Toward a complete sub-percent lattice determination of the muon $g-2$: Update from Fermilab Lattice, HPQCD and MILC

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Lattice 2026 - University of Maryland

Fermilab-HPQCD-MILC Acknowledgements

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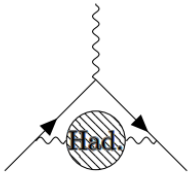
(Fermilab Lattice, HPQCD, and MILC Collaborations)

ACCESS ALCC DiRAC High Performance Computing Facility NERSC U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

Leadership-Class Computing Resource Allocations (LRAC) BIG RED 200 USQCD XSEDE

The HVP from Lattice QCD

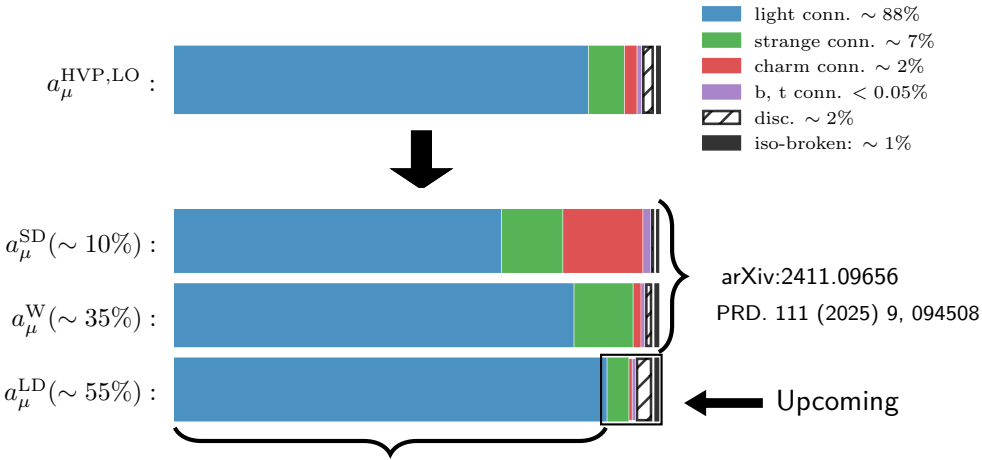


$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t)\, C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \, \langle J_i(x) J_i(0) \rangle$$

$$J_i(x) = \sum_f Q_{q_f} \bar{q}_f(x) \gamma_i q_f(x), \quad Q_u = +\frac{2}{3}, \; Q_d = -\frac{1}{3}, \; Q_s = -\frac{1}{3}, \ldots$$

$C^{ff}_{\mathrm{conn.}}(t)$ & $C^{ff}_{\mathrm{disc.}}(t)$ are computed on the lattice.

Fermilab/HPQCD/MILC Status



This talk: Isospin symmetric update

Previous talk: Isospin breaking (Ethan Neil)

Outline

- I=0
  - Preliminary results based on new disconencted dataset at 0.06 fm
- I=1
  - Updating our light-quark connected contribution
  - Exclusive staggered two-pion reconstruction progress.
- Outlook

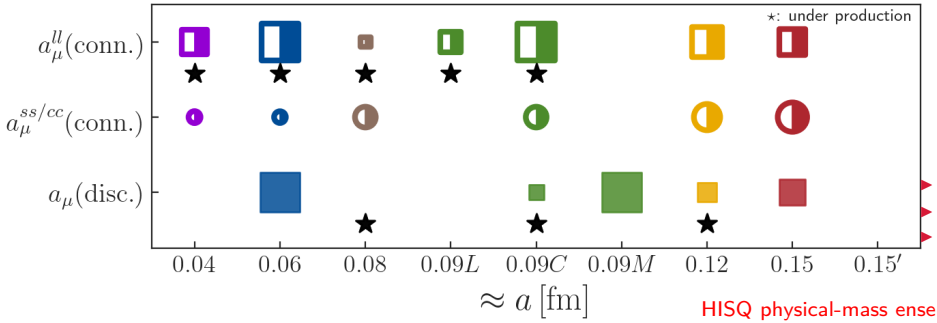
I=0 / Disconnected

$I = 0$

$$a_\mu^{I=1} : \frac{9}{10} a_\mu^{ll}$$
$$a_\mu^{I=0} : \frac{1}{10} a_\mu^{ll} + a_\mu^{ss} + a_\mu^{cc} + a_\mu(\text{disc.})$$

