Diffusion of Heavy Quarkonia on the Lattice

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Based on: Brambilla *et.al.* PRD112 2025 and ongoing work

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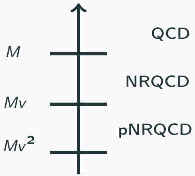
Motivation

- Quark Gluon Plasma described by set of transport coefficients
- Effective field theories give descriptions of transport as chromoelectric and -magnetic correlators



- These correlators can be extracted using lattice QCD
- Goal to understand quarkonia in the QGP
- Similar to well studied heavy quark diffusion
- Nuclear modification factor R_{AA} and elliptic flow v_2 described by diffusion coefficients

See e.g. Brambilla *et.al.*PRD96 2017, Brambilla *et.al.*PRD97 2018

Quarkonium in medium

- Scales hierarchy for the QGP (environment energy scale πT)

$$M \gg \frac{1}{a_0} \gg \pi T \gg E\,, \quad \tau_R \gg \tau_E \sim 1/\pi$$

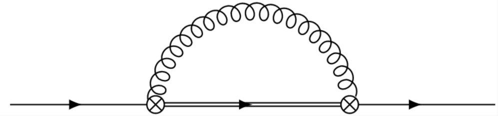
- HQ mass M, Bohr radius a_0, binding energy E, correlation time τ_E
- Quarkonium in fireball can be described with Open Quantum System approach

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -i[H,\rho] + \sum_{n,m} h_{nm}\left(L_i^n \rho L_i^{m\dagger} - \frac{1}{2}\{L_i^{m\dagger} L_i^n, \rho\}\right)$$

- Quarkonium suppression defined with two transport coefficients κ_{X} and γ_{X}

  Decay width $\Gamma(1S) = 3a_0^2\kappa\,,$ Mass shift $\delta M(1S) = 2a_0^2\gamma/3$

$$\kappa_{\mathrm{X}} + i\gamma_{\mathrm{X}} = \frac{g^2}{6N_{\mathrm{C}}}\int_0^\infty dt D_{\mathrm{X}}^{>\dagger}$$



Brambilla *et.al.*PRD97 2018, Brambilla *et.al.*PRD100 2019

Heavy Quarkonium Diffusion

- Three possible interactions for quarkonia (plus heavy adjoint quark diffusion) with respective transport coefficients and chromo-electric correlators

singlet-octet	octet-singlet	octet-octet	adjoint quark
$-\langle E_i^a(0)U^{ab}(0,\tau)E_i^b(\tau)\rangle$	$-\langle E_i^a(0)E_i^b(\tau)U^{ab}(\tau,\frac{1}{T})\rangle$	$-\langle E_i^{ab}(0)U^{bc}(0,\tau)E_i^{cd}(\tau)U^{da}(\tau,\frac{1}{T})\rangle$	$-\langle E_i^{ab}(0)U^{bc}(0,\tau)E_i^{cd}(\tau)U^{da}(\tau,\frac{1}{T})\rangle$

- Real time correlators can be analytically continued to Euclidean ones
- Derivation with Limblad equations gives correlators with proper normalization
- Singlet-to-octet (and os) have asymmetry arising from the vacuum part
- All κ_X equal at NLO

Brambilla *et.al.*PRD96 (2017), Brambilla *et.al.*PRD97 (2018), Brambilla *et.al.*PRD100 (2019), Eller *et.al.*PRD99 (2019)
Scheihing-Hitschfeld & Yao PRD108 (2023), Brambilla *et.al.*PRD112 (2025), Brambilla et.al. TUM-EFT 191/24, FERMILAB-PUB-24-0451-T

