Spectral density reconstruction from lattice QCD correlators: an approach via numerical anti-Laplace transform.

Analytical characterisation.

José Alejandro Vidal Maxiá

University of Parma & INFN

In collaboration with F. Di Renzo, P. Dimopoulos, D. Gavriel & M. Aliberti

LATTICE 2026 contributed talk

College Park, Maryland – Jul. 29th

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Talk overview

José Alejandro Vidal Maxiá
University of Parma & INFN

- Numerical approach to the inverse problem
- Singular value decomposition (SVD) key aspects
- Relation to other approaches
- Conclusions/outlook

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Numerical approach to the inverse problem

José Alejandro Vidal Maxiá
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Euclidean correlators as Laplace transform of spectral densities
(Kallen-Lehmann representation)

$$C(na) = \int_0^\infty \rho(\omega) e^{-\omega n} d\omega = \omega_0 \int_0^\infty \rho(\omega_0 \xi) e^{-\omega_0 \xi n} d\xi$$

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$$n \in \{n_0, \cdots, N_t + n_0 - 1\}, n_0 \geq 1$$

$$k \in \{0, \cdots, N_\omega - 1\}$$

$$C(na) = \int_0^\infty \rho(\omega) e^{-\omega n} d\omega = \omega_0 \int_0^\infty \rho(\omega_0 \xi) e^{-\omega_0 \xi n} d\xi \approx \omega_0 \sum_{k=0}^{N_\omega - 1} w_k e^{-\xi_k(\omega_0 n - 1)} \rho(\omega_0 \xi_k).$$

Gauss-Laguerre (GL) quadrature rule applied for integrals of the form $\int_0^\infty f(x) e^{-x} dx$

(all quantities understood to be in lattice units)

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Gauss-Laguerre (GL) quadrature rule applied for integrals of the form $\int_0^\infty f(x) e^{-x} dx$

(all quantities understood to be in lattice units)

We can thus write

$$\boxed{C_n = \sum_k Z_{nk} \rho_k}$$

w/

$$C_n \equiv C(na)$$

$$Z_{nk} \equiv \omega_0 w_k e^{-\xi_k(\omega_0 n - 1)}$$

$$\rho_k \equiv \rho(\omega_0 \xi_k)$$

M. Aliberti, F. Di Renzo, P. Dimopoulos, D. Gavriel. arXiv:2603.00764

Numerical approach to the inverse problem

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This inverse problem needs regularisation
(in our case Tikhonov prescription)

$\boldsymbol{C} \in \mathbb{R}^{N_t}, \boldsymbol{\rho} \in \mathbb{R}^{N_\omega}$

Minimise $$\left(\boldsymbol{C} - \hat{Z}\boldsymbol{\rho}\right)^T \hat{\Sigma}^{-1}\left(\boldsymbol{C} - \hat{Z}\boldsymbol{\rho}\right) + \alpha||\boldsymbol{\rho}||_2^2.$$

$\hat{\Sigma}$ is the $N_t \times N_t$ correlation matrix for the observable

for now we set it to $\mathbb{I}_{N_t}$

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Tikhonov-regularised least-squares solution $$\boldsymbol{\rho}^{(\alpha)} = \left(\hat{Z}^T\hat{\Sigma}^{-1}\hat{Z} + \alpha\mathbb{I}_{N_\omega}\right)^{-1}\hat{Z}^T\hat{\Sigma}^{-1}\boldsymbol{C}.$$

Regularisation induces smearing $\longrightarrow$ regularised delta peaks for discrete spectrum

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Numerical approach to the inverse problem

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Synthetic noiseless correlator $C_n = e^{-0.2n} + 2e^{-0.29n} + 3.5e^{-0.35n} + 3e^{-0.41n}$

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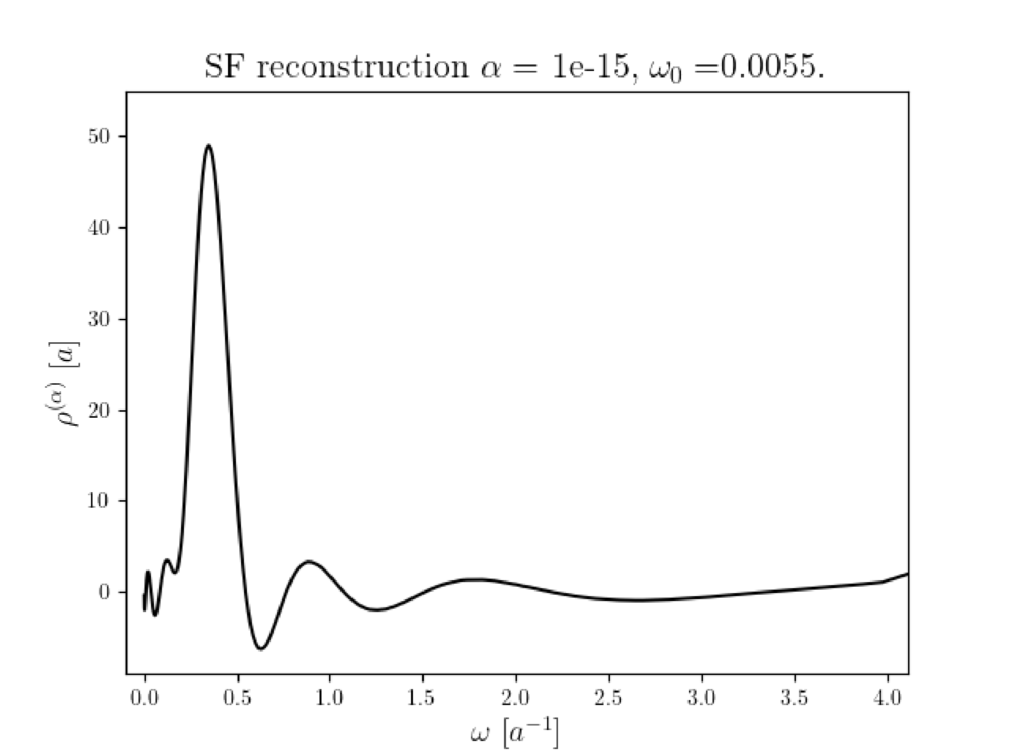

