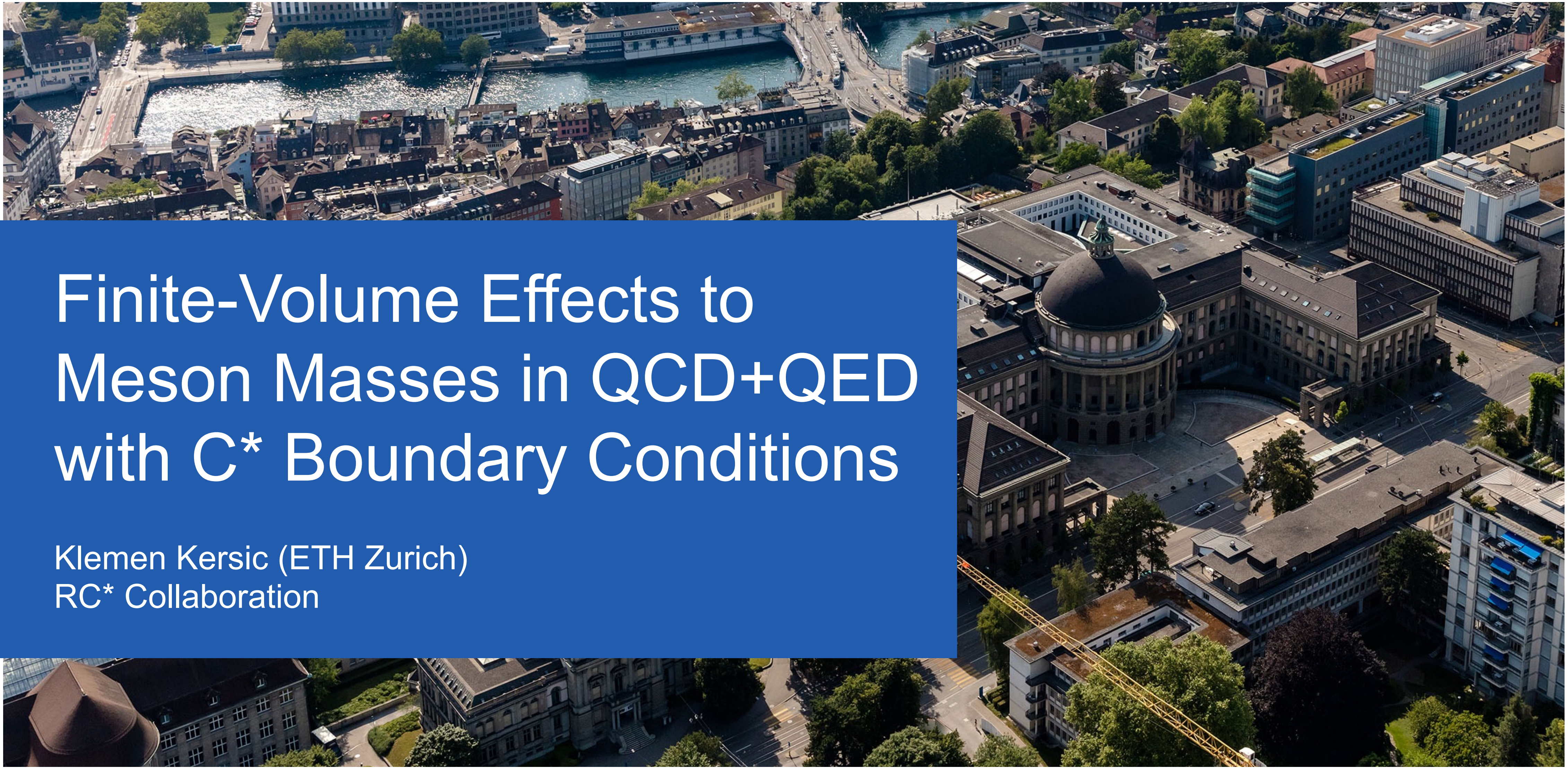
ETH zürich

HPCP Group, ITP



Finite-Volume Effects to Meson Masses in QCD+QED with C* Boundary Conditions

Klemen Kersic (ETH Zurich)
RC* Collaboration

The 43rd International Symposium on Lattice Field Theory

Overview

- Motivation
- The Setup and Methodology
- **Results**: $\pi^{\pm}$ Masses in Finite Volume
- **Discussion**: Analytical Predictions in the Continuum
- Conclusions

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Motivation

Precision Lattice Computations

$$\%$$

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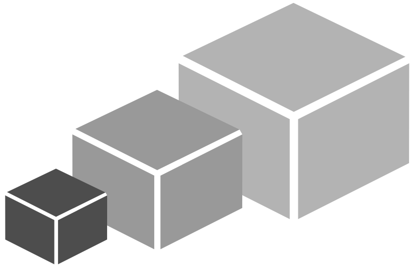
Control of of finite-volume effects (FVE)



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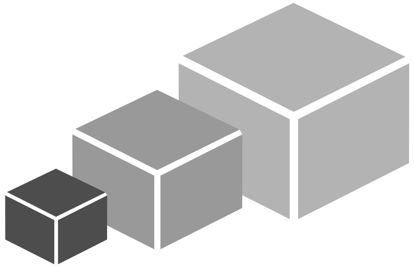

Inclusion of isospin-breaking effects (IBE)

$$m_u \neq m_d$$

$$e^2 \neq 0$$

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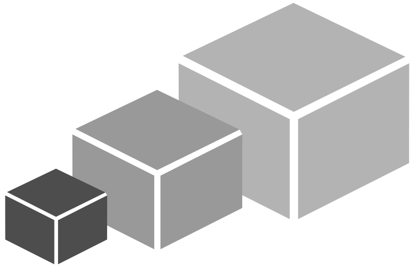

$$\mathcal{O}\left(e^{-mL}\right) \to \mathcal{O}\left(\tfrac{1}{L}\right)$$

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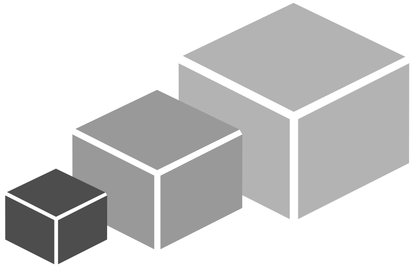

$\mathcal{O}\left(e^{-mL}\right) \to \mathcal{O}\left(\frac{1}{L}\right)$

QED regularisation dependence

Inclusion of isospin-breaking effects (IBE)

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$$e^2 \neq 0$$

The Setup and Methodology: QED in Finite Volume

Challenge of QED in finite volume:

Gauss' law constraint

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Photon's zero mode

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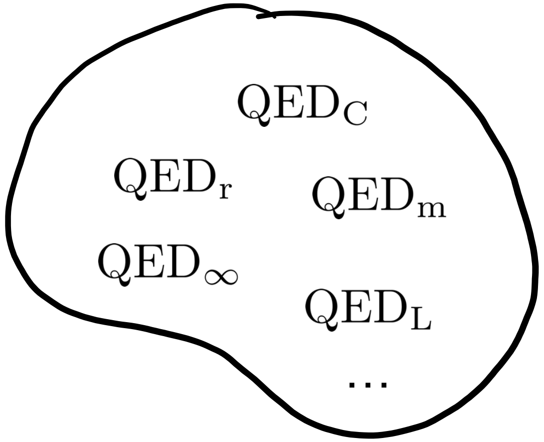
Approaches:



[$\mathrm{QED_L}$] S. Uno and M. Hayakawa, Prog Theor Phys **120**, 413 (2008).
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The Setup and Methodology: QED in Finite Volume

Challenge of QED in finite volume:

$\boxed{\text{Gauss' law constraint}}$

$\wr$

$\boxed{\text{Photon's zero mode}}$

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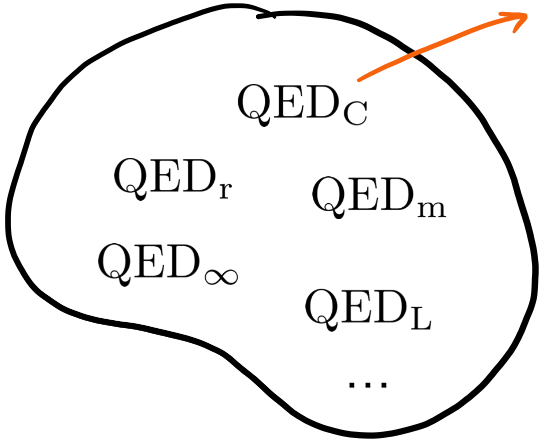

Setup with C* boundary conditions:

- $\mathcal{C}$-conjugate fields at the boundary
- zero mode absent by construction
- $\mathbf{k} \in \Pi_- = \left\{ \frac{\pi}{L}(2\mathbf{n} + \bar{\mathbf{n}}) \;\middle|\; \mathbf{n} \in \mathbb{Z}^3,\ \bar{\mathbf{n}} = (1,1,1) \right\}$

$\boxed{\begin{array}{c}\text{collaboration} \\ \texttt{openQxD}\text{ codebase} \\ \text{(https://gitlab.com/rcstar/openQxD)}\end{array}}$

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The Setup and Methodology: Rome 123

Isospin breaking effects via Rome 123 (RM123) method [G. M. De Divitiis, et. al., Phys. Rev. D **87**, 114505 (2013)].

- Perturbatively add IBE onto isospin-symmetric QCD (isoQCD):

isoQCD		strong		electromagnetic
$(e^2 = 0, m_u = m_d)$	$+$	$(m_u \neq m_d)$	$+$	$(e^2 \neq 0)$

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- IsoQCD defined as:

$$\sqrt{8t_0} = 0.415\,\mathrm{fm}$$

$$8t_0(M_{K^\pm}^2 - M_{\pi^\pm}^2) = 0$$

$$8t_0(M_{K^\pm}^2 + M_{\pi^\pm}^2 + M_{K^0}^2) = 2.11$$

$$\sqrt{8t_0}(M_{D_s^\pm} + M_{D^0} + M_{D^\pm}) = 12.1$$

[M. Bruno, T. Korzec, and S. Schaefer, Phys. Rev. D **95**, 074504 (2017).,
The RCstar collaboration, *J. High Energ. Phys.* **2025**, 158 (2025)]

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$$\underbrace{\text{isoQCD}}_{(e^2=0,\, m_u=m_d)} + \underbrace{\text{strong}}_{(m_u \neq m_d)} + \underbrace{\text{electromagnetic}}_{(e^2 \neq 0)}$$

- Observable expansion:

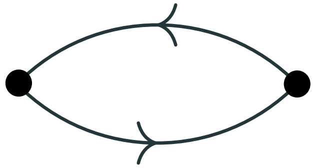
$$X = X^{\text{isoQCD}} + \sum_f \Delta m_f \partial_{m_f} X + (\Delta g_s)^2 \partial_{g_s^2} X + e^2 \partial_{e^2} X + \mathcal{O}\left(e^4, (\Delta g_s)^2\right)$$

Pseudoscalar Meson Mass with RM123

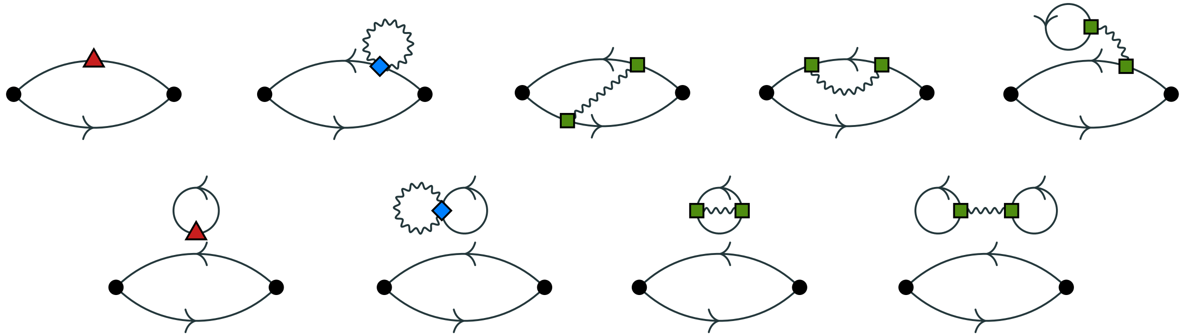
$\pi^{\pm}$ mass:

$$M = M_{\mathrm{isoQCD}} + e^2 \partial_e^2 M\Big|_{e^2=0} + \sum_f \Delta m_f \, \partial_{m_f} M\Big|_{\Delta m_f = 0}$$

effective mass:



effective $\partial_i M$:



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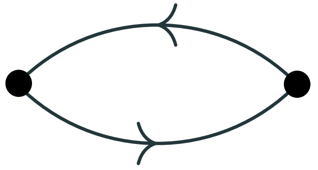

