Dissipative Preparation of Vacuum and Meson States in a 1+1D $\mathbb{Z}_2$ Lattice Gauge Theory

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Quantum Simulation

- Real-time dynamics of lattice gauge theories are largely inaccessible to conventional Euclidean lattice calculations
- Digital and analog quantum simulators offer a direct route to real-time dynamics, but require **state preparation**
- Preparing the vacuum and low-lying states with high fidelity is a prerequisite for any subsequent scattering or real-time experiment
- Goal of this talk: steps towards **dissipative preparation** of such states on quantum computers

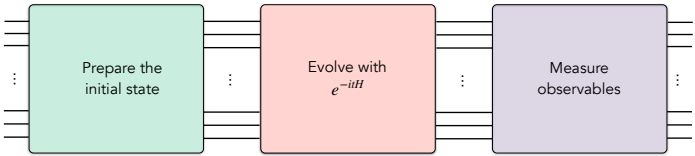

Figure from Davoudi, hep-lat:2507.15840

State Preparation Methods

- Adiabatic State Preparation
  - Performance guarantees
  - hindered by phase transitions, long ramp times are difficult for (current) hardware
- VQE Ansätze
  - Shallow circuits, classical optimization
  - No performance guarantees
- Spectral Filtering Algorithms
  - Performance guarantees
  - initial state overlap influences cost

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- **Dissipative State Preparation**
  - Potential for performance guarantees
  - Can start from **arbitrary initial states**

Dissipative Dynamics

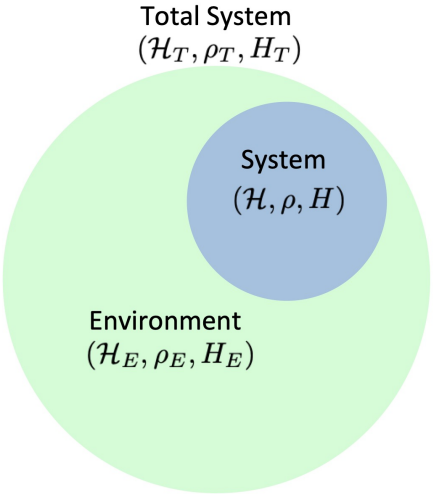

Figure from AIP Advances 1 February 2020; 10 (2): 025106

Dissipative Dynamics

The Lindbladian $\mathcal{L}$ provides the evolution of a system governed by Hamiltonian H in the presence of dissipation:

$$\dot{\rho} = \mathcal{L}[\rho] = -i[H, \rho] + \sum_{\alpha} \left(K_\alpha \, \rho \, K_\alpha^\dagger - \tfrac{1}{2}\{K_\alpha^\dagger K_\alpha, \rho\} \right)$$

- $\{K_\alpha\}$: Jump operators, not necessarily unitary
- **Steady state condition:** $|\psi\rangle$ is a steady state of the dynamics if

$$H\,|\psi\rangle = E\,|\psi\rangle\,, \qquad K_\alpha\,|\psi\rangle = 0 \;\; \forall \alpha$$

- Lindblad dynamics can be simulated on quantum computers

[1] In particular, we follow [Zhiyan Ding, Chi-Fang Chen, Lin Lin, PRR, arXiv:2308.15676]

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- Lindblad dynamics can be simulated on quantum computers
- **Engineer jump operators $\{K_\alpha\}$ such that the ground/meson state is the steady state of the dissipative dynamics**[1]

