Dimensional Decomposition of Eigenstates in Chiral Anderson Models

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31/07/26, University of Maryland

- Work with

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Andrei Alexandru (GWU)

Ivan Horváth (NPI, Řež)

Frank Lee (GWU)

Nilmani Mathur (TIFR)

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Disorder, Localization and Quantum States

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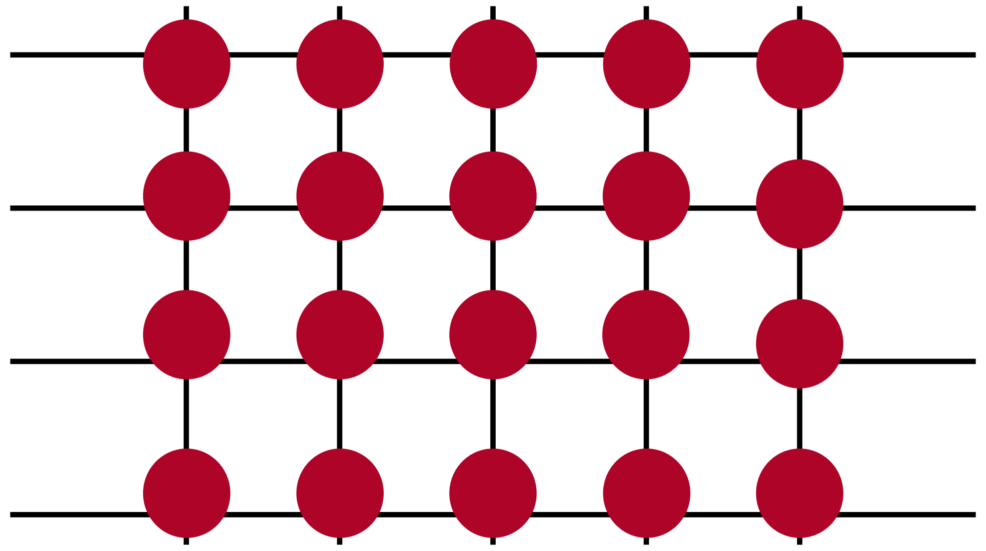
Ordered System



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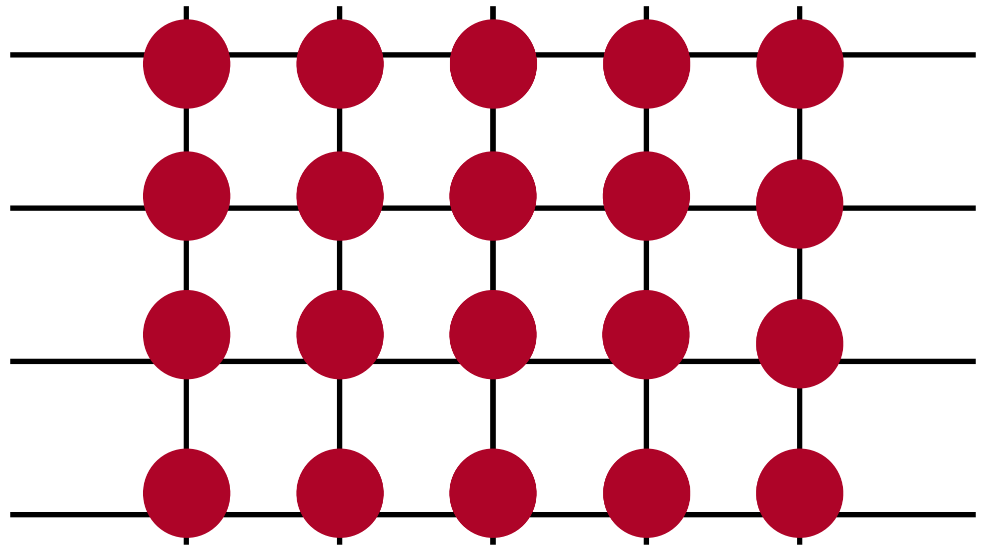

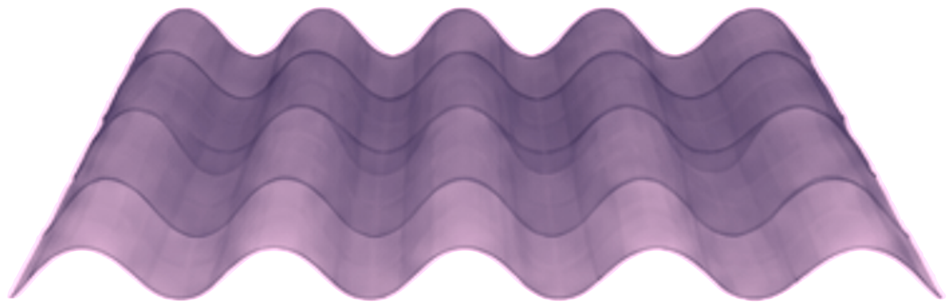

Extended state,
Occupies entire system

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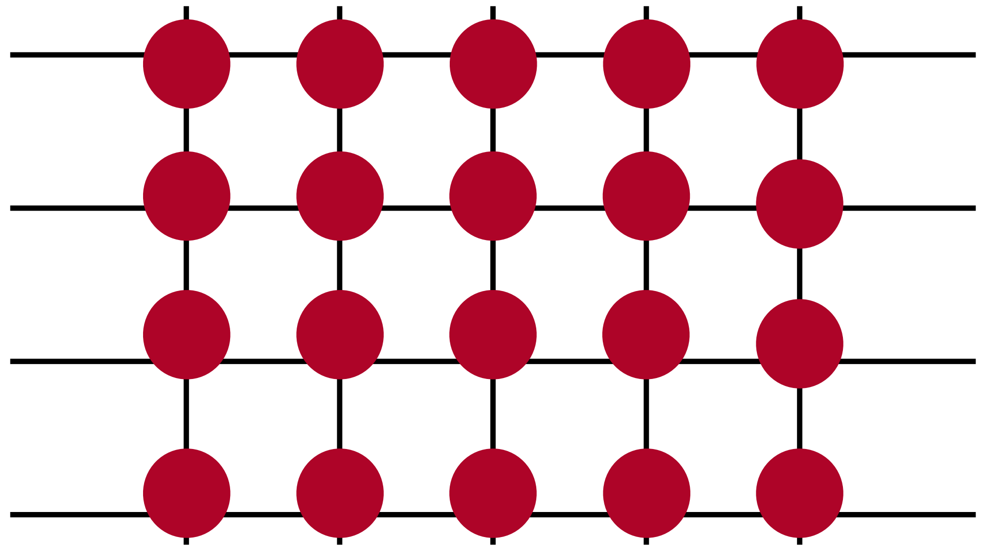

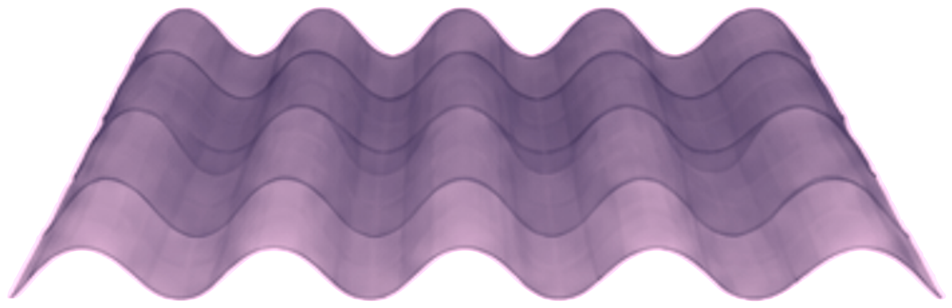

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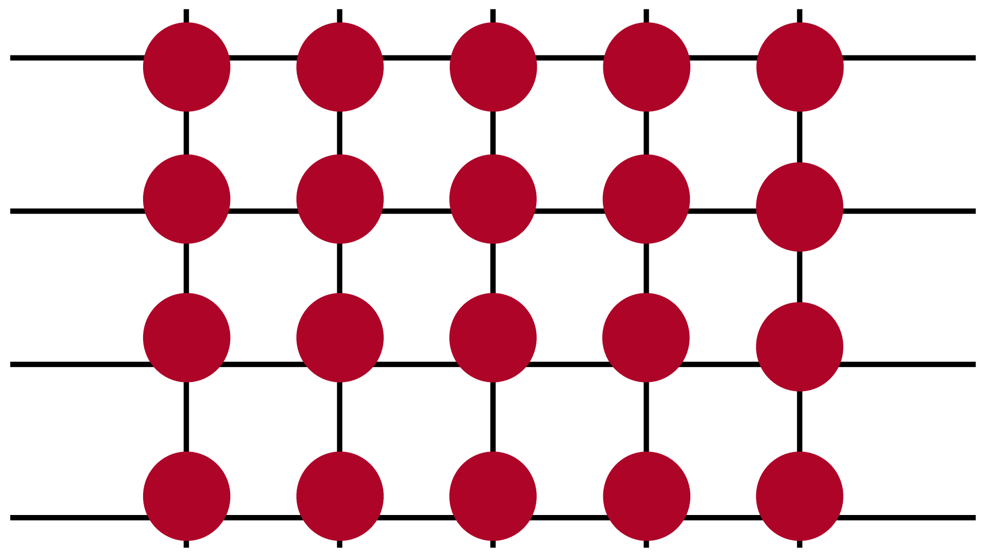

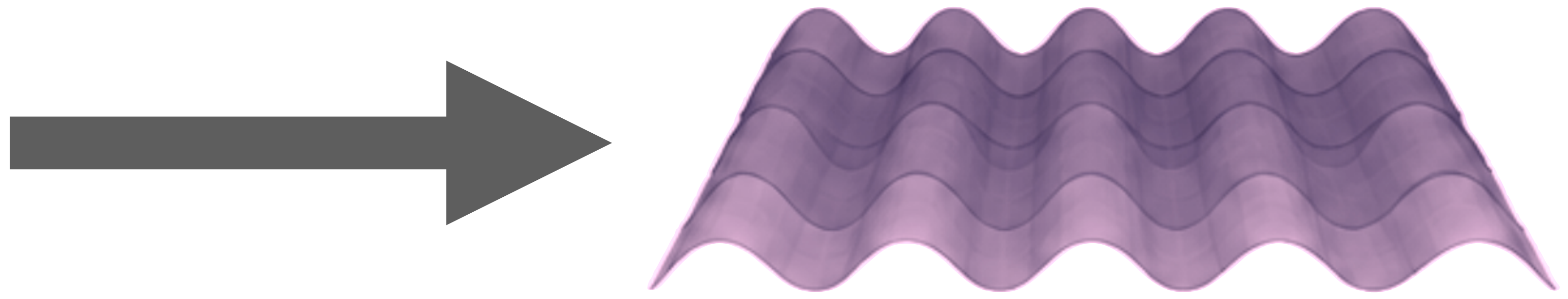

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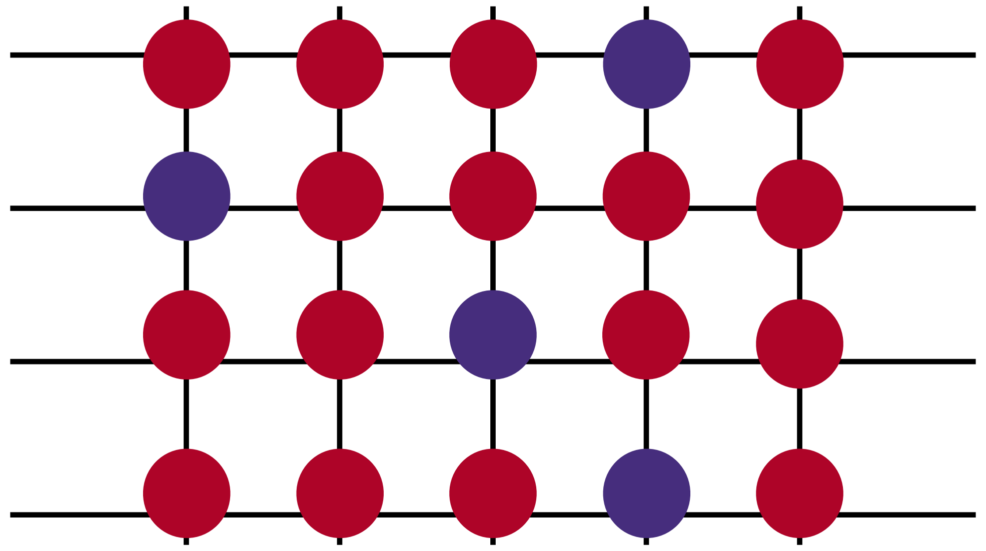

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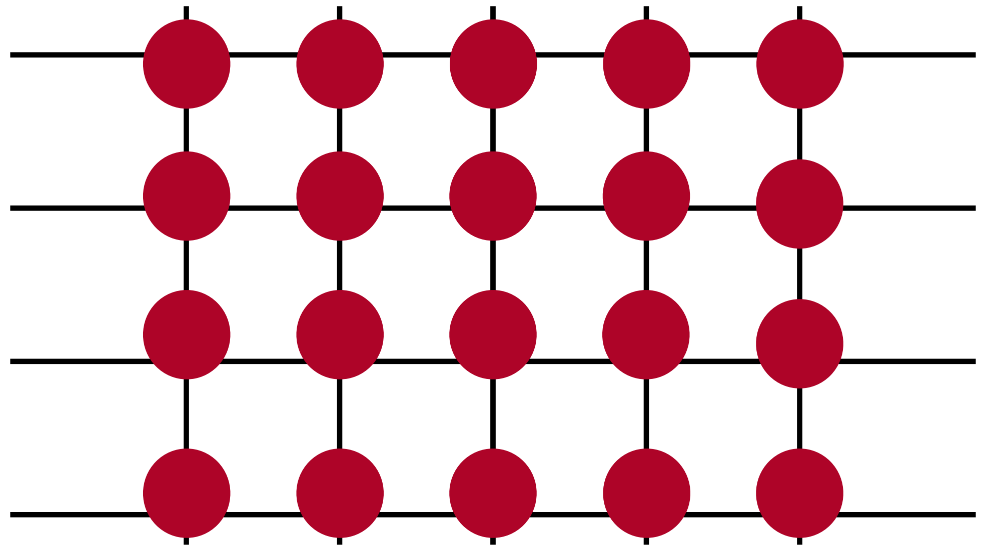

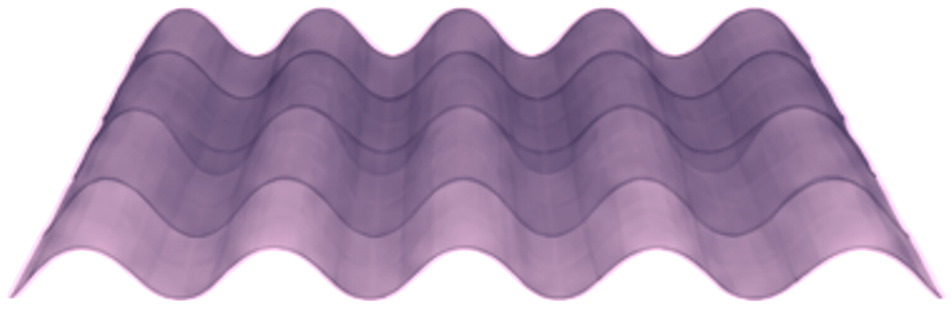

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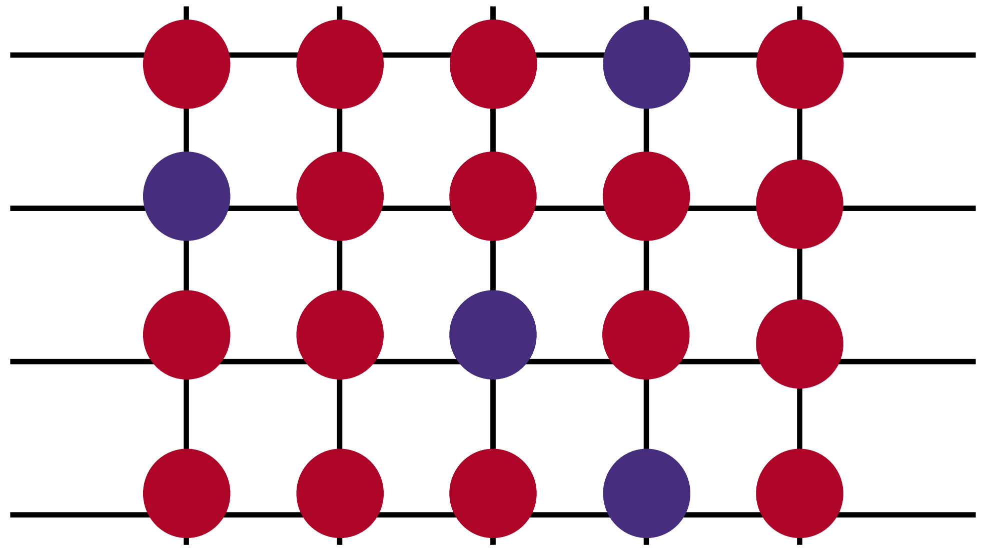

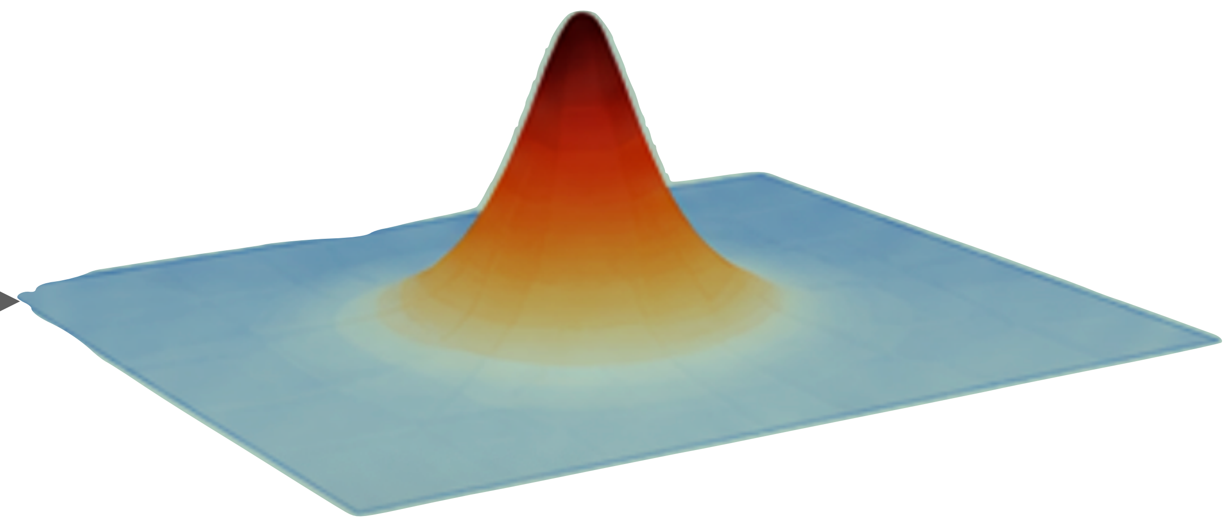

Localized state,
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- Disorder drives a transition between extended and localised states

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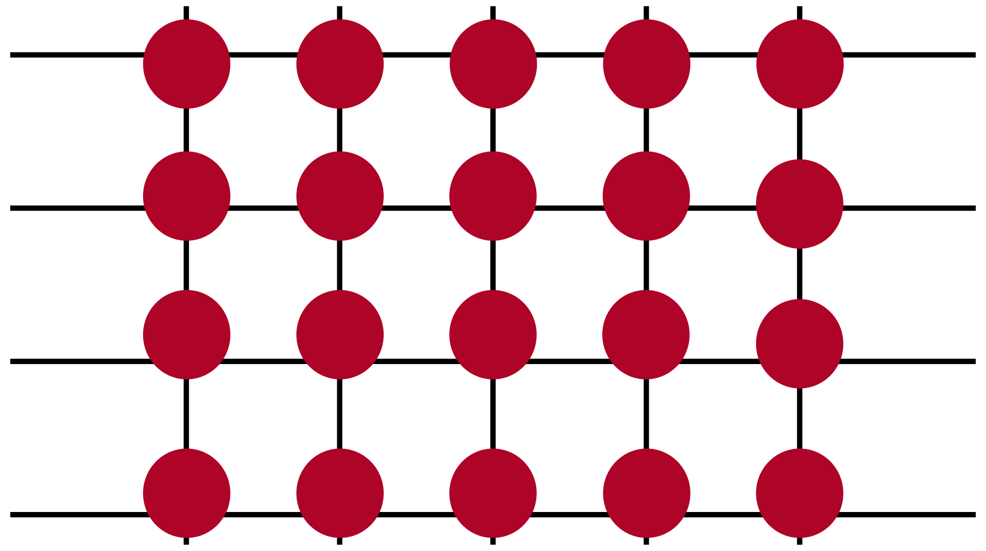

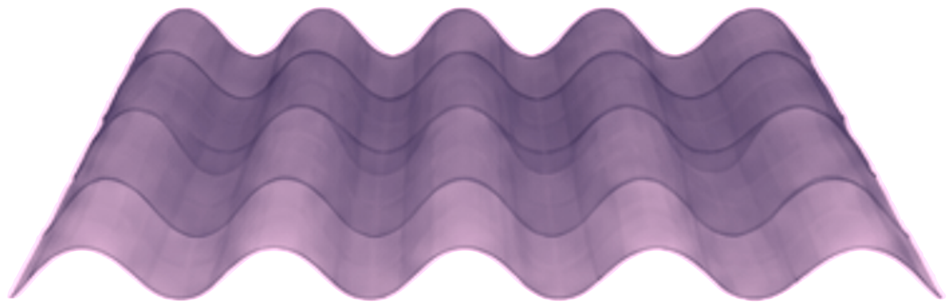

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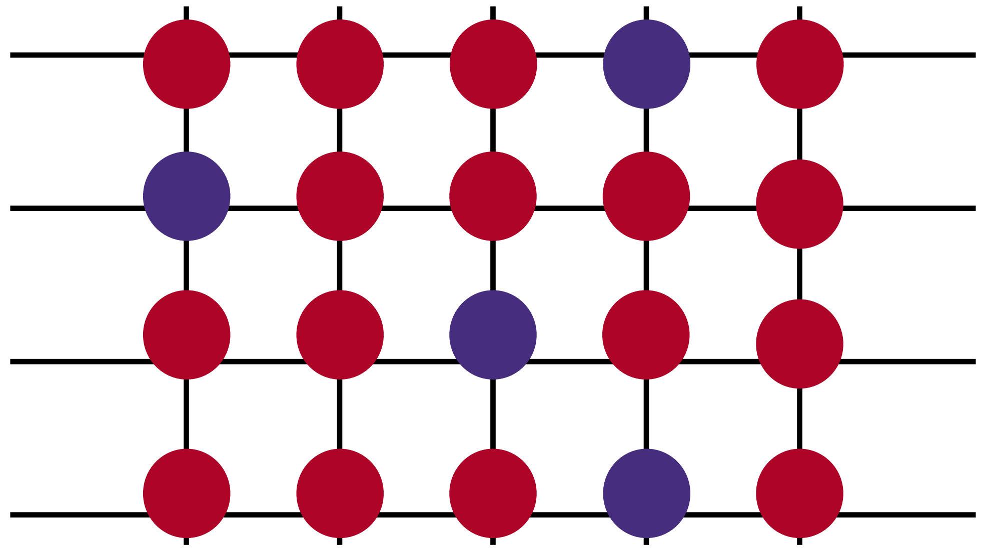

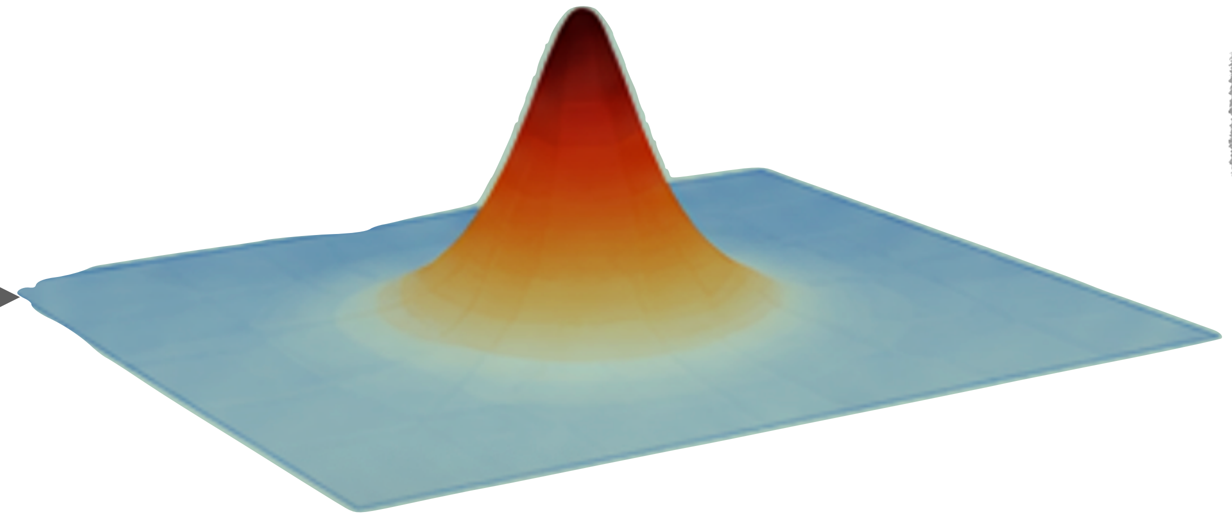

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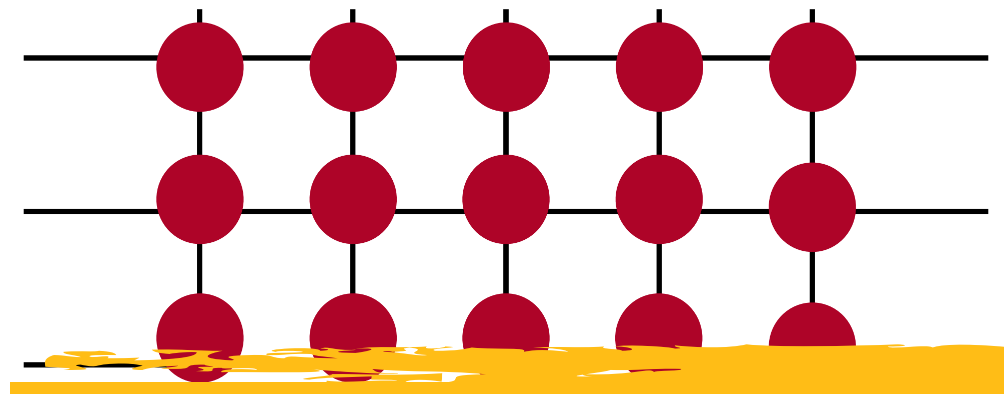

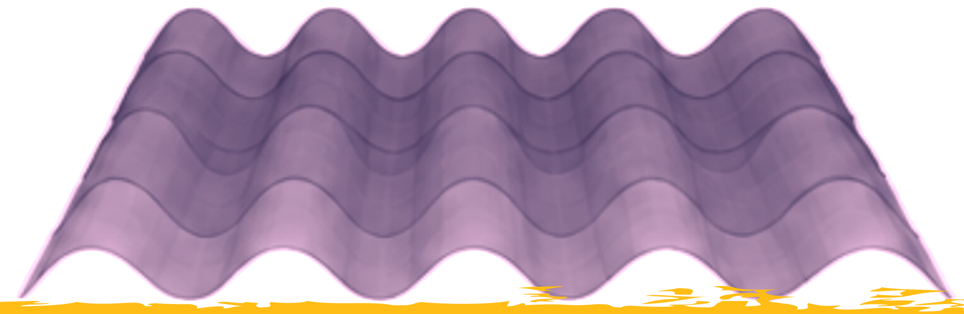

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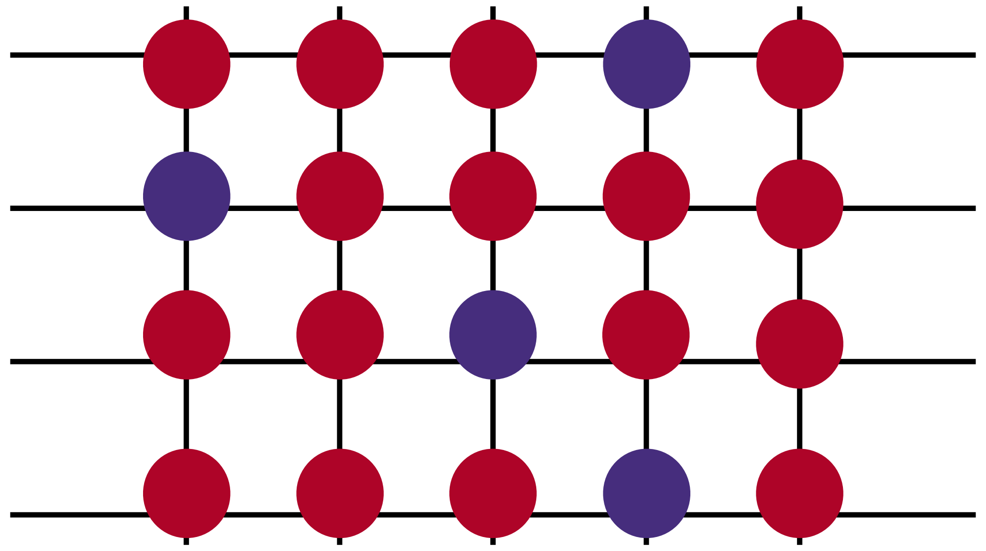
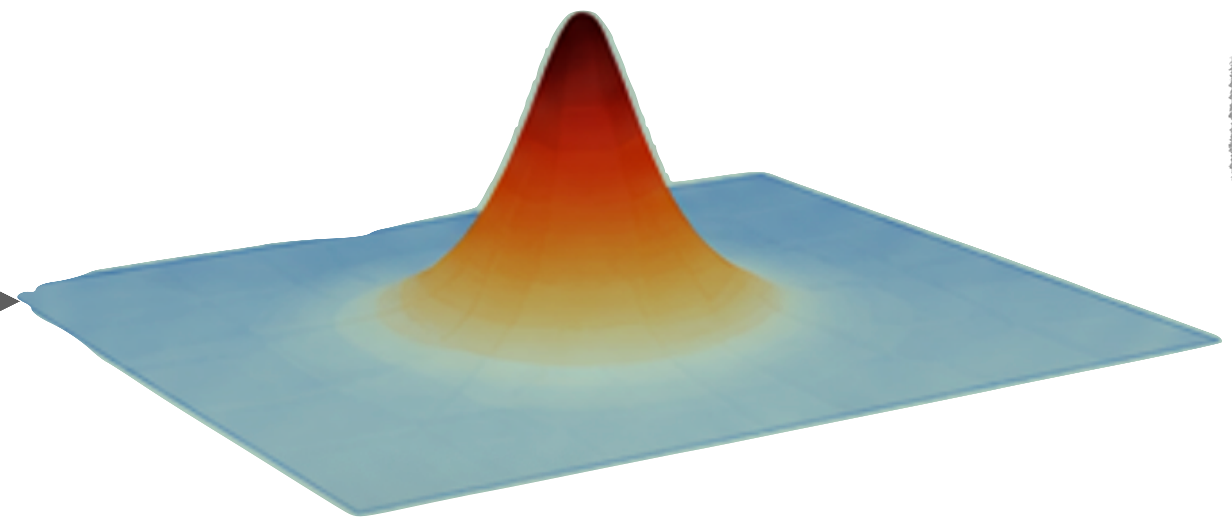
How does a quantum state occupy space?
Can we quantify its effective dimensions?

