IR Phase of QCD and Chiral Anderson Models

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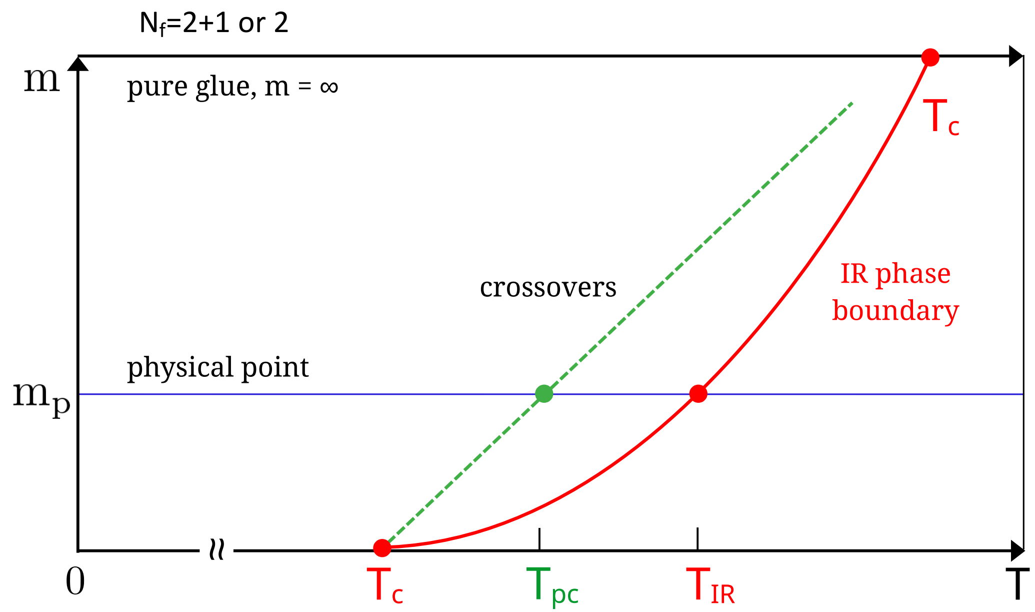
Related literature:

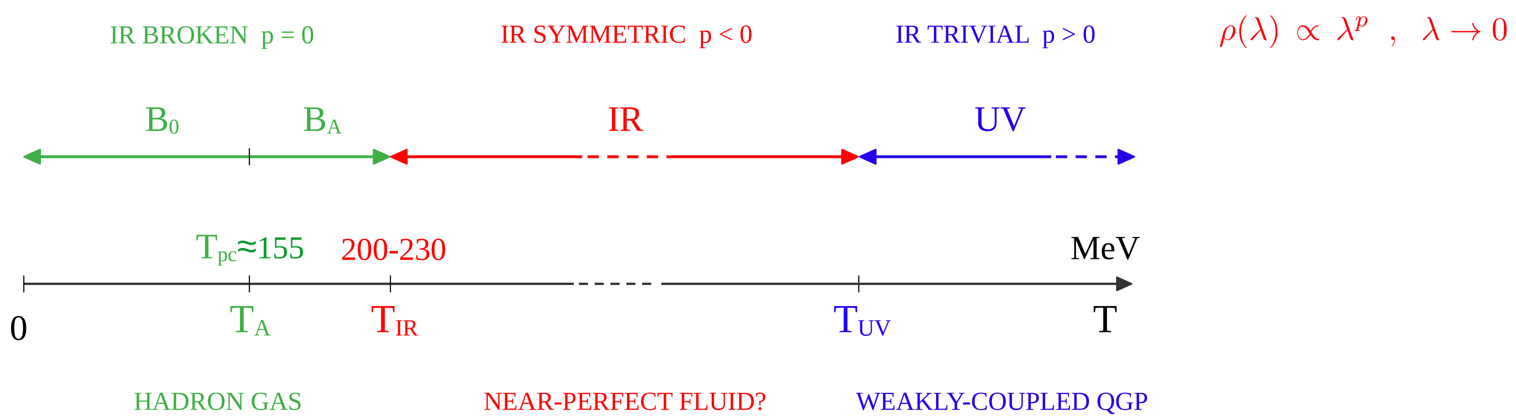
QCD	Applied math & Anderson
1906.08047	1807.03995
1502.07732	1809.07249
2103.05607	2110.11266
2110.04833	2205.11520
2305.09459	2207.13569
2310.03621	2212.09806

Everything grew out of Dirac mode & IR phase studies/considerations

IR Phase

1906.08047 1502.07732
2103.05607 2110.04833



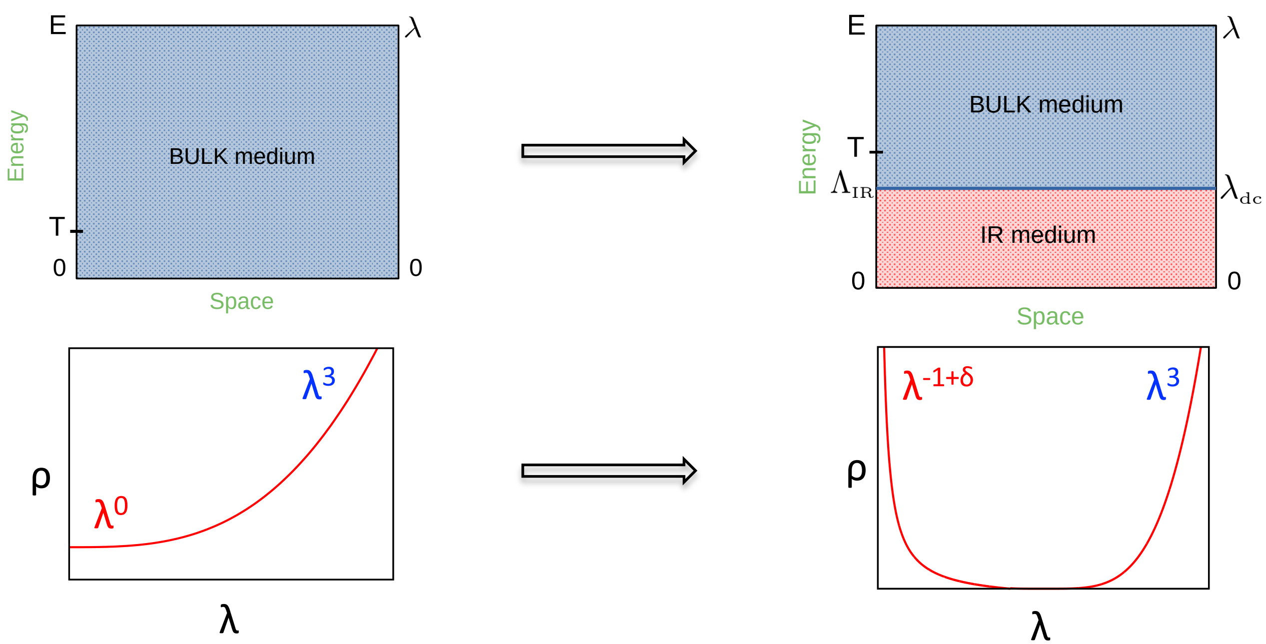


IR Phase… what happens?

