

Some Aspects of Quantum Simulations for Bosons and D-brane Scattering [†]

Emanuele Mendicelli

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LIVERPOOL

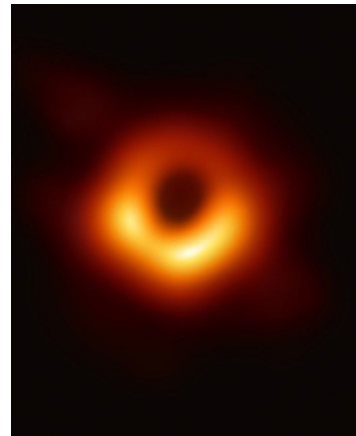
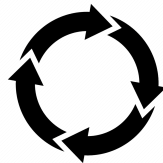
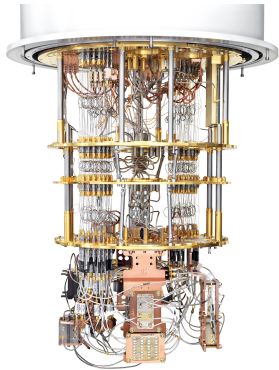


Lattice 2026, July 27th, University of Maryland, College Park, USA

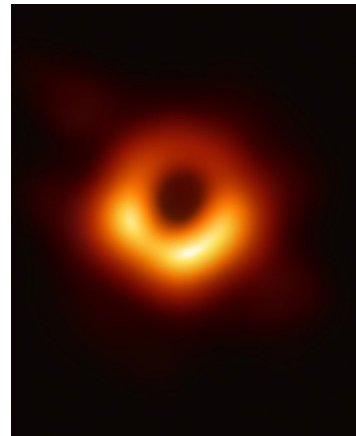
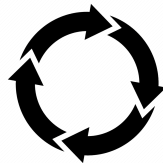
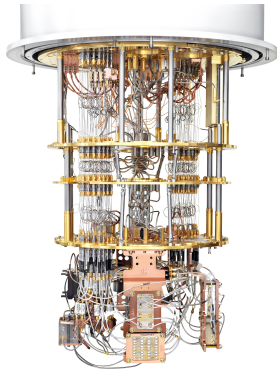


[†] In collaboration with: Masanori Hanada

The Dream: Using Quantum Computers to Study Black Holes



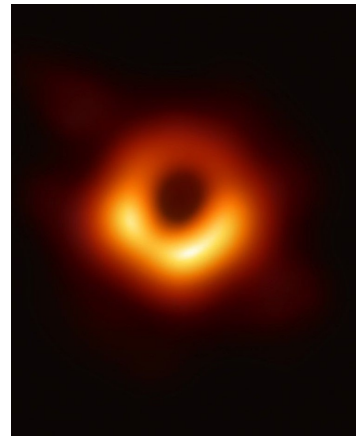
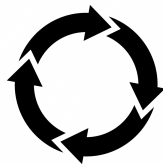
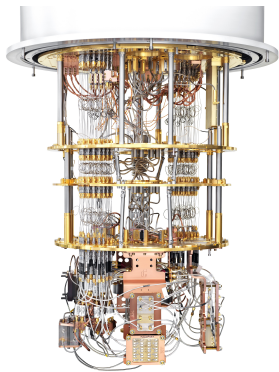
The Dream: Using Quantum Computers to Study Black Holes



Present goals:

- Explore quantum simulations of the BFSS matrix model and black hole dynamics.

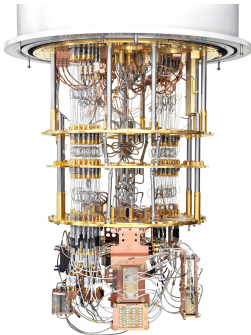
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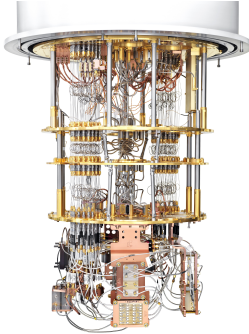
- Explore quantum simulations of the BFSS matrix model and black hole dynamics.
- Develop quantum algorithms for strongly coupled quantum systems

Challenges in using quantum computers: Hardware & Formalism



- Few qubits
- Low qubit connectivity
- Noisy gates
- No error correction $\Rightarrow$ **Error mitigation!**

Challenges in using quantum computers: Hardware & Formalism

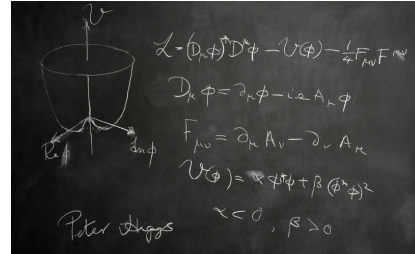
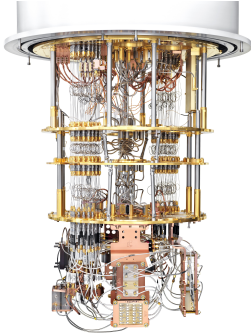


