Update on $B_s \to K\ell\nu$ and $B \to \pi\ell\nu$ semileptonic decay form factors from the ALPHA Collaboration

Antonino D'Anna,
A. Conigli, P. Fritzsch, A. Gérardin, J. Heitger, G. Herdoíza,
N. Husung, S. Kuberski, C. Pena, H. Simma

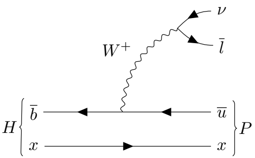
Instituto de Física Teórica UAM - CSIC

ALPHA Collaboration

Lattice 2026, University of Maryland, College Park, July 30

EXCELENCIA SEVERO OCHOA · MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES · Cofinanciado por la Unión Europea · AGENCIA ESTATAL DE INVESTIGACIÓN · CSIC Consejo Superior de Investigaciones Científicas · ift Instituto de Física Teórica UAM-CSIC · UAM Universidad Autónoma de Madrid

$B \to \pi, B_s \to K$ semileptonic decay form factors



$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |\eta_{\mathrm{EW}}|^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 |\mathbf{p}_P|}{q^4} \times \left[\left(1 + \frac{m_l^2}{2q^2}\right) |\mathbf{p}_P|^2 |f_+(q^2)|^2 + \frac{3m_l^2}{8q^2} \frac{(m_H^2 - m_P^2)^2}{m_H^2} |f_0(q^2)|^2 \right]$$

where $P = \pi, K,\ H = B, B_s,\ q = p_H - p_P$

$$\langle P|\hat{V}_\mu|H\rangle = f_+(q^2)\left(p_{H,\mu} + p_{P,\mu} - \frac{m_H^2 - m_P^2}{q^2} q_\mu\right) + f_0(q^2) \frac{m_H^2 - m_P^2}{q^2} q_\mu$$

We use the HQET[[1501.03060]] parametrization:

$$\langle P|\hat{V}_\mu|H\rangle = \sqrt{2m_H}(v^\mu h_\parallel(E_P) + p_\perp^\mu h_\perp(E_P))$$

where v^μ is the H-meson 4-velocity and $p_\perp^\mu = p_P^\mu - (p_P \cdot v)v^\mu$.

Strategy

Aim: Form factors at $m_h = m_b$ and $L \to \infty$

Strategy [2312.09811, PoS(EuroPLEx2023)005, 2603.25295] :

- Interpolating to the b-sector using static and relativistic theories via suitable observables $\to$ Neither matching nor renormalization to the HQET is required

$$\ln\left(L_{\text{ref}}^{1/2} h_{\parallel}(E_P)\right) = \ln\left(L_{\text{ref}}^{1/2} h_{\parallel}(m_P)\right) + \ln\left(\frac{h_{\parallel}(E_P)}{h_{\parallel}(m_P)}\right)$$

$$= \Phi_{\parallel} + \tau_{\parallel}(E_P)$$

$$\ln\left(E_P L_{\text{ref}}^{1/2} h_{\perp}(E_P)\right) = \ln\left(L_{\text{ref}}^{3/2} \hat{f}_{B^*}\right) + \ln\left(\frac{E_P h_{\perp}(E_P)}{L_{\text{ref}} \hat{f}_{B^*}}\right)$$

$$= \Phi_{\perp} + \tau_{\perp}(E_P)$$

where $L_{\text{ref}} = f_P^{-1}$ and

$$\hat{f}_{B^*} = \sqrt{2}\langle 0|V_k(0)|B^*(\mathbf{0}, k)\rangle$$

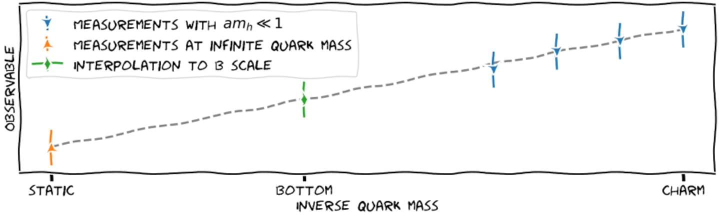
Strategy

Aim: Form factors at $m_h = m_b$ and $L \to \infty$

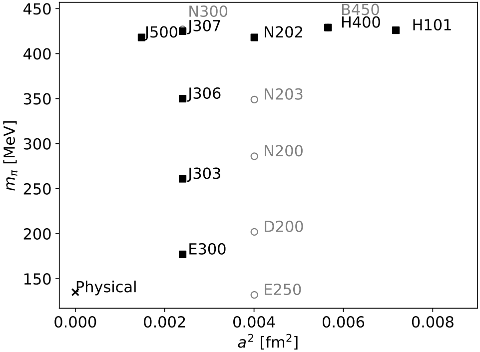
Strategy [2312.09811, PoS(EuroPLEx2023)005, 2603.25295] :

- Interpolating to the b-sector using static and relativistic theories via suitable observables → Neither matching nor renormalization to the HQET is required ($X = \perp, \parallel$)

$$\tau_X(E_P) = \tau_X^{\text{stat}}(E_P) + \mathcal{O}\left(m_b^{-1}\right)$$



Relativistic observables on CLS ensembles [1411.3982]