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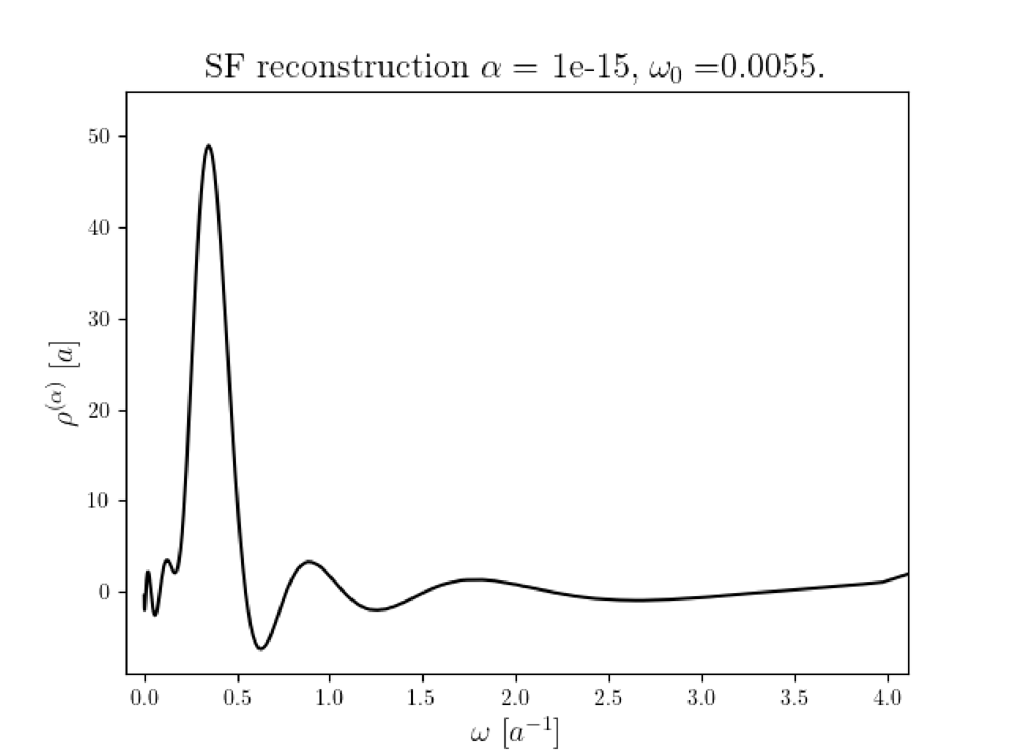

$$N_\omega = 195$$
$$N_t = 30$$
$$n_0 = 1$$

Many peaks clumped up

Structure **appears** ‘hidden’

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SVD key aspects

José Alejandro Vidal Maxiá
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Singular Value Decomposition (SVD) is a central component of Laplace (and others) numerical inversion

R.Tsuji, S.Hashimoto. arXiv:2605.15674v1

M. Bruno, L. Giusti, M. Saccardi, arXiv:2407.04141v1

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Continuous time: $t \in [0, \infty)\,, \quad \omega \in [0, \infty)$

SVD of Laplace kernel

$$\boxed{e^{-\omega t} = \int_{-\infty}^{+\infty} v_s(\omega)\lambda_s^* u_s^*(t)ds, \quad \text{w/} \quad |\lambda_s| = \sqrt{\frac{\pi}{\cosh(\pi s)}}}$$

the expression is actually real, it can be expressed in a different choice of basis similar to Fourier transform

$v_s(\omega)$ eigenfunctions of the Carlemann operator $\mathcal{H}(\omega, \omega') \equiv \int_0^\infty e^{-(\omega+\omega')t} dt = \frac{1}{\omega+\omega'}$

$u_s(t)$ eigenfunctions of the (conjugate) Carlemann operator $\mathcal{A}(t, t') \equiv \int_0^\infty e^{-(t+t')\omega} d\omega = \frac{1}{t+t'}$

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Simple relation among both (Mellin basis)

$$v_s(\omega) = u_s^*(\omega) = u_{-s}(\omega) = \frac{\omega^{-is}}{\sqrt{2\pi\omega}}$$

R.Tsuji, S.Hashimoto. arXiv:2605.15674v1

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equal (real) s. values but spectrum is halved

SVD of Laplace kernel

$$\boxed{e^{-\omega t} = \int_0^\infty \overline{v}_s(\omega)|\lambda_s|\overline{u}_s^*(n)ds, \quad \text{w/} \quad |\lambda_s| = \sqrt{\frac{\pi}{\cosh(\pi s)}}}$$

