Some Aspects of Quantum Simulations for Bosons and D-brane Scattering †

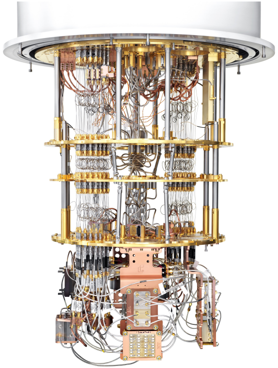
Emanuele Mendicelli

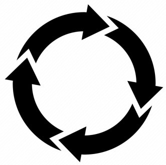
Department of Mathematical Sciences, Theoretical Physics Wing, University of Liverpool

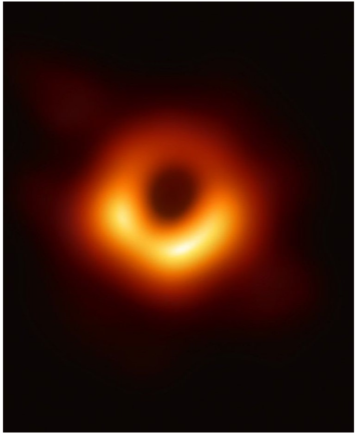
Lattice 2026, July 27th, University of Maryland, College Park, USA

† In collaboration with: Masanori Hanada

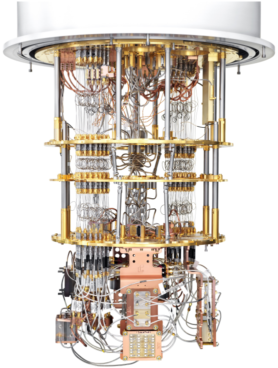
The Dream: Using Quantum Computers to Study Black Holes

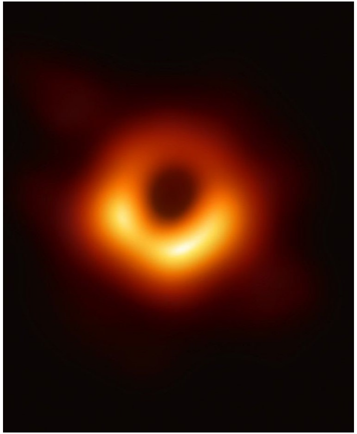






The Dream: Using Quantum Computers to Study Black Holes

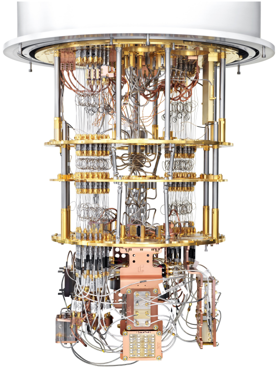


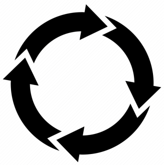


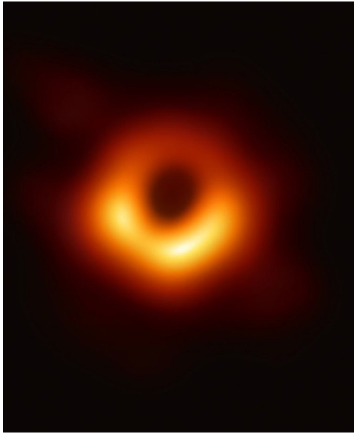
Present goals:

- Explore quantum simulations of the BFSS matrix model and black hole dynamics.

The Dream: Using Quantum Computers to Study Black Holes



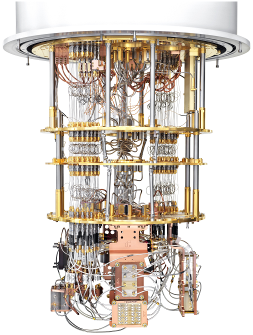




Present goals:

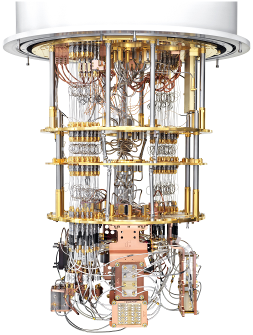
- Explore quantum simulations of the BFSS matrix model and black hole dynamics.
- Develop quantum algorithms for strongly coupled quantum systems

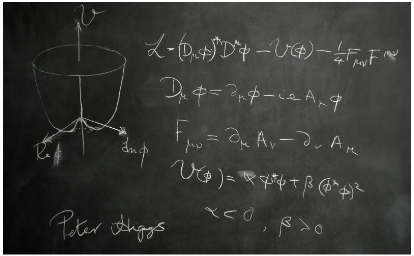
Challenges in using quantum computers: Hardware & Formalism



- Few qubits
- Low qubit connectivity
- Noisy gates
- No error correction $\Rightarrow$ **Error mitigation**!

Challenges in using quantum computers: Hardware & Formalism

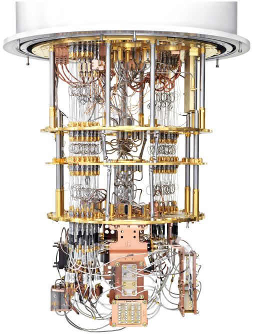


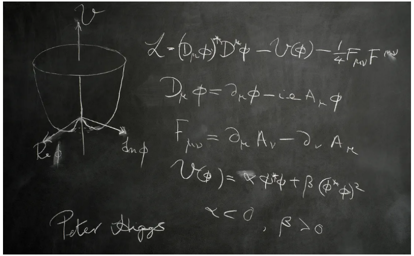


- Few qubits
- Low qubit connectivity
- Noisy gates
- No error correction $\Rightarrow$ **Error mitigation**!