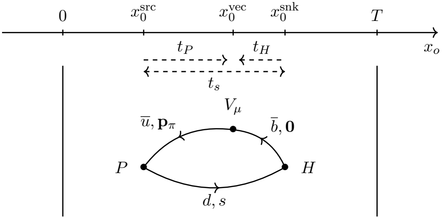


- $m_c^{\rm RGI} \lesssim m_h^{\rm RGI} \lesssim 0.64 m_b^{\rm RGI}$
- $0.039 \text{ fm} \leq a \leq 0.085 \text{ fm}$
- $\mathrm{Tr}\, M_q \approx \mathrm{Tr}\, M_q^{\rm phys}$

- $N_f = 2+1$, $\mathrm{O}(a)$-improved Wilson Fermions [Nucl. Phys. B 874 (2013) 188]
- Non-perturbative renormalization and $\mathrm{O}(a)$ improvement [2010.09539, 2510.06869, 1808.09236]
- Open Boundary Conditions in time for most ensembles
- Continuum limit on SU(3) symmetric point at $m_\pi = m_K \approx 410$ MeV
- approach to physical pion at $a = 0.05$ fm

Extracting the matrix element

$$C_\mu(t_H, t_P, \mathbf{p}_P) \underset{0 \ll t_H, t_P \ll t_s}{\longrightarrow} \frac{\alpha_P \langle P(\mathbf{p}_P)|V_\mu|H(\mathbf{0})\rangle \beta_H}{4E_P m_H} e^{-E_P t_P - m_H t_H}.$$



- Up to 6 different $t_s = x_0^{\mathrm{snk}} - x_0^{\mathrm{src}}$, with $1.36 \text{ fm} \lesssim t_s \lesssim 3.2 \text{ fm}$
- The insertion time of the current, x_0^{vec}, varies.
- $t_P + t_H = t_s$
- Vector current non-perturbatively O(a)-improved [2010.09539]

Extracting the matrix element

$$C_\mu(t_H, t_P, \mathbf{p}_P) \xrightarrow[0 \ll t_H, t_P \ll t_s]{} \frac{\alpha_P \langle P(\mathbf{p}_P)|V_\mu|H(\mathbf{0})\rangle \beta_H}{4E_P m_H} e^{-E_P t_P - m_H t_H}.$$

We isolate the matrix element employing the ratio:

$$\mathcal{R}^I_\mu = \sqrt{4E_P m_H} \frac{C_\mu(t_H, t_P, \mathbf{p}_P)}{\sqrt{C^{\mathrm{gs}}_H(t_H, \mathbf{0}) C^{\mathrm{gs}}_P(t_P, \mathbf{p}_P)}} e^{\frac{1}{2}E_P t_P + \frac{1}{2} m_H t_H}$$

The form factors are given by:

$$\begin{aligned} p^k_P h_\perp(E_P) =& Z_V \frac{\langle P(\mathbf{p}_P)|V_k|H(\mathbf{0})\rangle}{\sqrt{2m_H}} = \frac{Z_V}{\sqrt{2m_H}} \lim_{0 \ll t_H, t_P \ll t_s} \mathcal{R}^I_k(t_H, t_P, \mathbf{p}_P) \\ h_\parallel(E_P) =& Z_V \frac{\langle P(\mathbf{p}_P)|V_0|H(\mathbf{0})\rangle}{\sqrt{2m_H}} = \frac{Z_V}{\sqrt{2m_H}} \lim_{0 \ll t_H, t_P \ll t_s} \mathcal{R}^I_0(t_H, t_P, \mathbf{p}_P) \end{aligned}$$

Summed Ratio [1108.3774]

Signal-to-Noise problem and the excited state contamination → ground state hard to resolve:

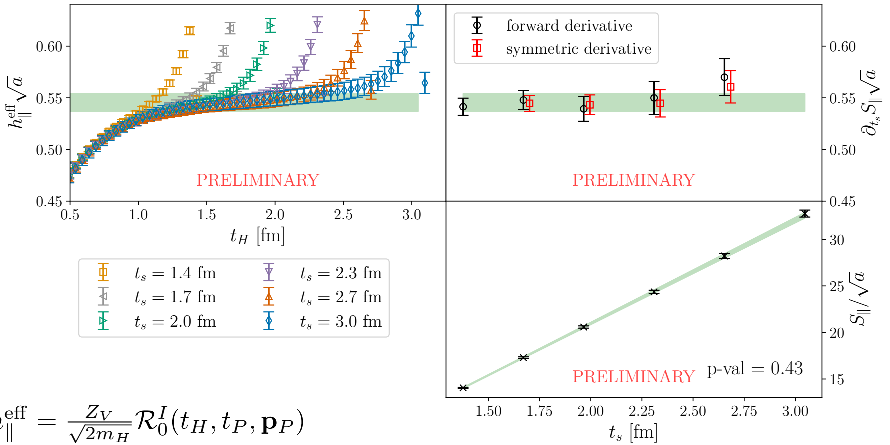
$$S^I_\perp(t_s) = \frac{Z_V}{p^k_P\sqrt{2m_H}} a \sum_{t_H=0}^{t_s-a} \mathcal{R}^I_k(t_H, t_s - t_H, \mathbf{p}_P) = K + h_\perp t_s + \text{excited states},$$