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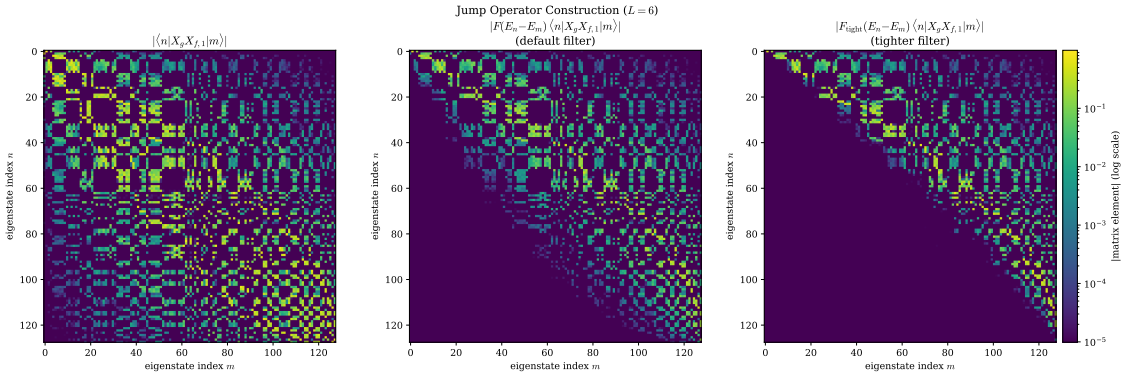
Engineering Jump Operators

- Choose a **coupling operator** A_α acting on the system
- Apply an **energy filter** f in the eigenbasis of H:

$$K_\alpha = \sum_{i,j} \hat{f}(E_i - E_j)\ |i\rangle \langle i|A_\alpha|j\rangle \langle j|$$

  Where $\hat{f}(E_i - E_j) = 0$ if $E_i \geq E_j$
- Consequence: K_α can only *decrement* energy
- In practice, implemented via a smooth approximation to a step function

Engineering Jump Operators



Engineering Jump Operators

We can construct K_α without the spectrum of H, via the inverse Fourier transform of $\hat{f}$, f:

$$\begin{aligned} K_\alpha &= \int_{-\infty}^{\infty} \mathrm{d}s\, f(s) e^{iHs} A_\alpha e^{-iHs} \\ &\approx \sum_{l=-M}^{M} f(s_l) e^{iHs_l} A_\alpha e^{-iHs_l} w_l, \qquad s_l = l t_s, \quad w_{\pm M} = \frac{t_s}{2}, w_l = t_s \end{aligned}$$

We truncate the bounds of the integral, and then approximate it as a discrete sum $\rightarrow$ allows for representation of K_α as a quantum circuit.

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Algorithm couples system register to ancilla qubit $\rightarrow$ measure and reset ancilla qubit $\rightarrow$ non-unitary dynamics on system qubits

In particular, we follow [Zhiyan Ding, Chi-Fang Chen, Lin Lin, PRR, arXiv:2308.15676]

Algorithm Overview

1. **Choose** a set of $\{A_\alpha\}$ coupling operators
2. Set filter parameters (based on Δ, $||H||$)
3. Determine the approximation parameters for the integral (bounds, number of samples)
4. Apply quantum circuit that approximates Lindbladian evolution

The choice of coupling operators is fundamental to the performance of the algorithm, not all choices of $\{A_\alpha\}$ will converge to the target state.

In this talk, we will consider the exact Lindblad dynamics, rather than the circuit dynamics.

In particular, we follow [Zhiyan Ding, Chi-Fang Chen, Lin Lin, PRR, arXiv:2308.15676]

$\mathbb{Z}_2$ LGT Hamiltonian

Staggered fermions ξ_n on matter sites, $\mathbb{Z}_2$ gauge qubits $X_{g,n}, Z_{g,n}$ on links, with periodic boundary conditions:

$$H = \underbrace{\frac{1}{2a}\sum_n \left(\xi_n^\dagger Z_{g,n}\xi_{n+1} + \text{h.c.}\right)}_{H_{\text{kin}}} + \underbrace{m\sum_n (-1)^n \xi_n^\dagger \xi_n}_{H_{\text{mass}}} + \underbrace{\varepsilon \sum_n X_{g,n}}_{H_{\text{el}}}$$

- We work in the subspace of physical (Gauss Law preserving), total charge 0 states.
- Charge conjugation C commutes with H; eigenstates carry momentum k and $c = \pm 1$: $C\,|k, c\rangle = c\, e^{-ik}\,|k, c\rangle$
- **Vacuum**: unique ground state, $c = +1$
- **Vector meson**: unique lowest energy state $|k, c = -1\rangle$

