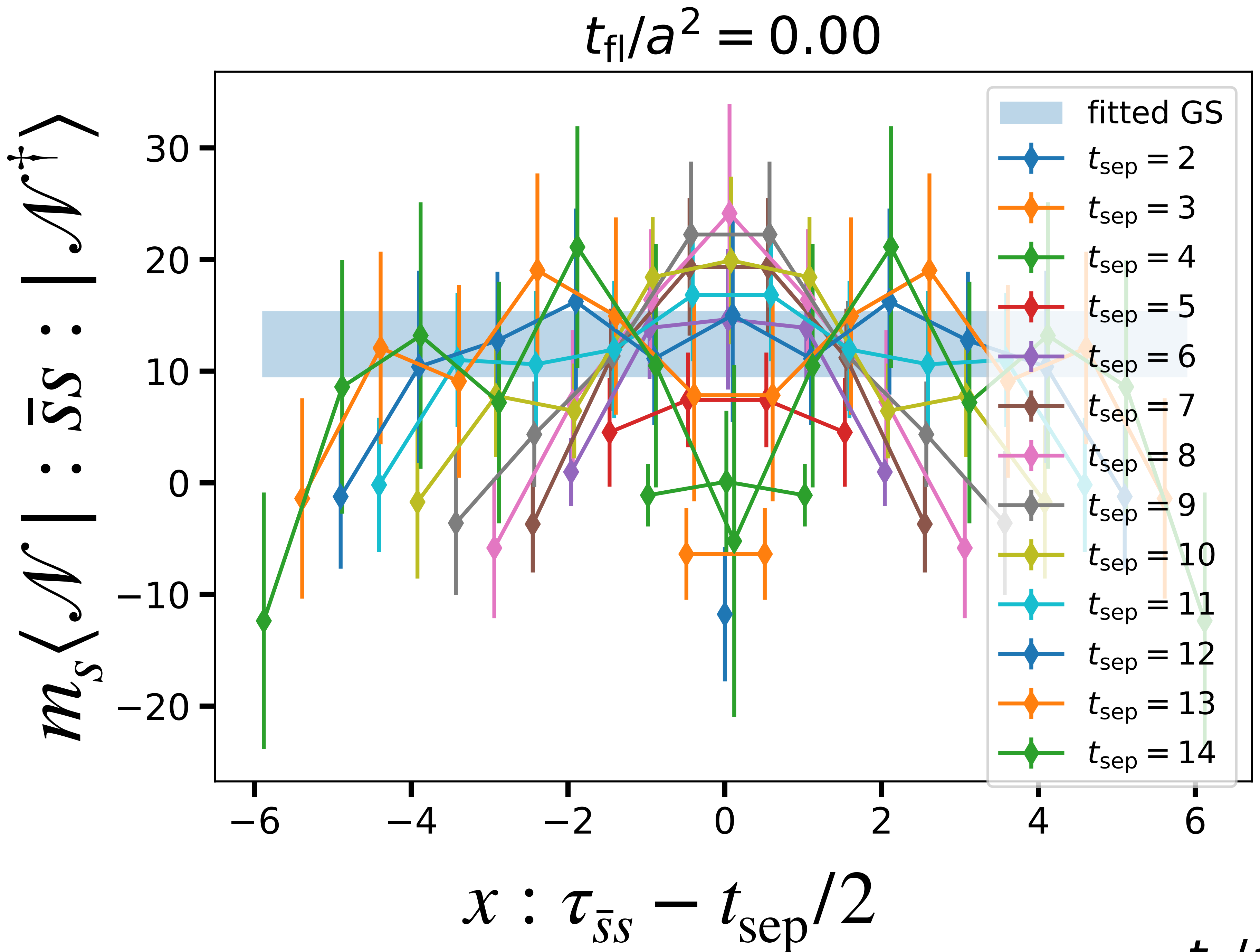
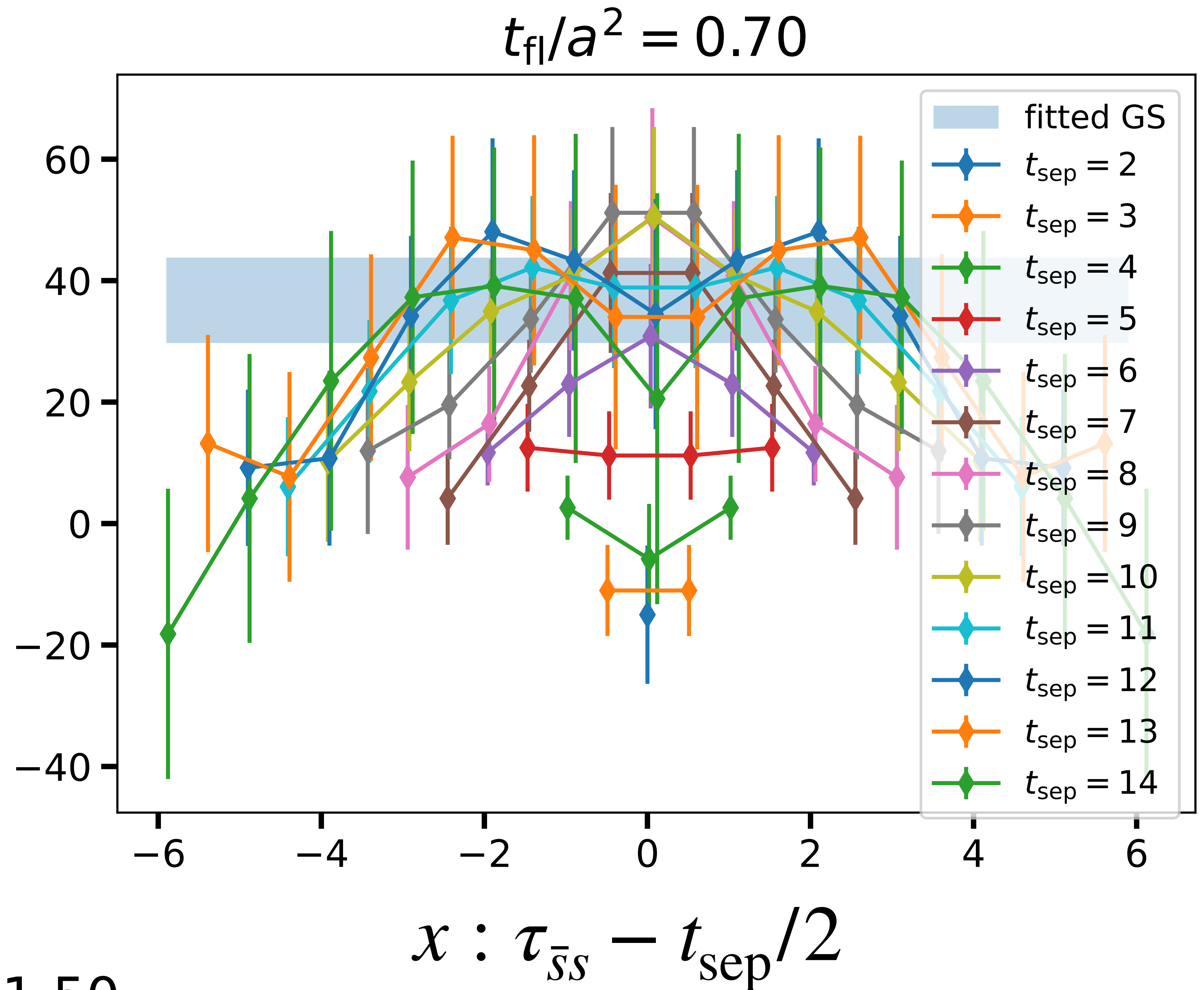
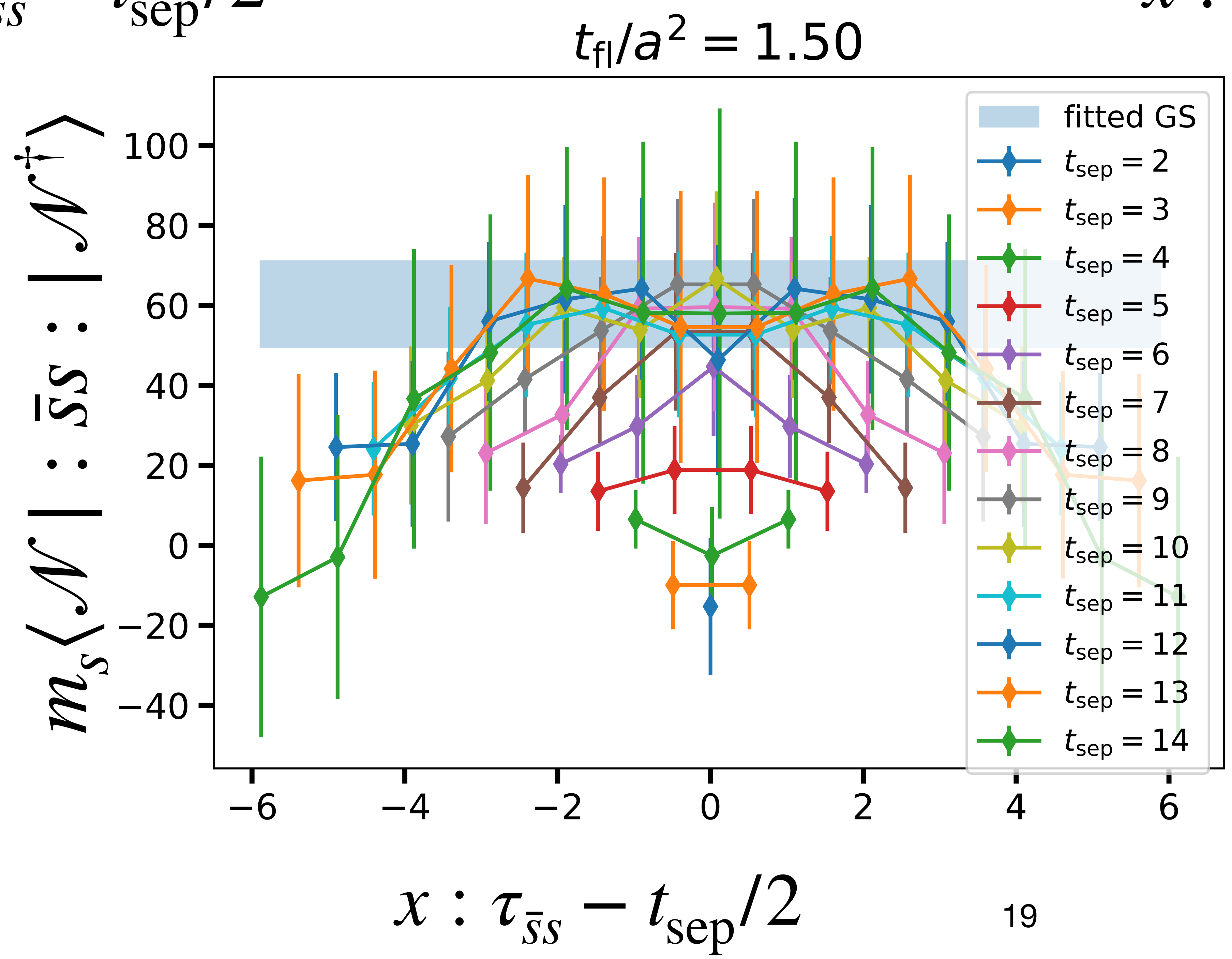