Random Disordered System





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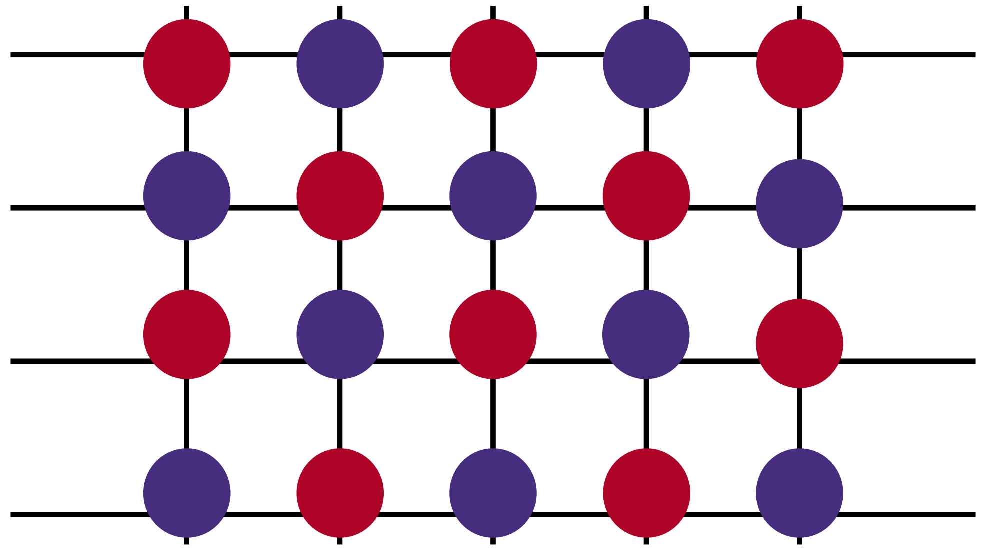
Chiral Symmetry

Bipartite Lattice



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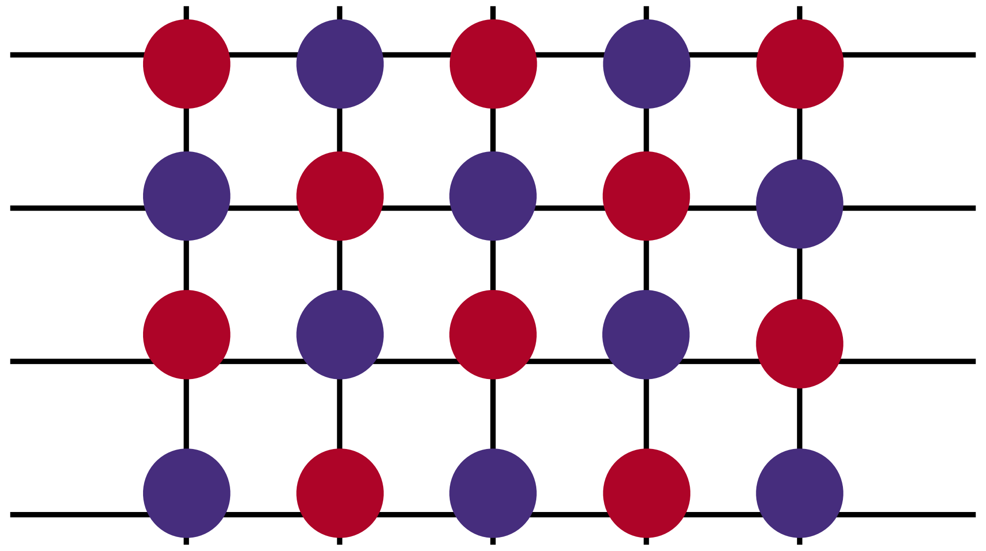

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$$H = \begin{pmatrix} 0 & T \\ T^\dagger & 0 \end{pmatrix}$$

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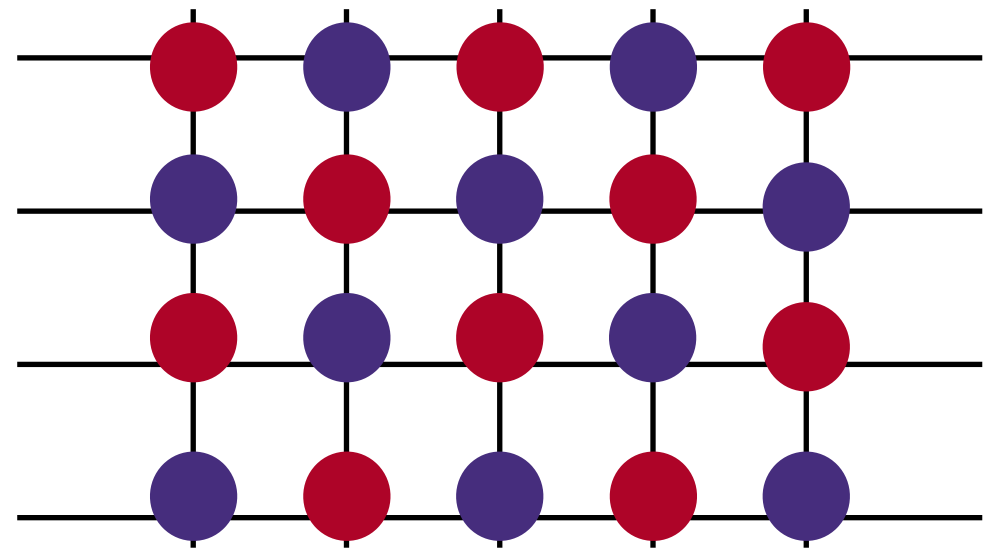

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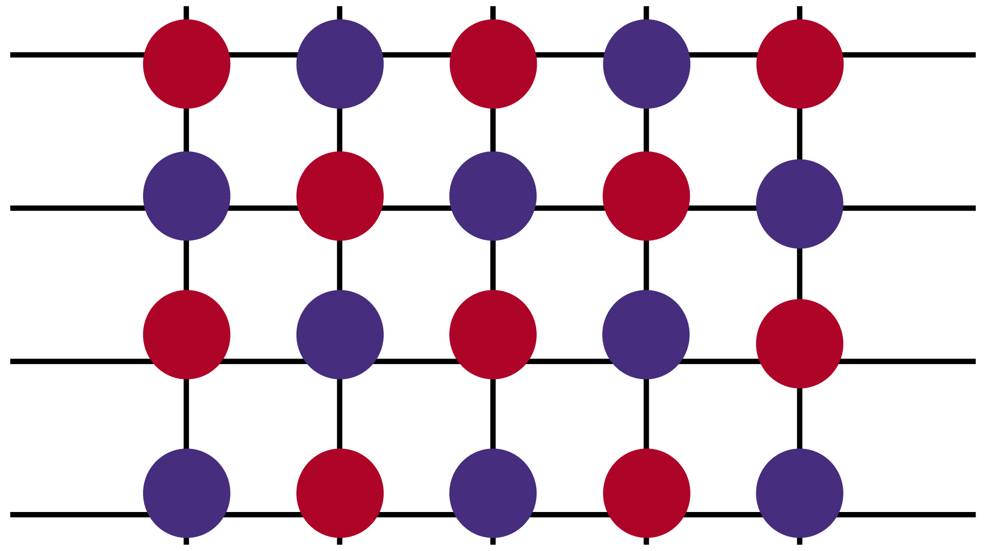

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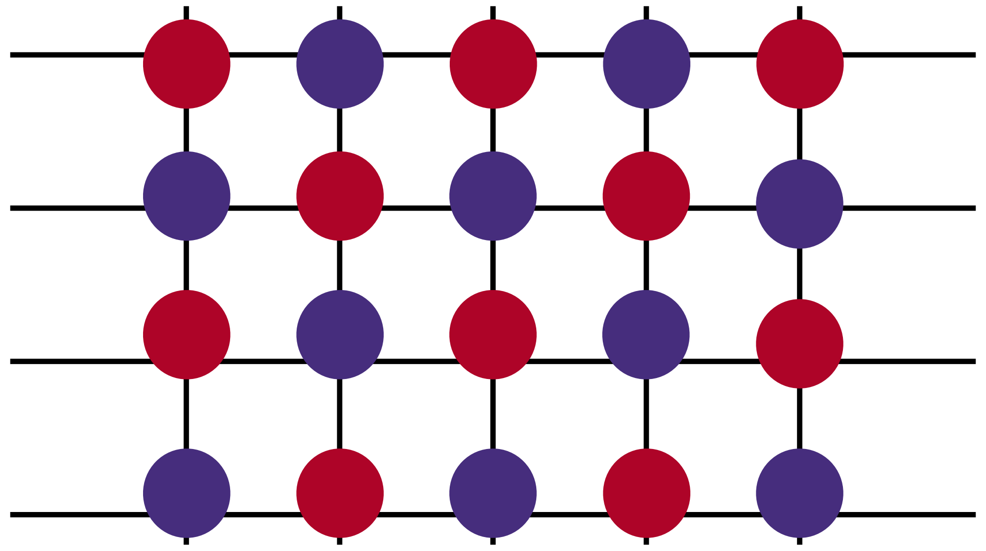

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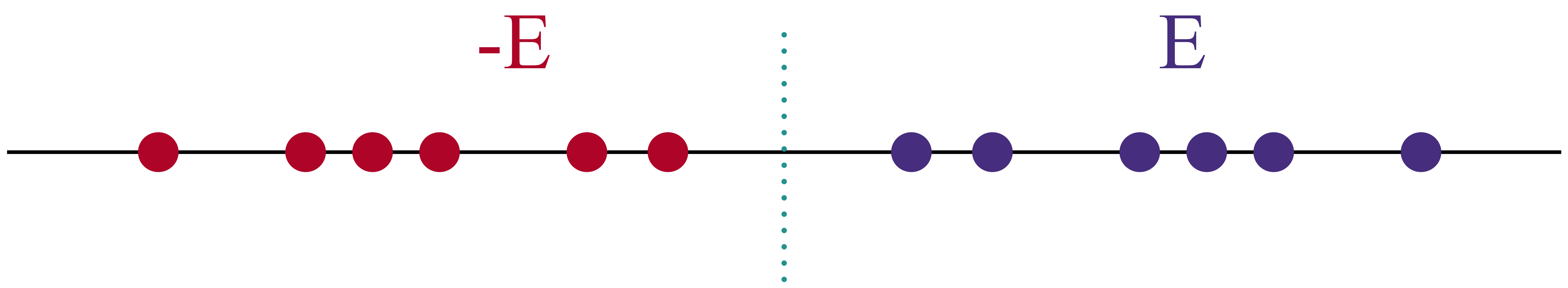

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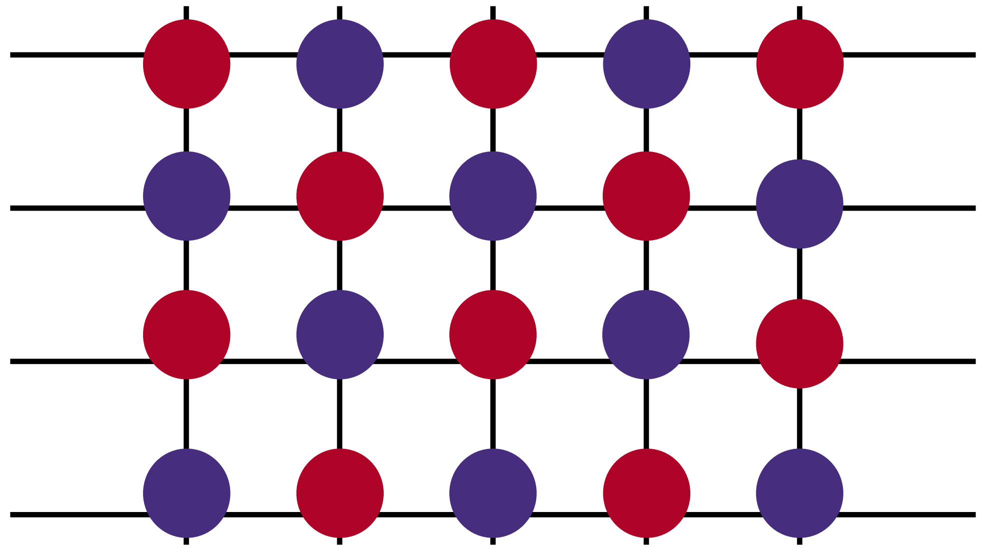

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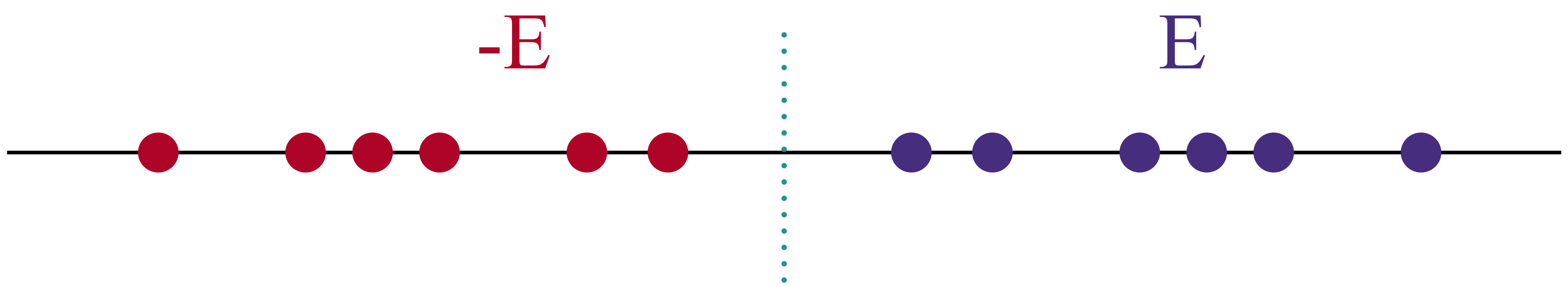

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(Qualitative Statement!)

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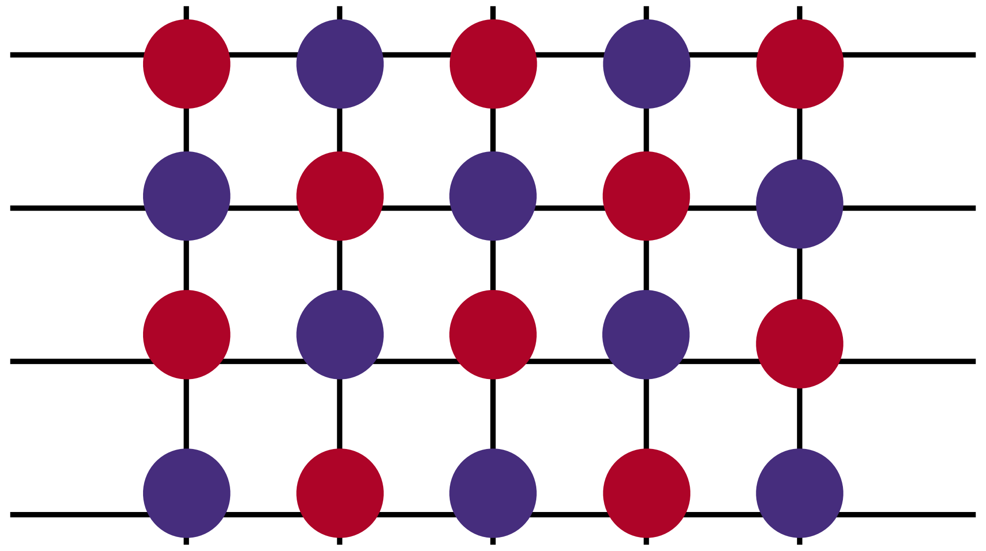

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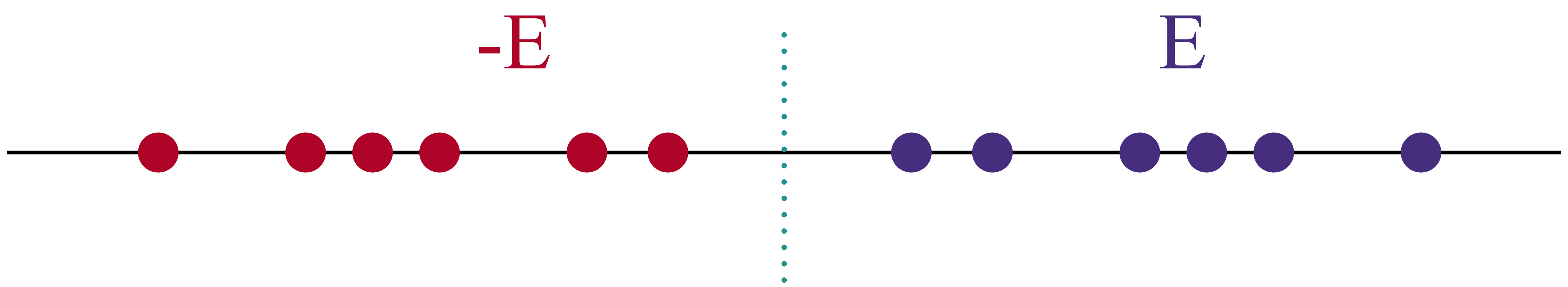

- Near E = 0, DOS develops singular behaviour (Qualitative Statement!)
- Chiral symmetry introduces a fundamental constraint on the spectrum leading to new localisation phenomenon at E = 0.

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Quark propagation through a highly fluctuating gluonic background *(disordered medium)*

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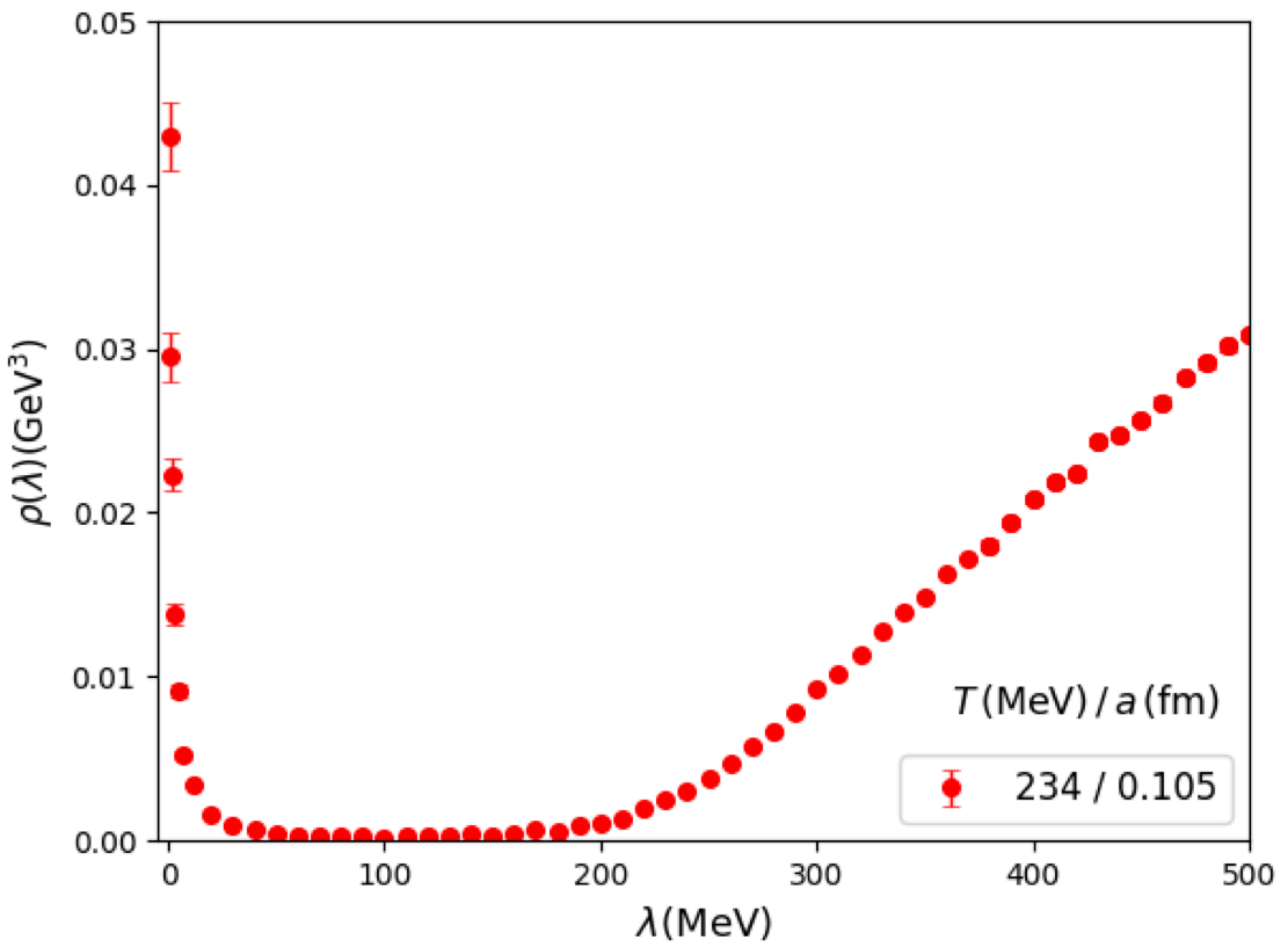
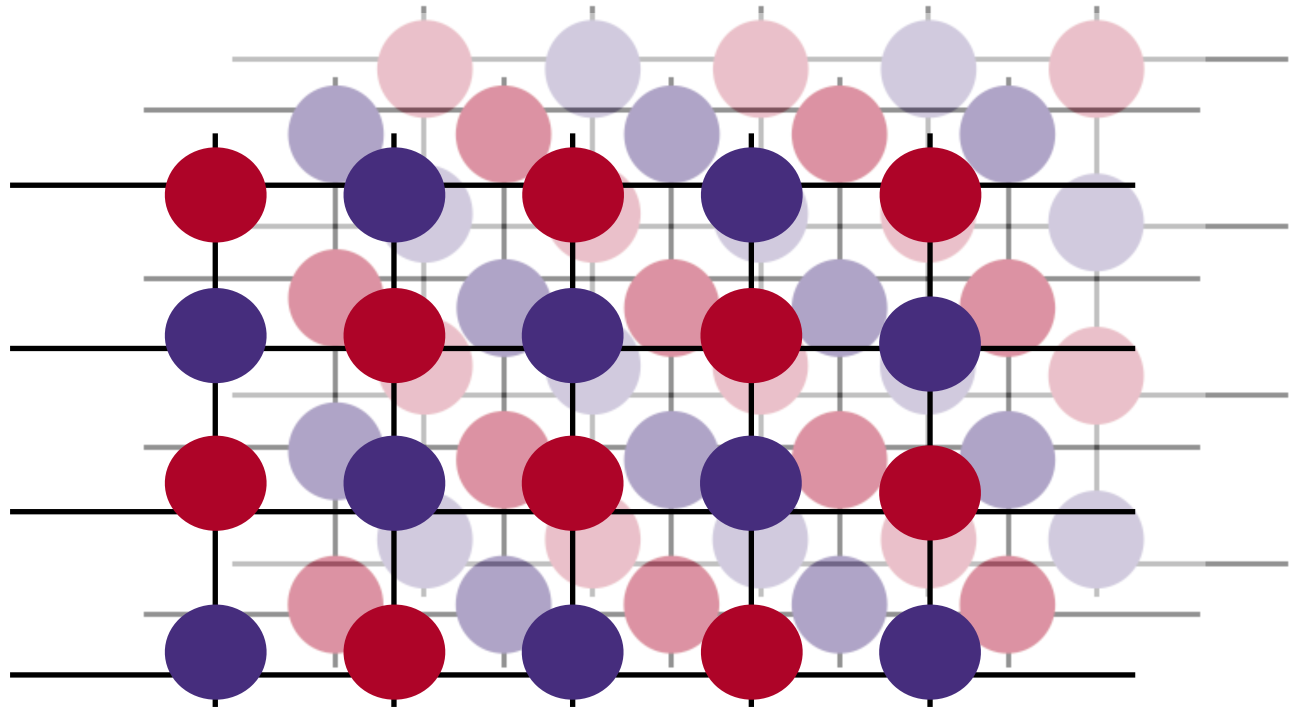
Rather than classifying eigenmodes simply as localised or extended, we want to quantify the geometry of the space they occupy



χQCD and CLQCD (JHEP 12 (2024) 101)

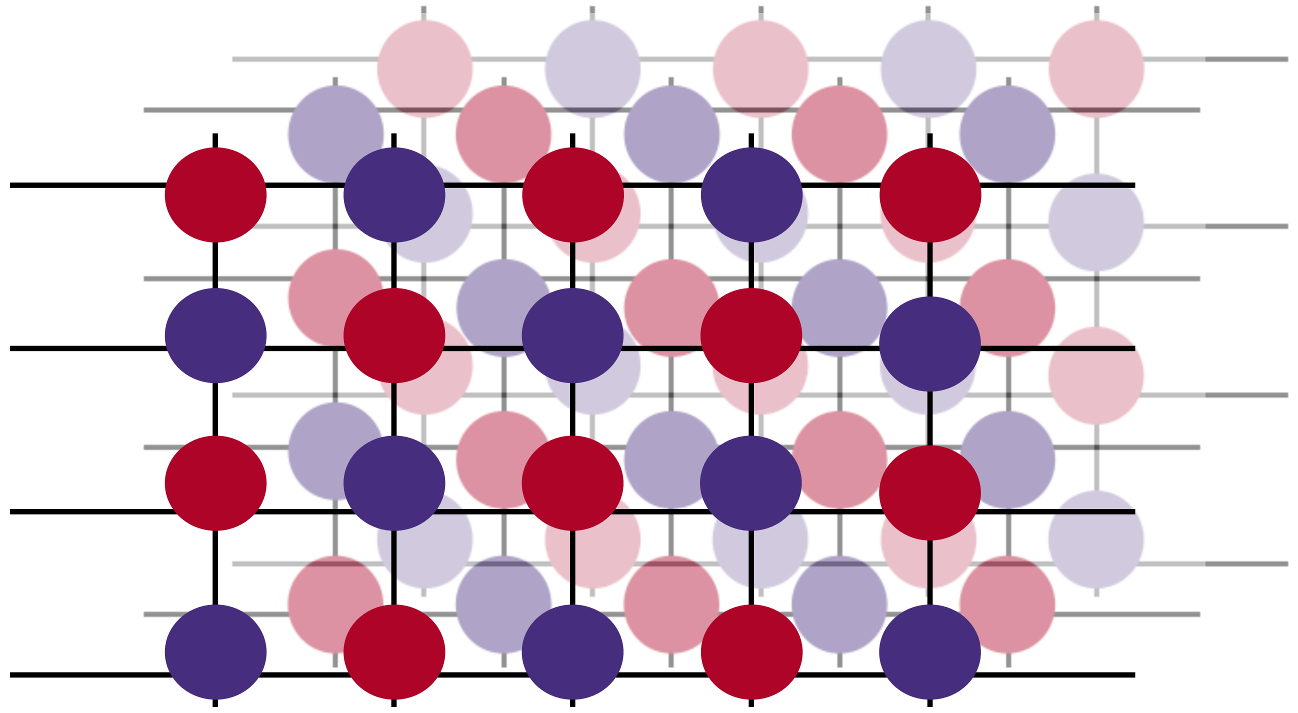
$N_f = 2+1$, *Clover*

Model



Model

Nearest neighbour hopping only, no on site disorder



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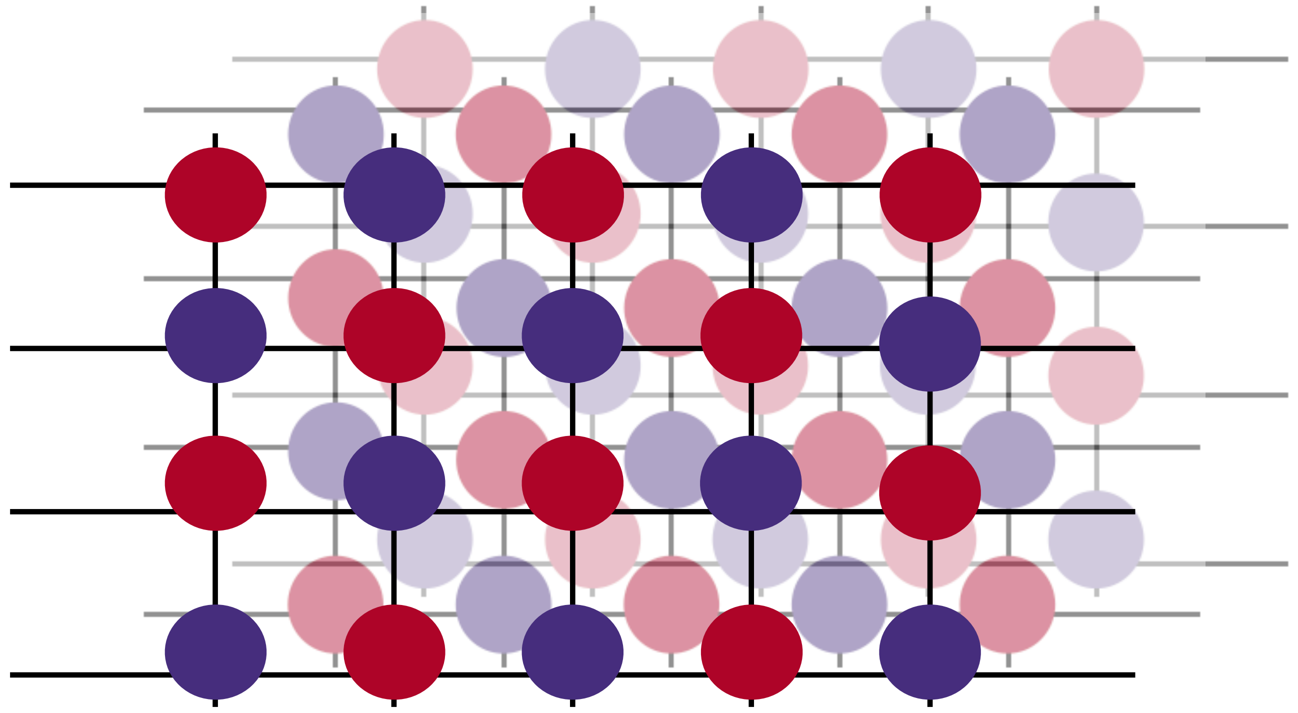

