Towards Gluon PDFs in the Coulomb Gauge

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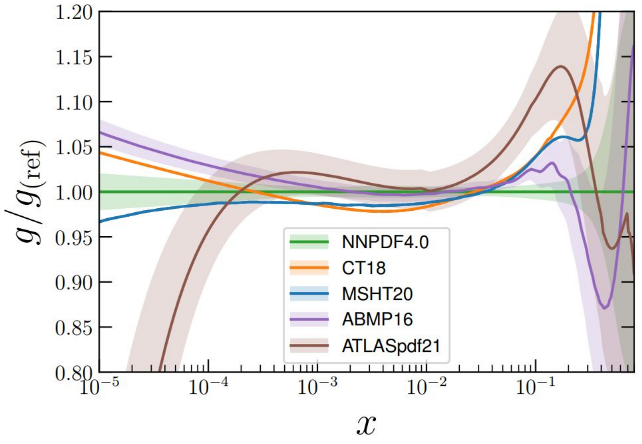

Amaroso, *et al.* (Snowmass) Acta Physica Polonica. B, 53, 12 (2023)

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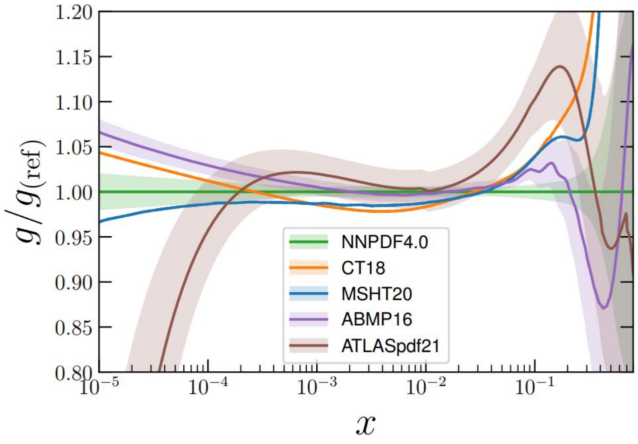

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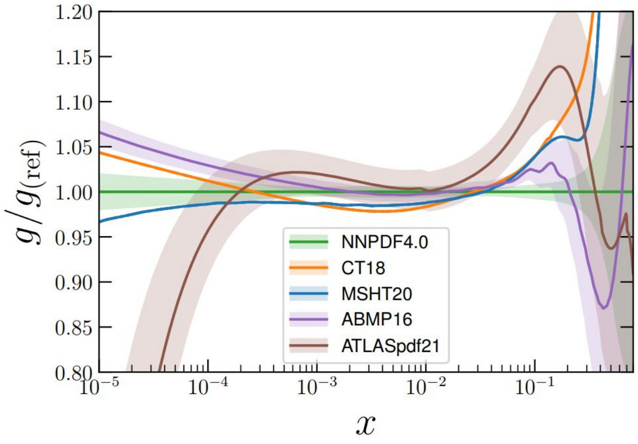

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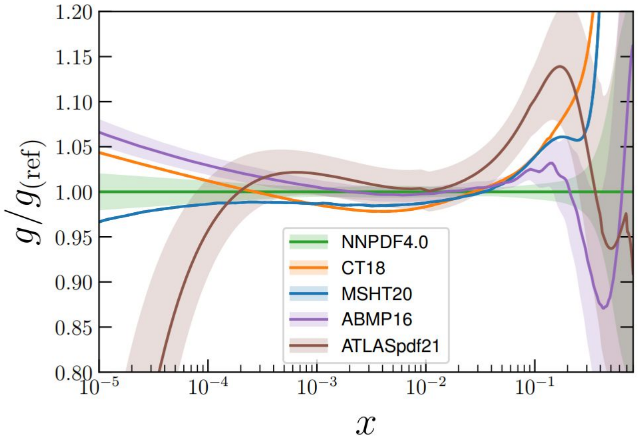
- Coulomb gauge PDFs promise to reduce statistical errors and in turn allow us to go to higher momenta, lower pion mass, etc.



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Ji, PRL 110:262002 (2013)
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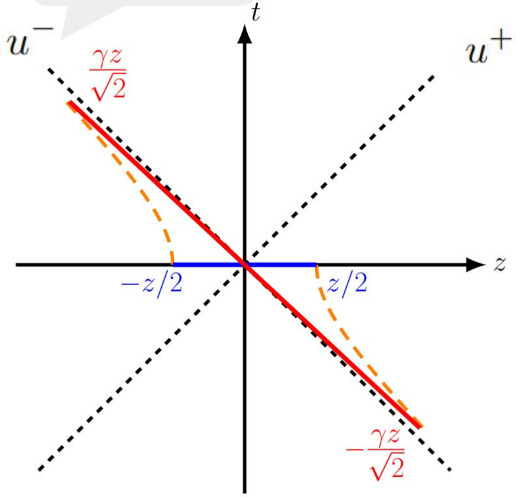
Light Cone Gluon PDFs

$$u^{\pm} = \frac{u^0 \pm u^3}{\sqrt{2}}$$



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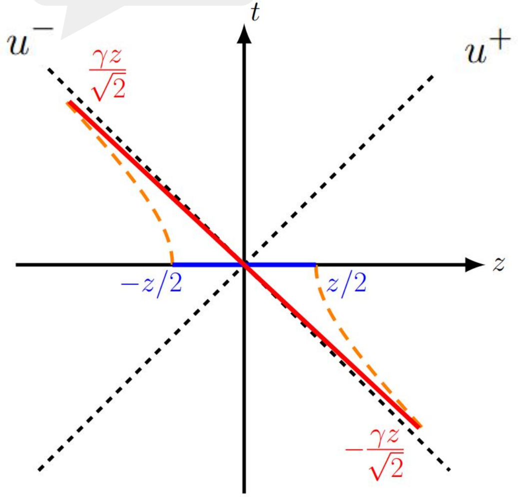