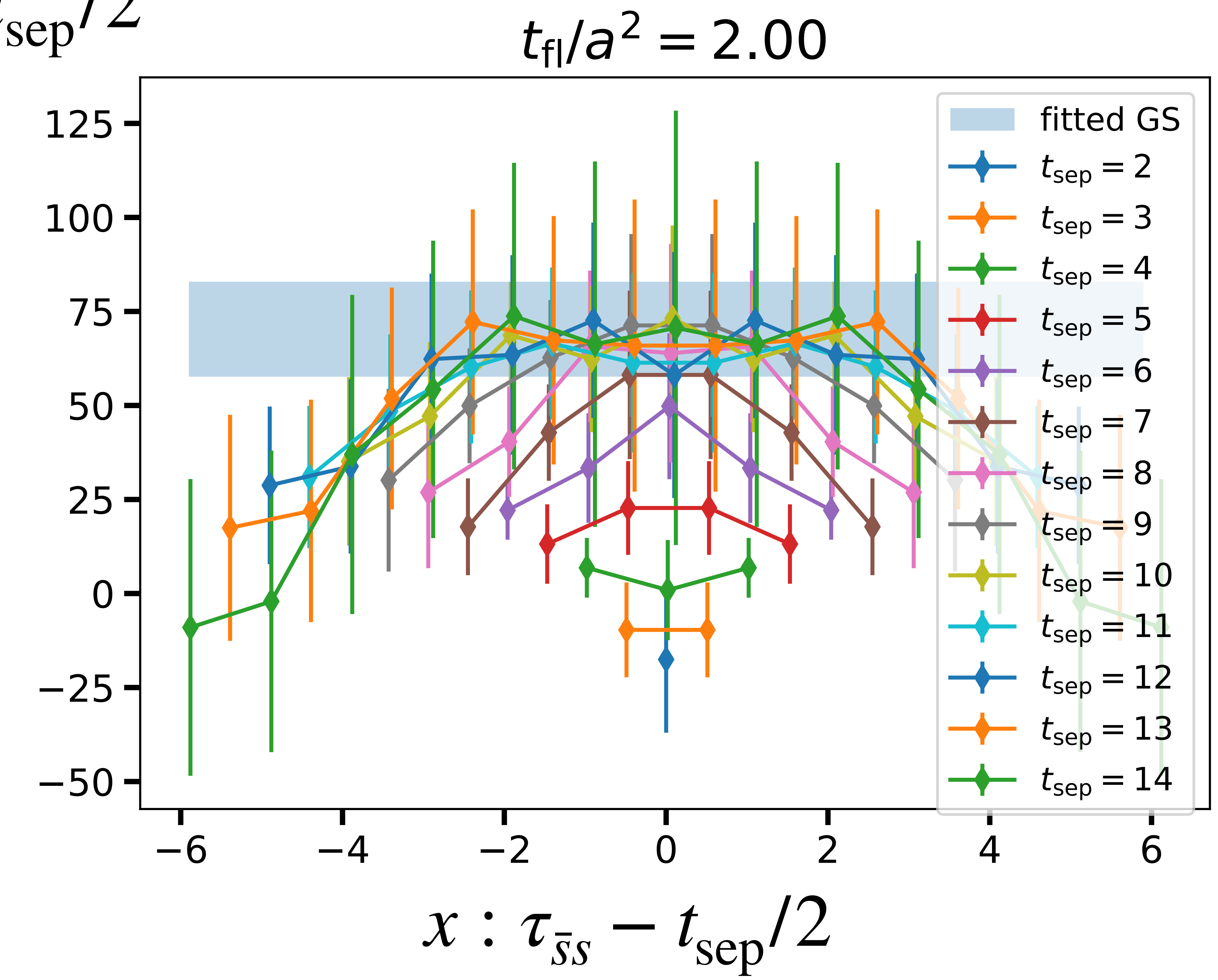




Preliminary

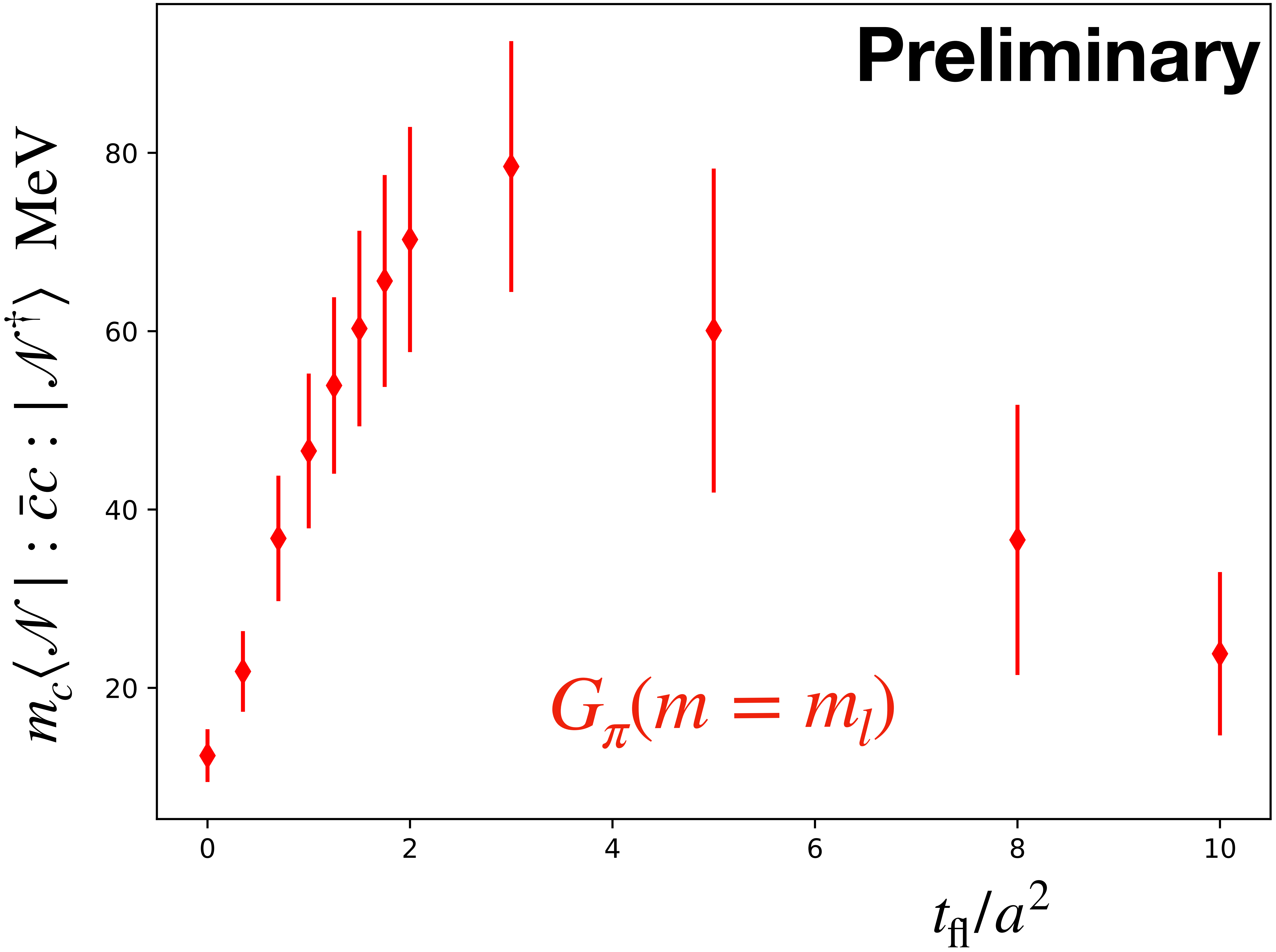
$R_{\bar{c}c}$ plots $G_\pi(m = m_l)$ used





τ and t_{sep} given in lattice units