Targeting Excited States

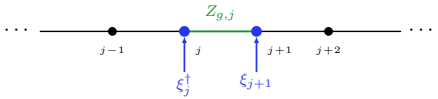
- To target excited states, we target ground states in specific sectors of the Hamiltonian
- If $[A_\alpha, S] = 0$, then $[K_\alpha, S] = 0$, and therefore we can choose $\{A_\alpha\}$ such that symmetries are not broken $\rightarrow$ dissipative dynamics will not mix states across sectors
- Starting state with definite quantum numbers $\rightarrow \{K_\alpha\}$ that respect symmetries $\rightarrow$ Ground state of sector

To obtain the vector meson state, we restrict the space of allowed coupling operators to translationally invariant, charge conjugation invariant operators.

Vacuum Coupling Operators

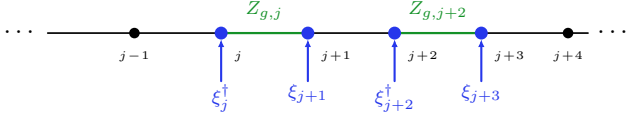
Single hop

$$K_j \sim \xi_j^\dagger \, Z_{g,j} \, \xi_{j+1} + \text{h.c.}$$



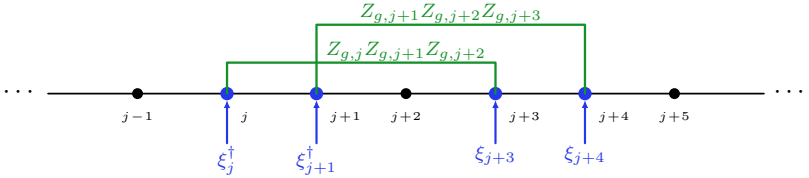
Double hop

$$\text{DH}_{j,2} \sim K_j K_{j+2} + \text{h.c.}$$



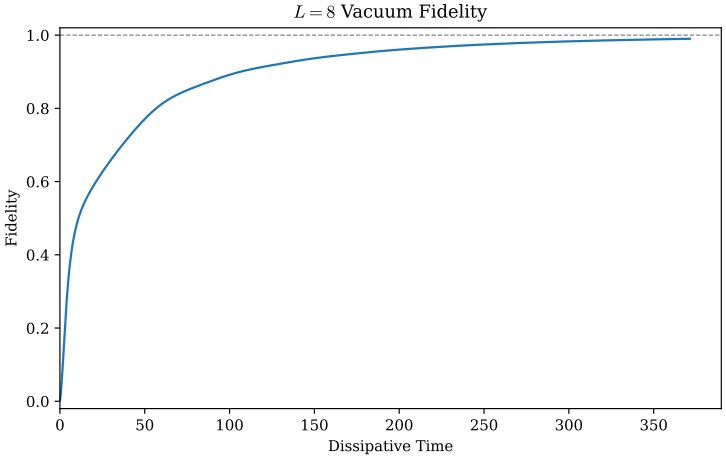
Pair hop

$$\text{PH}_{j,3} \sim \xi_j^\dagger \xi_{j+1}^\dagger \left(\prod_{m=j}^{j+2} Z_{g,m}\right)\left(\prod_{m=j+1}^{j+3} Z_{g,m}\right) \xi_{j+3}\xi_{j+4} + \text{h.c.}$$