$\overline{v}_s(\omega)$ eigenfunctions of the Carlemann operator $\overline{\mathcal{H}}(\omega,\omega') \equiv \sum_n e^{-n(\omega+\omega')} = \frac{e^{-(\omega+\omega')n_0}}{1-e^{-(\omega+\omega')}}$

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No longer simple relation between both bases

$\overline{u}_s(n)$ and $\overline{v}_s(\omega)$ given by different Hypergeometric functions ${}_qF_p$

R.Tsuji, S.Hashimoto. arXiv:2605.15674v1
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Discrete time & frequency: $n \in \{n_0, \cdots, n_0 + N_t - 1\}\,, \quad \mathrm{k} \in \{0, \cdots, N_\omega - 1\}$

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Recall our initial inverse problem

$$\boxed{C_n = \sum_k Z_{nk}\rho_k}$$

SVD of Laplace kernel

$$\boxed{Z_{nk} = \omega_0 w_k e^{-\xi_k(\omega_0 n - 1)} = \sum_s U_s(n)\lambda_s V_s(\xi_k)}$$

$V_s(\xi_k)$ eigenbasis of the 'Carlemann' operator $\widetilde{\mathcal{H}}(\xi_k, \xi_{k'}) \equiv \left(\hat{Z}^T \hat{Z}\right)_{kk'} = \sum_s V_s(\xi_k)|\lambda_s|^2 V_s(\xi_{k'})$

Noise modifies singular values and basis vectors but the treatment is the same overall

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General solution in terms of SVD

$$\rho_k = \sum_s V_s(\xi_k) \frac{\lambda_s}{\lambda_s^2 + \alpha} \sum_n U_s(n) C_n$$

$U_s(n)$ and $V_s(\xi_k)$ form two orthonormal bases of $\mathbb{R}^{N_t}$ and $\mathbb{R}^{N_\omega}$, resp.

SVD key aspects

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Characterisation of lattice correlator signals

In a lattice-regulated Euclidean theory, we get sums of exponentials

$$C_n = \sum_q A_q e^{-nE_q} = \sum_q A_q e^{-n\omega_0 \bar{E}_q}$$

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Our anti-transformed signal will thus be of the form

$$\rho_k = \sum_q A_q \underbrace{\sum_s V_s(\xi_k) \frac{\lambda_s}{\lambda_s^2 + \alpha} \sum_n U_s(n) e^{-n\omega_0 \bar{E}_q}}_{\delta_k^{(\alpha)}(\bar{E}_q) \equiv \delta_{qk}^{(\alpha)}}$$

Sums of regularised delta peaks $\delta_k^{(\alpha)}(\bar{E}_q)$

SVD key aspects

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Sums of regularised delta peaks $\delta_k^{(\alpha)}(\overline{E}_q)$

Take $\overline{E}_q = \xi_l$ from within the GL quadrature points

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By using SVD $e^{-\omega_0 n \xi_l} = \frac{e^{-\xi_l}}{\omega_0 \mathrm{w}_l} \sum_s U_s(n) \lambda_s V_s(\xi_l)$

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$\boxed{\text{Under } \sum_k \omega_0 w_k e^{\xi_k} \longrightarrow \int_0^\infty d\omega}$

effective cutoff in eigenvalue sum

$$\delta_{kl}^{(\alpha)} = \frac{e^{-\xi_l}}{\omega_0 \mathrm{w}_l} \sum_s V_s(\xi_k) \frac{\lambda_s^2}{\lambda_s^2 + \alpha} V_s(\xi_l) \underset{\alpha \to 0}{\longrightarrow} \frac{e^{-\xi_l}}{\omega_0 \mathrm{w}_l} \delta_{kl} \underset{N_\omega \to \infty}{\longrightarrow} \delta(\xi_k - \xi_l)$$

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$\delta_{kl}^{(\alpha)}$ continuous in $\overline{E}_q$ (sum of exponentials) $\longrightarrow$ $\delta_{kq}^{(\alpha)}$ continuously extrapolates δ behaviour