Less conclusive for charm



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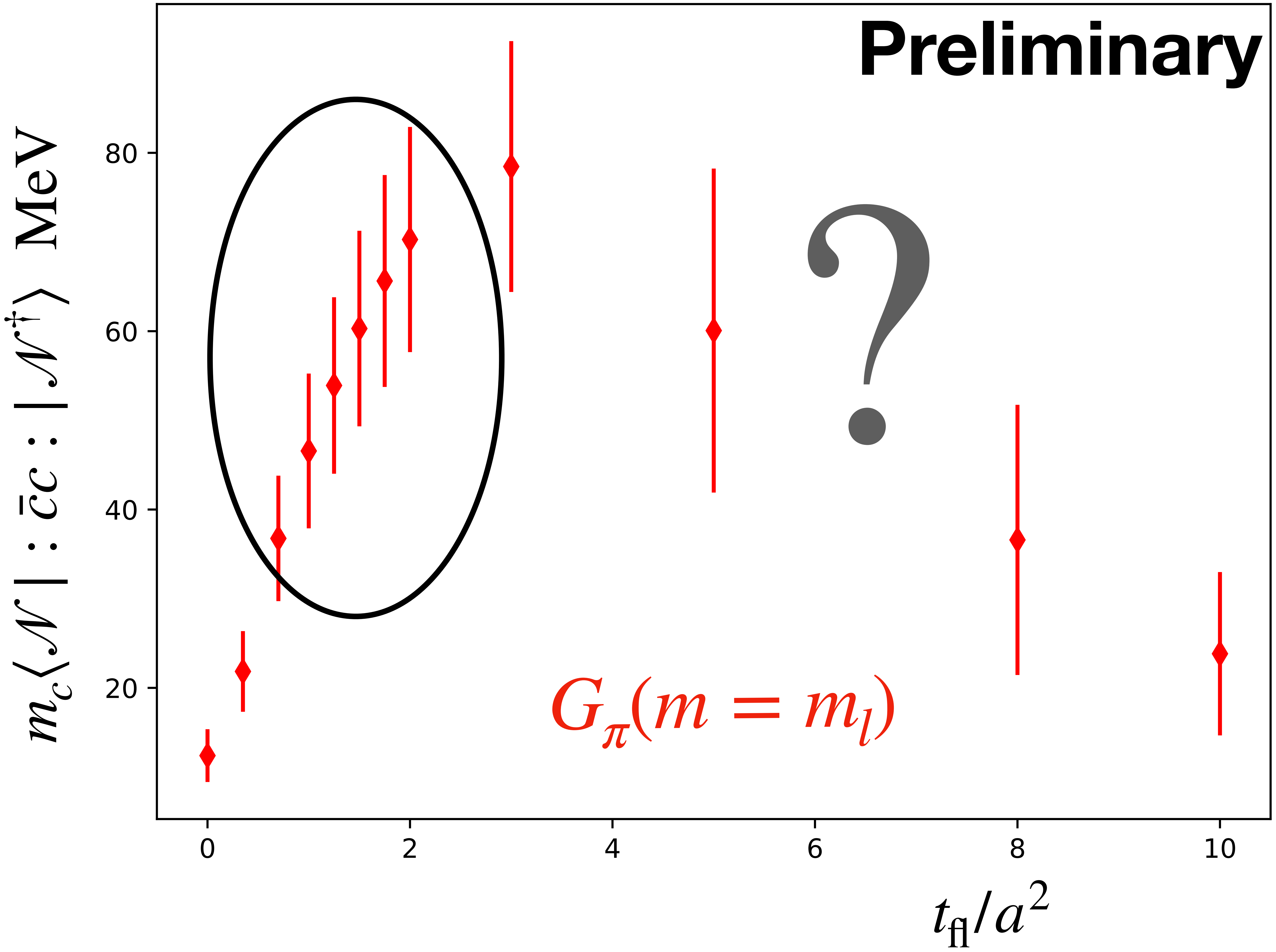
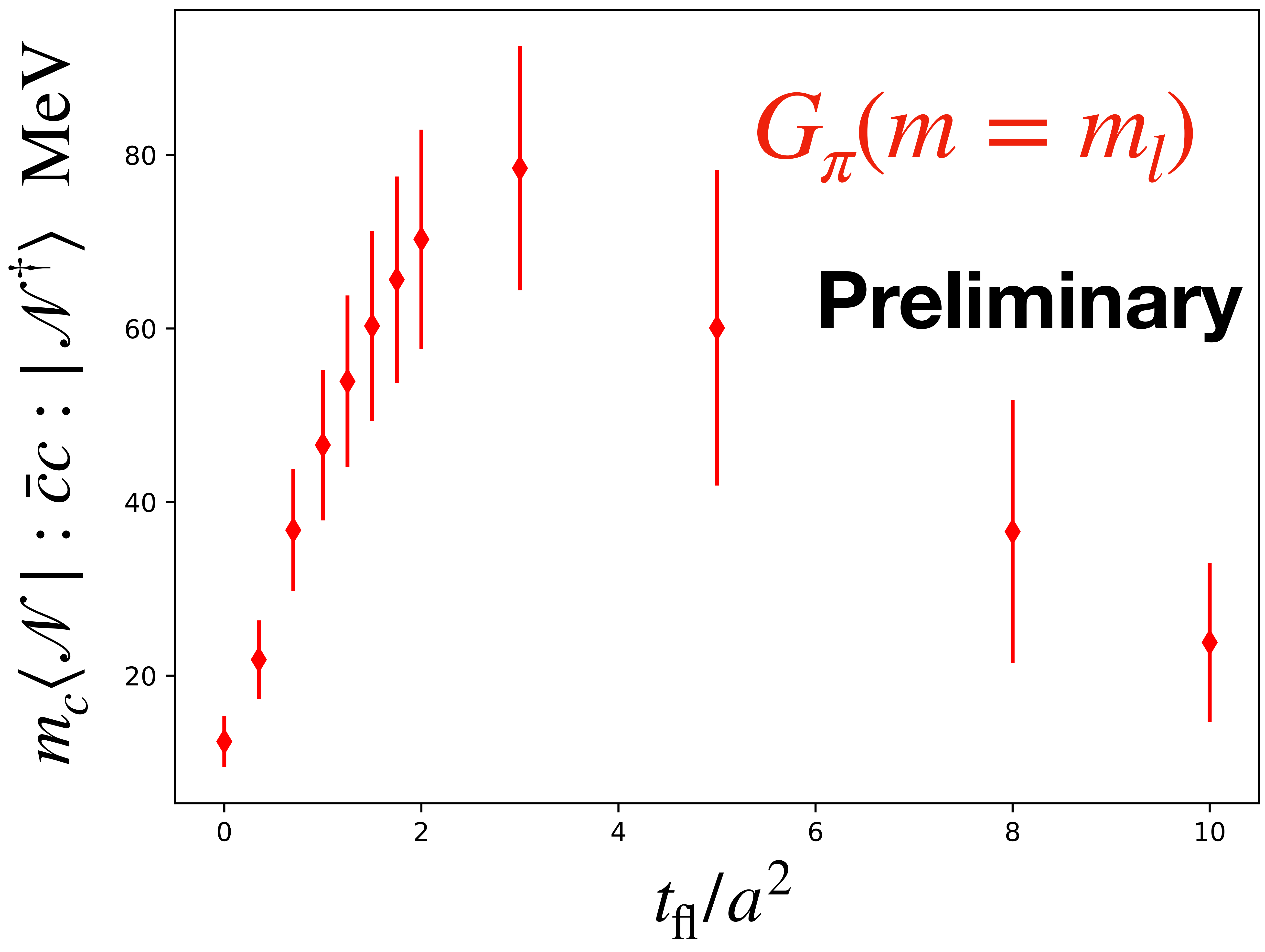