$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
 $D_\mu \phi = \partial_\mu \phi - i e A_\mu \phi$
 $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
 $V(\phi) = \frac{\alpha}{4} \phi^\dagger \phi + \frac{\beta}{4} (\phi^\dagger \phi)^2$
 Peter Higgs $\alpha < 0, \beta > 0$

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- Hamiltonian formulation
- Representation of the Hamiltonian
- Scalability of the continuum limit
- Scalability of the thermodynamic limit

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Encoding challenges

Where Do We Start?

The model

- Two-site model with two bosonic degrees of freedom

$$H = \frac{1}{2} (\hat{p}_y^2 + \hat{p}_x^2 + \hat{x}^2 \hat{y}^2)$$

$\hat{x}, \hat{p}_x$
●

$\hat{y}, \hat{p}_y$
●

where $[\hat{x}, \hat{p}_x] = i$ and $[\hat{y}, \hat{p}_y] = i$

The model

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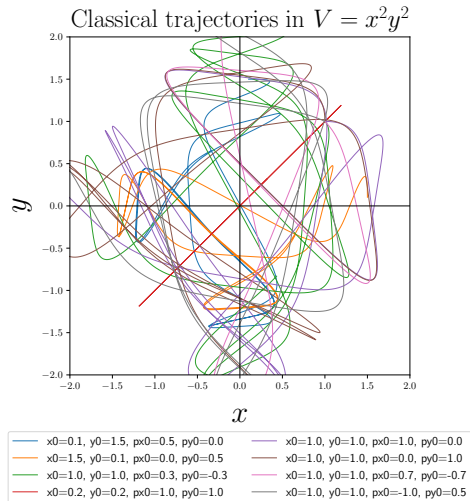
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Does it look too simple?

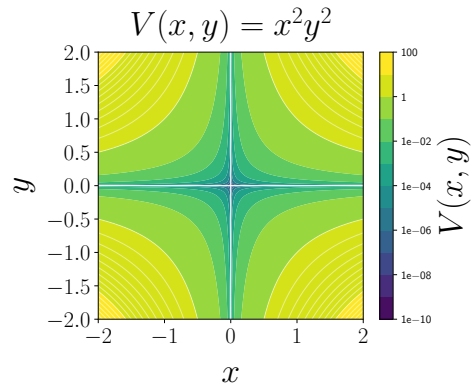
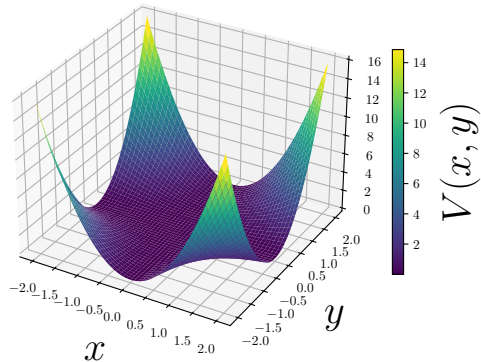
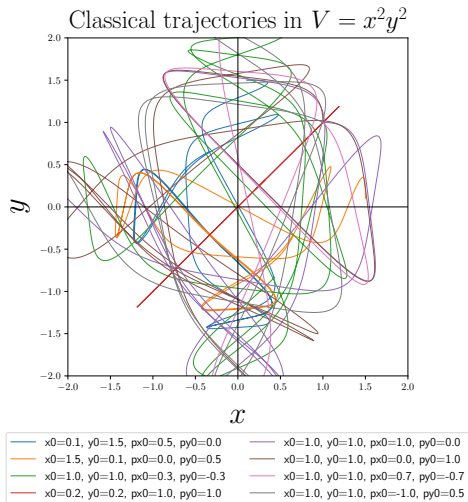
Classical Vs Quantum dynamics