where K is a constant. Similarly, we define $S_\parallel$. The form factors can be extracted by ($X = \perp, \parallel$)

$$\partial_{t_s} S^I_X(t_s) = h_X(E_P) + \mathrm{O}\left(\exp(-\Delta t_s)\right)$$

with $\Delta = \min\{\Delta_H, \Delta_P\}$ is the energy gap between the ground state and the first excited state.
Alternatively, h_X can be extracted from the slope of a linear fit to $S^I_X(t_s)$.

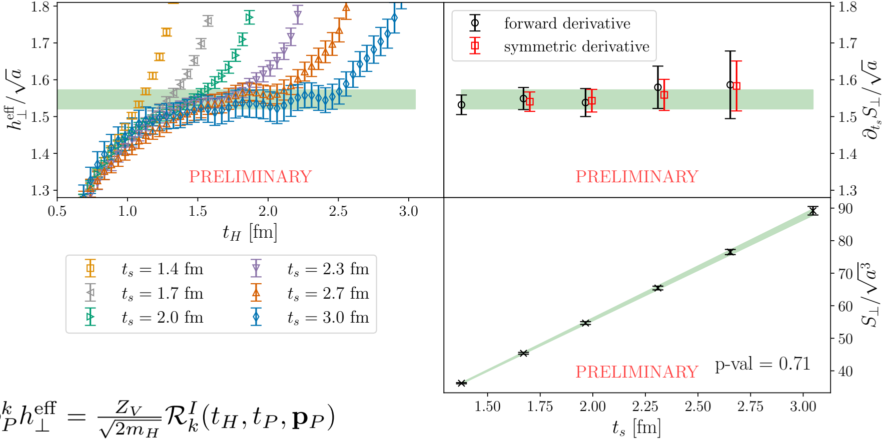
Summed Ratio [1108.3774]



$$
\begin{aligned}
h_\parallel^{\mathrm{eff}} &= \frac{Z_V}{\sqrt{2m_H}} \mathcal{R}_0^I(t_H, t_P, \mathbf{p}_P) \\
&= Z_V \sqrt{2E_P}\, \frac{C_0(t_H, t_P, \mathbf{p}_P)}{\sqrt{C_H^{\mathrm{gs}}(t_H, \mathbf{0})\, C_P^{\mathrm{gs}}(t_P, \mathbf{p}_P)}}\, e^{\frac{1}{2}E_P t_P + \frac{1}{2} m_H t_H}
\end{aligned}
$$

J307: $a = 0.049$ fm, $m_K = m_\pi = 422$ MeV, $m_h^{\mathrm{RGI}} \approx 0.22 m_b^{\mathrm{RGI}}$, $|\mathbf{p}_P| = 0$ MeV

Summed Ratio [1108.3774]



$$p_P^k h_\perp^{\rm eff} = \frac{Z_V}{\sqrt{2m_H}} \mathcal{R}_k^I(t_H, t_P, \mathbf{p}_P)$$
$$= Z_V \sqrt{2E_P} \frac{C_k(t_H, t_P, \mathbf{p}_P)}{\sqrt{C_H^{\rm gs}(t_H, \mathbf{0}) C_P^{\rm gs}(t_P, \mathbf{p}_P)}} e^{\frac{1}{2}E_P t_P + \frac{1}{2} m_H t_H}$$

J307: $a = 0.049$ fm, $m_K = m_\pi = 422$ MeV, $m_h^{\rm RGI} \approx 0.22 m_b^{\rm RGI}$, $|\mathbf{p}_P| = 480$ MeV

B^*P states: Static limit

The excited state contribution to the form factors can be estimated using HMχPT [2306.02703]

$$h_\perp^{\text{eff}}(t_H, t_P, \mathbf{p}_P) = h_\perp(E_P)\Big(1 + \Delta h_\perp(t_H, t_P, \mathbf{p}_P)\Big)$$

where in the static limit we have [A. Broll, PhD thesis, 2023]

$$\Delta h_\perp(t_H, t_P, \mathbf{p}_P) = c_0^{\text{stat}} e^{-\Delta E^{\text{stat}}\, t_H} = -\frac{1 + \beta_1 E_P/g}{1 - \beta_1 E_P/g} e^{-E_P t_H}$$

with $g = 0.492(29)$[1404.6951] a LO LEC, and $\beta_1 = 0.20(4)\ \text{GeV}^{-1}$ is a NLO LEC. [A. Gérardin, Lattice 2024]

