Machine-learned 4d SU(3) fixed-point action

Topology and thermodynamics

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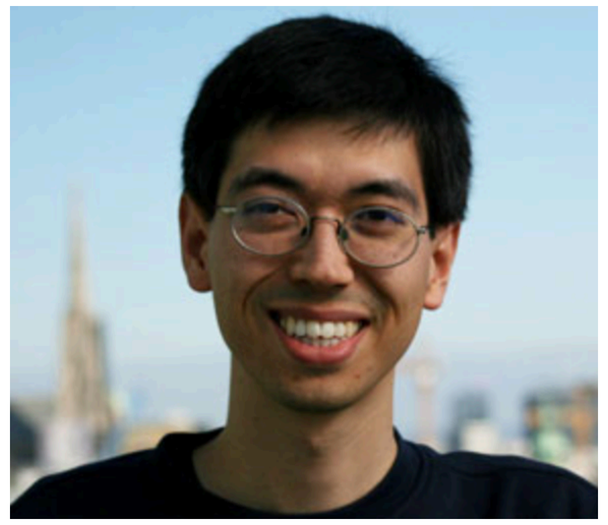
Swiss National Science Foundation

UNIVERSITÄT BERN



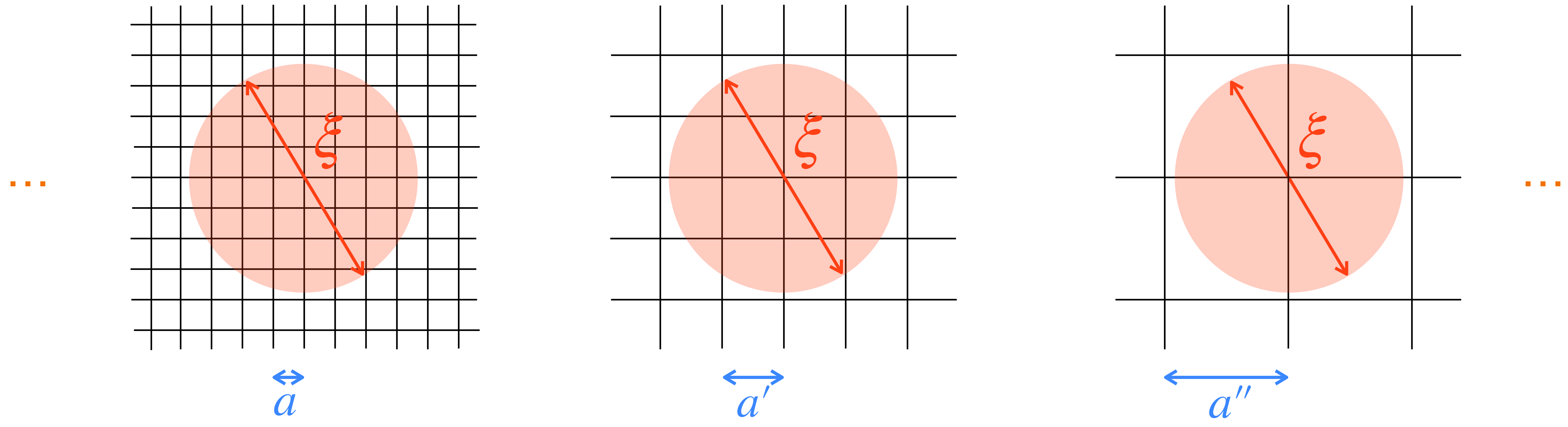
TU WIEN

in collaboration with Liane Backfried (University of Bern), Kieran Holland (University of Pacific), Andreas Ipp and David Müller (TU Wien)

PRD 110 (2024) 7, 074502 [arXiv:2401.06481], PRL 136 (2026) 3, 031901 [arXiv:2502.03315]

43rd Intl. Symposium on Lattice Field Theory, 31 July 2026 - University of Maryland, College Park (USA)