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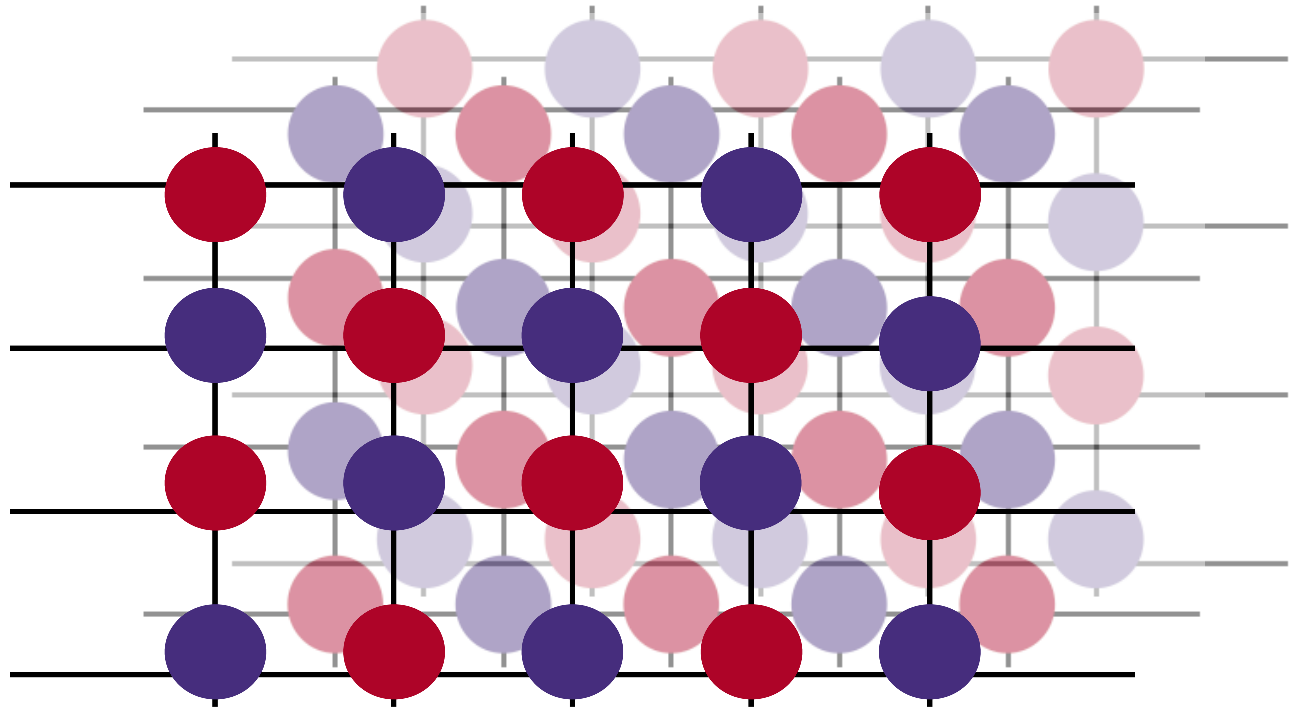

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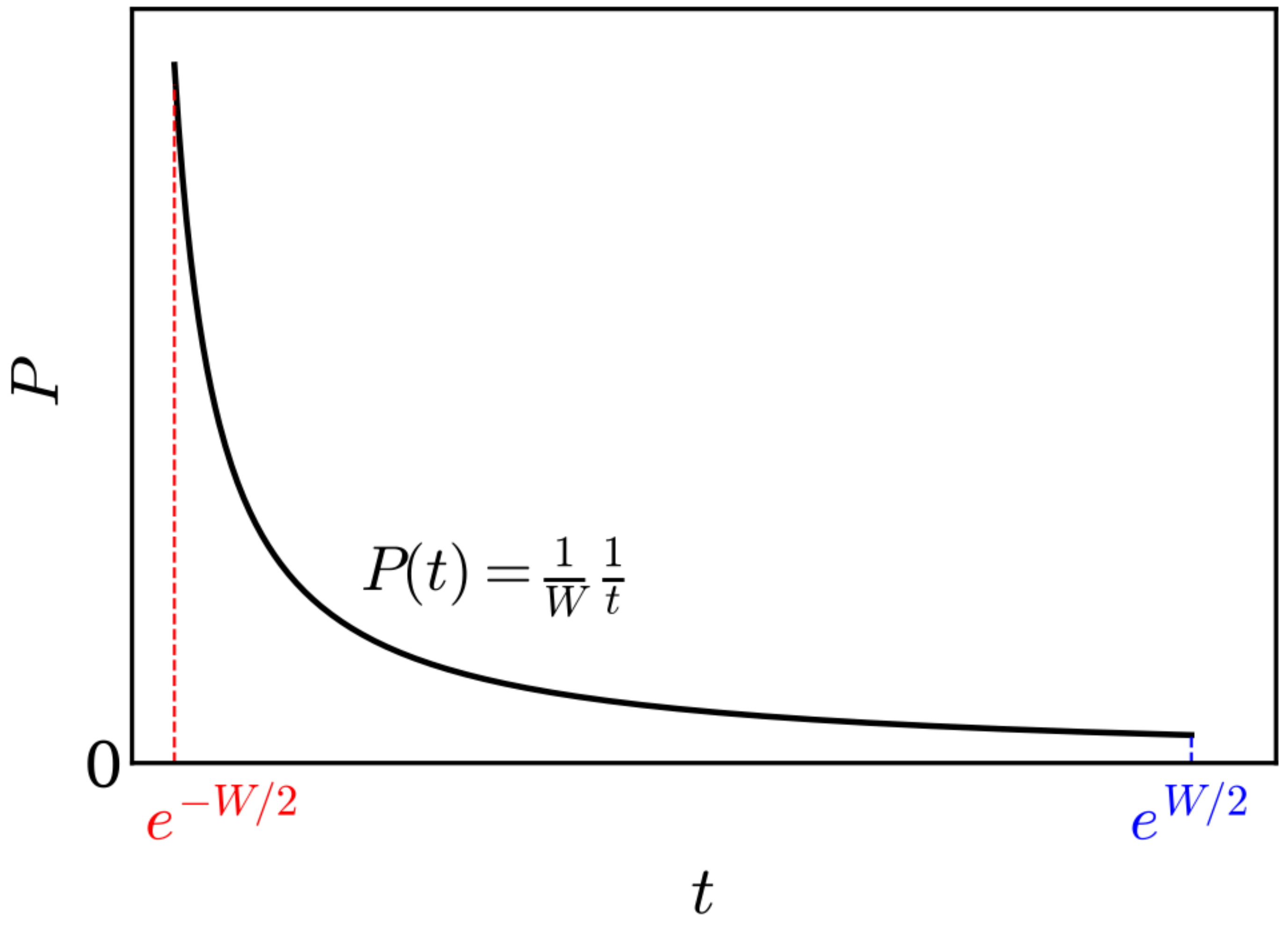
$$t_{\mathbf{r},\mathbf{r}\pm\hat{i}} \in \left[\exp\left(-\frac{W}{2}\right), \exp\left(\frac{W}{2}\right) \right]$$



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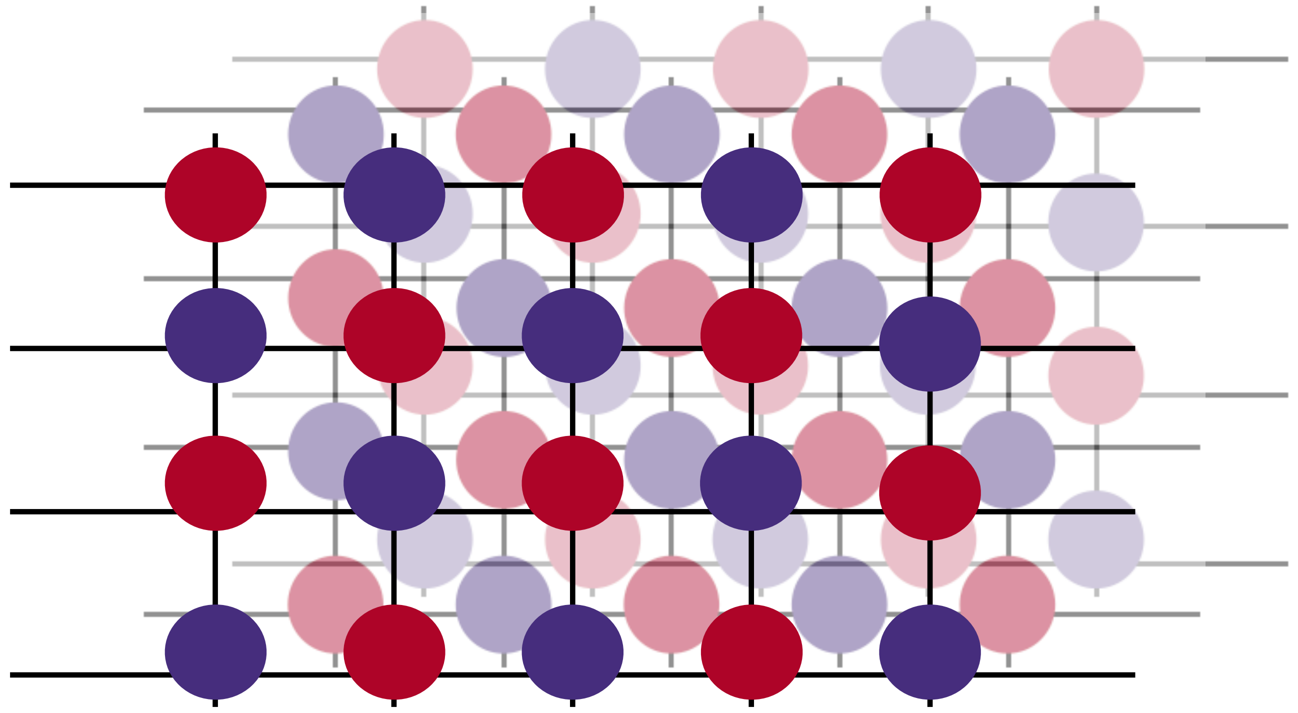

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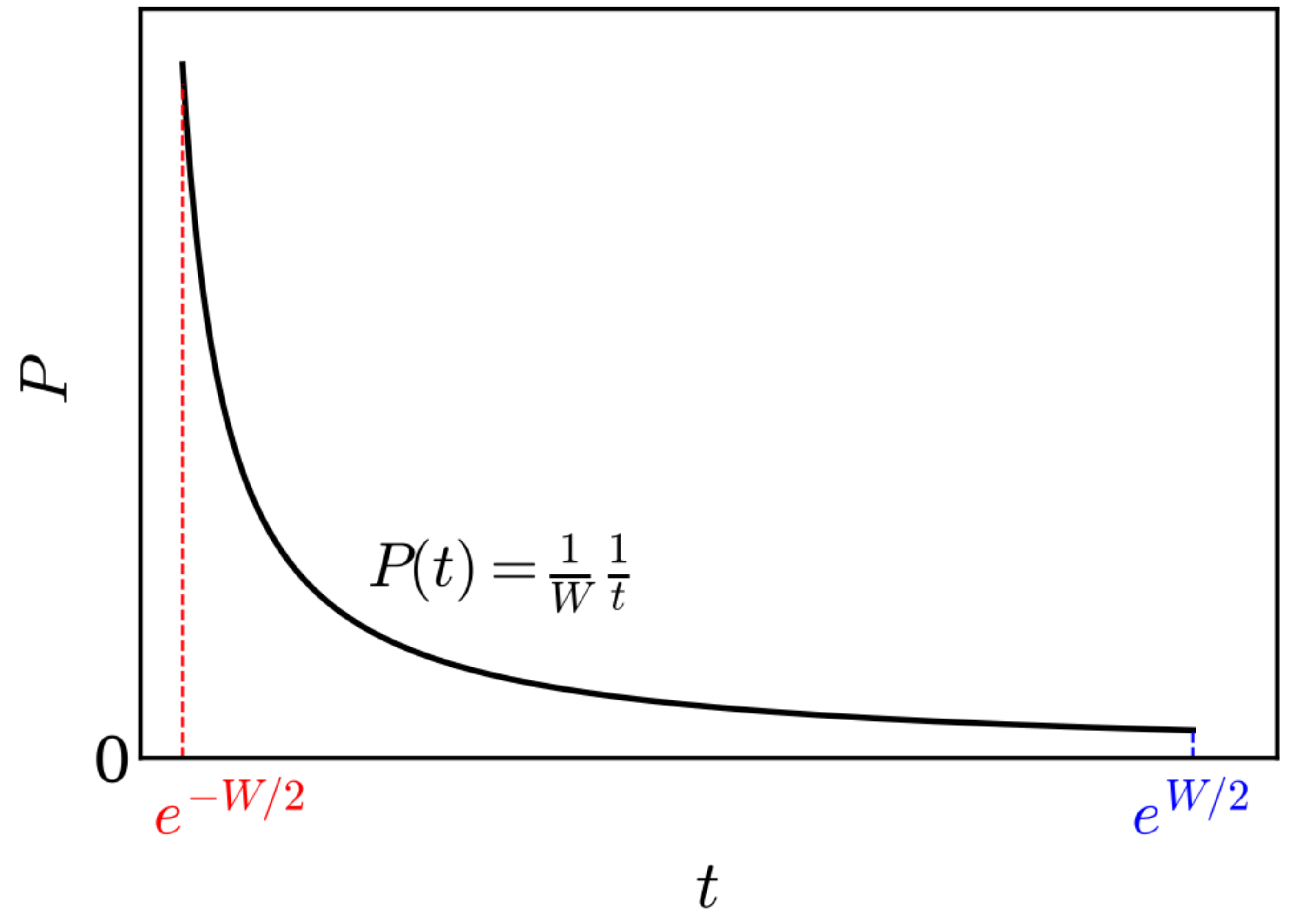

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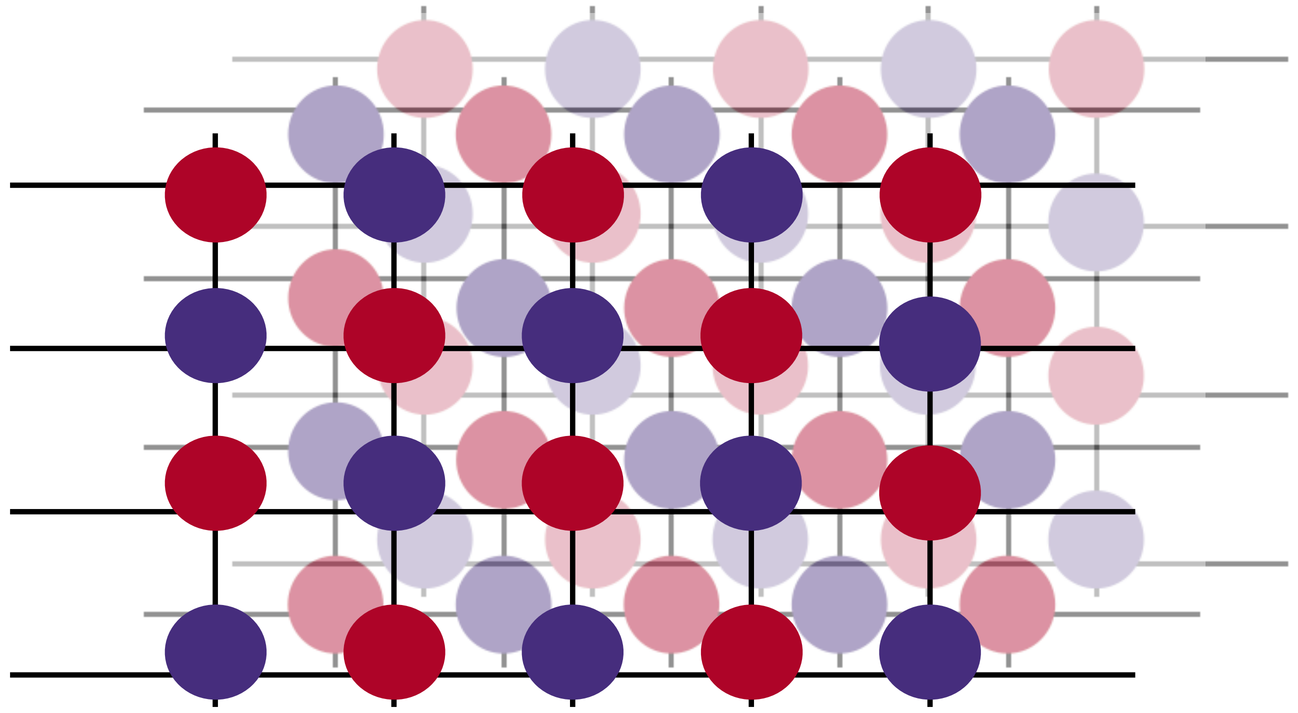

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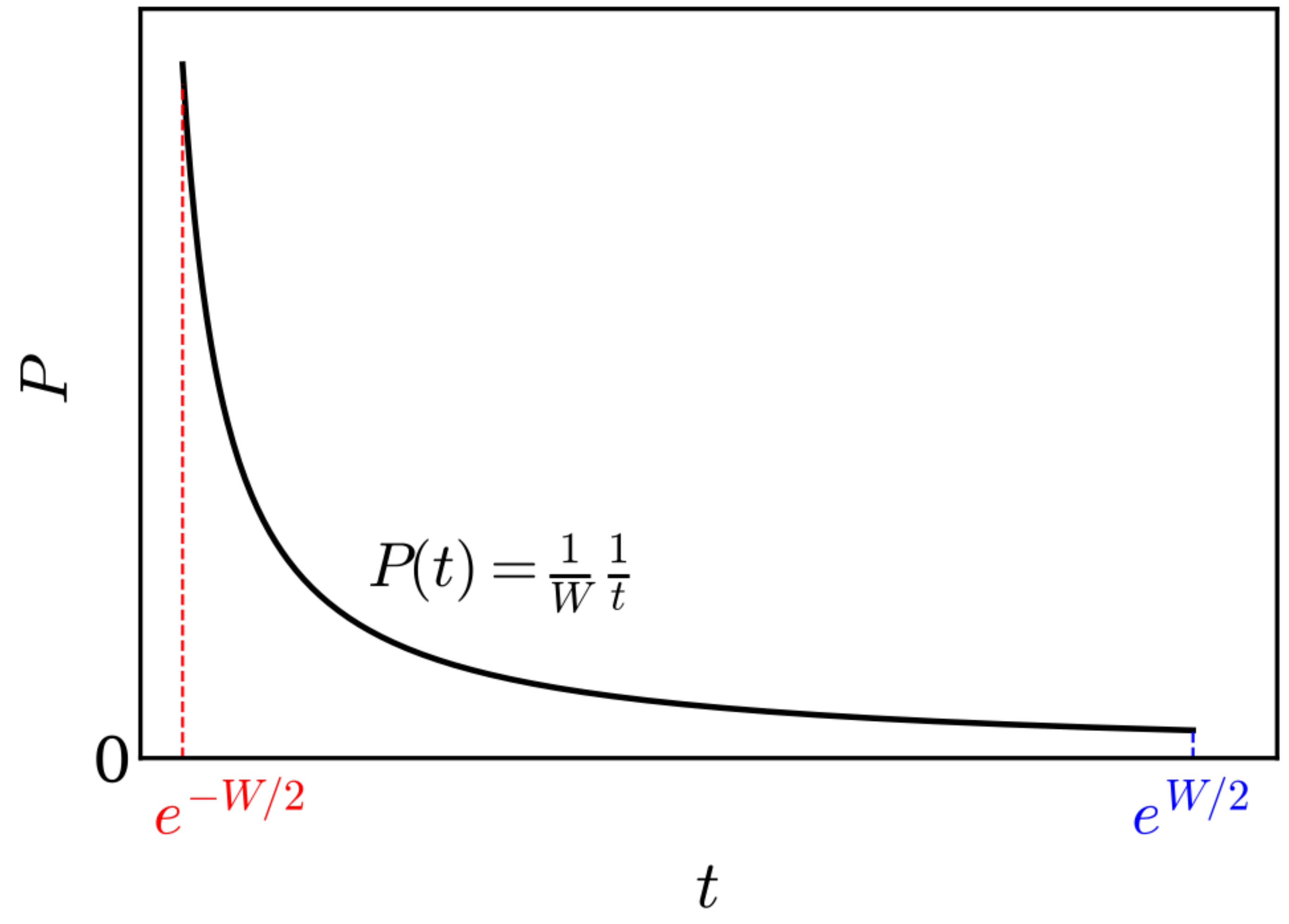

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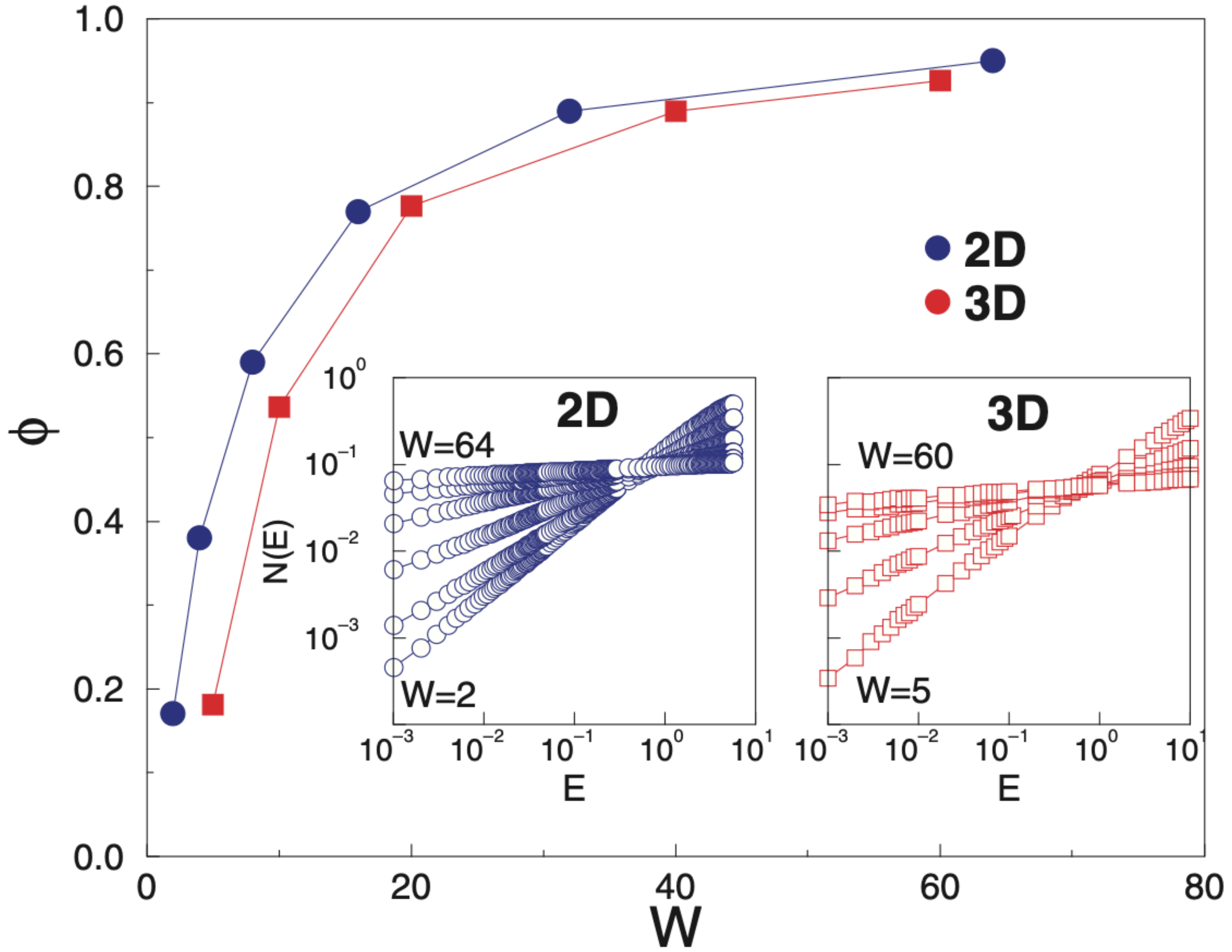

Exponent to power law for DOS

Evangelou and Katsanos
J. Phys. A: Math. Gen. 36 3237 (2003)

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In Ivan's talk we looked at

$$C_i = N|\psi_i|^2$$

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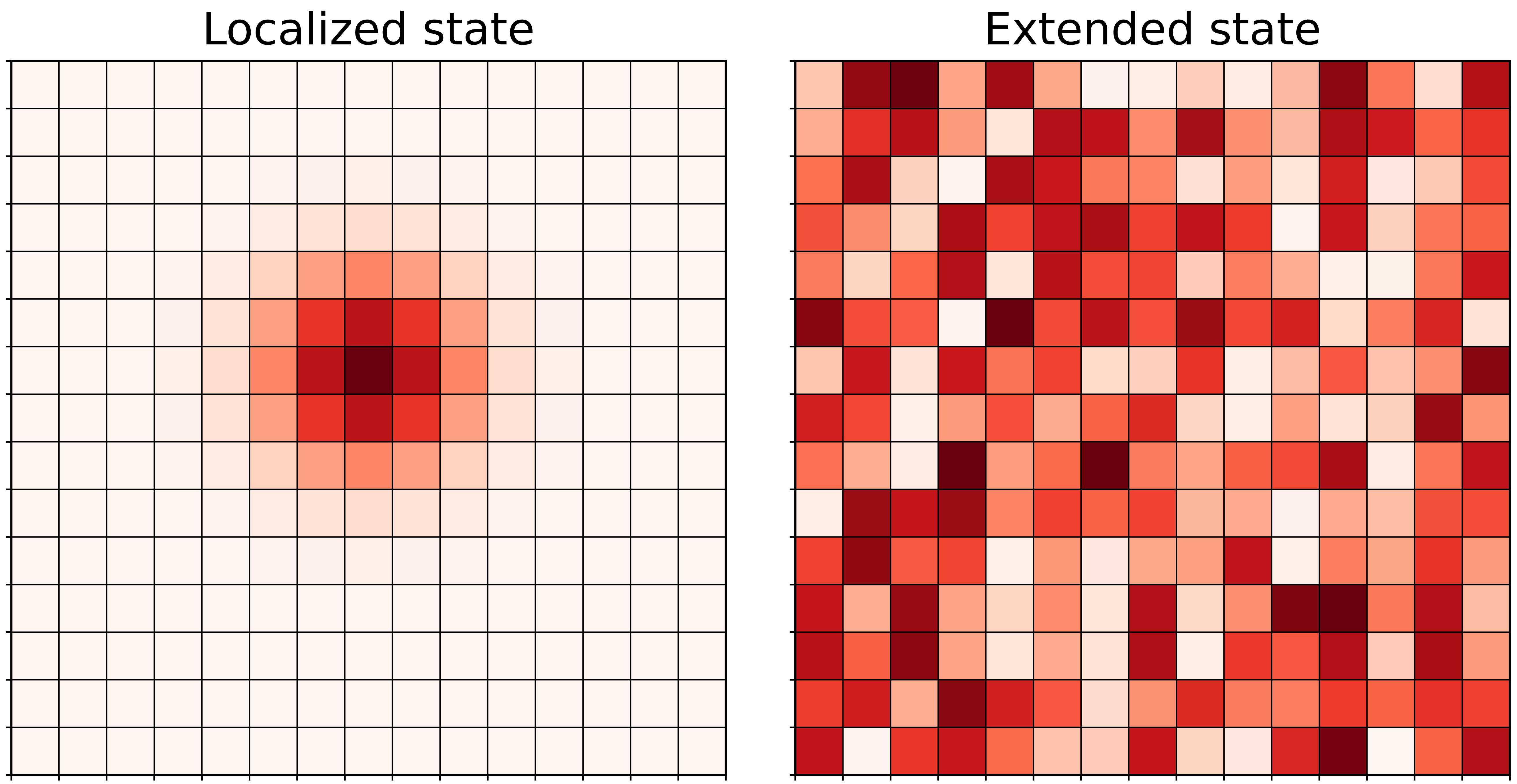