B^*P states: Beyond the static limit

Beyond the static limit, an expression for $\Delta h_\perp$ can be derived based on covariant Baryon χPT results [S. Szyperski, O. Bär, *in preparation*], by mapping the nucleon sector onto the heavy-light one ($N \to H$, $N\pi \to B^*\pi$, $g_A \to g$):

$$\Delta h_\perp(t_H, t_P, \mathbf{p}_P) = c_0^{\text{rel}} e^{-\Delta E^{\text{rel}}\, t_H}$$

where $\Delta E^{\text{rel}}(\mathbf{p}_P) = E_P(\mathbf{p}_P) + E_{B^*}(\mathbf{p}_P) - m_H$ and

$$c_0^{\text{rel}}(\mathbf{p}_P) = -\frac{\left(1 - \frac{E_P + E_{B^*} - m_H}{g(E_P + E_{B^*} + m_H)} + \frac{\beta_1}{g}\left(E_P + E_{B^*} - m_H\right)\right)}{\left(1 - 2\frac{\beta_1}{g} E_{B^*} \frac{E_P + E_{B^*} - m_H}{E_P + E_{B^*} + m_H}\right)}$$

- Reproduces the static HMχPT result [2306.02703, A. Broll, PhD thesis, 2023] for $m_H \to \infty$.
- the larger gap and the reduced $|c_0|$ suppress the excited-state contamination with respect to the static result

$B^*\pi$ states in $B \to \pi\ell\nu$

The excited state contribution to the form factors can be estimated using HMχPT

$$h_\perp^{\text{eff}}(t_H, t_\pi, E_\pi) = h_\perp(E_\pi)\left(1 + c_0^{\text{rel}} e^{-\Delta E^{\text{rel}}\, t_H}\right),$$
$$S_\perp(t_s) = K_\perp + h_\perp(E_\pi)\left(t_s - a c_0^{\text{rel}} \frac{e^{-\Delta E^{\text{rel}} t_s}}{1 - e^{-a\Delta E^{\text{rel}}}}\right).$$

Taking a derivative with respect to t_s

$$h_\perp^{\text{sub}}(E_\pi, t_s) = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + a c_0^{\text{rel}} \Delta E^{\text{rel}} \frac{\exp(-\Delta E^{\text{rel}} t_s)}{1 - \exp(-a\Delta E^{\text{rel}})}} = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + \delta h_\perp^{\text{sub}}}$$

At $t_s \approx 2.0$ fm,at $m_\pi = 135$ MeV and $|\mathbf{p}_\pi| = 480$ MeV

- Physical charm we have a $\delta h_\perp^{\text{sub}} \sim -0.1\%$ effect.
- Physical bottom we have a $\delta h_\perp^{\text{sub}} \sim -0.5\%$ effect.

$B^*\pi$ states in $B \to \pi\ell\nu$

The excited state contribution to the form factors can be estimated using HMχPT

$$h_\perp^{\rm eff}(t_H, t_\pi, E_\pi) = h_\perp(E_\pi)\Big(1 + c_0^{\rm rel} e^{-\Delta E^{\rm rel}\, t_H}\Big),$$
$$S_\perp(t_s) = K_\perp + h_\perp(E_\pi)\left(t_s - a c_0^{\rm rel}\frac{e^{-\Delta E^{\rm rel} t_s}}{1 - e^{-a\Delta E^{\rm rel}}}\right).$$

Taking a derivative with respect to t_s

$$h_\perp^{\rm sub}(E_\pi, t_s) = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + a c_0^{\rm rel}\Delta E^{\rm rel}\frac{\exp(-\Delta E^{\rm rel} t_s)}{1-\exp(-a\Delta E^{\rm rel})}} = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + \delta h_\perp^{\rm sub}}$$

At $t_s \approx 2.0$ fm, $m_h^{\rm RGI} \approx 0.22 m_b^{\rm RGI}$ and $|\mathbf{p}_\pi| = 480$ MeV

- $m_\pi = 422$ MeV $\to$ we have a $\delta h_\perp^{\rm sub} \sim -0.04\%$ effect.
- $m_\pi = 176$ MeV $\to$ we have a $\delta h_\perp^{\rm sub} \sim -0.11\%$ effect.

$B^*\pi$ states in $B \to \pi\ell\nu$

The excited state contribution to the form factors can be estimated using HMχPT

$$h_\perp^{\mathrm{eff}}(t_H, t_\pi, E_\pi) = h_\perp(E_\pi)\left(1 + c_0^{\mathrm{rel}} e^{-\Delta E^{\mathrm{rel}}\, t_H}\right),$$
$$S_\perp(t_s) = K_\perp + h_\perp(E_\pi)\left(t_s - a c_0^{\mathrm{rel}} \frac{e^{-\Delta E^{\mathrm{rel}} t_s}}{1 - e^{-a\Delta E^{\mathrm{rel}}}}\right).$$