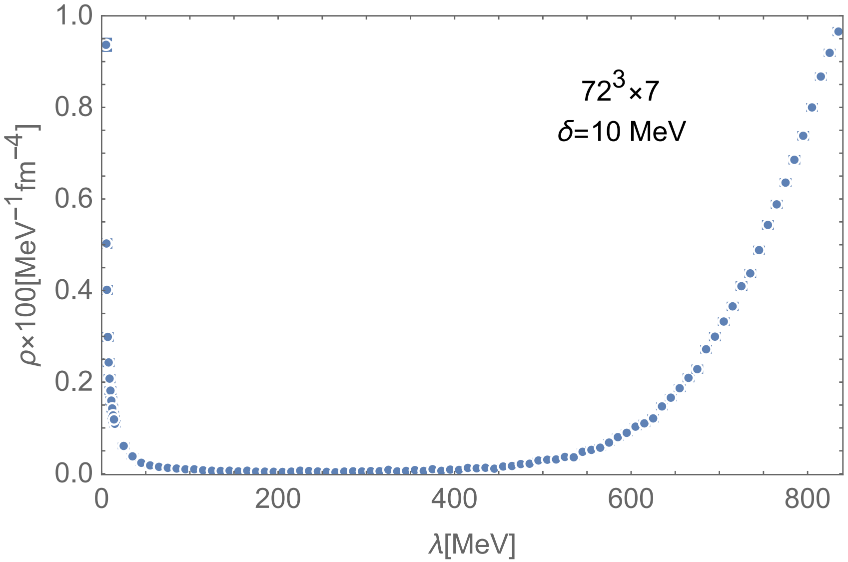
IR Transition at T_{IR}:

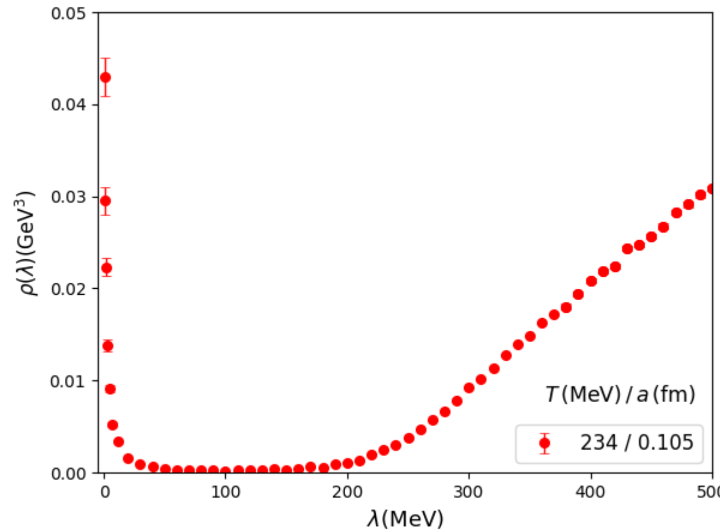
- single-component $\longrightarrow$ multi-component
- scale invariance restored in IR component

1906.08047 1502.07732
2103.05607 2110.04833



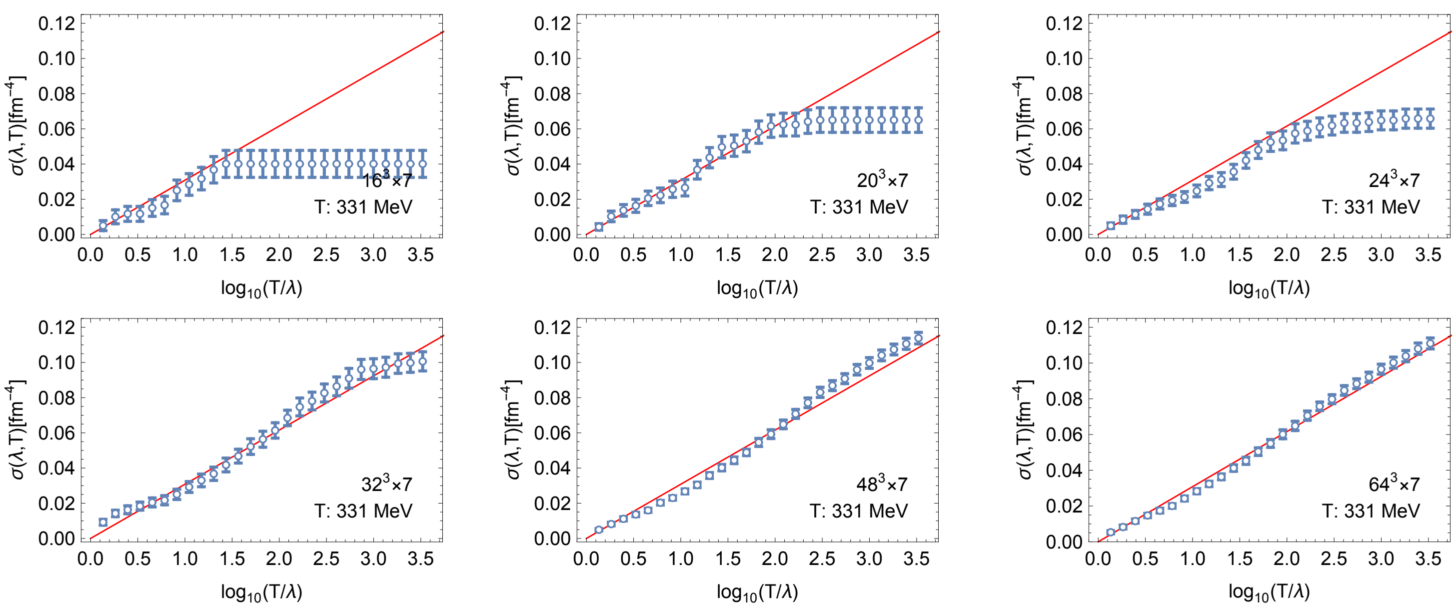
IR Phase… what does it look like?