- $a_\mu(\text{disc.}) \sim a_\mu^{I=0} - \frac{1}{9} a_\mu^{I=1}$
- Correlator reconstruction difficult & not easily testable ⇒ construct $I = 0$ instead.
- Removes dominant finite-volume & taste-breaking effects.
- Rest of talk: Charm is separate

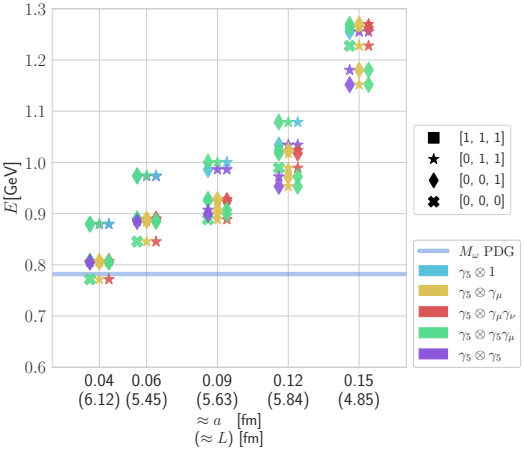
$I = 0$ / disconnected dataset



- $I = 0$ constrained by the disconnected dataset (taste-singlet discretization).
- New dataset generated at 0.06 fm with LMA:
  - 4000 exact low-modes (pairs)
  - 1000 random-volume tsm-improved deflated sources
  - Mass-split technique
- Coarser ensembles also under production: retuned $0.09C$ and new 0.08 ($a \approx 0.077$ fm).

$I = 0$ lattice spectrum

- Continuum IV: $\omega \to \pi\pi\pi$, $\phi \to KK$
- Lattice group theory restricts $\pi\pi\pi$ states coupling to vector current.
  eg. No $\pi(0,0,1)\pi(0,0,-1)\pi(0,0,0)$ coupling.
- Lowest state in $I = 0$ channel on HISQ physical-mass lattices is dominated by (stable) ω not $\pi\pi\pi$ [1]
- KK states also above ω (and ϕ).
- This is non-interacting picture but expect binding energies to be small [2].



$$E_{\pi_\xi,\text{free}} = \sqrt{\vec{p}^2 + M_\xi^2}$$

[1] Except at 0.04 fm

[2] Parrino/Lehner talk @ 8th g-2 TI workshop.

$I = 0$ & disconnected correlator reconstruction

$I = 0$: fit

$$C_{\text{fit}}^{I=0}(t) = \sum_{n=0}^{N_{\text{states}}} Z_n^2 \left(e^{-E_n t} + e^{-E_n (T-t)} \right) + (-1)^t \sum_{m=0}^{M_{\text{states}}} Z_{m,\text{osc}}^2 \left(e^{-E_{m,\text{osc}}\, t} + e^{-E_{m,\text{osc}} (T-t)} \right)$$

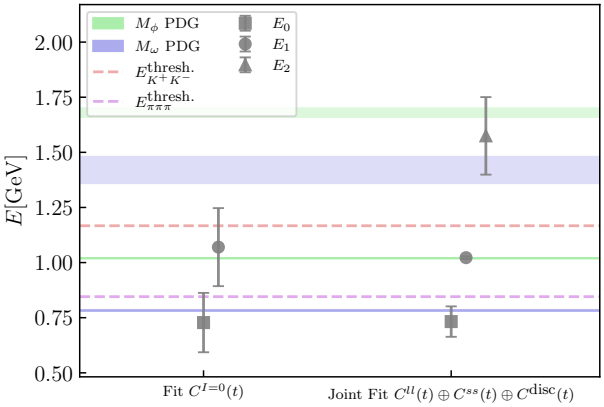
- Expect $E_{\text{fit},0} = M_\omega$
- Reconstruct tail with $C_{\text{recon}}^{I=0}(t) = Z_0^2 e^{-E_{\text{fit},0} t}$
- Also consider bounding $C^{I=0}(t)$ with $E_{\text{upper}} = M_\omega$, $E_{\text{lower}} = E_{\pi\pi\pi,0,\text{free}}$