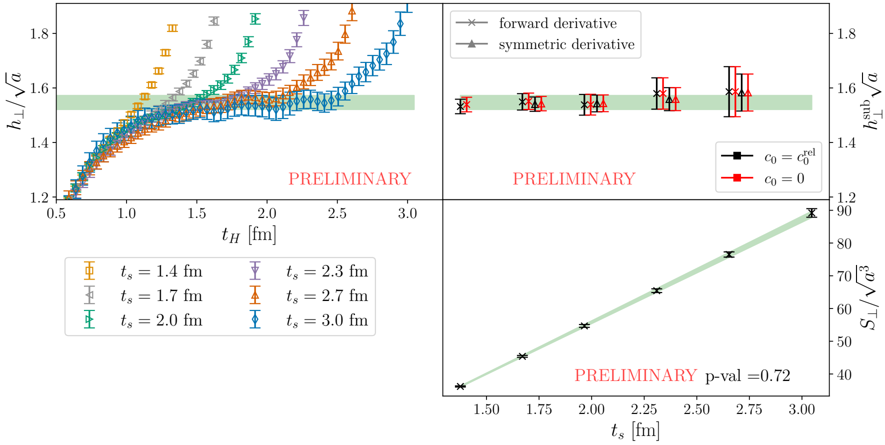
Taking a derivative with respect to t_s

$$h_\perp^{\mathrm{sub}}(E_\pi, t_s) = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + a c_0^{\mathrm{rel}} \Delta E^{\mathrm{rel}} \frac{\exp(-\Delta E^{\mathrm{rel}} t_s)}{1 - \exp(-a\Delta E^{\mathrm{rel}})}} = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + \delta h_\perp^{\mathrm{sub}}}$$

At $t_s \approx 2.0$ fm, $m_h^{\mathrm{RGI}} \approx 0.64 m_b^{\mathrm{RGI}}$ and $|\mathbf{p}_\pi| = 480$ MeV

- $m_\pi = 422$ MeV $\to$ we have a $\delta h_\perp^{\mathrm{sub}} \sim -0.11\%$ effect.
- $m_\pi = 176$ MeV $\to$ we have a $\delta h_\perp^{\mathrm{sub}} \sim -0.26\%$ effect.

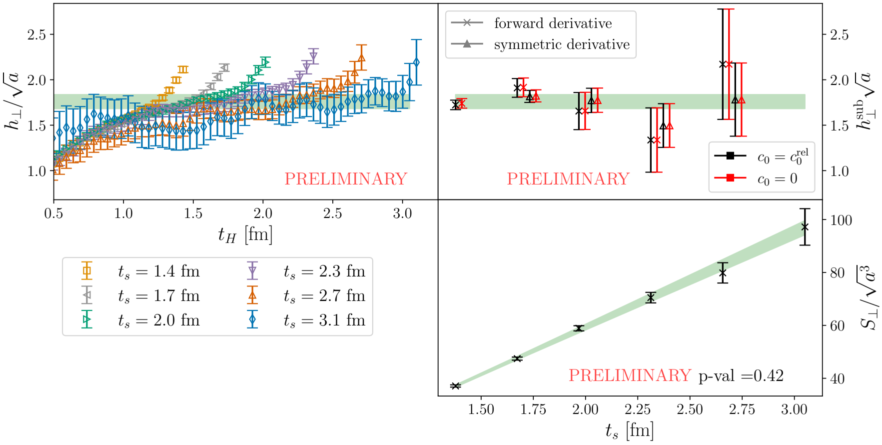
$B^*\pi$ states in $B \to \pi\ell\nu$



J307: $a = 0.049$ fm, $m_\pi = 422$ MeV, $m_h^{\rm RGI} = 0.22 m_b^{\rm RGI}$, $|\mathbf{p}_\pi| = 480$ MeV

$$h_\perp^{\rm sub}(E_P, t_s) = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + a c_0 \Delta E^{\rm rel} \frac{\exp(-\Delta E^{\rm rel} t_s)}{1-\exp(-a\Delta E^{\rm rel})}}.$$

$B^*\pi$ states in $B \to \pi\ell\nu$

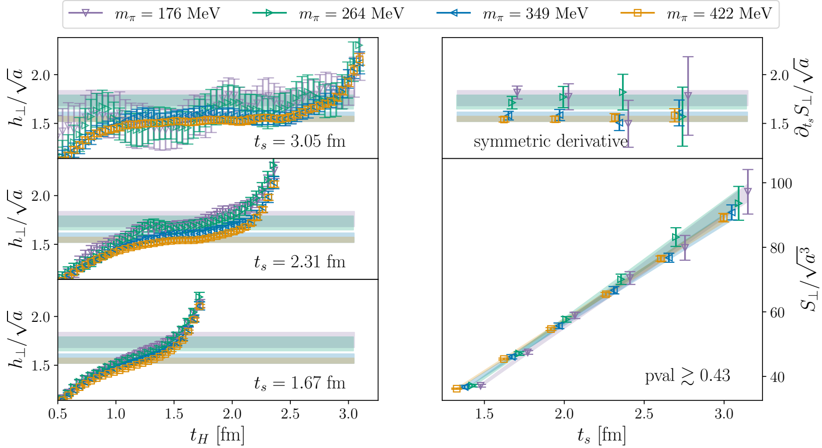


E300: $a = 0.049$ fm, $m_\pi = 176$ MeV, $m_h^{\mathrm{RGI}} = 0.22 m_b^{\mathrm{RGI}}$, $|\mathbf{p}_\pi| = 480$ MeV

$$h_\perp^{\mathrm{sub}}(E_P, t_s) = \frac{\partial S_\perp(t_s)/\partial t_s}{1 + a c_0 \Delta E^{\mathrm{rel}} \frac{\exp(-\Delta E^{\mathrm{rel}} t_s)}{1-\exp(-a\Delta E^{\mathrm{rel}})}}.$$