- Hamiltonian formulation
- Representation of the Hamiltonian
- Scalability of the continuum limit
- Scalability of the thermodynamic limit

Challenges in using quantum computers: Hardware & Formalism





- Few qubits
- Low qubit connectivity
- Noisy gates
- No error correction $\Rightarrow$ **Error mitigation**!

- Hamiltonian formulation
- Representation of the Hamiltonian
- Scalability of the continuum limit
- Scalability of the thermodynamic limit

Encoding challenges

Where Do We Start?

The model

- Two-site model with two bosonic degrees of freedom

$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$

$\hat{x},\ \hat{p}_x$ $\qquad\qquad$ $\hat{y},\ \hat{p}_y$

where $[\hat{x}, \hat{p}_x] = i$ and $[\hat{y}, \hat{p}_y] = i$

The model

- Two-site model with two bosonic degrees of freedom

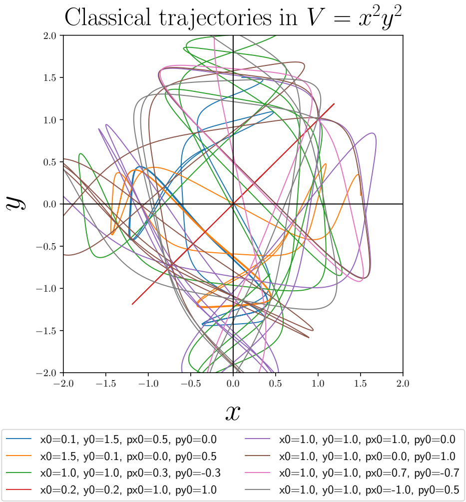
$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$

where $[\hat{x}, \hat{p}_x] = i$ and $[\hat{y}, \hat{p}_y] = i$

Does it look too simple?