Introduction

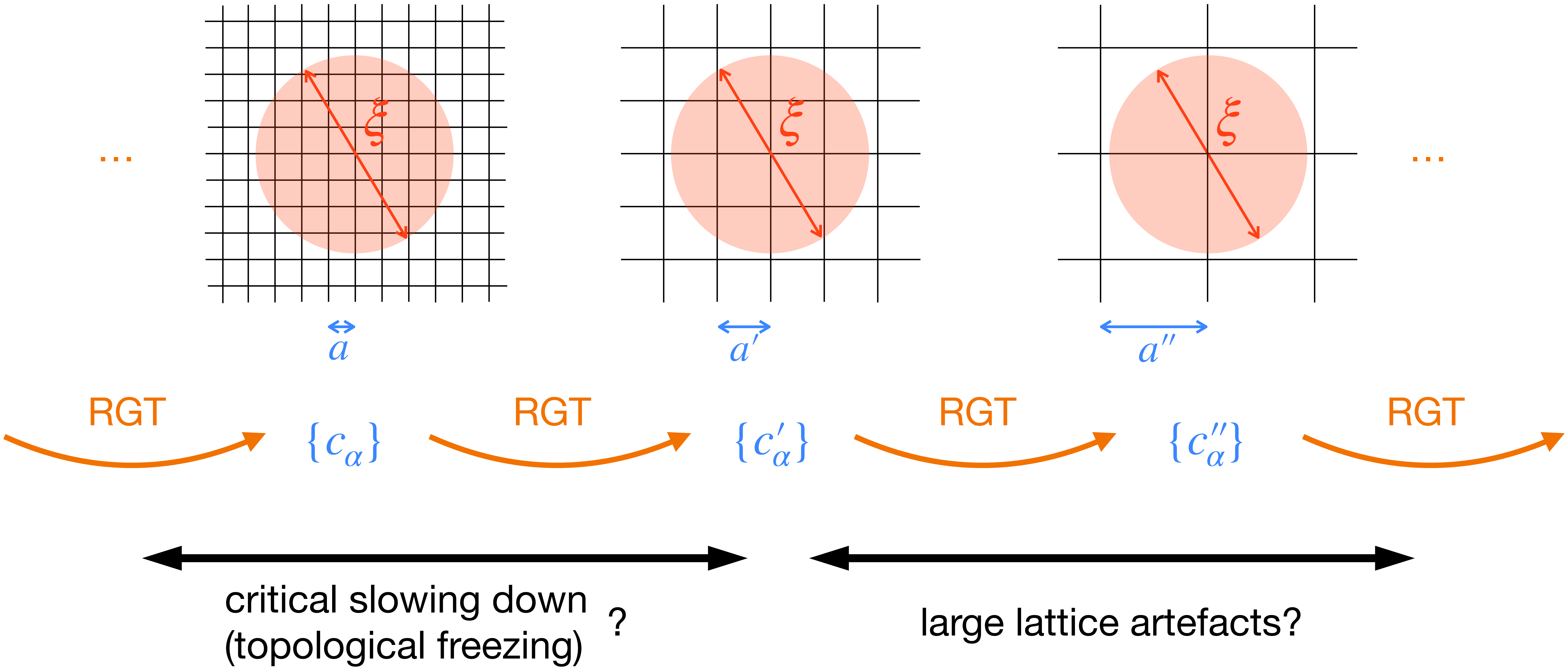


continuum limit (2nd order phase transition $\xi/a \to \infty$)

critical slowing down
(topological freezing) ?

large lattice artefacts?

