Nucleon Electromagnetic Form Factors at the Physical Pion Mass from PNDME Collaboration

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Acknowledgements

PNDME and NME Collaborations
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J. Yoo (Korea U.), H.-W. Lin (MSU), F. Winter (JLab)

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- The calculations used the CHROMA software suite.
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Electromagnetic Form Factors, G_E and G_M

- Dirac/Pauli Form factors, $\boldsymbol{F_{1,2}}$ parametrize the matrix element of the electromagnetic current $V_\mu^{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d$ within nucleon ground states

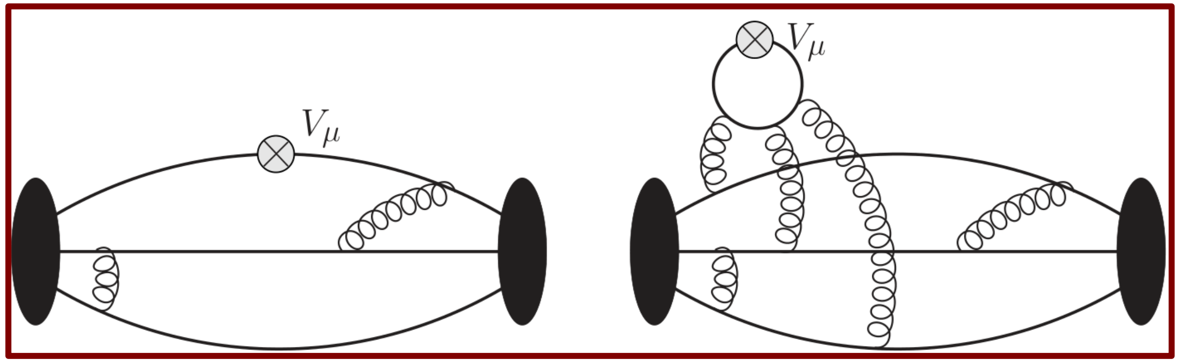
$$\langle N(\boldsymbol{p}', s')|V_\mu^{\mathrm{em}}|N(\boldsymbol{p}, s)\rangle = \bar{u}(\boldsymbol{p}', s')\left(\boldsymbol{F_1(q^2)}\gamma_\mu + \frac{i\sigma_{\mu\nu}}{2M_N}\boldsymbol{F_2(q^2)}\right)u(\boldsymbol{p}, s)$$

- Sachs form factors, $\boldsymbol{G_E} = F_1 - \frac{Q^2}{4M_N^2}F_2, \quad \boldsymbol{G_M} = F_1 + F_2$

- Isovector form factors $\boldsymbol{G_{E,M}^{u-d} = G_{E,M}^{p} - G_{E,M}^{n}}$ assuming flavor SU(2) isospin symmetry,

$$\langle p|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|p\rangle = \langle p|V_\mu^{\mathrm{em}}|p\rangle - \langle n|V_\mu^{\mathrm{em}}|n\rangle$$

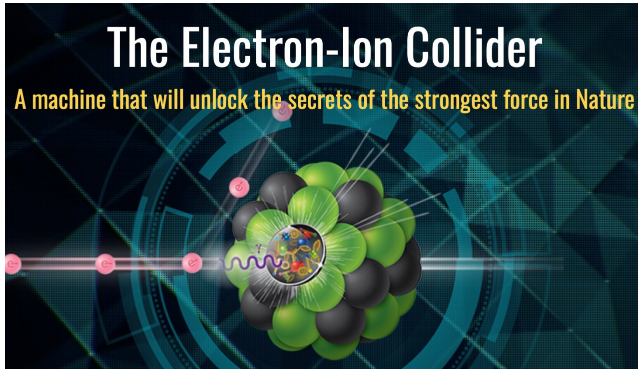
Isovector Charge Radii

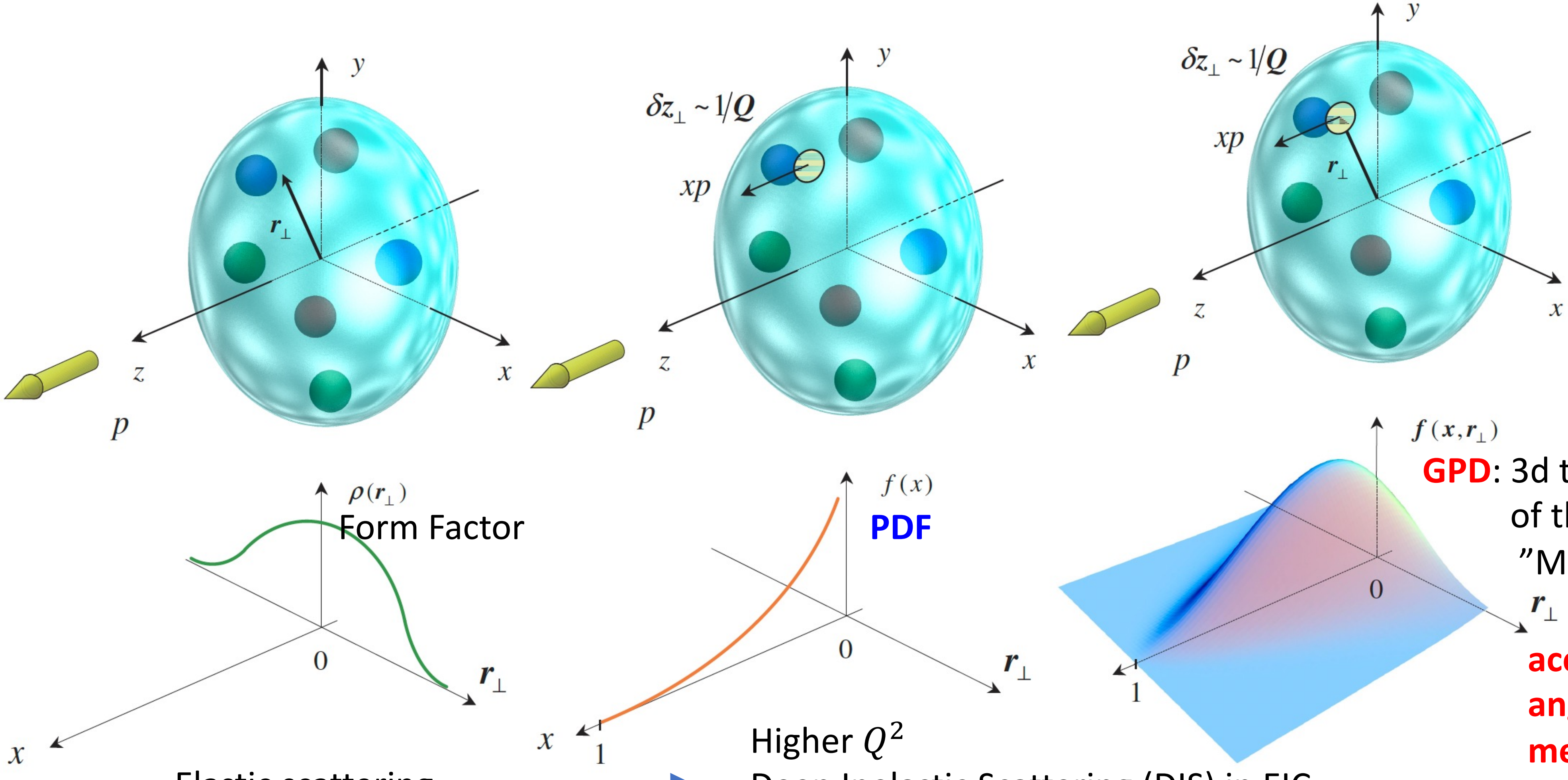


- Proton electromagnetic form factor $\boldsymbol{G^{p}_{E,M}}$ has contribution from both connected and disconnected diagrams
- We focus on the isovector form factor $\boldsymbol{G^{u-d}_{E,M} = G^{p-n}_{E,M}}$ where the disconnected contribution cancels in the isospin symmetric limit.
- Charge radii squared

$$\langle r_E^2 \rangle^{u-d} = -6 \frac{d}{dQ^2} \left(\frac{G^{u-d}_{E,M}(Q^2)}{G^{u-d}_{E,M}(0)} \right) \Big|_{Q^2=0}$$

Beyond the nucleon form factor:
Parton Distribution Function (PDF)
Generalized Parton Distribution (GPD)





GPD: 3d tomographic image of the nucleon "Main Goal of EIC"

access to quark/gluon angular momentum and mechanical structure

Elastic scattering → Higher Q^2 Deep Inelastic Scattering (DIS) in EIC

Three 2+1+1-flavor physical pion mass HISQ Ensembles

Ensemble ID	a [fm]	M_π [MeV]	$M_\pi L$	$L^3\ T$	N_{conf}
a09m135	**0.0873**	**135**	**3.95**	$64^3 96$	**1290 -> 6060 / 8000**
a06m135	**0.0564**	**135**	**3.95**	$96^3 192$	**675 -> 8269 / 10000**
a04m135	0.0426	135	3.95	$144^3 288$	0 / 2000

- Ensembles generated by MILC/CalLat/LANL Collaborations
- Improved gauge link smearing
- Precise quark mass parameter tuning
- Larger source-sink time separations to remove excited states
- Doubled range of momentum transfer $\vec{q} = 2\pi\vec{n}/L$ Largest spacelike 4-momentum transfer $Q^2 = -q^2)$ increased from $0.45\ \text{GeV}^2$ to $0.82\ \text{GeV}^2$

- **Will improve all nucleon results in the continuum limit very significantly (2-3X)**

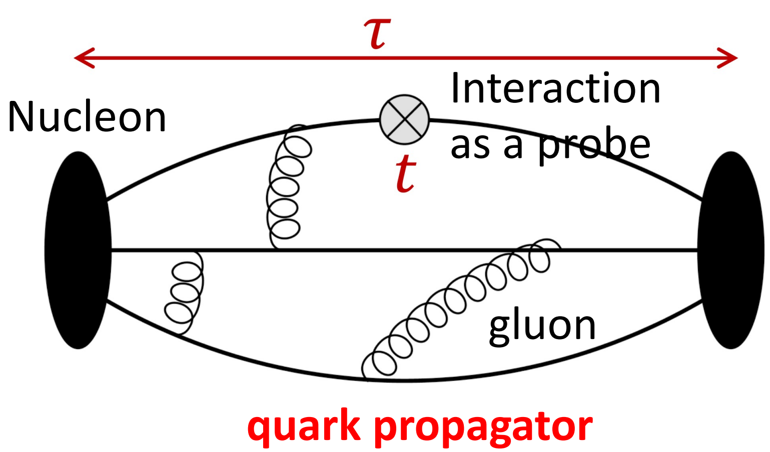
[PNDME Collab.]