Disconnected: Joint fit

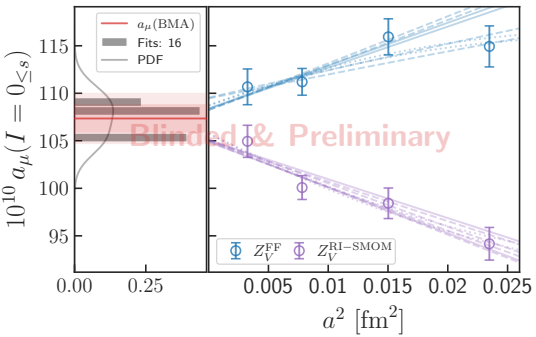
- $C^{\text{disc}}(t) = C^{I=0}(t) - \frac{1}{9} C^{I=1}(t)$
- Two channel fit to $C^{\text{disc}}(t)$ unconstrained $\Rightarrow$ Joint fit to $C^{\text{ll}}(t) \oplus C^{\text{ss}}(t) \oplus C^{\text{disc}}(t)$
- $I = 1$ amplitudes shared between light & disc (up to charge factors)

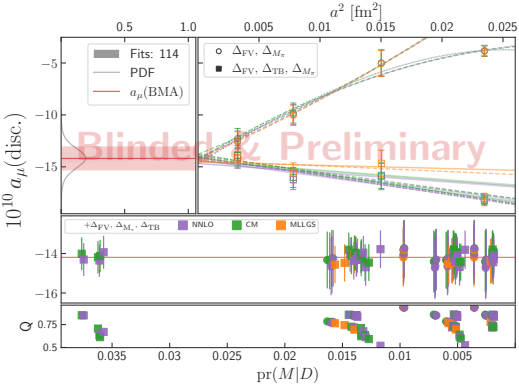
$I = 0$ spectrum (0.06 fm)

- Ground state E_0 consistent with ω, below the $\pi\pi\pi$ & K^+K^- thresholds.
- Only using the ground state, not doing spectroscopy!
- Final a_μ result cross-checked against bounding method



$I = 0$ & disconnected continuum extrap.



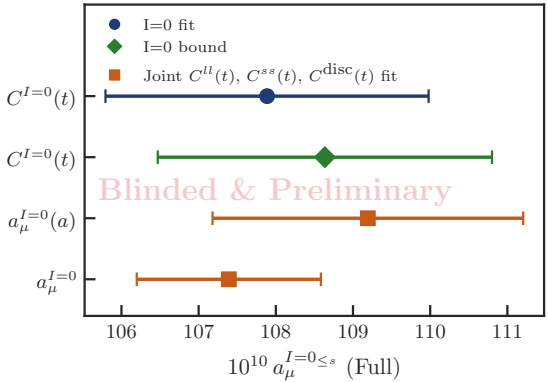


- $a_\mu^{I=0}$: FV & taste-breaking effects cancel by construction $\Rightarrow$ cleaner continuum, no TB correction.
- a_μ(disc.): LD FV & taste-breaking effects (from $I = 1$) $\Rightarrow$ corrections: ($\Delta_{\rm FV}$, $\Delta_{\rm TB}$, Δ_{M_π}).

$I = 0$ & disconnected cross-checks

Cross-check: $a_\mu^{I=0}$ consistent if constructed from:

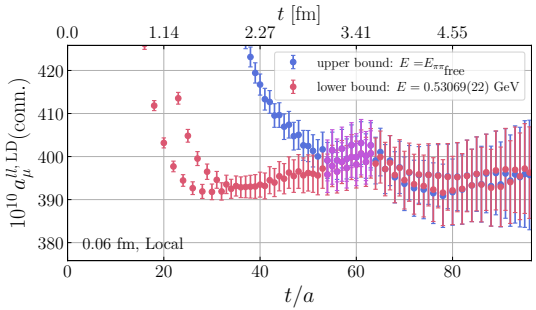
- $I = 0$ correlator w/ fit
- $I = 0$ correlator w/ bound
- flavor components on the lattice
- flavor components in the continuuum

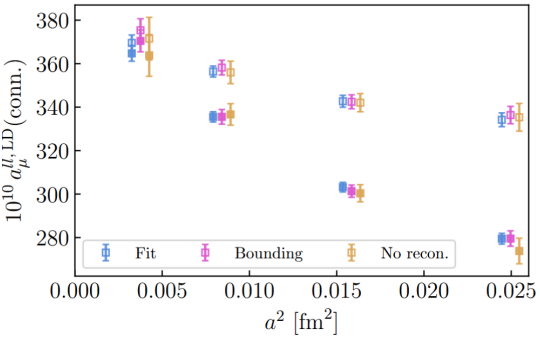


Smaller error if constructed in continuum but increased correlations with a_μ^{ll} ($a_\mu^{I=1}$)
$\Rightarrow a_\mu(\mathrm{iso}) = a_\mu^{I=0} + a_\mu^{I=1}$ has similar errors regardless of approach. (backup slide)

