Gluon Collins-Soper Kernel with Coulomb-Gauge Operators

Speaker: Yang Fu (MIT)

Collaborators: Artur Avkhadiev (Argonne), Phiala Shanahan (MIT), Michael Wagman (Fermilab), Yong Zhao (Argonne)

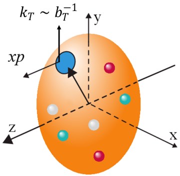
July 26 - Aug 1, 2026

Lattice 2026, University of Maryland

Collins-Soper evolution kernel

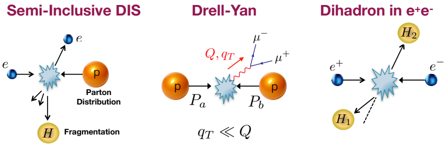
- Transverse-momentum-dependent PDF (TMD PDF):

$$f_{i/h}(x, \vec{k}_{\mathrm{T}}), \quad \text{or coordinate-space} \quad f_{i/h}(x, \vec{b}_{\mathrm{T}}) = \int \mathrm{d}^2 \vec{k}_{\mathrm{T}} \, e^{i \vec{k}_{\mathrm{T}} \cdot \vec{b}_{\mathrm{T}}} f_i(x, \vec{k}_{\mathrm{T}})$$



- parton i in hadron h
- momentum fraction x
- transverse momentum $\vec{k}_{\mathrm{T}}$

hadron 3D structure



- Collins-Soper kernel: governs TMD rapidity evolution

$$\gamma_i(\mu, \vec{b}_{\mathrm{T}}) = \frac{d}{d \ln \sqrt{\zeta}} \ln f_i(x, \vec{b}_{\mathrm{T}}, \mu, \zeta), \qquad \text{independent of hadron } h$$

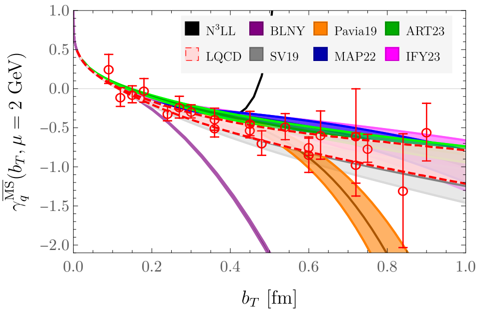
- Key to global fit: ability to relate different scales
- Nonperturbative for $b_{\mathrm{T}}^{-1} \lesssim \Lambda_{\mathrm{QCD}}$ even when scales μ, ζ are large $\Rightarrow$ lattice QCD

- Quark CS kernel: reached the level of systematic control

  $\Rightarrow$ physical pion mass, operator mixing, continuum extrapolation, $\cdots$

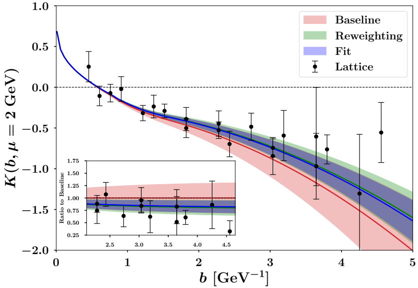
- First such calculation by our group

  PRL 132 (2024) 23, 231901



- Joint exp. + lattice analysis (with MAP)

  PRL 136 (2026) 17, 171902



- Results are precise enough to discriminate between different pheno. models

- Inclusion of lattice data can reduce the uncertainties by $\sim 40 - 50\%$

Also quark kernel results from W. Morris & J. He's talk!

Gluon CS kernel

- Gluon CS kernel: remains largely unknown

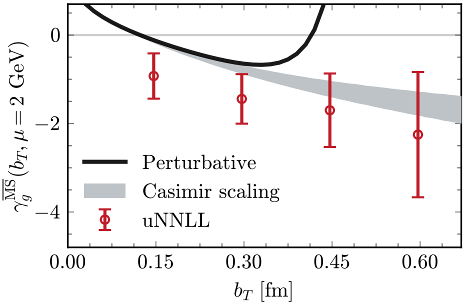
- Experiment: no gluon TMD data yet (future EIC?)