Introduction



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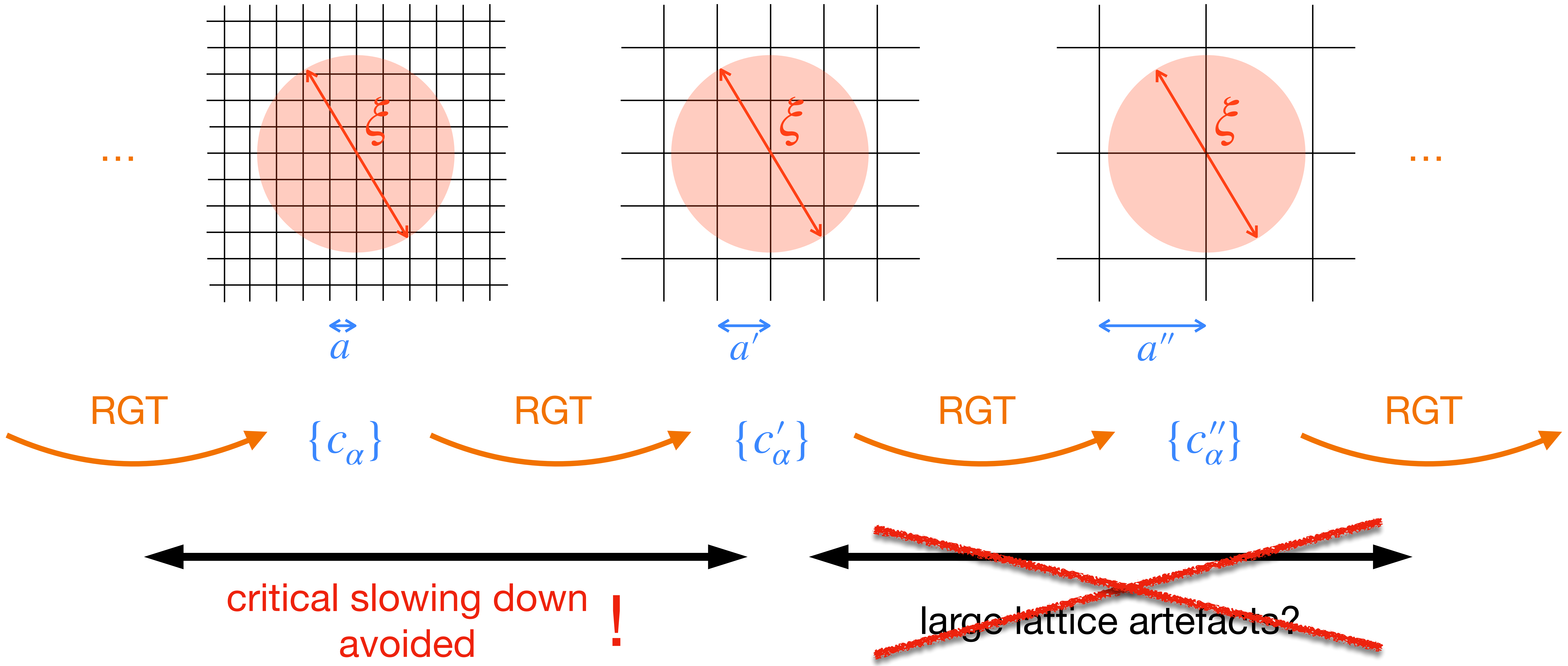