I=1 / Light

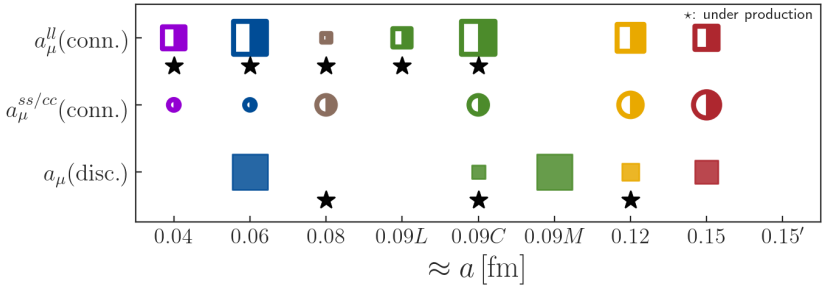
Correlator reconstruction recap





- Fit vs. bounding at 0.06 fm needs further investigation
  (fake-data test confirmed fit approach (backup slides), but plan to revisit with more statistics.)

Light update plans



- M: Mistuned
- C: Corrected
- L: Large volume

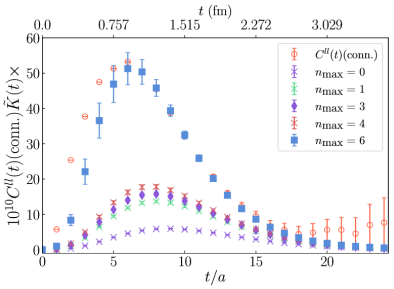
- Extensive light data generation ongoing on Aurora, Frontier & Perlmutter:
  - 300 $\rightarrow$ 1k configs @ 0.04
  - 1k $\rightarrow$ 3k configs @ 0.06
  - 0 $\rightarrow$ 1k configs @ 0.08
  - 1k $\rightarrow$ 3k configs @ $0.09C$ (primarily for FV study)
- Near-term: update the light contribution with $2\times$ statistics @ 0.06 fm.

Staggered two-pion reconstruction

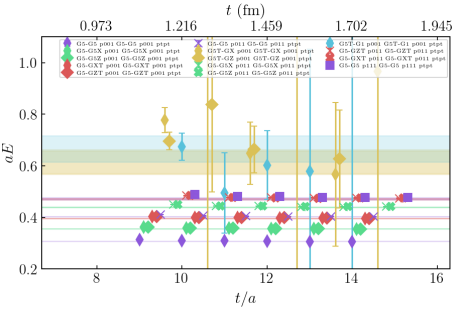
$$\mathbf{C}(t) = \begin{pmatrix} C_{\rho,\tilde{\rho}\to\rho,\tilde{\rho}}(t) & C_{\rho,\tilde{\rho}\to\pi\pi}(t) \\ C_{\pi\pi\to\rho,\tilde{\rho}}(t) & C_{\pi\pi\to\pi\pi}(t) \end{pmatrix}$$

$$C^{ll}(t)(\text{conn.}) = \frac{10}{9}\sum_{n=0} \left|\langle 0|\rho^0|n\rangle\right|^2 e^{-E_n t}$$

Established at 0.15 **fm** (8×8 basis):



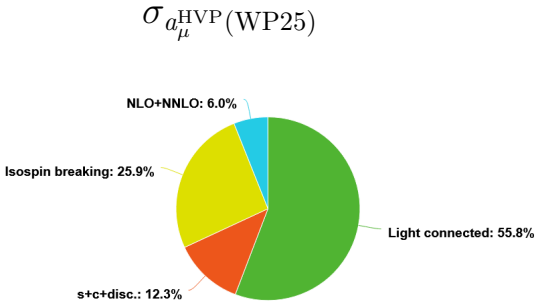
Extending to 0.12 **fm:** More momenta, more tastes $\Rightarrow$ basis: $8 \times 8 \to 16 \times 16$.



- $\gamma_5\gamma_0 \otimes \xi_A$ ops used over $\gamma_5 \otimes \xi_A$ to remove staggered time-links
- $\gamma_5\gamma_0$: bad ground-state overlap $\Rightarrow$ noisy, no clean plateau,
- GEVP won't diagonalise with full basis (not complete).
- **Plan:** Investigate time-link ops; modify matrix fit / GEVP.