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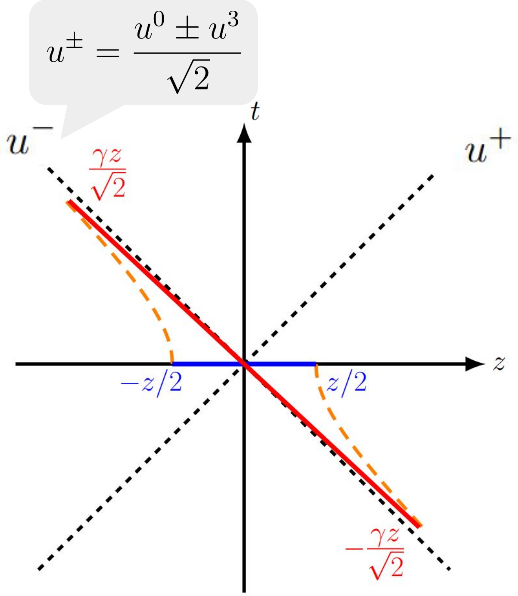

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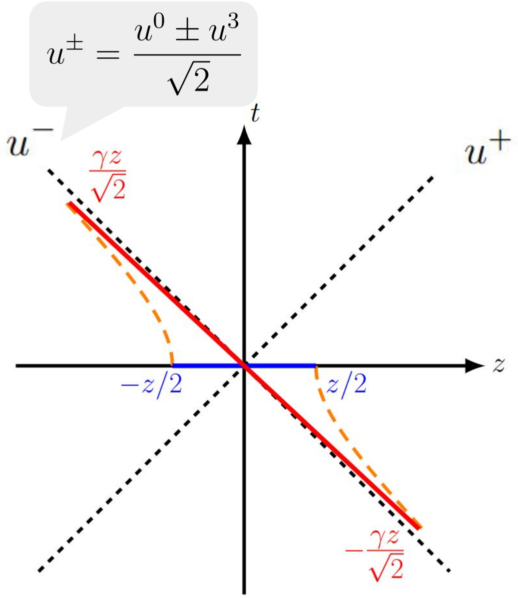

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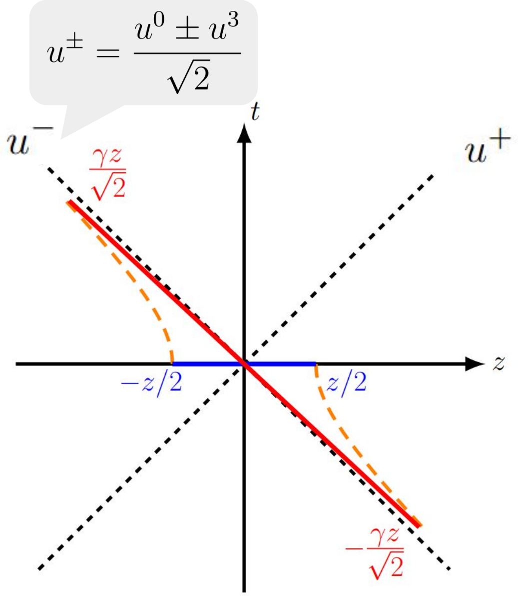

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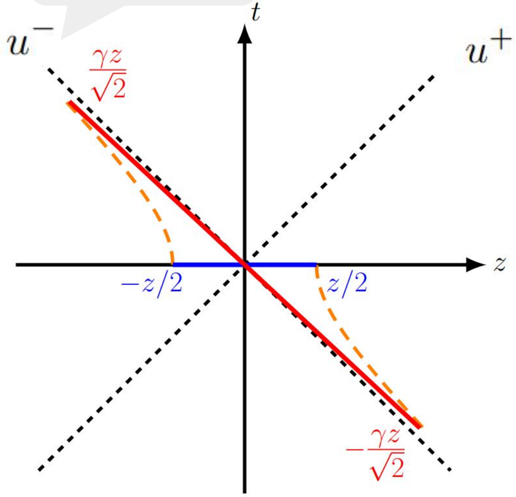

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$$\boxed{xg(x,\mu^2) = \int dy\, C_{gg}(\frac{x}{y}, \frac{\mu^2}{P_z^2}) y\tilde{g}(y,P_z) + C_{gq}(\frac{x}{y}, \frac{\mu^2}{P_z^2})\tilde{q}(y,P_z) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P_z)^2}\right)}$$

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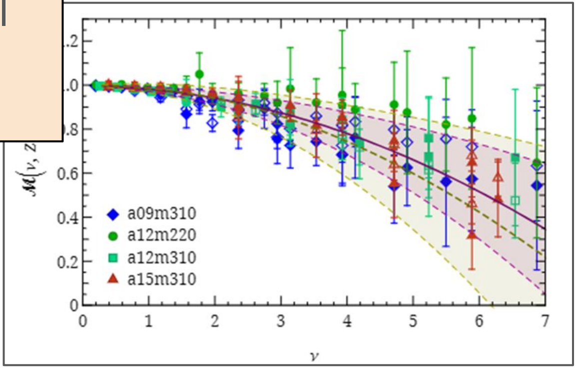
[MSULat] First physical continuum lattice nucleon gluon PDFs

PRD 108:014508(1), (2023)



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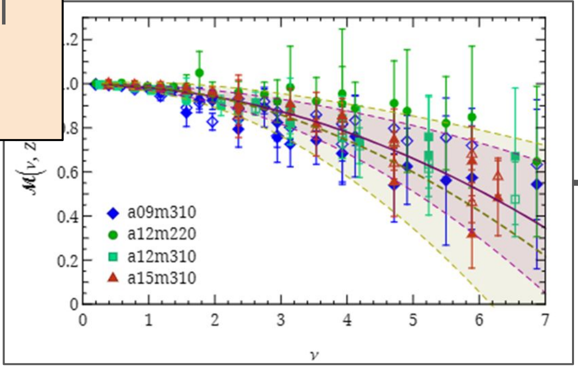