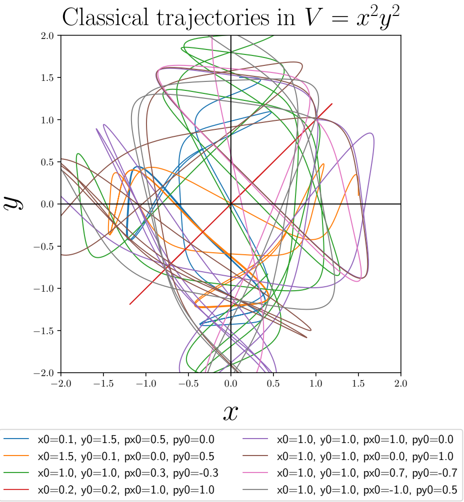
Classical Vs Quantum dynamics

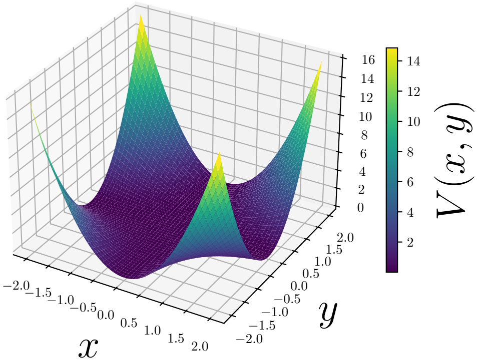
Classical

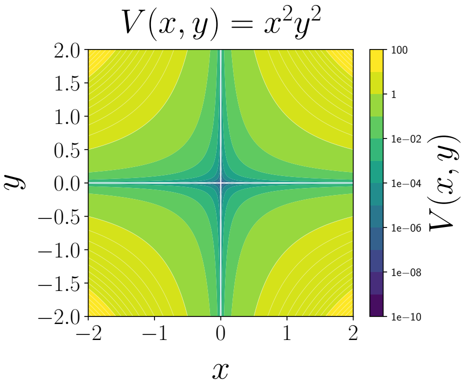


Classical Vs Quantum dynamics

Classical

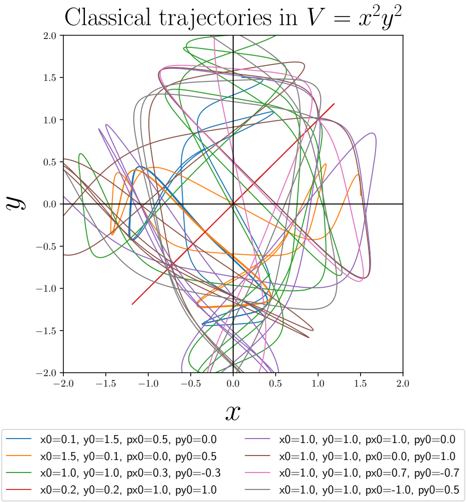


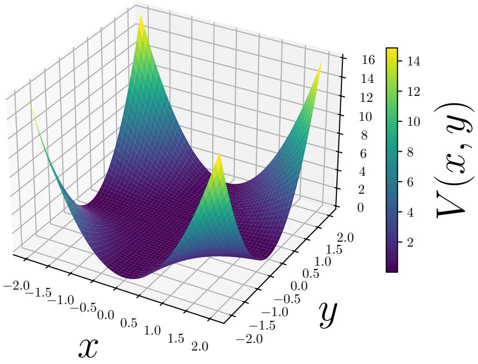


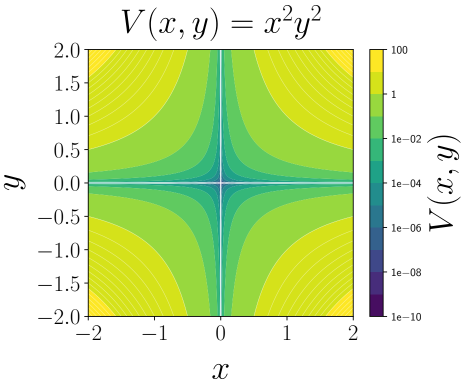


Classical Vs Quantum dynamics

Classical







Quantum

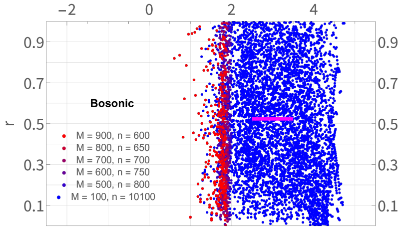
- The zero point energy generate an extra effective potential $V_{\text{eff}} = |x|/2$ lifts classical flat directions!
- Quantum zero-point energy $\Rightarrow$ dynamical confinement and discrete energy levels [arXiv:0810.3275]

Recent study of quantum chaos by P. Buividovich arXiv:2205.09704

Bosonic model

$$H_B = \hat{p}_x^2 + \hat{p}_y^2 + \hat{x}^2\hat{y}^2$$

Quantum chaotic ($\hat{r} = 0.523 \pm 0.006$)
[Wigner $r = 0.53$]



Recent study of quantum chaos by P. Buividovich arXiv:2205.09704

Bosonic model

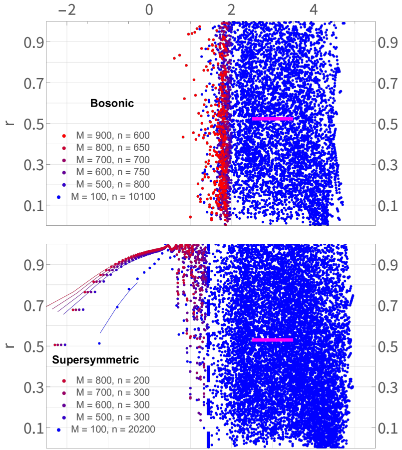
$$H_B = \hat{p}_x^2 + \hat{p}_y^2 + \hat{x}^2\hat{y}^2$$

Quantum chaotic ($\hat{r} = 0.523 \pm 0.006$)
[Wigner $r = 0.53$]

Supersymmetric model

$$H_S = H_B \otimes I + \hat{x} \otimes \sigma_x + \hat{y} \otimes \sigma_y$$

Quantum chaotic ($\hat{r} = 0.530 \pm 0.004$)



Recent study of quantum chaos by P. Buividovich arXiv:2205.09704

Bosonic model

$$H_B = \hat{p}_x^2 + \hat{p}_y^2 + \hat{x}^2\hat{y}^2$$

Quantum chaotic ($\hat{r} = 0.523 \pm 0.006$)
[Wigner $r = 0.53$]

Supersymmetric model

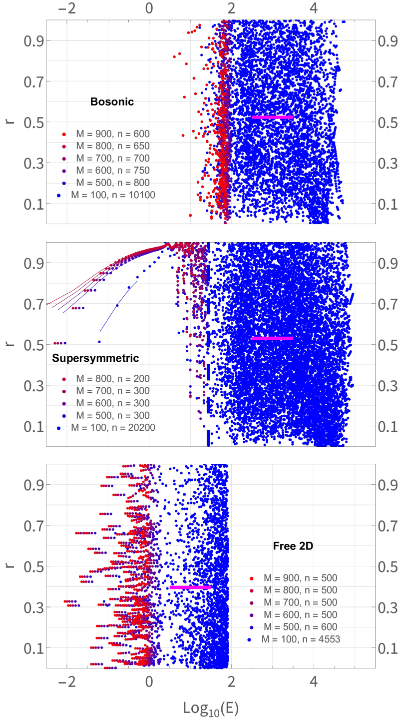
$$H_S = H_B \otimes I + \hat{x} \otimes \sigma_x + \hat{y} \otimes \sigma_y$$

Quantum chaotic ($\hat{r} = 0.530 \pm 0.004$)

Free model

$$H = \hat{p}_x^2 + \hat{p}_y^2$$

Non-chaotic ($\hat{r} = 0.40 \pm 0.01$)
[Poission $r = 0.39$]



How we view and use this two-boson model

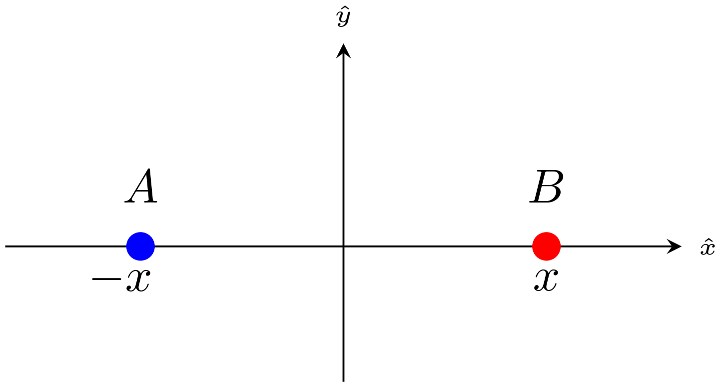
- Two-site model with two bosonic degrees of freedom