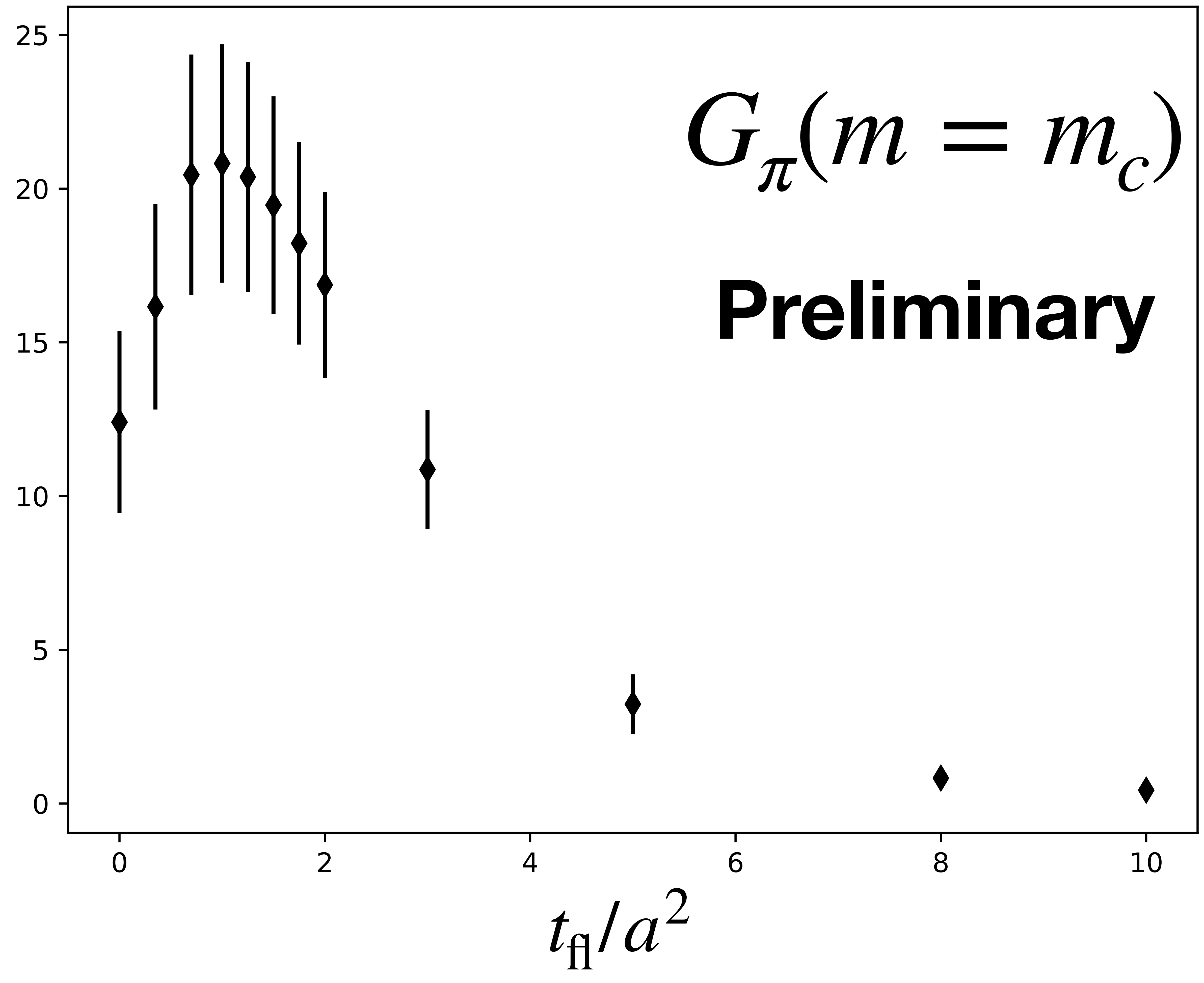
Where's the window?

All this while, G_π was constructed with light quarks
Maybe try to vary the quark in the creation of G_π?