N_f=0 a=0.085 fm T=1.12 T_c AA & IH unpublished

N_f=2+1 "real world" L=5.0 fm Clover quarks 2305.09459



1906.08047

$\rho(\lambda)\propto 1/\lambda$

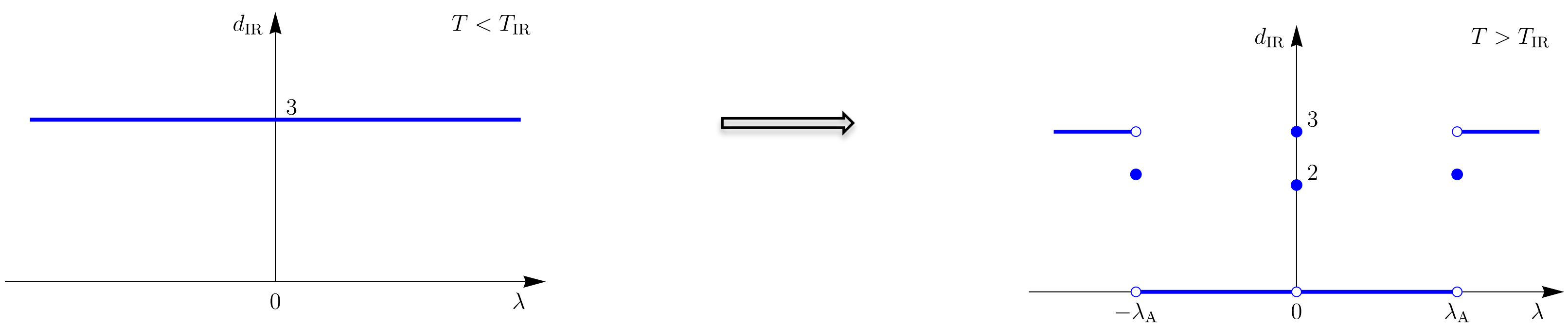
IR Phase… distinct features relevant here

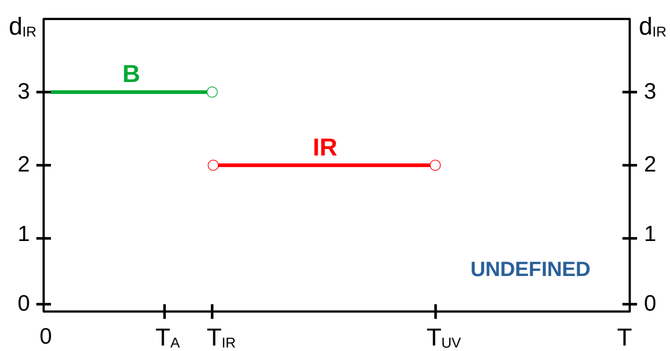
(A) $$\rho(\lambda) \propto \lambda^{-1+\delta} \;,\; \lambda \to 0 \;,\; \delta \text{ small}$$

1906.08047
p < 0 and close to -1

(B) dimensional properties – dimensional collapse

2103.05607 2110.04833 2310.03621





-- d_{IR} is the infrared spatial dimension

-- dimensional collapse: source of non-analyticity in T

- Here: take (A) and (B) as blueprints for modeling the deep IR of IR phase and seek a model…

Dimensions: Topological & Non

"Of all the theorems of analysis situs, the most important is that which we express by saying that the space has three dimensions. … we shall put the question in these terms: <u>when we say that space has three dimensions, what do we mean?</u>"

"…if to divide a continuum it suffices to consider as cuts a certain number of elements all distinguishable from one another, we say that this continuum is of *one dimension*; if, on the contrary, to divide a continuum it is necessary to consider as cuts a system of elements themselves forming one or several continua, we shall say that this continuum is of *several dimensions*."

"If to divide a continuum C, cuts which form one or several continua of one dimension suffice, we shall say that C is a continuum of *two dimensions;* if cuts which form one or several continua of at most two dimensions suffice, we shall we shall say that that the continuum is of *three dimensions* and so on."

Poincaré, 1912, def of <u>topological dimension</u> in philosophy paper ☺

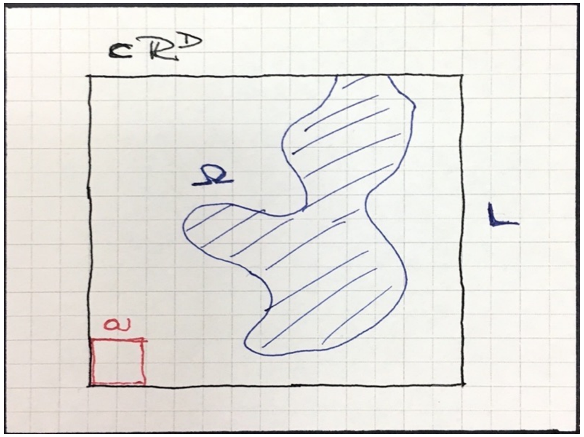
Brouwer (1913) → Menger (1922) , Urysohn (1922) completed
Arguably the most important topological invariant there is.

BUT: unusual sets (Cantor and such) brough to the forefront the <u>measure ↔ volume</u> as the heart for the notion of dimension

- Minkowski-Bouligand [box-counting] measure
- Hausdorff measure

<u>measure-based dimensions</u>

Dimensions: Probabilistic Setting



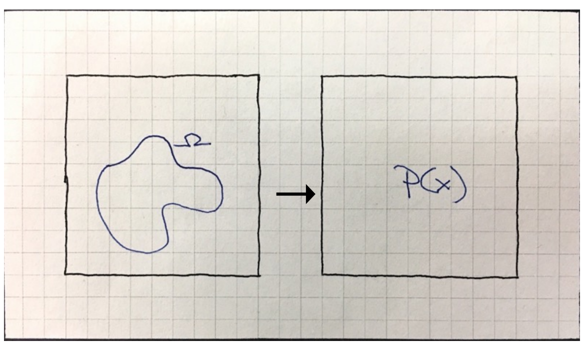
characterize fine [UV] and global [IR] features of fixed sets

all points in regularized space: $N \propto (L/a)^D$

points covering Ω: N_+

UV: $N_+(a,L) \propto a^{-d_{\rm UV}(L)}$, $a \to 0$

IR: $N_+(a,L) \propto L^{d_{\rm IR}(a)}$, $L \to \infty$



$P(x) \implies \Omega_{\rm eff}$

BUT: If instead of fixed Ω we have P(x)???

measure: Effective-Number Theory 1807.03995 [all measures]

dims: Effective-Dimension Theory 2205.11520 [unique d]

UV: $\langle \mathcal{N}_\star \rangle_{a,L} \propto a^{-d_{\rm UV}(L)}$, $a \to 0$

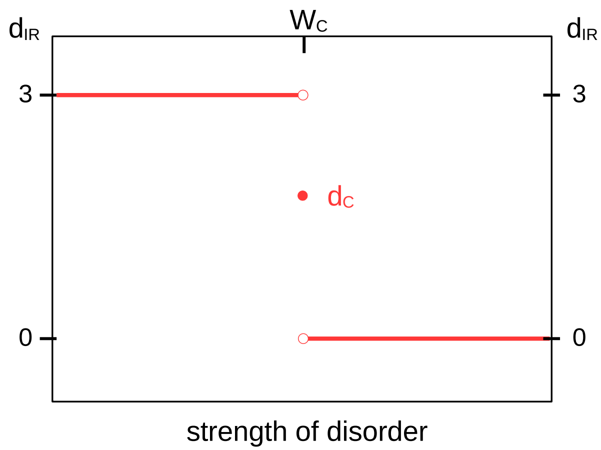
IR: $\langle \mathcal{N}_\star \rangle_{a,L} \propto L^{d_{\rm IR}(a)}$, $L \to \infty$