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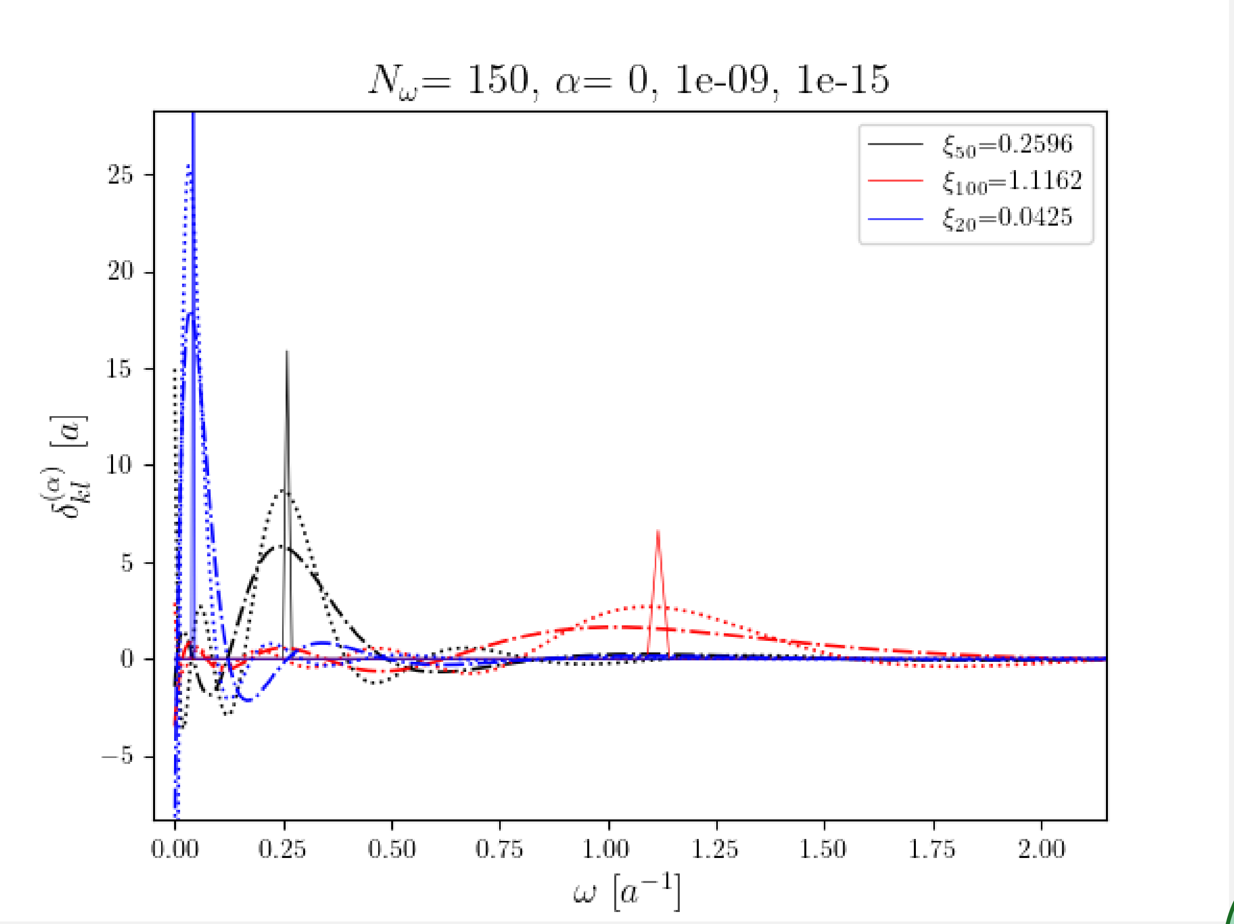
Sums of regularised delta peaks.. **graphically**

$\delta^{(\alpha)}$ at quadrature points

- Continuous lines are $\delta_{kl}^{(0)} = \frac{e^{-\xi_l}}{\omega_0 w_l}\delta_{kl}$ at different ξ_l
- GL quadrature points **not** evenly spaced
- Smaller α leads to more pronounced peaks



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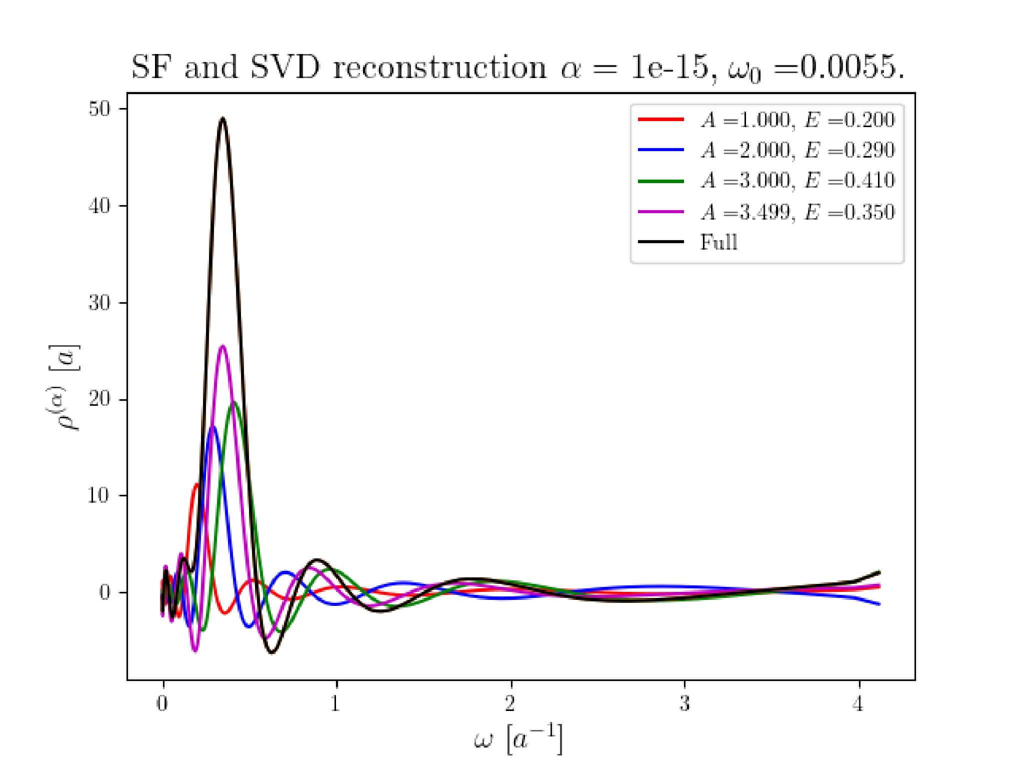