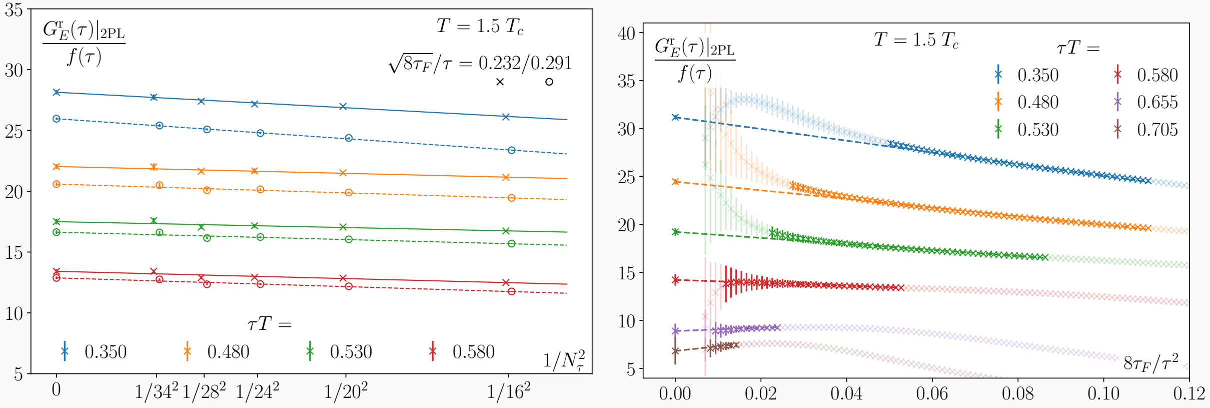
Lattice simulations

- SU(3) pure gauge Yang-Mills simulations at temperatures $T = 1.5\,T_\mathrm{c}$ and $T = 10^4\,T_\mathrm{c}$
- Two different discretizations of the E-field (clover and 2-plaquette)
- Re-express adjoint correlators with fundamental links via Fierz relations
- Measurements with both gradient flow and multilevel algorithm
- Tree-level improve the correlators
- All correlators have the same LO behavior up to color factor C_X

$$G^{\mathrm{LO}}_{\mathrm{EE}} = g^2 C_\mathrm{X} f(\tau)\,, \quad f(\tau) = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$

- Wilson line has $1/a$ divergent self-energy
- Divide symmetric correlators by Polyakov loop,
  for asymmetric ones multiply by $\left(L^\mathrm{A}_\mathrm{r}/L^\mathrm{A}_\mathrm{bare}\right)^{\tau T}$ where L_r given by Gupta *et.al.*PRD77 2008

Continuum and zero flow time limits

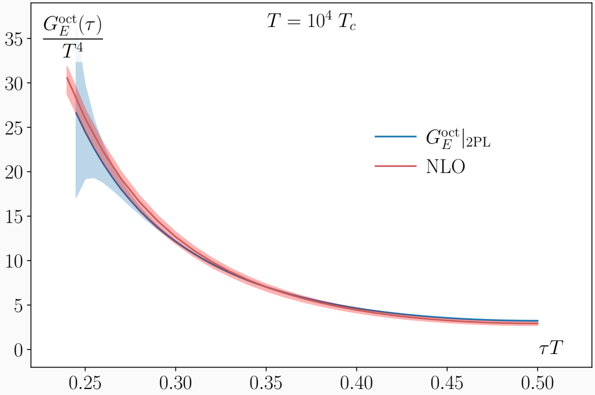


Continuum limit

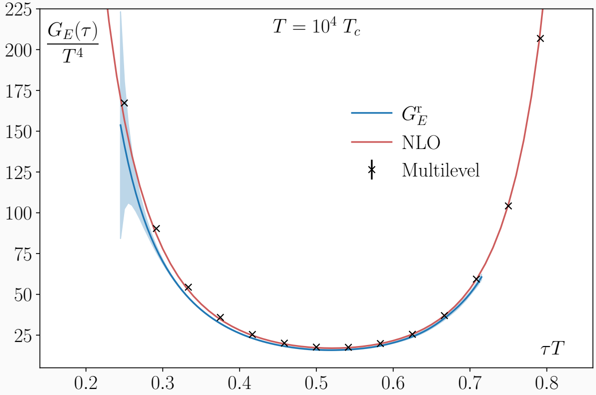
Zero flow time limit

- To compare to perturbation theory, we need to take continuum and zero flow time limits
- Order of limits matter