Vacuum State Preparation

Starting state is the maximally mixed state in the physical Hilbert space



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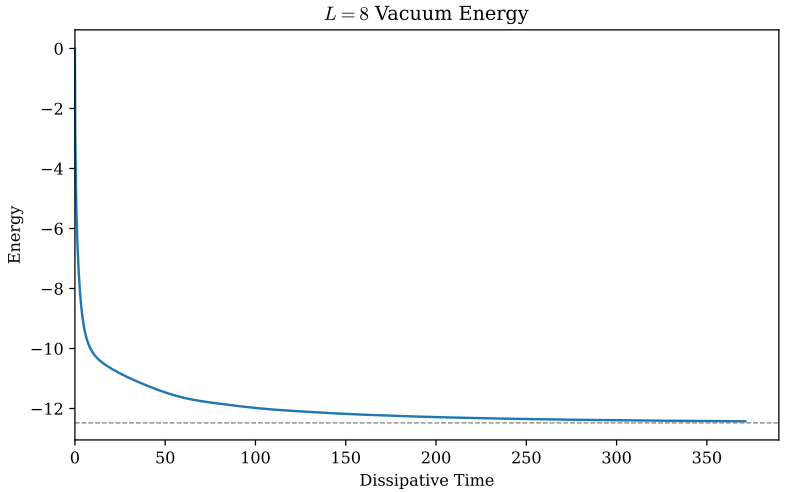
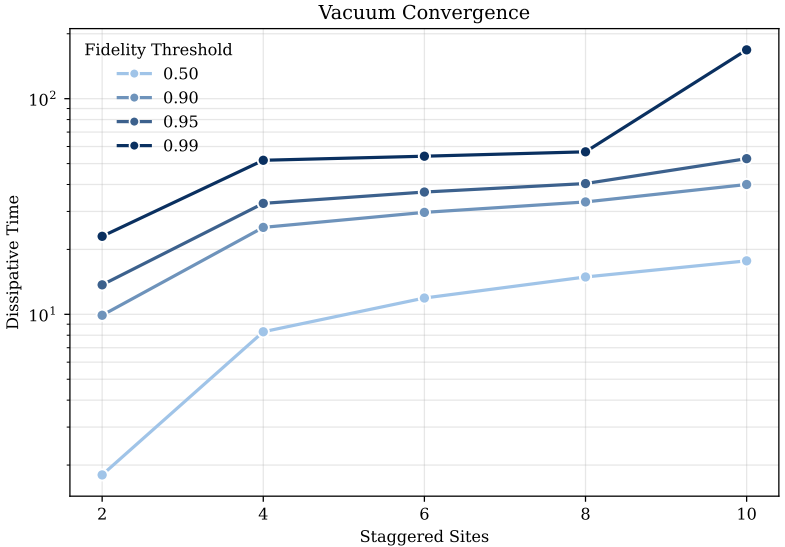


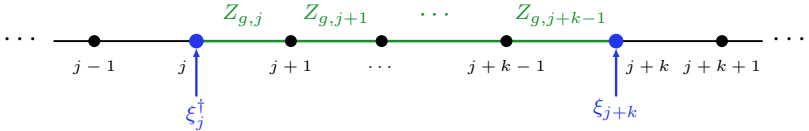
Vacuum State Preparation



Vector Meson Jump Operators

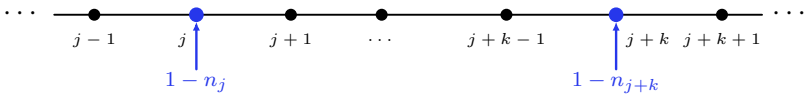
Extended fermion-hopping operators

$$W_k \sim \sum_{j=1}^{L} \left(\xi_j^\dagger \left(\prod_{m=j}^{j+k-1} Z_{g,m} \right) \xi_{j+k} + \text{h.c.} \right)$$