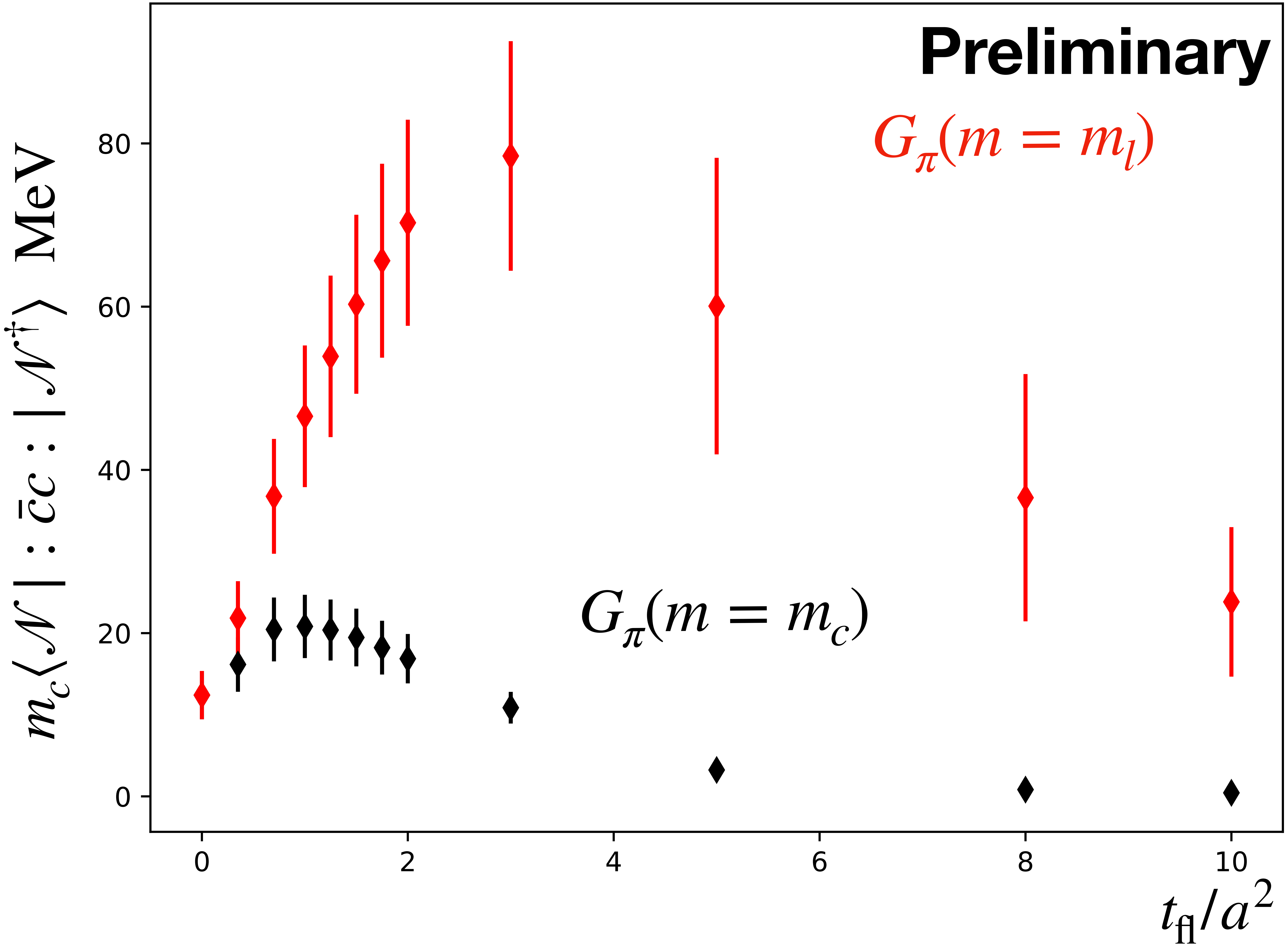
Still not immediately obvious



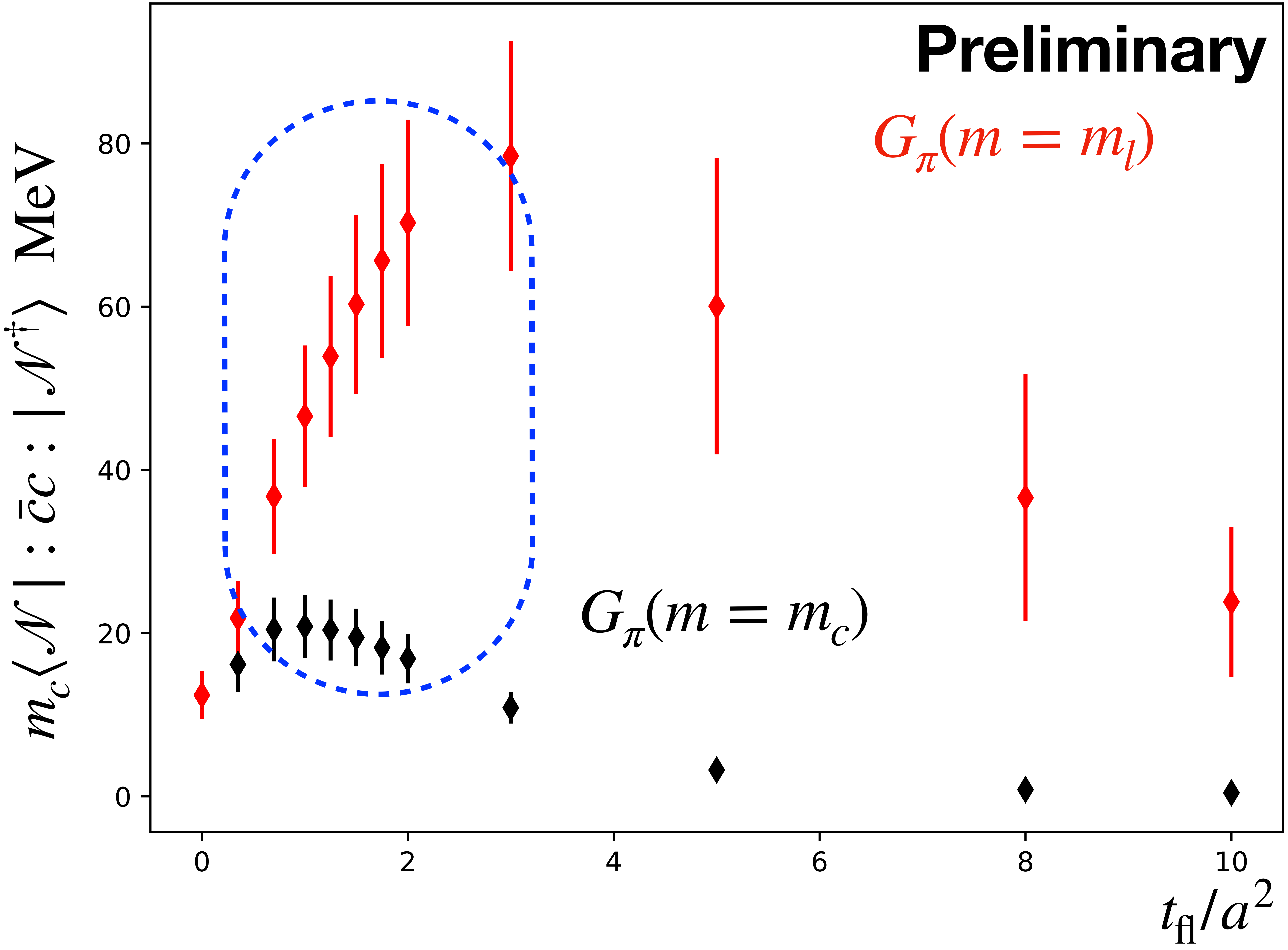
Maybe overlay both?