High temperature correlators



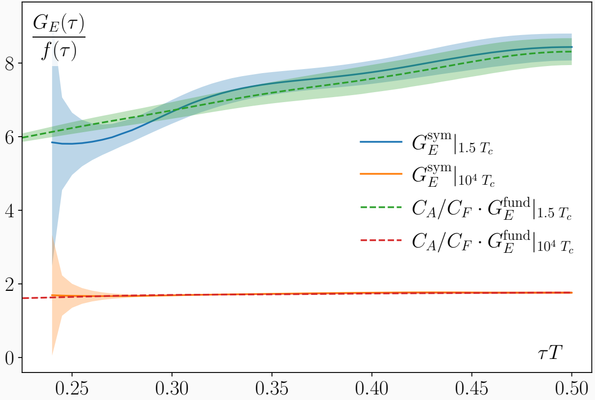
Octet-Octet



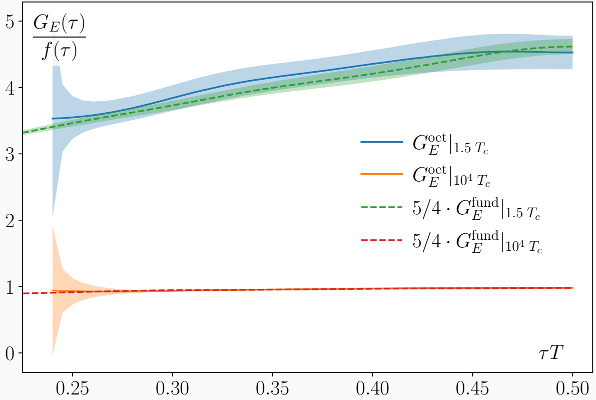
Singlet-to-Octet

- Perturbation theory works at high temperatures

Symmetric correlators $T = 1.5\,T_c$



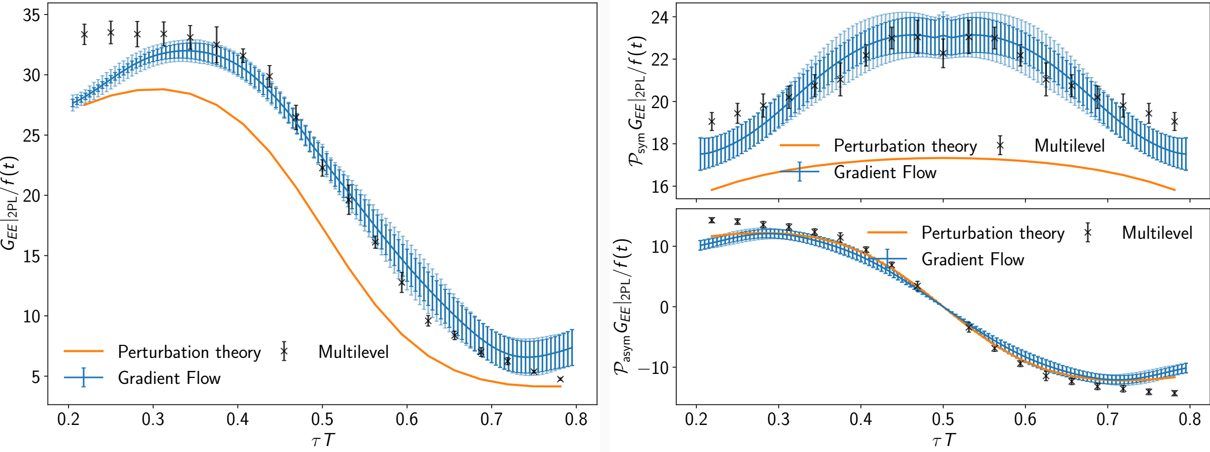
Adjoint quark



Octet-Octet

- For symmetric correlators we observe Casimir scaling
- Indication that for symmetric correlators κ color scales trivially