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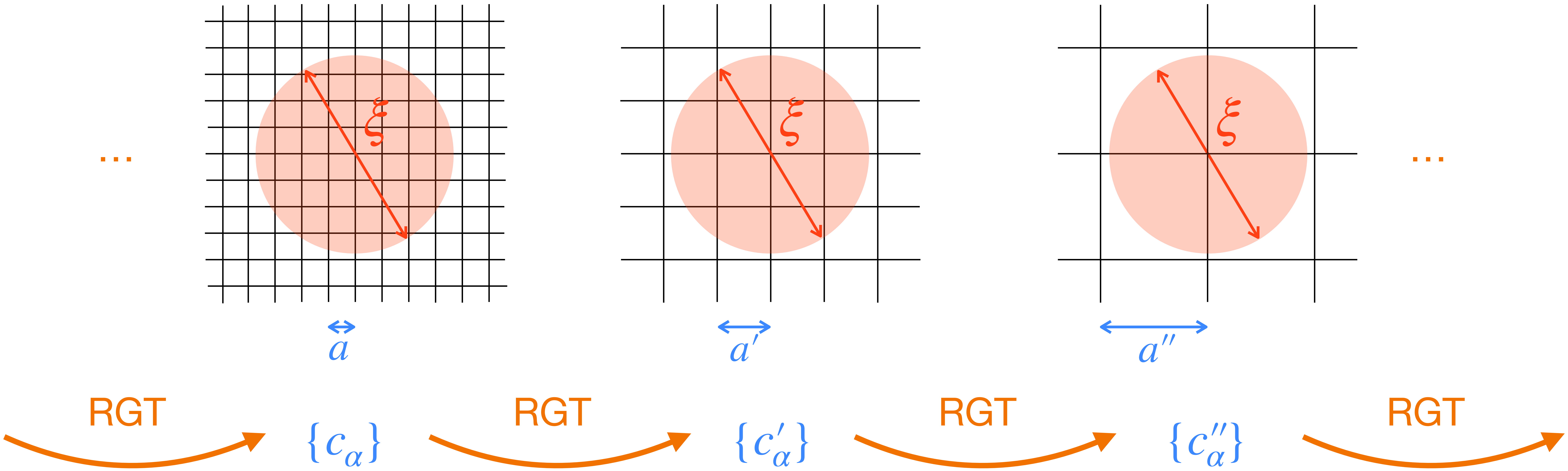
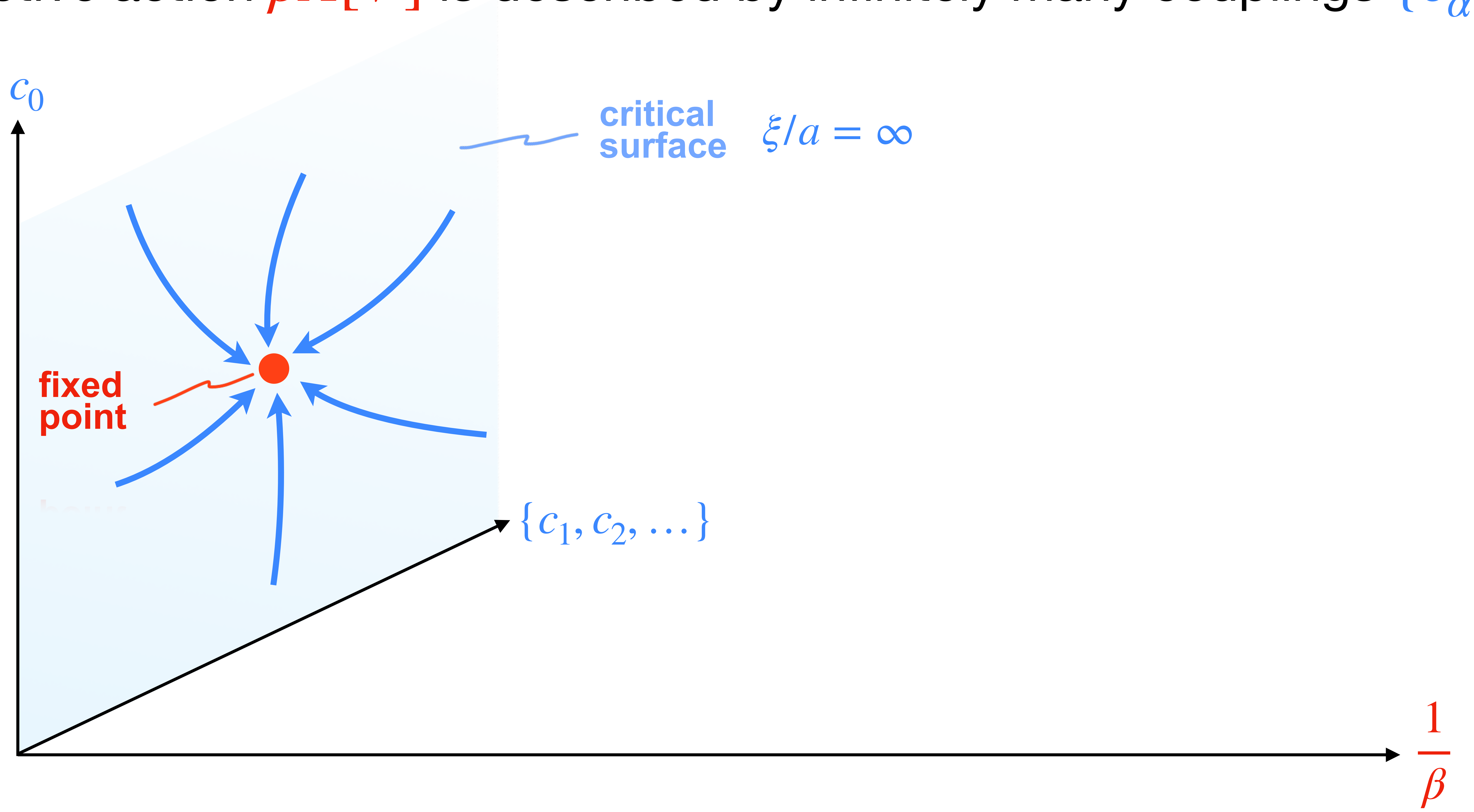