- Theory:

  · perturbative region: 4-loop available and

$$\gamma_\zeta(\mu, b_T) = -\frac{\alpha_s}{\pi} \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} \times \begin{cases} C_F, & \text{quark} \\ C_A, & \text{gluon} \end{cases} + \cdots$$

  Casimir scaling holds through all known orders (C_A vs C_F)



  · nonperturbative region: first proof-of-principle LQCD calculation (A. Avkhadiev's talk ∼ 20 mins ago)

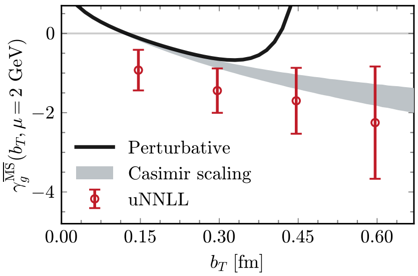
  (near physical pion mass, systematics not fully quantified)

- Full systematic control is underway!

  ⇒ it will provide the first nonperturbative constraints for future experiments

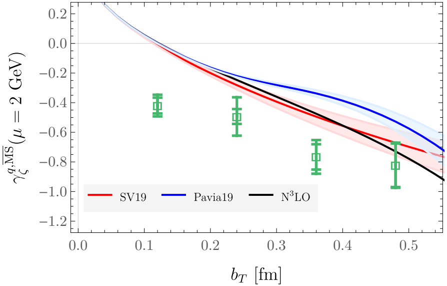
- Compare to our group's quark CS kernel $\sim$ 5 years ago

- Gluon CS kernel



- close-to-physical $m_\pi \approx 170$ MeV ✓
- NNLL matching ✓
- $\sim$6M measurements !

- Quark $\sim$ 5 years ago 2107.11930



- unphysical $m_\pi \approx 500$ MeV
- NLO matching
- $\sim$16K measurements

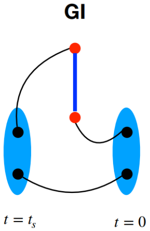
Similar stage, but significant statistics challenge $\Rightarrow$ need new ideas/methods

Coulomb gauge method

- Coulomb gauge operators: eliminates the need for Wilson lines

Not the same operator, but same universality class/IR physics $\Rightarrow$ same light-cone distribution

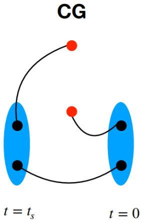
X. Gao et al. PRD 2024, Y. Zhao, PRL 2024



$$\langle h|\bar{\psi}(z)\Gamma W(z,0)\psi(0)|h\rangle \sim e^{-\delta m\cdot|z|}$$

$\delta m(a) \sim \frac{1}{a}$, linear divergence

$\Rightarrow$ z-dependent renormalization and StN decay at long distance



$$\langle h|\bar{\psi}(z)\Gamma\psi(0)\big|_{\nabla\cdot A=0}|h\rangle$$

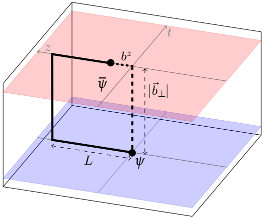
No linear divergence

- Several advantages:

- No Wilson line self-energy, less signal-to-noise degradation, simpler renormalization
- Use the same quark propagators, almost free to calculate both

Coulomb gauge method

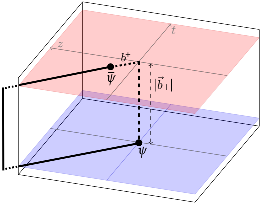
- For TMD operators, the absence of the Wilson line provides even more convenience

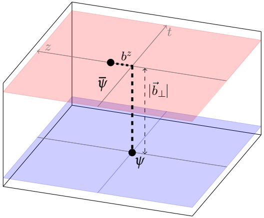


GI: staple-shaped
need length $\ell \to \infty$

light-cone TMD

$\xRightarrow{P^z \to \infty}$





CG: no Wilson line
$\nabla \cdot A = 0 \to A^+ = 0$
when $P^z \to \infty$

Figs based on Ebert, Stewart, Zhao, 1901.03685

- No staple length $\ell \to \infty$ extrapolation, simpler data analysis and systematic control

