Light and strange quark masses in $N_f = 2 + 1$ QCD with Wilson-like fermions.

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ALPHA Collaboration

Lattice 2026, Maryland

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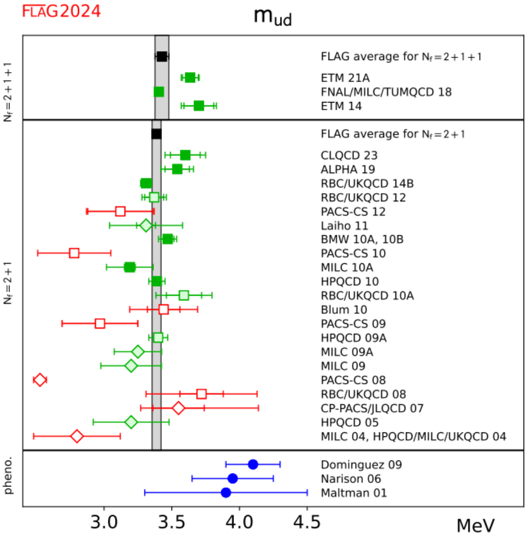
Outline

- Status of m_q in FLAG [2411.04268]
- Strategy of the computation:
  - ➔ $N_f = 2+1$ CLS ensembles
  - ➔ Non-perturbative renormalization
  - ➔ Wilson fermions and Mixed action setup
- Preliminary results : $\mathbf{m_{ud}}$, $\mathbf{m_s}$ and $\mathbf{m_s/m_{ud}}$
- Conclusions

Status of m_{ud}



- FLAG Average:

$$m_{ud}(2\text{GeV}) = 3.387(39)\ \text{MeV}\ [1.2\%]$$

$$M_{ud}^{RGI} = 4.714(55)_m(46)_\Lambda\ \text{MeV}\ [1.5\ \%]$$

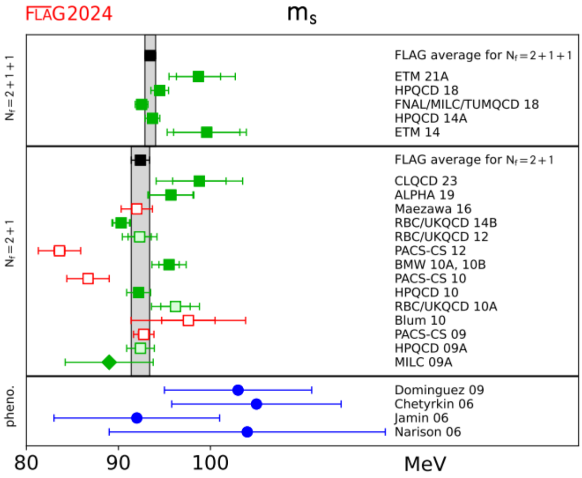
- [ALPHA 19]:

$$m_{ud}(2\text{GeV}) = 3.54(15)\ \text{MeV}\ [4.2\ \%]$$

$$M_{ud}^{RGI} = 4.70(19)\ \text{MeV}\ [4.1\ \%]$$

$\overline{MS}$-scheme, $N_f = 4$ flavour theory

Status of m_s



- FLAG Average:

$$m_s(2\text{GeV}) = 92.4(1.0)\ \text{MeV}\ [1.1\%]$$

$$M_s^{RGI} = 128.5(1.4)_m(1.2)_\Lambda\ \text{MeV}\ [1.4\ \%]$$

- `[ALPHA 19]`:

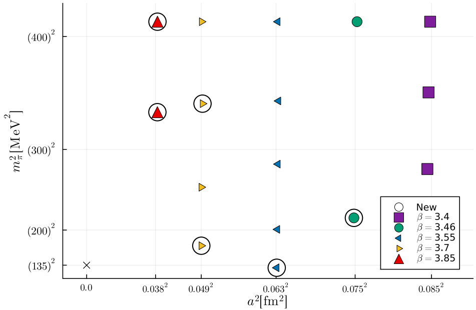
$$m_s(2\text{GeV}) = 95.7(3.5)\ \text{MeV}\ [3.6\ \%]$$

$$M_s^{RGI} = 127.0(4.5)\ \text{MeV}\ [3.5\ \%]$$

$\overline{MS}$-scheme, $N_f = 4$ flavour theory

Setup

- Update on [ALPHA 19] [1911.08025]. Six new ensembles including
  - One ensemble with physical m_π
  - Two ensembles at a finer lattice spacing
  - 16 ensembles in total



