Institute for Theoretical Physics
High Performance Computational Physics

Anomalous Thermalization from Generalized Symmetries

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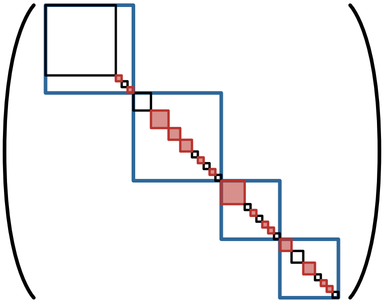
Lattice 2026

arXiv:2604.12907



Outline

1. Thermalization and Ergodicity Breaking

2. Hilbert Space Fragmentation from Symmetry

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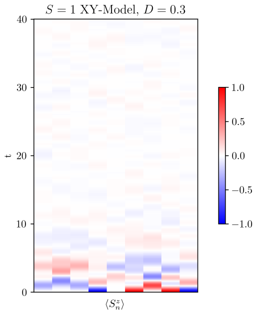
Thermalization in isolated quantum systems

Non-integrable local many-body Hamiltonians are expected to be ergodic and thermalize:

- After some time, *local* observables have only small fluctuations
- Expectation values do not depend on details of initial state
- They can be predicted by an appropriate statistical ensemble

$$\langle \mathcal{O} \rangle_{\substack{t\to\infty \\ V\to\infty}} = \frac{1}{Z} \mathrm{Tr}\left(\mathcal{O}\rho_{\text{thermal}}\right)$$

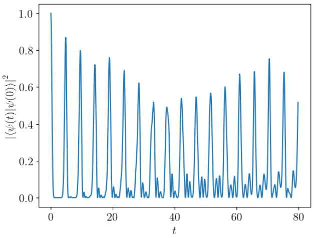


Does this always happen?

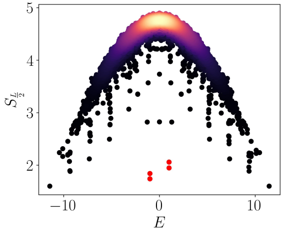
Deutsch, PRA 43, 2046 (1991); Srednicki, PRE 50, 888 (1994)

Ergodicity breaking

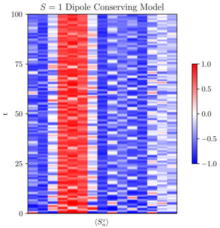
Many counterexamples have now been found:



Revivals



Simple eigenstates



Localization

But did we consider the right ensembles?

Ensembles from Ergodicity

The dynamics explore possible states uniformly subject to conserved quantities, e.g.

$$\rho_{\mathrm{Gibbs}} = \frac{1}{Z} e^{-\beta H} e^{-\sum_i \mu_i G_i}, \qquad \langle \psi_0 | G_i | \psi_0 \rangle \equiv \mathrm{Tr}\left(G_i \rho_{\mathrm{Gibbs}} \right)$$

But we should only consider G_i that affect local observables, not all $|E\rangle \langle E|$.

Which ones are these?

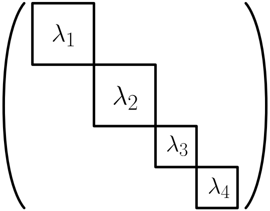
Conventional symmetries

U is relevant if it is a local observable itself

$$U = \prod_n u_n,$$

for example magnetization $U = e^{iM} = \prod_n e^{i\alpha\sigma_n^z}$.

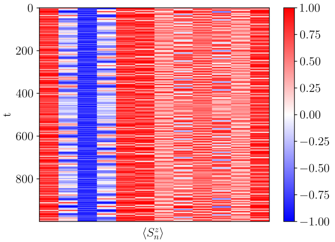
$\Rightarrow \mathcal{O}(\mathrm{poly}(V))$ symmetry sectors $U_i \left|\phi\right\rangle = \lambda_{i,\phi} \left|\phi\right\rangle$.

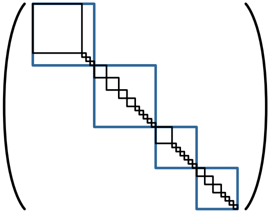


Hilbert Space Fragmentation

But for some models, the number of sectors in the product basis grows exponentially with volume:

$$H = \sum_n S_n^+ (S_{n+1}^-)^2 S_{n+2}^- + h.c.$$





This can lead to localization

S. Moudgalya et al., Rept.Prog.Phys. 85 (2022) 8, 086501

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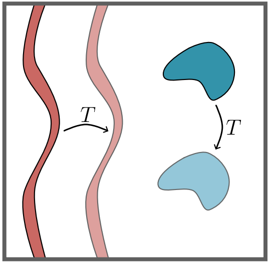
Subsystem symmetries

There can be symmetries that are products over only a subsystem $\mathcal{M}$

$$U(g) = \prod_{n \in \mathcal{M}} u_n(g)$$

Any disjoint translation is also an independent symmetry

$$[H, T] = 0 \quad \Rightarrow \quad [TUT^{-1}, H] = 0$$

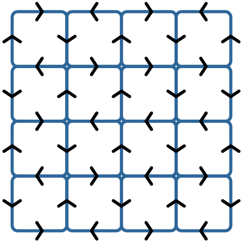


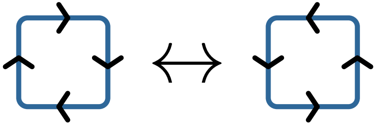
The sectors are now labeled by $|T|$ independent eigenvalues