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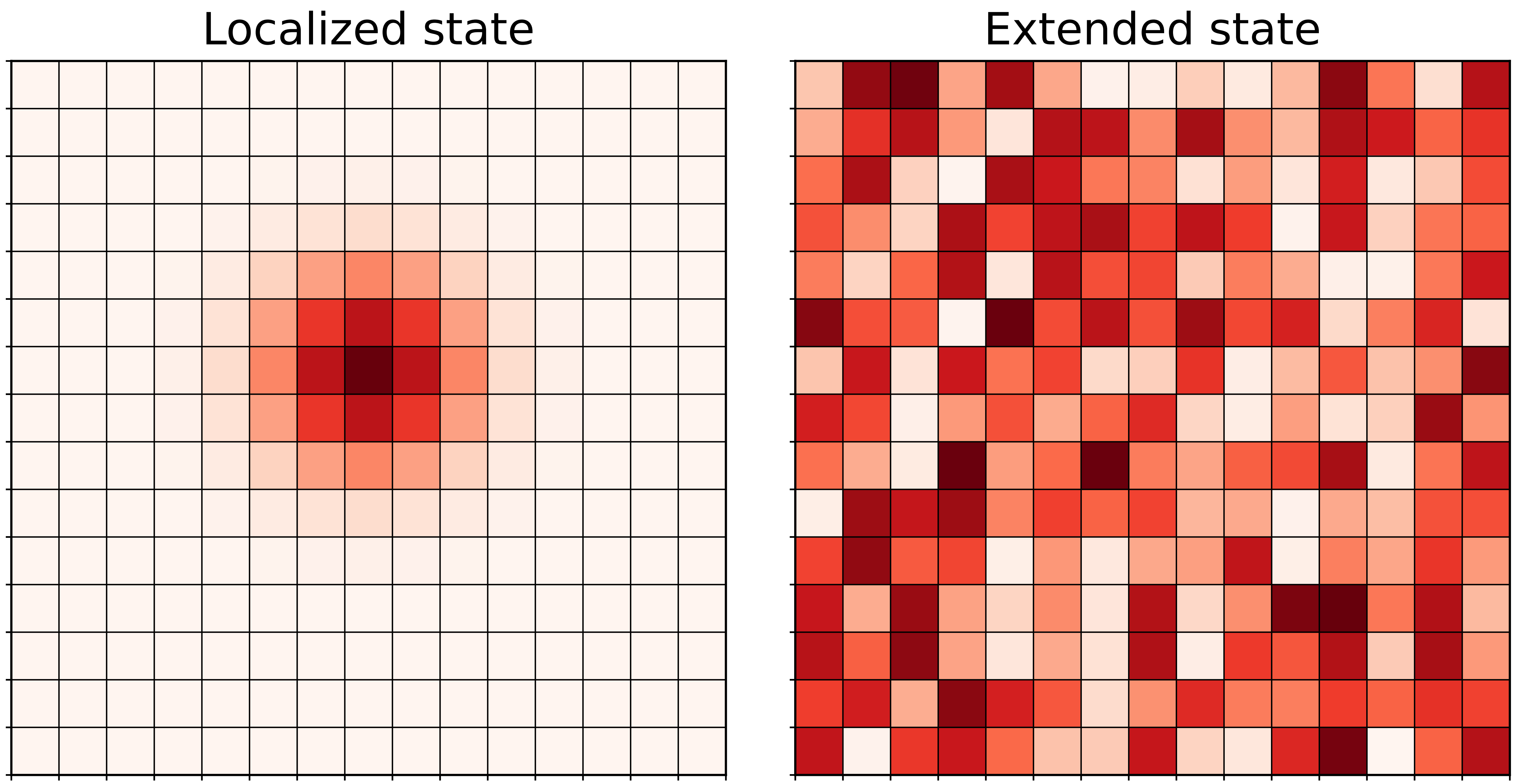

Can we extract more information than a number?
Spectral decomposition (discussed next)

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$$p_{(1)} \geq p_{(2)} \geq \cdots \geq p_{(N)}$$

and define the cumulative probability

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Horváth and Markoš

Entropy 25 (2023) 11, 1557

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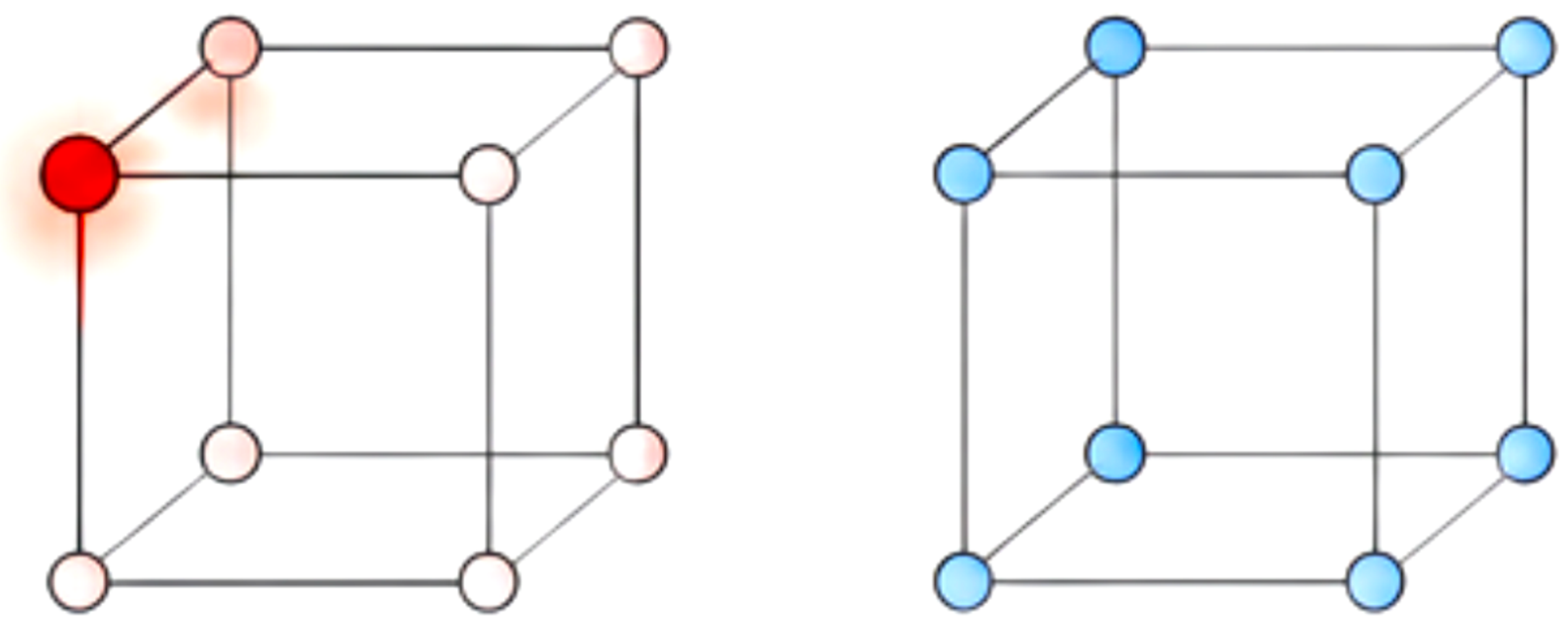

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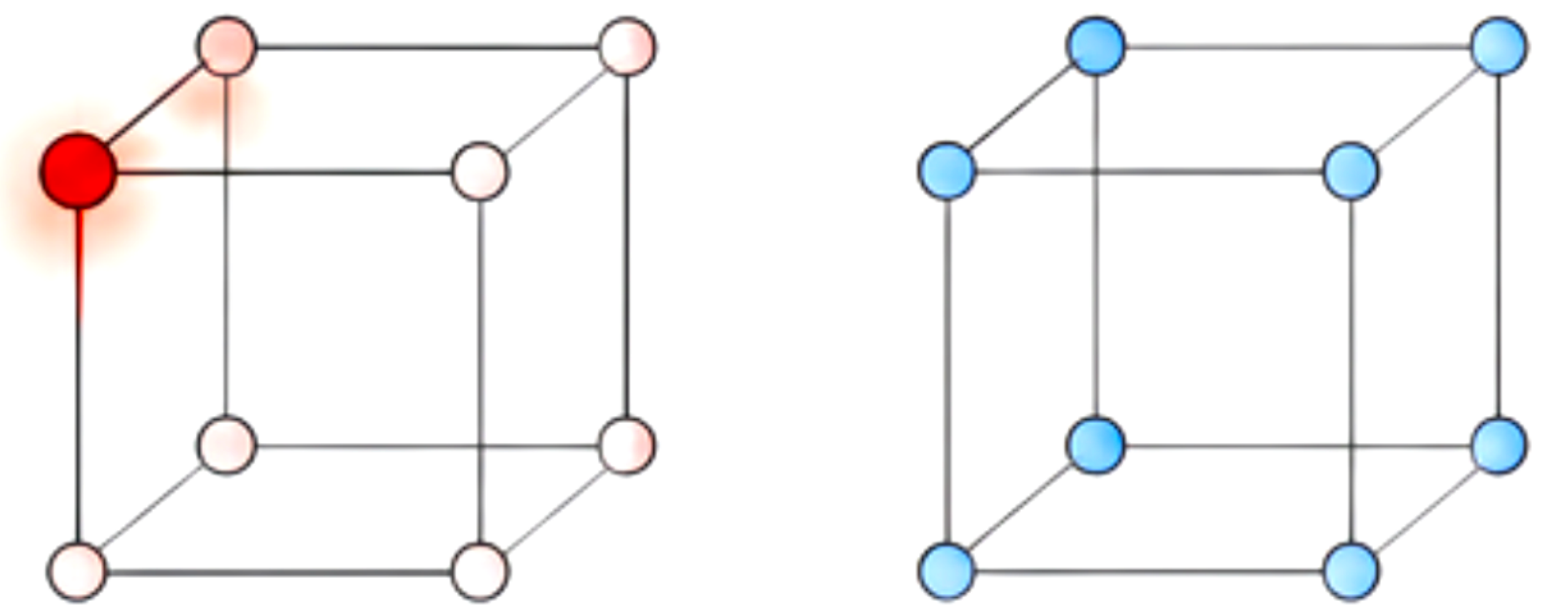

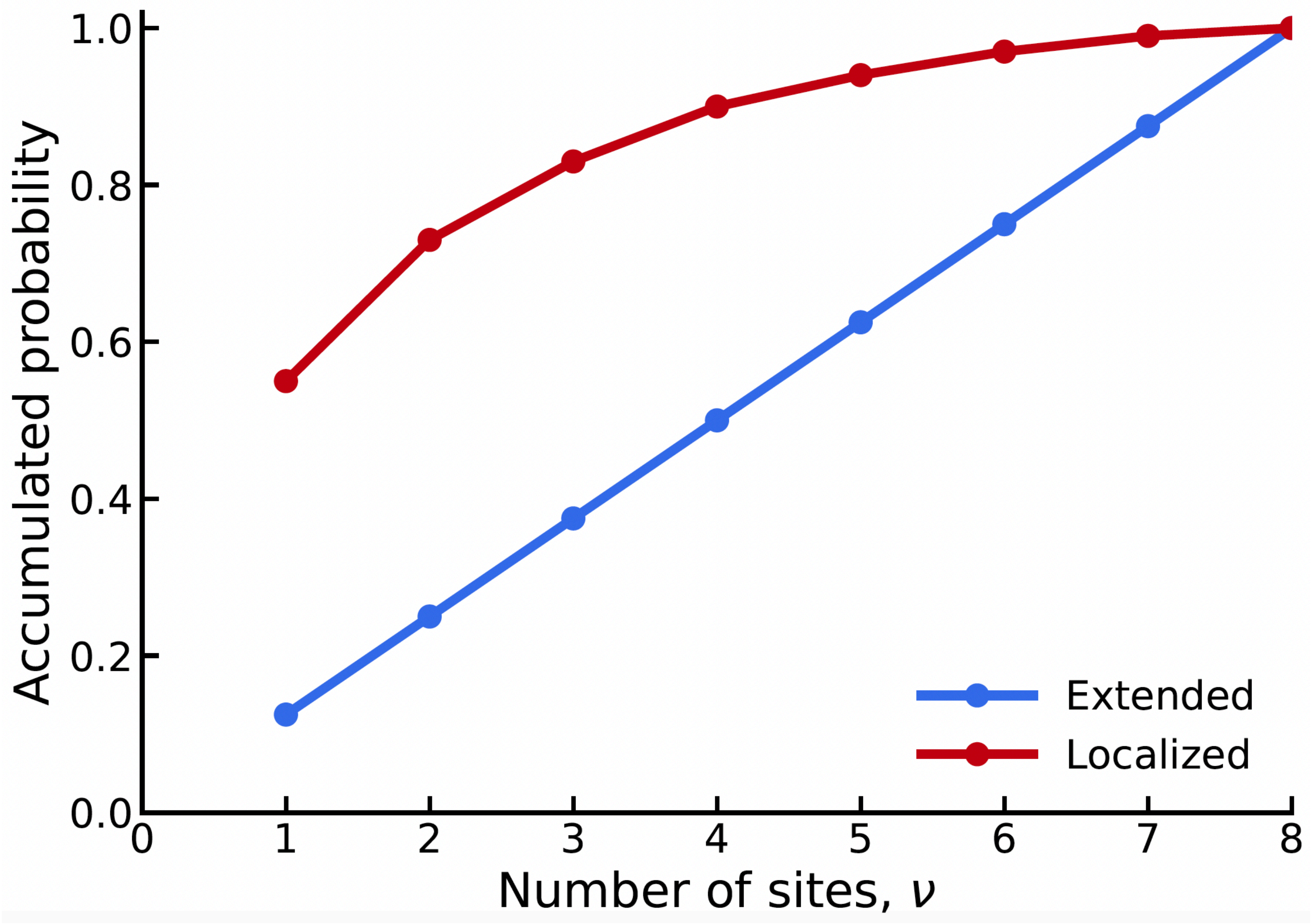

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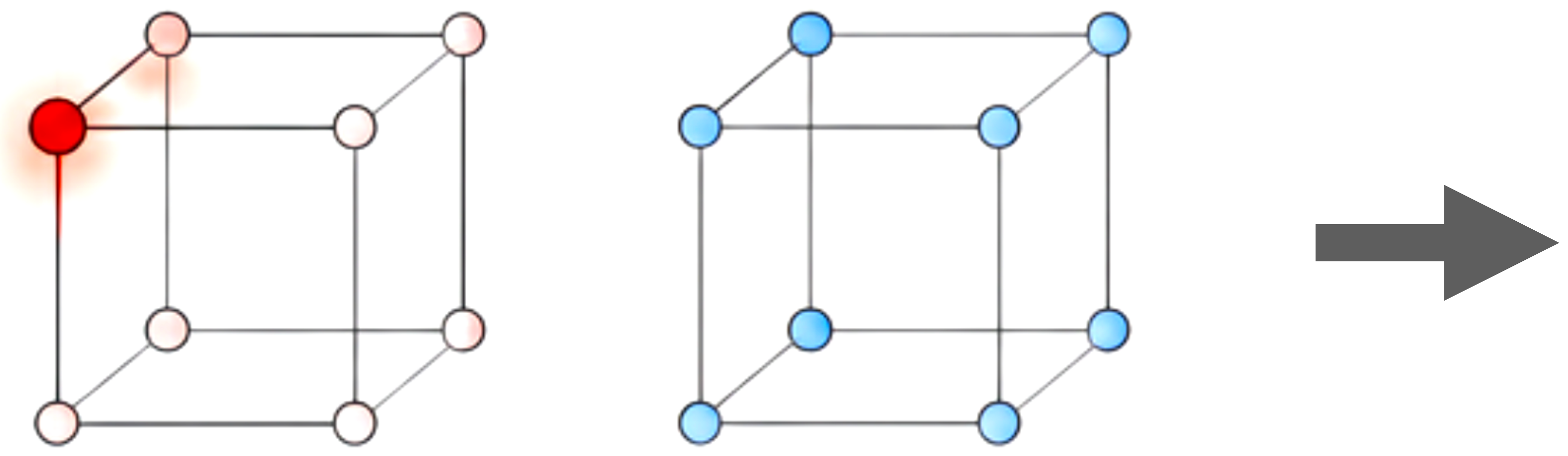

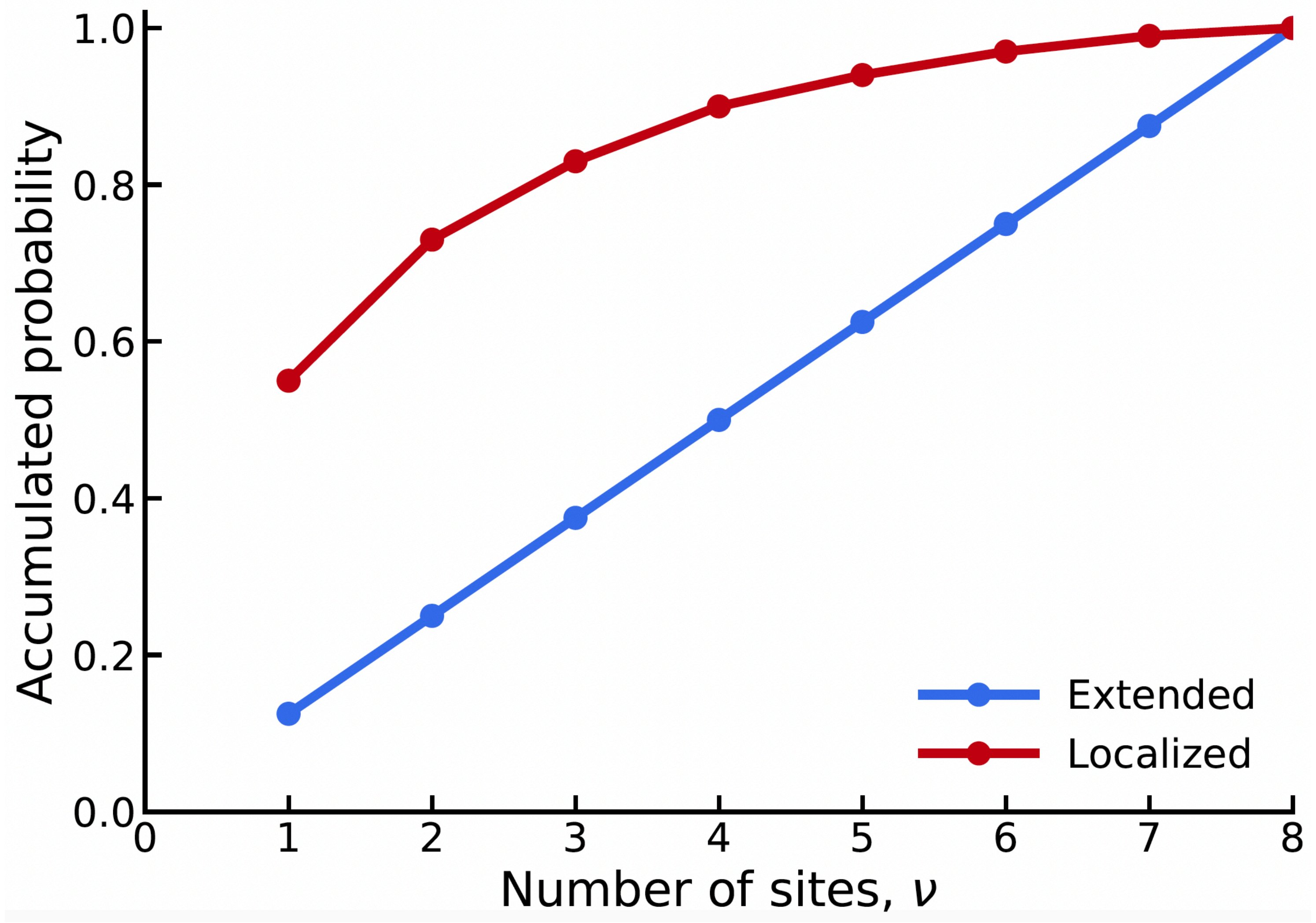

$$\Delta\nu(q) = \nu(q + \Delta q) - \nu(q),$$

$$\Delta\nu(q, L) = A(q)\, L^{d(q)} e^{-c(q)/L},$$

Fitted using various lattice sizes

Counting 'ν'

Let us look at density of ν

against q and E

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For $V = 24^3, W = 10$

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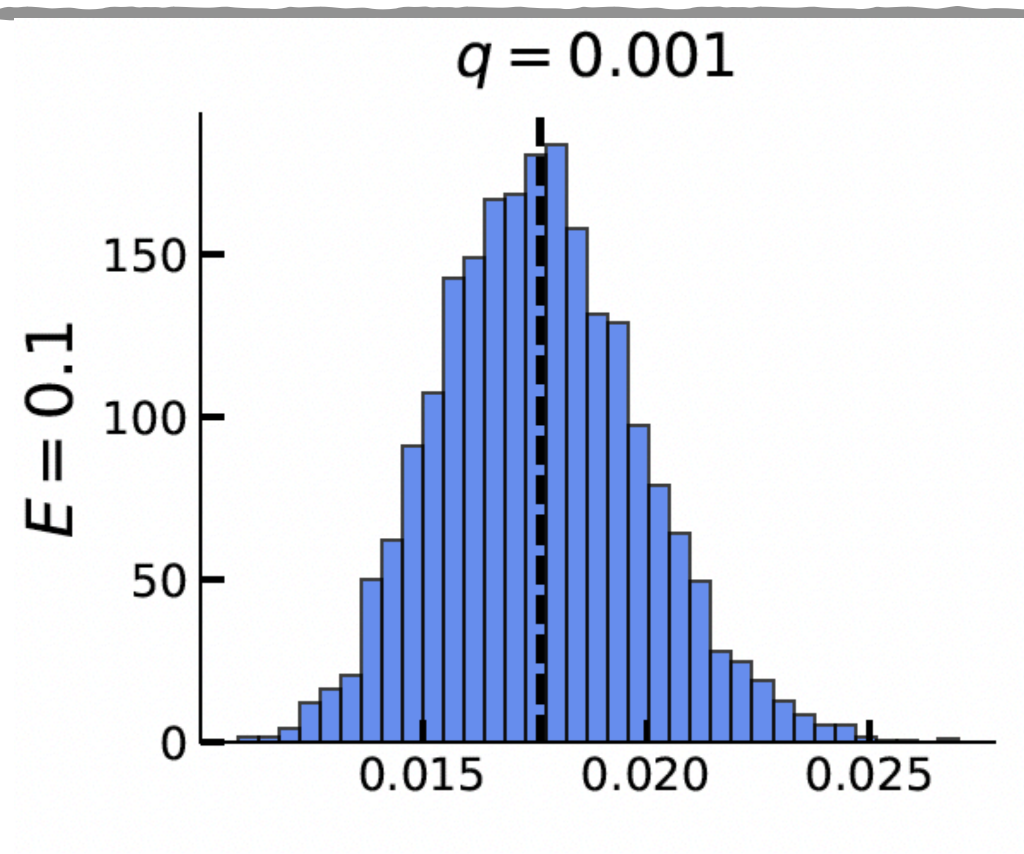

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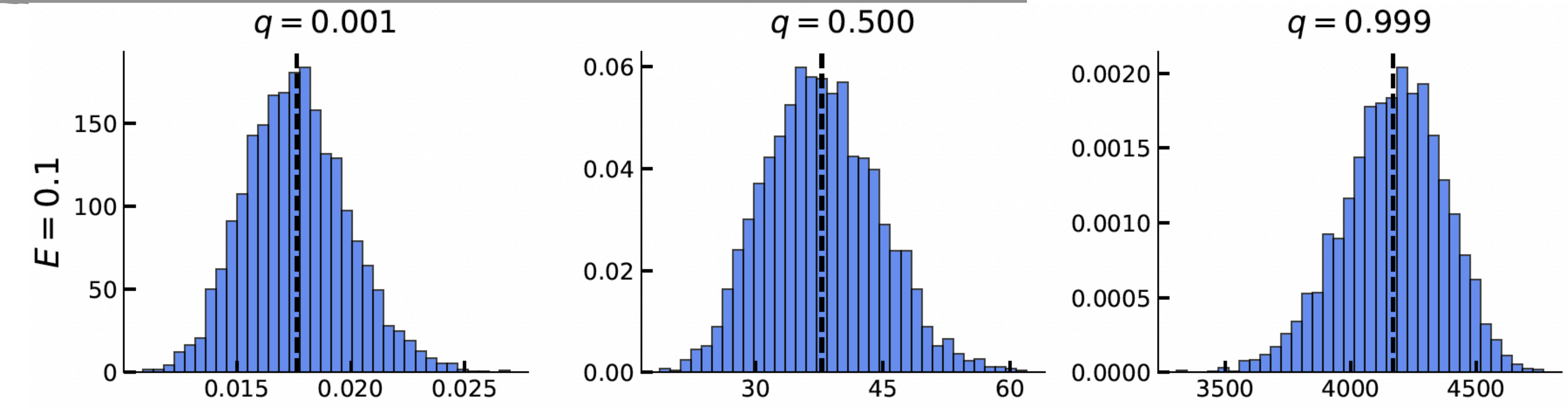

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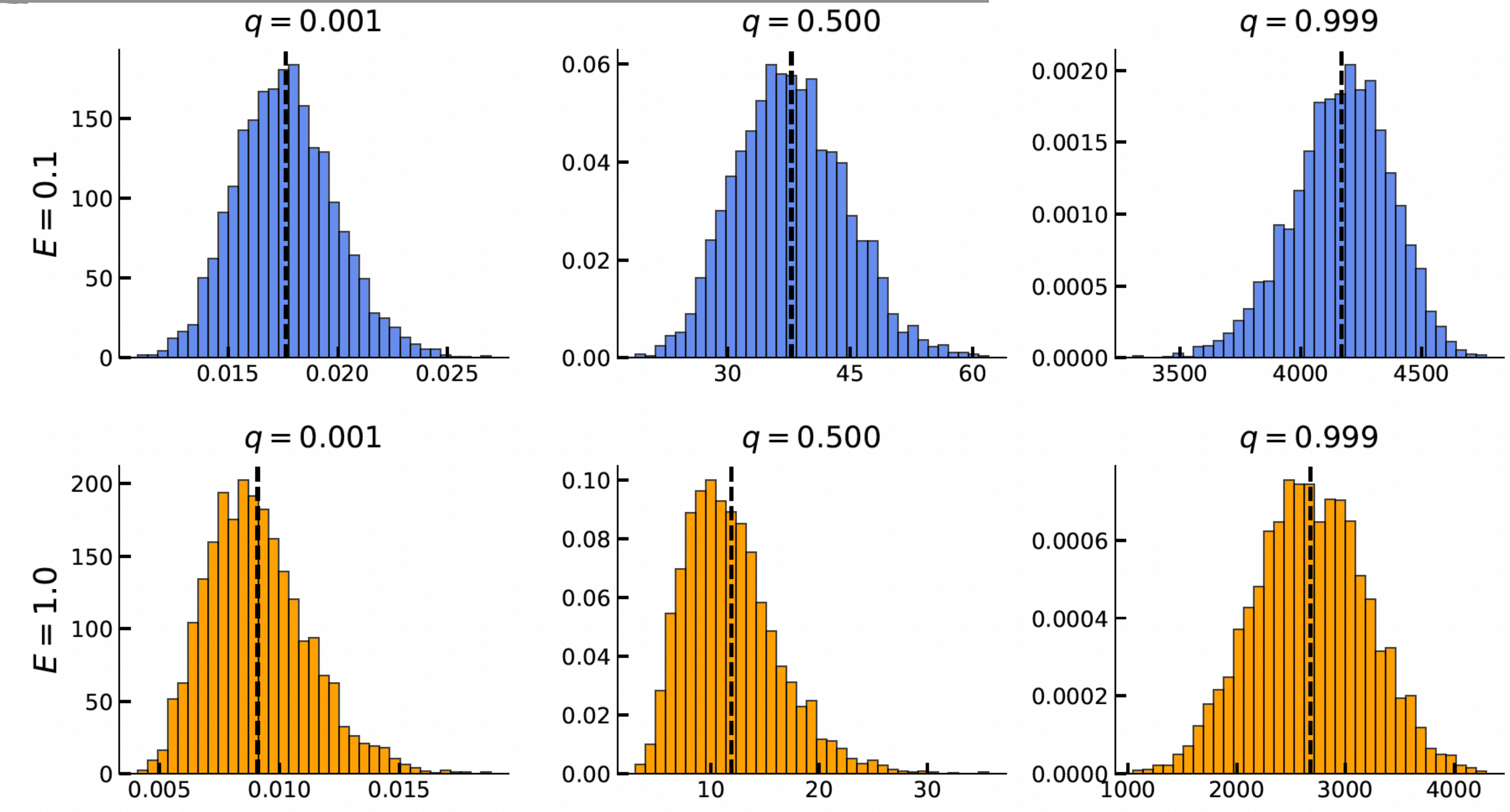