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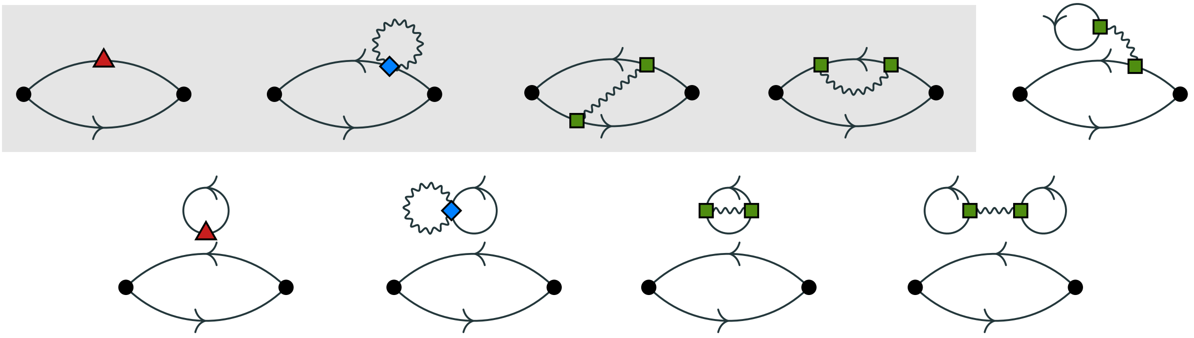

The Setup and Methodology: Ensembles

Three (four) ensembles with by the RC* collaboration [The RCstar collaboration, *J. High Energ. Phys.* **2023**, 12 (2023)]:

- C* B.C
- isoQCD ($e^2 = 0\,, m_u = m_d$)
- $N_f = 3 + 1$
- $m_\pi = m_K \sim 400\,\mathrm{MeV}$
- equal bare parameters
- $a = 0.054\,\mathrm{fm}$
- $L = 1.7 - 3.5\,\mathrm{fm}$

A400	64×32^3	$m_\pi L \sim 3.5$	with 2000 configurations
AB400	72×36^3	$m_\pi L \sim 4.0$	with 512 configurations (preliminary)
B400	80×48^3	$m_\pi L \sim 5.2$	with 540 configurations
D400	128×64^3	$m_\pi L \sim 7.0$	with 709 configurations

(see also, https://rcstar.gitlab.io/ensembles)

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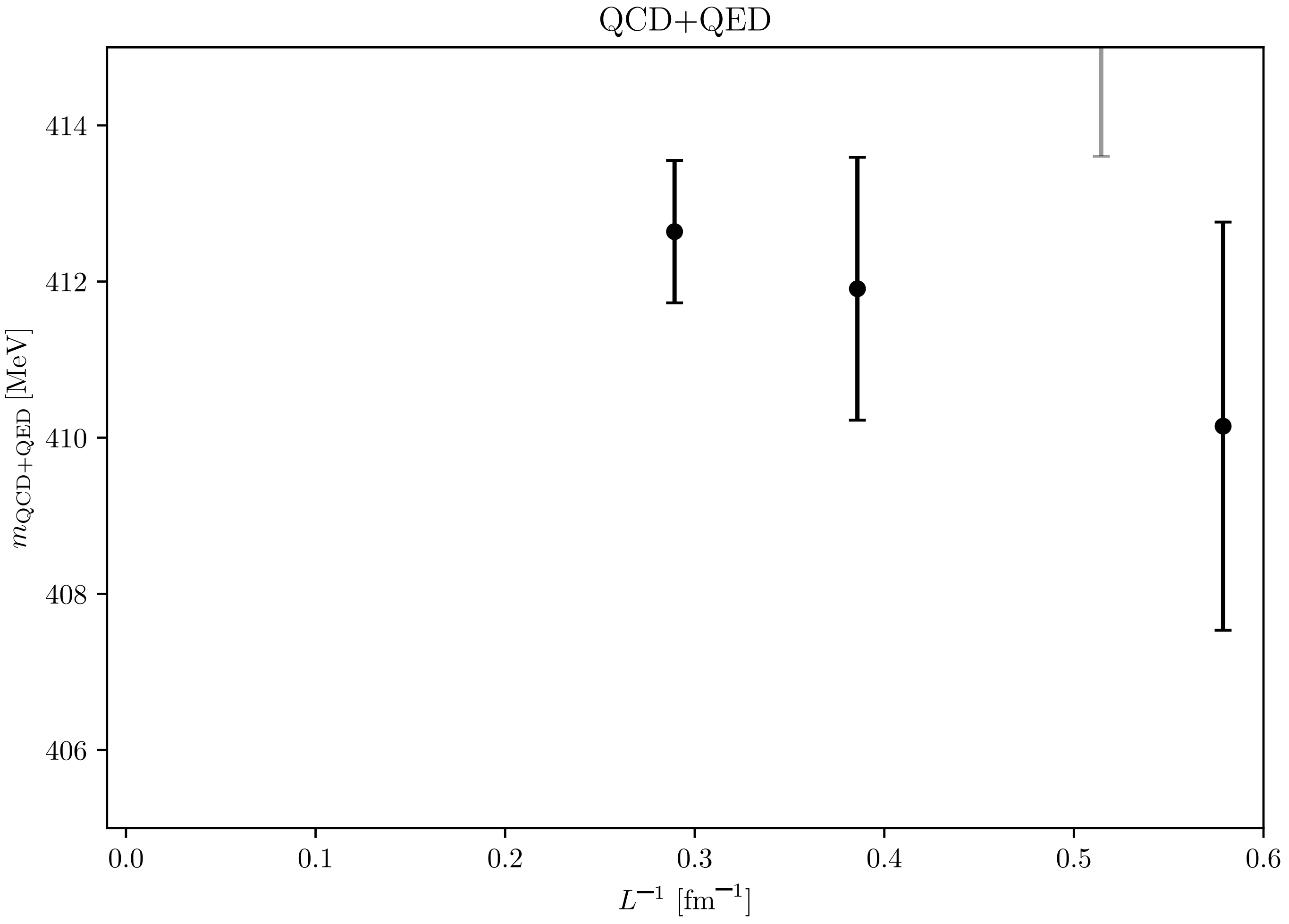
$\pi^{\pm}$ Masses in QCD+QED



Statistics:

- A400 with 14×2000 sources
- AB400 with 12×512 sources
- B400 with 4×540 sources
- D400 with 9×709 sources

Analytical Prediction of Finite-Volume Scaling

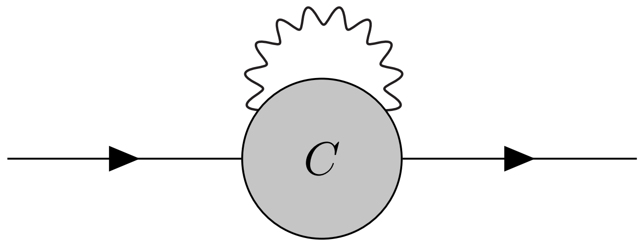
F.V. corrections to meson masses arise as a momentum sum-integral difference:

[e.g. M. Di Carlo, M. T. Hansen, A. Portelli, and N. Hermansson-Truedsson, Phys. Rev. D **105**, 074509 (2022)]

$$\Delta M^2(L) = -\frac{e^2}{2}\left[\frac{1}{L^3}\sum_{\mathbf{k}} - \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\right]\int \frac{\mathrm{d}k_0}{2\pi}\,\frac{C_{\mu\mu}(k)}{k^2}$$

set of discrete momenta in the FV QED formulation

particle self energy:



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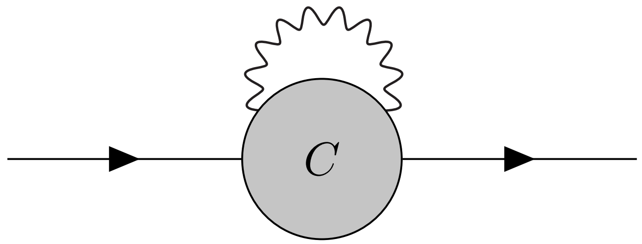


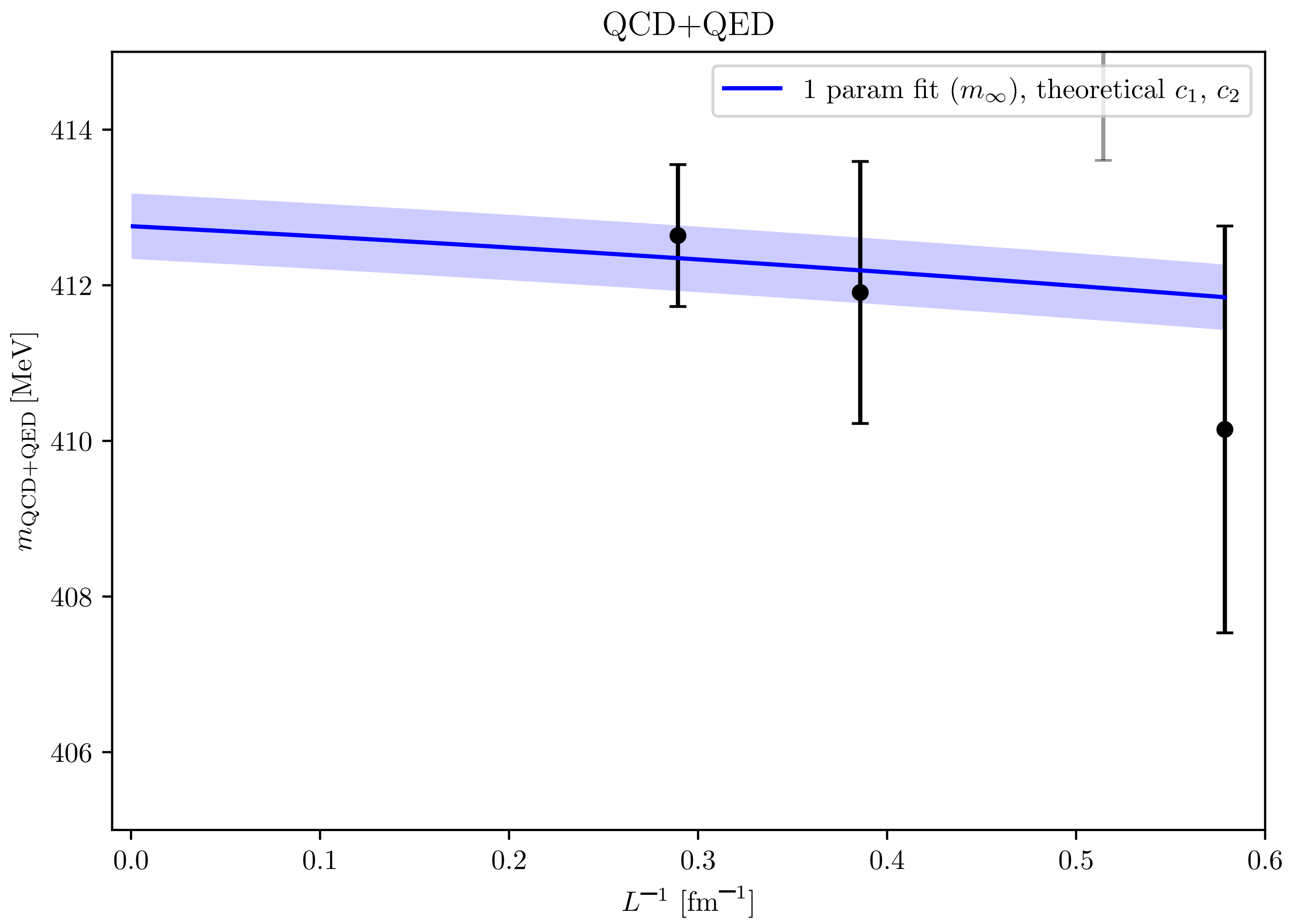
QED$_C$ Result:

$$M(L) = M_\infty + c_1 L^{-1} + \frac{c_2}{M_\infty} L^{-2} + \mathcal{O}(L^{-4})$$

NB: $\mathcal{O}\left(L^{-3}\right) = 0$

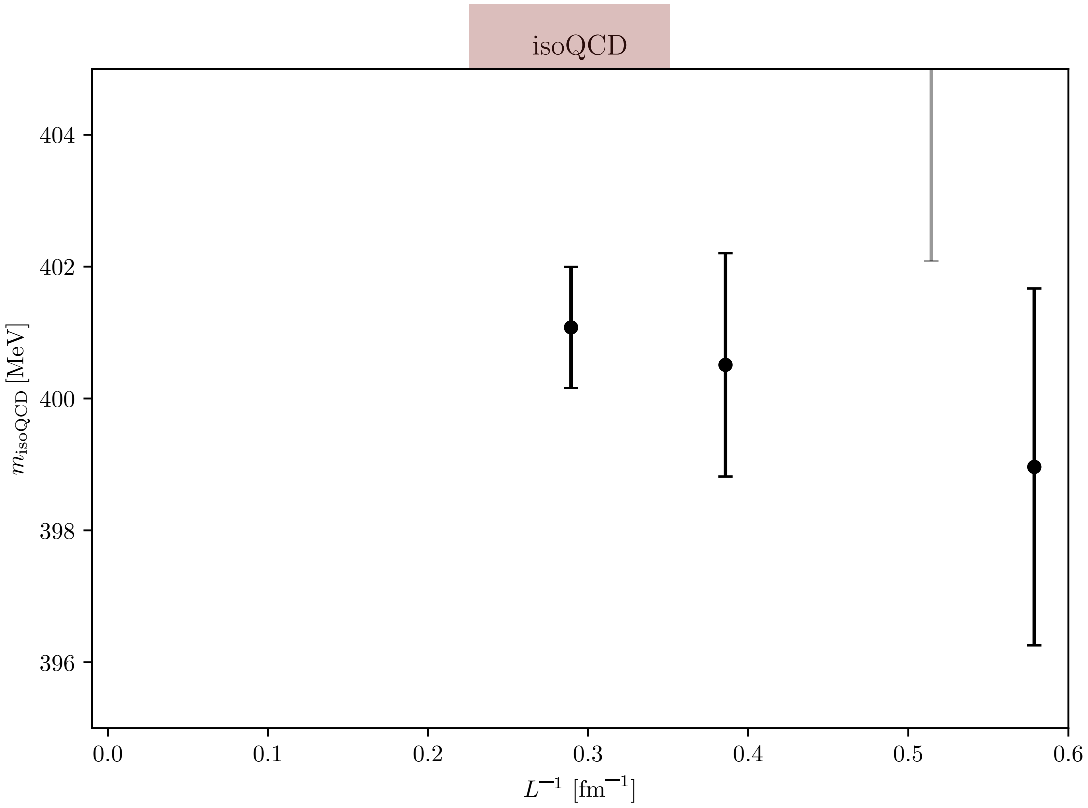
[B. Lucini, A. Patella, A. Ramos, and N. Tantalo, J. High Energ. Phys. **2016**, 76 (2016)]