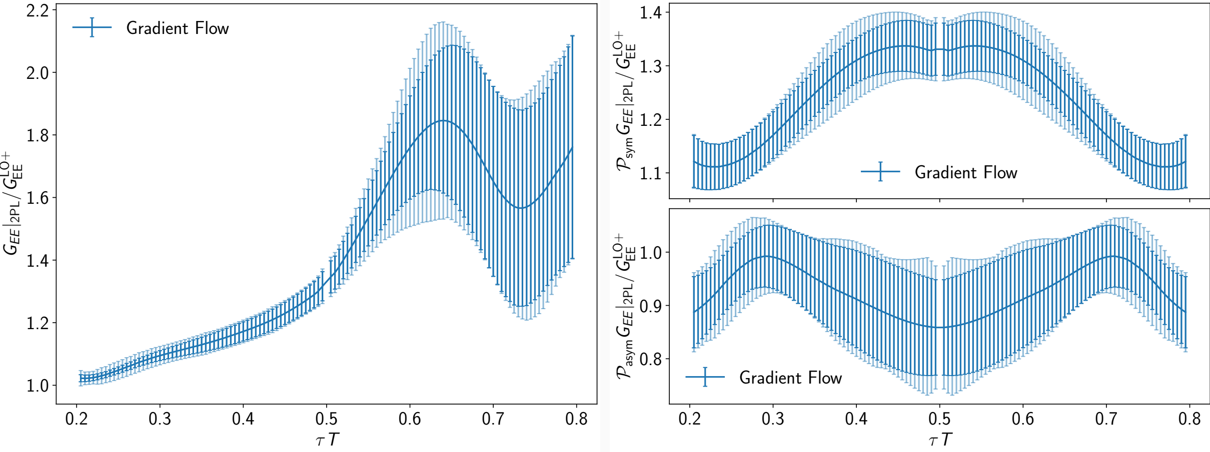
Asymmetric correlator $T = 1.5\,T_c$



- Clearly asymmetric, can split to symmetric and asymmetric parts
- At high temperatures excellent match to perturbation theory

Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023)

Asymmetric correlator normalized



- Normalize with the perturbative curve
- Focus on GF data for now

Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023)

Spectral function

- Relate Euclidean correlators to spectral functions with symmetric and asymmetric parts

$$G_{\mathrm{EE}}(\tau) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{S}}(\omega) \frac{\cosh\left(\frac{\omega}{T}[\frac{1}{2} - \tau T]\right)}{\sinh\frac{\omega}{2T}} + \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{A}}(\omega) \frac{\sinh\left(\frac{\omega}{T}[\frac{1}{2} - \tau T]\right)}{\sinh\frac{\omega}{2T}}$$

- NLO spectral function

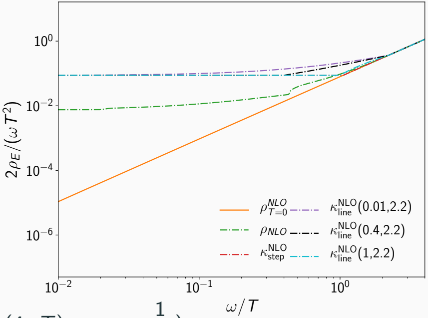
$$\rho_{\mathrm{S}}^{\mathrm{T}=0}(\omega) = \frac{g^2 C_{\mathrm{F}} \omega^3}{6\pi} \left\{ 1 + \frac{g^2}{(4\pi)^2} \left[N_{\mathrm{c}} \left(\frac{11}{3} \ln\frac{\mu^2}{4\omega^2} + \frac{149}{9} - \frac{2\pi^2}{3} \right) \right] \right\},$$

$$\rho_{\mathrm{A}} = 2\frac{g^4 C_{\mathrm{F}} \omega^3}{96\pi^3} N_{\mathrm{c}} \pi^2 \mathrm{sgn}(\omega)$$

- Extract transport coefficients

$$\kappa_{\mathrm{X}} = \lim_{\omega\to 0} \frac{2T}{\omega} \rho(\omega), \qquad \gamma_{\mathrm{X}} = -\frac{1}{3N_c} \int_0^\infty \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega}$$

Scales and spectral fit



- Assume simple behavior on IR ($\omega \ll T$):

$$\rho_{\mathrm{IR}}(\omega) = \frac{\kappa\omega}{2T}$$

- Perturbative behavior in UV ($\omega \gg T$) by LO with running coupling from NLO:

$$\rho^{\mathrm{LO}}_{\mathrm{UV}}(\omega) = \frac{g^2(\mu_\omega) C_F \omega^3}{6\pi}$$