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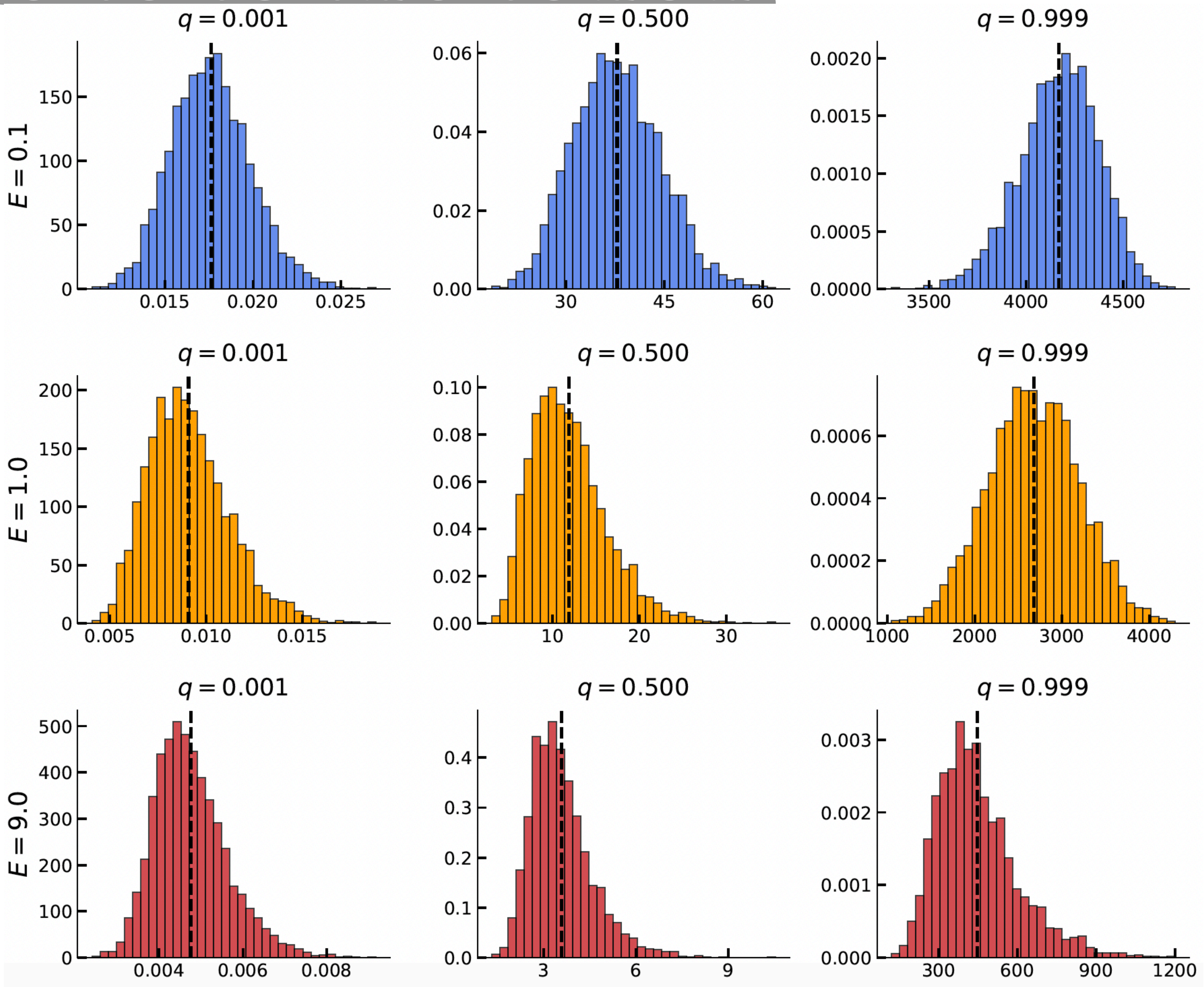

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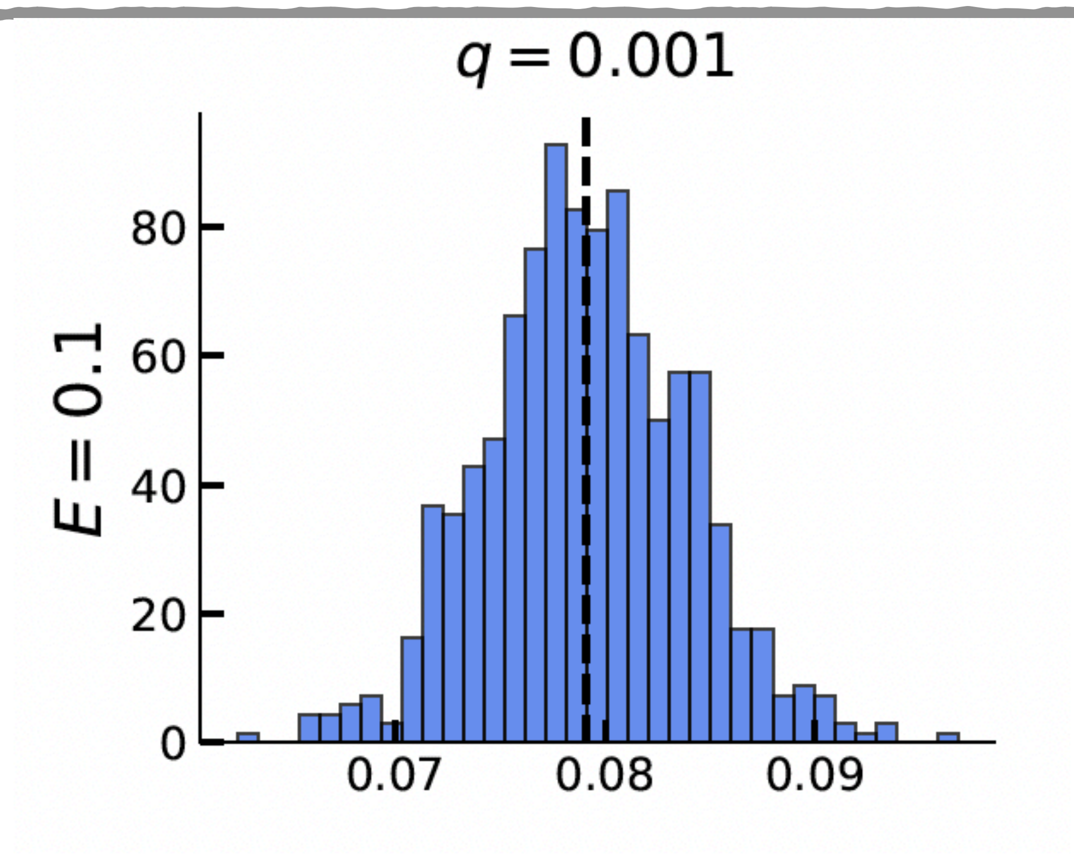

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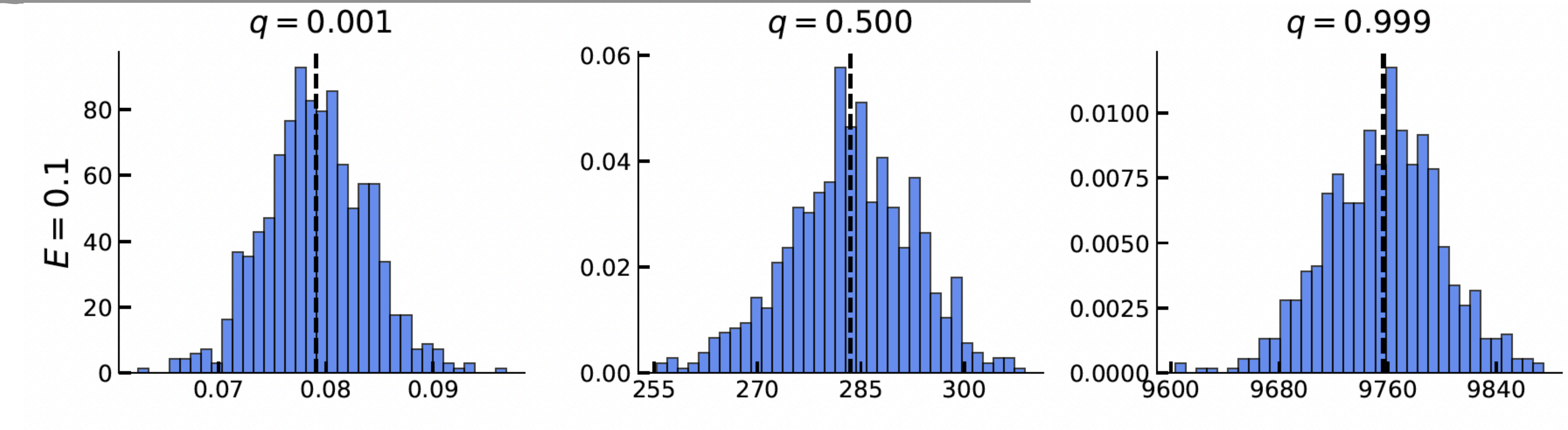

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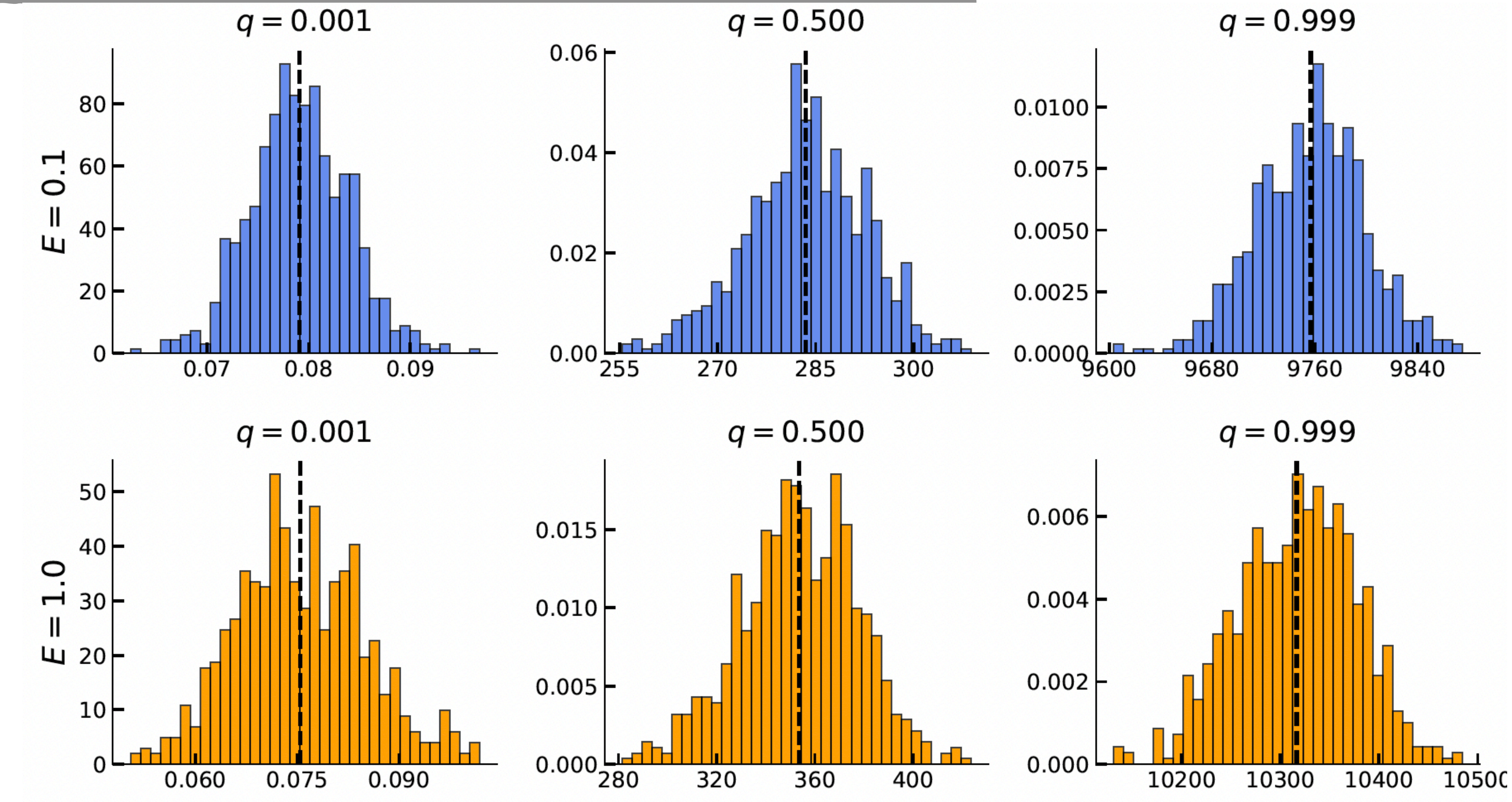

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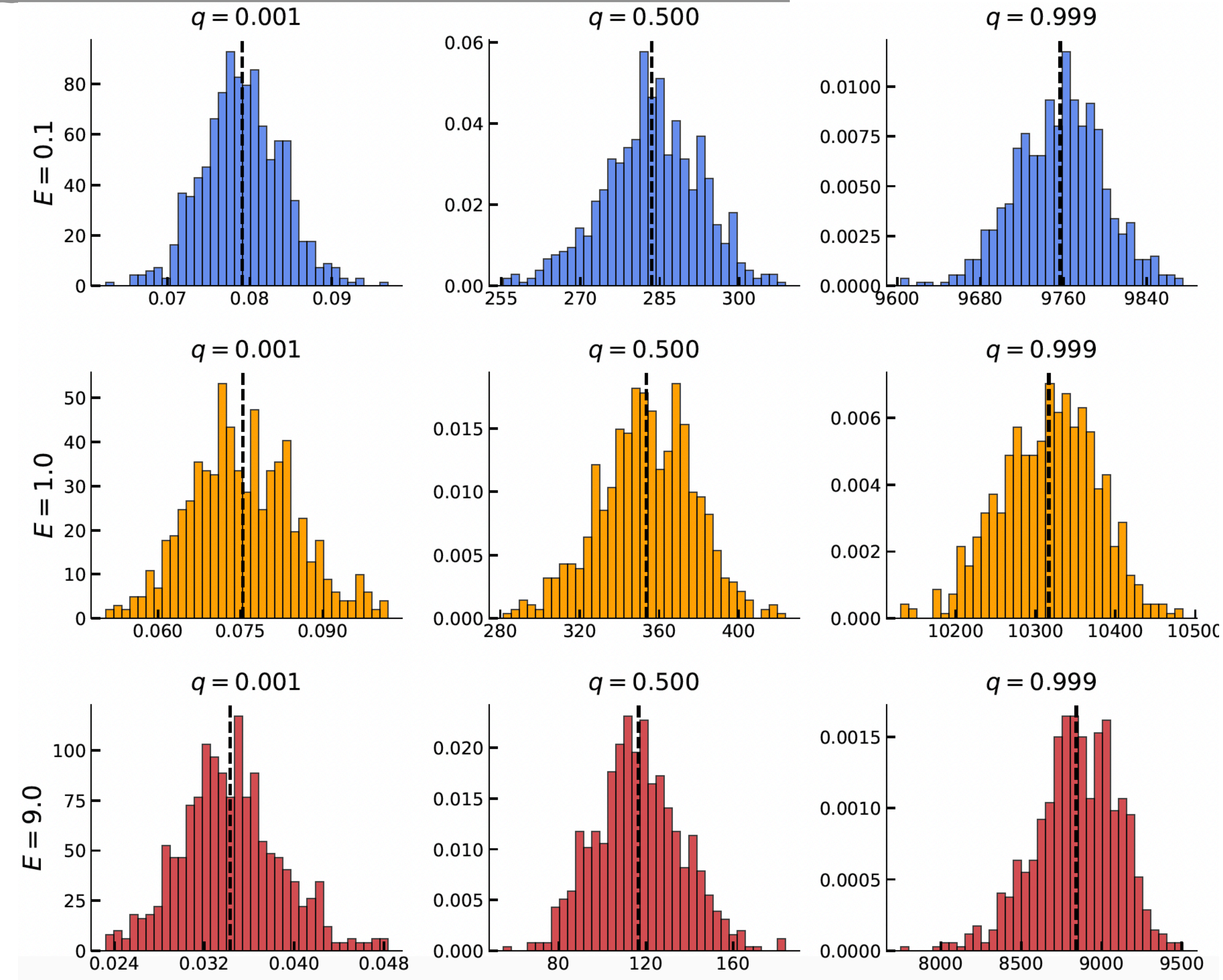

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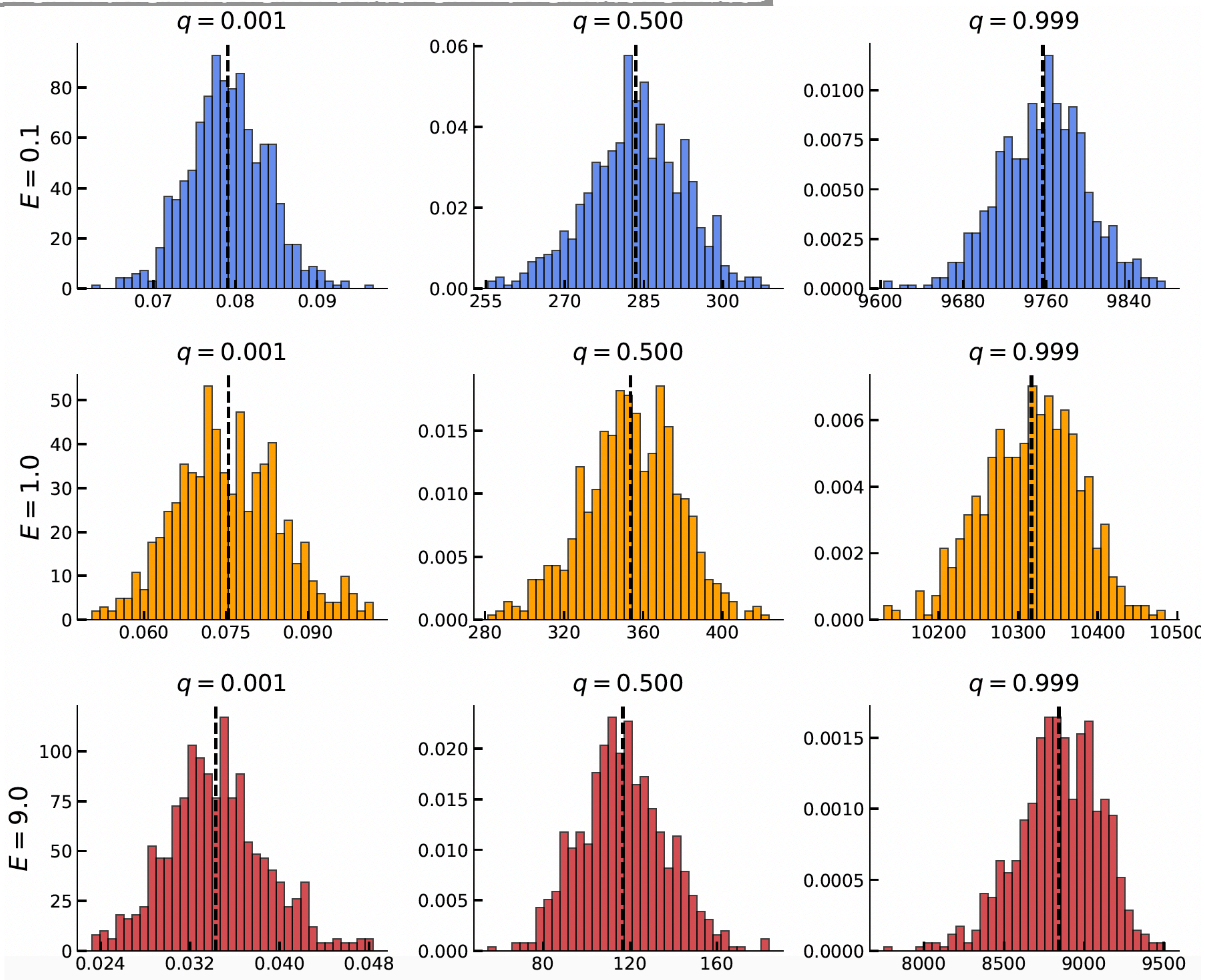


Limited statistics

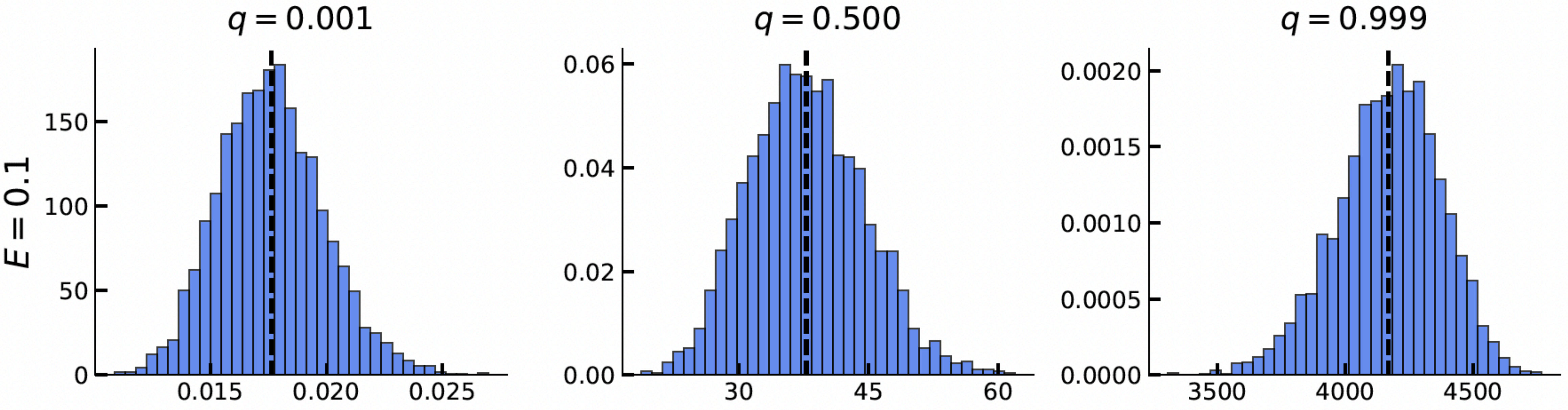
+

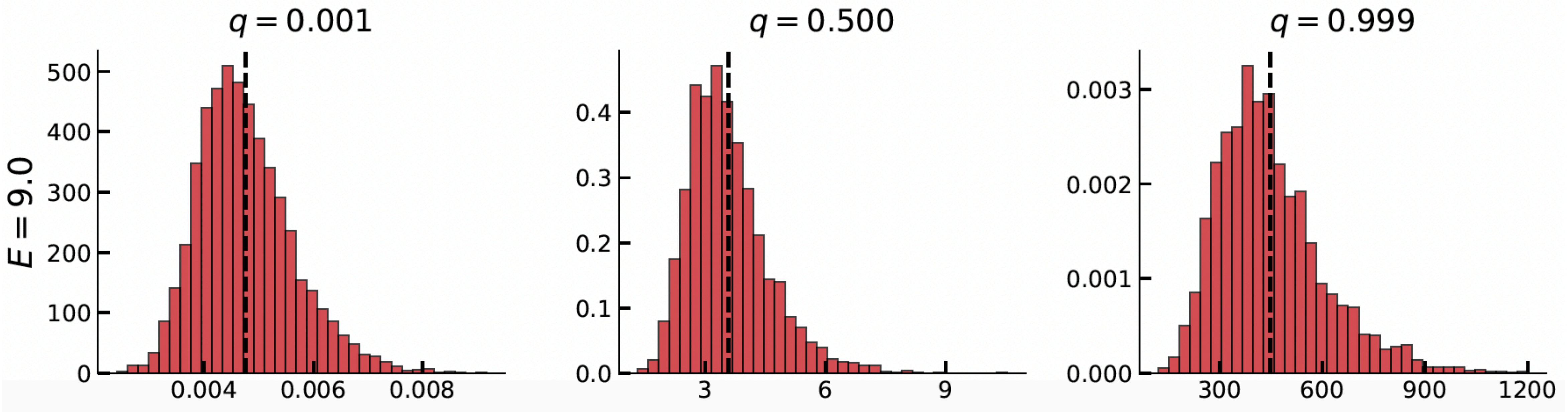
Finite Volume

Counting ‘ν’

Counting ‘ν’

W = 10





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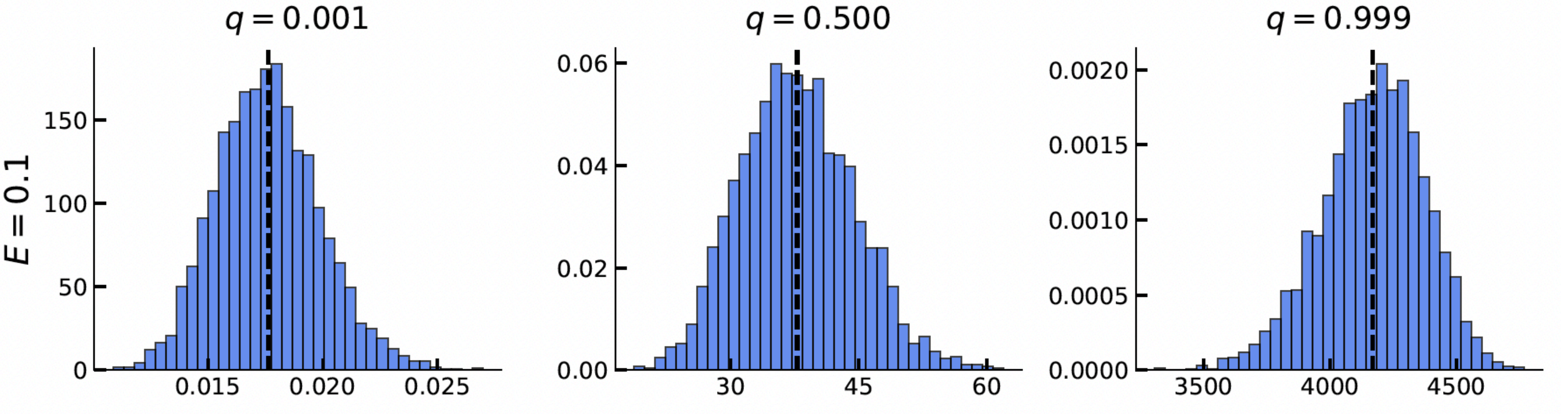

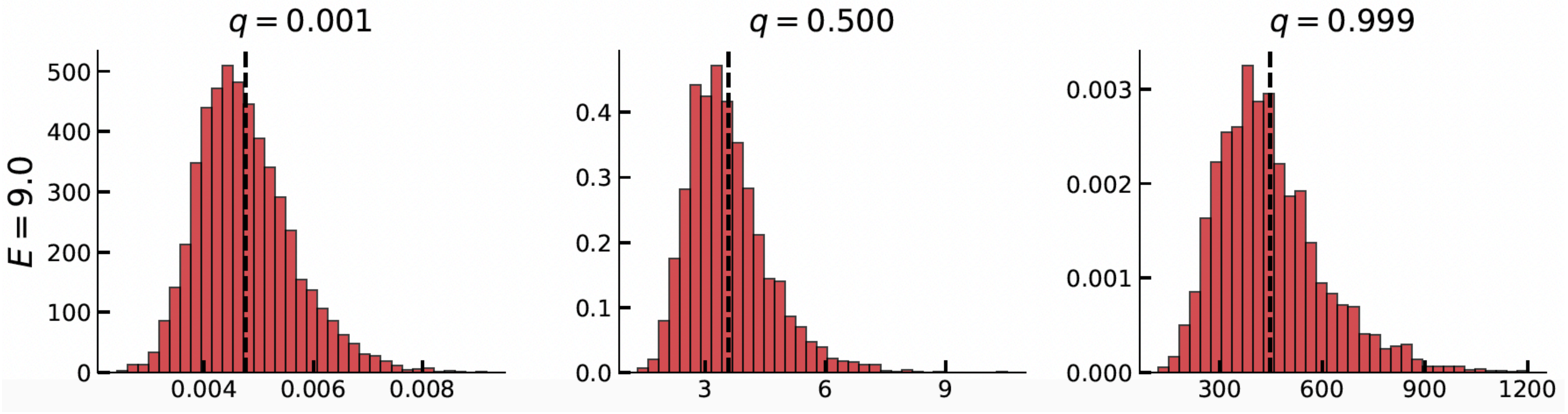

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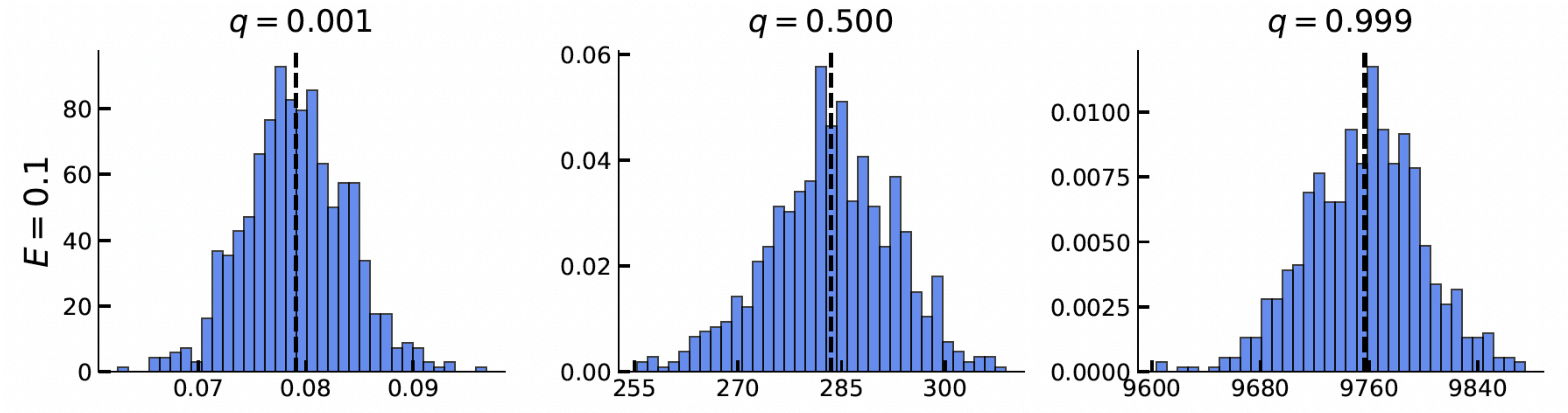