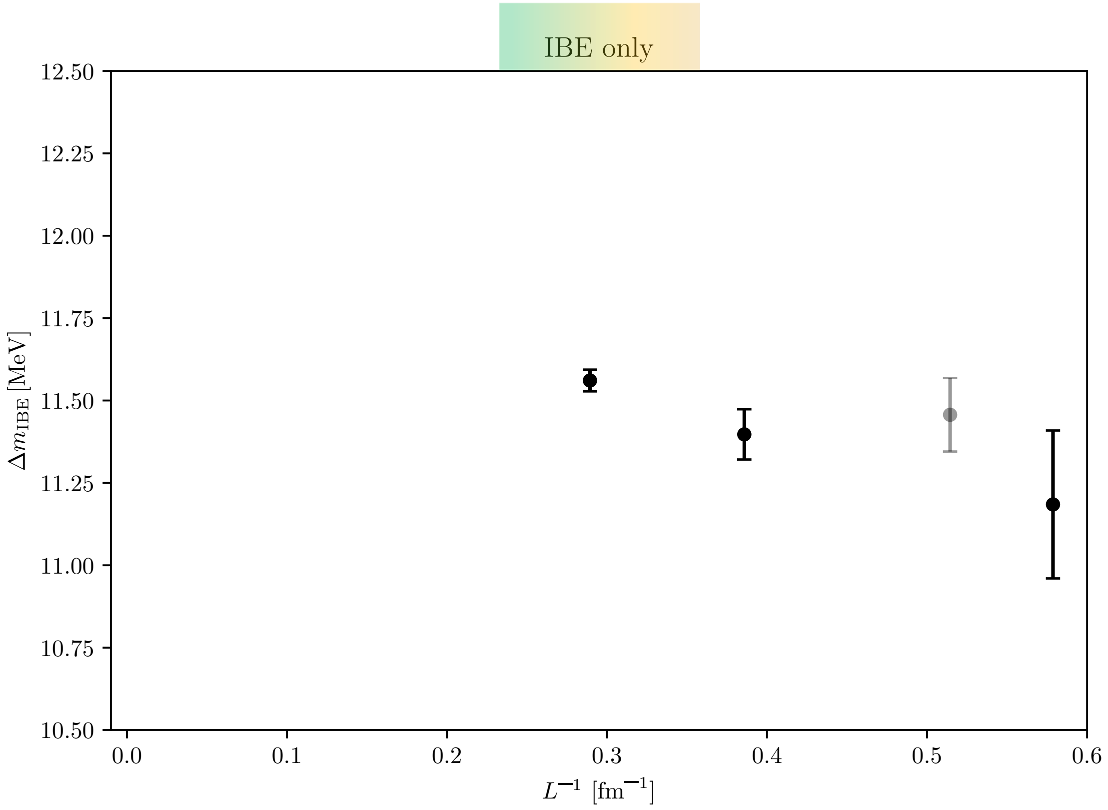
$\pi^{\pm}$ Masses in QCD+QED



$\pi^{\pm}$ Masses Separated into Contributions

$$M_{\mathrm{QCD+QED}} = \boxed{M_{\mathrm{isoQCD}}} + e^2 \partial_e^2 M\Big|_{e^2=0} + \sum_f \Delta m_f \, \partial_{m_f} M\Big|_{\Delta m_f=0}$$





Interlude: Rome 123 and the isoQCD ↔ QCD+QED Separation

1. Isospin breaking effects via Rome 123 (RM123) method [G. M. De Divitiis, et. al., Phys. Rev. D **87**, 114505 (2013)].

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$$X = \underbrace{X^{\text{isoQCD}}}_{\mathcal{O}\left(e^{-mL}\right)} + \underbrace{\sum_f \Delta m_f \partial_{m_f} X + (\Delta g_s)^2 \partial_{g_s^2} X + e^2 \partial_{e^2} X}_{\mathcal{O}\left(\frac{1}{L}\right)} + \mathcal{O}\left(e^4, (\Delta g_s)^2\right)$$

$\pi^{\pm}$ Masses Separated into Contributions

IBE component:

$$\Delta M_{\mathrm{IBE}}(L) = \Delta M_{\infty}^{\mathrm{IBE}} + c_1 L^{-1} + \frac{c_2}{M_\infty} L^{-2} + \mathcal{O}(L^{-4}) + \mathcal{O}\left(e^{-m_\infty L}\right)$$
$$= \Delta M_{\infty}^{\mathrm{IBE}} + c_1 L^{-1} + \frac{c_2}{M_{\infty}^{\mathrm{isoQCD}} + \Delta M_{\infty}^{\mathrm{IBE}}} L^{-2} + \mathcal{O}(L^{-4}) + \mathcal{O}\left(e^{-M_\infty L}\right).$$