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- $N_f = 2 + 1$ CLS ensembles [1411.3982]
  - → Open boundary conditions in time on most ensembles [1206.2809]
  - → $\mathrm{Tr(M)} = \mathrm{const.}$ chiral trajectory [1608.08900]
  - → Five lattice spacings $[0.038, 0.085]\ \mathrm{fm}$ and $m_\pi \in [130, 420]\ \mathrm{MeV}$

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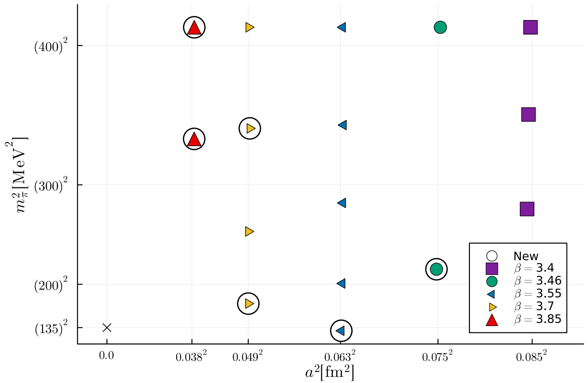
- Pure Wilson + Wilson twisted mass action [2309.14154]
  - Unitary setup with $\mathcal{O}(a)$ improved Wilson fermions [2603.26623]
  - Mixed-action setup with twisted mass fermions in the valence [hep-lat/0101001]
  - Same strategy as for $\sqrt{t_0}$ [2510.20450]. Correlations $\rightarrow$ `ADerrors.jl` [1809.01289]

Ensembles

Name	β	a[fm]	T/a	L/a	$\frac{m_\pi}{\mathrm{MeV}}$	$\frac{m_K}{\mathrm{MeV}}$
H101	3.4	0.085	96	32	426	426
H102			96	32	360	446
H105			96	32	286	470
H400	3.46	0.075	96	32	429	429
D450			128	64	221	483
N202	3.55	0.063	128	48	418	418
N203			128	48	350	448
N200			128	48	288	470
D200			128	64	204	488
E250			192	96	131	497
N300	3.70	0.049	128	48	427	427
N302			128	48	350	457
J303			192	64	261	481
E300			192	96	177	499
J500	3.85	0.038	192	64	418	418
J501			192	64	339	453



[1411.3982][1608.08900][2510.20450]

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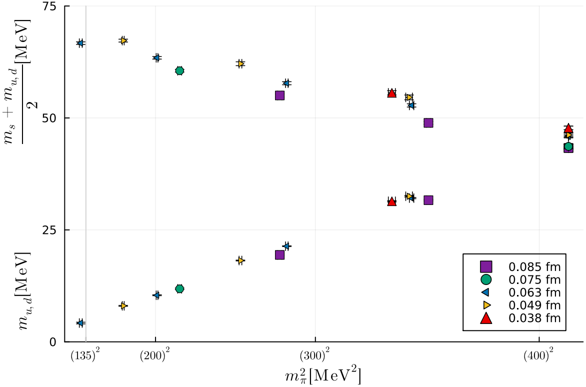

[1411.3982][1608.08900][2510.20450]

Computation of m_q

Dimensionless quantities for the masses

$$\phi_2 = 8t_0 m_\pi^2, \quad \phi_K = 8t_0 m_K^2, \quad \phi_4 = 8t_0(m_K^2 + \frac{1}{2}m_\pi^2)$$

$$\phi_{12} = \sqrt{8t_0}m_{12} = \sqrt{8t_0}m_{ud}, \qquad \phi_{13} = \sqrt{8t_0}m_{13} = \sqrt{8t_0}\frac{(m_{ud} + m_s)}{2}$$

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We want $\phi_4 = \mathrm{const.}$ to have a pricise line of constant physics $\rightarrow$ Mass shifts for all observables

$$\mathcal{O}\left(\{m_i'^{(\mathrm{sea})}\}_{i=u,d,s}\right) = \mathcal{O}\left(\{m_i^{(\mathrm{sea})}\}_{i=u,d,s}\right) + \sum_{i=u,d,s}\left(m_i'^{(\mathrm{sea})} - m_i^{(\mathrm{sea})}\right)\boxed{\frac{d\mathcal{O}}{dm_i^{(\mathrm{sea})}}}.$$