Outlook

- **Near term**:
  - Increasing statistics on light datasets
  - FV study on lattice with $2\times$volume.
  - Lattice calculation of IB effects (Complete SIB & partial QED)
  - NLO calculation
  - Plan: Total HVP paper by end of the year
    - $a_\mu(\mathrm{iso.})$ uncertainty $\sim$ WP25 average
    - IB errors less well known at this point.

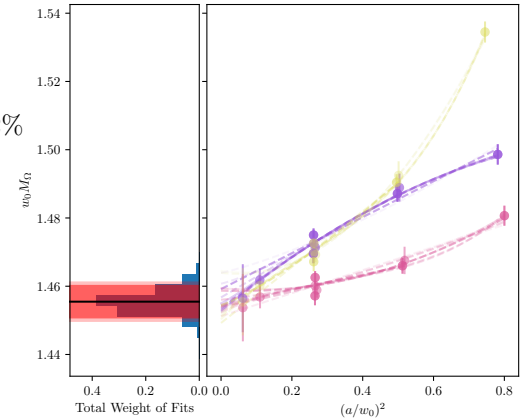


- **Longer term**:
  - Complete lattice calculation of QED contributions.
  - Light-quark tail using two-pion operators.

Thank you

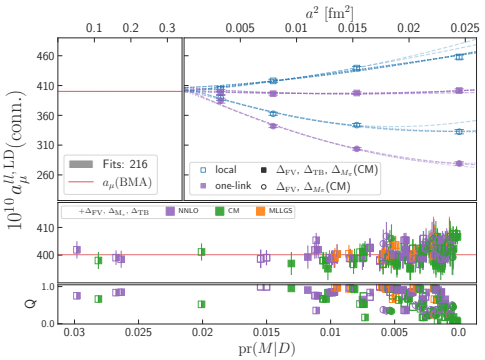
Updated scale setting

- New scale setting based on M_Ω[3]
  - w_0 fm $= 0.17187(68)$, $w_0/a < 0.05\%$
  - $M_\Omega(\mathrm{QCD}) = 1671.01(43)$ MeV, $aM_\Omega < 0.2\%$
- Quantification of:
  - QED effects
  - Quark-mass dependence
  - Finite volume
  - w_0/a definition
- Updated cross-check with f_π determination soon.

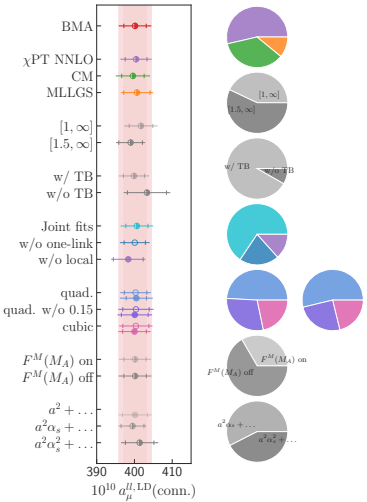


[3] High-Precision Scale Setting with the Omega-Baryon Mass and Gradient Flow, arXiv:2509.14367

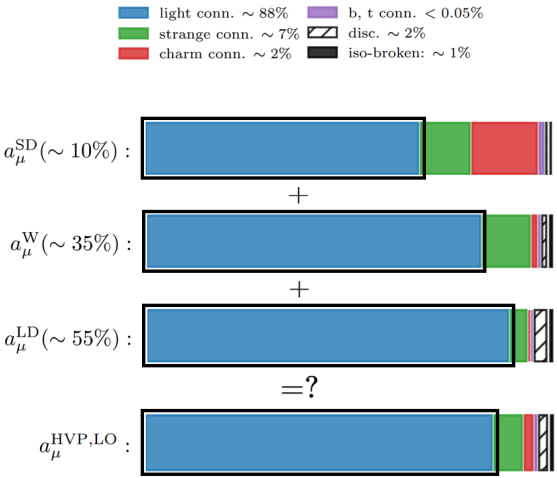
Light quark: Long-distance window (arXiv:2412.18491)



- Only four spacings (not enough statistics at 0.04).
- 'Taste-breaking corrections' to remove long-distance discretization effects (LQCD formulation specific).