$\delta^{(\alpha)}$ at arbitrary points

- Actual decomposition of previous reconstructed density
- Behaviour of δ peaks indeed like at quadrature points
- Subtle cancellations lead to overall shape

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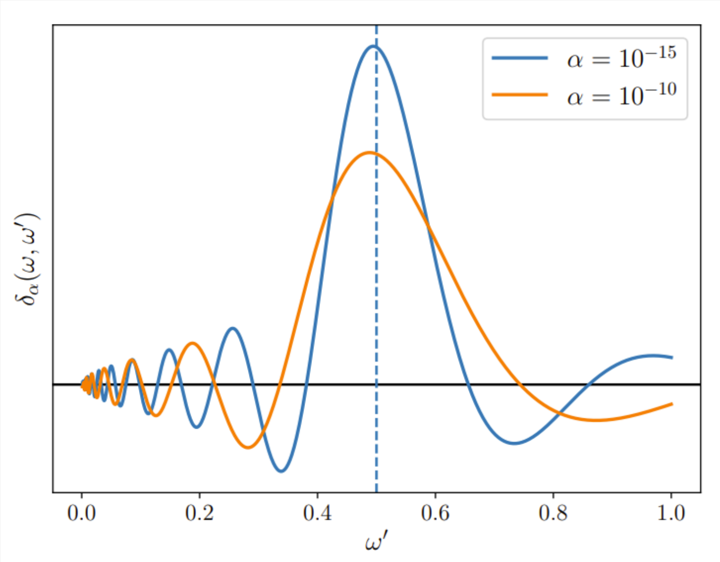
Relation to other approaches

This kind of profile (rather families) already encountered in other occasions

Mellin transforms



M. Saccardi, *Methods for extracting spectral densities from Euclidean lattice correlators,* 2024 (thesis) https://boa.unimib.it/handle/10281/541185

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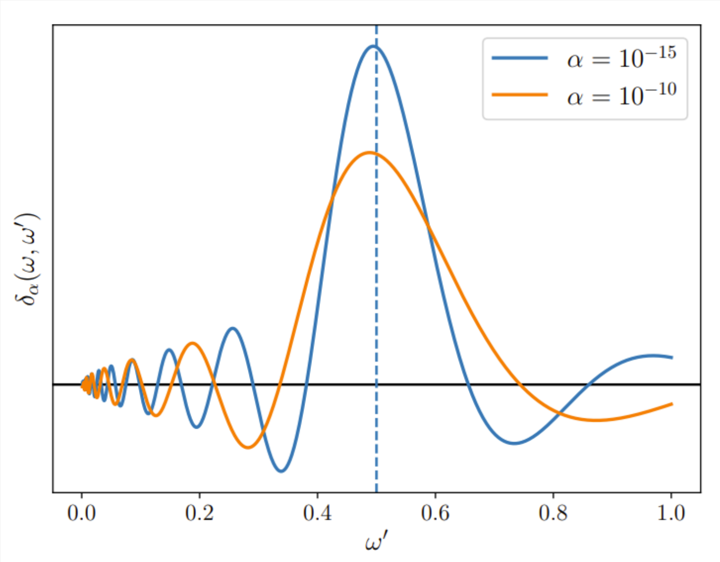

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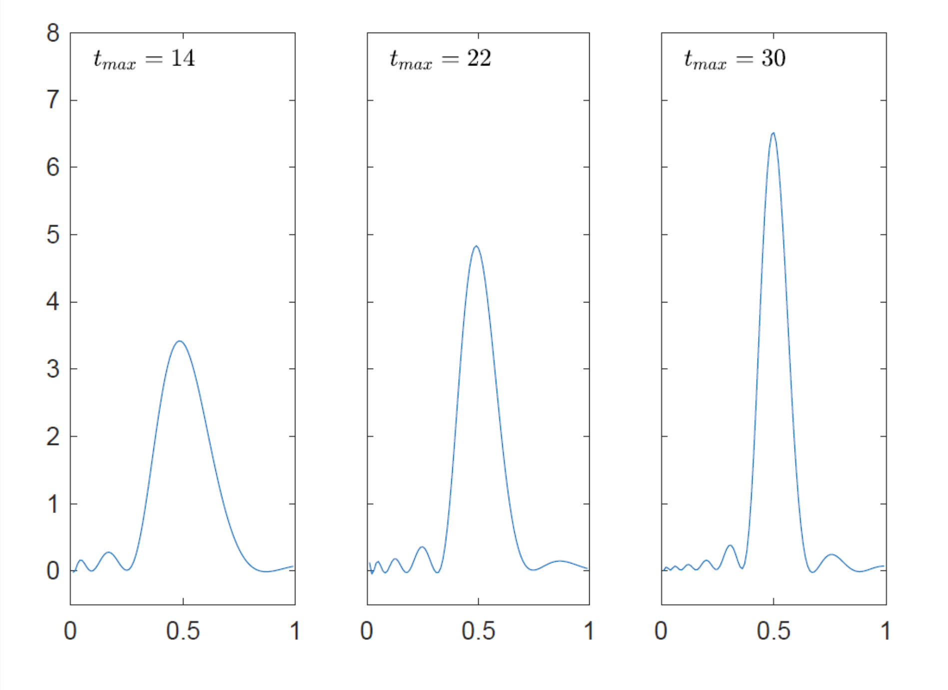
Backus-Gilbert method