Introduce (coordinate space) renormalization group transformation (RGT):

$$\exp\left\{-\beta' A'[V]\right\} = \int \mathscr{D}U \exp\left\{-\beta\left(A[U] + T[U,V]\right)\right\}$$

Renormalization group transformation

The effective action $\beta A[V]$ is described by infinitely many couplings $\{c_\alpha\}$:



$\Rightarrow$ fixed point of RGT iterations (when $\xi/a \to \infty$): $\{c_\alpha^{FP}\} \xrightarrow{\text{RGT}} \{c_\alpha^{FP}\}$

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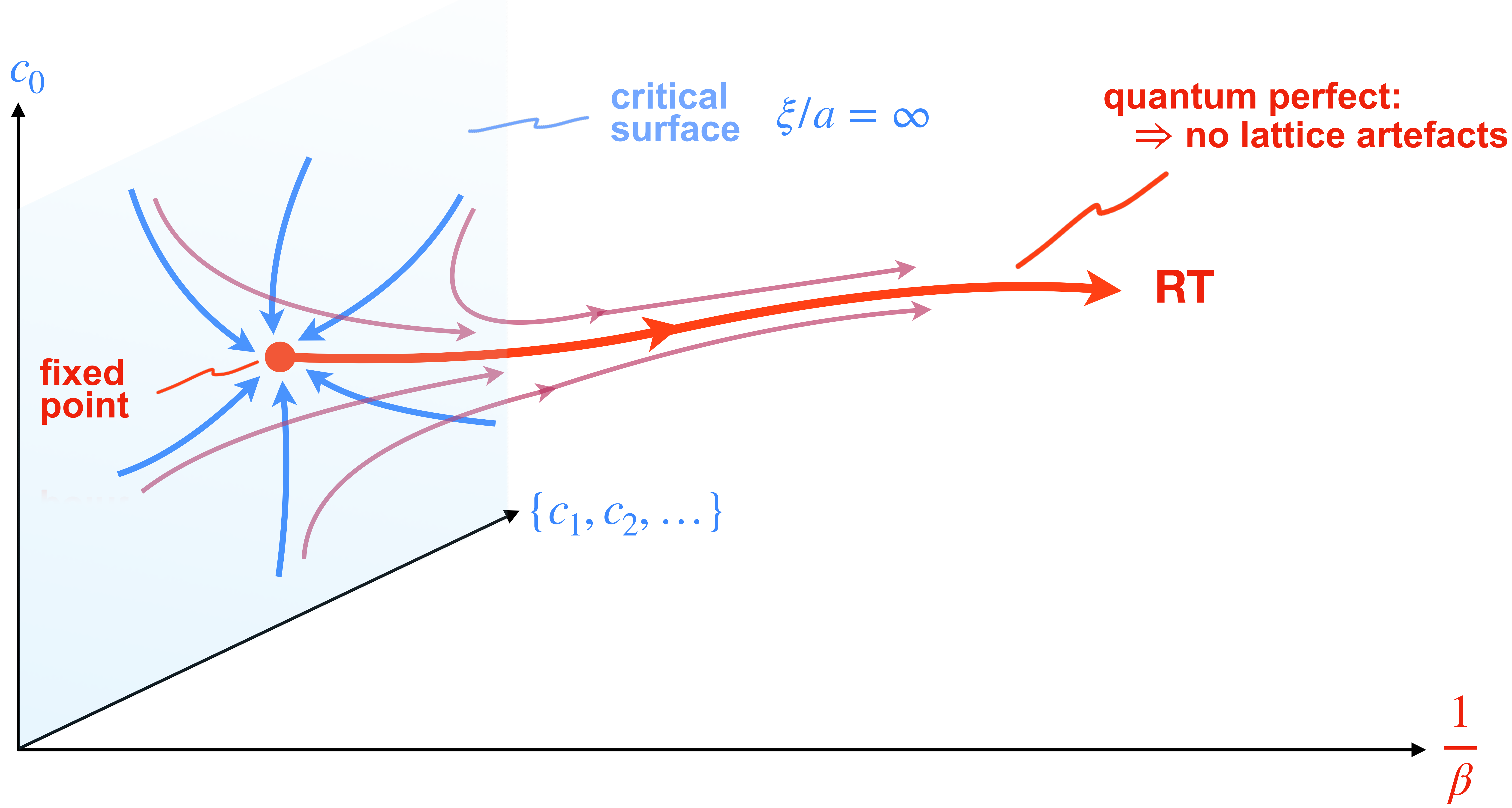