$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$

How we view and use this two-boson model

- Two-site model with two bosonic degrees of freedom

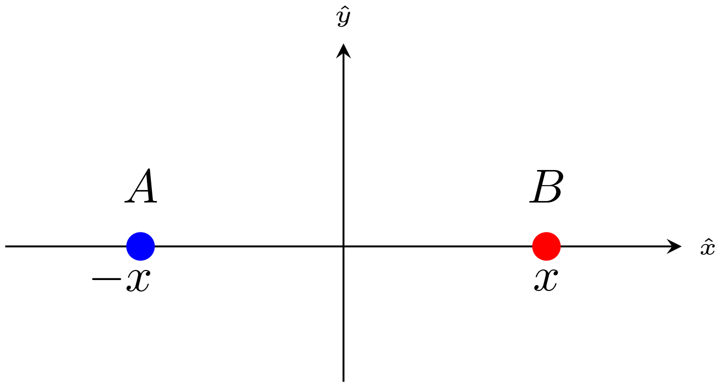
$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$



How we view and use this two-boson model

- Two-site model with two bosonic degrees of freedom

$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$

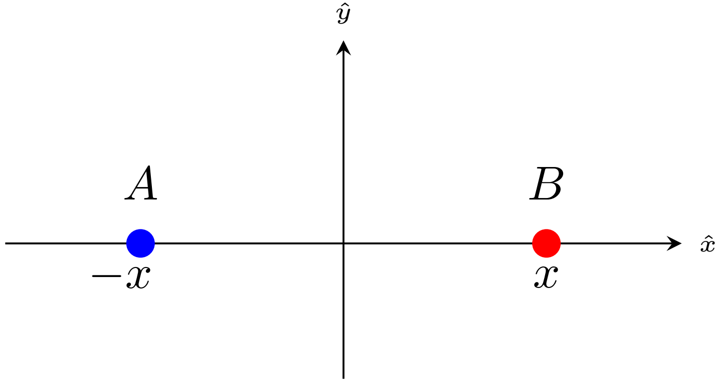


- x represent then distance between the two object, branes, **A** and **B**

How we view and use this two-boson model

- Two-site model with two bosonic degrees of freedom

$$H = \frac{1}{2}\left(\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2\hat{y}^2\right)$$



- x represent then distance between the two object, branes, **A** and **B**
- y is the fluctuation of the string connecting the two branes in the y-direction

Digitization

Digitization in the x-p-basis + QFT

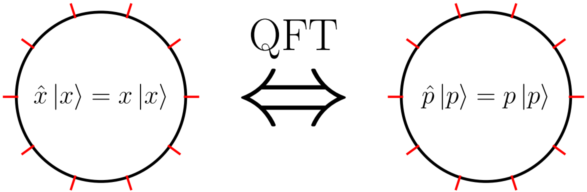
- Q qubits per boson $\Longrightarrow$ truncation level $\Lambda = 2^Q$ grid point (JPL [arXiv:1112.4833])

Λ points on a circle of radius $2R$ in the x- and p-basis.

Digitization in the x-p-basis + QFT

- Q qubits per boson $\Longrightarrow$ truncation level $\Lambda = 2^Q$ grid point (JPL [arXiv:1112.4833])

Λ points on a circle of radius $2R$ in the x- and p-basis.



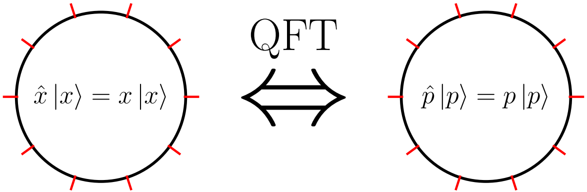
$$\delta_x = 2R/\Lambda \qquad \delta_p = \pi/R$$

$$\hat{x}_a = -\frac{\delta_x}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right) \qquad \hat{p}_a = -\frac{\delta_p}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right)$$

Digitization in the x-p-basis + QFT

- Q qubits per boson $\Longrightarrow$ truncation level $\Lambda = 2^Q$ grid point (JPL [arXiv:1112.4833])

Λ points on a circle of radius $2R$ in the x- and p-basis.



$$\delta_x = 2R/\Lambda \qquad \delta_p = \pi/R$$

$$\hat{x}_a = -\frac{\delta_x}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right) \qquad \hat{p}_a = -\frac{\delta_p}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right)$$

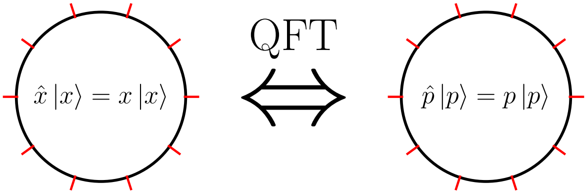
Benefits:

- Hamiltonian written directly in Paulis
- Polynomial scaling of the gate count

Digitization in the x-p-basis + QFT

- Q qubits per boson $\Longrightarrow$ truncation level $\Lambda = 2^Q$ grid point (JPL [arXiv:1112.4833])

Λ points on a circle of radius $2R$ in the x- and p-basis.



$\delta_x = 2R/\Lambda$ $\qquad\qquad$ $\delta_p = \pi/R$

$$\hat{x}_a = -\frac{\delta_x}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right) \qquad \hat{p}_a = -\frac{\delta_p}{2}\left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1}\hat{\sigma}_{z;a,Q}\right)$$

Benefits:

- Hamiltonian written directly in Paulis
- Polynomial scaling of the gate count
- **but...** requires the tuning of two parameters, Λ and R!