M. Hansen, A. Lupo, N. Tantalo, *Extraction of spectral densities from lattice correlators,* Phys. Rev. D **99,** 094508 (2019)
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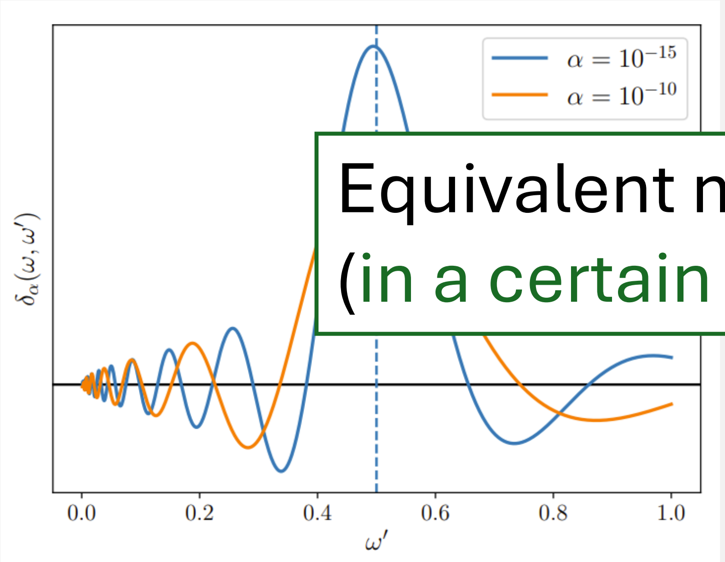

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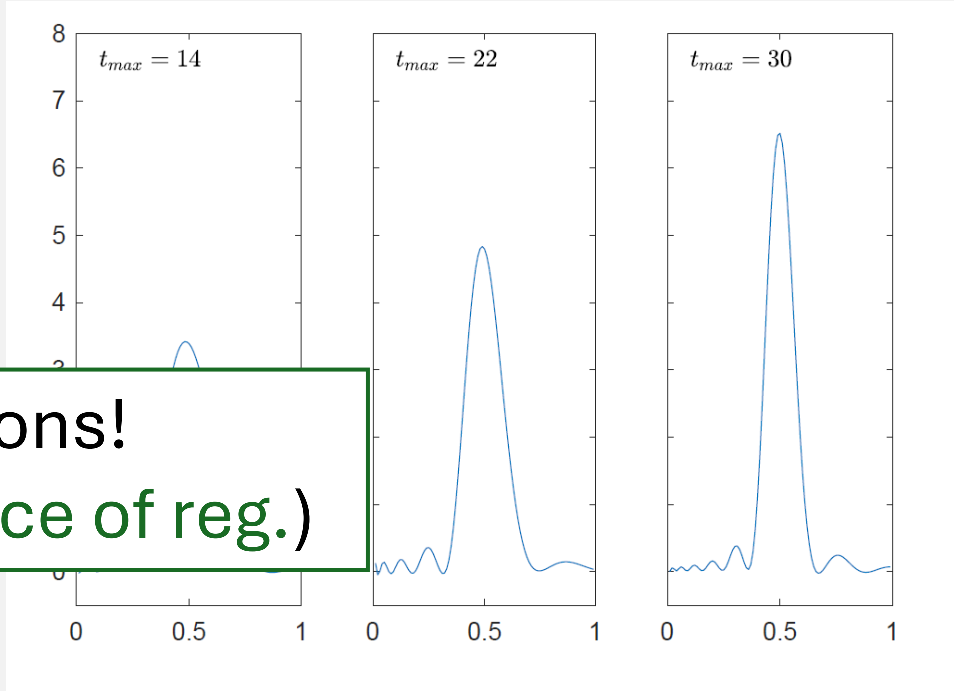

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Equivalent minimizations!
(in a certain limit/choice of reg.)

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Conclusions/outlook

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Conclusions

- Laplace inversion problem of correlators yields reg. induced smearing
- Regulated delta peaks can be easily characterized
- SVD powerful tool → equivalent to our inverse problem
- Relation extends to other linear reconstruction approaches

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Outlook

- Errors need to be accounted for next
- Adequate minimization procedure underway for spectral reconstruction (energies, matrix elements)
- Interesting to consider generalizing to linear programming

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Goodbye

José Alejandro Vidal Maxiá
University of Parma & INFN

THANK YOU FOR THE ATTENTION!

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Extra: equivalence MI vs SVD