$B^*\pi$ states in $B \to \pi\ell\nu$

PRELIMINARY

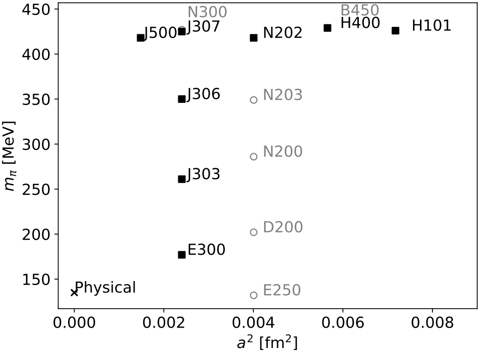


$a = 0.049$ fm, $m_h^{\mathrm{RGI}} = 0.22 m_b^{\mathrm{RGI}}$, $|\mathbf{p}_\pi| = 480$ MeV

Continuum limit: $\tau_\perp$ and $\tau_\parallel$

- Suitable observables to interpolate with HQET static point theory to the physical b-quark mass:

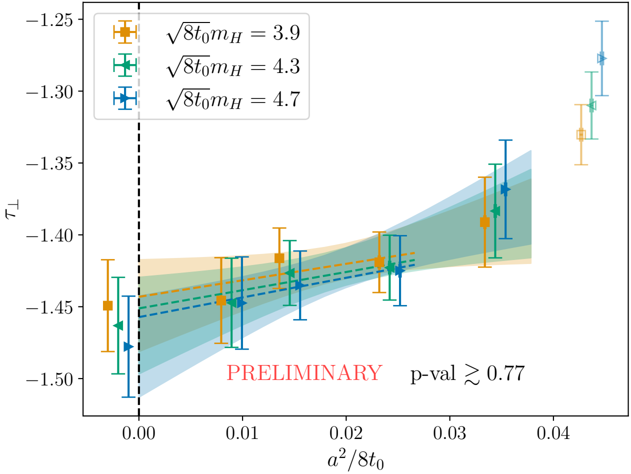
$$\tau_\parallel(E_P) = \ln\left(\frac{h_\parallel(E_P)}{h_\parallel(m_P)}\right), \quad \tau_\perp(E_P) = \ln\left(\frac{E_P h_\perp(E_P)}{L_{\text{ref}}\hat{f}_{B^*}}\right)$$



- Continuum limits on the SU(3)-symmetric point ($m_\pi = m_K \approx 410$ MeV) in range $a = 0.039 - 0.076$ fm.
- $B_s \to K\ell\nu$ and $B \to \pi\ell\nu$ are equivalent

Continuum limit on SU(3)-symmetric point: $\tau_\perp$

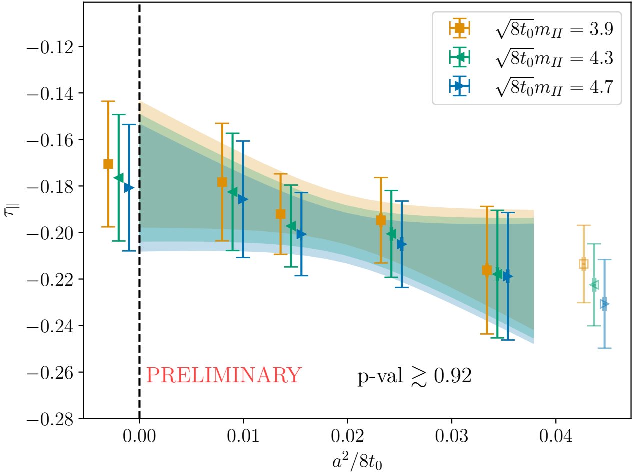
$m_K = m_\pi \approx 410$ MeV



$|\mathbf{p}_P| = 480$ MeV, dashed line: continuum limit in range $a = 0.039 - 0.063$ fm