Naive encoding of the theory in qubits

- Q qubits per boson $\implies$ Fock basis, $\hat{q}$ and $\hat{p}$ from the harmonic oscillator truncated to $\Lambda = 2^Q$ modes

$$\hat{q} \doteq \frac{1}{\sqrt{2m}} \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & \sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & \sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix} \qquad \hat{p} \doteq i\sqrt{\frac{m}{2}} \begin{pmatrix} 0 & -\sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & -\sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & -\sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix}$$

Naive encoding of the theory in qubits

- Q qubits per boson $\Longrightarrow$ Fock basis, $\hat{q}$ and $\hat{p}$ from the harmonic oscillator truncated to $\Lambda = 2^Q$ modes

$$\hat{q} \doteq \frac{1}{\sqrt{2m}} \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & \sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & \sqrt{\Lambda - 1} \\ 0 & 0 & \cdots & \sqrt{\Lambda - 1} & 0 \end{pmatrix} \qquad \hat{p} \doteq i\sqrt{\frac{m}{2}} \begin{pmatrix} 0 & -\sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & -\sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & -\sqrt{\Lambda - 1} \\ 0 & 0 & \cdots & \sqrt{\Lambda - 1} & 0 \end{pmatrix}$$

Benefits:

- Only one parameter to tune Λ!

Naive encoding of the theory in qubits

- Q qubits per boson $\Longrightarrow$ Fock basis, $\hat{q}$ and $\hat{p}$ from the harmonic oscillator truncated to $\Lambda = 2^Q$ modes

$$\hat{q} \doteq \frac{1}{\sqrt{2m}} \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & \sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & \sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix} \qquad \hat{p} \doteq i\sqrt{\frac{m}{2}} \begin{pmatrix} 0 & -\sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & -\sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & -\sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix}$$

Benefits:

- Only one parameter to tune Λ!

but...

- Pauli decomposition of the Hamiltonian (worst-case $\mathcal{O}(4^Q)$ [arXiv:2310.13421])
- Exponential scaling of the gate count [arXiv:2505.02553]

Toward Scattering

Setup for Scattering

- Prepare an initial state:

$$\psi(x,y) = \frac{1}{\sqrt{2\pi\sigma_x\sigma_y}} \exp\left(-\frac{(x-a)^2}{4\sigma_x^2} - \frac{y^2}{4\sigma_y^2}\right)$$

Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

Setup for Scattering

- Prepare an initial state:

$$\psi(x, y) = \frac{1}{\sqrt{2\pi\sigma_x\sigma_y}} \exp\left(-\frac{(x-a)^2}{4\sigma_x^2} - \frac{y^2}{4\sigma_y^2}\right)$$

Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

- Real-time evolution

Setup for Scattering

- Prepare an initial state:

$$\psi(x, y) = \frac{1}{\sqrt{2\pi\sigma_x\sigma_y}} \exp\left(-\frac{(x-a)^2}{4\sigma_x^2} - \frac{y^2}{4\sigma_y^2}\right)$$

Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

- Real-time evolution

- Measurement

Setup for Scattering

- Prepare an initial state:

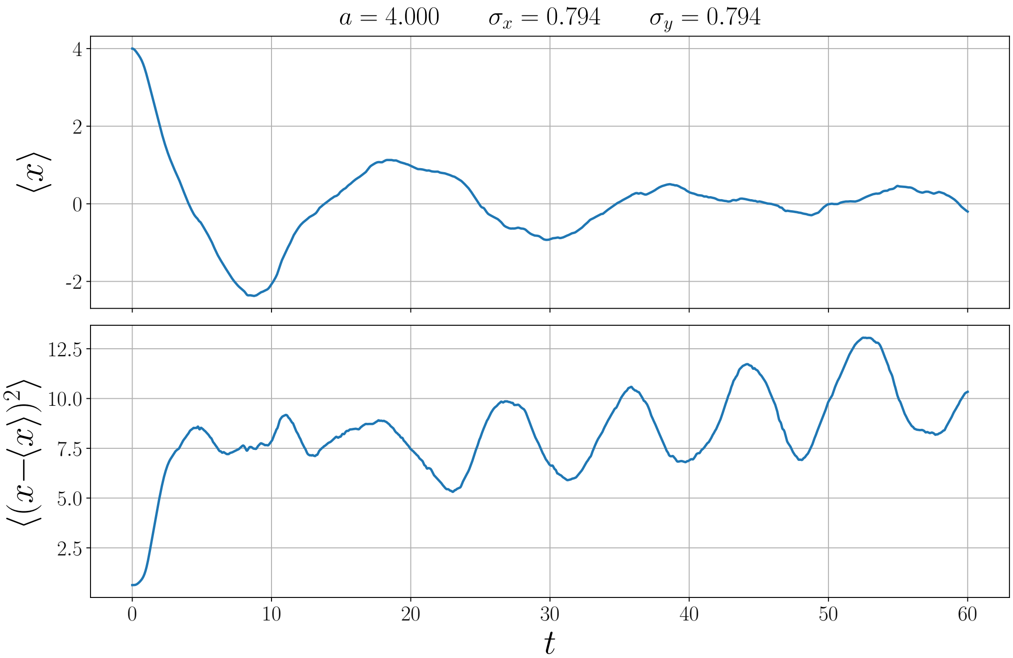
$$\psi(x,y) = \frac{1}{\sqrt{2\pi\sigma_x\sigma_y}} \exp\left(-\frac{(x-a)^2}{4\sigma_x^2} - \frac{y^2}{4\sigma_y^2}\right)$$