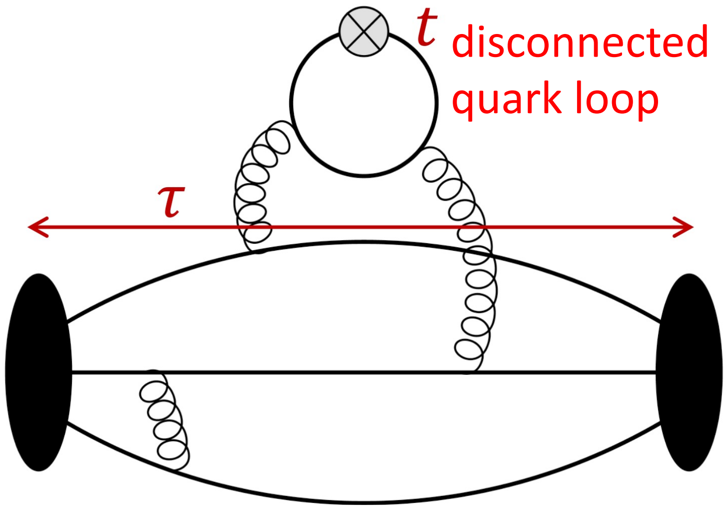
Nucleon structures from LQCD correlation functions:

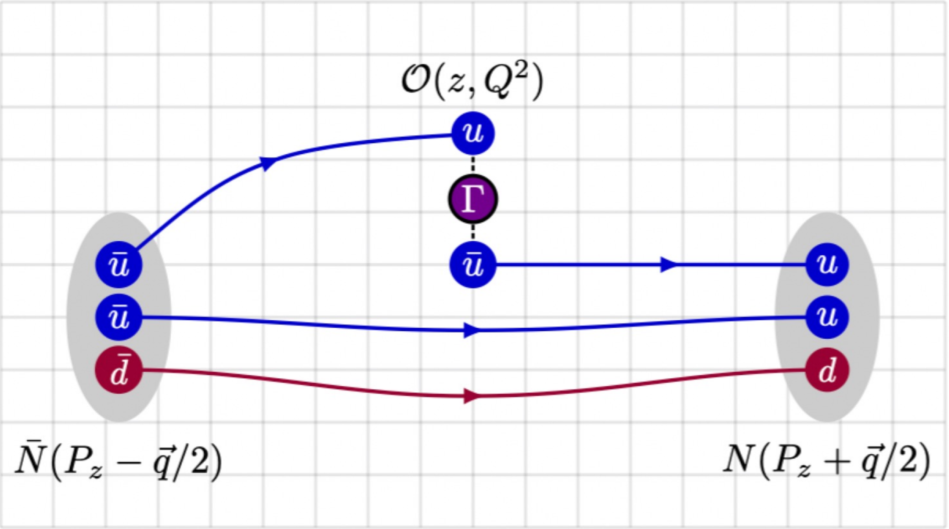
$$C_\Gamma^q(t,\tau) = C_\Gamma^{q,\mathrm{conn}}(t,\tau) + C_\Gamma^{q,\mathrm{disc}}(t,\tau)$$

$\rightarrow$ ***Charges, form factors, moments, PDFs and GPDs***

$$\mathcal{R}_\mathcal{O}(t,\tau,\boldsymbol{p},\boldsymbol{0}) = \frac{C_\mathcal{O}^{3\mathrm{pt}}(t,\tau;\boldsymbol{p},\boldsymbol{0})}{C^{2\mathrm{pt}}(\tau,\boldsymbol{p})} \times \left[\frac{C^{2\mathrm{pt}}(t,\boldsymbol{p})C^{2\mathrm{pt}}(\tau,\boldsymbol{p})C^{2\mathrm{pt}}(\tau-t,\boldsymbol{0})}{C^{2\mathrm{pt}}(t,\boldsymbol{0})C^{2\mathrm{pt}}(\tau,\boldsymbol{0})C^{2\mathrm{pt}}(\tau-t,\boldsymbol{p})}\right]^{1/2}$$

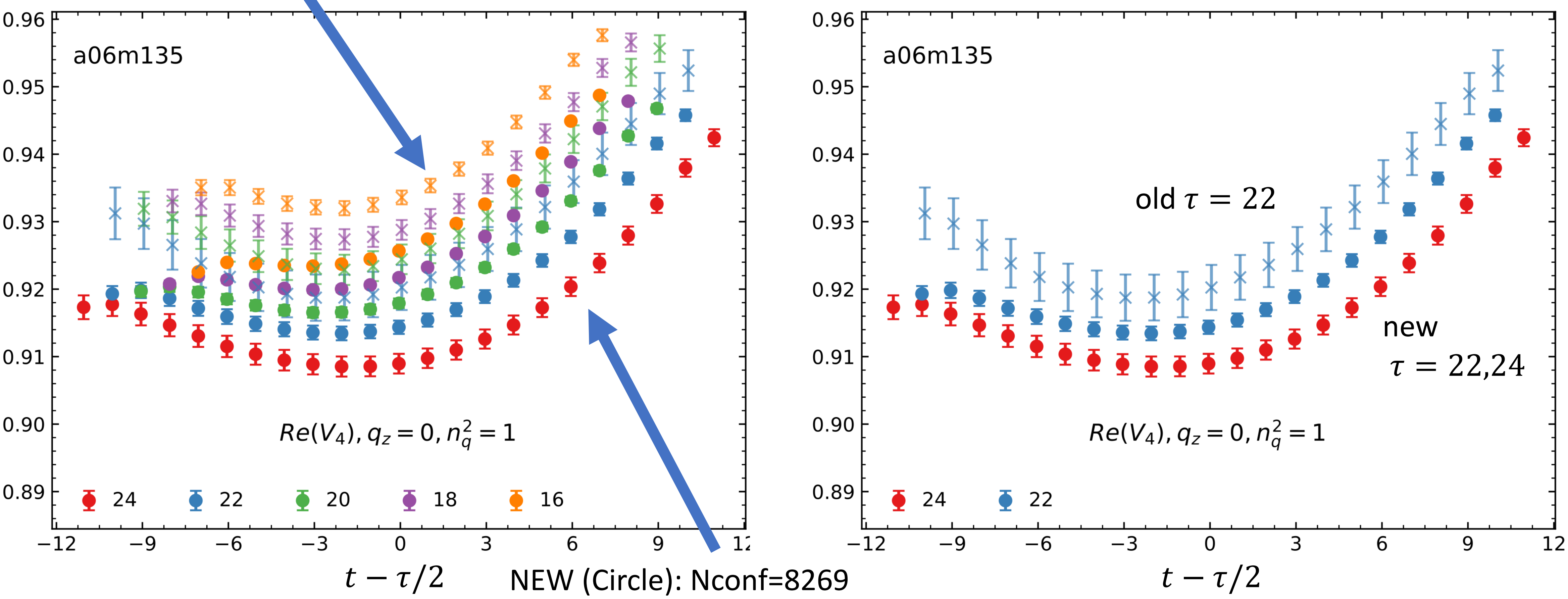






G_E: improvement in signal at $a \sim 0.06\,\mathrm{fm}$, $|\vec{n}|^2 = 1$

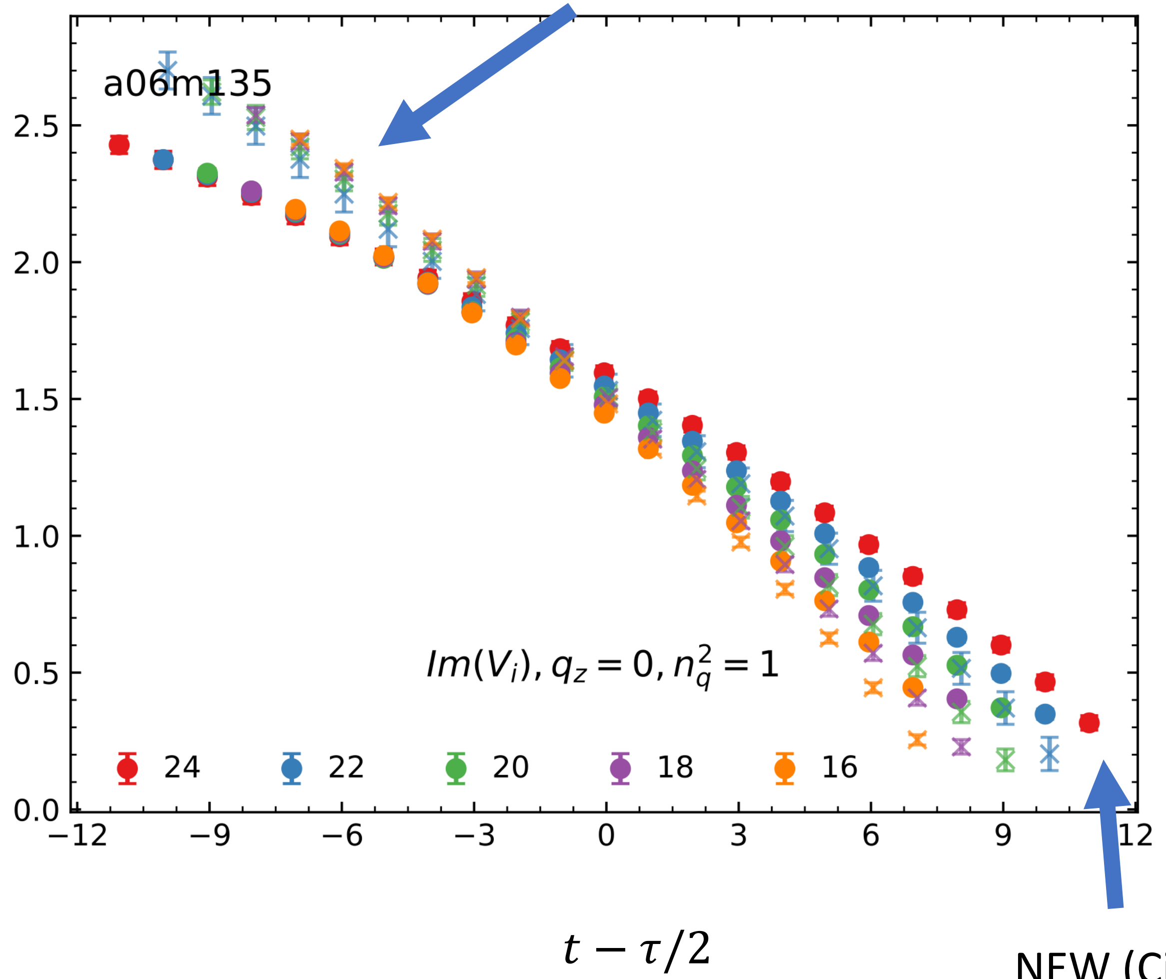
OLD (Cross): Nconf=1074 (PNDME 20 used 675)