Computation of m_q

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PCAC quark masses improvement $(\tilde{b}_A - \tilde{b}_P) \rightarrow$ [1906.03445] (LCP-0), $c_A \rightarrow$[1502.04999], $Z_A \rightarrow$[1808.09236]

$$m_{ij}^I = \frac{Z_A}{Z_P} m_{ij} \left(1 + (\tilde{b}_A - \tilde{b}_P) a m_{ij} + \left(\frac{\overline{b}_A - \overline{b}_P}{Z r_m} - (\tilde{b}_A - \tilde{b}_P)\frac{r_m - 1}{r_m N_f}\right) a Z r_m Tr(M)\right)$$

$$m_{ij} = \frac{\tilde{\partial}_0 \sum_{\vec{y},\vec{x}} \left(\langle A_0(x)P(y)\rangle + a c_A \tilde{\partial}_0 \langle P(x)P(y)\rangle\right)}{2\sum_{\vec{y},\vec{x}}\langle P(x)P(y)\rangle}$$

Mass shift and matching

→ Matching between Wilson and Twisted mass sectors for each ensemble

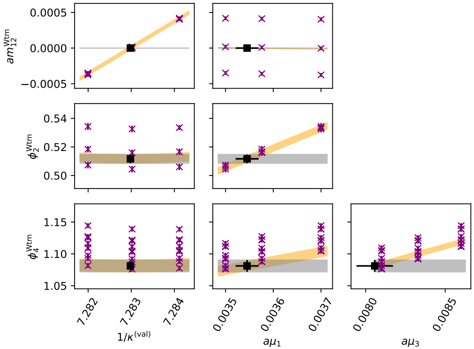
- $m_{PCAC}^{Wtm} = 0 \rightarrow$ Automatic $\mathcal{O}(a)$-improvement

  $\Longrightarrow$ Hopping parameter $\kappa^{Wtm} = \kappa^*$
- $\phi_2^{Wil} = \phi_2^{Wtm}$

  $\Longrightarrow$ Light twisted mass μ_1
- $\phi_4^{Wil} = \phi_4^{Wtm} =$ physical value

  $\Longrightarrow$ Strange twisted mass μ_3

Matching for N203



[1411.3982][1608.08900][2510.20450]

Renormalization: RGI masses

$$M_i = \frac{M_i}{\overline{m}_i(\mu_{\rm pt})} \frac{\overline{m}_i(\mu_{\rm pt})}{\overline{m}_i(\mu_{\rm had})} \lim_{a\to 0} Z(g_0^2, a\mu_{\rm had}) m_i(g_0^2)$$

- $M_i/\overline{m}_i(\mu_{\rm had})$: RG running to a hadronic scale, $\mu_{\rm had} \sim 230$ MeV. [1802.05243]
  - $\mu_{\rm pt} \sim M_W \rightarrow$ safe contact with perturbation theory
  - Rel. precision $\sim 1\%$, improvements are planned
- $Z(g_0^2, a\mu_{\rm had})$: Renormalization factor in the same scheme as the RG running
  - Wilson $\rightarrow Z_A(g_0^2)/Z_P(g_0^2, a\mu_{\rm had})$. Twisted-mass $\rightarrow 1/Z_P(g_0^2, a\mu_{\rm had})$
- $m_i(g_0^2)$: Bare masses in large volume simulations

Chiral-continuum extrapolations

NLO $\mathrm{SU}(3)$ χPT

$$\phi_{12} = \frac{\phi_2}{2B_0\sqrt{8t_0}}\left(1 - \frac{16}{8t_0 f_0^2}(2L_8 - L_5)\phi_2 - \frac{32}{8t_0 f_0^2}(2L_6 - L_4)\phi_4 - \frac{1}{24\pi^2 8t_0 f_0^2}\left[\frac{3}{2}\phi_2 ln\left(\frac{\phi_2}{8t_0\Lambda_\chi^2}\right) - \frac{1}{2}\phi_\eta ln\left(\frac{\phi_\eta}{8t_0\Lambda_\chi^2}\right)\right]\right)$$

$$\phi_{13} = \frac{\phi_K}{2B_0\sqrt{8t_0}}\left(1 - \frac{8}{8t_0 f_0^2}(L_5 - 2L_8)\phi_2 - \frac{16}{8t_0 f_0^2}(4L_6 - 2L_8 - 2L_4 - L_5)\phi_4 - \frac{1}{24\pi^2 8t_0 f_0^2}\phi_\eta ln\left(\frac{\phi_\eta}{8t_0\Lambda_\chi^2}\right)\right)$$