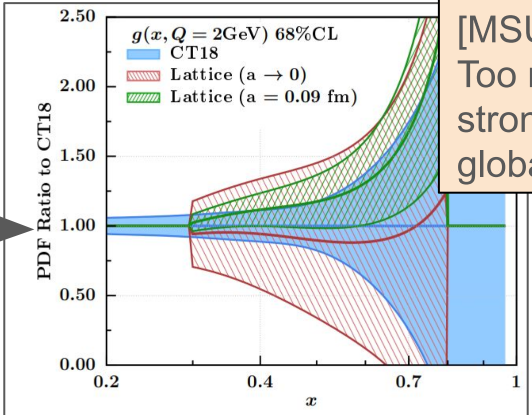

[MSULat+CTEQ] Too noisy to strongly impact global fit arXiv:2502.10630

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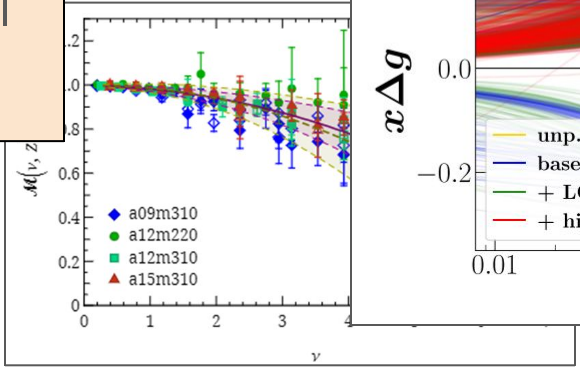

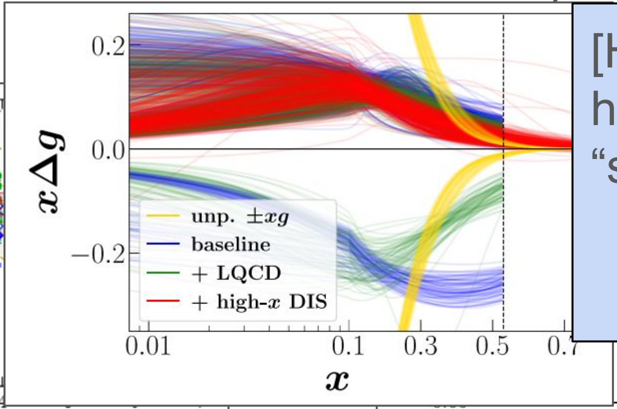

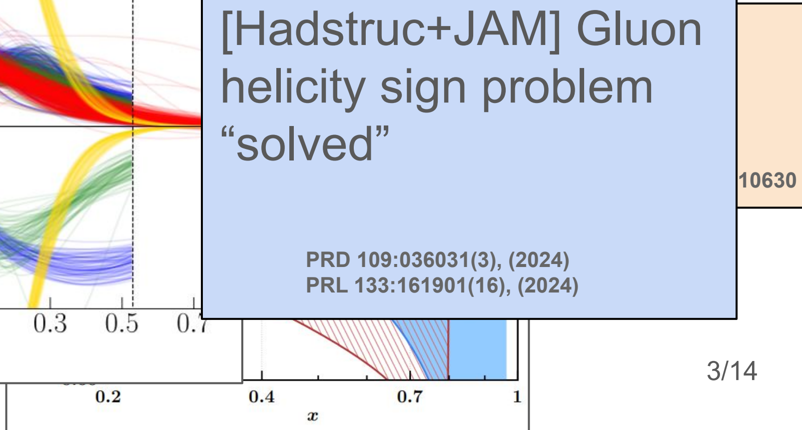

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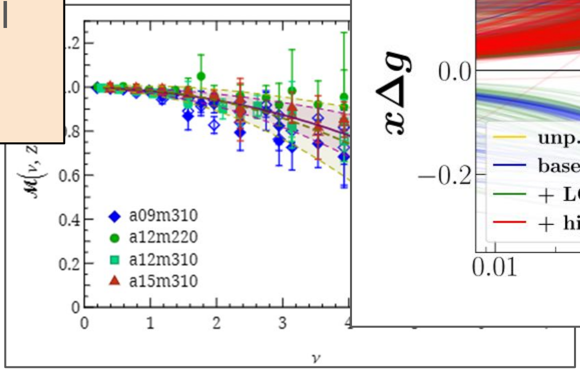