$\Rightarrow$ Exponentially growing number of sectors

Example: U(1) gauge symmetry

Take a lattice with a $S = 1/2$ spin at every link representing the electric field





$$H = \sum_{\square} S_1^+ S_2^+ S_3^- S_4^- + h.c.$$

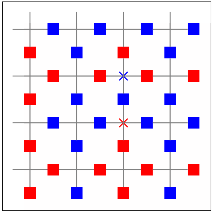
$U(1)$ gauge theory conserves the Gauss law at each site:

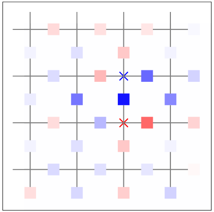
$$G_n = \nabla E_n, \qquad U_n = e^{-i\alpha_n G_n}$$

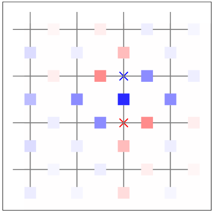
G_n measures local static charge, which can have exponentially many configurations

Gauge-symmetric Hamiltonians outside the physical sector

Non-physical sectors break translation invariance


$\langle S_n^z \rangle_{t=0}$


$\langle S_n^z \rangle_{t=10^4}$


$\langle S_n^z \rangle_{\beta=0}$

The thermal ensemble constrained by G_n replicates this

Disorder-free localization: Since most of the Hilbert space is non-physical and frozen, even the translation-invariant state $\frac{1}{\sqrt{2}^V} \left(|\uparrow\rangle + |\downarrow\rangle\right)^{\otimes V}$ results in anomalous correlations

A. Smith et al., Phys. Rev. Lett. 118, 266601 (2017)

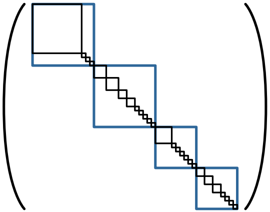
Partial isometries

Individual symmetry sectors may have additional symmetry

This can be described with a conserved operator

$$D = UP$$

- where $[D, H] = 0$
- U is unitary, but $[U, H] \neq 0$
- P is a projector onto a sector of H



Example: Non-invertible symmetry of the critical Ising model

G. Ortiz et al., Phys. Rev. B 113, 165126 (2026)

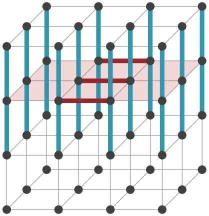
N. Seiberg et al., SciPost Phys. 16 (2024) 064

Higher-form symmetry in one symmetry sector

Something similar happens in the gauge theory:

- The $(3+1)$d $U(1)$ pure gauge QLM has 1-form symmetries over planes

$$W^{(1)}_{xy}(z) = \sum_{x,y} S^z_{r,\hat{z}},$$



J. Steinegger et al., Phys.Rev.D 112 (2025) 11, 114512
TB, M. Marinkovic, J. Pinto Barros, arXiv:2604.12907

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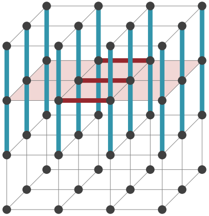

- The $(3+1)$d $U(1)$ pure gauge QLM has 1-form symmetries over planes

$$W_{xy}^{(1)}(z) = \sum_{x,y} S_{r,\hat{z}}^z,$$

- Only in the $\pm\frac{L^2}{2}$ sectors, winding numbers are also conserved

$$W_x^{(2)}(y,z) = \sum_{x} S_{r,\hat{y}}^z$$

$\Rightarrow$ exponentially many sub-fragments even though $[H, W_x^{(2)}] \neq 0$

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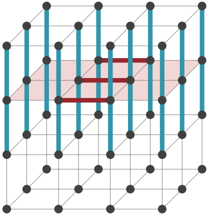
- $D = W_x^{(2)} P$ does commute with H and can be considered a generalized symmetry.



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Spin-1 dipole-conserving model

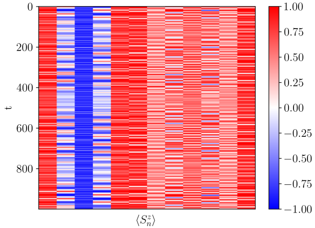
The Spin-1 dipole-conserving model

$$H = \sum_n S_n^+ (S_{n+1}^-)^2 S_{n+2}^- + h.c.$$

has many sectors that have gauge symmetry

$$G_n = |0\rangle \langle 0|_n \,, \quad \tilde{G}_n = S_n^z + 2S_{n+1}^z + S_{n+2}^z$$

for a subset of sites.



This explains the localization, but not all fragments.

TB, J. Pinto Barros, M. Marinkovic, arXiv:2604.15820

Conclusion

- Generalized symmetries can lead to phenomena that resemble ergodicity breaking
- Subsystem symmetry: low-dimensional support leads to exponentially many symmetry sectors
- Partial isometries: Quantities may be conserved in some symmetry sectors and not in others
- This leads to the phenomenology of Hilbert space fragmentation and can also explain some quantum many-body scars [K. Pakrouski et al., PRL 125 (2020) 23, 230602]

The High Performance Computational Physics Group at ETH

Thank you!