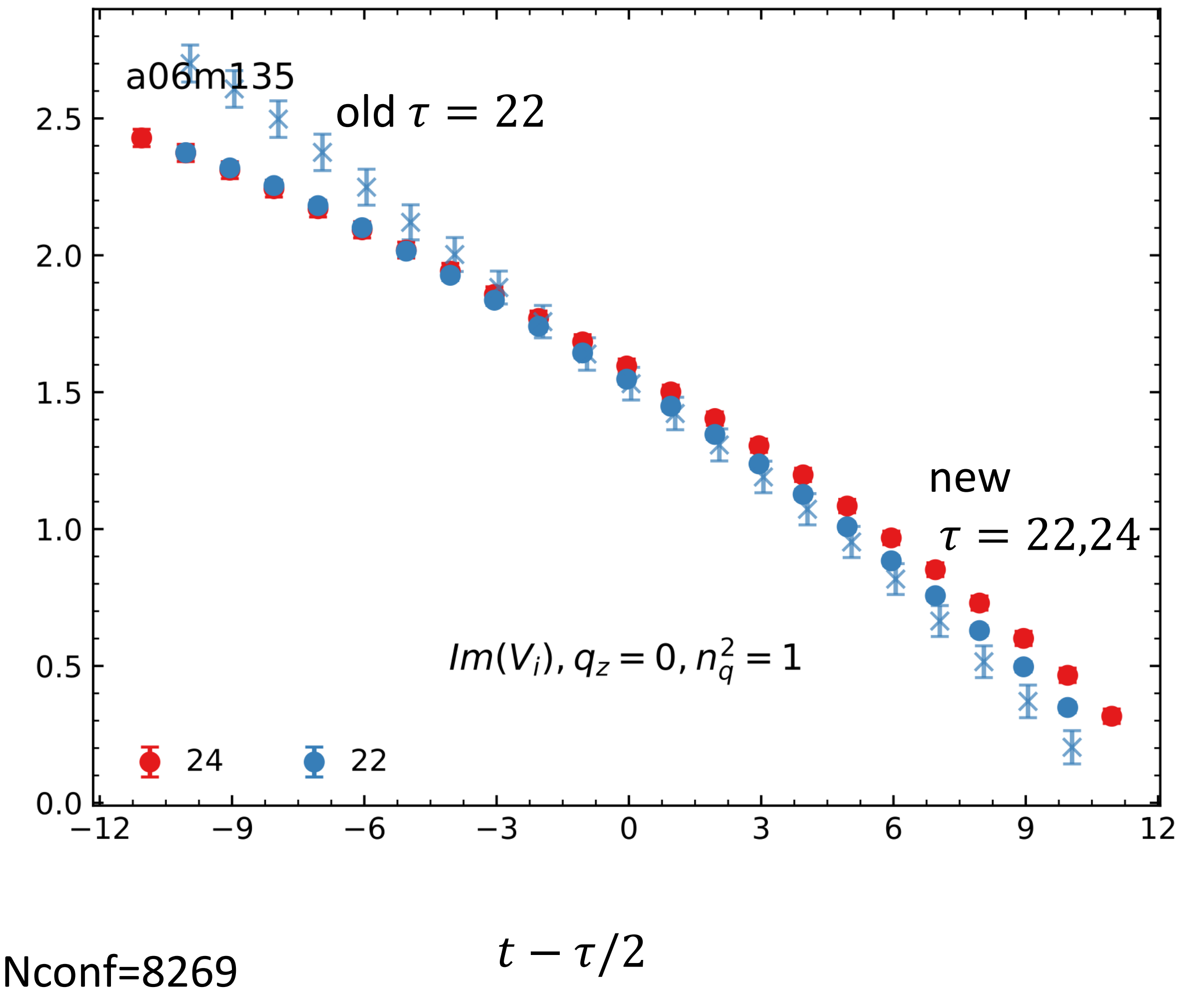


NEW (Circle): Nconf=8269

G_E: improvement in signal at $a\sim 0.06\mathrm{fm}$, $|\vec{n}|^2 = 1$

OLD (Cross): Nconf=1074 (PNDME 20 used 675)

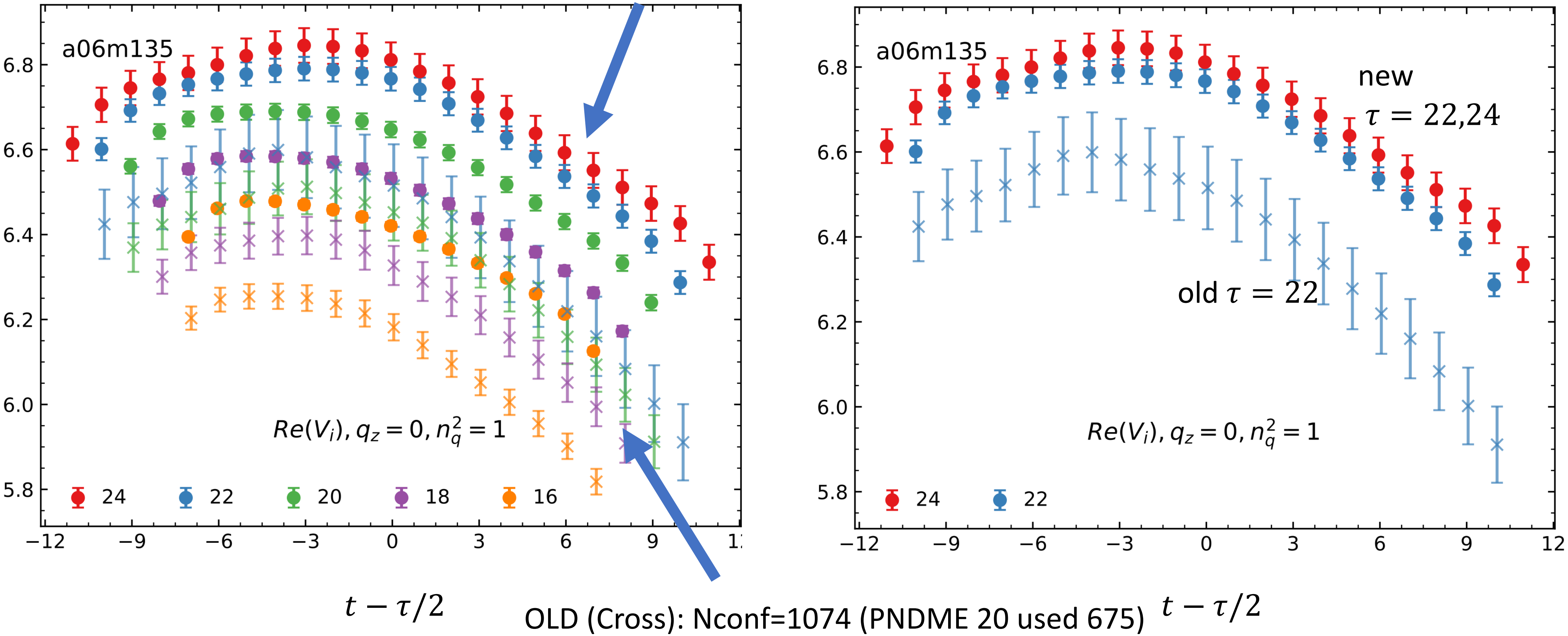




NEW (Circle): Nconf=8269

G_M: improvement in signal at $a \sim 0.06\text{fm}$, $|\vec{n}|^2 = 1$

NEW (Circle): Nconf=8269

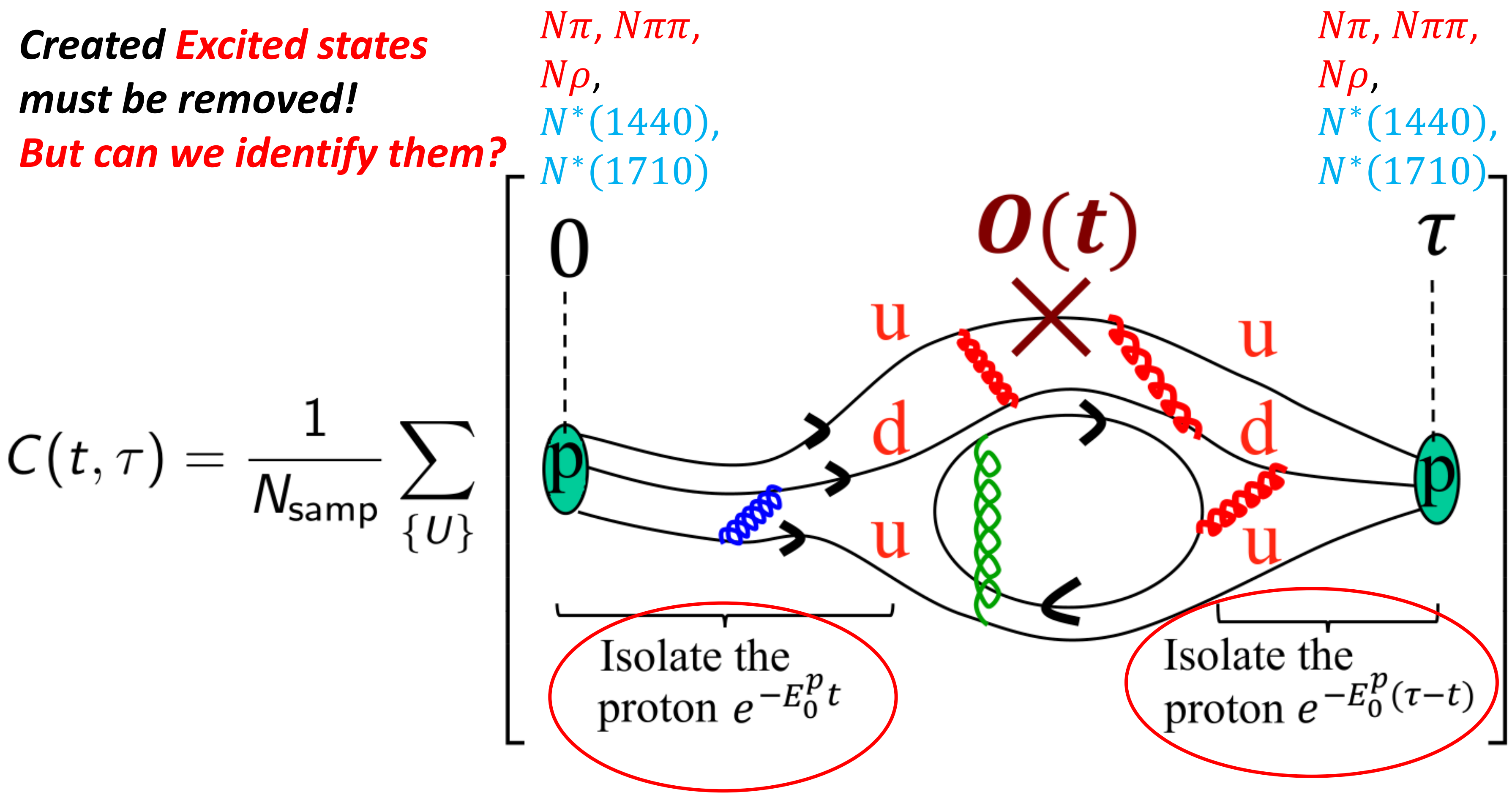


OLD (Cross): Nconf=1074 (PNDME 20 used 675)