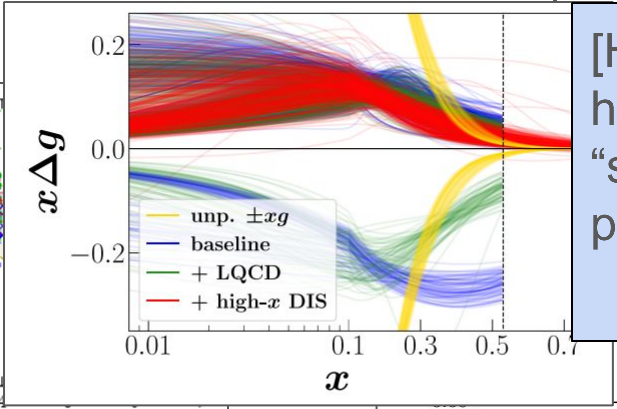

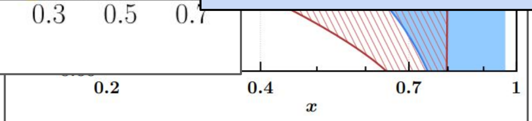

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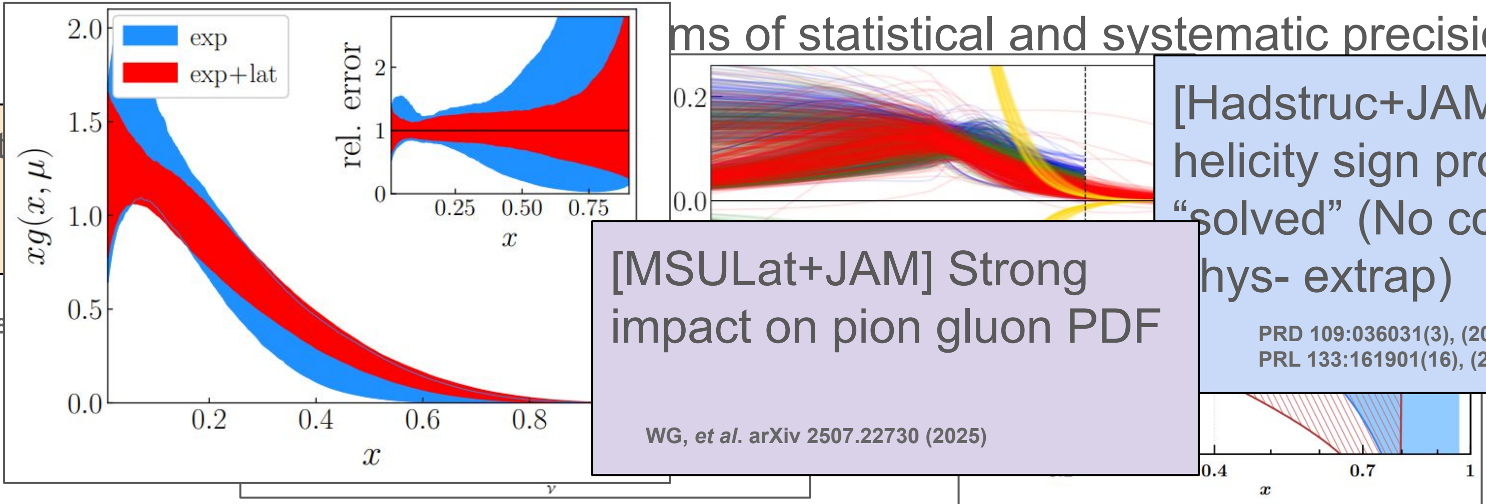

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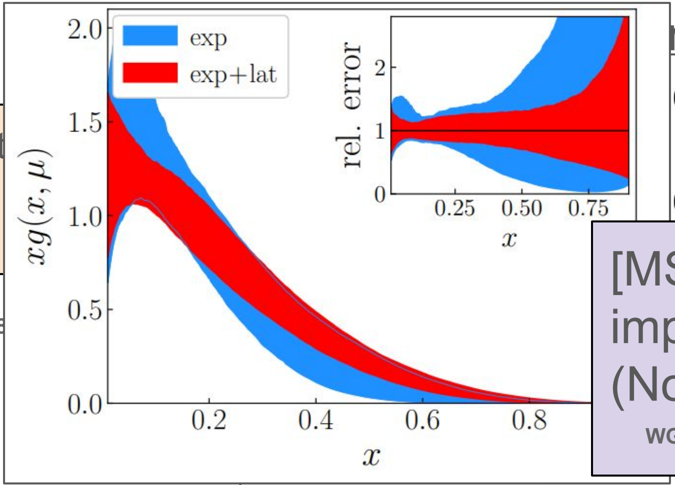

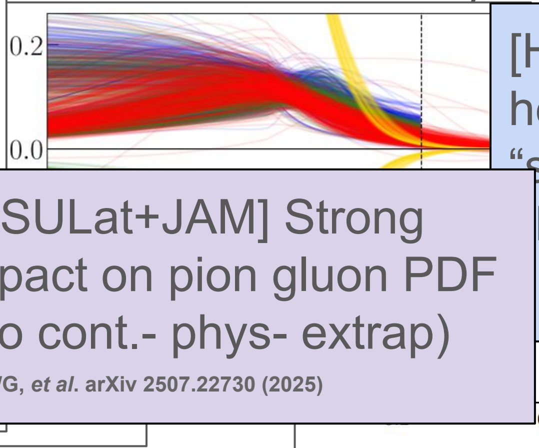

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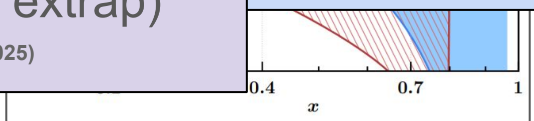

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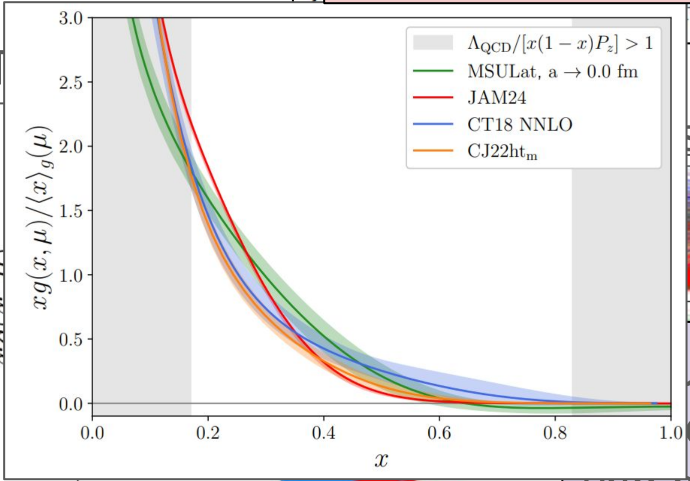