Classical



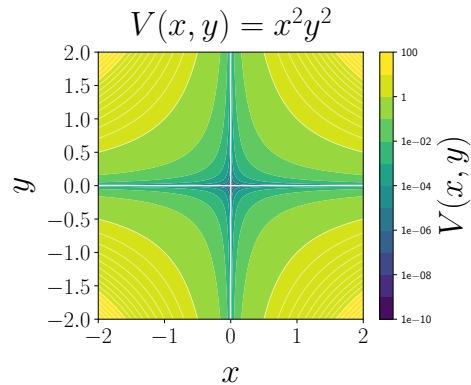
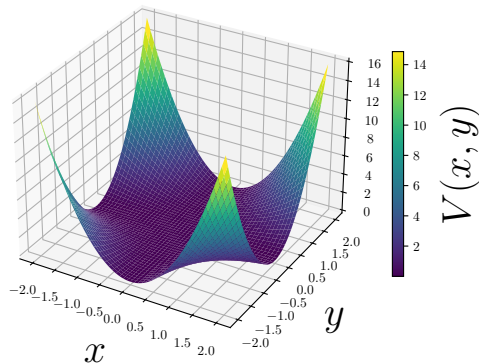
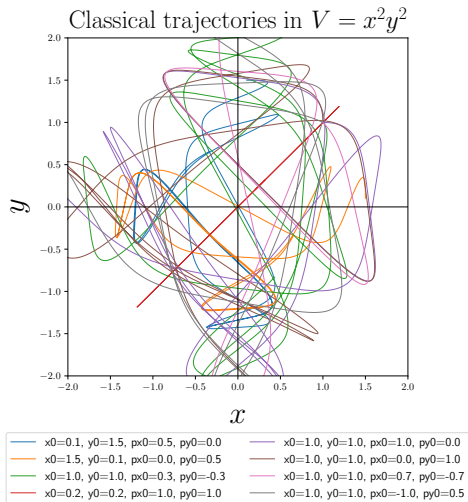
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Quantum

- The zero point energy generate an extra effective potential $V_{\text{eff}} = |x|/2$ lifts classical flat directions!
- Quantum zero-point energy $\Rightarrow$ dynamical confinement and discrete energy levels [[arXiv:0810.3275](https://arxiv.org/abs/0810.3275)]

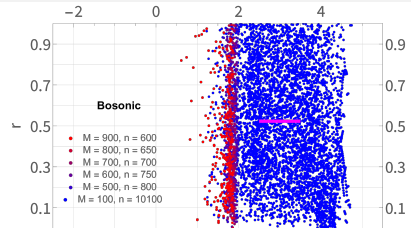
Recent study of quantum chaos by P. Buividovich [arXiv:2205.09704](https://arxiv.org/abs/2205.09704)

Bosonic model

$$H_B = \hat{p}_x^2 + \hat{p}_y^2 + \hat{x}^2 \hat{y}^2$$

Quantum chaotic ($\hat{r} = 0.523 \pm 0.006$)

[Wigner $r = 0.53$]



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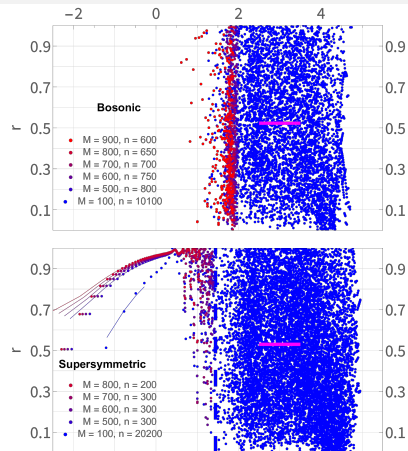
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Supersymmetric model

$$H_S = H_B \otimes I + \hat{x} \otimes \sigma_x + \hat{y} \otimes \sigma_y$$

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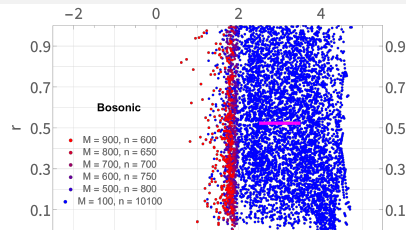
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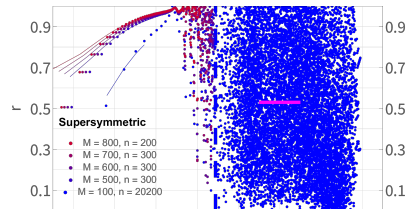
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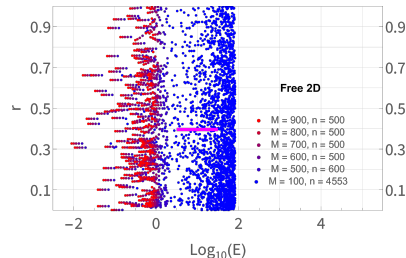


Free model

$$H = \hat{p}_x^2 + \hat{p}_y^2$$

Non-chaotic ($\hat{r} = 0.40 \pm 0.01$)

[Poisson $r = 0.39$]