Remove Excited State (ES) effects from Nucleon Matrix Element

- Nucleon signal/noise decays $\propto e^{-(E-1.5M_\pi)\tau}$ with Euclidean time τ. Cannot go to large $\boldsymbol{\tau}$

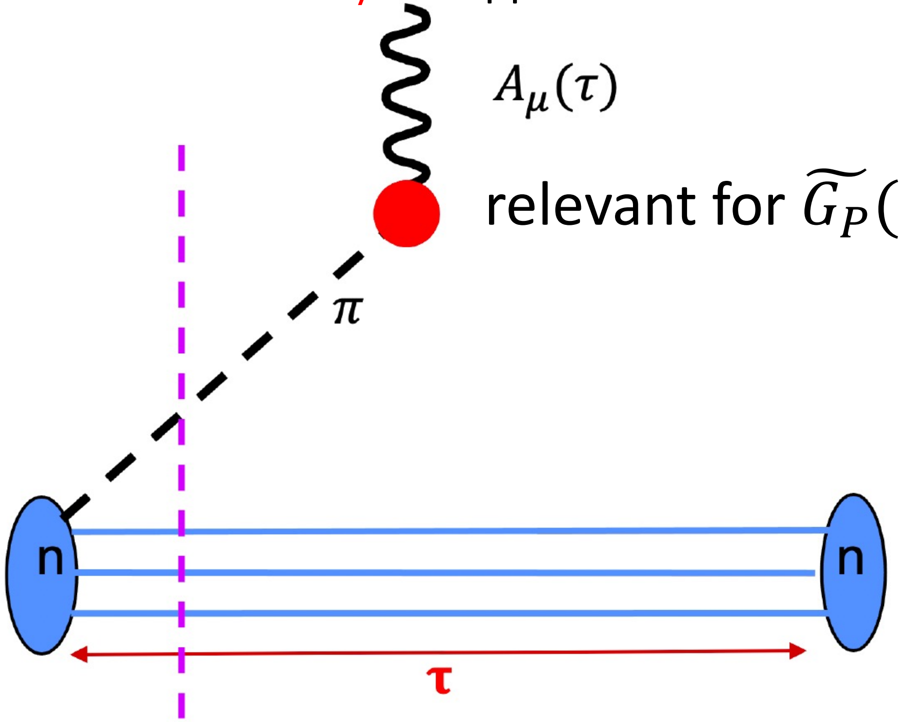
Created Excited states must be removed!
But can we identify them?



Charges,
form factors,
PDFs,
GPDs

$N\pi$-Excited state contribution can be enhanced

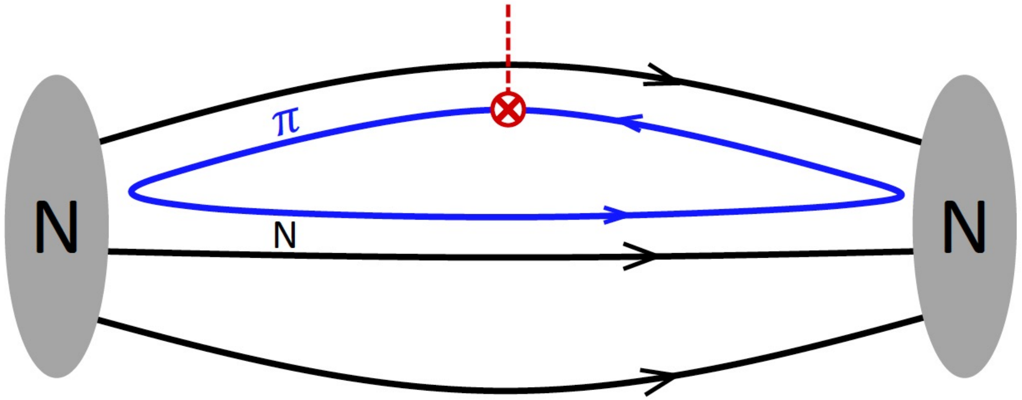
- To control ESC, we need to first determine the spectrum
- In nucleon 2pt functions, $1/L^3$-suppressed coupling to $N\pi$-state makes it challenging to extract its spectrum
- In certain 3pt functions (matrix elements) , ChPT suggests $N\pi$-state contributions are enhanced and compensate for the $1/L^3$-suppression



relevant for $\widetilde{G}_P(Q^2)$

ChiPT: O.Bär, PRD99 054506 (2019), PRD 100 054507 (2019)
Lattice: Jang et al. PRL [PNDME 2020]
Park et al. PRD 105 054505 [NME 2022]

Scalar operator
relevant for $g_S^{u,d}$ and $\sigma_{\pi N} = m_{ud} g_S^{u+d}$

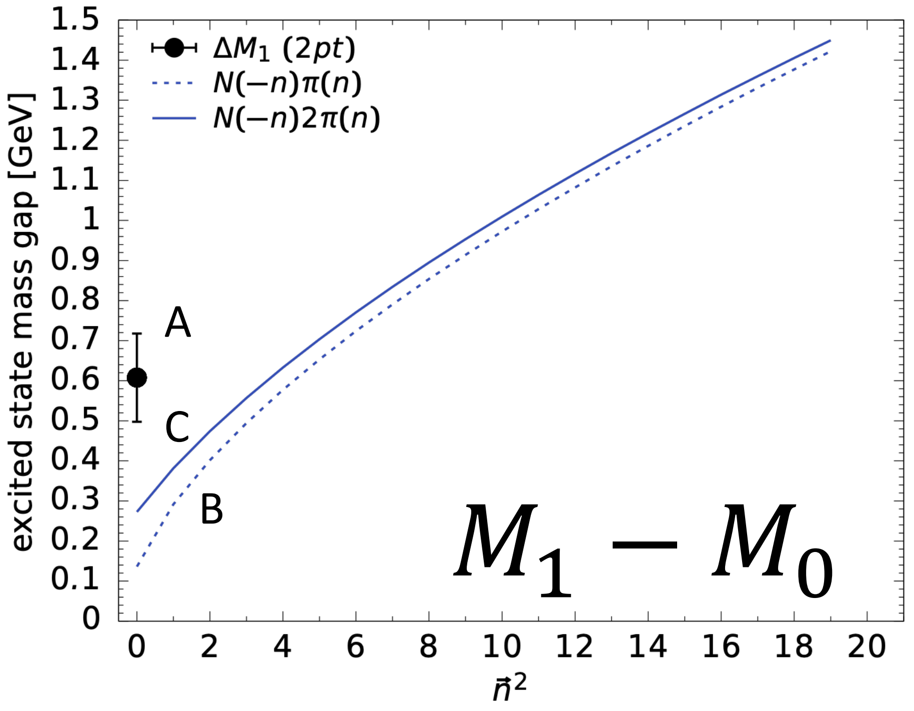


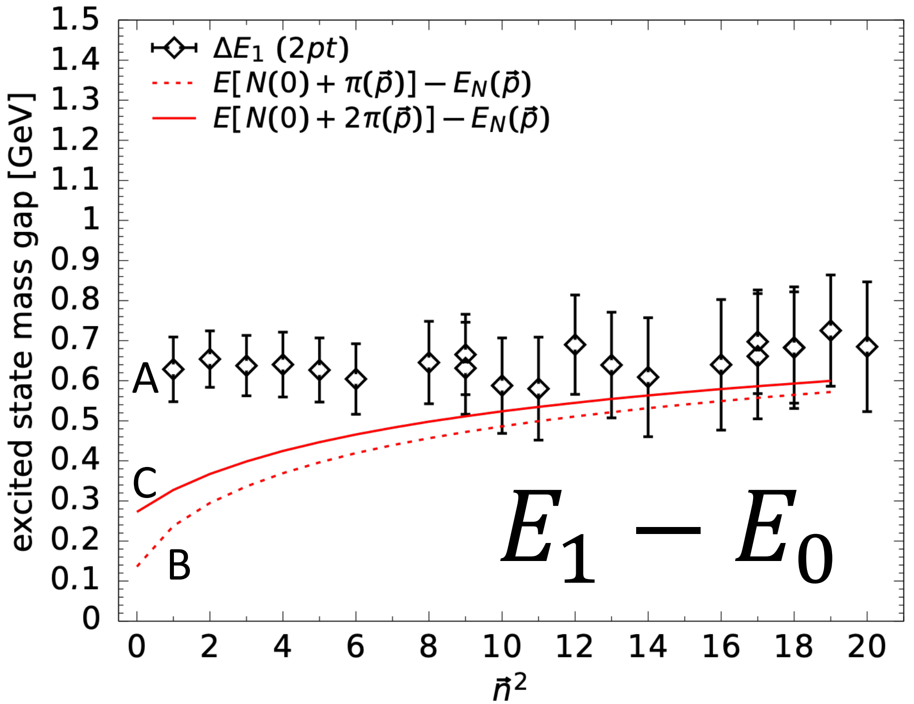
ChiPT and Lattice:
Gupta et al., [PNDME 2021]

3 Excited State Fit Strategies for $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$

M_0 taken from 4-state fit

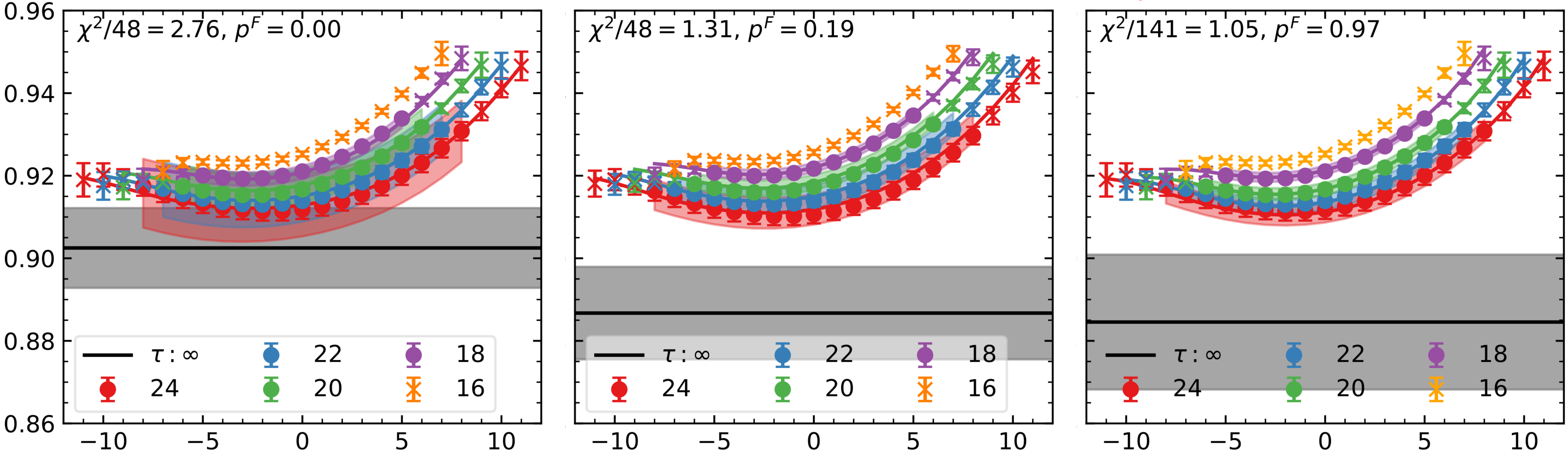
Strategy	$M_1 - M_0$, $E_1 - E_0$	$M_2 - M_0$, $E_2 - E_0$	Note
A	**0.6 GeV**	**2.0 GeV**	* Spectrum determined from 2pt fits with empirical Bayesian priors
B	Noninteracting $N - \pi$ ~ **0.2GeV**	**1.5 GeV**	
C	Noninteracting $N - 2\pi$ ~ **0.3GeV**	**0.6 GeV**	* Vector-meson dominance model, * Simultaneous fits to ReV_4, ImV_i, ReV_i