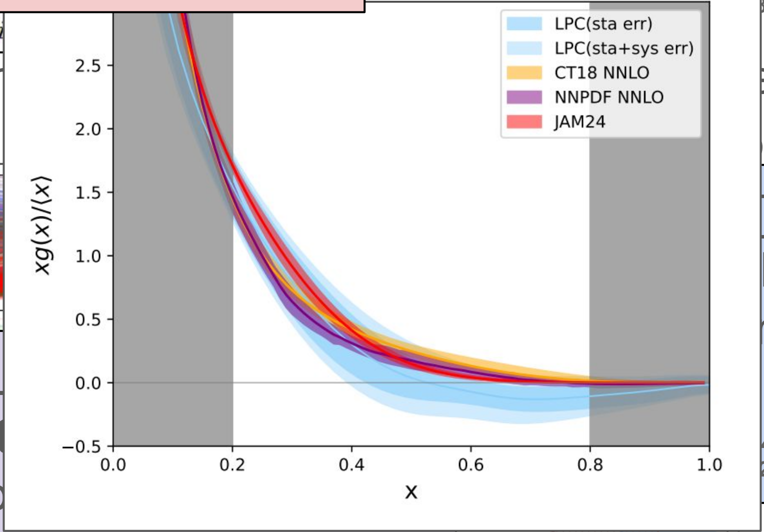

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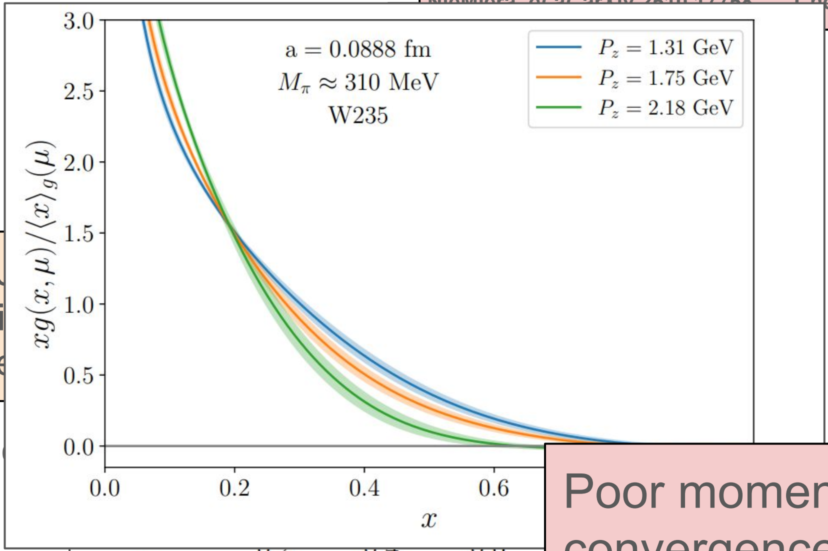

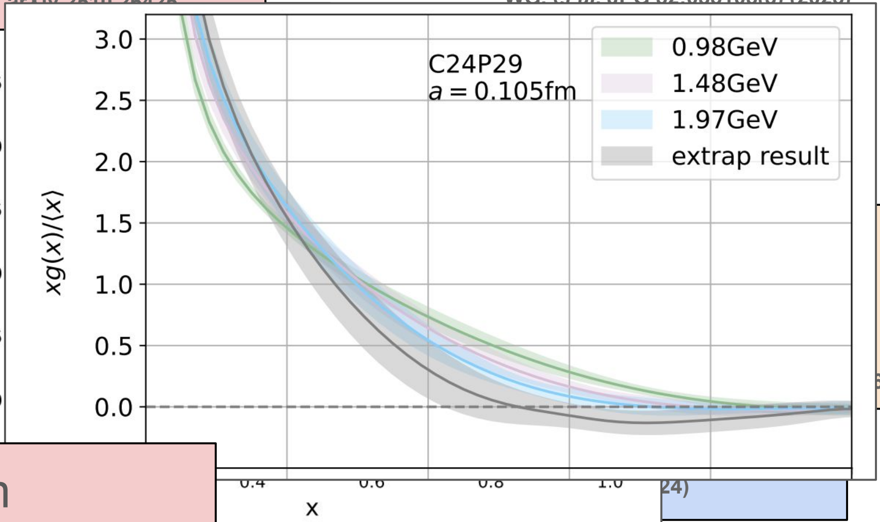

Poor momentum convergence!!

Coulomb Gauge to the Rescue

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Gao, *et al*. PRD 109(0):094506 (2024)

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- I will give a quick demo showing that the GI gluon operators have no linear divergence on the lattice and show our progress towards calculating the perturbative matching

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Follana *et al*. PRD 75:054502, 2007.
A. Bazavov, *et al*. [MILC], PRD 82:074501, 2010.
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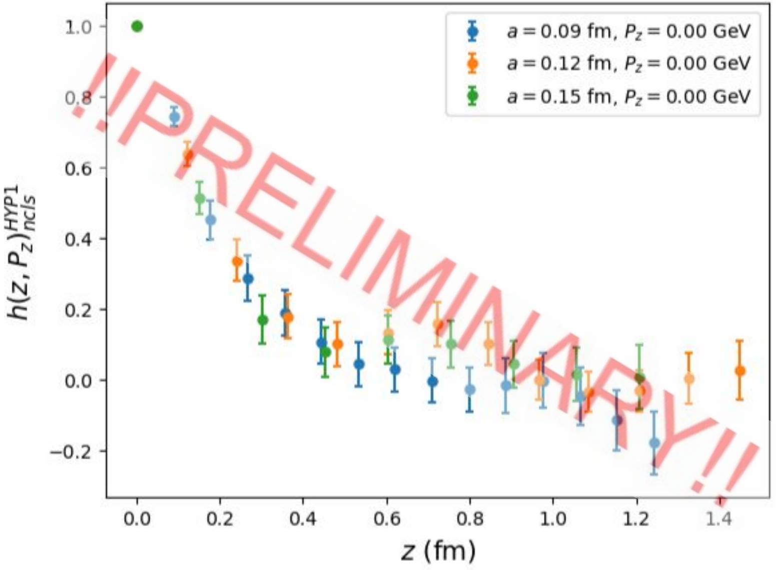

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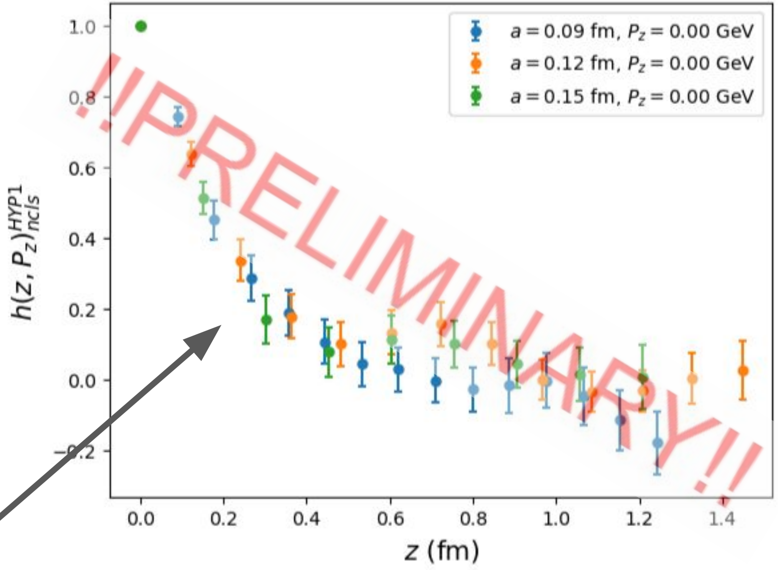