Now clearer



Preliminary
$G_\pi(m = m_l)$
$G_\pi(m = m_c)$
$m_c \langle \mathcal{N} | : \bar{c}c : | \mathcal{N}^\dagger \rangle$ MeV
t_{fl}/a^2
0
20
40
60
80
0
2
4
6
8
10



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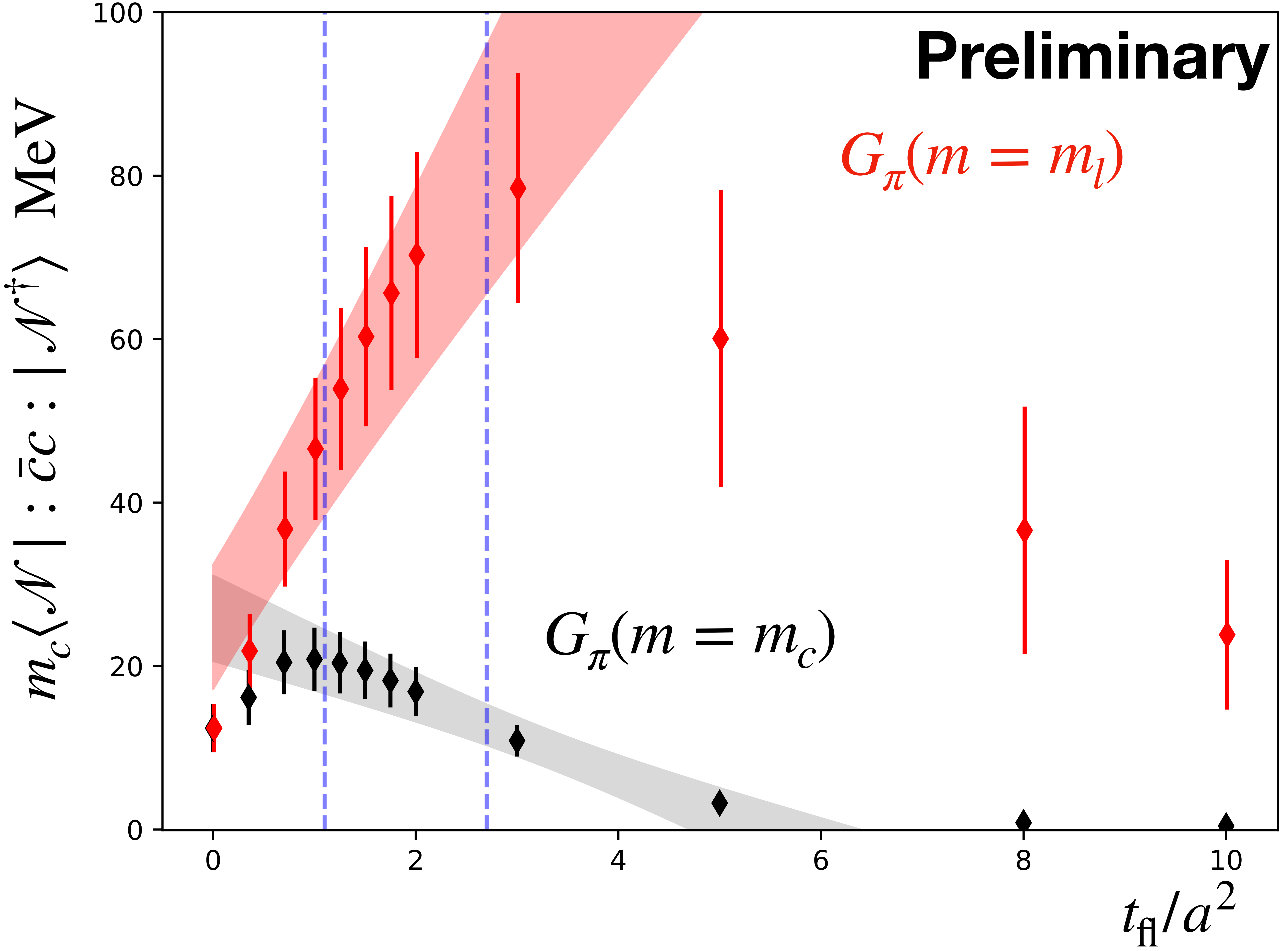

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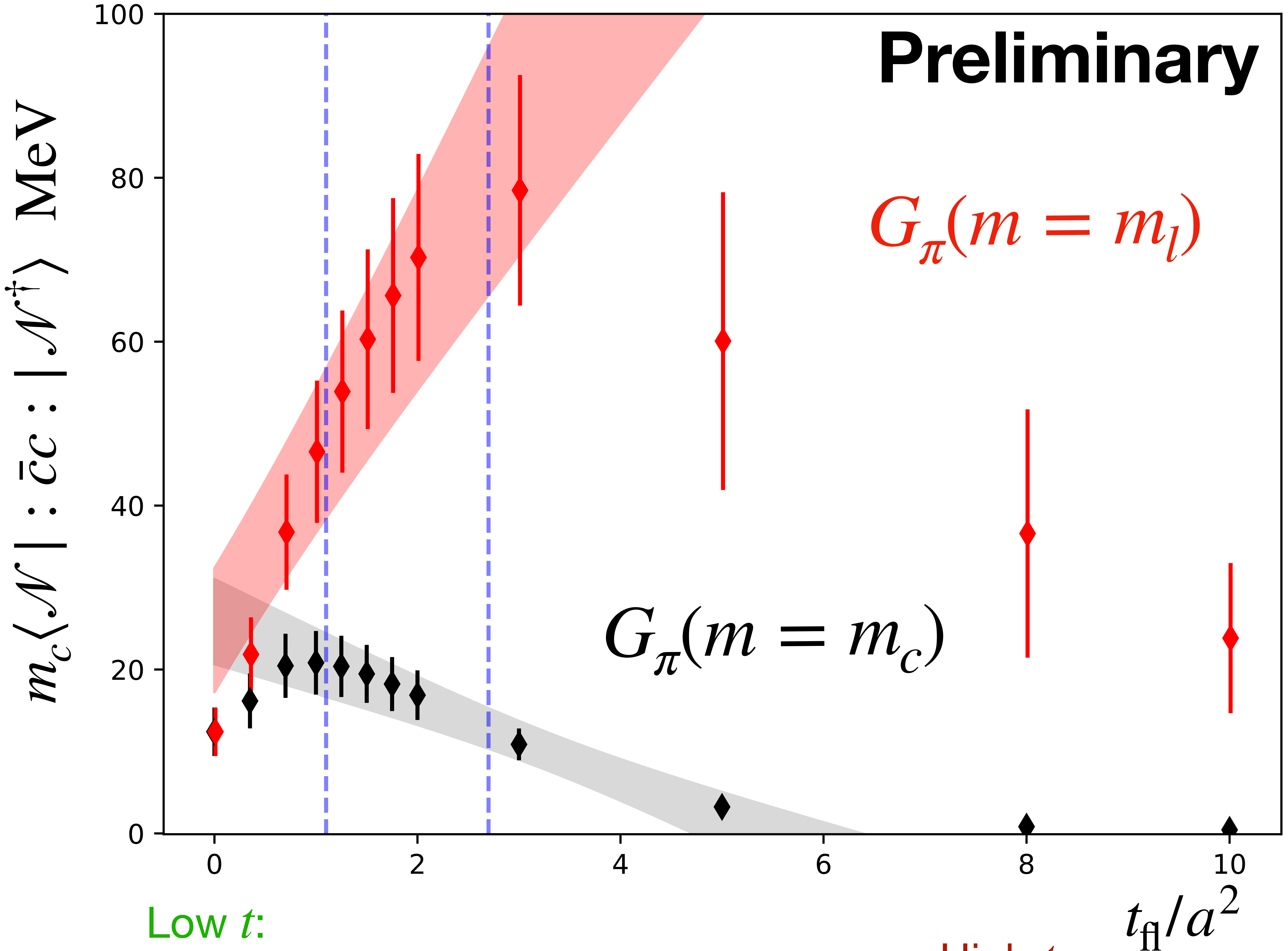
$m_c\langle \mathcal{N} | : \bar{c}c : | \mathcal{N}^\dagger \rangle$ MeV
t_{fl}/a^2
0
20
40
60
80
100
0
2
4
6
8
10

$$G_{\pi,t} = c_3(t)G_\pi + Am^2tG_\pi + \mathcal{O}(t)$$

$$\Sigma = \Sigma_t \frac{G_\pi}{G_{\pi,t}}$$

$$\Sigma = \frac{\Sigma_t}{c_3(t)}\left(1 - \frac{Am^2t}{c_3(t)} + \mathcal{O}(t)\right)$$