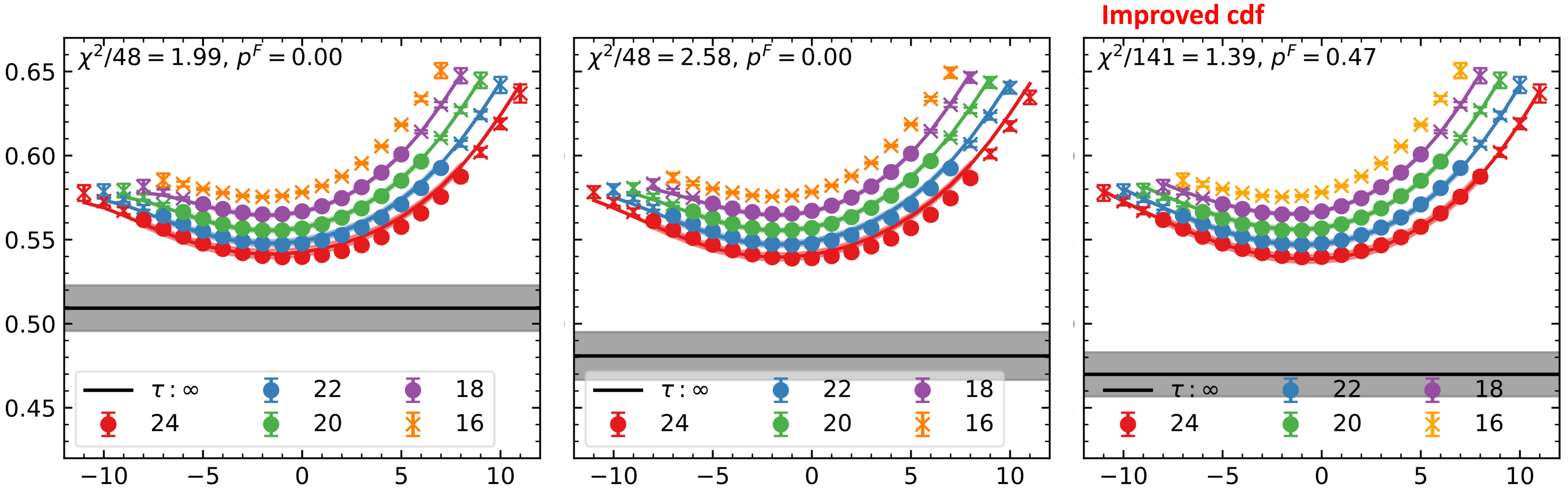
G_E $from\ Re(V_4)$
: χ^2 for the 3 strategies at $a \sim 0.06\text{fm}$, $|\vec{n}|^2 = 1$

Improved cdf



Strategy	A	*B*	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

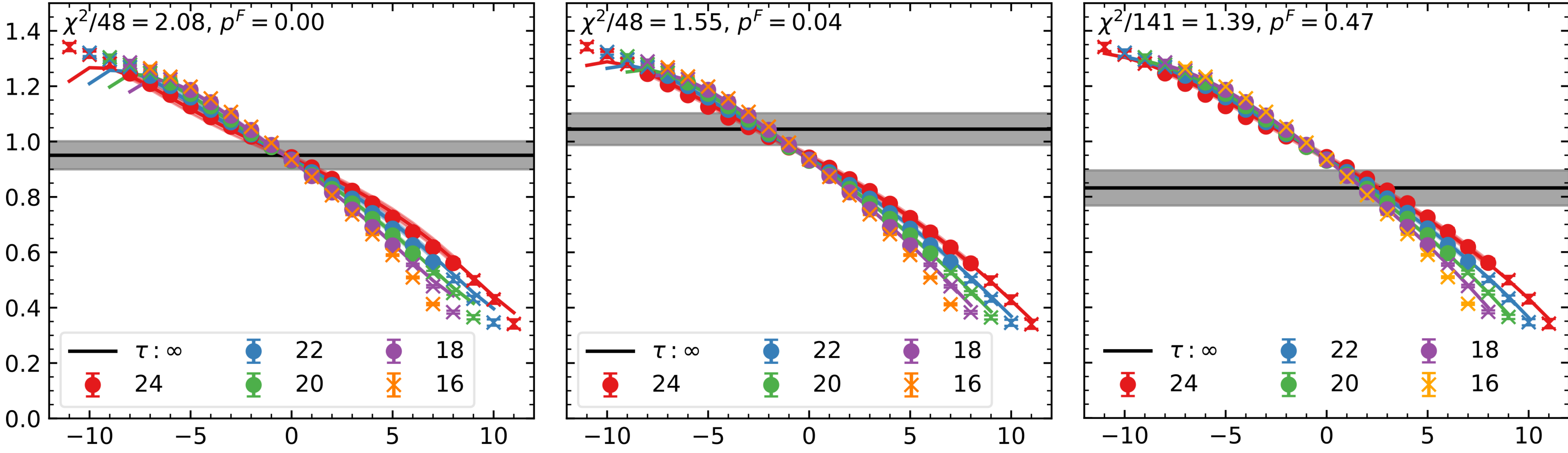
G_E $from\ Re(V_4)$
: χ^2 for the 3 strategies at $a\sim 0.06\mathrm{fm}$, $|\vec{n}|^2 = 6$



Strategy	A	*B*	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

G_E from $Im(V_i)$
: χ^2 for the 3 strategies at $a \sim 0.06\text{fm}$, $|\vec{n}|^2 = 6$

Improved cdf



GE(ReV4)=0.528(14)
GE(ImVi)=0.577(31) **inconsistent**

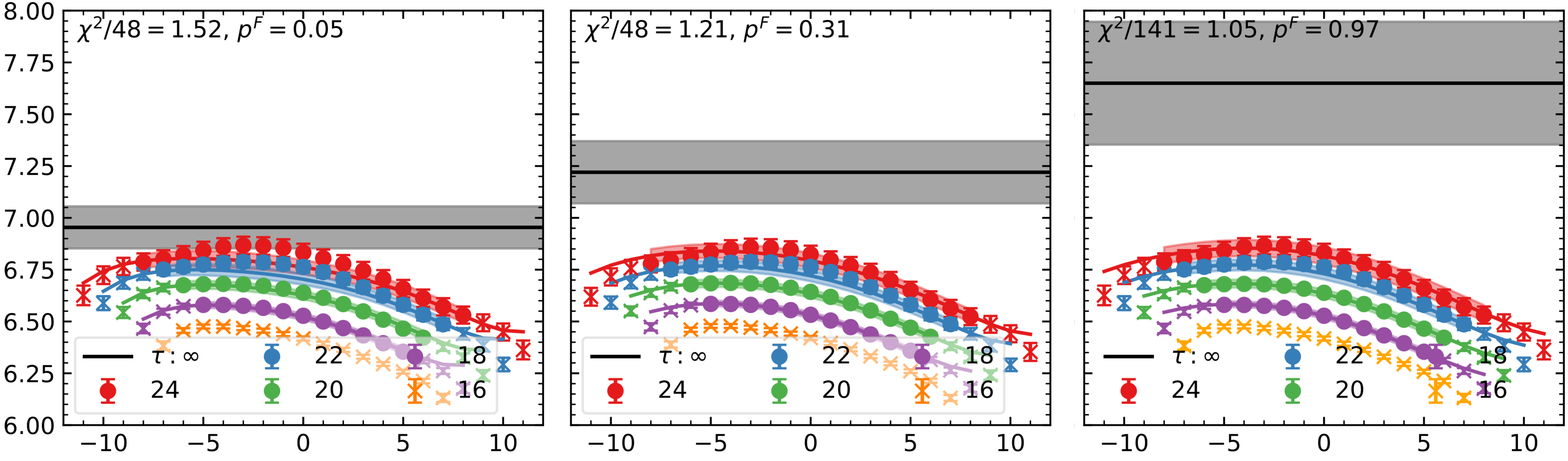
GE(ReV4)=0.500(15)
GE(ImVi)=0.629(32) **inconsistent**