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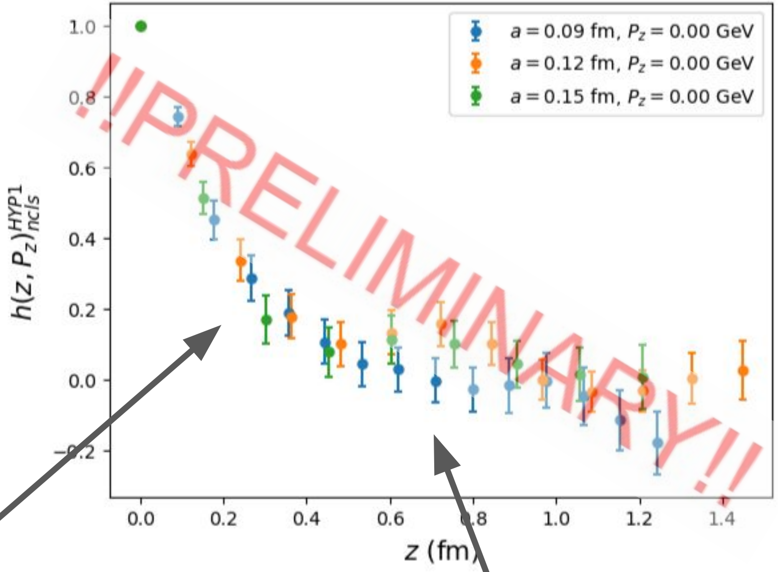

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Slight tension is likely from my sloppy preliminary matrix element fits

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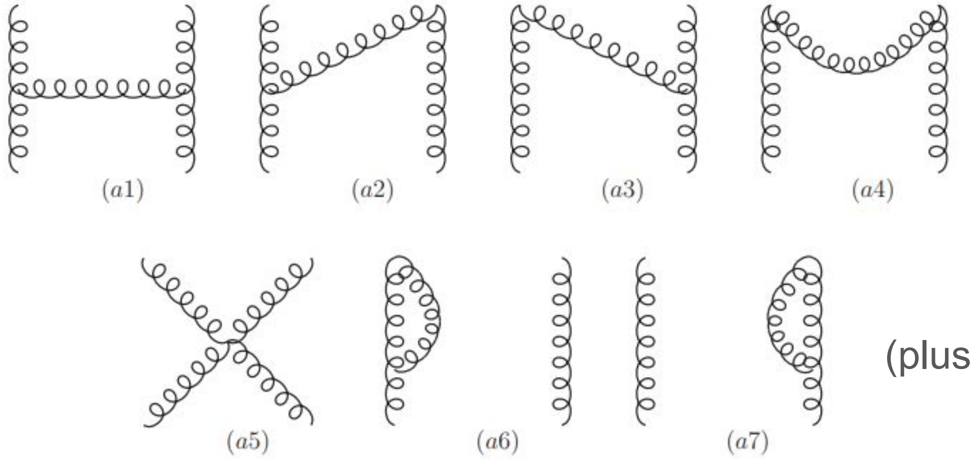

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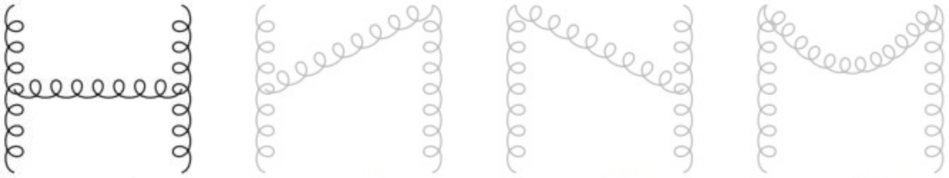

$(a1)$ $(a2)$ $(a3)$ $(a4)$

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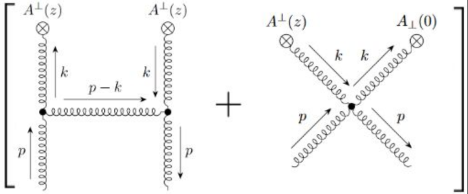

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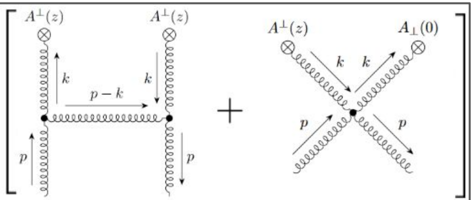

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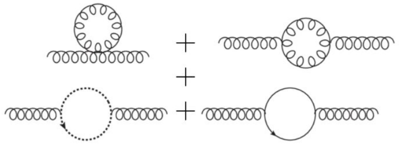

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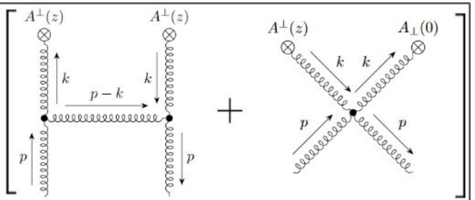

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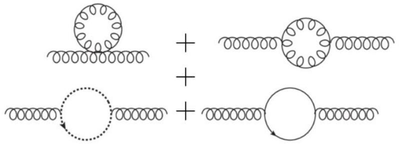

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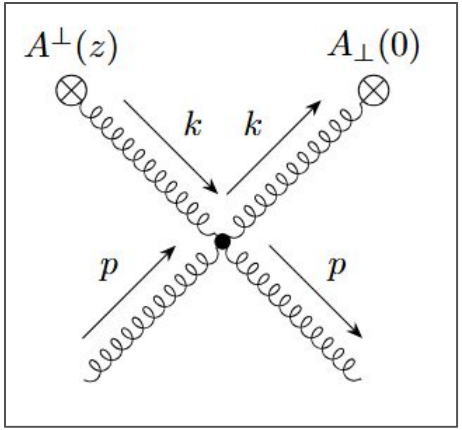