Continuum limit on SU(3)-symmetric point: $\tau_\parallel$

$m_K = m_\pi \approx 410$ MeV



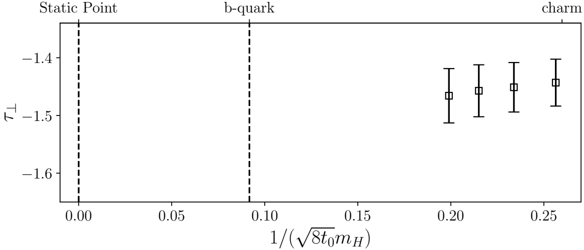
$|\mathbf{p}_P| = 480$ MeV

Conclusion

- Excited states are under theoretical control: in the summation method the $B^*\pi$ contamination is suppressed as $e^{-\Delta E\, t_s}$, with $\Delta E = E_P + E_{B^*} - m_H$. For source-sink separations, $t_s \gtrsim 2.0\,\mathrm{fm}$, the subtraction of $B^*\pi$ from HMχPT leads to a negligible shift of $h_\perp$ at $|\mathbf{p}_\pi| = 480\,\mathrm{MeV}$, down to $m_\pi = 176\,\mathrm{MeV}$. Theory and data agree.
- On the SU(3)-symmetric point, $h_\perp$ and $h_\parallel$ can be extracted with statistical precision of 1.2–2% in the considered kinematic regime.
- First results for the continuum limits of $\tau_\perp$, $\tau_\parallel$: good scaling behaviour in the range $a = 0.039 - 0.076$ fm for various heavy masses at and above the charm sector. We observe a statistical precision of 3–4% on $\exp(\tau_\parallel)$ and $\exp(\tau_\perp)$
- Systematic effects are being further explored in excited-state contamination and continuum limit.

Outlook - 1

- $B_s \to K\ell\nu$ is the first target: better statistical precision and excited states more strongly suppressed since the heavier kaon opens the gap ΔE with, furthermore, ongoing experimental effort on the branching fraction.
- Towards the physical pion mass and the b scale: physical point ensemble in production; interpolation to m_b between relativistic and static data, with step scaling in volume supplying the normalisation Φ_X and thus the form factors.



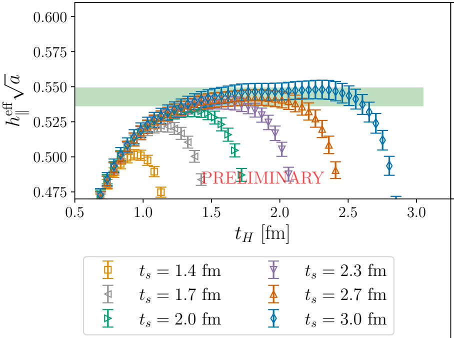
The Static Point will be crucial for the interpolation

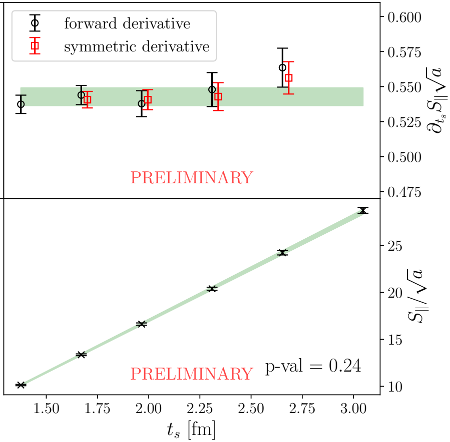
Outlook - 2

- Treatment of multi-hadron states:
  - Computation of HMχPT LECs
  - Test of HMχPT against GEVP with explicit inclusion of $B^*\pi$ interpolating operators.
- Target: $|V_{ub}|$

BACKUP SLIDES

Summed Ratio [1108.3774] with full two-point0 functions

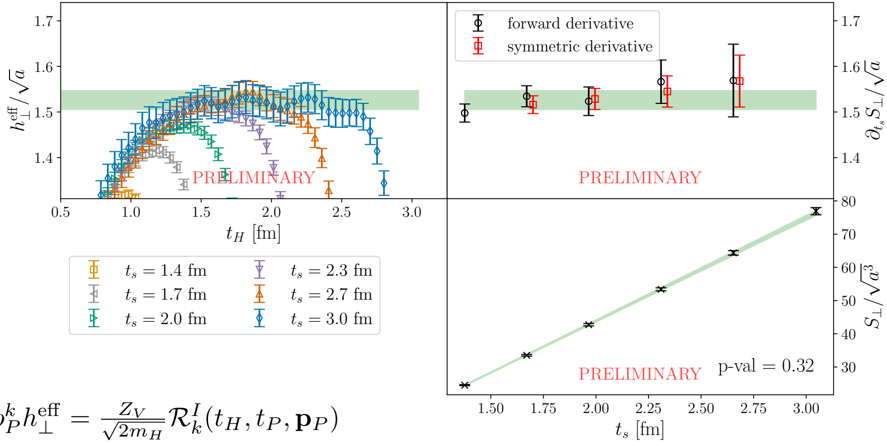




$$
\begin{aligned}
h_\parallel^{\mathrm{eff}} &= \frac{Z_V}{\sqrt{2m_H}} \mathcal{R}_0^I(t_H, t_P, \mathbf{p}_P) \\
&= Z_V \sqrt{2E_P} \frac{C_0(t_H, t_P, \mathbf{p}_P)}{\sqrt{C_H(t_H, \mathbf{0}) C_P(t_P, \mathbf{p}_P)}} e^{\frac{1}{2}E_P t_P + \frac{1}{2} m_H t_H}
\end{aligned}
$$