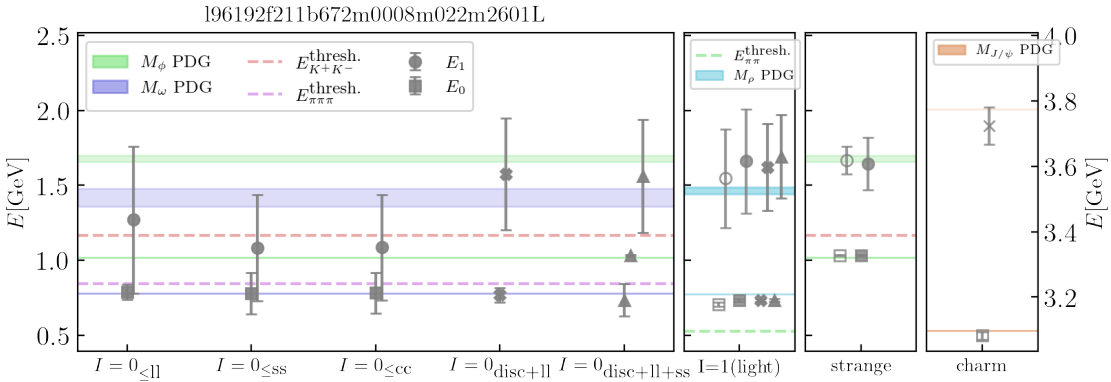


$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$ (arXiv:2412.18491)

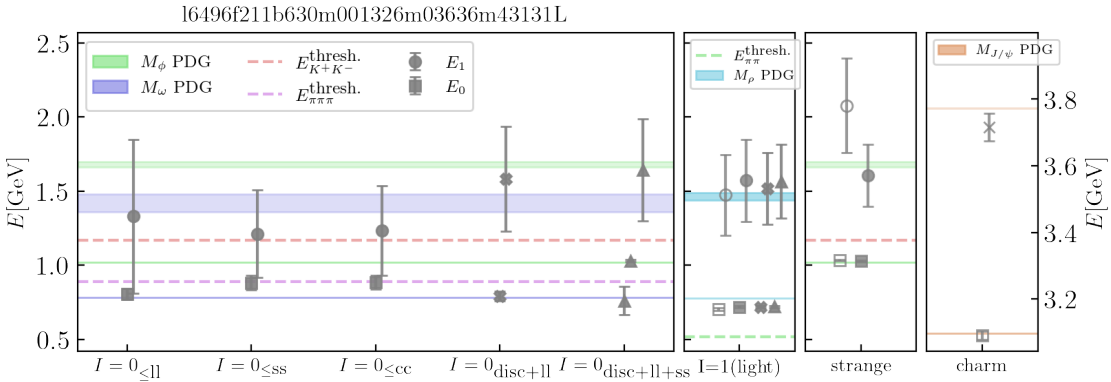


- Full integrand from: $a_\mu^{ll,\,\rm SD} + a_\mu^{ll,\,\rm W} + a_\mu^{ll,\,\rm LD}$
  - including stat. and syst. correlations.
- Performed independent analysis of $a_\mu^{ll,\,\rm Full}$
  - Have shown:
    $a_\mu^{ll,\,\rm Full} - \left(a_\mu^{ll,\,\rm SD} + a_\mu^{ll,\,\rm W} + a_\mu^{ll,\,\rm LD}\right) = 0$

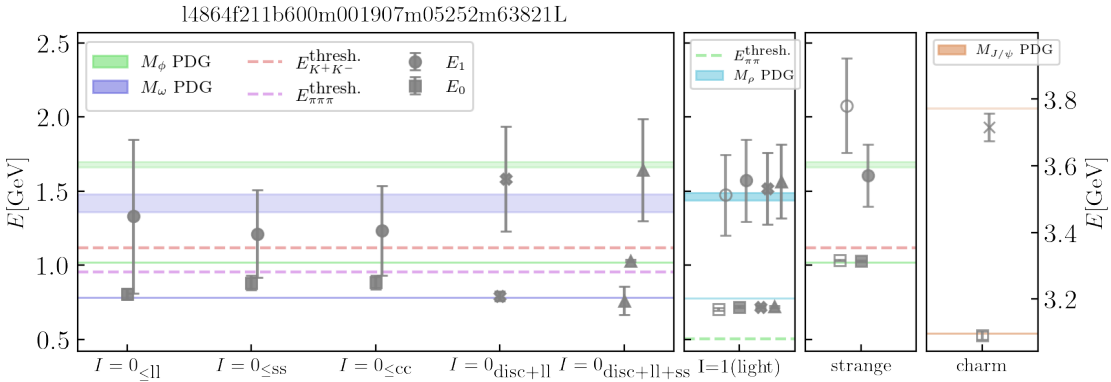
$I = 0$ spectrum (0.06 fm) — full detail