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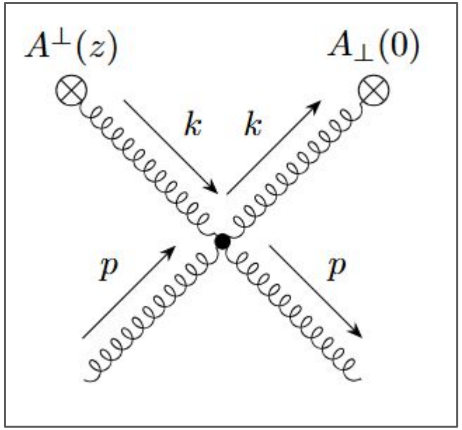

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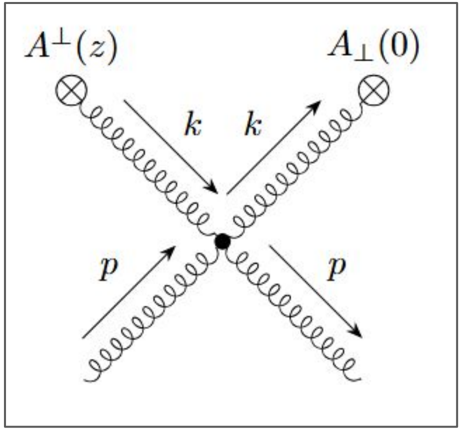

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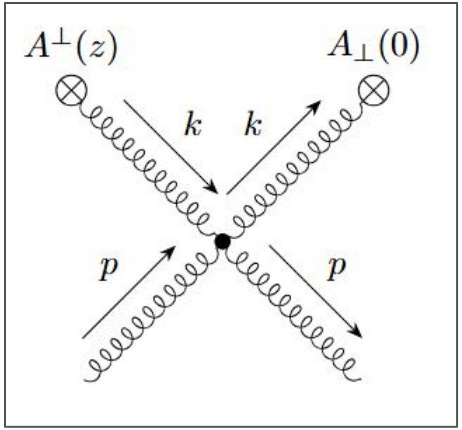

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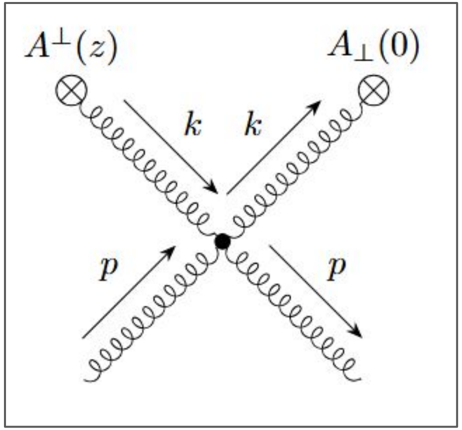

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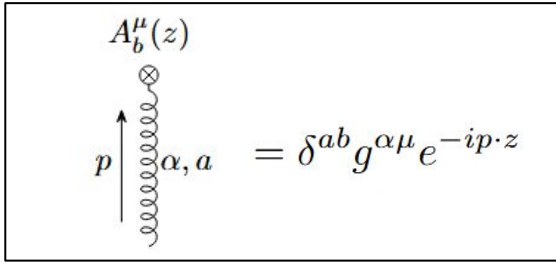

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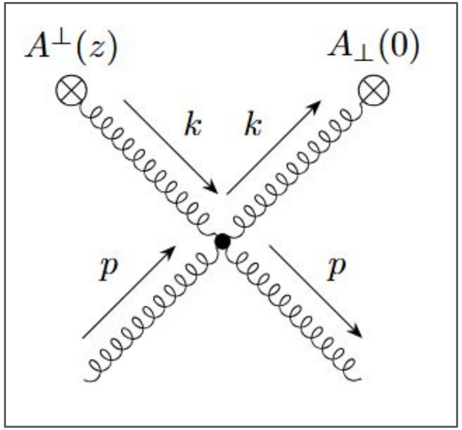

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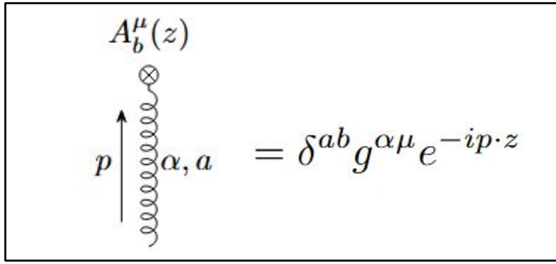

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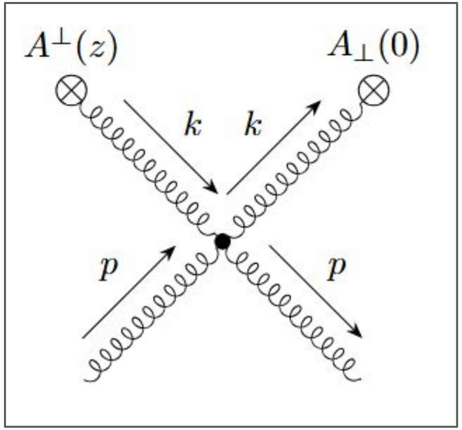

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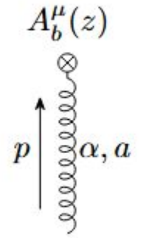

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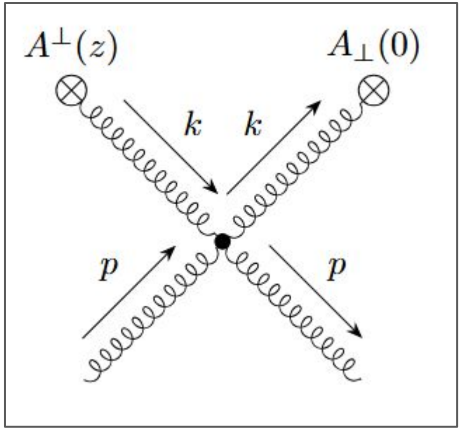

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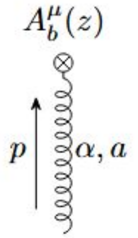

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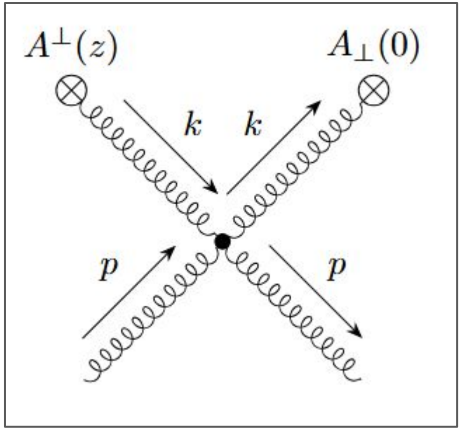