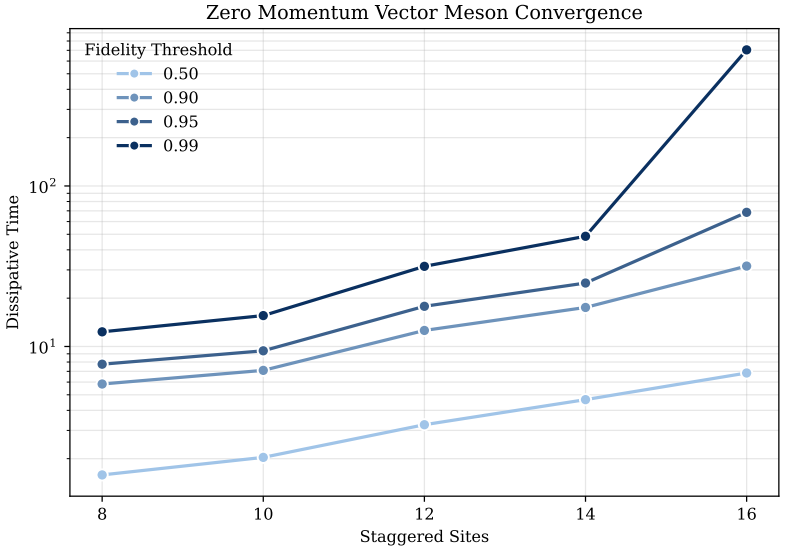


Extended density-density operators

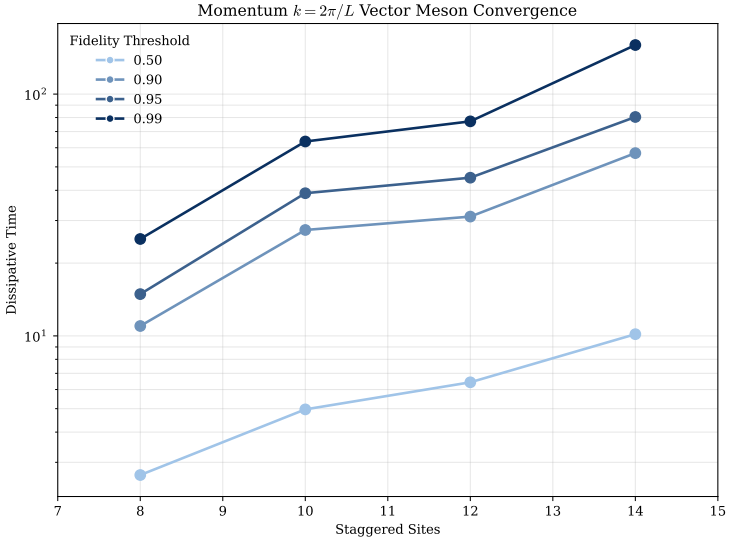
$$D_k \sim \sum_{j=1}^{L} (1 - n_j)(1 - n_{j+k}), \qquad n_j = \xi_j^\dagger \xi_j$$



Vector Meson at $k = 0$



Vector Meson at $k \neq 0$



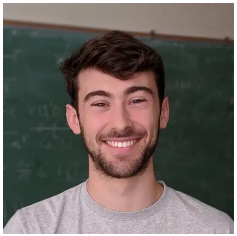
Future Work

- Extend Lindblad dynamics to larger systems
- Explicit circuit costs and dynamics

Open Questions:

- Given a set of coupling operators, can we provably converge to ground states of LGTs?
- Are there *optimal* choices of coupling operators for LGTs?
- Can we use dissipative state preparation in conjunction with other state preparation methods?

Acknowledgements



Chris Kane

Zohreh Davoudi

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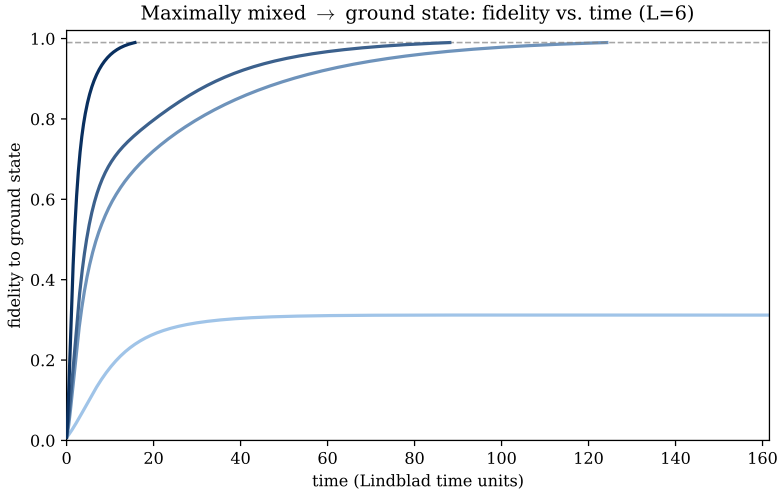
Questions?