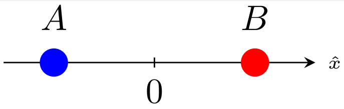
Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

- Real-time evolution
- Measurement
- Calculate $\langle x \rangle(t)$ and $\langle (x - \langle x \rangle)^2 \rangle(t)$ to visualize the scattering process

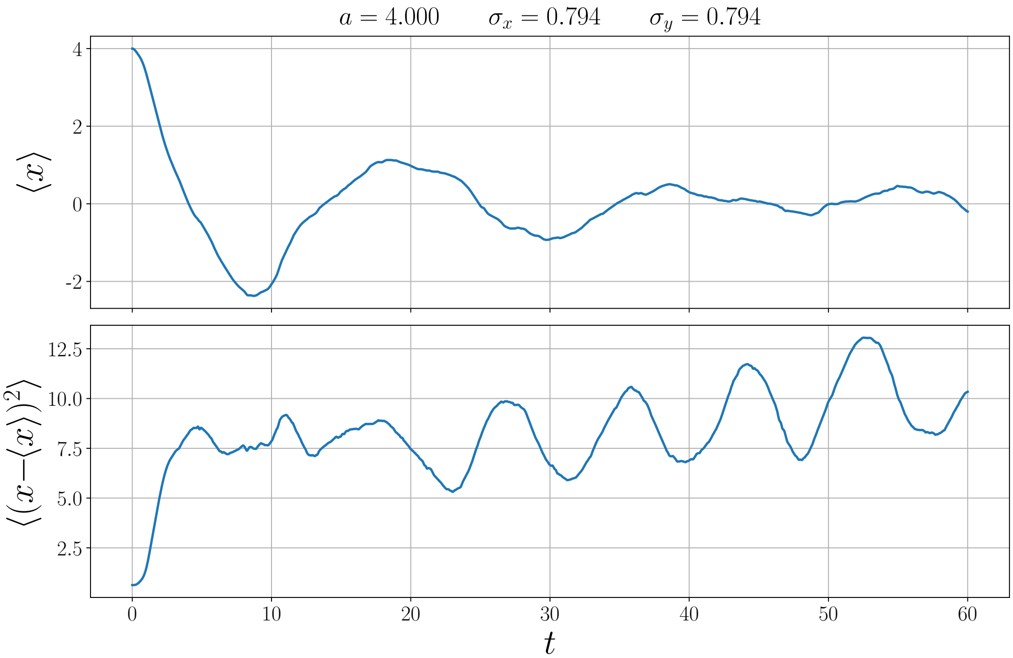
Preliminary Results

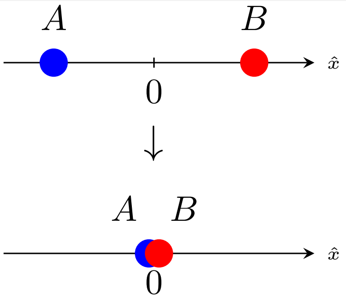
Preliminary results: Toward the formation of a "bound state"



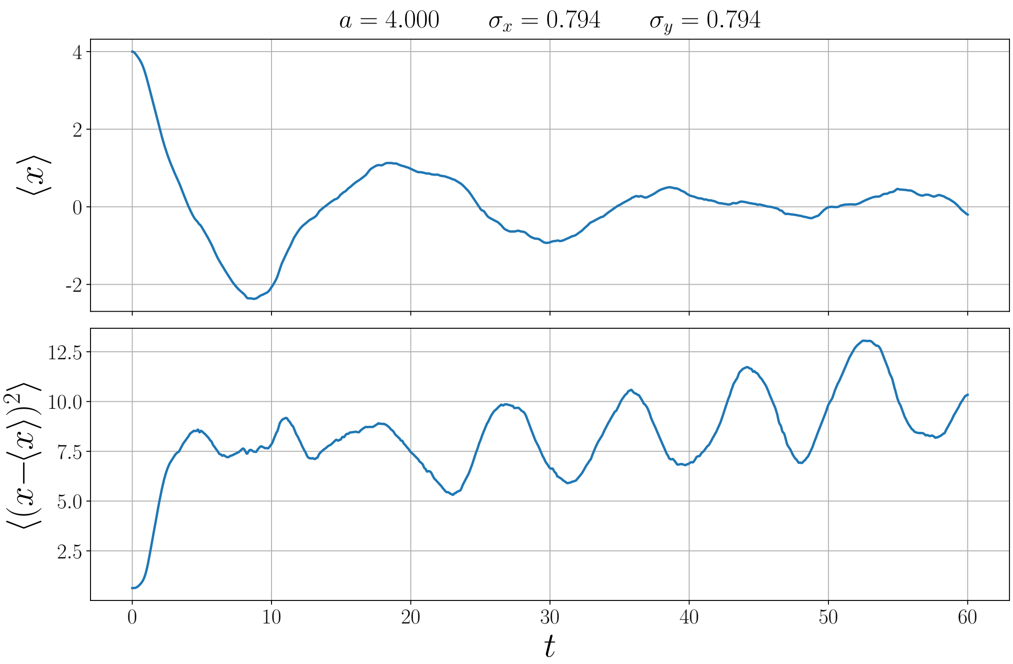


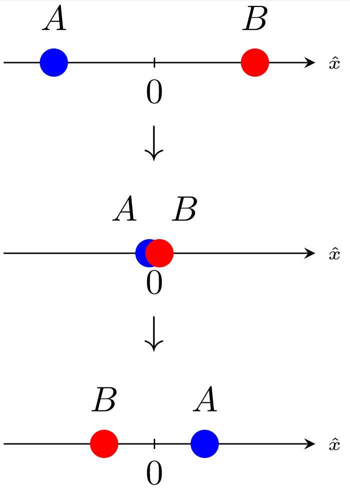
Preliminary results: Toward the formation of a "bound state"