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Fit ansatze: $\mathcal{L}_2 = \phi_2 log\phi_2, \quad \mathcal{L}_\eta = \phi_\eta log\phi_\eta, \quad \phi_\eta = \frac{4}{3}\phi_4 - \phi_2$

$$[F0] \qquad \frac{\phi_{12}}{\phi_2} = \left[p_1 + p_2\phi_2 + p_3\left(\mathcal{L}_2 - \frac{1}{3}\mathcal{L}_\eta\right)\right] + \mathcal{O}(a^2), \qquad \frac{\phi_{13}}{\phi_K} = \left[p_1 + p_2\phi_K + \frac{2}{3}p_3\mathcal{L}_\eta\right] + \mathcal{O}(a^2)$$

$$[F\pi K] \qquad \frac{\phi_{12}}{\phi_2} = q_1\left(1 - \frac{1}{16\pi^2 8t_0 f_{\pi K}^2}\left(\mathcal{L}_2 - \frac{1}{3}\mathcal{L}_\eta\right)\right) + q_2\frac{1}{8t_0 f_{\pi K}^2}\phi_2 + q_3\frac{1}{8t_0 f_{\pi K}^2}\phi_4 + \mathcal{O}(a^2),$$

$$\frac{\phi_{13}}{\phi_K} = q_1\left(1 - \frac{1}{16\pi^2 8t_0 f_{\pi K}^2}\frac{2}{3}\mathcal{L}_\eta\right) + q_2\frac{1}{8t_0 f_{\pi K}^2}\phi_2 + q_3\frac{1}{8t_0 f_{\pi K}^2}\phi_4 + \mathcal{O}(a^2)$$

Chiral-continuum extrapolations

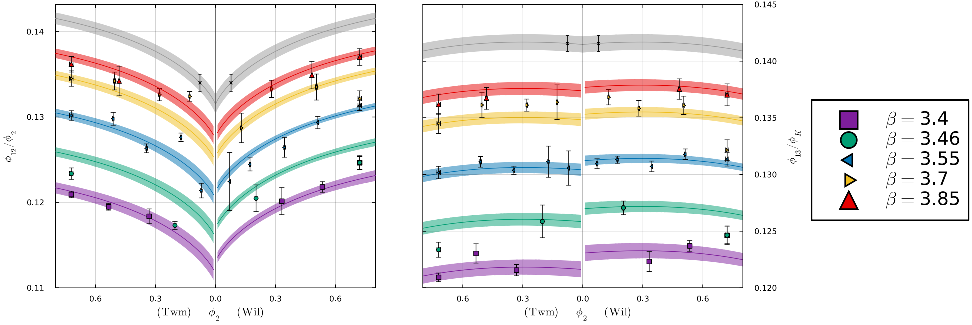
We consider different combinations of quark masses to assess systematic effects:

- $[\chi ptm]$: $\phi_{12}\,,\phi_{13}$
- $[\chi ptr]$: $\phi_{12}/\phi_{13}\,,2\phi_{13}/\phi_K + \phi_{12}/\phi_2$
- $[\chi ptc]$: $\phi_{12}/\phi_2 - \phi_{13}/\phi_K\,,2\phi_{13} + \phi_{12}$

Chiral-continuum extrapolations

We consider different combinations of quark masses to assess systematic effects:

- $[\chi ptm]$: Direct fit to the masses $\rightarrow$ $\phi_{12} = F(\phi_2, a^2/t_0)$ $\qquad$ $\phi_{13} = G(\phi_2, a^2/t_0)$



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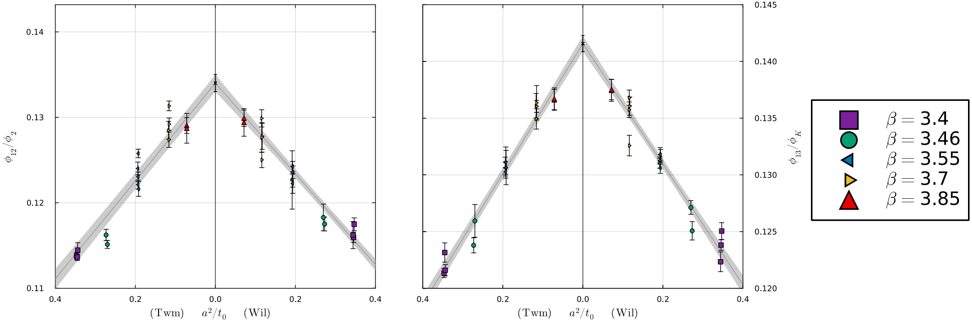

Chiral-continuum extrapolations

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- $[\chi ptr]$:

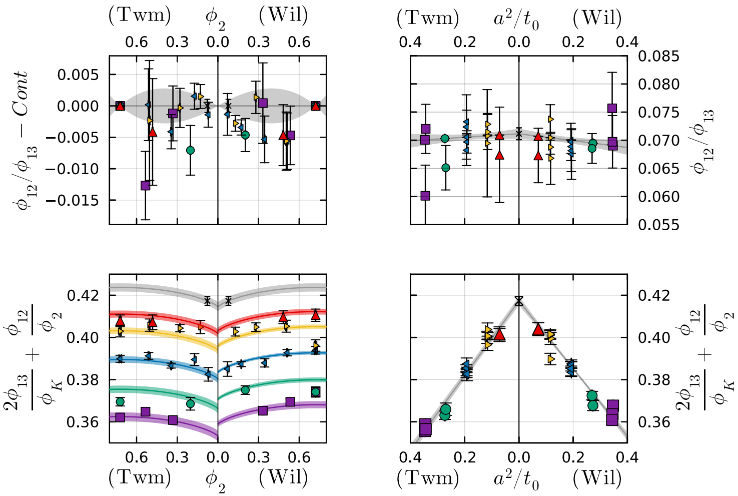
LO cutoff effects cancel. Mostly chiral effects.

$$\boxed{\frac{\phi_{12}}{\phi_{13}}} = \frac{2\phi_2}{2\phi_4 - \phi_2}\left[1 + \frac{p_2}{p_1}\left(\frac{3}{2}\phi_2 - \phi_4\right) - K(\mathcal{L}_2 - \mathcal{L}_\eta)\right] + \mathcal{O}\left(a^2(m_\pi^2 - m_K^2)\right)$$

$$\boxed{\frac{2\phi_{13}}{\phi_K} + \frac{\phi_{12}}{\phi_2}} = 3p_1 + 2p_2\phi_4 + p_3 K(\mathcal{L}_2 + \mathcal{L}_\eta) + \mathcal{O}(a^2)$$

Chiral effects highly suppressed. Mostly cutoff effects.

Chiral-continuum extrapolations



ϕ_{12}/ϕ_{13}

- Small cutoff effects
- Exact at the symmetric point

$2\phi_{13}/\phi_K + \phi_{12}/\phi_2$

- Suppressed chiral dependence
- Quark/meson masses correlation

Chiral-continuum extrapolations

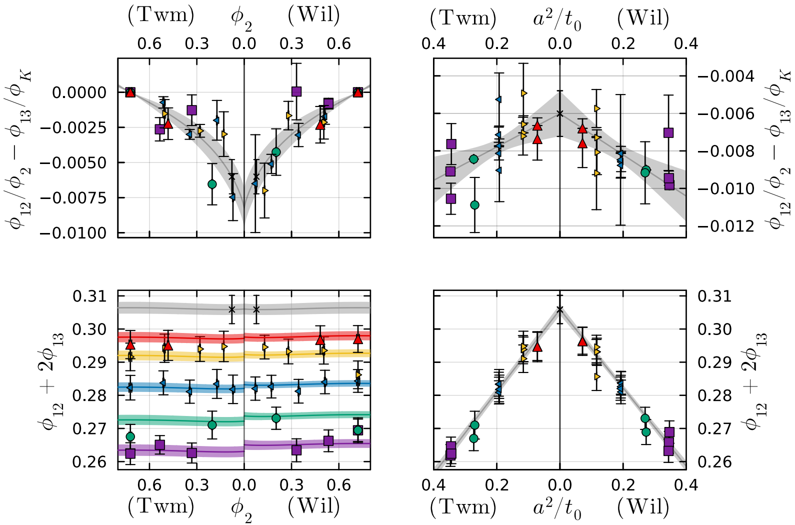
We consider different combinations of quark masses to assess systematic effects:

- $[\chi ptm]$: $\phi_{12}\,,\phi_{13}$
- $[\chi ptr]$: $\phi_{12}/\phi_{13}\,, 2\phi_{13}/\phi_K + \phi_{12}/\phi_2$
- $[\chi ptc]$:

$$\frac{\phi_{12}}{\phi_2} - \frac{\phi_{13}}{\phi_K} \to \text{Vanishes at } \phi_2 = \phi_K + \text{very little cutoff effects}$$

$$2\phi_{13} + \phi_{12} \to \text{Small chiral dependence, due to } \phi_4 = \mathrm{cnst.}$$

Chiral-continuum extrapolations



Chiral-continuum extrapolations

Quark mass combinations:

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- $[\chi ptr]$: $\phi_{12}/\phi_{13}\,,2\phi_{13}/\phi_K + \phi_{12}/\phi_2$
- $[\chi ptc]$: $\phi_{12}/\phi_2 - \phi_{13}/\phi_K\,,2\phi_{13} + \phi_{12}$