Workflow

CS kernel: evolution kernel of TMD / quasi-TMD

$$\tilde{B}(b^z, b_T, P^z) = \langle h(P^z)|O_g(b^z, b_T)|h(P^z)\rangle$$

$$\tilde{f}(x, b_T, \mu, P^z) = \frac{1}{x}\int \frac{P^z \mathrm{d}b^z}{2\pi}\, e^{ixP^z b^z} N(P^z)\tilde{B}(b^z, b_T, P^z)$$

$$\frac{\tilde{f}(x, b_T, \mu, P^z)}{H(x, b_T, \mu, P^z)} = c(x, b_T)\, \exp\left[\gamma(b_T, \mu)\, \ln P^z\right] + \text{power corrections}$$

1. Position-space MEs
   no ℓ extrap. for CG
2. x-space MEs
   integral range b^z_{cut}
3. Evolution of MEs
   with pert. matching

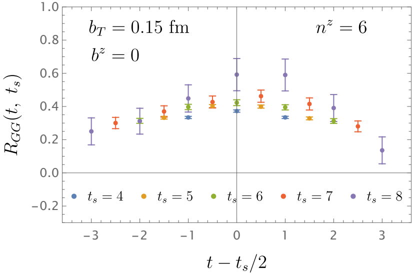
$\Longrightarrow$ CS kernel $\gamma_\zeta(b_T, \mu)$ ($x \sim 0.5$ with p.c. under control) $\Longrightarrow$ Repeat for each b_T

TMD operators

- In Coulomb gauge, two types of operators can be used

1. $O_g^{GG}(b^z, b_T) = 2\,\mathrm{Tr}\left[G^{\mu\nu}(b)G^{\rho\sigma}(0)\right]\Big|_{\nabla\cdot A=0}$

with $\mu, \rho = \{t, z\}$, $\nu = \sigma = \{x, y\}$

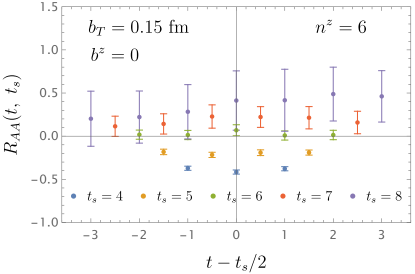


much better signal: $O_g^{GG} \sim |P^z|^2 O_g^{AA}$ (✓)

but $G^{\mu\nu}$ can mix with operators like $[A^\mu, A^\nu]$ (!)

2. $O_g^{AA}(b^z, b_T) = 2\,\mathrm{Tr}\left[A^{\mu}(b)A^{\nu}(0)\right]\Big|_{\nabla\cdot A=0}$

with $\mu = \nu = \{x, y\}$



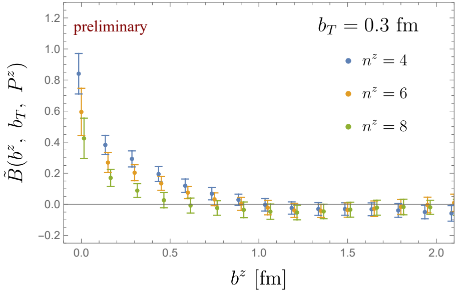
multip. renormalizable up to discretization

but much worse signal (×)

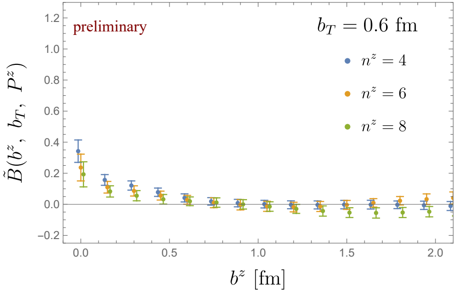
- With current 6M measurements, we can only use O_g^{GG}

- CG quasi-TMDs in position space:

- $b_T = 0.3$ fm



- $b_T = 0.6$ fm

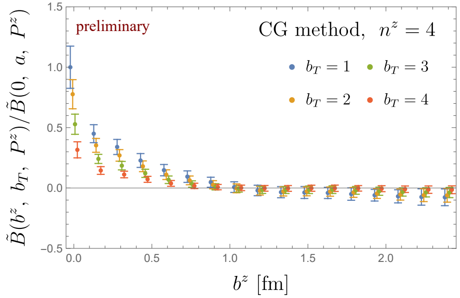


- Numerically real and symmetric after averaging over all orientations
- The magnitude of the MEs decreases at larger b_T, the absolute noise doesn't increase

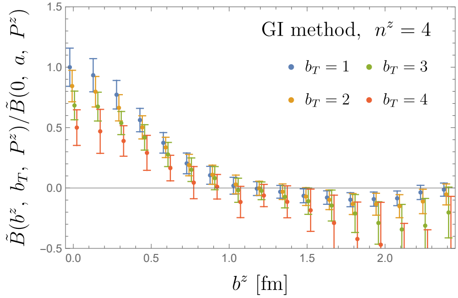
Position-space MEs

- Compare with the gauge-invariant MEs

- Coulomb-gauge



- Gauge-invariant



Note: 1. For comparison, both normalized to $b^z = 0,\ b_T = 1a$

2. CG and GI are different operators

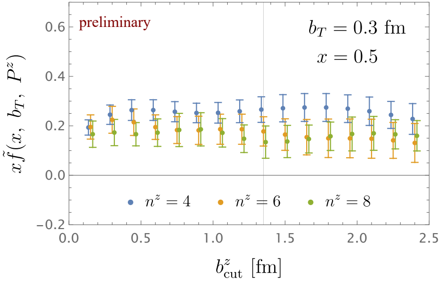
- Much slower signal-to-noise decay with increasing b_T and b^z compared to the GI cases

- Fourier transform to x-space

$$x\tilde{f}(x, b_T, \mu, P^z) = \int \frac{P^z \mathrm{d}b^z}{2\pi}\, e^{ixP^z b^z} N(P^z)\tilde{B}(b^z, b_T, P^z)$$

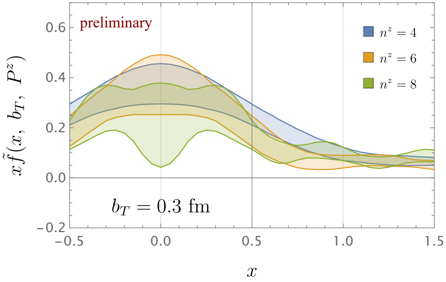
- Dependence on b^z_{cut}

Fourier transformation saturated at small b^z_{cut}, same choice of $b^z_{\text{cut}} = 1.35$ fm used



- MEs as a function of x

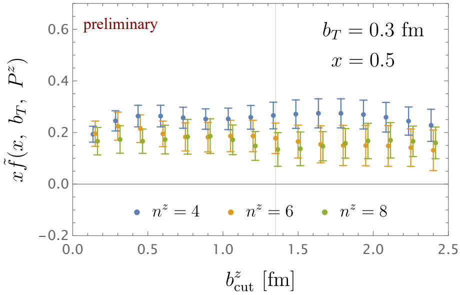
Symmetric under $x \to -x$, gluons are their own anti-particles