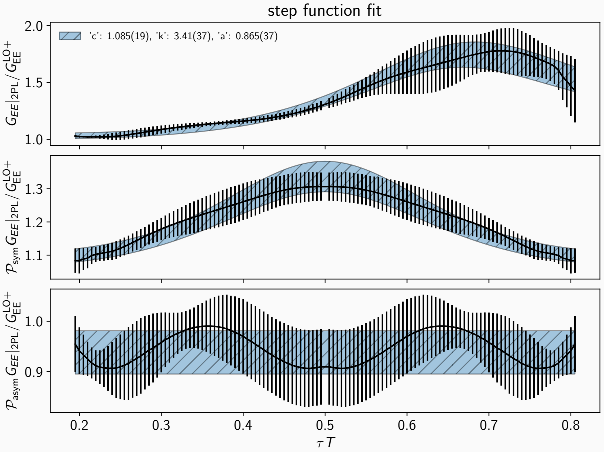
$$\ln(\mu_\omega) = \max\left(\ln(2\omega) + \frac{6\pi^2 - 149}{66}, \ln(4\pi T) - \gamma_{\mathrm{E}} - \frac{1}{22}\right)$$

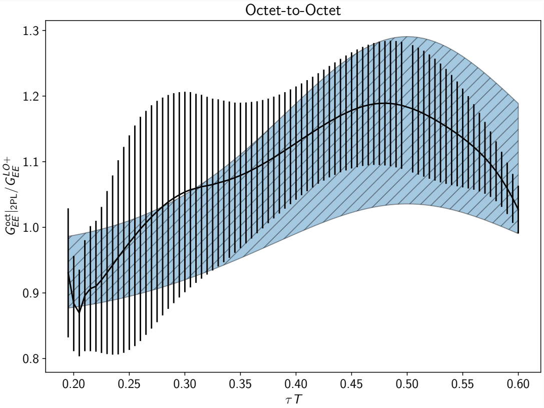
- Model simple shapes for intermediate ω and compare to lattice correlator
- General fit ansatz

$$G_{\mathrm{EE}}(\tau) = C\left[\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{S}}(\omega,\kappa) \frac{\cosh\left(\frac{\omega}{T}\left[\frac{1}{2} - \tau T\right]\right)}{\sinh \frac{\omega}{2T}} + A \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{A}}(\omega) \frac{\sinh\left(\frac{\omega}{T}\left[\frac{1}{2} - \tau T\right]\right)}{\sinh \frac{\omega}{2T}}\right]$$

- Different normalizations for symmetric and asymmetric parts to gauge higher order effects

Fits to the correlator

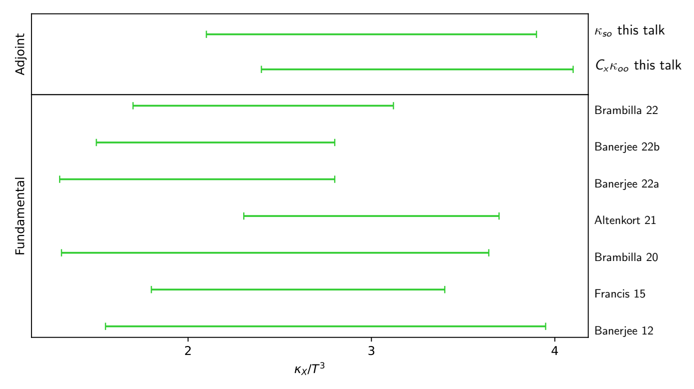




- Fit ansatz work well

κ_{so} and κ_{oo}

- Vary scale by factor of 2 over different ansatze
- Preliminary results
- Perturbation theory expectation: NLO: $\kappa_{so} = \kappa_{\mathrm{fund}}$
- κ_{oo} needs color scaling
- Results within 25% of perturbative NLO



γ

- Relying on the spectral function for γ would be tricky with the inversion precision
- Reformulate γ in terms of the correlator

$$\gamma_{\mathrm{U}} = -\frac{1}{6N_{\mathrm{c}}}\int_0^{1/T} \mathcal{P}_{\mathrm{symm}} G_{\mathrm{EE}}(\tau) d\tau + \frac{1}{6N_{\mathrm{c}}}\int_0^{1/T} 2\tau T \mathcal{P}_{\mathrm{asymm}} G_{\mathrm{EE}}(\tau) d\tau$$

- On the lattice the correlator diverges in UV as $G_{\mathrm{EE}} \sim \tau^{-3}$
- Subtract the vacuum