Cuts in data:

- $[m_\pi < 420\mathrm{MeV}]$
- $[\beta > 3.4]$: $a \leq 0.075$ fm
- $[\beta > 3.46]$: $a \leq 0.063$ fm

Cutoff effects:

- $[a^2]$: a^2/t_0
- $[a^2\alpha^\Gamma]$: $a^2\alpha^\Gamma(1/a)/t_0$ [2206.03536]
- $[a^2 + a^3]$: $a^2/t_0 + a^3/t_0^{3/2}$
- $[a^2\phi]$: $\phi a^2/t_0$

Model Average

$$\langle \mathcal{O} \rangle_{MA} = \sum_i \mathcal{O}^{(i)} W^{(i)} \qquad \sigma^2_{syst} = \langle \mathcal{O}^2 \rangle_{MA} - \langle \mathcal{O} \rangle^2_{MA}$$

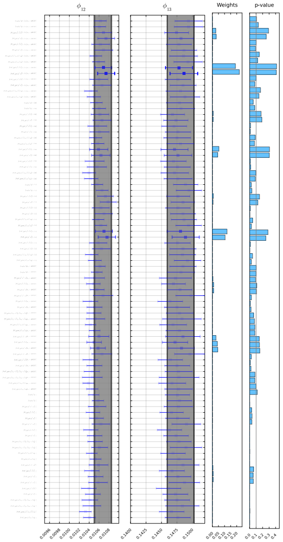
Takeuchi Information Criterion
[Takeuchi, 76][2209.14188]

$$W^{(i)} = \mathcal{N} exp\left(-\frac{1}{2}(\chi^2 - 2\langle \chi^2 \rangle) \right)$$

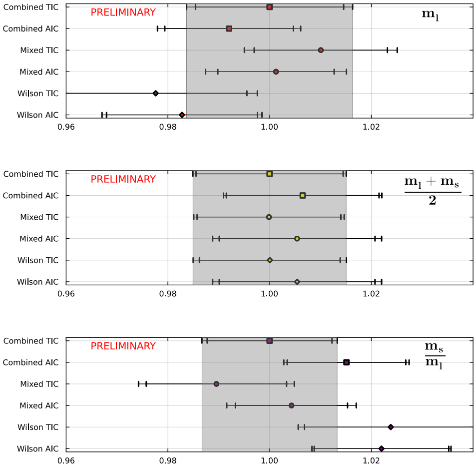
Akaike Information Criterion [Akaike, 76]

$$W^{(i)} = \mathcal{N} exp\left(-\frac{1}{2}(\chi^2 + 2n_{\mathrm{param}}) \right)$$

[2008.01069][2302.06550][2510.20450]



$N_f = 2 + 1$ results



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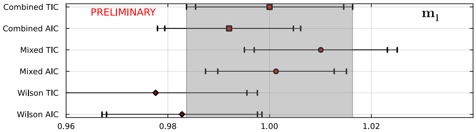
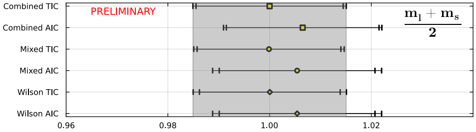
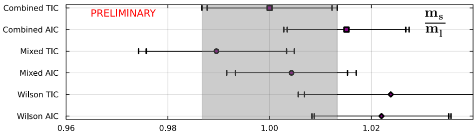
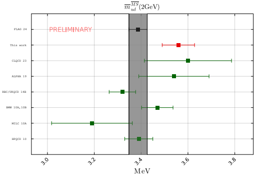
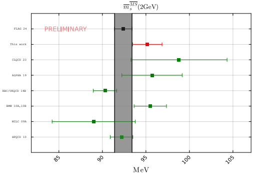
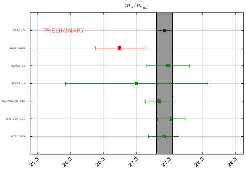