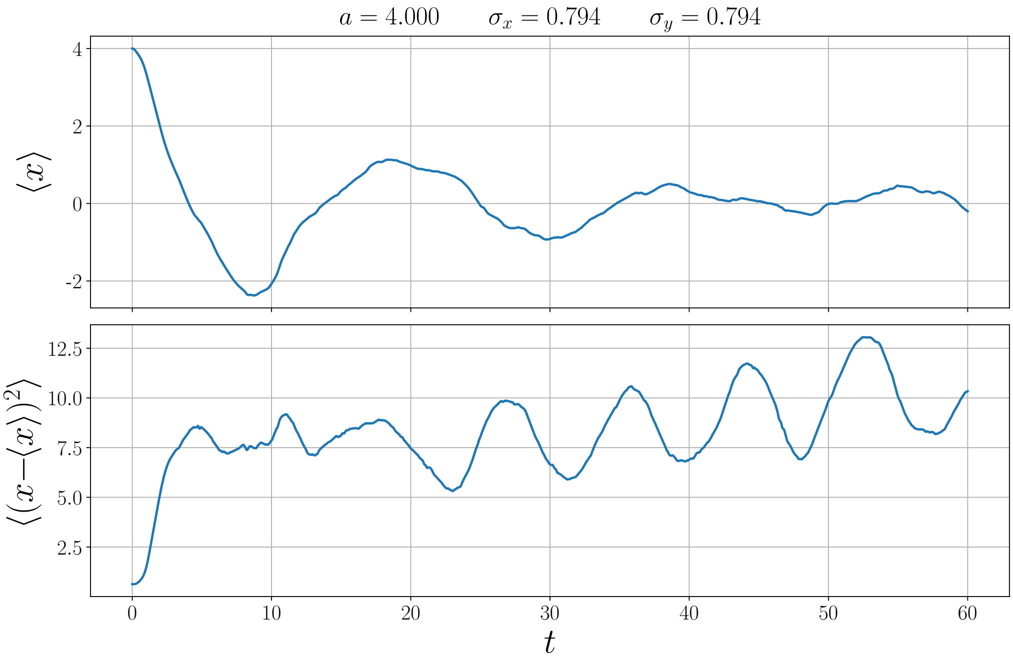


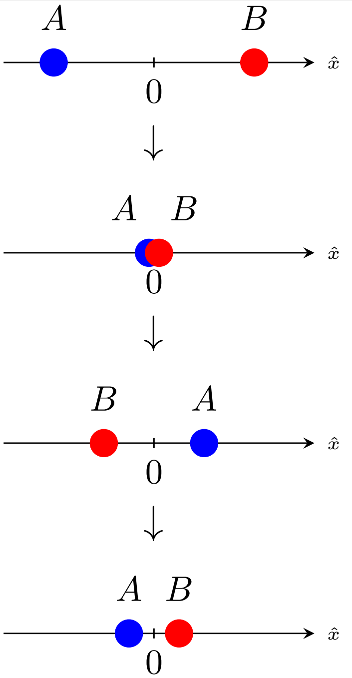
Preliminary results: Toward the formation of a "bound state"





Preliminary results: Toward the formation of a "bound state"





Challenges we are facing:

- Hamiltonian Encoding

Challenges we are facing:

- Hamiltonian Encoding

$$\downarrow$$

- Hilbert Space Truncation

Challenges we are facing:

- Hamiltonian Encoding

$\downarrow$

- Hilbert Space Truncation

$\downarrow$

- State preparation

Challenges we are facing:

- Hamiltonian Encoding

$\downarrow$

- Hilbert Space Truncation

$\downarrow$

- State preparation

$\downarrow$

- Trotterizzation

Challenges we are facing:

- Hamiltonian Encoding

$\downarrow$

- Hilbert Space Truncation

$\downarrow$

- State preparation

$\downarrow$

- Trotterizzation

$\downarrow$

- Hardware noise

Challenges we are facing:

- Hamiltonian Encoding

$\downarrow$

- Hilbert Space Truncation

$\downarrow$

- State preparation

$\downarrow$

- Trotterizzation

$\downarrow$

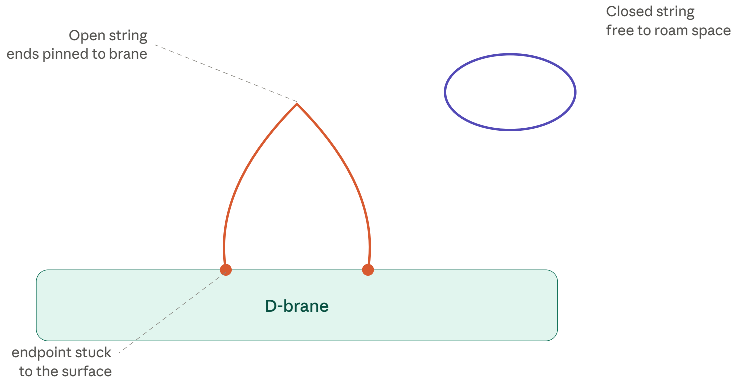
- Hardware noise

Each step introduces error which we need to address!