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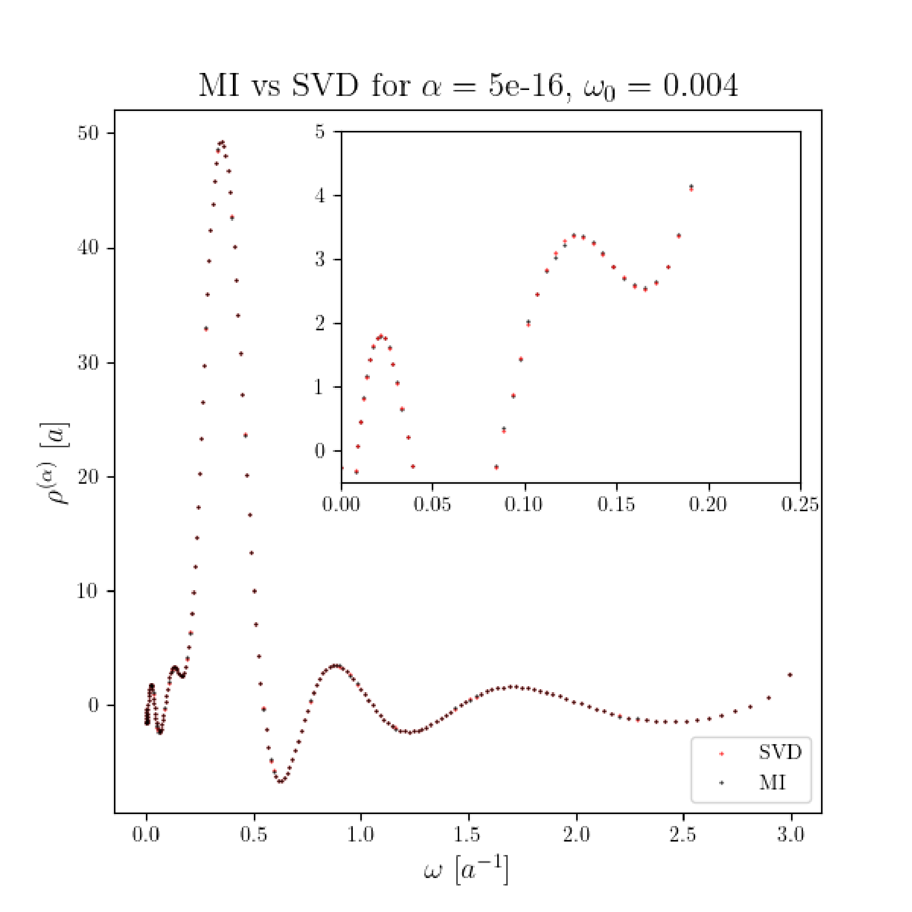
Up to certain errors, direct matrix inversion (MI) of Laplace kernel is equivalent to SVD approach



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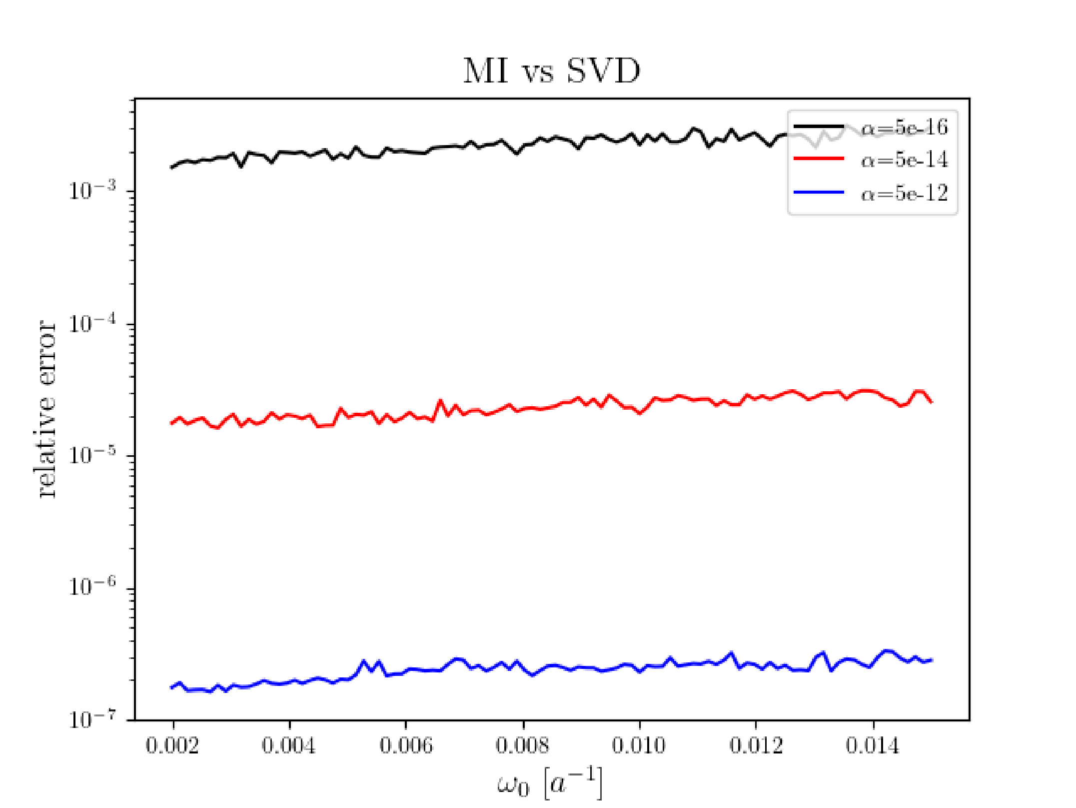
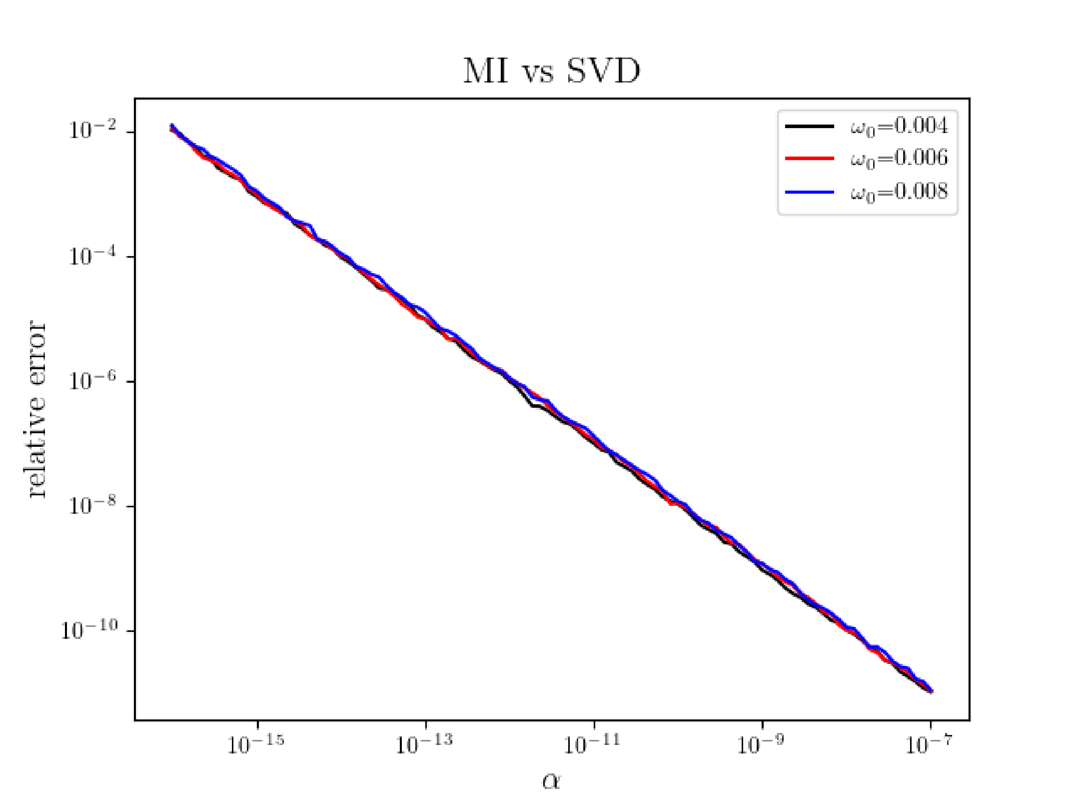
Up to certain errors, direct matrix inversion (MI) of Laplace kernel is equivalent to SVD approach