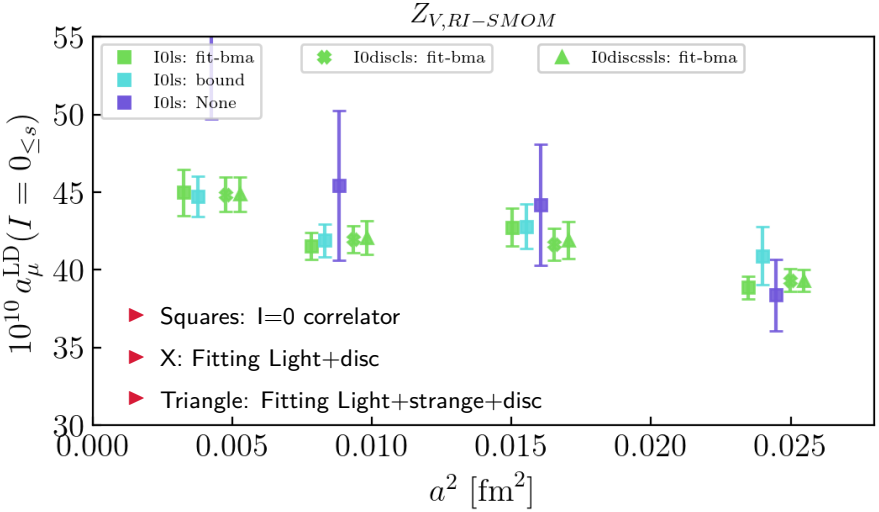
$I = 0$ spectrum (0.09 fm) — full detail



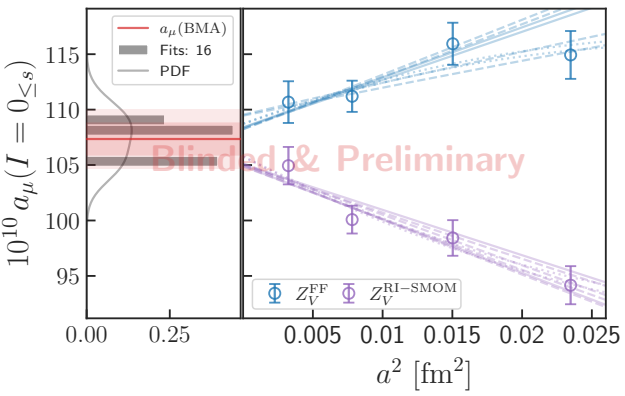
$I = 0$ spectrum (0.12 fm) — full detail

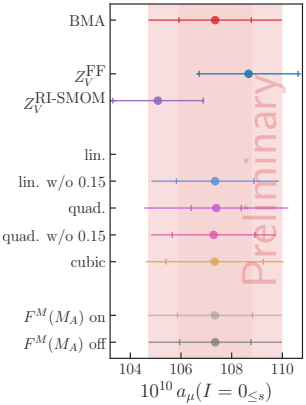


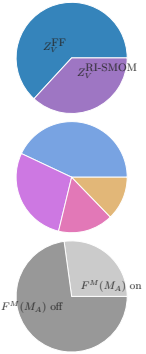
$I = 0$ noise reduction: fit vs. bounding



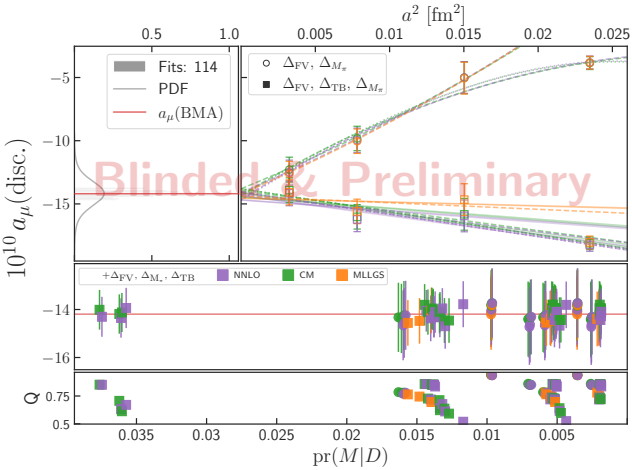
$I = 0$ continuum extrap. & model average