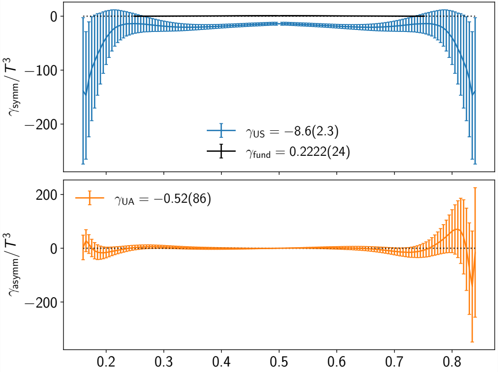
$$G_{\mathrm{EE}}^{\mathrm{reduced}} = G_{\mathrm{EE}}^{\mathrm{lat}} - G_{\mathrm{EE}}^{\mathrm{LO+}},$$

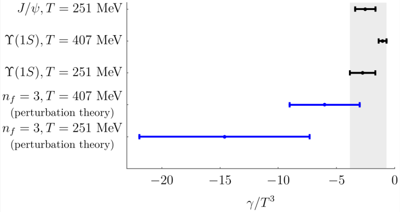
$$G_{\mathrm{EE}}^{\mathrm{LO+}} = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{UV}}^{\mathrm{LO}}(\omega) \frac{\cosh\left(\frac{\omega}{T}[\frac{1}{2} - \tau T]\right)}{\sinh\frac{\omega}{2T}} + \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_{\mathrm{A}}(\omega) \frac{\sinh\left(\frac{\omega}{T}[\frac{1}{2} - \tau T]\right)}{\sinh\frac{\omega}{2T}}$$

- The asymmetric part of $G_{\mathrm{EE}}^{\mathrm{LO+}}$ includes the perturbative gamma

Brambilla *et.al.*JHEP 08 (2025). See also: Scheihing-Hitschfeld & Yao PRD108 (2023).

- Proper vacuum subtraction still work in progress
- Currently: subtracting $G_{\rm EE}^{\rm LO+}$ with matching coefficients from κ_{s0} fit
- We get same result for both upper and lower correlators
- Asymmetric part is purely perturbative
- Symmetric part much harder, currently large single digit contribution (very Preliminary)
- Fundamental heavy quark γ very small





Lower fig from: Brambilla *et.al.*PRD100 2019

Summary

- Measure chromo-electric field correlators for quarkonium decay in GQP inspired by pNRQCD
- All quarkonia κ seem to be equivalent withing errors with the heavy quark diffusion constant as expected by NLO perturbation theory
- Octet-to-octet diffusion coefficient color scales trivially from the heavy quark one
- γ can be extracted directly from the correlators
- γ extraction still preliminary
- γ is related also to static potential screening

Summary

- Measure chromo-electric field correlators for quarkonium decay in GQP inspired by pNRQCD
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Thank you for your attention!

Adjoint correlators with fundamental links

- Euclidean correlators similar to HQ-case, but with adjoint Wilson line
- κ_{so} and κ_{os} given by

$$G_{\mathrm{E}}(\tau) = -\frac{1}{3}\sum_{i=1}^{3}\langle \mathrm{Re}\,\mathrm{Tr}\,[gE_i(\tau,0)\Phi(\tau,0)gE_i(0,0)]\rangle = -\frac{2}{3}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)]\rangle\,,$$

- Separating κ_{so} and κ_{os} on lattice still work in progress
- κ_{oo} similar to HQ-diffusion

$$G_{\mathrm{EE}}^{\mathrm{oct}} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{\mathrm{A}}_{xa}(N_T,t)d_{abc}E^{i,c}(t)\Phi^{\mathrm{A}}_{bz}(t,0)d_{zxg}E^{i,g}(0)\rangle$$

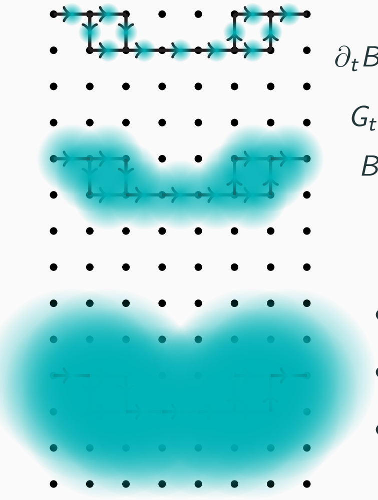
$$= \frac{-1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^\dagger]\mathrm{Tr}\,[E_i(0)P(0)] + \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] - \frac{4}{3}\mathrm{Tr}\,[E_i(\tau)U(\tau,0)E_i(0)U(0,\tau)] + \mathrm{h.c.}$$