$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \;\; , \;\; \mathfrak{n}_\star(c) = \min\{c,1\} \;\; , \;\; P=(p_1,\dots,p_N)$$

bottom line: $V_{\rm eff} \propto L^{d_{\rm IR}}$ is a well-defined notion for studying dimensional collapses

Prototypes of Dimensional Collapse: Anderson Transitions

In CM: metal-to-insulator via disorder

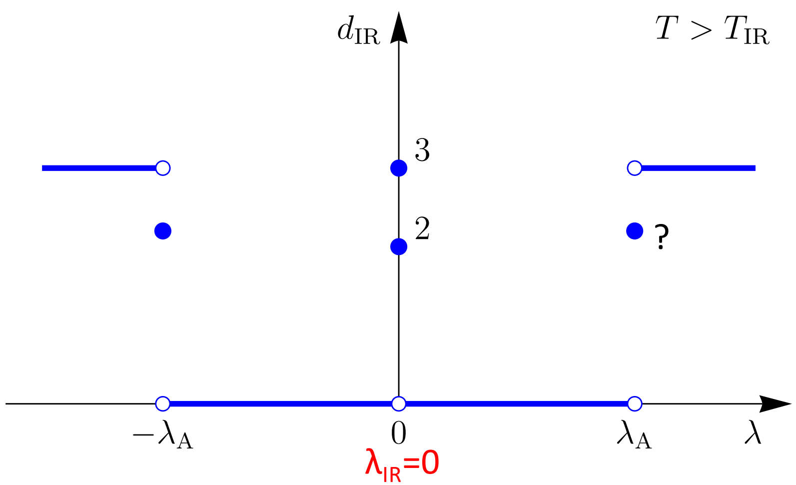


$$\mathcal{H} = \sum_r \epsilon_r \, c_r^\dagger \, \sigma_{\rm diag} \, c_r + \sum_{r,j} c_r^\dagger \, t_{r,e_j} c_{r-e_j} + h.c.$$

Exponential localization is, among other things, a collapse of space available to a particle to pointlike speck(s).

EXTENDED STATE $\Longrightarrow$ LOCALIZED STATE

Space-wise most interesting is the transitional state between the two: CRITICAL STATE [ENT & EFT essential]



$\lambda_{IR}=0$

AA & IH: 2103.05607 2110.04833 2310.03621

IR transition as metal-to-critical

singular mobility edge

essential to IR phase

$\lambda_A > 0$

Garcia-Garcia, Osborn 2007

chiral transition as metal-to-insulator

Kovacs, Pitler 2010 Giordano, Kovacs, Pitler 2014

2103.05607

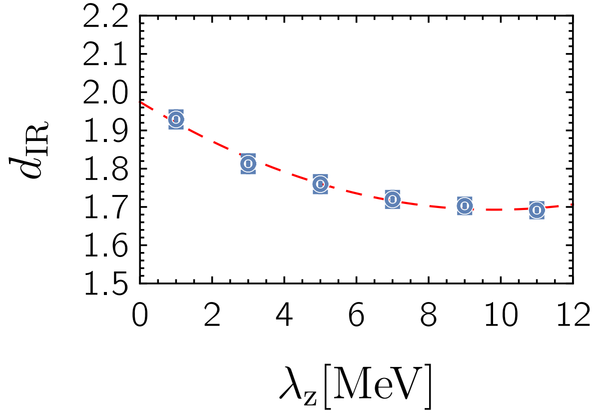
fits the IR phase picture (decoupling)

Dimensional collapses of IR phase Anderson-like but to what extent? Particularly for λ_{IR}.

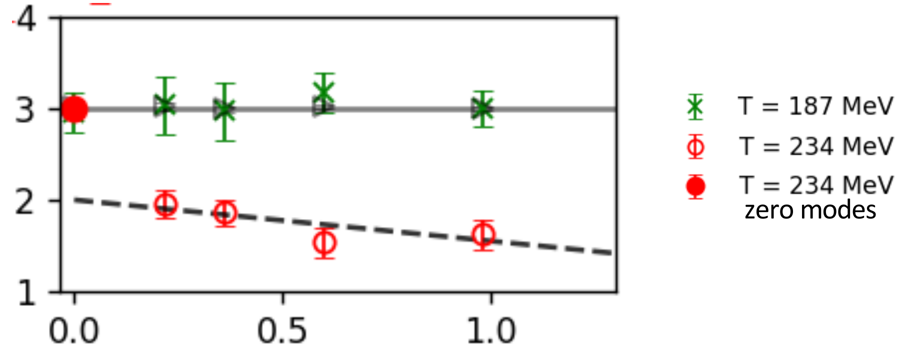
QCD & Anderson-like

$d_{IR}(0^+)=2$ in QCD:

$N_f=0$, $T=1.12\ T_c$ 2103.05607



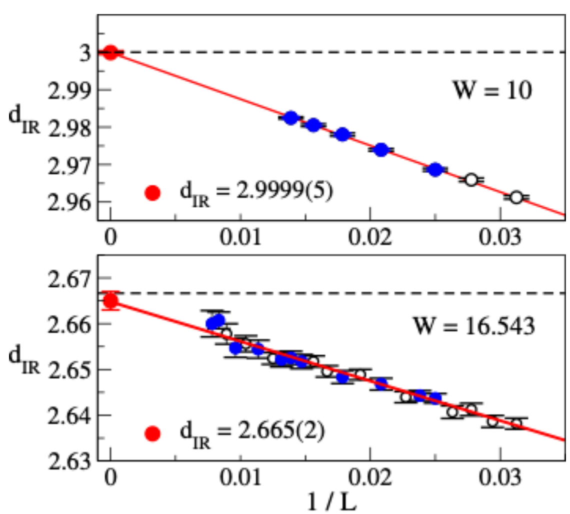
$N_f=2+1$, physical point 2305.09459

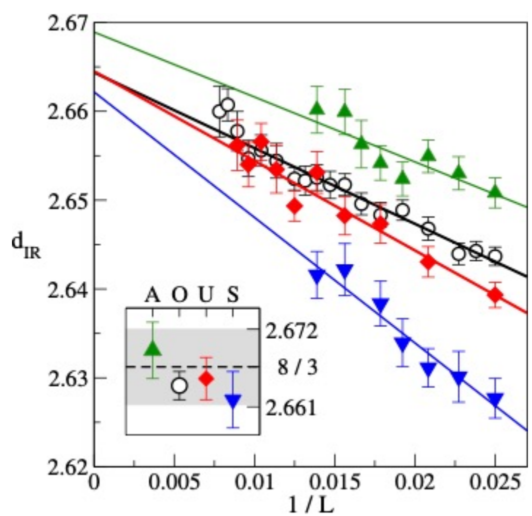


"diagonal disorder" "off-diagonal disorder"