J307: $a = 0.049$ fm, $m_K = m_\pi = 422$ MeV, $m_h^{\mathrm{RGI}} \approx 0.22 m_b^{\mathrm{RGI}}$, $|\mathbf{p}_P| = 0$ MeV

Summed Ratio [1108.3774] with full two-point functions

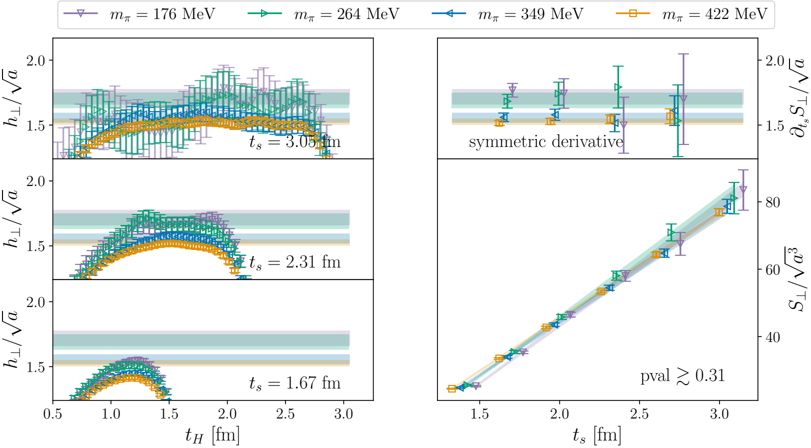


$$p_P^k h_\perp^{\rm eff} = \frac{Z_V}{\sqrt{2m_H}} \mathcal{R}_k^I(t_H, t_P, \mathbf{p}_P)$$
$$= Z_V \sqrt{2E_P} \frac{C_k(t_H, t_P, \mathbf{p}_P)}{\sqrt{C_H(t_H, \mathbf{0}) C_P(t_P, \mathbf{p}_P)}} e^{\frac{1}{2}E_P t_P + \frac{1}{2} m_H t_H}$$

J307: $a = 0.049$ fm, $m_K = m_\pi = 422$ MeV, $m_h^{\rm RGI} \approx 0.22 m_b^{\rm RGI}$, $|\mathbf{p}_P| = 480$ MeV

$B^*\pi$ states in $B \to \pi\ell\nu$ with full two-point functions

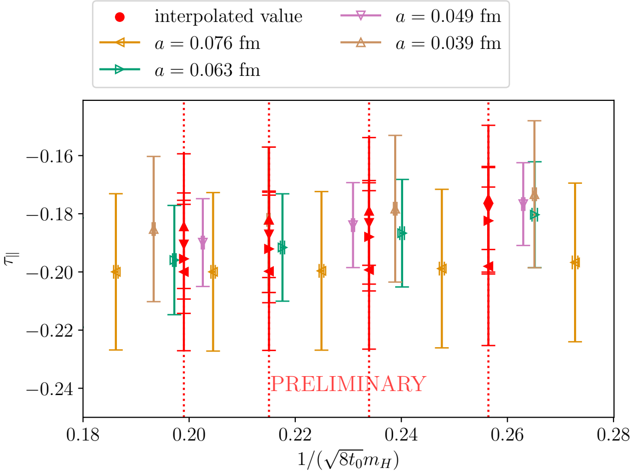
PRELIMINARY



$a = 0.049$ fm, $m_h^{\mathrm{RGI}} = 0.22 m_b^{\mathrm{RGI}}$, $|\mathbf{p}_\pi| = 480$ MeV

Interpolating $\tau_\parallel$ to target m_H on SU(3)-symmetric point

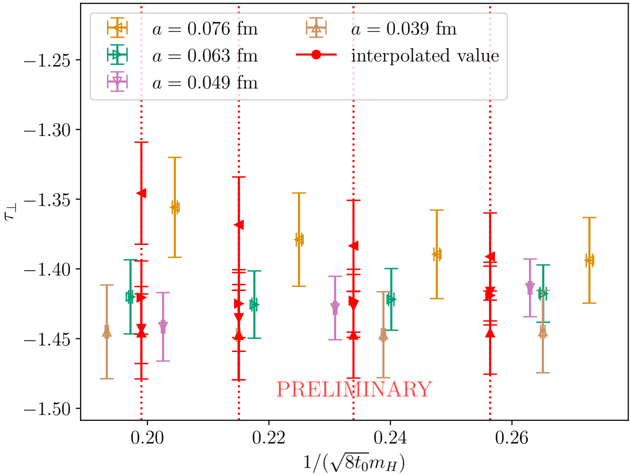
$m_K = m_\pi \approx 410$ MeV



$|\mathbf{p}_\pi| = 480$ MeV

Interpolating $\tau_\perp$ to target m_H on SU(3)-symmetric point

$m_K = m_\pi \approx 410$ MeV



$|\mathbf{p}_\pi| = 480$ MeV