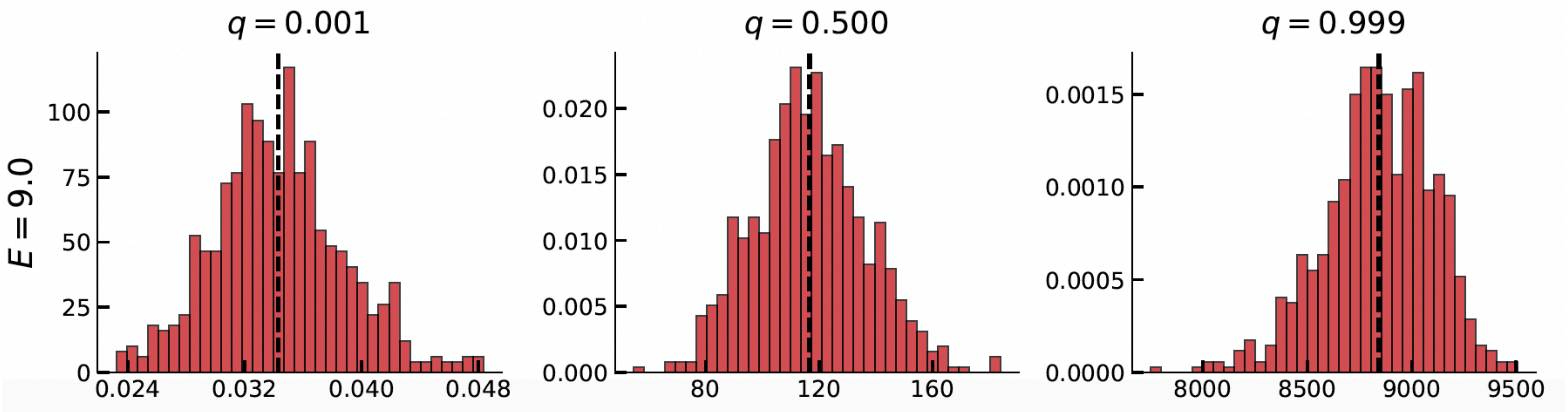

$d(q)$

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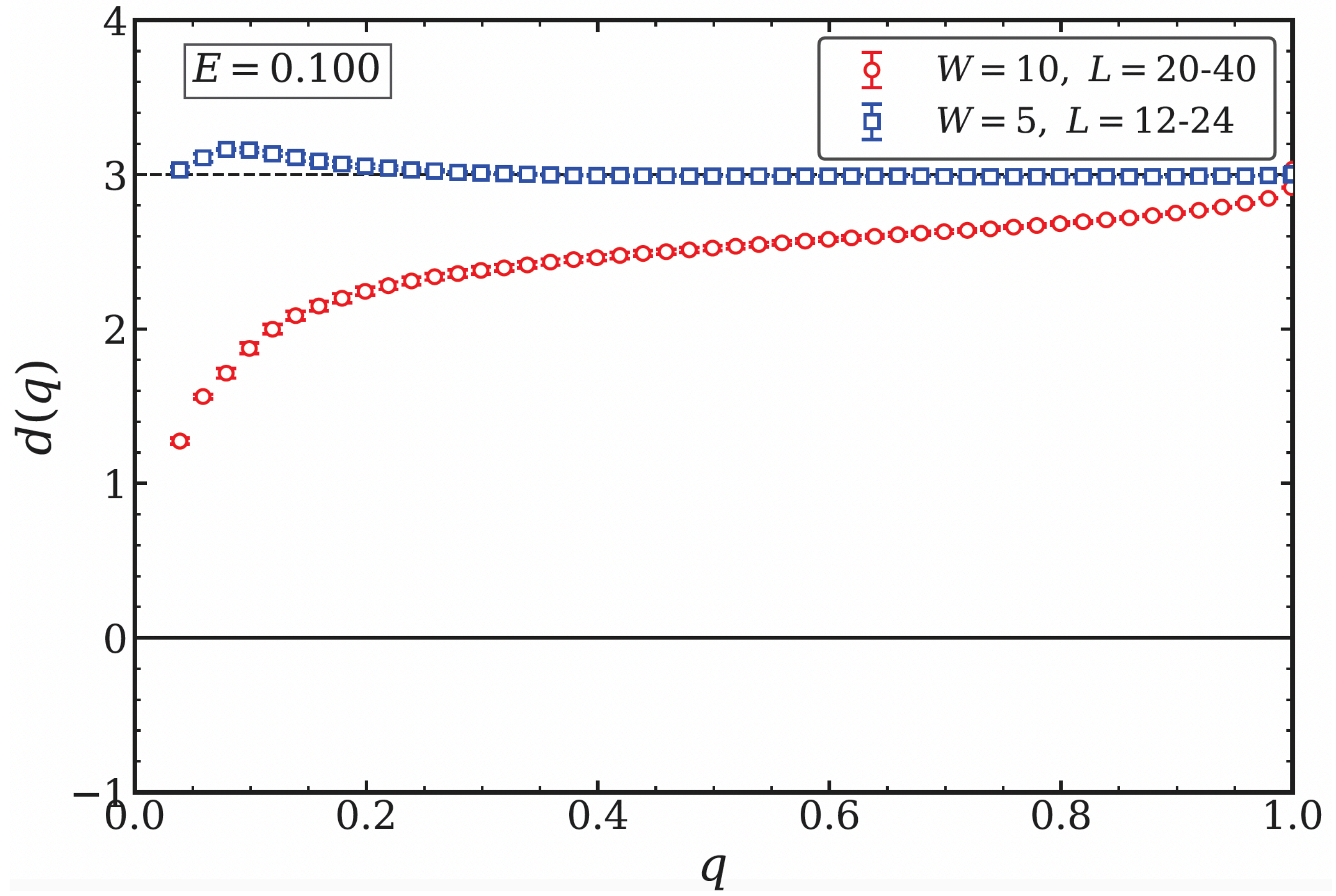
- Extracting the effective dimension $d(q)$ from the scaling behaviour of $\nu(q)$



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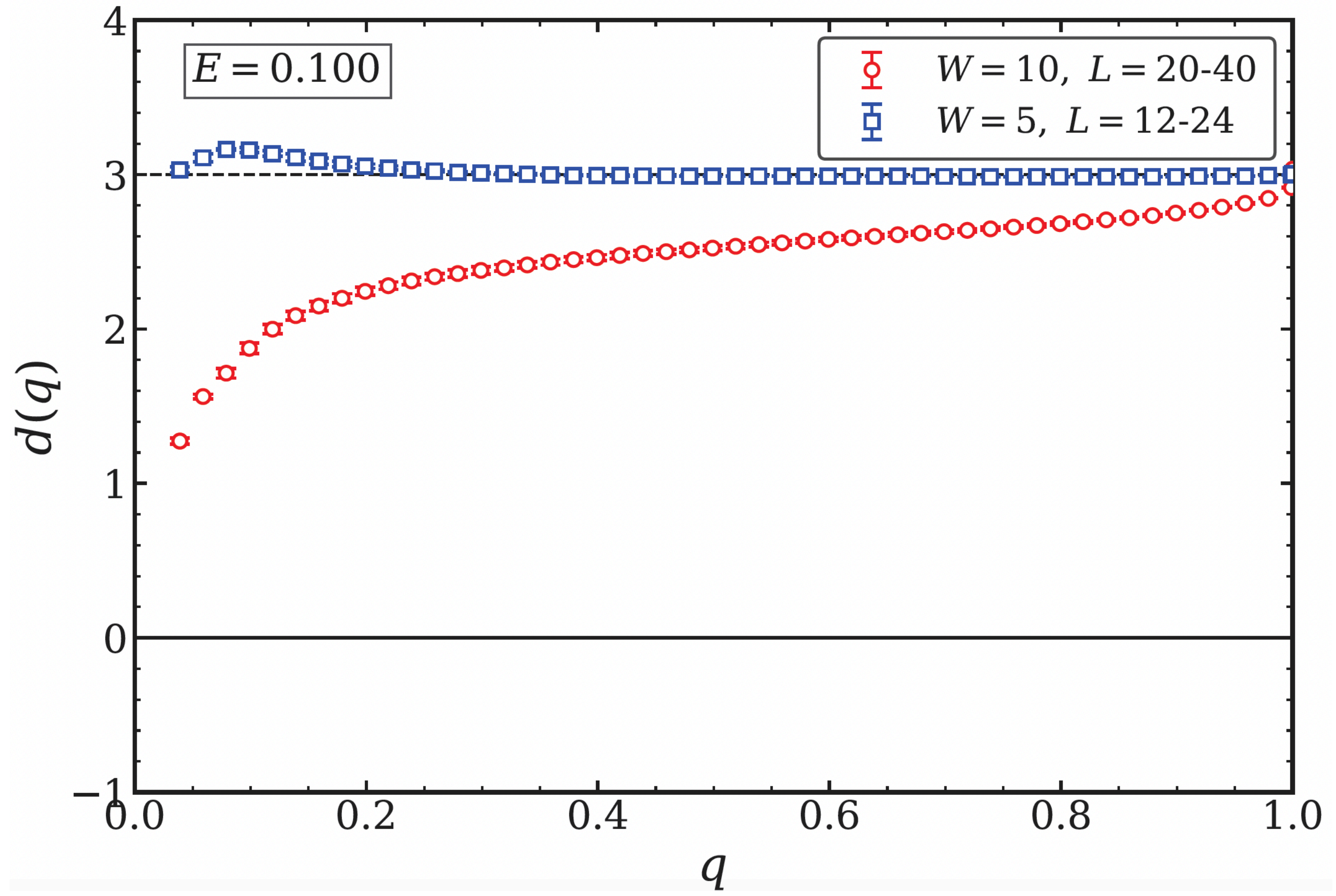

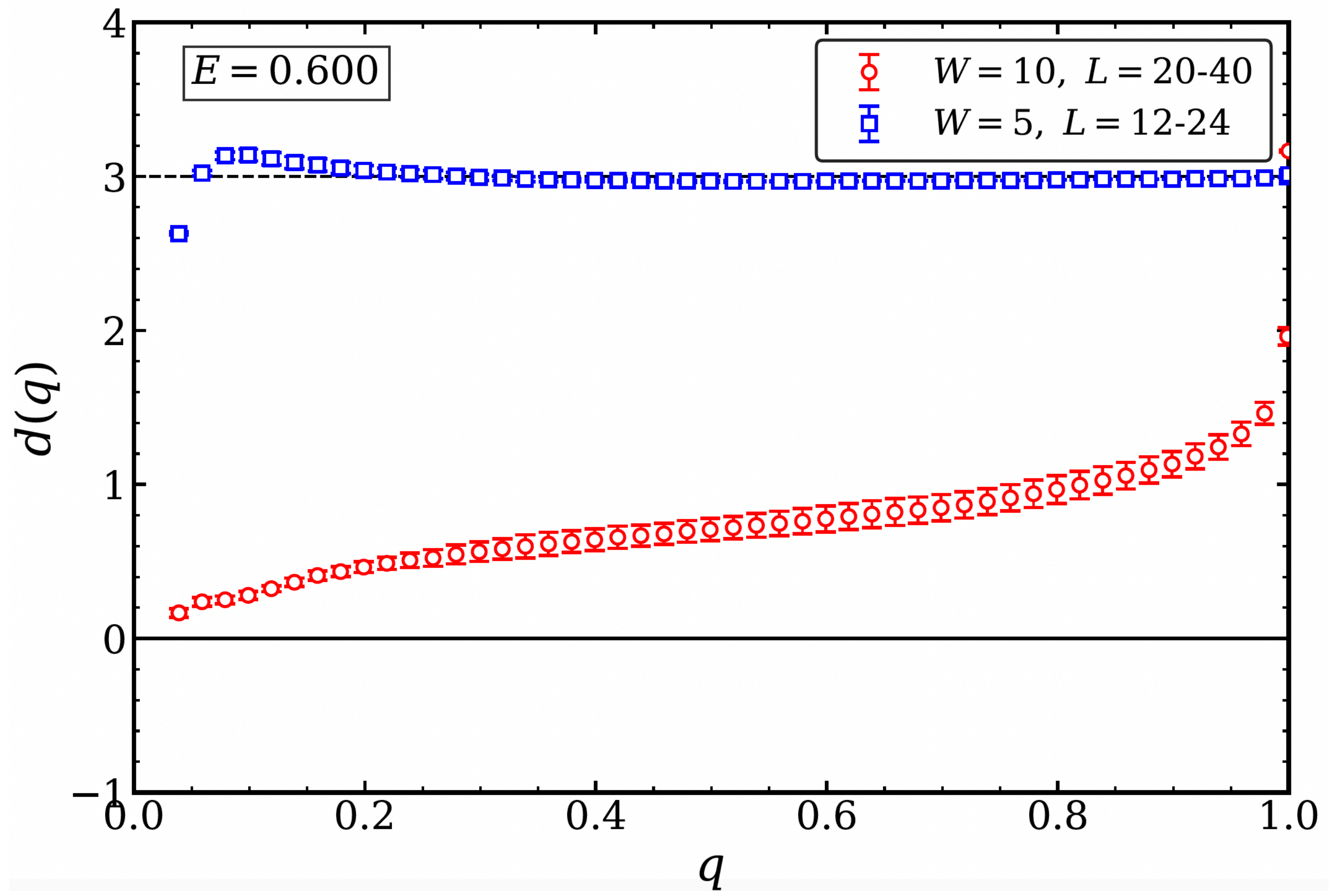

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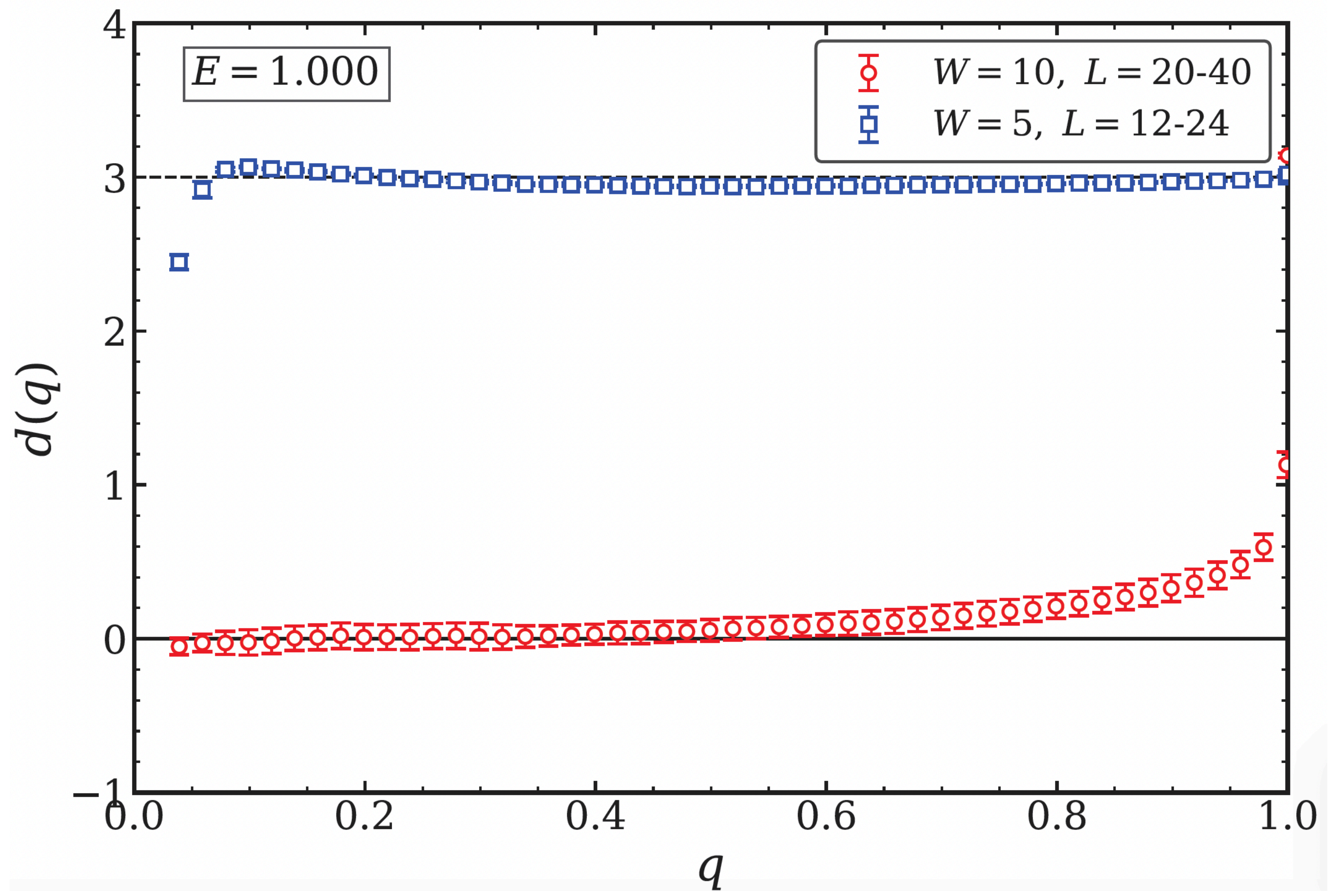

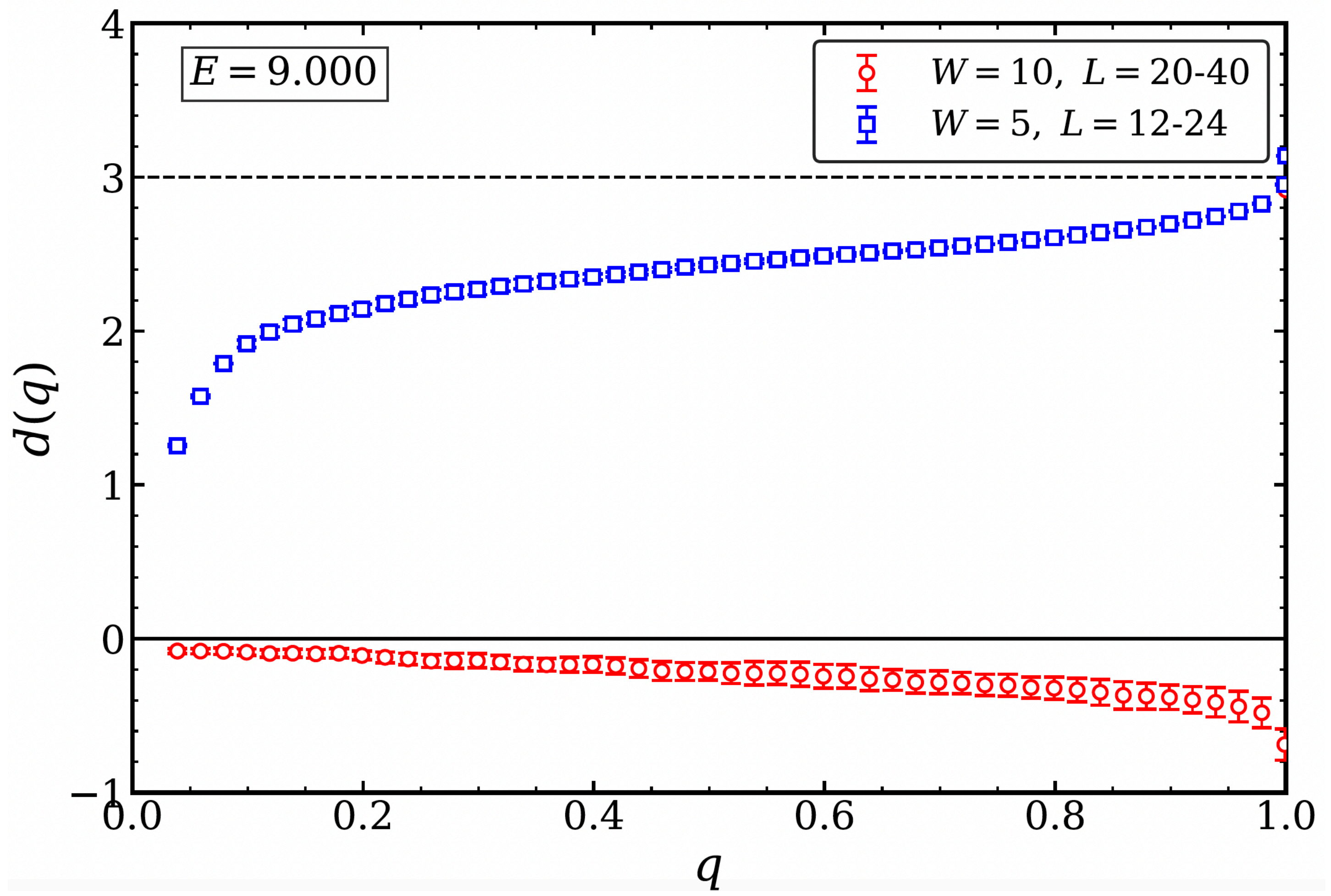

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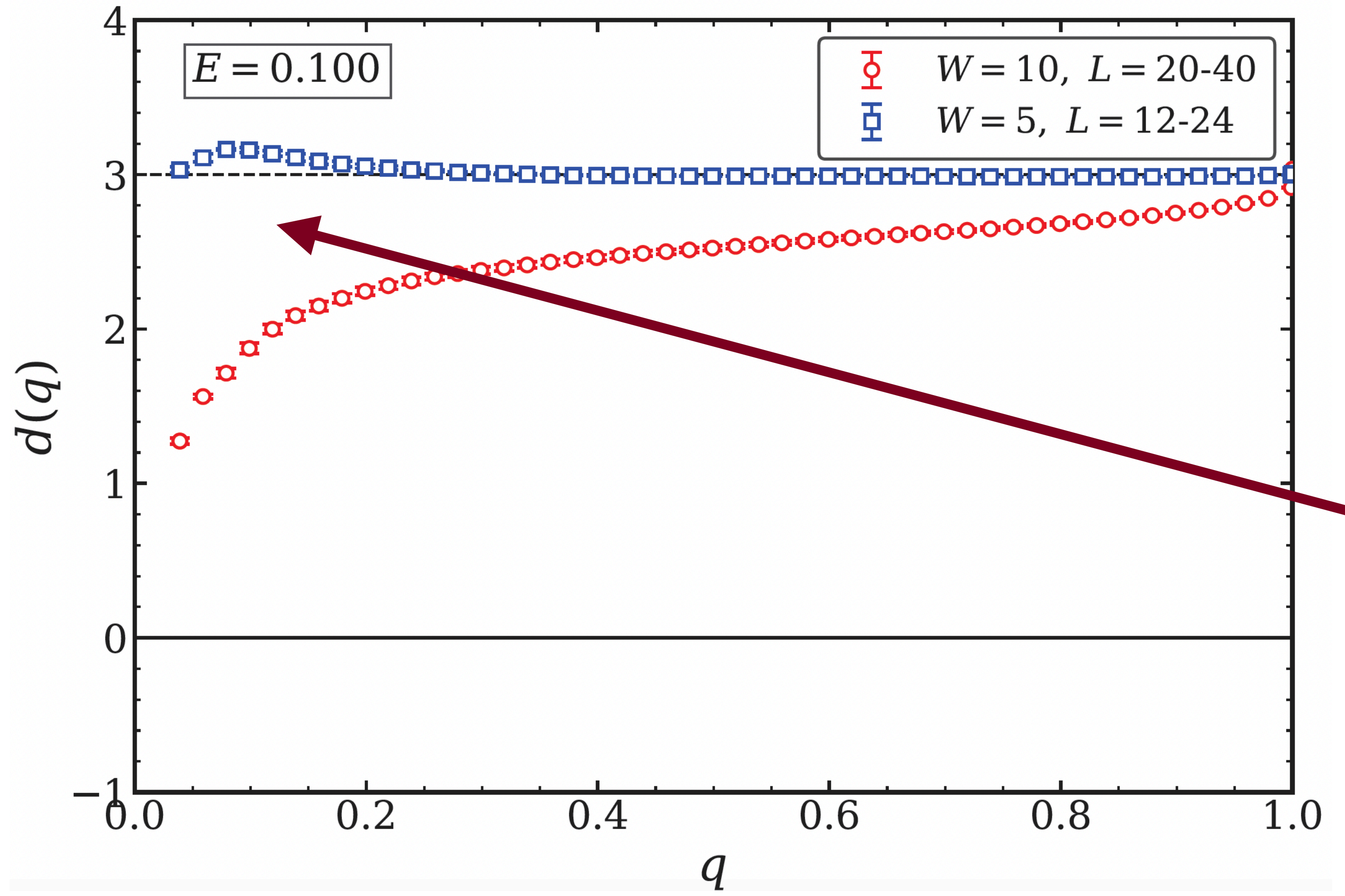

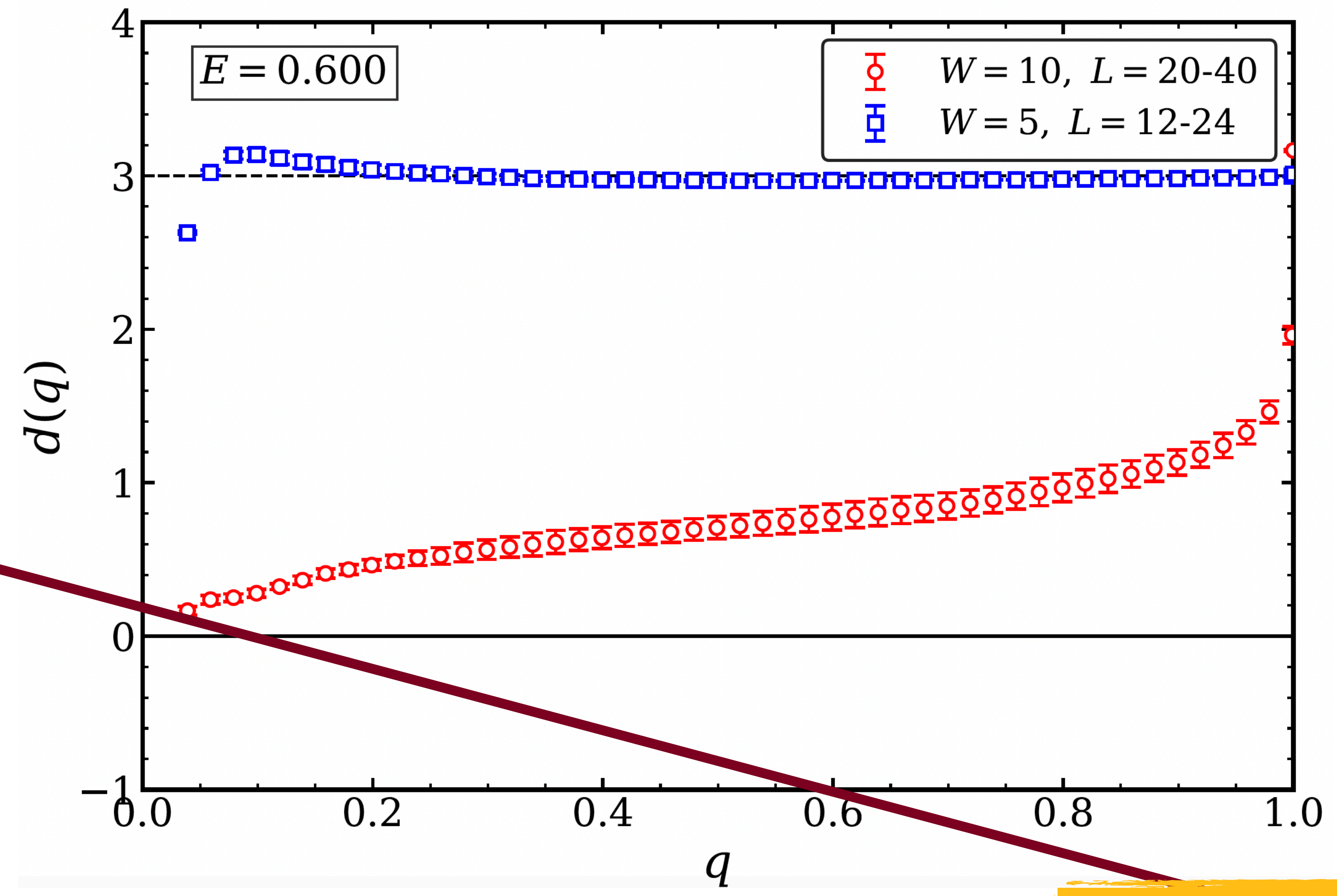

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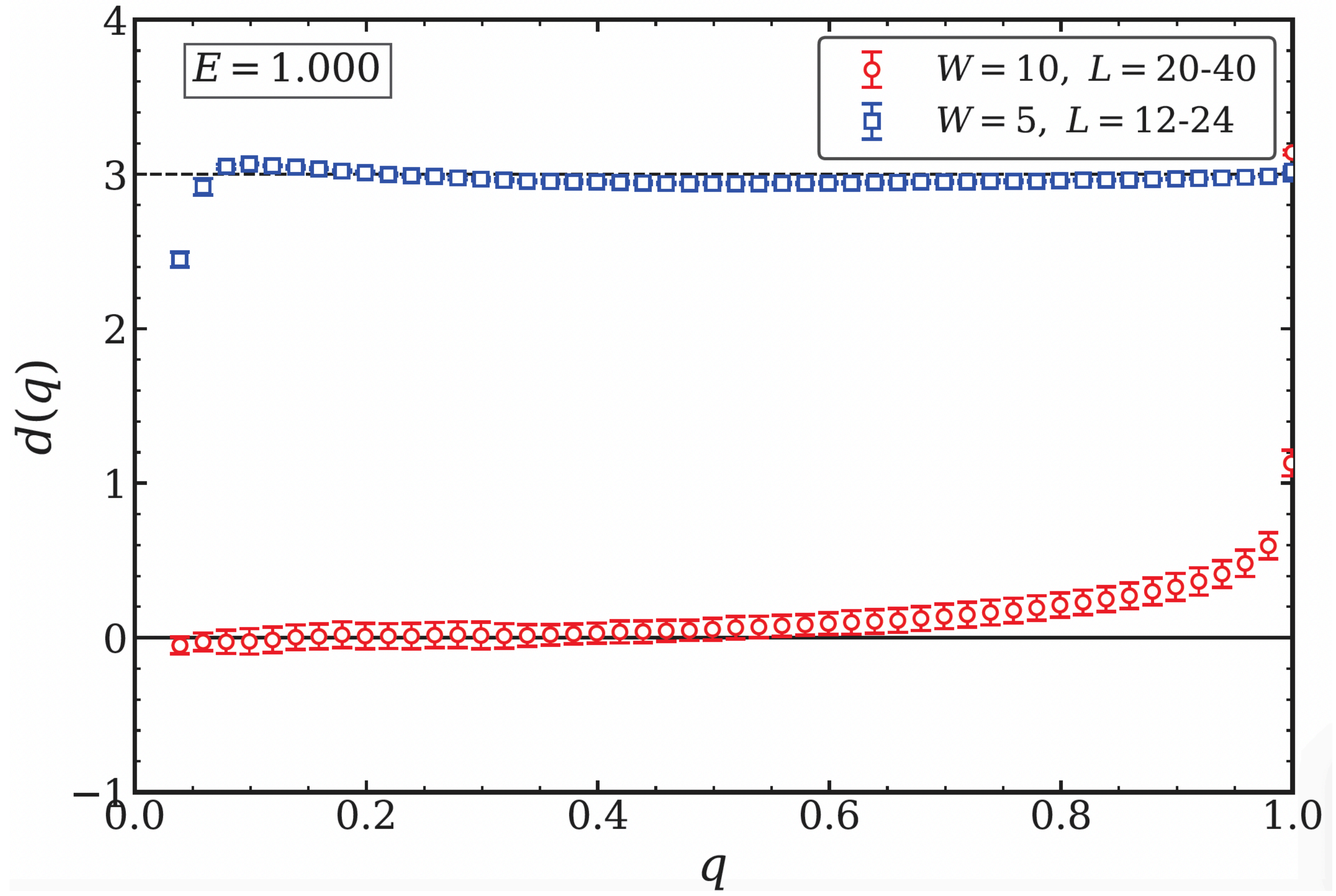