But standard models with diagonal disorder: 2110.11266

$$\mathcal{H} = \sum_r \epsilon_r\, c_r^\dagger\, \sigma_{\mathrm{diag}}\, c_r + \sum_{r,j} c_r^\dagger\, t_{r,e_j}\, c_{r-e_j} + h.c.$$





- super-universal $d_{IR} \approx 8/3$
- non-singular density of states

→ will not describe deep IR of IR phase: neither (A) nor (B)

→ check chiral off-diagonal

Chiral-Orthogonal Disordered Systems

Evangelou & Katsanos
cond-mat/0206089

Bipartite system with off-diagonal disorder:

$$H = \sum_{r\in A, r'\in B} t_{r,r'}\, c_r^\dagger c_{r'} + t^*_{r,r'}\, c^\dagger_{r'} c_r \qquad A \cap B = \emptyset \qquad A \cup B = \mathcal{L}$$

$t_{r,r'} = t^*_{r,r'}$ and even-odd bipartition: $\quad H = \sum_{r\in\mathcal{L},i} t_{r,r+i}\,(c_r^\dagger c_{r+i} + c^\dagger_{r+i} c_r)$

zero-mean disorder: $\quad p(w) = \dfrac{1}{W} \quad w \in [-W/2, W/2]$

logarithmic disorder: $\quad t = e^w$

claims about the model: $\quad \rho(E) \propto E^{\,p(W)} \;,\; E \to 0 \;,\; p < 0$

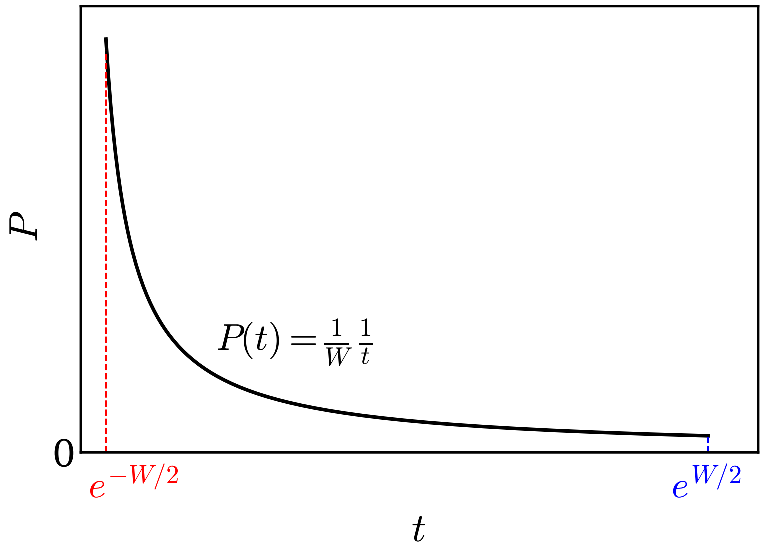
$$\lim_{W\to\infty} p(W) = -1$$

Sounds like (A) in QCD!

Chiral-Orthogonal Disordered Systems… moreover

$$p(w) = \frac{1}{W} \qquad w \in [-W/2, W/2]$$

$$t = e^{w} \qquad \Longrightarrow \qquad P(t) = \frac{1}{W}\frac{1}{t} \qquad t \in \left[\, e^{-W/2}, e^{W/2} \,\right]$$



This is scale invariant 1/x noise with UV cutoff exp(W/2) and IR cutoff exp(-W/2) in disguise!

<u>Note</u>: $\rho(\lambda) \propto 1/\lambda$ of IR phase was attributed to scale-invariant glue backgrounds! 1906.08047

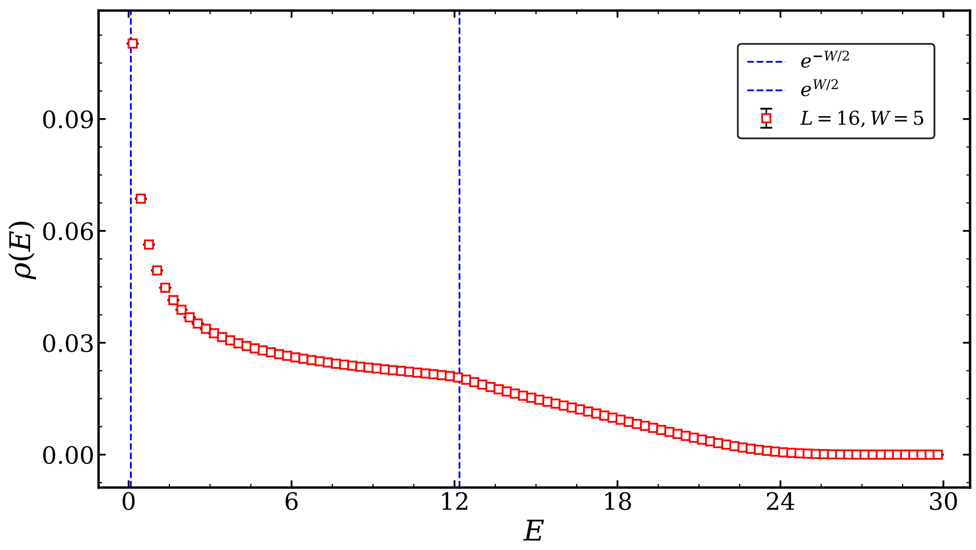
regularized noise $\Longleftrightarrow$ regularized glue background

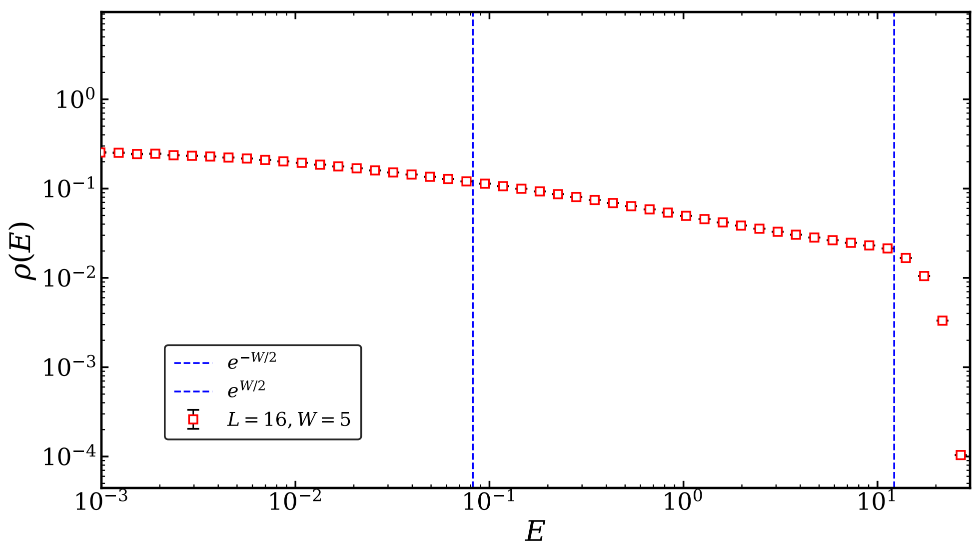
This should indeed be approached with the above perspective and vocabulary.