Conclusions

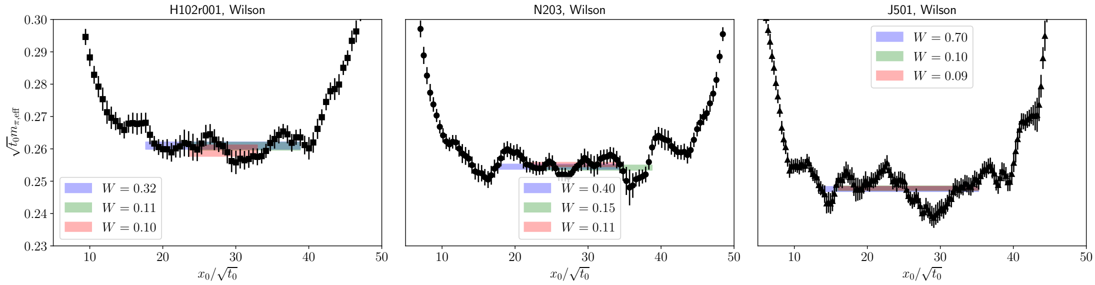
- We performed a combined chiral-continuum extrapolation with Wilson and Twisted mass regularizations. $\rightarrow$ Agreement with our previous determination [1911.08025].
- Five lattice spacings $(0.038, 0.085)\ \mathrm{fm}$ with $m_\pi L \geq 3.9$.
- Ongoing work on systematic analysis for ground state extraction.
- Current relative precision:

	m_{ud}	m_s	m_s/m_{ud}
[ALPHA 19]	4.2%	3.6%	4.0%
This work [PRELIMINARY]	1.9%	1.8%	1.3%
FLAG24	1.2%	1.1%	0.4%

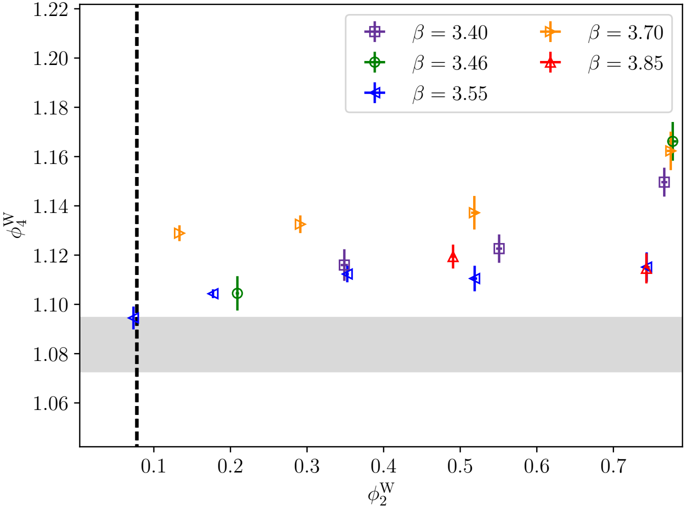
Thank you!

Backup

Plateau determination

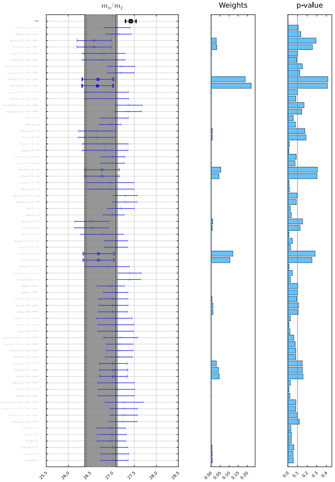


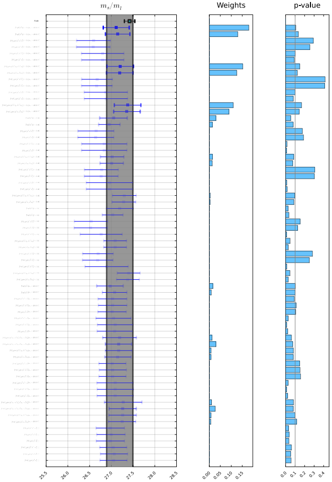
ϕ_4 shift



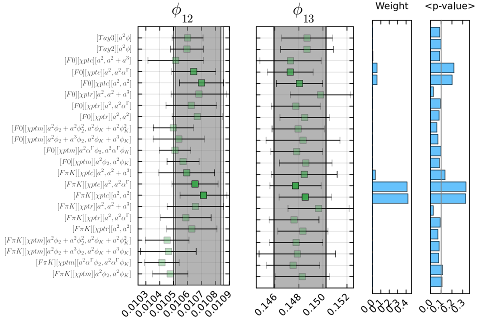
1.22
1.20
1.18
1.16
1.14
1.12
1.10
1.08
1.06
ϕ_4^W
0.1
0.2
0.3
0.4
0.5
0.6
0.7
ϕ_2^W
$\beta = 3.40$
$\beta = 3.46$
$\beta = 3.55$
$\beta = 3.70$
$\beta = 3.85$