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Extra: SVD spectrum decay

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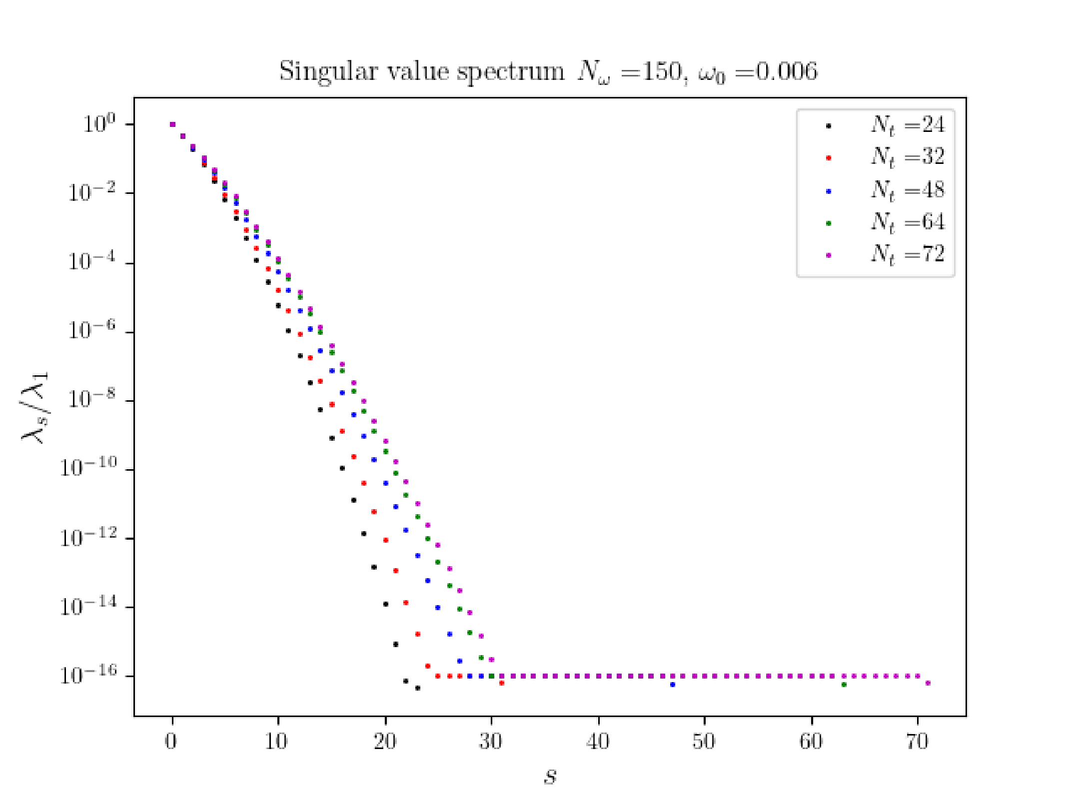
Maximum s available determines spectral function resolution



It does depend on N_t …

but not dramatically so