Check further…

Chiral-Orthogonal Disordered Systems…

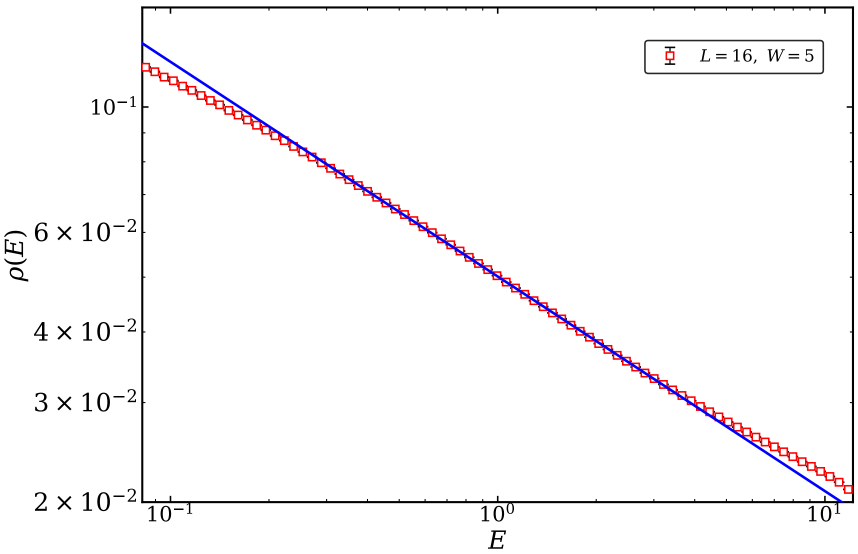
relevant stuff expected within the cutoffs