Toward D-brane

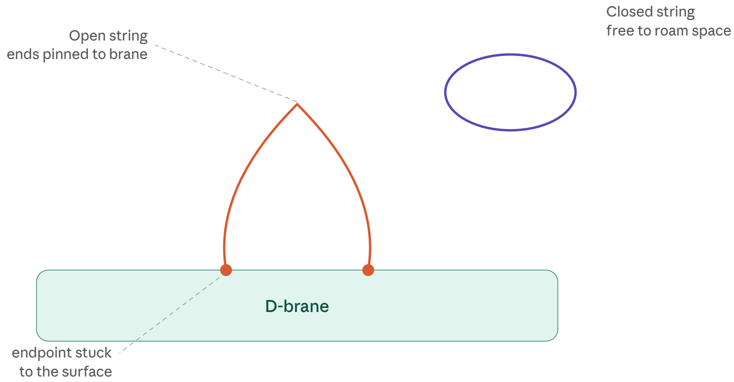
What D-brane are and why are they important?

D-branes are fundamental objects in string theory that link gravity and quantum field theory.
Dp-branes have p spatial dimensions: D0 (point), D1 (string), D2 (membrane), and so on.



What D-brane are and why are they important?

D-branes are fundamental objects in string theory that link gravity and quantum field theory.
Dp-branes have p spatial dimensions: D0 (point), D1 (string), D2 (membrane), and so on.



- Microscopic derivation of the Bekenstein–Hawking black hole entropy (Strominger & Vafa)
- Foundation of the AdS/CFT correspondence (Maldacena)
- Recognition of D-branes as fundamental objects, paving the way toward M-theory (Polchinski)
- Applications to string phenomenology and brane-world cosmology

How D0-brane are connected with a 2-boson model

Banks-Fischler-Shenker-Susskind (BFSS) matrix model describes the dynamics of interacting D0-brane

$$H = \mathrm{Tr}\left[\frac{1}{2}P_i^2 - \frac{1}{4}[X^i, X^j]^2\right]$$

where $X^i(t)$ and $P_i(t)$ are $N \times N$ Hermitian matrices satisfying $[X^i, P_j] = i\delta_{ij}$.

How D0-brane are connected with a 2-boson model

Banks-Fischler-Shenker-Susskind (BFSS) matrix model describes the dynamics of interacting D0-brane

$$H = \mathrm{Tr}\left[\frac{1}{2}P_i^2 - \frac{1}{4}[X^i, X^j]^2\right]$$

where $X^i(t)$ and $P_i(t)$ are $N \times N$ Hermitian matrices satisfying $[X^i, P_j] = i\delta_{ij}$.

In the regime of well-separated D0-branes, where ℓ_s is the fundamental string length scale:

$$|x_a - x_b| \gg \ell_s.$$

we can split the matrices as $X^i_{ab} = x^i_a \delta_{ab} + Y^i_{ab}$, assuming $|Y^i_{ab}| \ll |x^i_a - x^i_b|$

x^i_a are D0-brane positions
Y^i_{ab} are the off-diagonal fluctuations

How D0-brane are connected with a 2-boson model

Banks-Fischler-Shenker-Susskind (BFSS) matrix model describes the dynamics of interacting D0-brane

$$H = \mathrm{Tr}\left[\frac{1}{2}P_i^2 - \frac{1}{4}[X^i, X^j]^2\right]$$

where $X^i(t)$ and $P_i(t)$ are $N \times N$ Hermitian matrices satisfying $[X^i, P_j] = i\delta_{ij}$.

In the regime of well-separated D0-branes, where ℓ_s is the fundamental string length scale:

$$|x_a - x_b| \gg \ell_s.$$

we can split the matrices as $X^i_{ab} = x^i_a \delta_{ab} + Y^i_{ab}$, assuming $|Y^i_{ab}| \ll |x^i_a - x^i_b|$

x^i_a are D0-brane positions
Y^i_{ab} are the off-diagonal fluctuations

For two D0-branes ($N = 2$), expanding and keeping relevant terms, BFSS becomes:

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2}x^2y^2$$

Conclusions

What we have seen:

- A two-boson model with **very rich physics**: Classical Vs Quantum dynamics, quantum chaos etc
- It is a **toy-model for Matrix Model and D0-brane** dynamics on a quantum hardware
- Preliminary results show the formation of a bound state a "D0-black-hole".

What we have seen:

- A two-boson model with **very rich physics**: Classical Vs Quantum dynamics, quantum chaos etc
- It is a **toy-model for Matrix Model and D0-brane** dynamics on a quantum hardware
- Preliminary results show the formation of a bound state a ”D0-black-hole”.

Ongoing work:

- **Quantify the errors** from Encoding, Truncation, State preparation, Trotterization and Hardware noise
- Run simulations on **gate based quantum hardware**.
- Explore the use of **trapped ion quantum computer**, it should be more natural and easier!
- Extend the study to systems with **multiple bosons (D0-particles)**

Thank you all for your time!

Time for Questions!

Please feel free to get in touch with any questions or curiosities!

Emanuele Mendicelli

Department of Mathematical Sciences
Theoretical Physics Wing
University of Liverpool
United Kingdom

e.mendicelli@liverpool.ac.uk

Alternative contact:
emanuelemendicelli@hotmail.it

Backup Slides

Preliminary results: Previous Slide with $\langle p \rangle$