Backup Slides

Operator Sets

Vacuum fidelity given different choices of coupling operators.



Filter Function

$$\hat{f}(\omega) = \frac{1}{2}\left(\text{erf}\left(\frac{\omega + a}{\delta_a}\right) - \text{erf}\left(\frac{\omega + b}{\delta_b}\right)\right)$$

Where

$$\text{erf}(\omega) = \frac{2}{\sqrt{\pi}} \int_0^{\omega} e^{-x^2}\, \mathrm{d}x$$

The inverse Fourier transform of the filter function is given by:

$$f(s) = \frac{\exp\left(-\frac{\delta_a^2 s^2}{4}\right)\exp\left(ias\right) - \exp\left(-\frac{\delta_b^2 s^2}{4}\right)\exp(ibs)}{2\pi i s}$$

We choose $a = 2.5\|H\|$, $\delta_a = .5\|H\|$, and $b = \delta_b = E_1 - E_0$.

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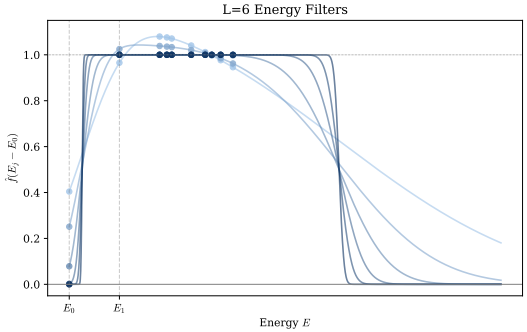
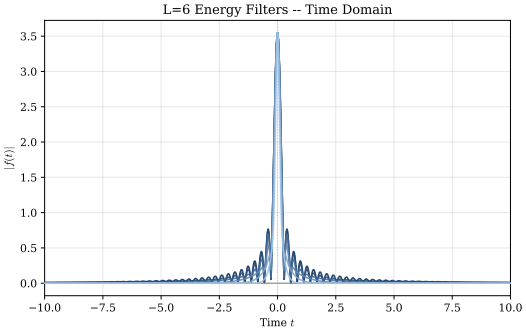
After gauge-fixing and Jordan-Wigner transforming onto qubits:

$$H_{\text{qubit}} = -\frac{1}{2}\sum_{j=1}^{L-1}\left(\sigma_j^+\sigma_{j+1}^- + \text{h.c.}\right) - \frac{1}{2}\left(\sigma_{\text{L}}^+ \text{X}_{\text{g}}\left(\prod_{\text{k}=2}^{\text{L}-1}\text{Z}_{\text{k}}\right)\sigma_1^- + \text{h.c.}\right)$$
$$+ m\sum_{j=1}^{L}(-1)^{j-1}\frac{1-Z_j}{2} + 2\varepsilon\, Z_g \sum_{J=1}^{L}\prod_{k=1}^{J}\left[(-1)^{k-1}Z_k\right]$$

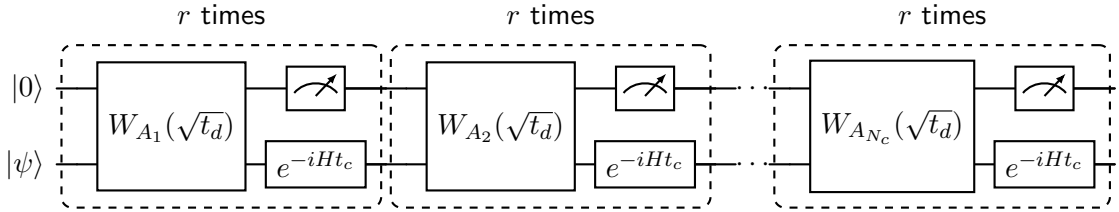
Filter Function





Quantum Circuit

- N_c coupling operators
- Dissipative time step t_d
- Coherent time step t_c



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