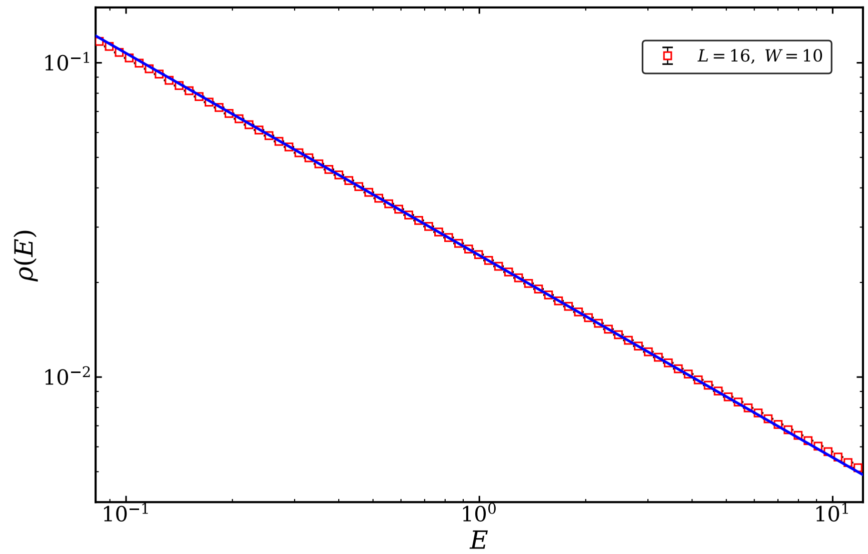


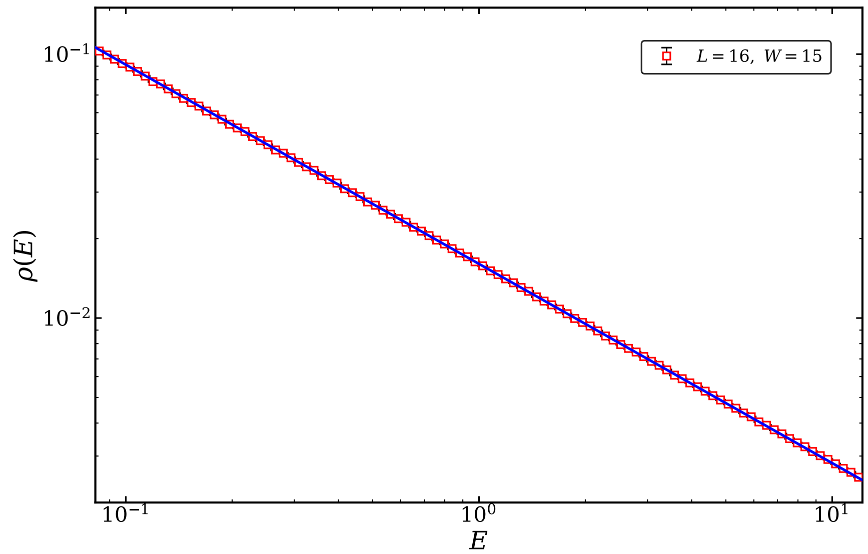


Chiral-Orthogonal Disordered Systems…

removing the cutoffs
this means increasing W

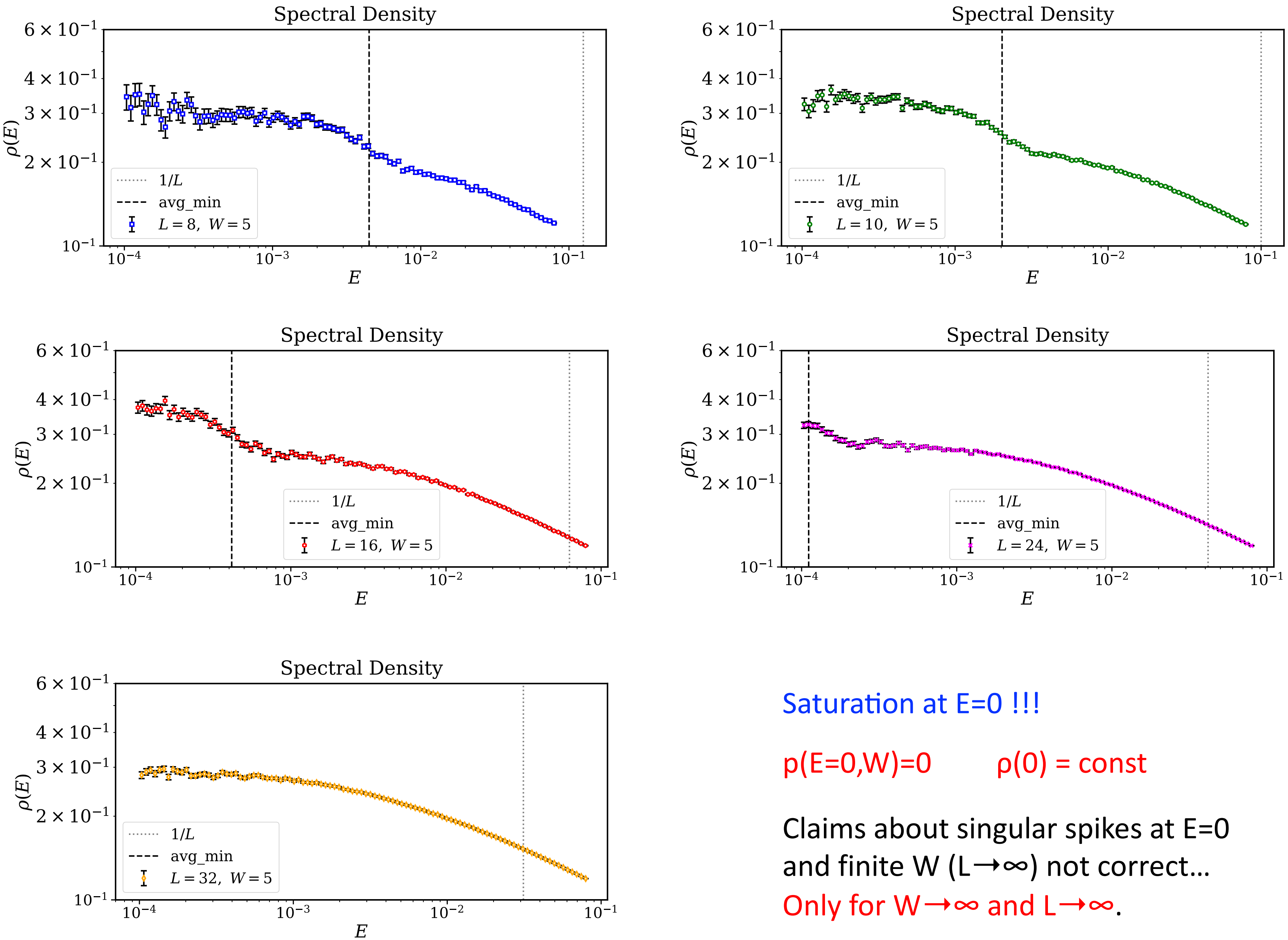






Chiral-Orthogonal Disordered Systems…

What about near E=0?



Saturation at E=0 !!!

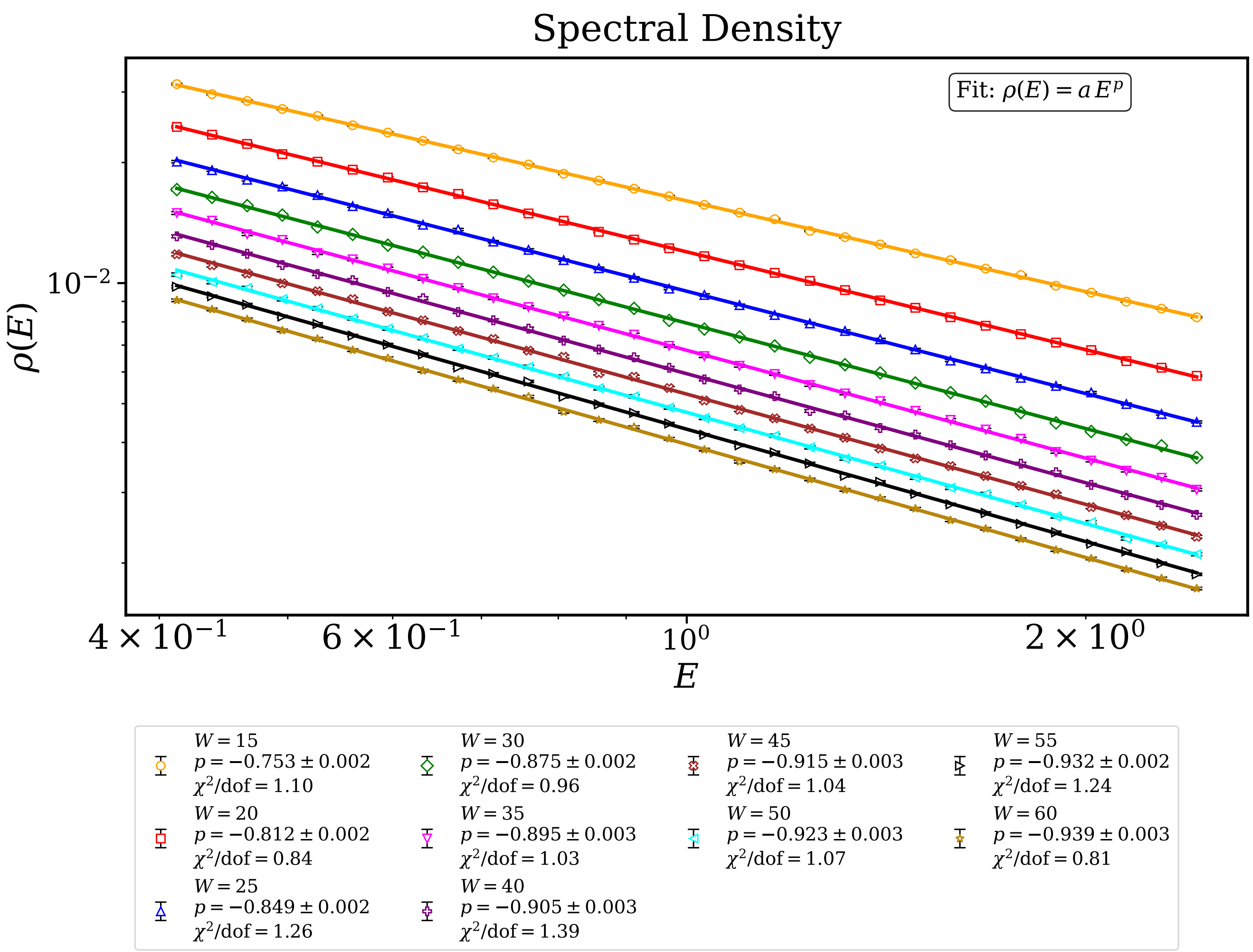
ρ(E=0,W)=0   ρ(0) = const

Claims about singular spikes at E=0 and finite W (L→∞) not correct…

Only for W→∞ and L→∞.