$c_3(t)$ evolves independently of quark mass used to construct the pseudoscalar, so we can use a different mass than m_ℓ



Very low t:
Discretization effects and divergences from $c_3(t)$ (**Bad**)

Low t:
$\langle \mathcal{N} | : \bar{q}q : | \mathcal{N}^\dagger \rangle(t)$ varies like low order polynomial
Can extrapolate to $t \to 0$

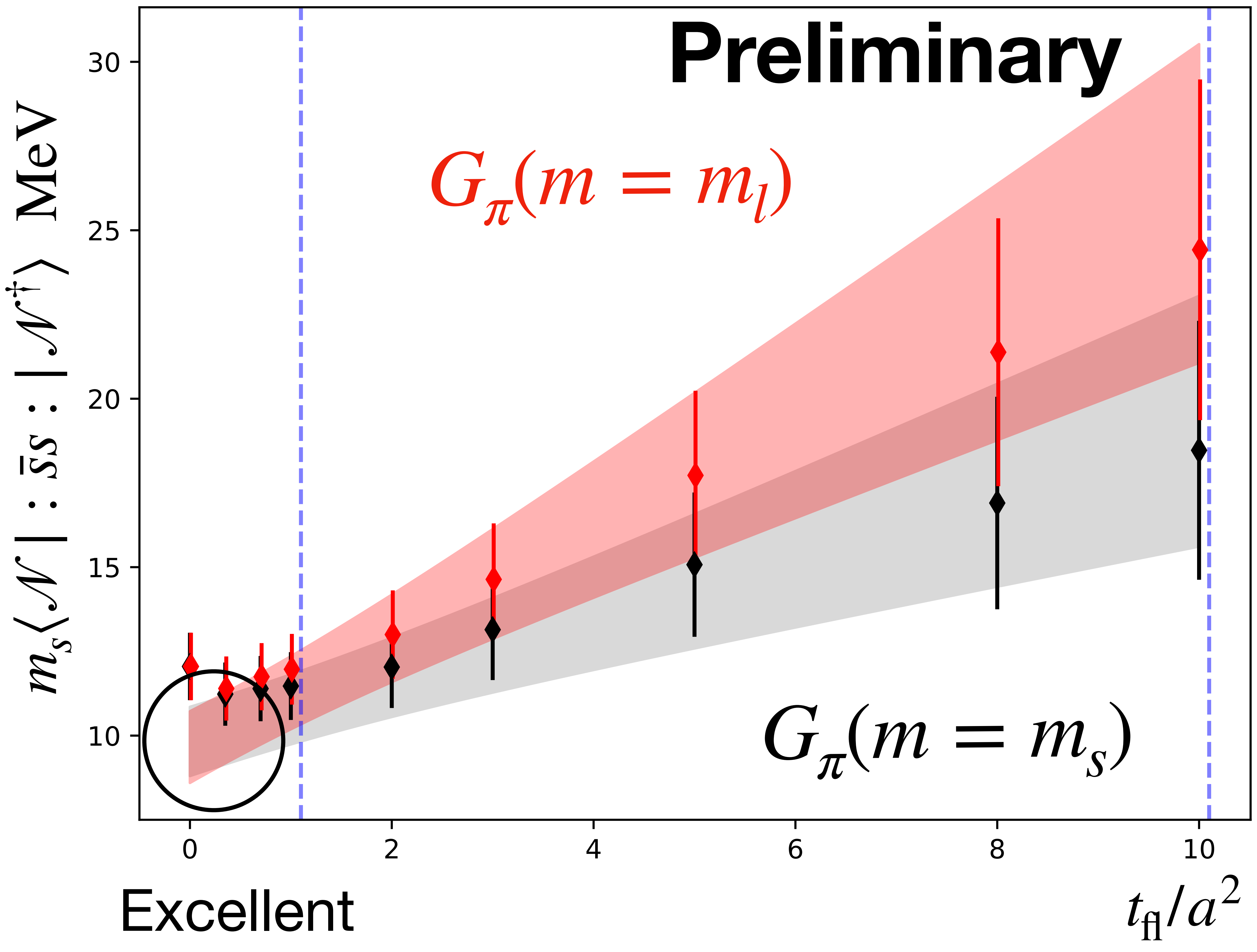
High t:
Bad

a $\longrightarrow$ $\sqrt{8t}$

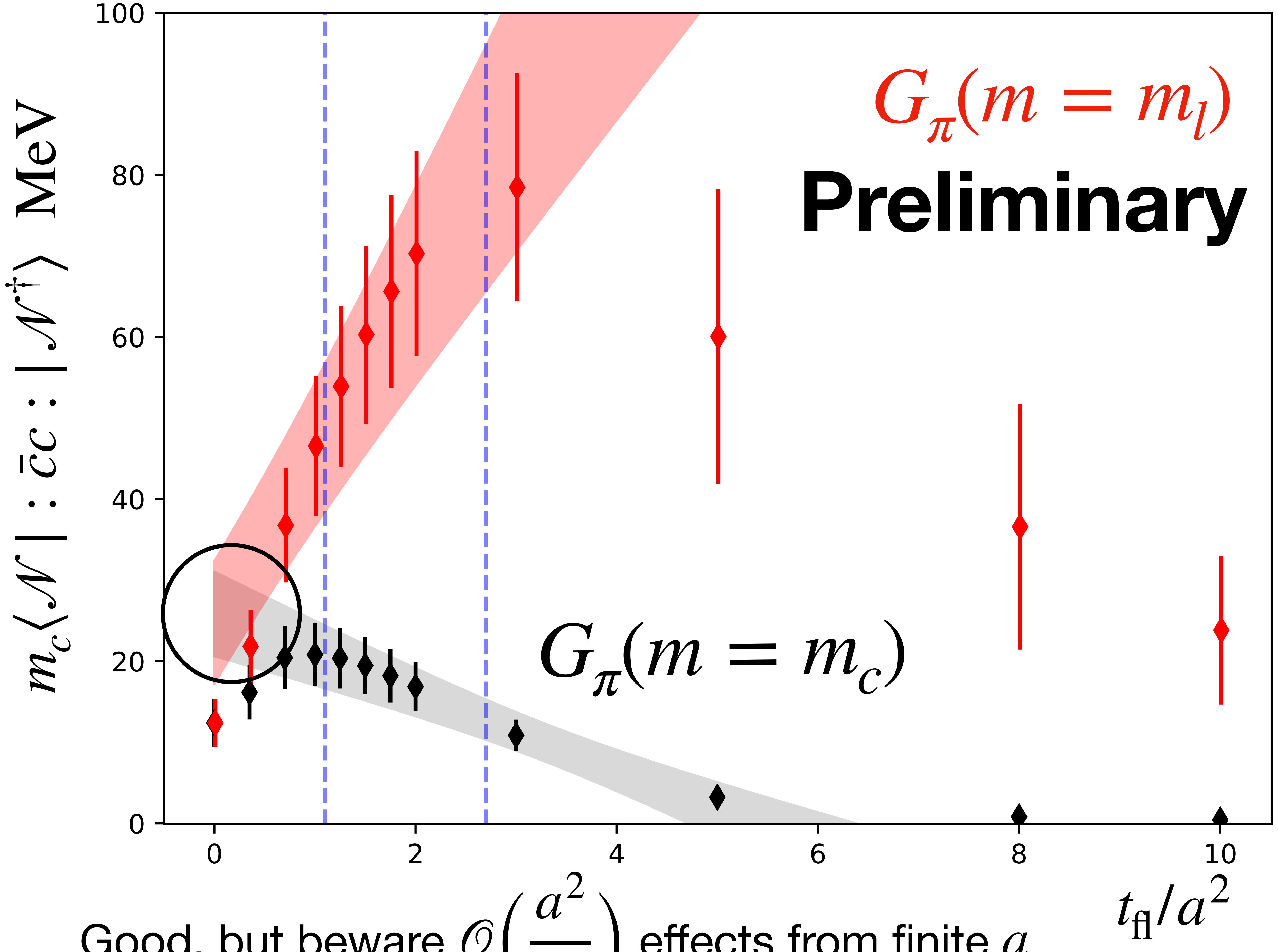
$\frac{1}{m_q}$ $\Lambda_{\text{QCD}}^{-2}$ L_T