GE(ReV4)=0.487(14)
GE(ImVi)=0.505(39) **consistent**

Strategy	A	*B*	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

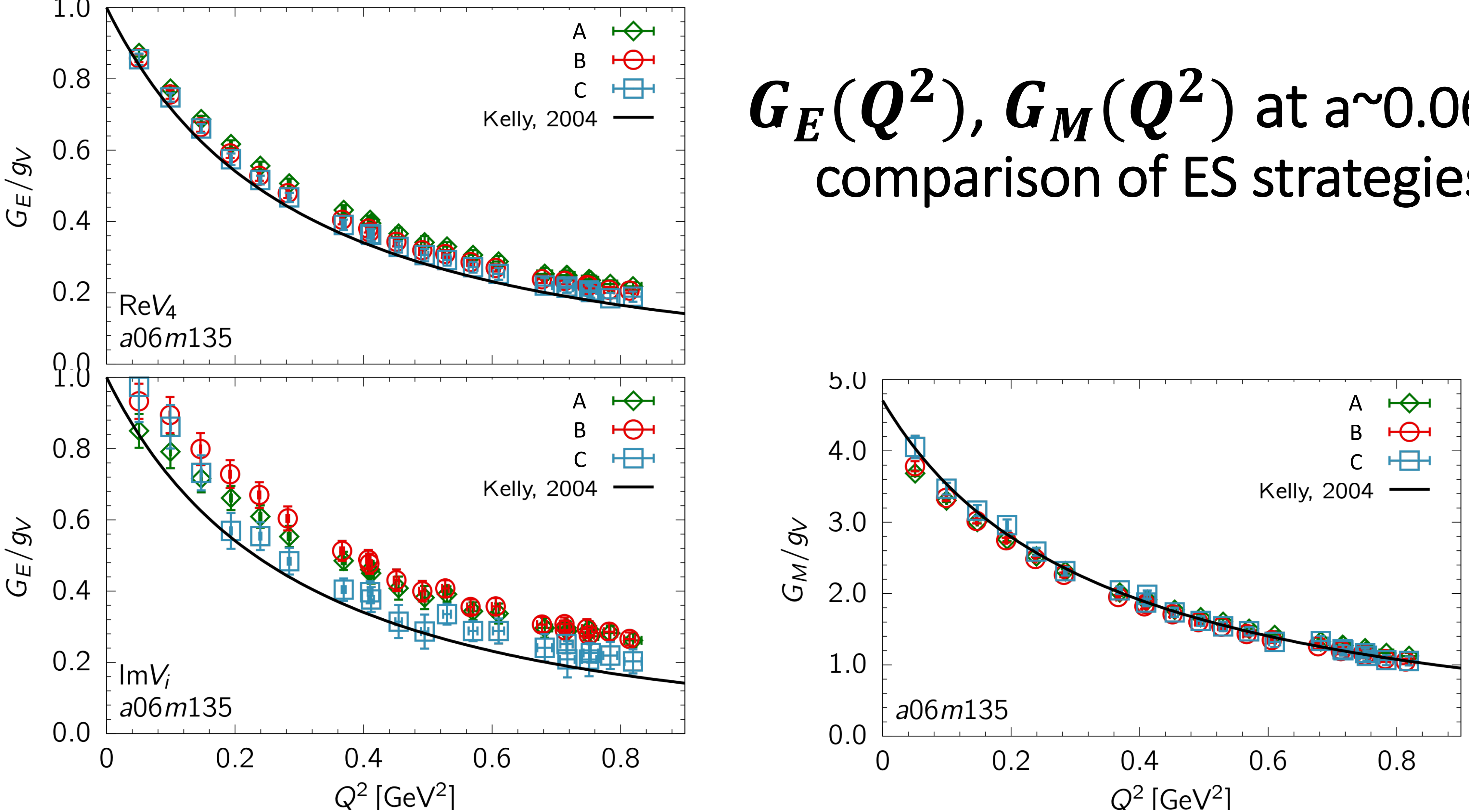
G_M $from$ $Re(V_i)$
: χ^2 for the 3 strategies at $a\sim 0.06\text{fm}$, $|\vec{n}|^2 = 1$

Improved cdf



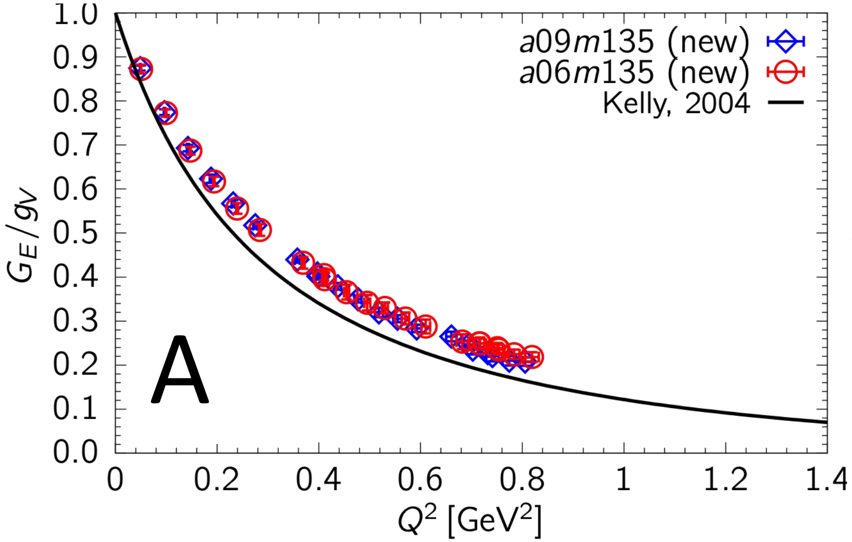
Strategy	A	B	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

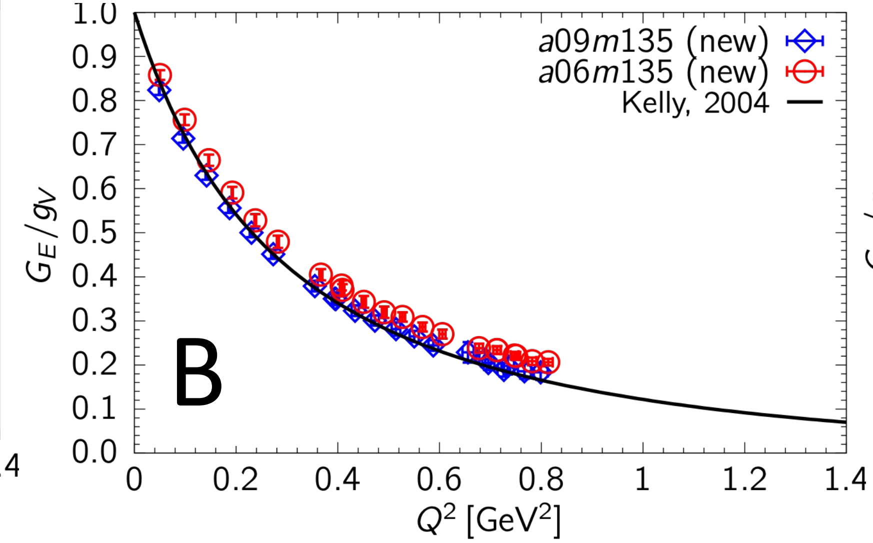
$G_E(Q^2)$, $G_M(Q^2)$ at a~0.06fm comparison of ES strategies

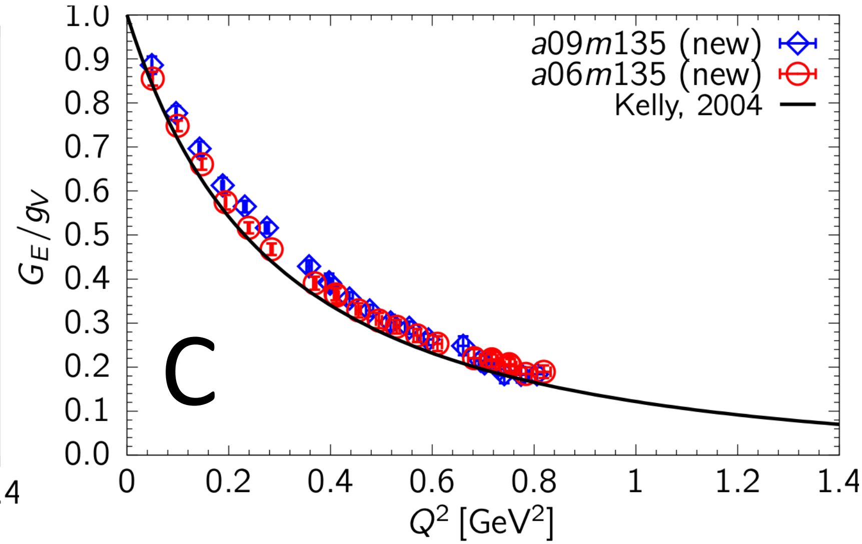


Strategy	A	B	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

$G_E(Q^2)$ a-dependence

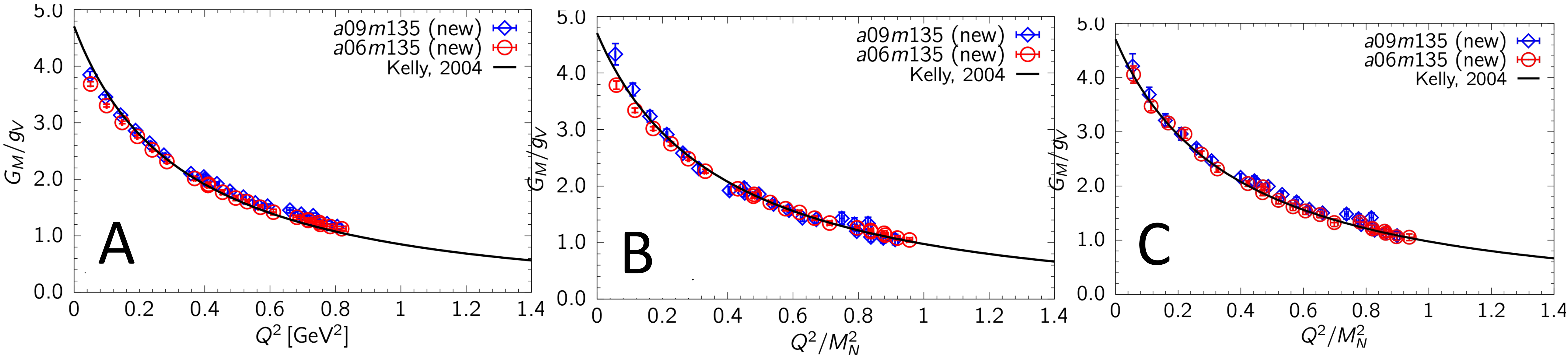






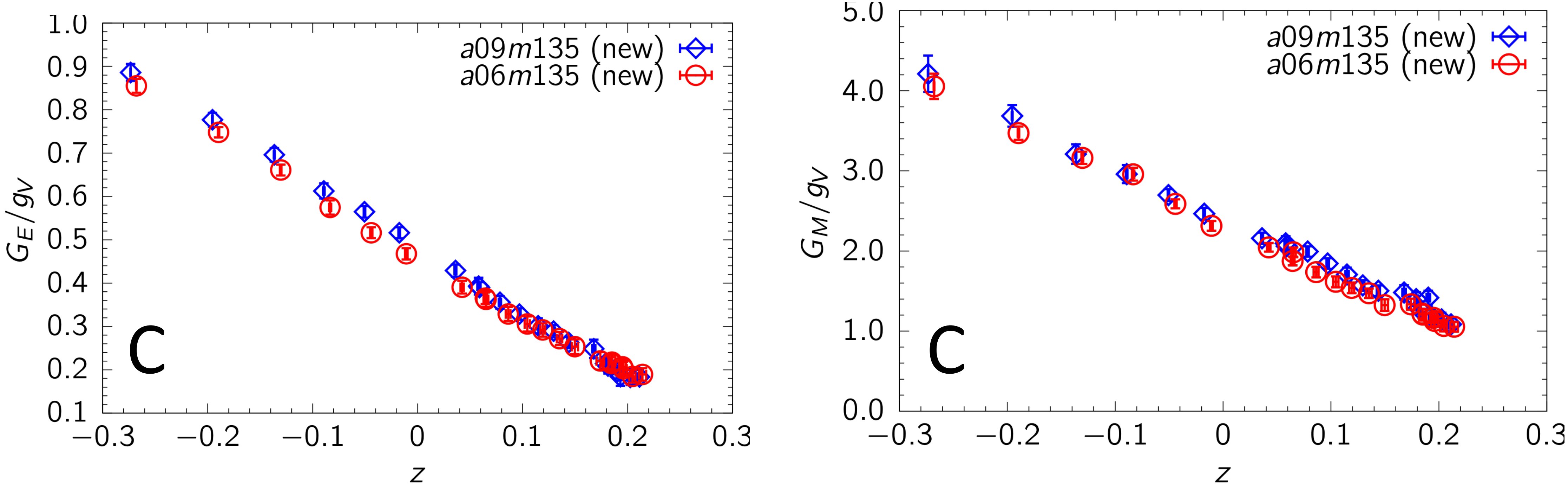
Strategy	A	*B*	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