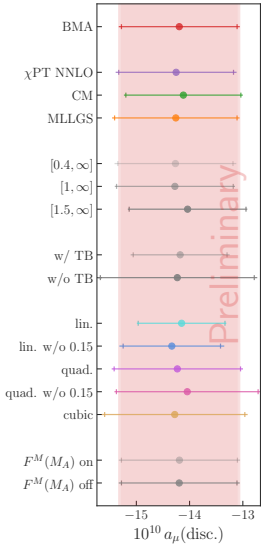


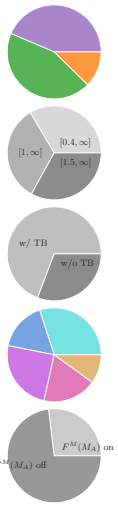




Disconnected continuum extrap. & model average



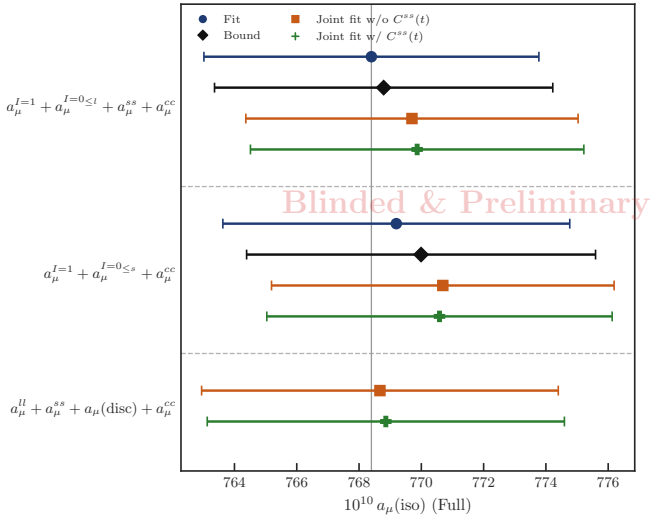




$a_\mu(\mathrm{iso})$ cross-checks

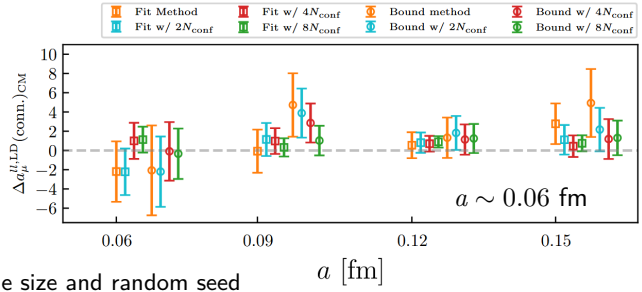
Cross-check: $a_\mu(\mathrm{iso}) = a_\mu^{I=0} + a_\mu^{I=1}$ consistent across every construction of the $I = 0$ piece:

- Constructing $a_\mu^{I=0}$ from $C^{I=0}$ w/ or w/o strange
- Constructing $a_\mu^{I=0}$ flavour components on the lattice
- Summing flavour components in the continuum



Fake data test

- Generate spectrum from EFT and EFT-inspired models (CM, MLLGS)
- Draw N_{conf} samples using (real) data covariance
- Reconstruct correlator with fit or bounding method

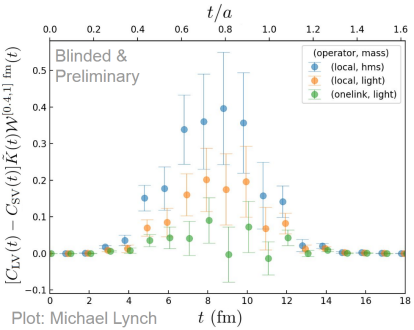


- Varied sample size and random seed
- Fit method consistent with truth
- Bounding method less robust to stat fluctuations

See [2412.18491] for details

FV Study

- Currently: FV effects estimated (χPT, CM, MLLGS, HP)
- Upcoming: Lattice FV determination with large volume (L$\sim$11 fm) ensemble at a$\sim$0.09 fm
- Data generation ongoing: light (local and one-link), "harmonic mean square[4]" mass (local)



- Light quark connected integrand on W
- Current number of configs (LMA):
  - $\sim$70 large volume (LV)
  - $\sim$1000 small volume (SV)

[4] S. Borsanyi et al., Nature, 593.7857, 51–55 (2021).