Varying m could potentially identify SFTX windows decisively

Now trying same procedure on strange



Excellent



Good, but beware $\mathcal{O}\left(\frac{a^2}{t}\right)$ effects from finite a

Both display at least 1σ deviation from unflowed sigma term

Conclusion

Gradient flow is a promising prescription for systematically removing UV artifacts from disconnected loop calculations on the lattice

Choice of mass in creating G_π is helpful in identifying SFTX viability window

Efficient adjoint flow algorithms necessary to effectively flow scalar condensate

Things to do

Run analysis on many ensembles at different M_π + lattice spacing and conduct $a \to 0$/physical $M_\pi$extrapolation

Assess excited state contamination (using summed correlators,…etc)

Further investigate impact of varying mass of G_π on analysis (simultaneous fits,…etc)

Explore different techniques for calculating trace for light quarks

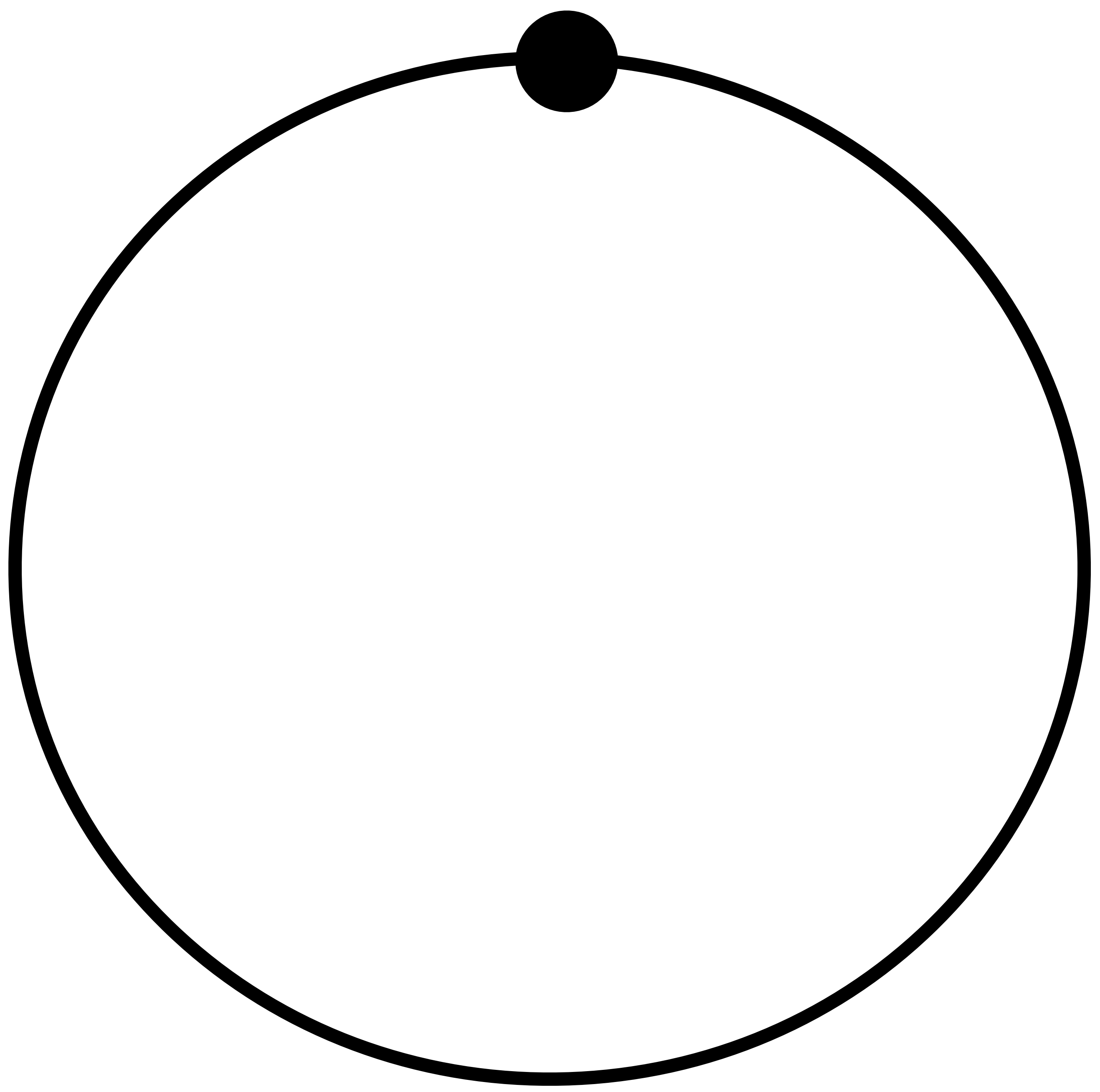
BACKUP SLIDES

Gradient Flow: Disconnected loops

η: stochastic source

$$\langle \bar{q}q\rangle(t) \longleftarrow -\langle \eta \,|\, R_t(U_t)\ldots R_0(U_0)D^{-1}(U_0)R_0^{\dagger}(U_0)\ldots R^{\dagger}_{-t}(U_{-t})\,|\, \eta\rangle$$

Every flow time measurement of $\langle \bar{q}q\rangle(t)$ requires a new initial η



Adjoint flow

Starting Normal Fermion

$$\chi_1 \xrightarrow{\text{Normal flow}} \chi_n$$

Starting gauge field

$$U_n \qquad U_1 \qquad U_n$$

$$\xi_n \xrightarrow{\text{Adjoint flow}} \xi_1$$

Starting Adjoint Fermion

Flow Operator

$$R_t^{\epsilon}$$

Time step size

Time coordinate

$$R_t^{\epsilon\dagger}(U_t)\xi^{\epsilon}_{t+\epsilon} = \xi_t \qquad R_t^{\epsilon}(U_t)\chi_t^{\epsilon} = \chi^{\epsilon}_{t+\epsilon}$$

Adjoint flow

Starting Normal Fermion

$\chi_0 \xrightarrow{\text{Normal flow}} \chi_n$

Starting gauge field

$U_n \qquad U_0 \qquad U_n$

$\xi_n \xrightarrow{\text{Adjoint flow}} \xi_0$

Starting Adjoint Fermion

$$\langle \phi \,|\, R_n R_{n-1} \cdots R_1 \,|\, \phi \rangle$$

Arbitrary Demarcation

$$\langle \phi \,|\, R_n \cdots R_{m+1} \,|\, R_m \cdots R_1 \,|\, \phi \rangle$$

$$\langle R^\dagger_{m+1} \cdots R^\dagger_n \phi \,|\, R_m \cdots R_1 \phi \rangle$$