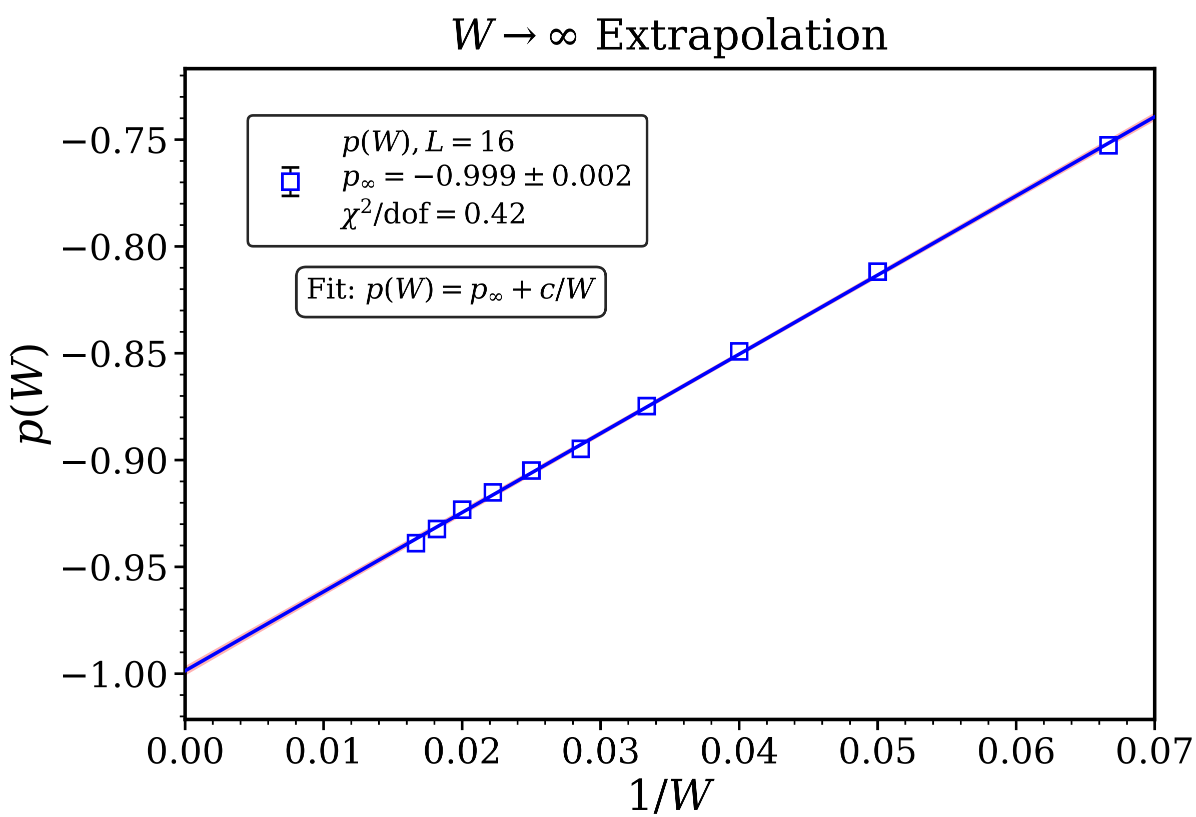
Chiral-Orthogonal Disordered Systems…

What about in the regularized region?



Chiral-Orthogonal Disordered Systems...

remove the cutoff & extract the power



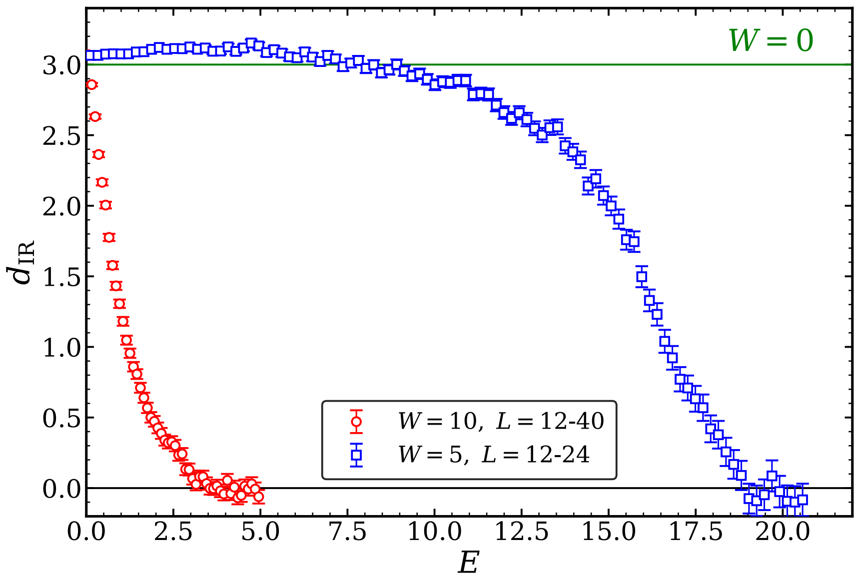
$$\lim_{W \to \infty} p(W) = -1$$

indeeed, but the first actual quantitative analysis

$$\frac{1}{t} \text{ distributed hopping noise } \longrightarrow \frac{1}{E} \text{ distributed quantum states}$$

Chiral-Orthogonal Disordered Systems…

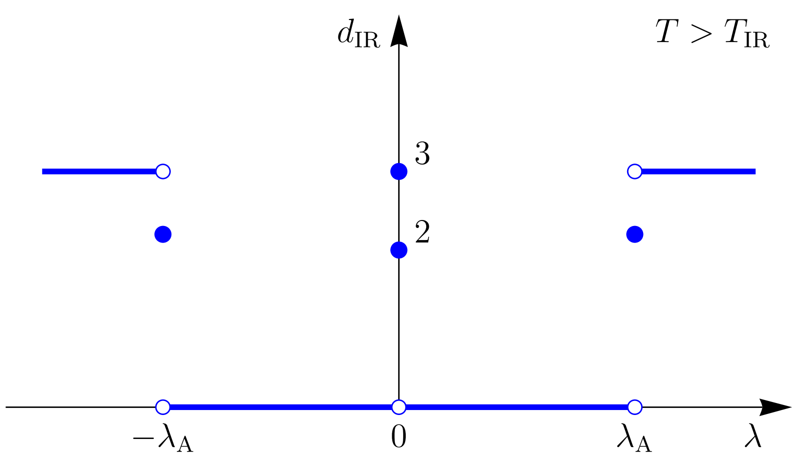
What about the IR dimensions?



Follow the trend of increasing W.

This has a chance to work in this aspect.

But $d_{IR}(0^+)$=?



TAKEAWAYS

Question: Can Anderson-like features of IR phase be described by actual Anderson models?

Question: Which IR Phase features are driven by randomness, in what way, and to what extent?

Focus: $p = -1+\delta$ & dimensional properties (A) & (B)

- Vicinity of $\lambda_A > 0$ may be described by Anderson model with diagonal disorder but $d_{IR}(\lambda_A)$ unknown in QCD
- For $\lambda_{IR}=0$ we study of chiral-orthogonal models w off-diagonal disorder
  - We view these systems as regularizations of scale-invariant disorder
  - Only $W \to \infty$ limit has connection to IR phase (regularization removal)
  - (A) has been ascertained and (B) has good chance in that case
- New things found about these models themselves & more in store
- Topology not explicitly involved: some combination of mixing models [Edwards et al 2000, Kovacs 2025] and chiral Anderson models w off-diagonal disorder may describe the vicinity of $\lambda_{IR}=0$ surprisingly faithfully