How we view and use this two-boson model

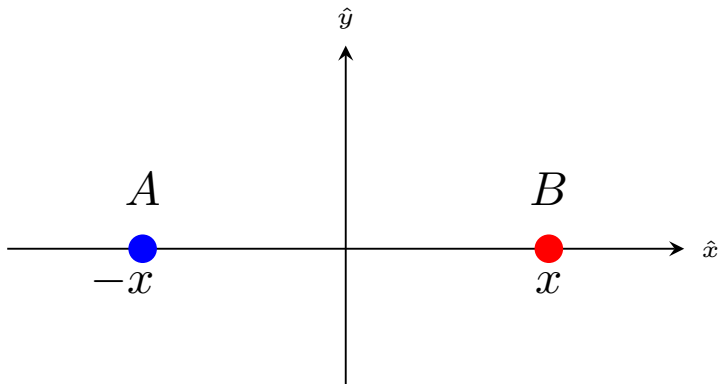
- Two-site model with two bosonic degrees of freedom

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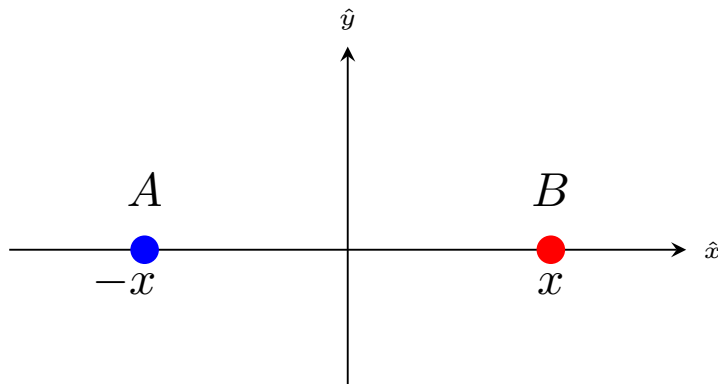
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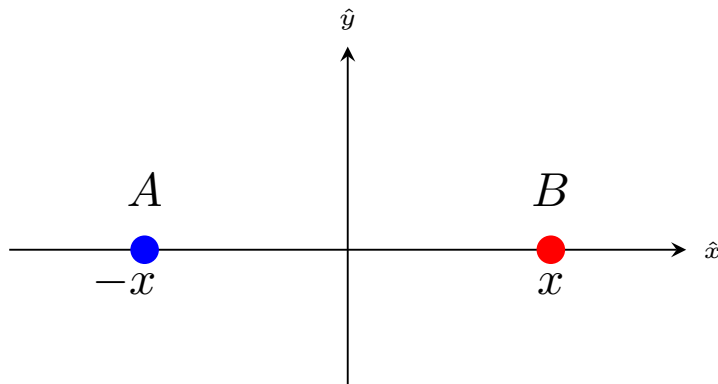


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- x represent then distance between the two object, branes, **A** and **B**
- y is the fluctuation of the string connecting the two branes in the y -direction

Digitization

Digitization in the x - p -basis + QFT

- Q qubits per boson $\implies$ truncation level $\Lambda = 2^Q$ grid point (JPL [[arXiv:1112.4833](#)])

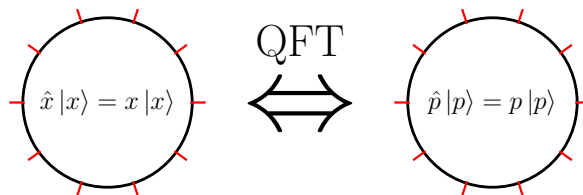
Λ points on a circle of radius $2R$ in the x - and p -basis.

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$$\delta_x = 2R/\Lambda$$

$$\delta_p = \pi/R$$

$$\hat{x}_a = -\frac{\delta_x}{2} \left(\hat{\sigma}_{z;a,1} + \cdots + 2^{Q-1} \hat{\sigma}_{z;a,Q} \right)$$

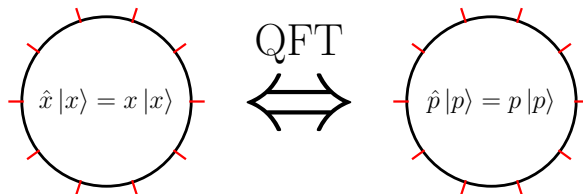
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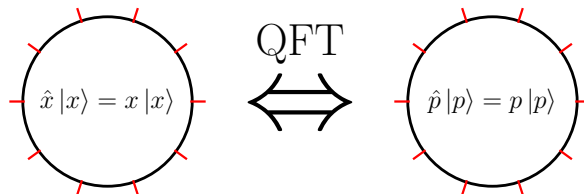
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Benefits:

- Hamiltonian written directly in Paulis
- Polynomial scaling of the gate count
- **but...** requires the tuning of two parameters, Λ and R !