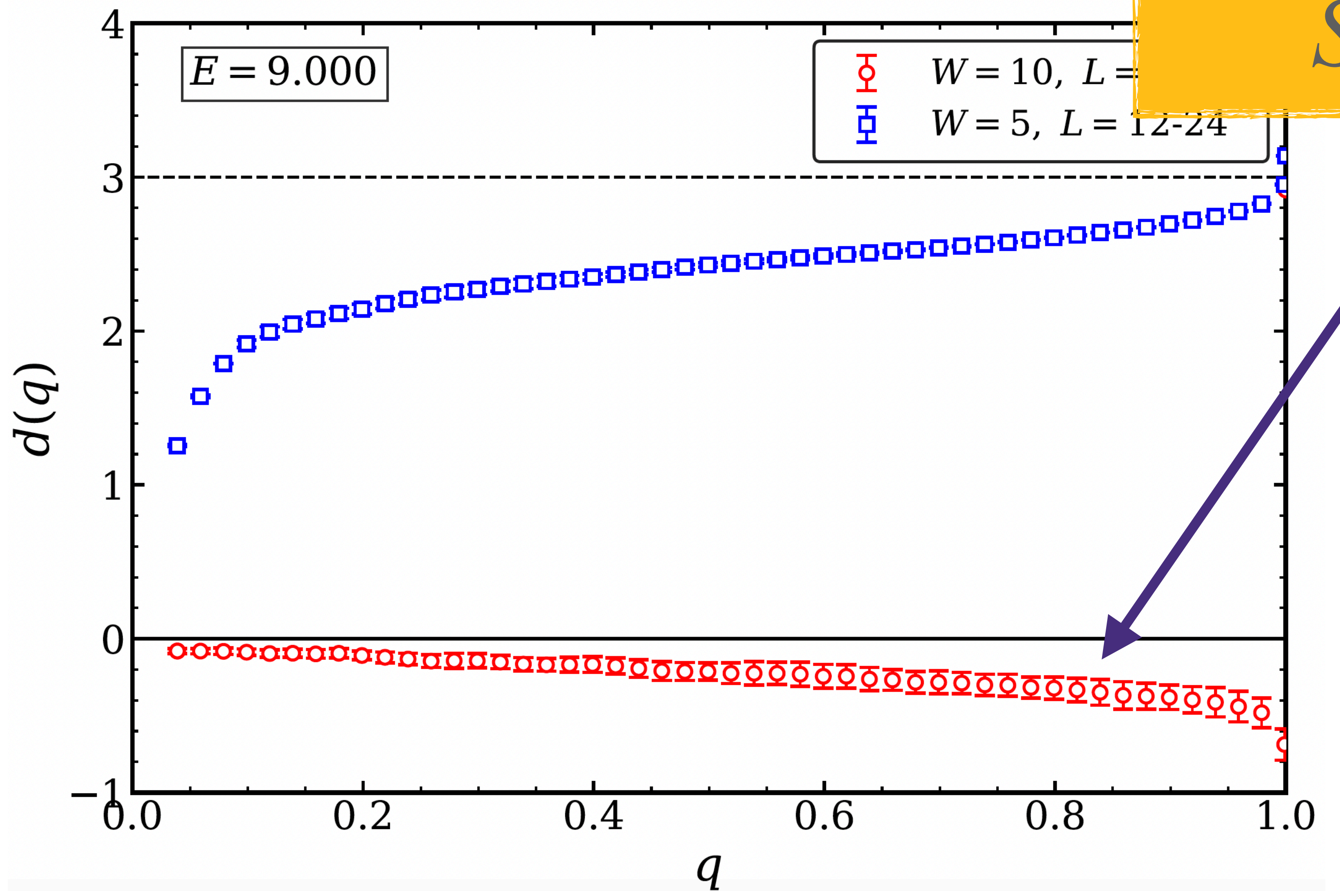

More data…
Sensitive fits…

$P(d)$

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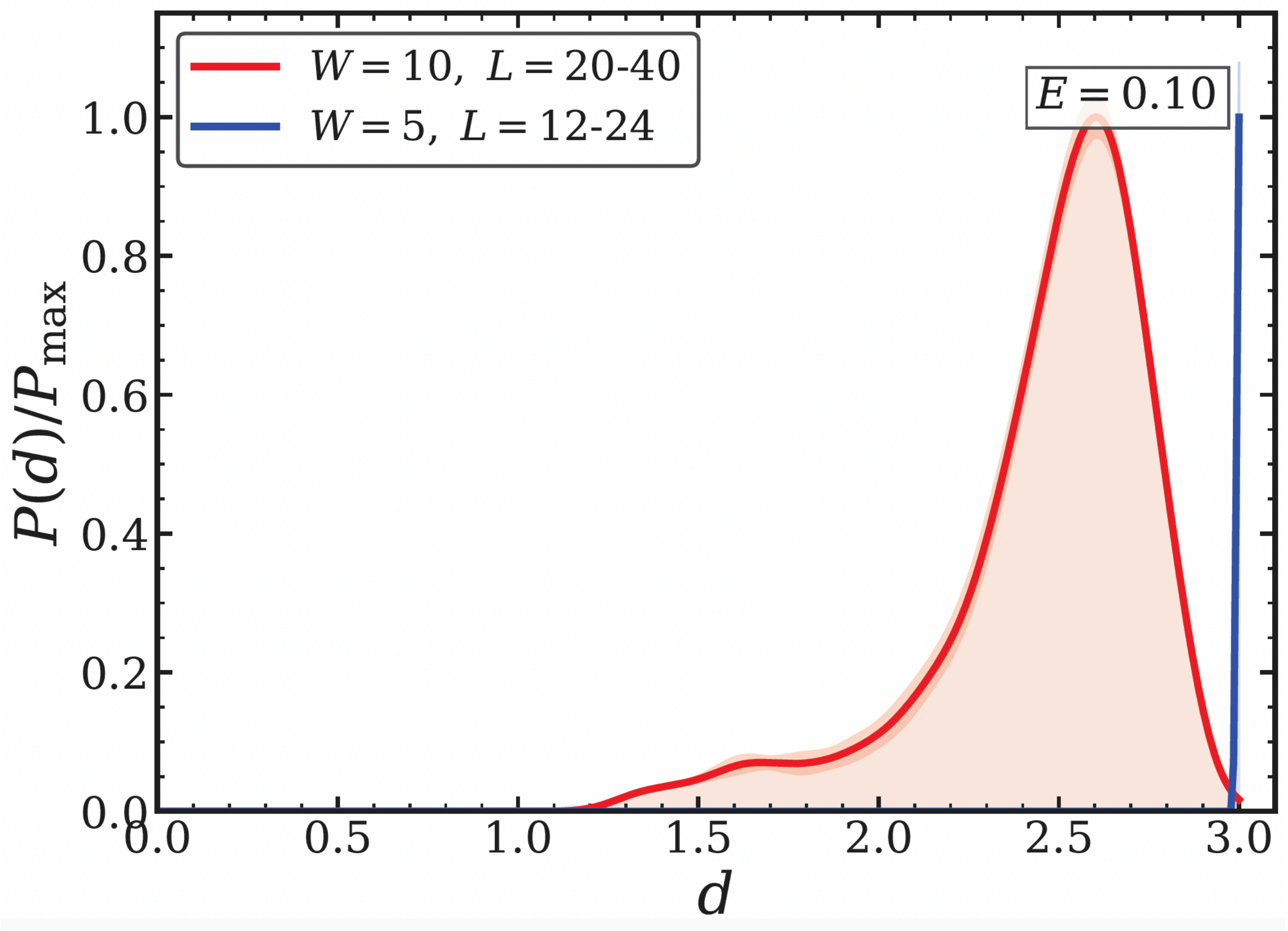
- Constructing the probability distribution $P(d)$ using Kernel Density Estimation (KDE)



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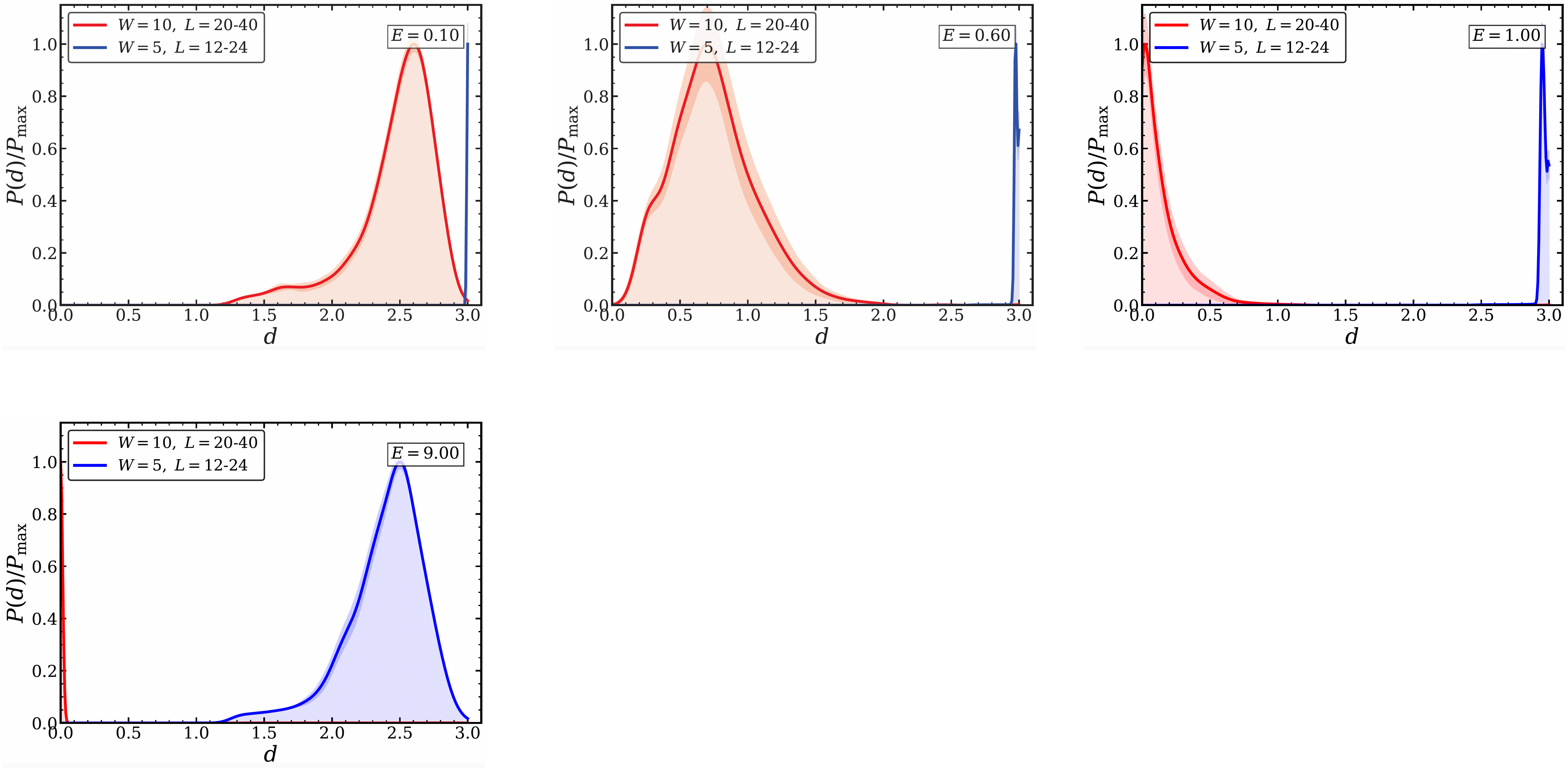

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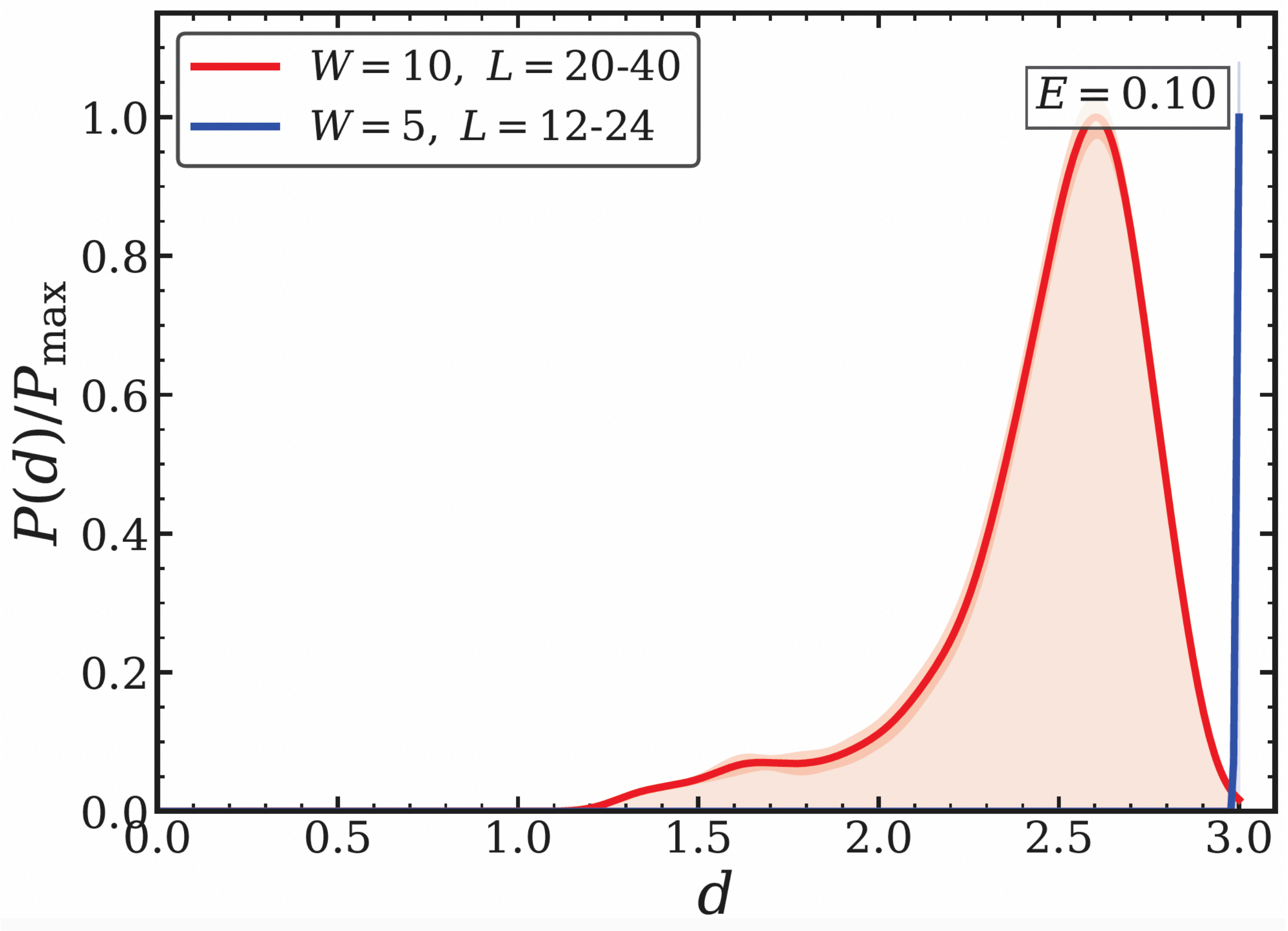

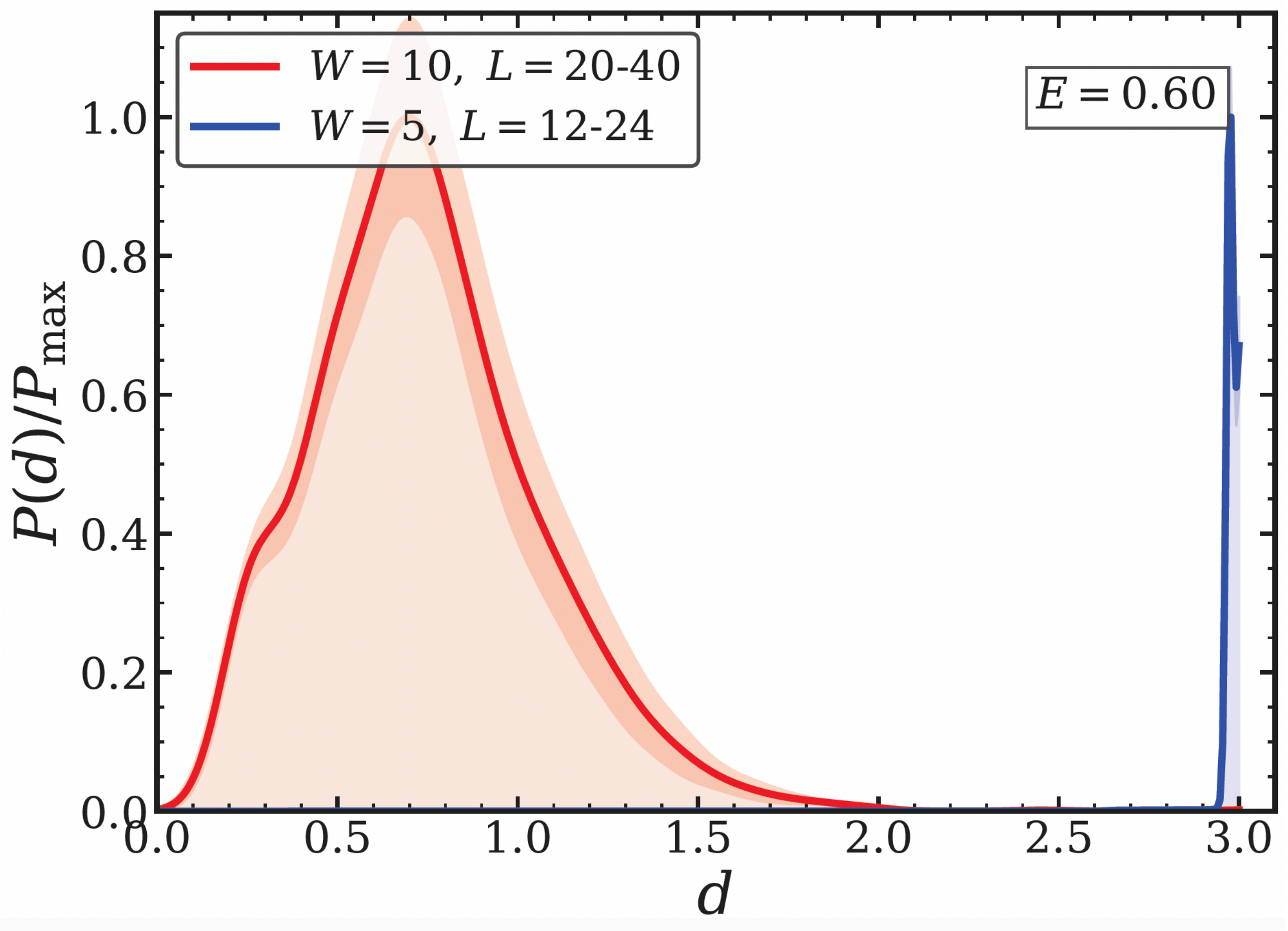

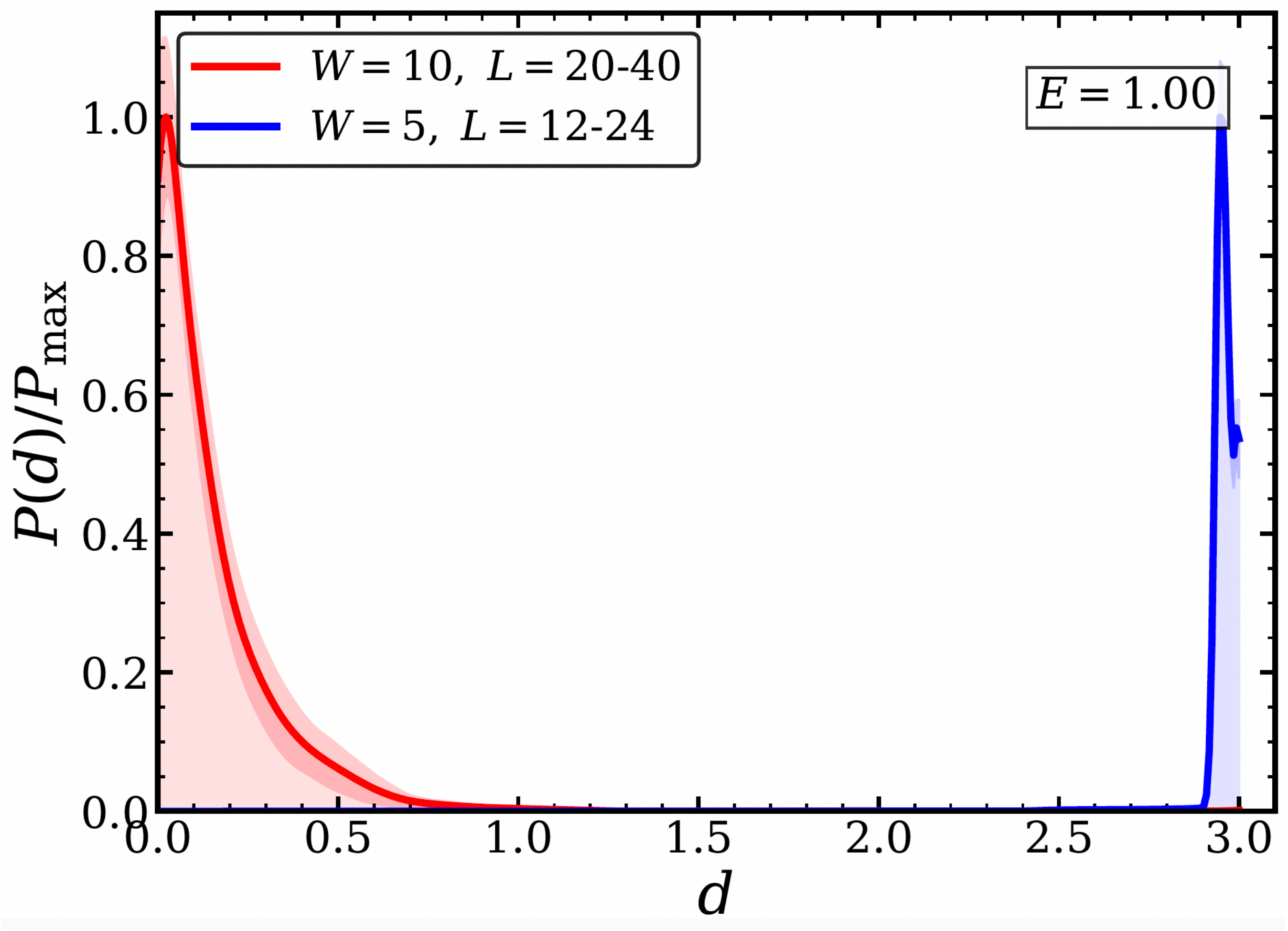

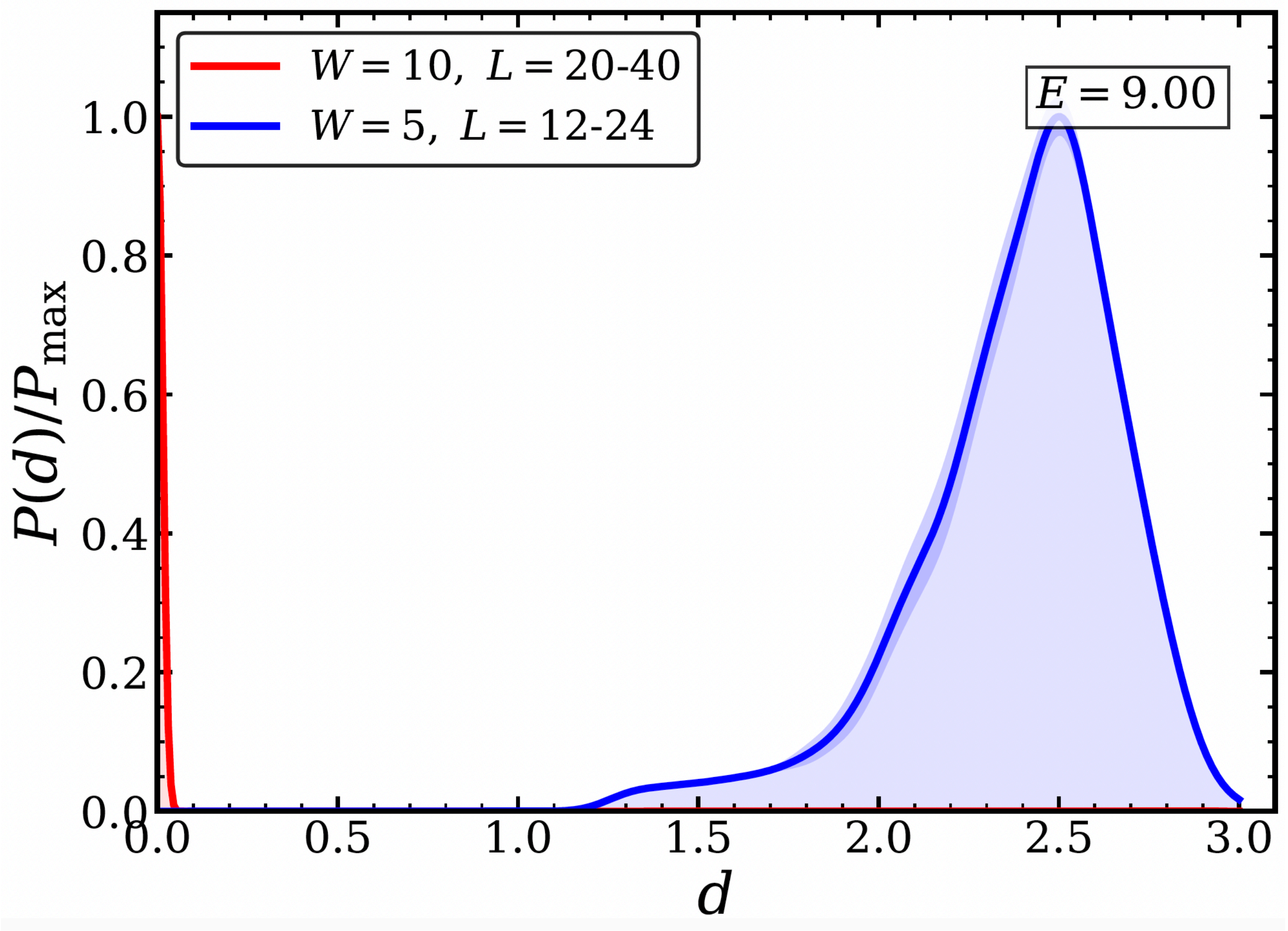

KDE summarizes the full d(q) spectrum into a characteristic dimension. The peak shifts from d ~ 3 to d ~ 0 as localization develops

Maximal and Probable Dimension

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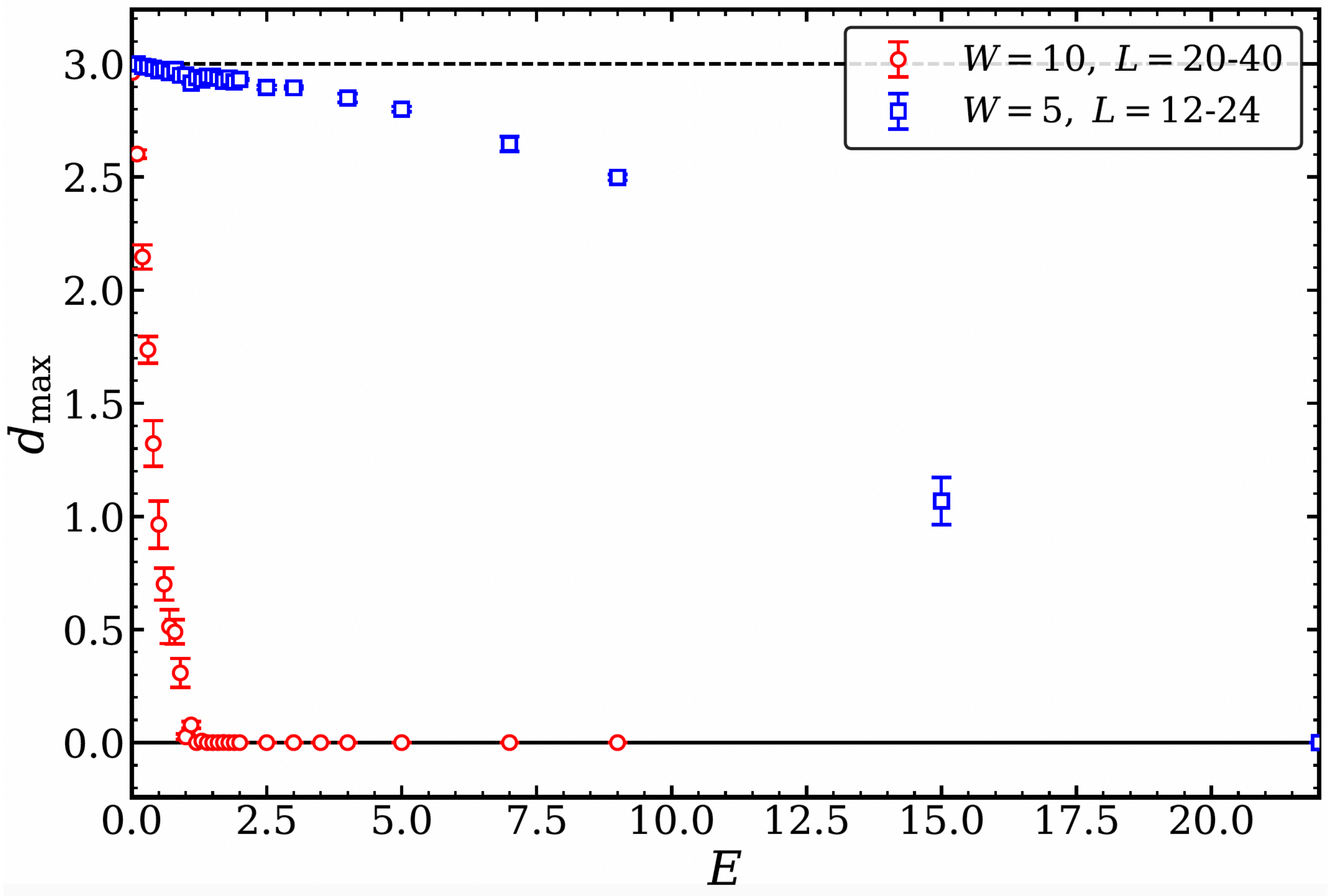
- $d_{max}(q = 0.999)$: the dimension obtained after capturing 99.9% of the eigenstate probability, characterizing the geometry of the full support.