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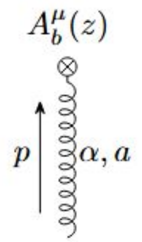

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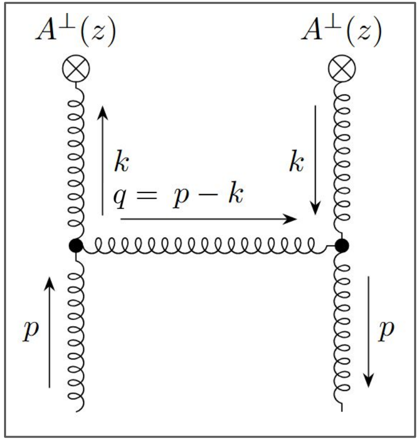

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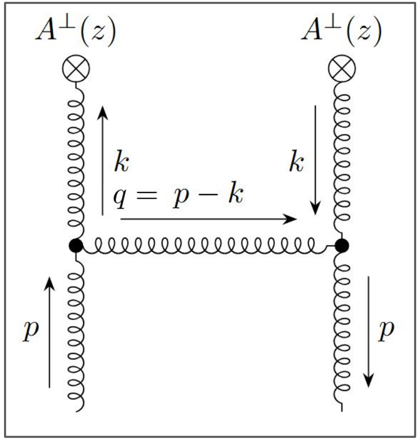

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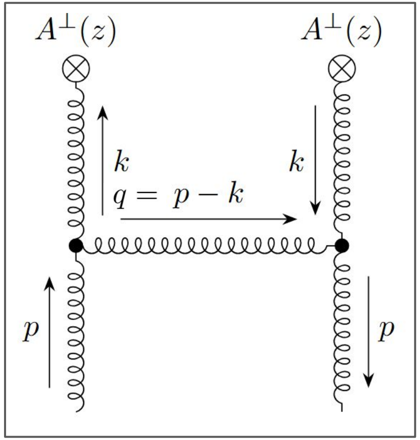

- If we do the temporal integration carefully, we can reduce everything to a nice set of 2-2ϵ integrals, which have closed forms

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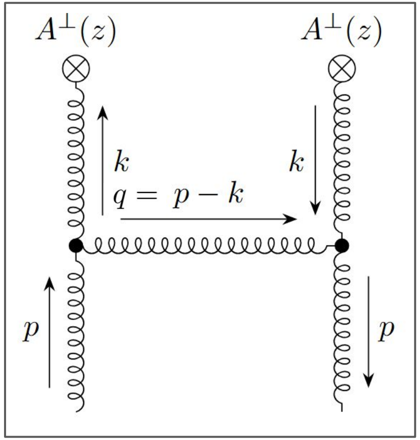

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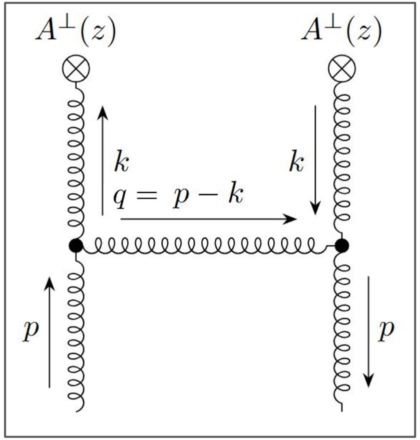

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- Expanding like this carefully and adding the contribution from the gluon wavefunction renormalization, we recover exactly the DGLAP kernel for our IR divergence, which nearly proves NLO factorization!

$$-\frac{P_{gg}(x)}{\epsilon_{\mathrm{IR}}} = \frac{1}{\epsilon_{\mathrm{IR}}}\left(-2C_A\left(\frac{x}{(1-x)_+} + x(1-x) + \frac{1-x}{x}\right) + \frac{\beta_0}{2}\delta(1-x)\right)$$

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- This suggests that the matching, convolution does not produce convergent results for any gluon PDF that does not approach 0 as y → 0

$$\widetilde{g}(x)=\int_{-1}^{1}\frac{dy}{|y|}C_{gg}\left(\frac{x}{y}\right)g(y)$$

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This observation and the explicit expression given by Eq. (7.28) may be used to check the gluon-gluon matching kernels in Refs. [17,18]. Our check shows that $Z_1^{gg}(y,1,p_3)|_{y>1}$ corresponding to Eq. (64) of Ref. [18] does not satisfy the constraint (7.27). The difference is by a constant term (-2/3) that leads to a linear divergence in the integral of $Z_1^{gg}(y,1,p_3)|_{y>1}$ over y. The same difference (with the opposite sign) appears in the $Z_1^{gg}(y,1,p_3)|_{y<0}$ term in Eq. (64) of Ref. [18]. Apparently, these differences result from the use of the off-shell external gluon fields in the calculations of Ref. [18], but a discussion of this topic is outside of the scope of our paper.

because we have chosen a different Dirac structure in the quark bilinear operator. Secondly, our ratio scheme result for the unpolarized gluon PDF is in agreement with that derived in eq. (7.28) of ref. [44], which also starts from coordinate space and then Fourier transforms to momentum space. However, both the results of ref. [44] and our results show a discrepancy with earlier calculations performed directly in momentum space [43] in the unphysical region. In the ratio scheme, the contribution of the unphysical region in momentum space comes entirely from the Fourier transform of the $\ln z_{12}^2\mu^2$ term in coordinate space. From this perspective, it is obvious that the momentum space matching in the unphysical region is entirely determined by the evolution kernel in coordinate space, where the latter is well-known in the literature. Nevertheless, it is highly desirable to further investigate and understand what leads to the mismatch between these two approaches.

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A. V. Radyushkin, PRD 96:034025, 2017.
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An Issue Persistent in the Literature

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- We will also complete the more straightforward calculation of the quark-in-gluon matching once the glue-glue calculation is resolved