- Also related: Diffusion of an adjoint static quark

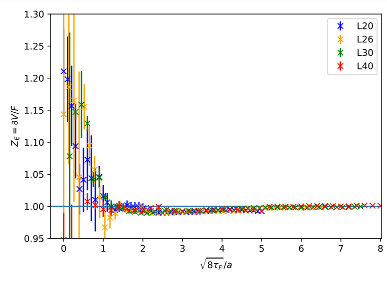
$$G_{\mathrm{EE}}^{\mathrm{symm}} \equiv \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \Phi^{\mathrm{A}}_{xa}(N_T,t)f_{abc}E^{i,c}(t)\Phi^{\mathrm{A}}_{bz}(t,0)f_{zxg}E^{i,g}(0)\rangle$$

$$= \frac{1}{3\langle l_8\rangle}\sum_{i=1}^{3}\langle \mathrm{Tr}\,[E_i(\tau)P(\tau)^\dagger]\mathrm{Tr}\,[E_i(0)P(0)] - \mathrm{Tr}\,[E_i(\tau)U(\tau,1/T)E_i(0)U(0,\tau)]\mathrm{Tr}\,[P(0)] + \mathrm{h.c.}$$

Gradient Flow



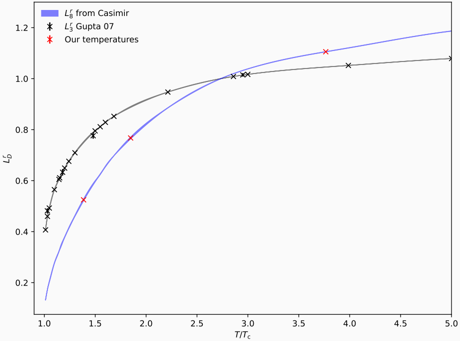
$$\partial_t B_{t,\mu} = -\frac{\delta S_{YM}}{\delta B} = D_{t,\mu} G_{t,\mu\nu} \,,$$
$$G_{t,\mu\nu} = \partial_\mu B_{t,\nu} - \partial_\nu B_{t,\mu} + [B_{t,\mu}, B_{t,\nu}] \,.$$
$$B_{0,\mu} = A_\mu \;\leftarrow \text{ the original gauge field}$$

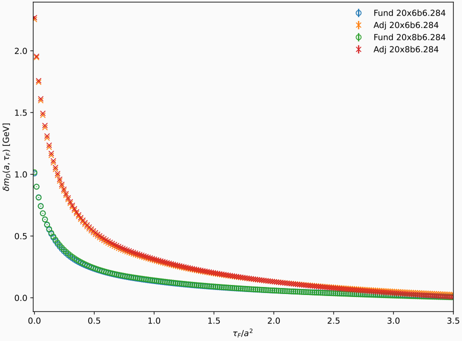


- Smearing the Wilson lines
- Avoid overlap $\sqrt{8\tau_f} < \tau/2$
- Automatically renormalizes the E-fields $\sqrt{8\tau_f} > a$

See e.g. M. Lüscher JHEP 08 (2010). Z_e figure from Brambilla *et.al.*PRD 105 (2022)

Removing Wilson line self-energy





- For symmetric correlators: same procedure as fundamental $\Rightarrow$ divide by Polyakov loop
- Non-symmetric correlator:

  $\Rightarrow$ Wilson line has divergence $\exp(\delta m \tau)$ with $\delta m \propto 1/\sqrt{8\tau_F}$
- Use renormalized Polyakov loops from Gupta *et.al.*PRD77 2008

$$L_8^{\mathrm{r}} = e^{\delta m(\tau_F)/T} L_8(\tau_F)\,, \quad G_E^{\mathrm{r}}(\tau,\tau_F) = e^{\delta m(\tau_F)\tau} G_E(\tau,\tau_F) = \left(\frac{L_8^{\mathrm{r}}}{L_8(\tau_F)}\right)^{\tau T} G_E(\tau,\tau_F)$$