TIC vs AIC

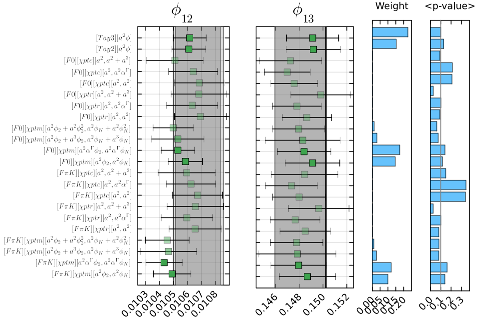




TIC vs AIC

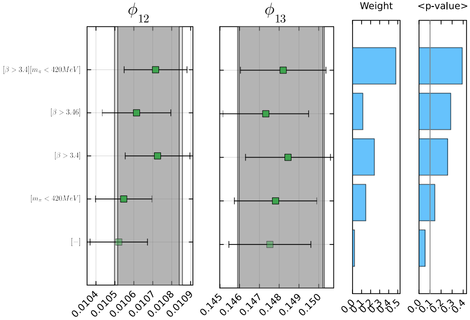


ϕ_{12}
ϕ_{13}
Weight
<p-value>
$[Tay3][a^2\phi]$
$[Tay2][a^2\phi]$
$[F0][\chi ptc][a^2, a^2+a^3]$
$[F0][\chi ptc][a^2, a^2\alpha^\Gamma]$
$[F0][\chi ptc][a^2, a^2]$
$[F0][\chi ptr][a^2, a^2+a^3]$
$[F0][\chi ptr][a^2, a^2\alpha^\Gamma]$
$[F0][\chi ptr][a^2, a^2]$
$[F0][\chi ptm][a^2\phi_2+a^2\phi_2^2, a^2\phi_K+a^2\phi_K^2]$
$[F0][\chi ptm][a^2\phi_2+a^3\phi_2, a^2\phi_K+a^3\phi_K]$
$[F0][\chi ptm][a^2\alpha^\Gamma\phi_2, a^2\alpha^\Gamma\phi_K]$
$[F0][\chi ptm][a^2\phi_2, a^2\phi_K]$
$[F\pi K][\chi ptc][a^2, a^2+a^3]$
$[F\pi K][\chi ptc][a^2, a^2\alpha^\Gamma]$
$[F\pi K][\chi ptc][a^2, a^2]$
$[F\pi K][\chi ptr][a^2, a^2+a^3]$
$[F\pi K][\chi ptr][a^2, a^2\alpha^\Gamma]$
$[F\pi K][\chi ptr][a^2, a^2]$
$[F\pi K][\chi ptm][a^2\phi_2+a^2\phi_2^2, a^2\phi_K+a^2\phi_K^2]$
$[F\pi K][\chi ptm][a^2\phi_2+a^3\phi_2, a^2\phi_K+a^3\phi_K]$
$[F\pi K][\chi ptm][a^2\alpha^\Gamma\phi_2, a^2\alpha^\Gamma\phi_K]$
$[F\pi K][\chi ptm][a^2\phi_2, a^2\phi_K]$
0.0103 0.0104 0.0105 0.0106 0.0107 0.0108 0.0109
0.146 0.148 0.150 0.152
0.0 0.1 0.2 0.3 0.4
0.0 0.1 0.2 0.3

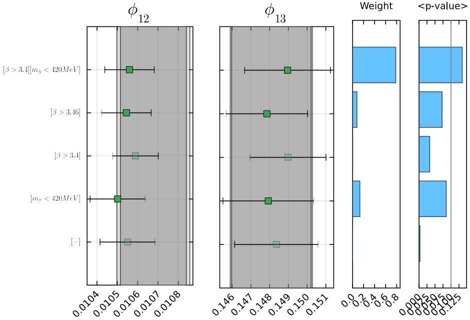


ϕ_{12}
ϕ_{13}
Weight
<p-value>
0.0103 0.0104 0.0105 0.0106 0.0107 0.0108
0.146 0.148 0.150 0.152
0.00 0.05 0.10 0.15 0.20
0.0 0.1 0.2 0.3

TIC vs AIC



ϕ_{12}
ϕ_{13}
Weight
<p-value>
$[\beta > 3.4][m_\pi < 420MeV]$
$[\beta > 3.46]$
$[\beta > 3.4]$
$[m_\pi < 420MeV]$
$[-]$



ϕ_{12}
ϕ_{13}
Weight
<p-value>
$[\beta > 3.4][m_\pi < 420MeV]$
$[\beta > 3.46]$
$[\beta > 3.4]$
$[m_\pi < 420MeV]$
$[-]$