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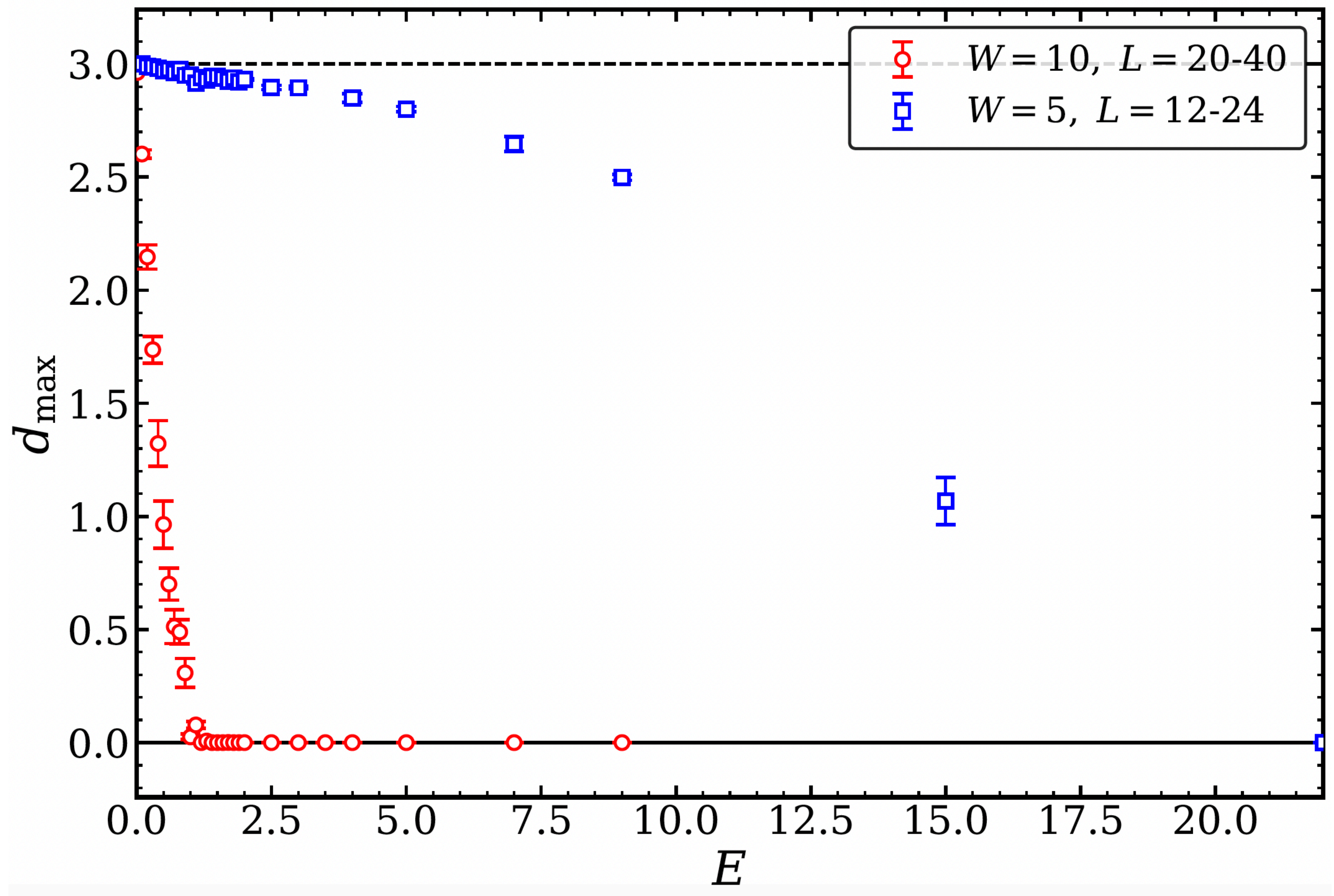

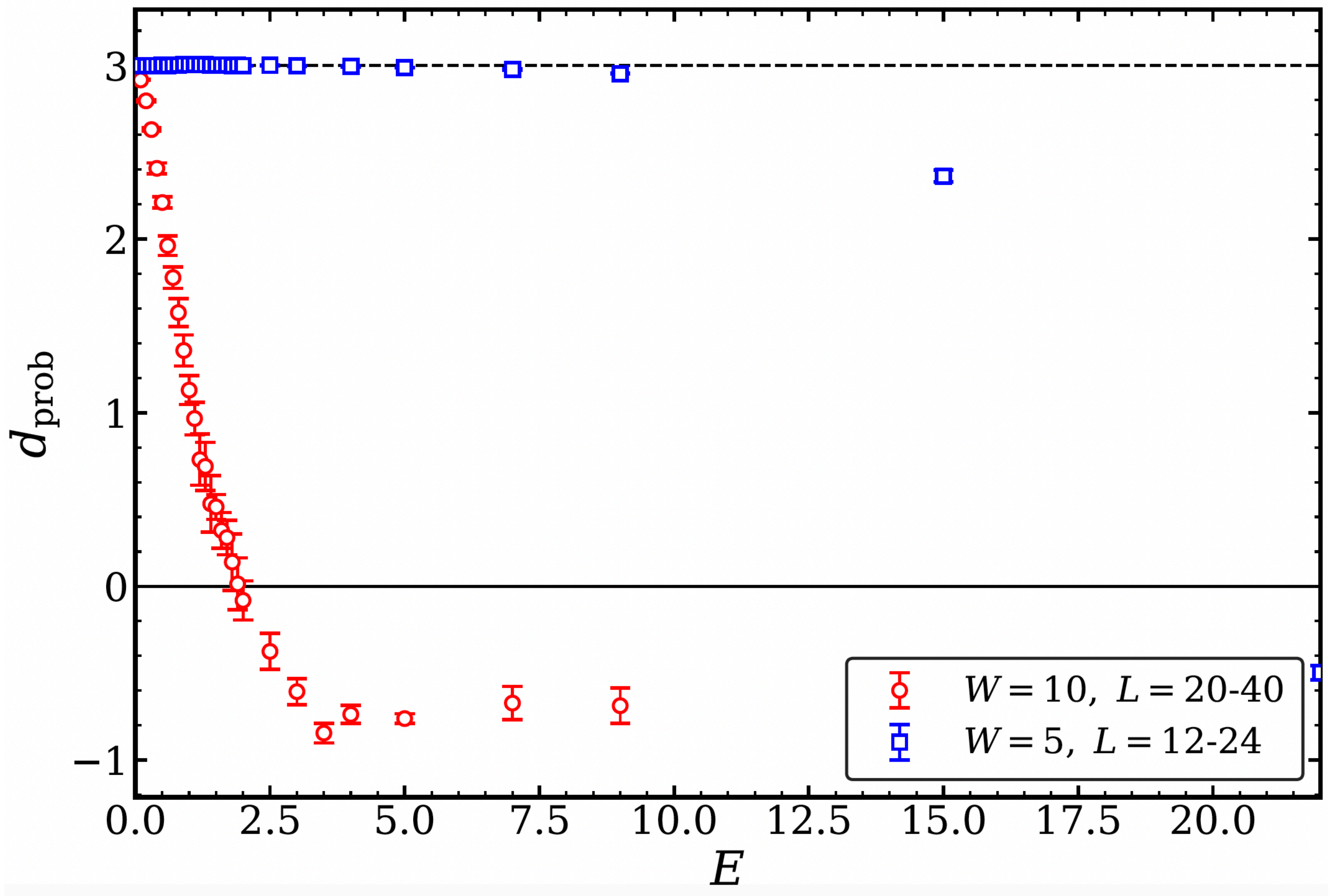

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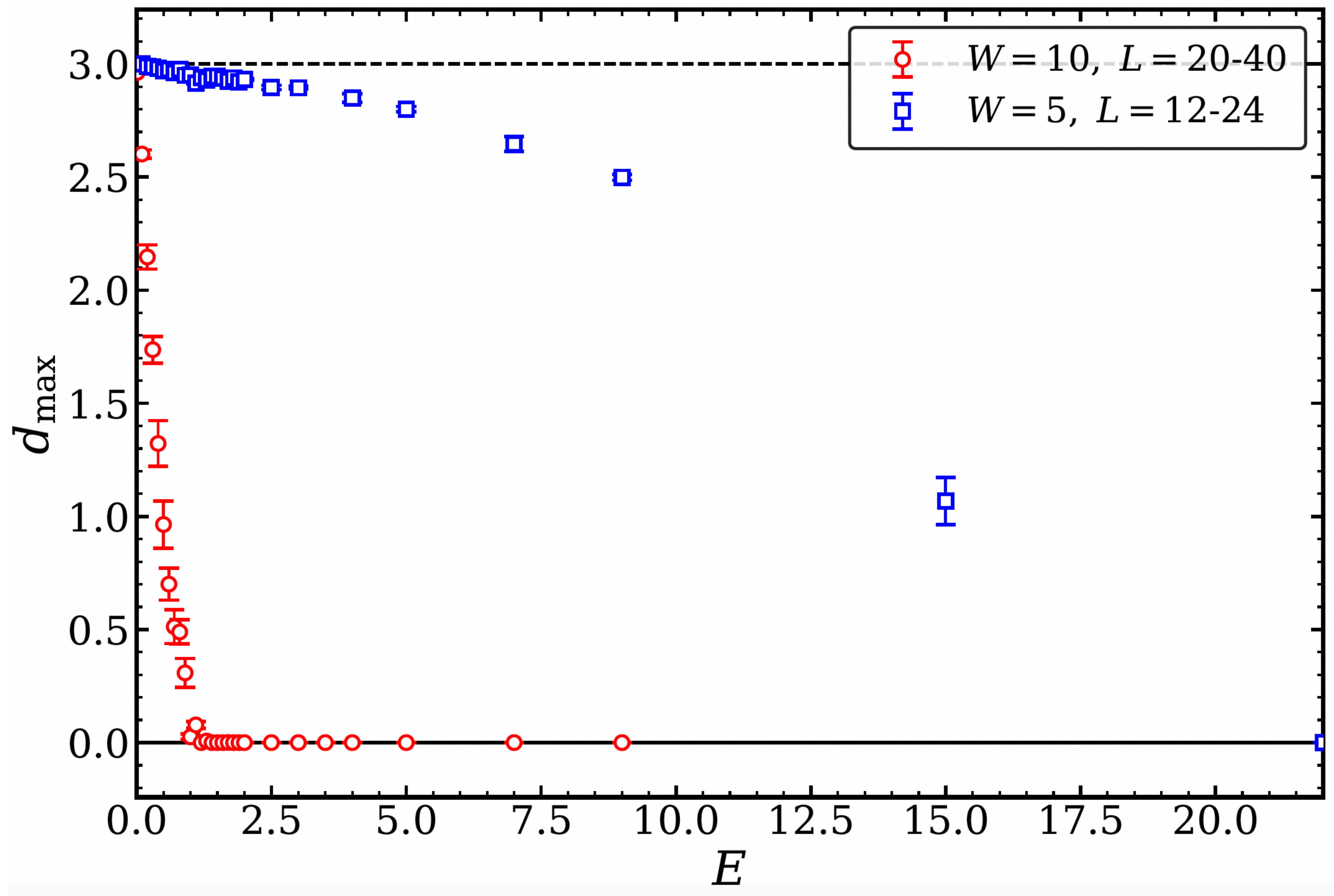

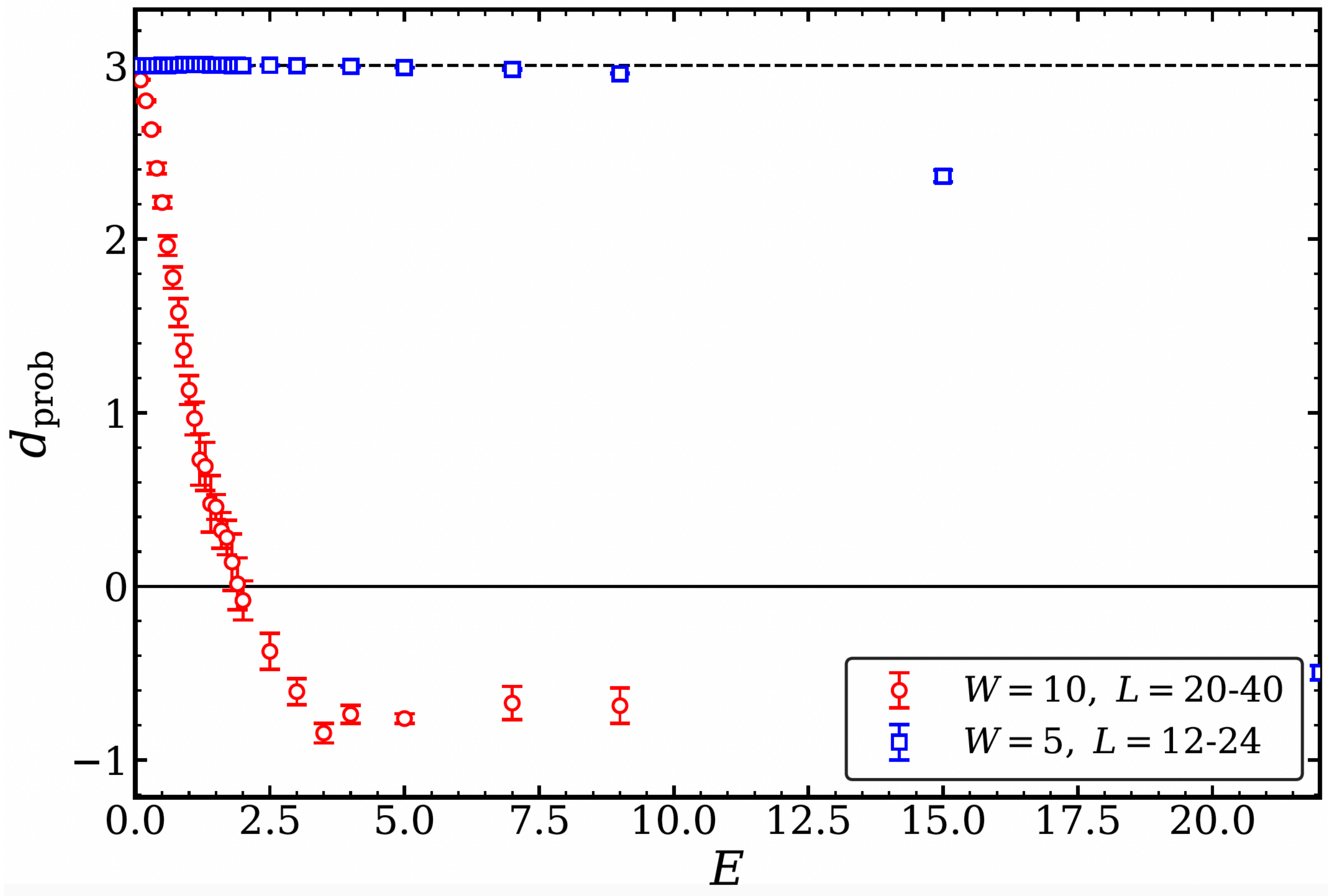

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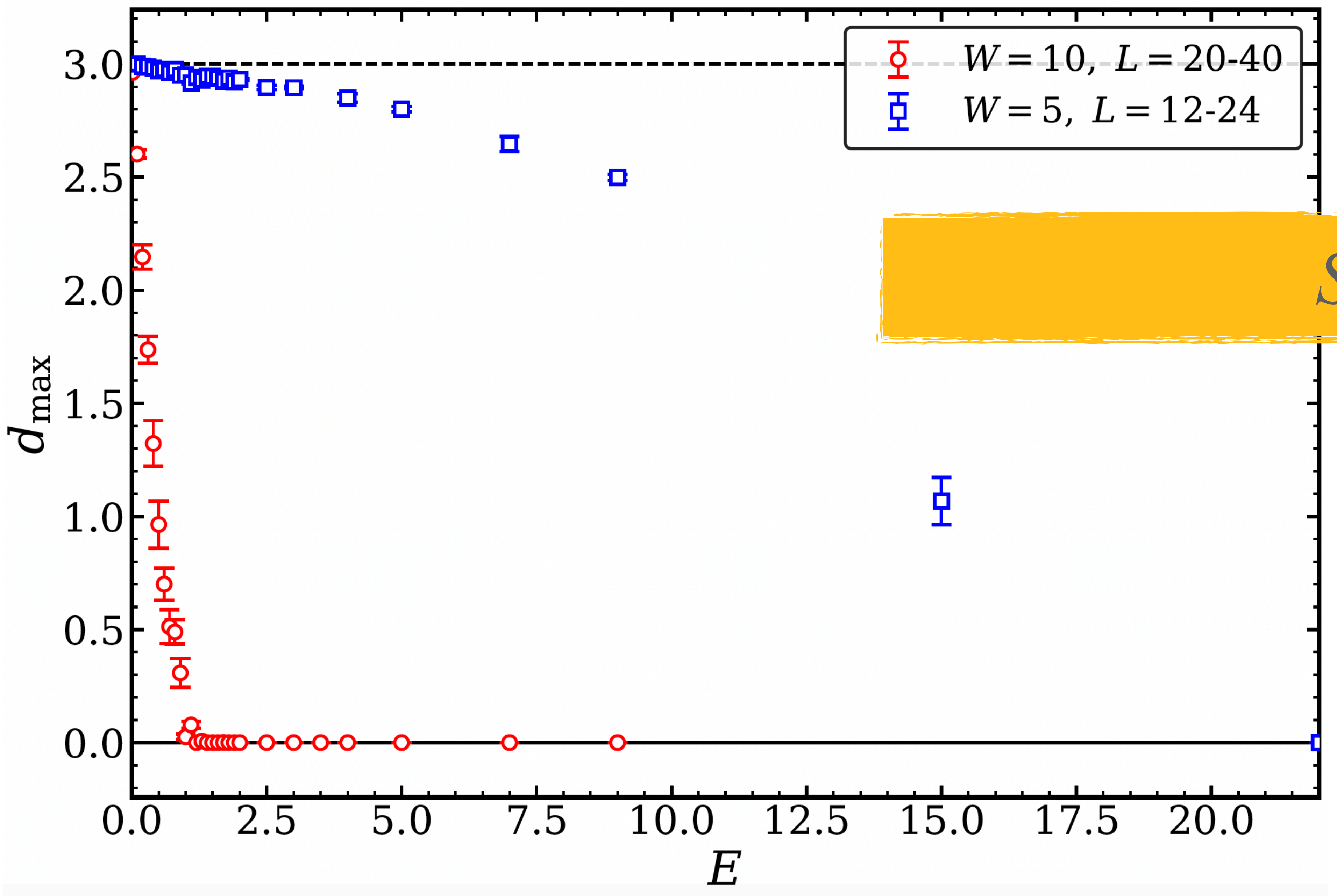
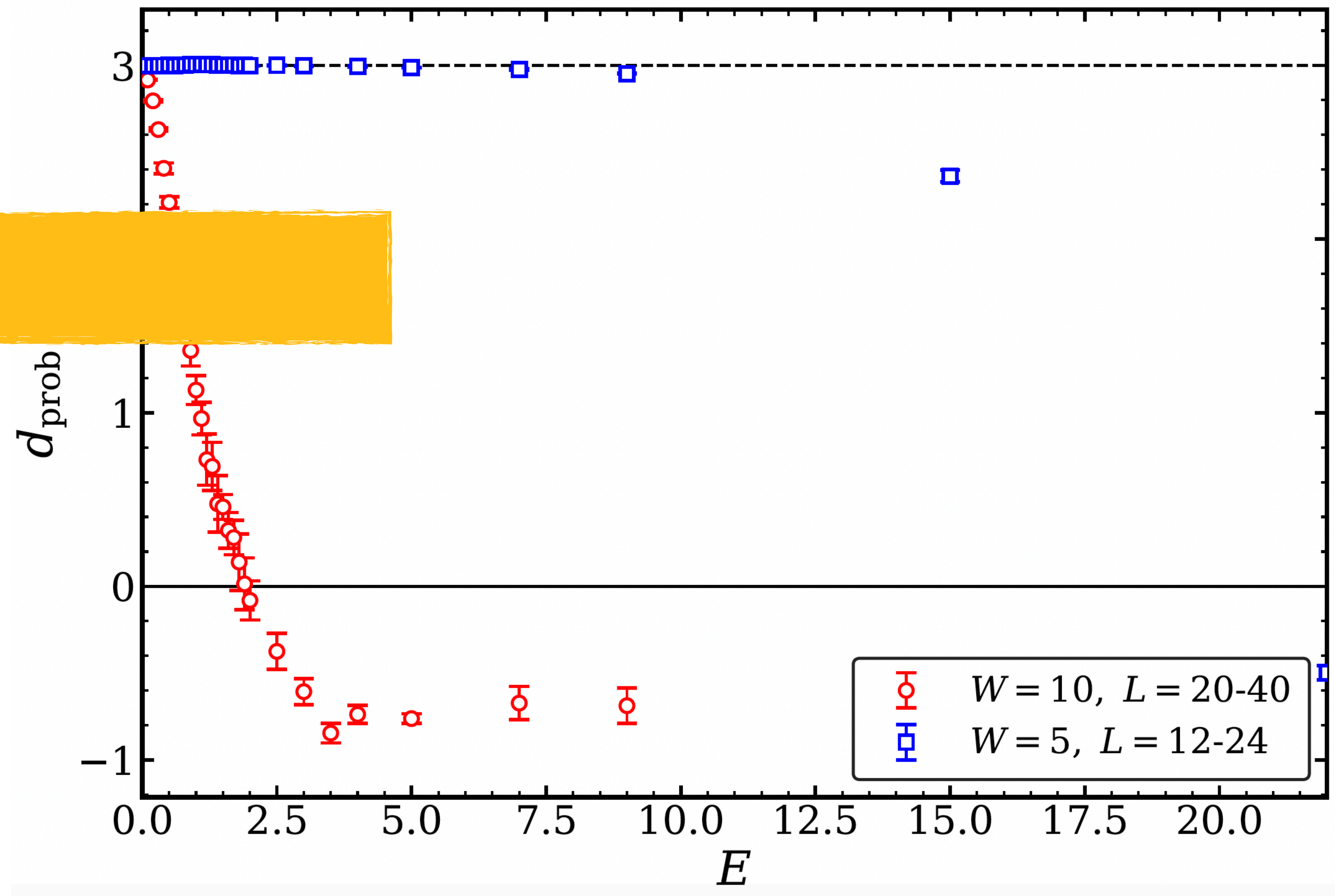


Still exploring



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$W \to \infty\,?$

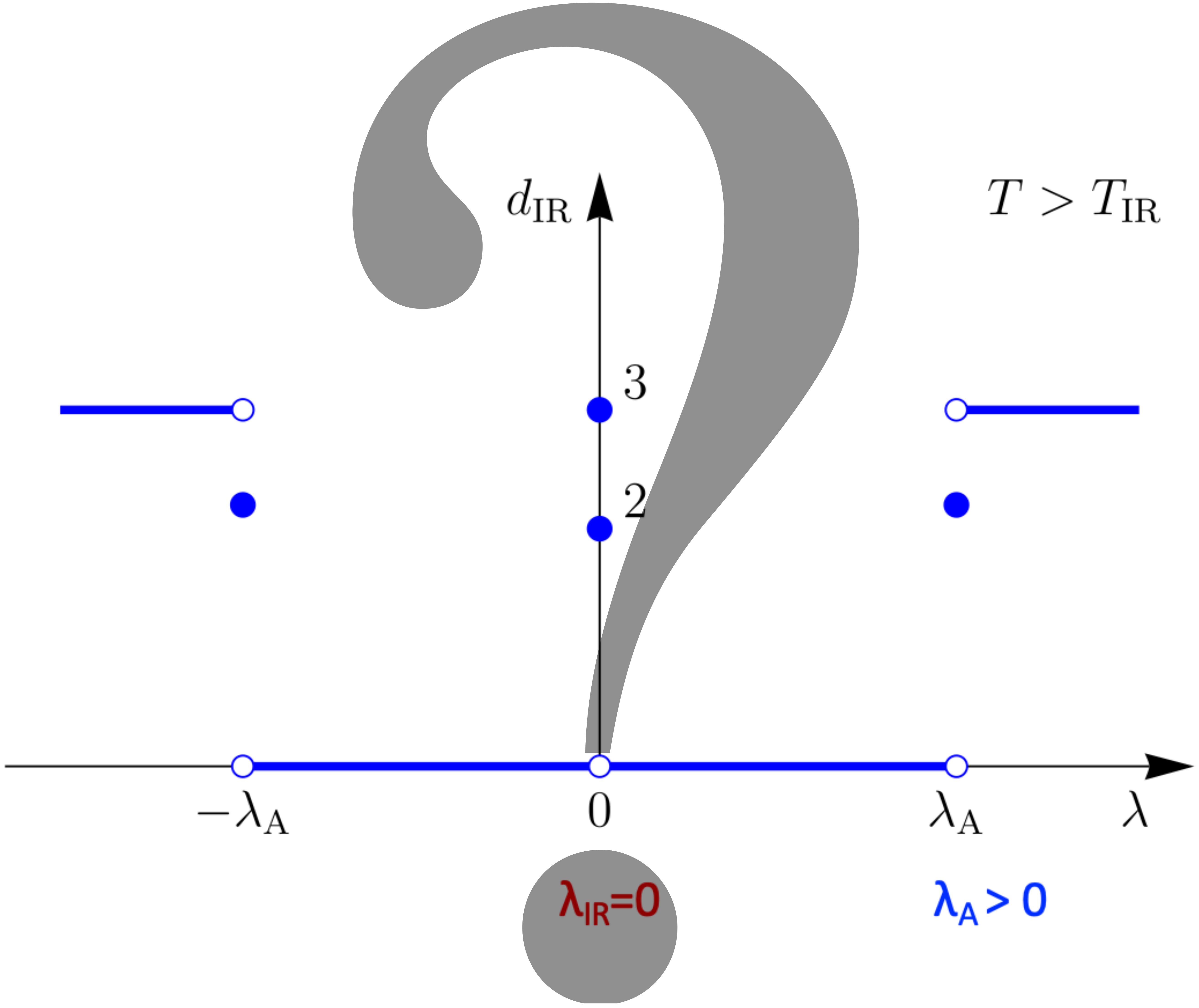

Figure from Ivan

Summary

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- ☆ And more as mentioned in Ivan's talk

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tata institute of
fundamental
research

THANK YOU

LATTICE 2026

Navdeep Singh Dhindsa

31/07/26

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