two-parameter fit

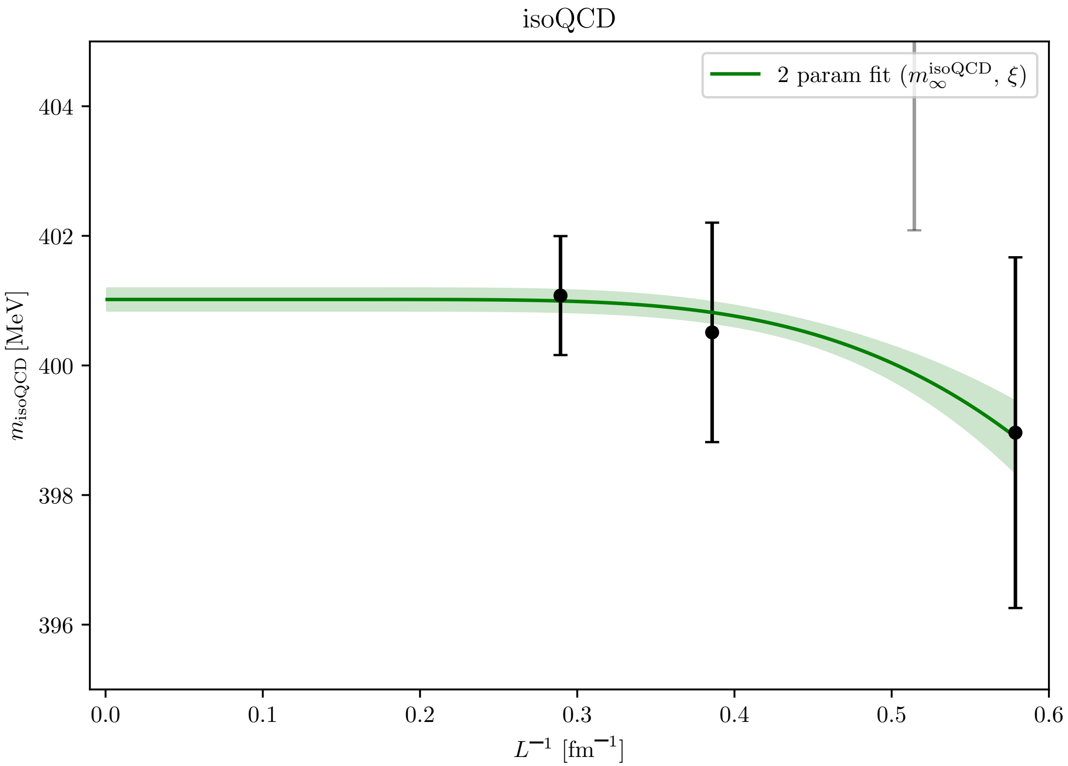
isoQCD component:

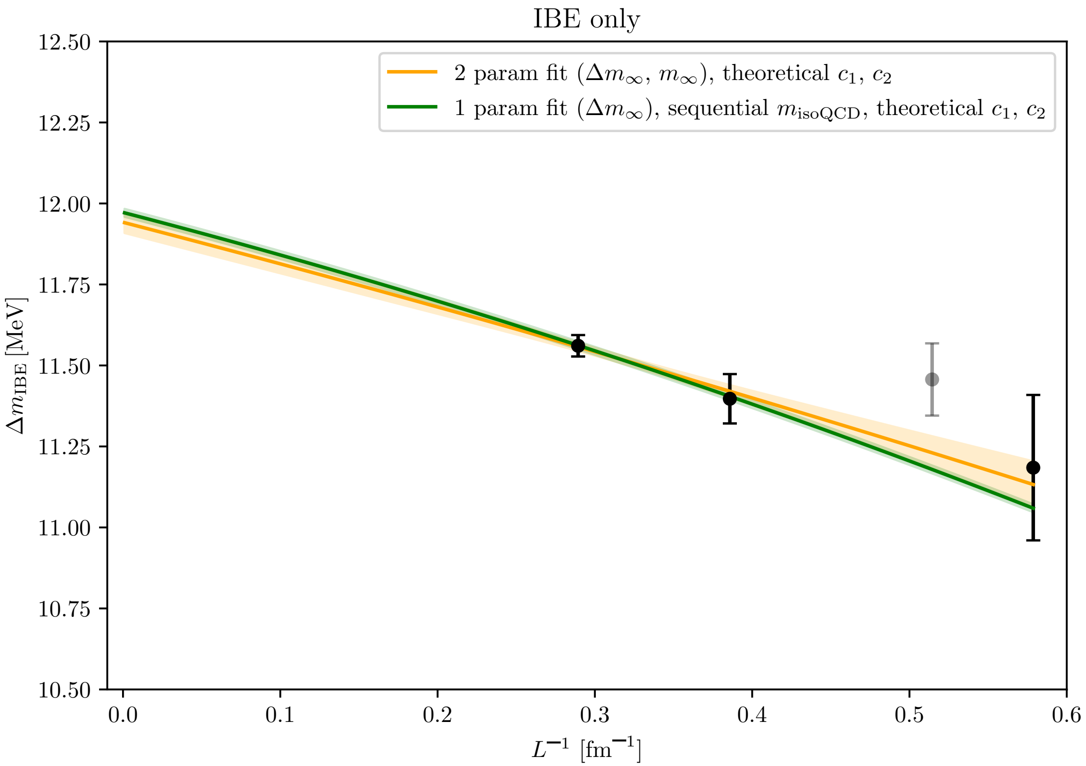
$$M_{\mathrm{isoQCD}}(L) = M_{\infty}^{\mathrm{isoQCD}} - \frac{\xi}{3} \sum_{\vec{n} \in \mathbb{Z}^3 \setminus \{0\}} \frac{1 - 3(-1)^{\sum_k n_k}}{|n| L} K_1(|n| M_{\infty}^{\mathrm{isoQCD}} L)$$

sequential fit

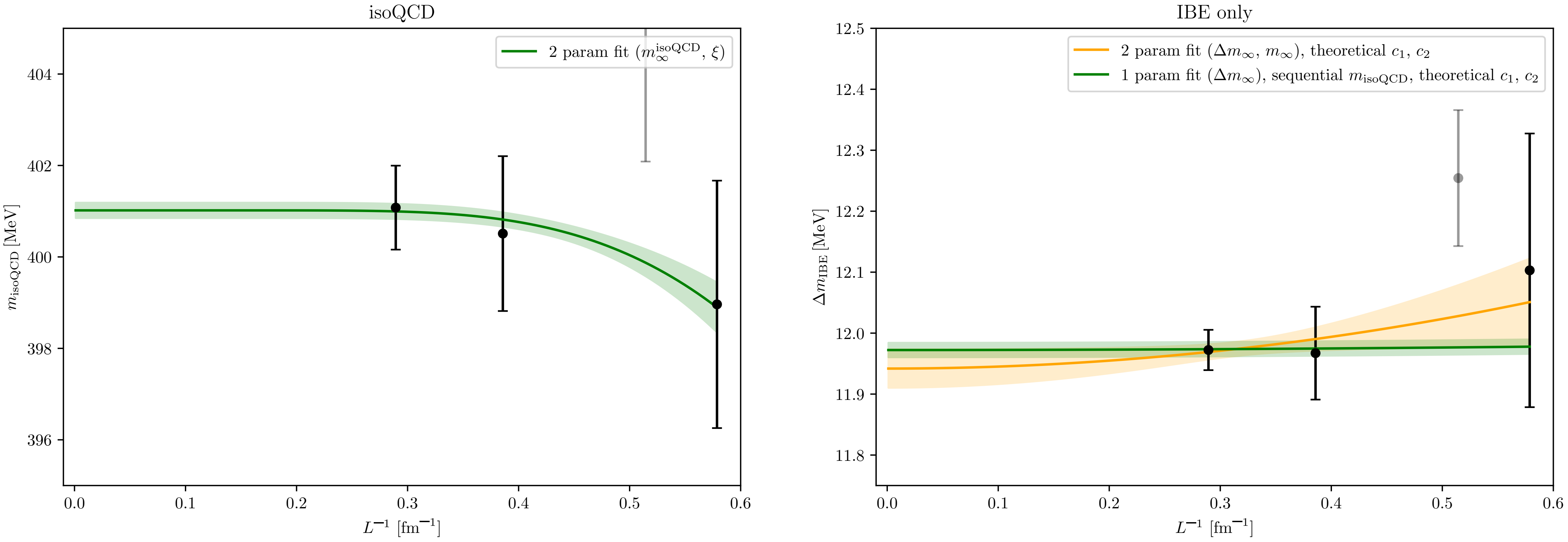
[The RCstar collaboration, *J. High Energ. Phys.* **2023**, 12 (2023)]