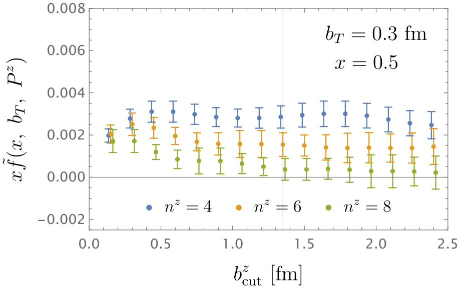
- Compare with the gauge-invariant MEs

1. Dependence on b^z_{cut}, at $b_T = 0.3$ fm:

- Coulomb-gauge



- Gauge-invariant

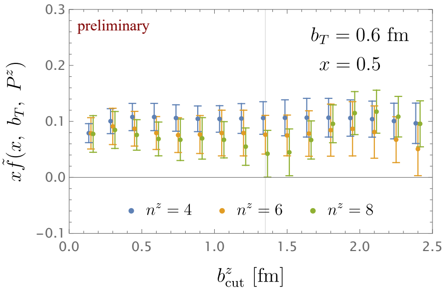


- Mild signal-to-noise degradation for both methods at small b_T

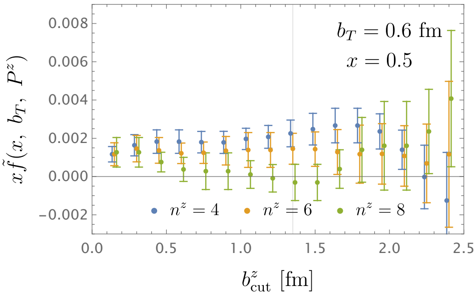
- Compare with the gauge-invariant MEs

2. Dependence on b^z_{cut}, at $b_T = 0.6$ fm:

- Coulomb-gauge



- Gauge-invariant



- Faster signal-to-noise degradation for GI at large b_T

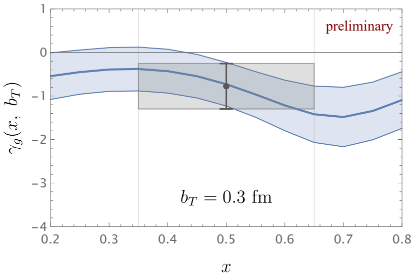
$\Rightarrow$ also potentially smaller b^z truncation systematics with CG as it has clearer plateaus

- CS kernel extracted by fitting

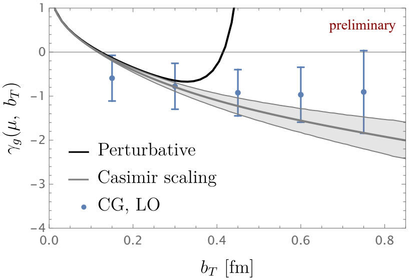
$$\tilde{f}\,(x, b_T, P_z)\,/\,H(x, b_T, \mu, P_z) = c(x, b_T)\exp\left[\gamma_g(x, b_T, \mu)\ln P_z\right]$$

Caution! Matching for CG operator is not available yet. Using LO $H = 1$ for now

- effective γ_g extracted at each x



- γ_g as a function of b_T

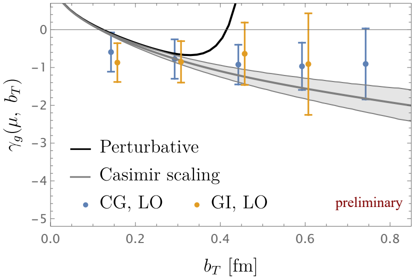


- Preliminary leading order result consistent with PT and Casimir scaling

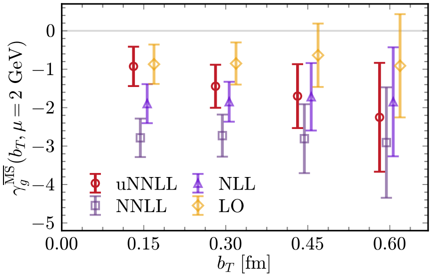
Gluon CS kernel

- Compare with the gauge-invariant results

- Coulomb-gauge



- Gauge-invariant



- LO CG consistent with GI, with smaller statistical errors at large b_T

There are $\sim 1 - 2\sigma$ difference between different GI matchings $\Rightarrow$ CG matching also important

Summary

- Gluon TMDs and CS kernel remain largely unknown, both experimentally and in LQCD
- Coulomb-gauge method helps reduce the noise in the long-distance region

  - larger b_T will be accessible
- Having another set of operators can help better control the systematics

  - no finite staple length and potentially smaller truncation effects
  - need higher order matching for CG method
- Calculation at a finer lattice spacing ongoing, both GI and CG operators will be calculated

Thank you!