Naive encoding of the theory in qubits

- Q qubits per boson $\implies$ Fock basis, $\hat{q}$ and $\hat{p}$ from the harmonic oscillator truncated to $\Lambda = 2^Q$ modes

$$\hat{q} \doteq \frac{1}{\sqrt{2m}} \begin{pmatrix} 0 & \sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & \sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & \sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix} \quad \hat{p} \doteq i\sqrt{\frac{m}{2}} \begin{pmatrix} 0 & -\sqrt{1} & 0 & \cdots & 0 \\ \sqrt{1} & 0 & -\sqrt{2} & \cdots & 0 \\ 0 & \sqrt{2} & \ddots & \ddots & 0 \\ 0 & 0 & \ddots & 0 & -\sqrt{\Lambda-1} \\ 0 & 0 & \cdots & \sqrt{\Lambda-1} & 0 \end{pmatrix}$$

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Benefits:

- Only one parameter to tune Λ !

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Benefits:

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but...

- Pauli decomposition of the Hamiltonian (worst-case $\mathcal{O}(4^Q)$ [[arXiv:2310.13421](https://arxiv.org/abs/2310.13421)])
- Exponential scaling of the gate count [[arXiv:2505.02553](https://arxiv.org/abs/2505.02553)]

Toward Scattering

Setup for Scattering

- Prepare an initial state:

$$\psi(x, y) = \frac{1}{\sqrt{2\pi\sigma_x\sigma_y}} \exp\left(-\frac{(x-a)^2}{4\sigma_x^2} - \frac{y^2}{4\sigma_y^2}\right)$$

Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

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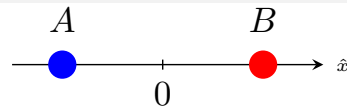
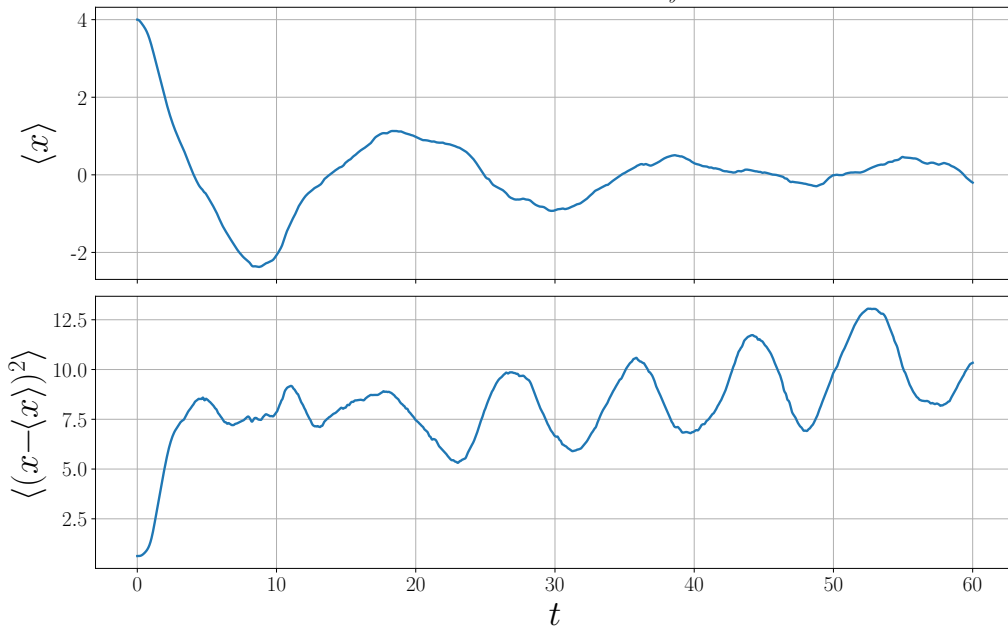
Gaussian wave packets centered at $x = a$, with σ_x and σ_y optimized by energy minimization.

- Real-time evolution
- Measurement
- Calculate $\langle x \rangle(t)$ and $\langle (x - \langle x \rangle)^2 \rangle(t)$ to visualize the scattering process