$\pi^{\pm}$ Masses Separated into Contributions





$\pi^{\pm}$ Masses Separated into Contributions



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Conclusions & Outlook

Summary:

- Numerically studied finite-volume effects to $\pi^{\pm}$ mass in C* B.C.
- Separate the power-law scaling in the IBE from the exponential scaling.
- Consistent with analytical prediction (universal coefficients)

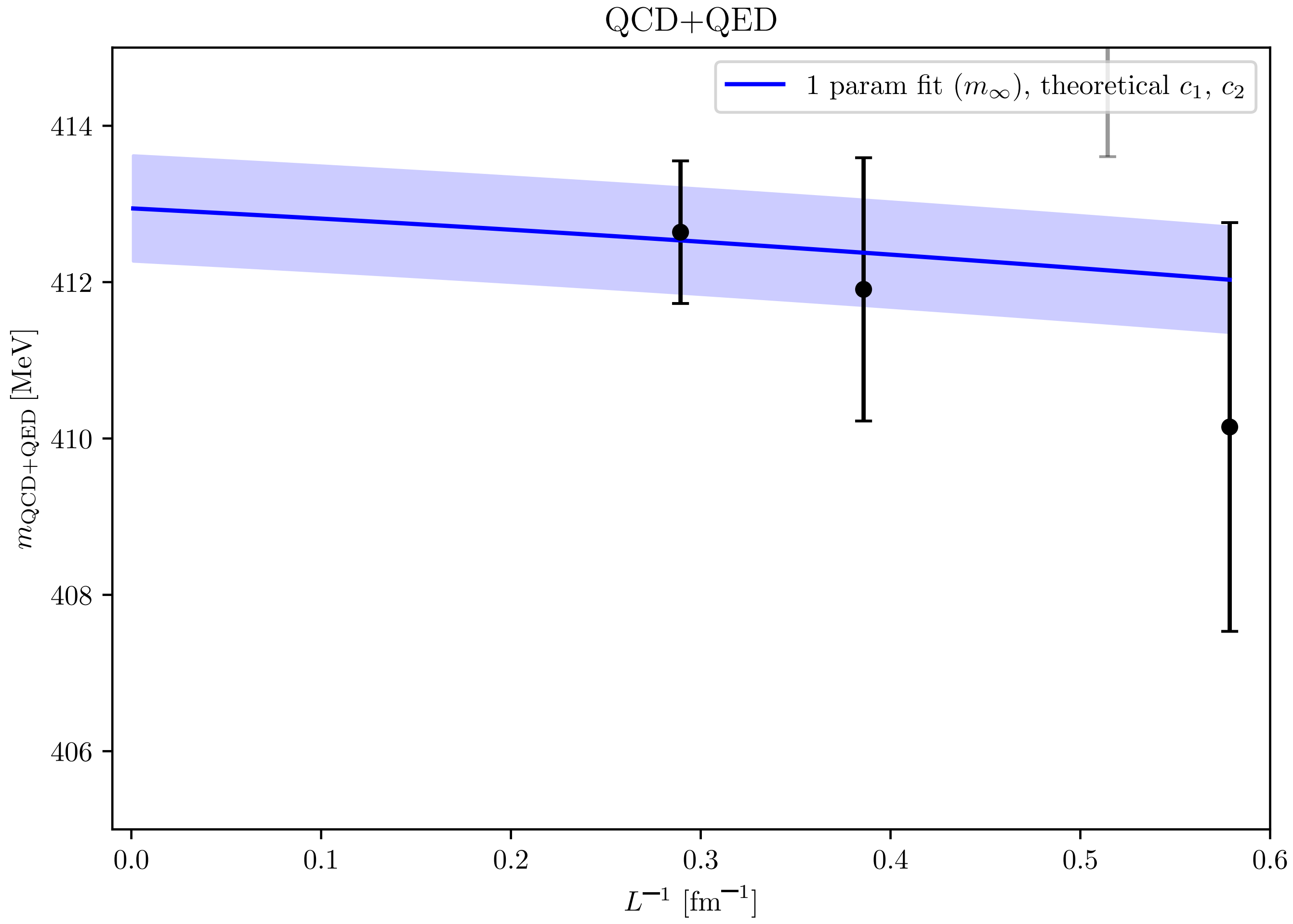
Next steps:

- HVP contribution to $g-2$ with C* B.C. in QCD+QED
- → QCD: P. Tavella’s talk Tue, 16:30 (now)