$G_M(Q^2)$ a-dependence

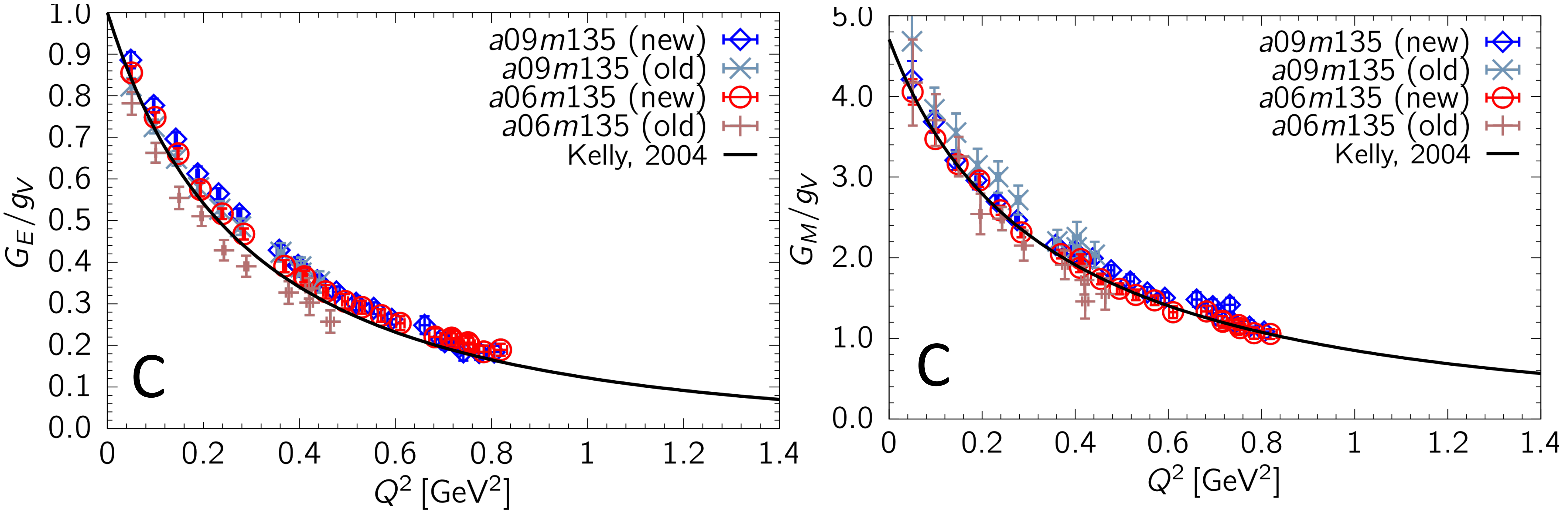


Strategy	A	B	C
$M_1 - M_0, M_2 - M_0$	0.6 GeV, 2.0 GeV	0.2 GeV, 1.5 GeV	0.3 GeV, 0.6 GeV

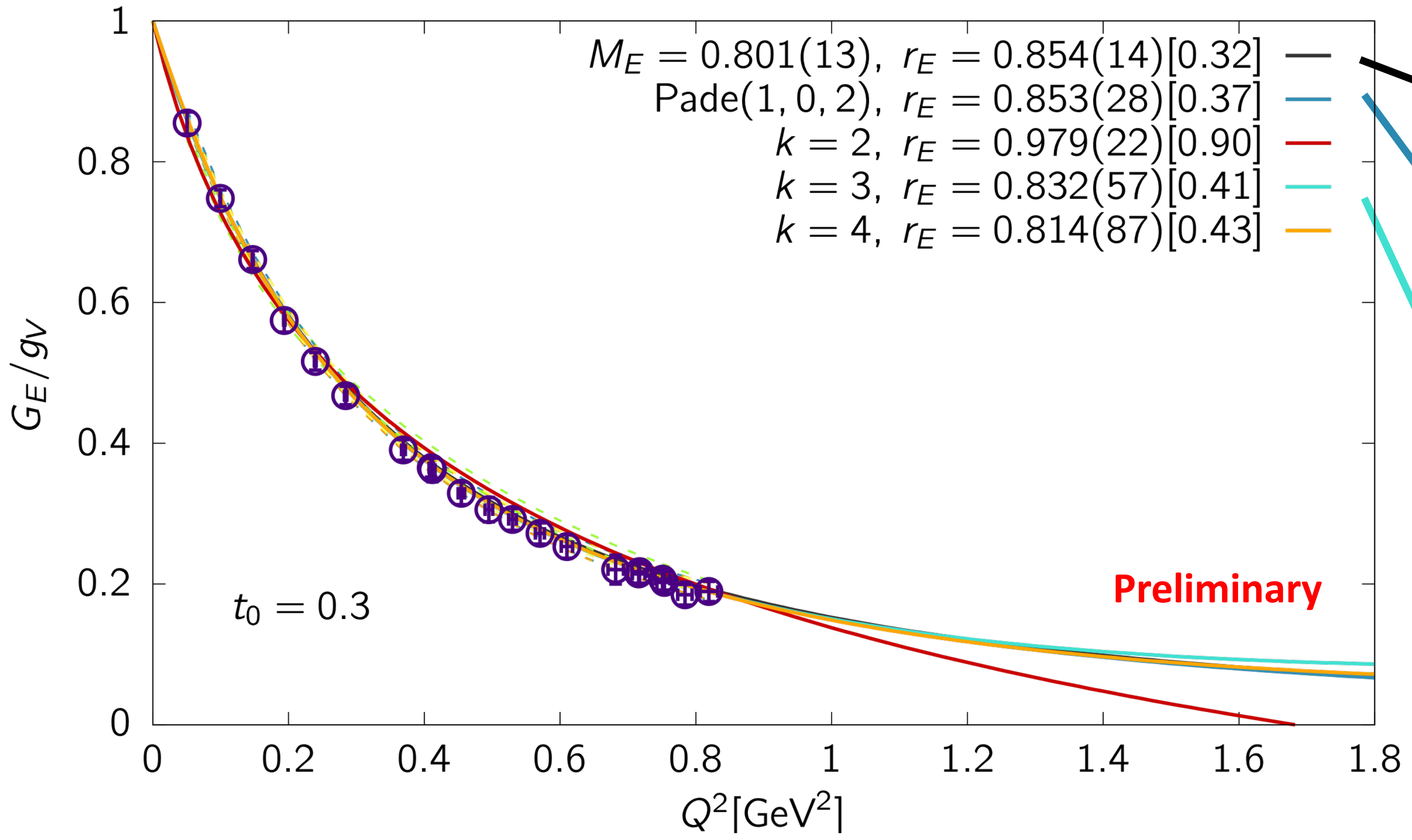
$G_E(Q^2)$, $G_M(Q^2)$ and z-expansion parameter



G_E^{u-d} , G_M^{u-d}: improvement



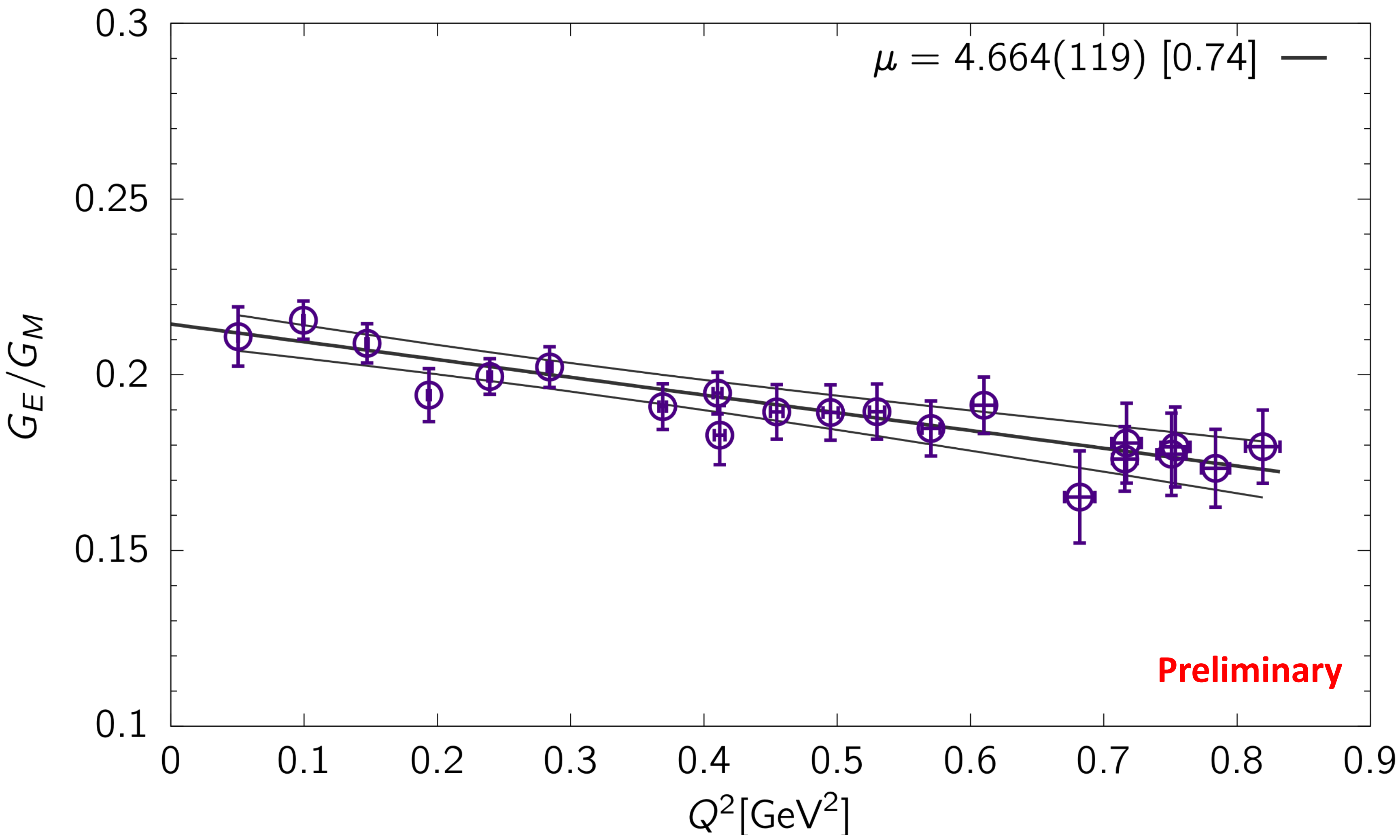
G_E^{u-d} : Examined Dipole, Pade and z-expansion fits