Preliminary Results

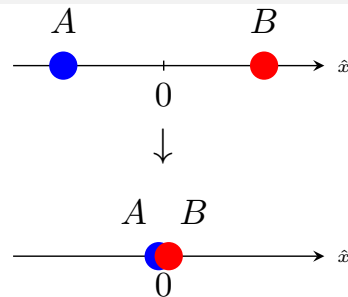
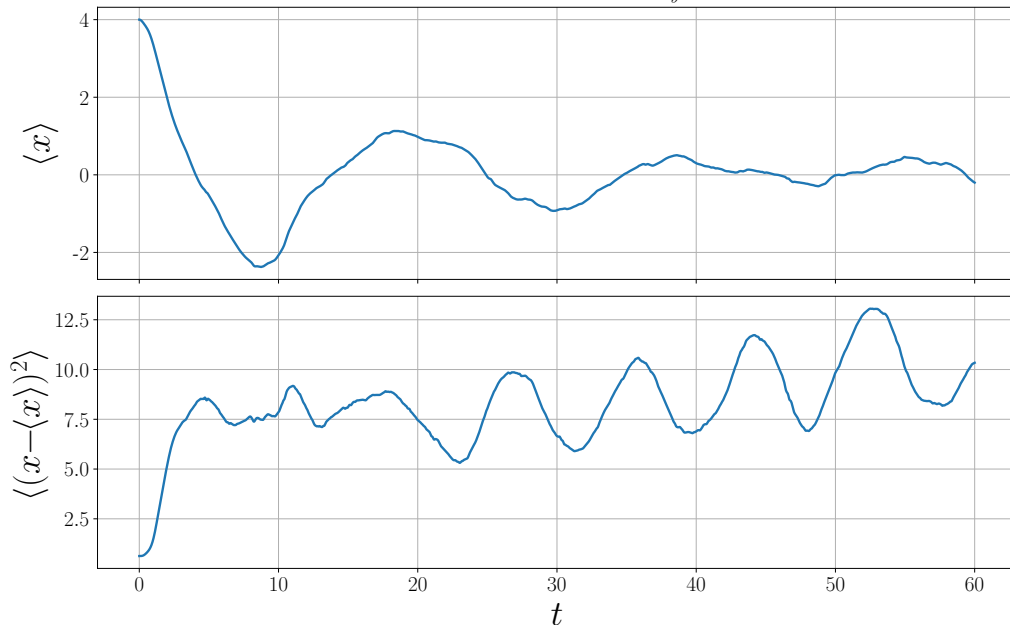
Preliminary results: Toward the formation of a "bound state"

$$a = 4.000 \quad \sigma_x = 0.794 \quad \sigma_y = 0.794$$



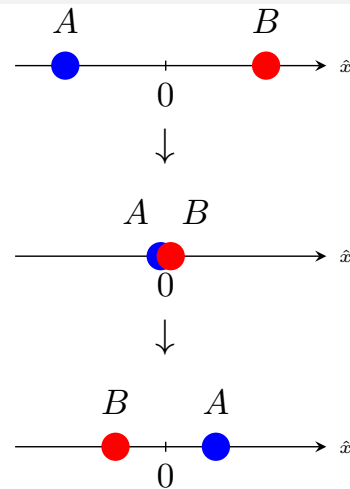
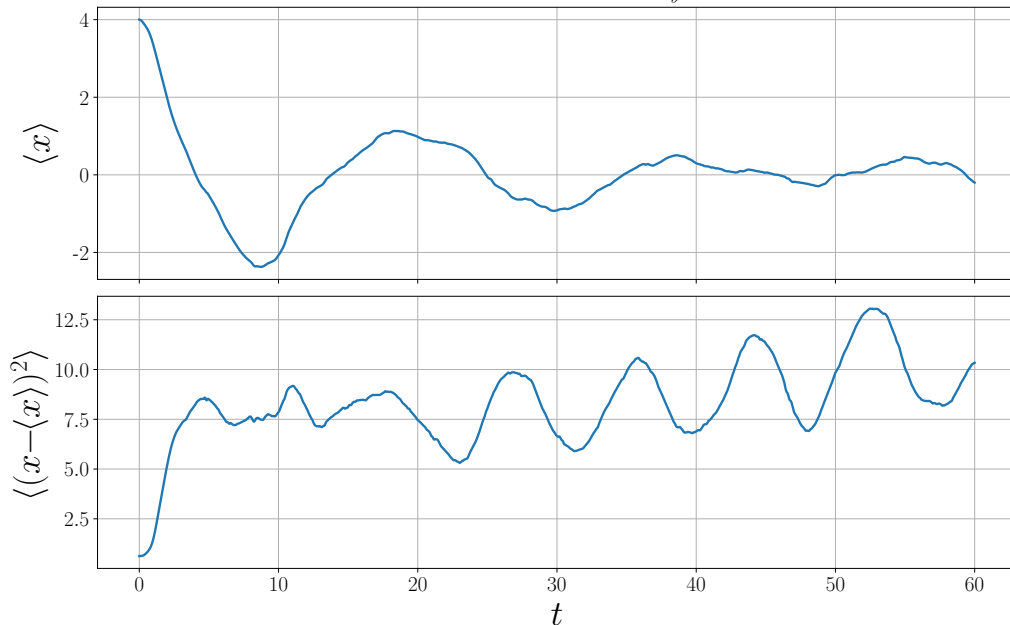
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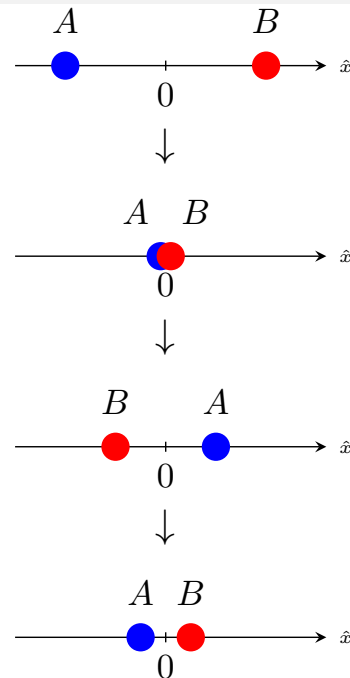
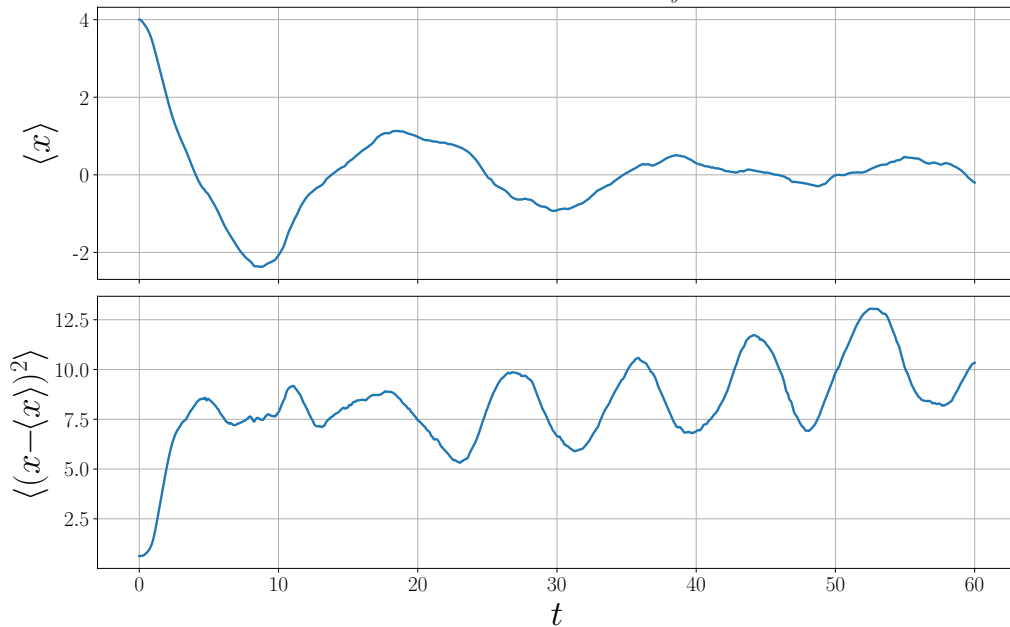
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Challenges we are facing:

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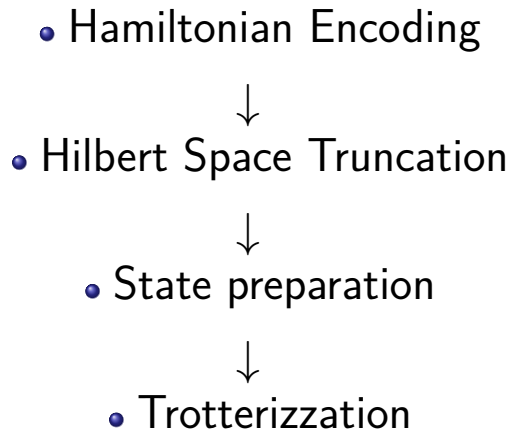
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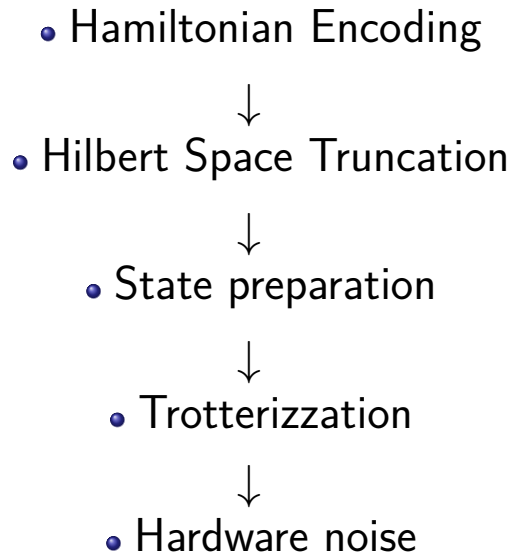
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- Hamiltonian Encoding
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- State preparation