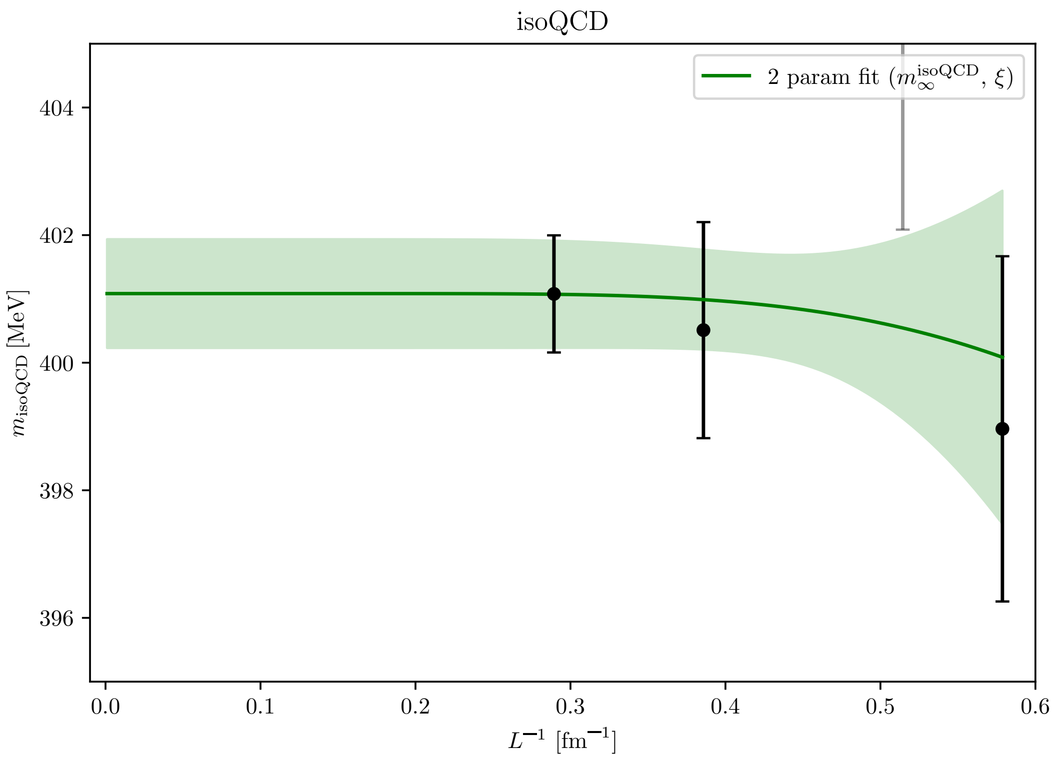
Thank you

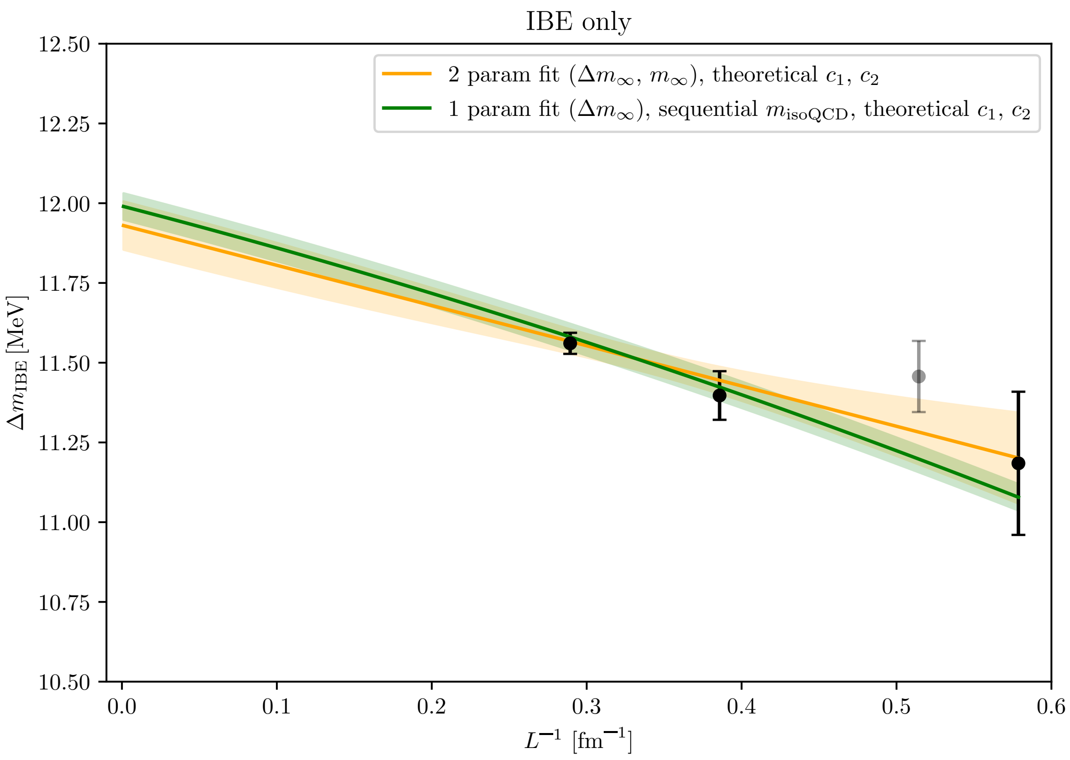
Backup Slides

With AB400 Fitted



With AB400 Fitted

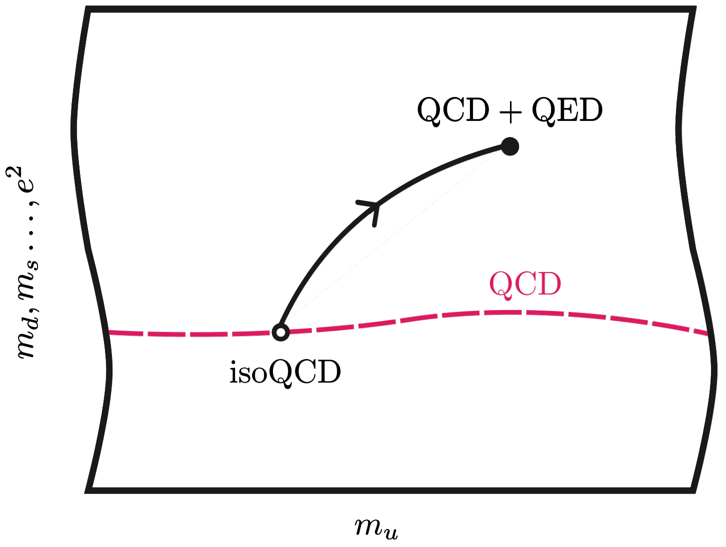




Interlude: Rome 123 and the isoQCD $\leftrightarrow$ QCD+QED Separation

$$\mathrm{QCD} + \mathrm{QED} \quad = \quad \underbrace{\mathrm{isoQCD} + \mathrm{IBE}_{\mathrm{S}}}_{\mathcal{O}\left(e^{-mL}\right)} + \underbrace{\mathrm{IBE}_{\mathrm{EM}}}_{\mathcal{O}\left(\frac{1}{L}\right)}$$

The separation is scheme-dependent



→ How do we know our scheme separates power-law corrections?

→ Pure QCD ($e^2 = 0$) is sufficient

Subtracting the Universal Component

$$
\begin{aligned}
\Delta m_{\mathrm{IBE}}(L) &= \Delta m_\infty^{\mathrm{IBE}} + c_1 L^{-1} + \frac{c_2}{m_\infty^{\mathrm{isoQCD}} + \Delta m_\infty^{\mathrm{IBE}}} L^{-2} + \mathcal{O}(L^{-4}) + \mathcal{O}\left(e^{-m_\infty L}\right) \\
&= \Delta m_\infty^{\mathrm{IBE}} + \underbrace{\left\{ c_1 L^{-1} + \frac{c_2}{m_\infty^{\mathrm{isoQCD}}} L^{-2} \right\}}_{\text{subtracted universal component}} - \frac{c_2}{m_\infty^{\mathrm{isoQCD}}} L^{-2} \mathcal{O}\left(\frac{\Delta m_\infty^{\mathrm{IBE}}}{m_\infty^{\mathrm{isoQCD}}} \right) + \mathcal{O}(L^{-4}) + \mathcal{O}\left(e^{-m_\infty L}\right)
\end{aligned}
$$