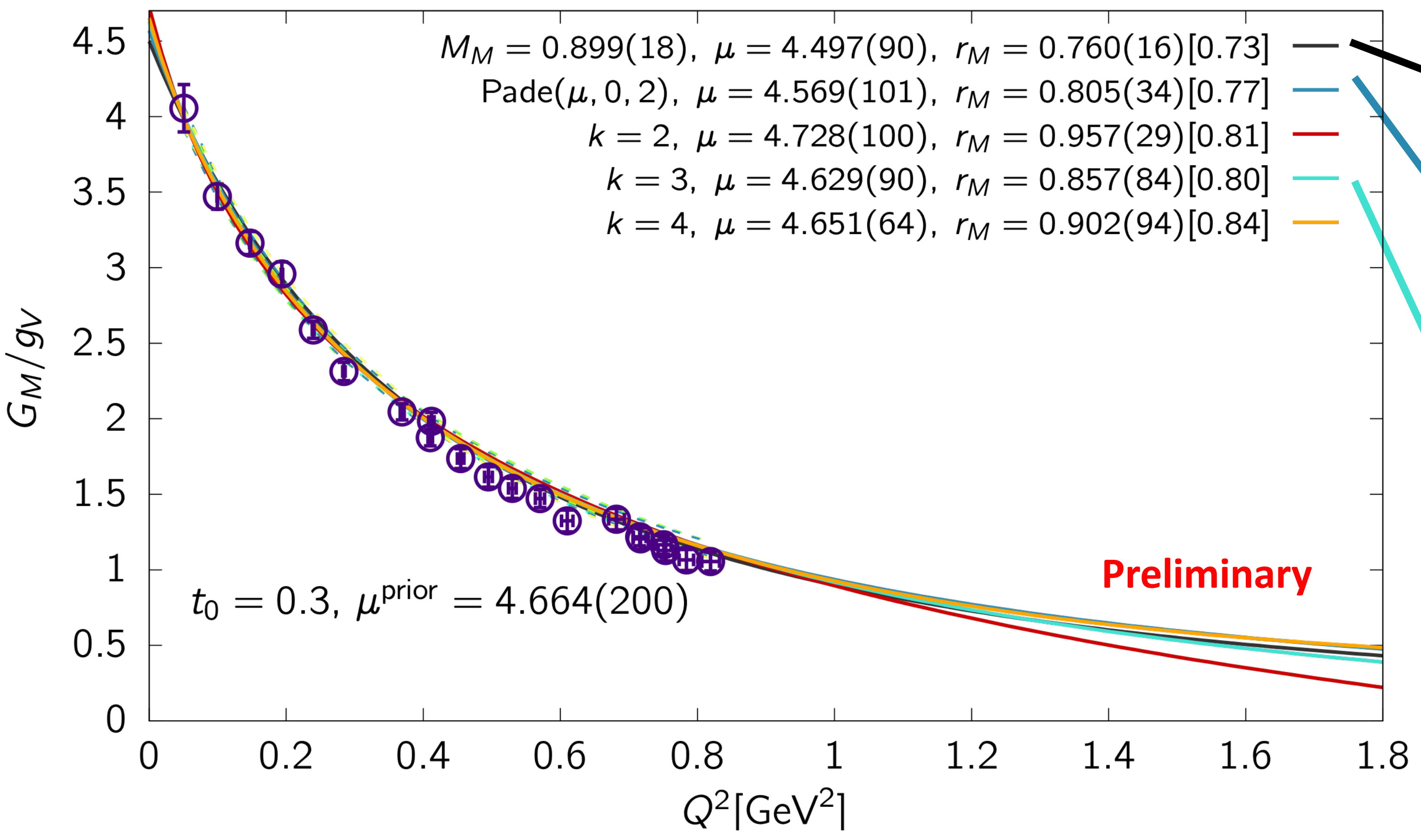
- Dipole: $\frac{G_E(0)/g_V}{(1+Q^2/M_E^2)}$
- Pade: $\frac{g}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

  Need z^3 to fit the data

G_E/G_M : linear fit



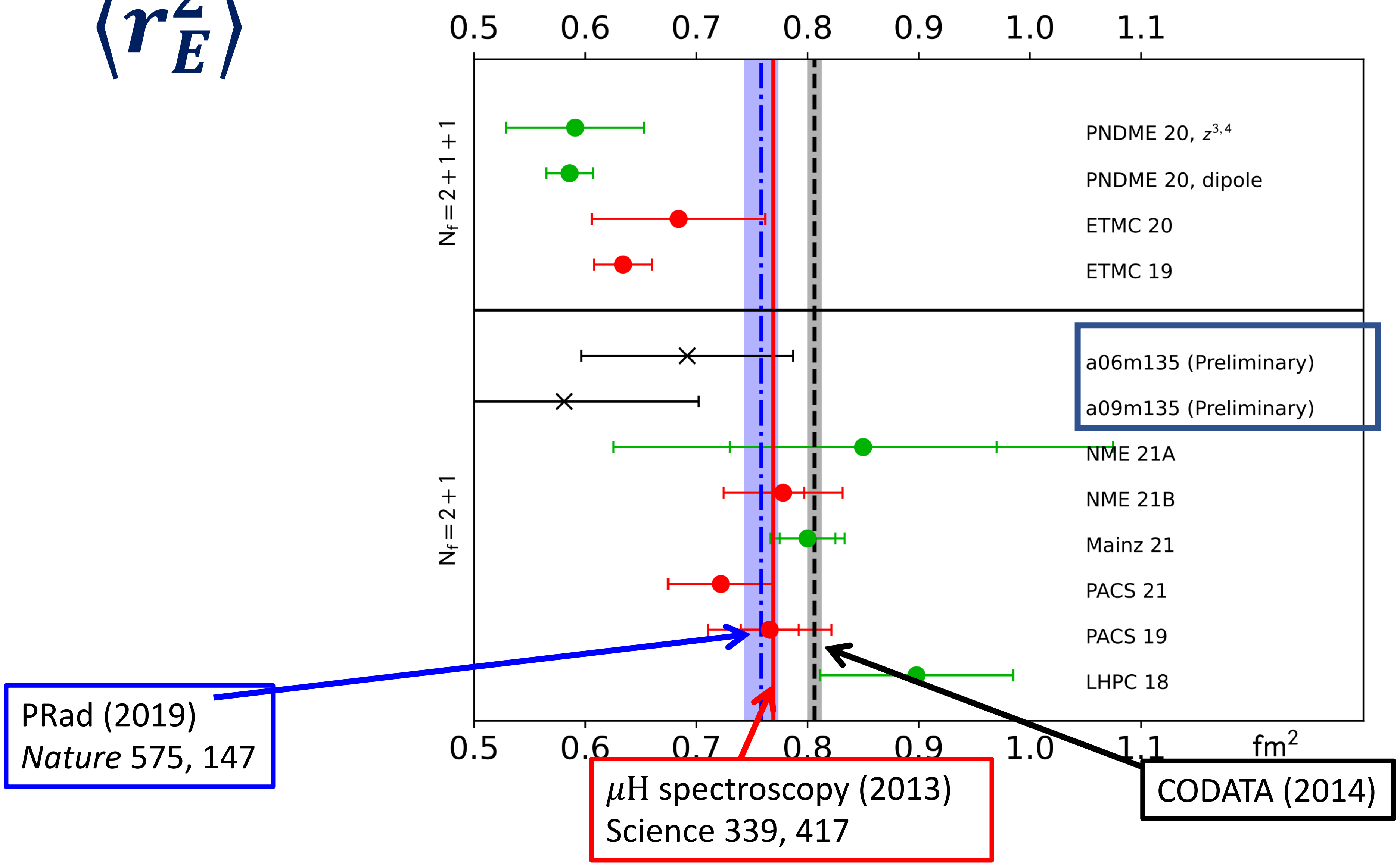
G_M^{u-d} : Examined Dipole, Pade and z-expansion fits



- Dipole: $\frac{G_E(0)/g_V}{(1+Q^2/M_E^2)}$
- Pade: $\frac{g}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

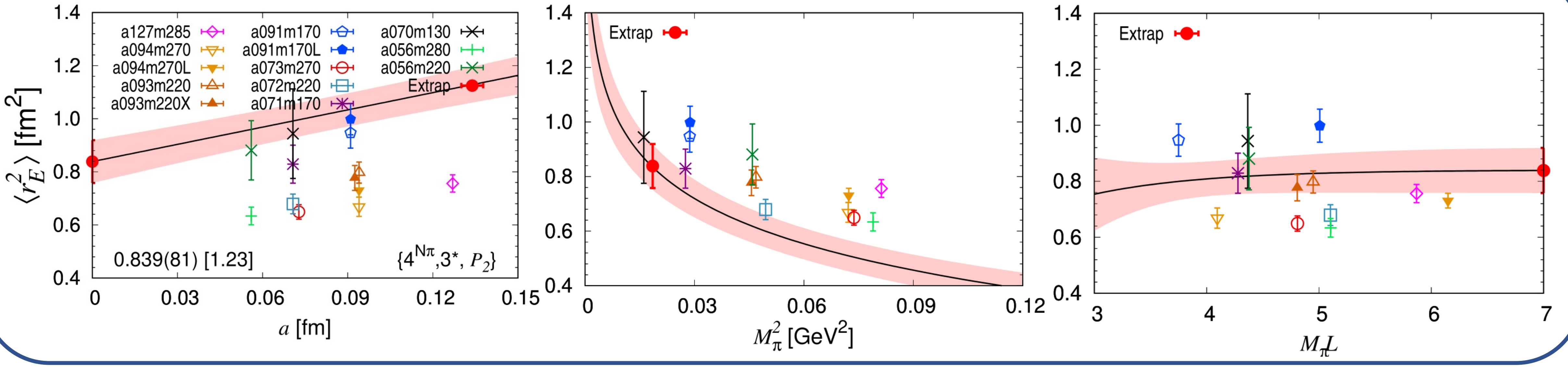
Need z^3 to fit the data

$\langle r_E^2 \rangle^{u-d}$



$\langle r_E^2 \rangle^{u-d}$: Chiral-Continuum-Finite Volume extrapolation

NME (2022), preliminary



Leading corrections included in the fit ansatz

- $c_1 + c_2 a + c_3 \log\frac{M_\pi^2}{\lambda^2} + c_4 \log\frac{M_\pi^2}{\lambda^2} e^{-M_\pi L}$

Nucl.Phys.A635, 121 (1998)
Nucl.Phys.A679, 698 (2001)
Phys.Rev.D71, 034508 (2005)

Summary

- We are calculating isovector electromagnetic form factors $\boldsymbol{G_{E,M}^{u-d}(Q^2)}$ using high statistics MILC HISQ ensembles at M_{Phys} and $a \approx 0.06$ and 0.09 as part of a comprehensive analysis of nucleon structure.
- Form factors are presented as a function of Q^2 over $0.04 < Q^2 < 0.8\ \mathrm{GeV}^2$.
- Using the high-statistics data, we examined excited state (ES) fit strategies for different M_1 and M_2 based on χ^2, and the result supports vector meson dominance i.e., the insertion of 2π(q) by the current.