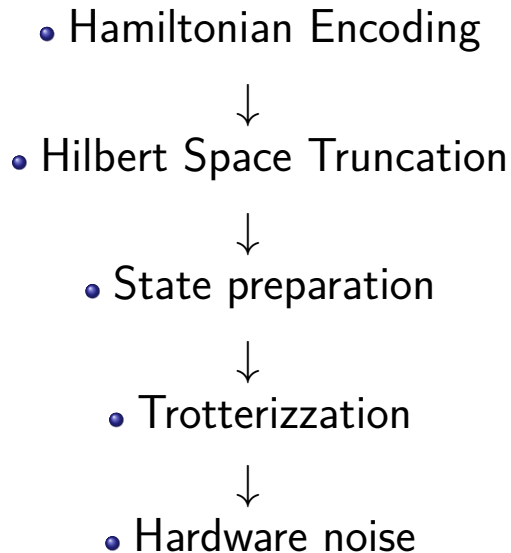
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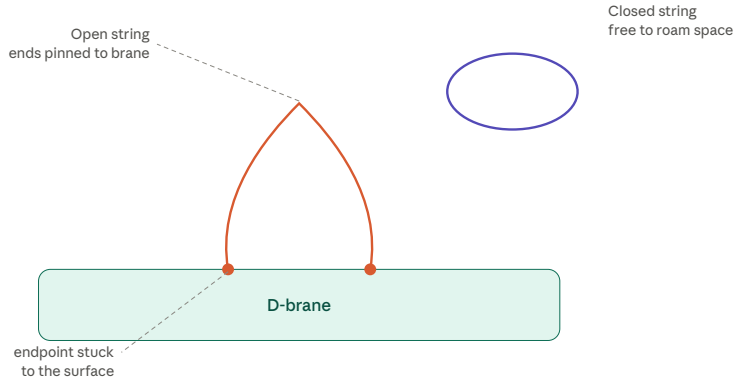


Each step introduces error which we need to address!

Toward D-brane

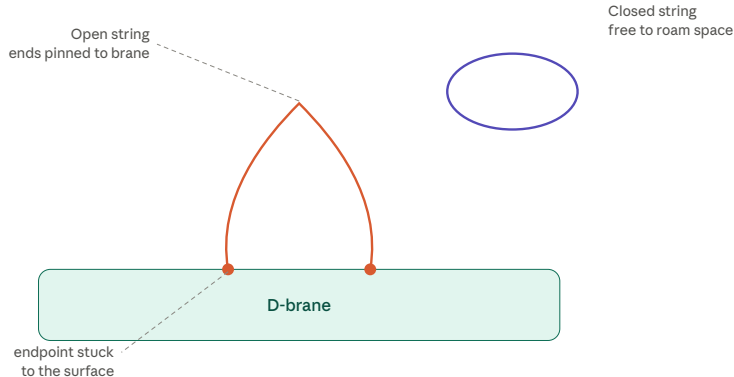
What D-brane are and why are they important?

D-branes are fundamental objects in string theory that link gravity and quantum field theory. D p -branes have p spatial dimensions: D0 (point), D1 (string), D2 (membrane), and so on.



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- Microscopic derivation of the Bekenstein–Hawking black hole entropy (Strominger & Vafa)
- Foundation of the AdS/CFT correspondence (Maldacena)
- Recognition of D-branes as fundamental objects, paving the way toward M-theory (Polchinski)
- Applications to string phenomenology and brane-world cosmology

How D0-brane are connected with a 2-boson model

Banks-Fischler-Shenker-Susskind (BFSS) matrix model describes the dynamics of interacting D0-brane

$$H = \text{Tr} \left[\frac{1}{2} P_i^2 - \frac{1}{4} [X^i, X^j]^2 \right]$$

where $X^i(t)$ and $P_i(t)$ are $N \times N$ Hermitian matrices satisfying $[X^i, P_j] = i\delta_{ij}$.

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In the regime of well-separated D0-branes, where ℓ_s is the fundamental string length scale:

$$|x_a - x_b| \gg \ell_s.$$

we can split the matrices as $X_{ab}^i = x_a^i \delta_{ab} + Y_{ab}^i$, assuming $|Y_{ab}^i| \ll |x_a^i - x_b^i|$

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For two D0-branes ($N = 2$), expanding and keeping relevant terms, BFSS becomes:

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{1}{2} x^2 y^2$$

Conclusions

What we have seen:

- A two-boson model with **very rich physics**: Classical Vs Quantum dynamics, quantum chaos etc
- It is a **toy-model for Matrix Model and D0-brane** dynamics on a quantum hardware
- Preliminary results show the formation of a bound state a "D0-black-hole".

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Ongoing work:

- **Quantify the errors** from Encoding, Truncation, State preparation, Trotterization and Hardware noise
- Run simulations on **gate based quantum hardware**.
- Explore the use of **trapped ion quantum computer**, it should be more natural and easier!
- Extend the study to systems with **multiple bosons (D0-particles)**

Thank you all for your time!

Time for Questions!

Backup Slides

Preliminary results: Previous Slide with $\langle p \rangle$