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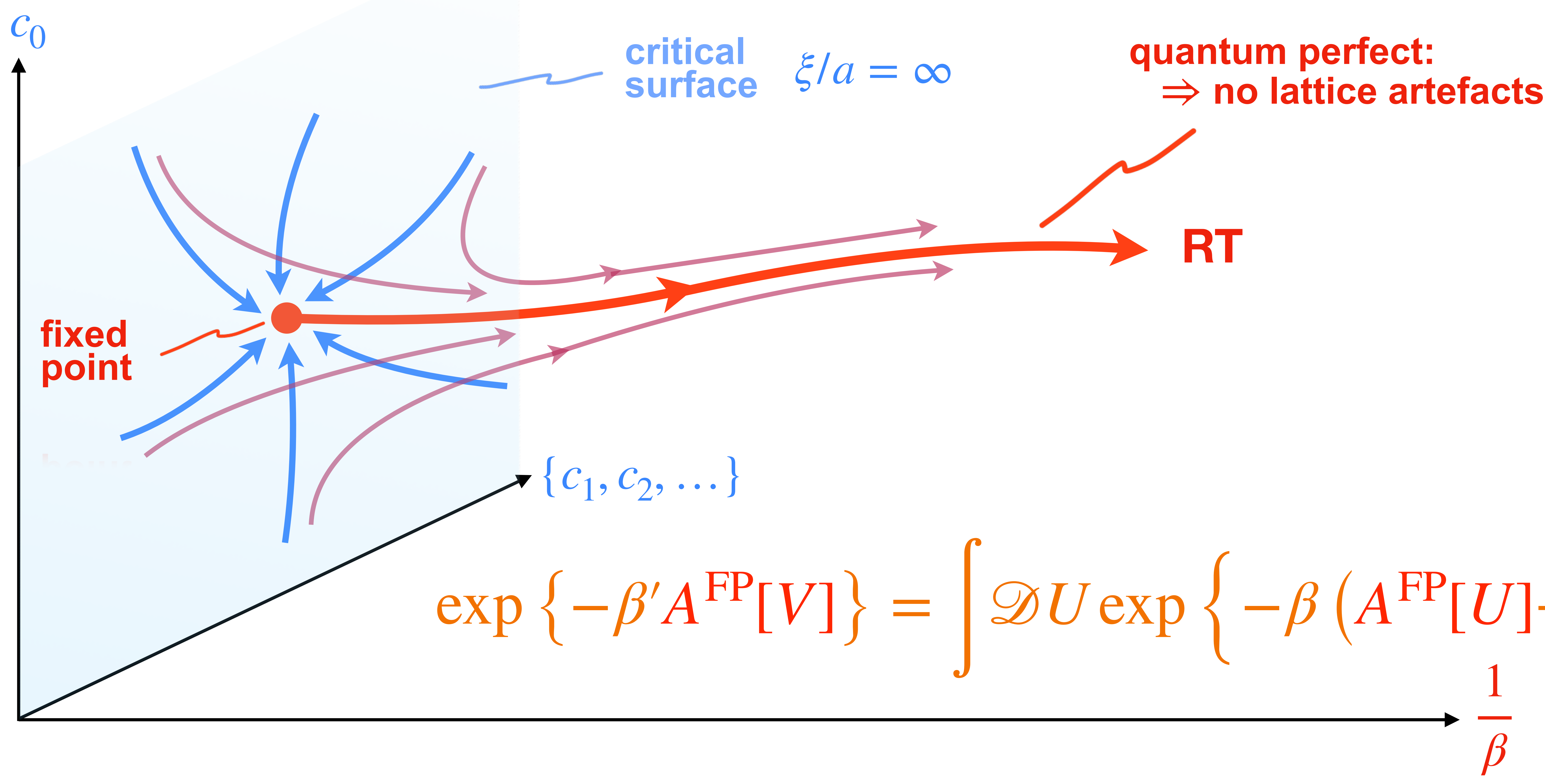

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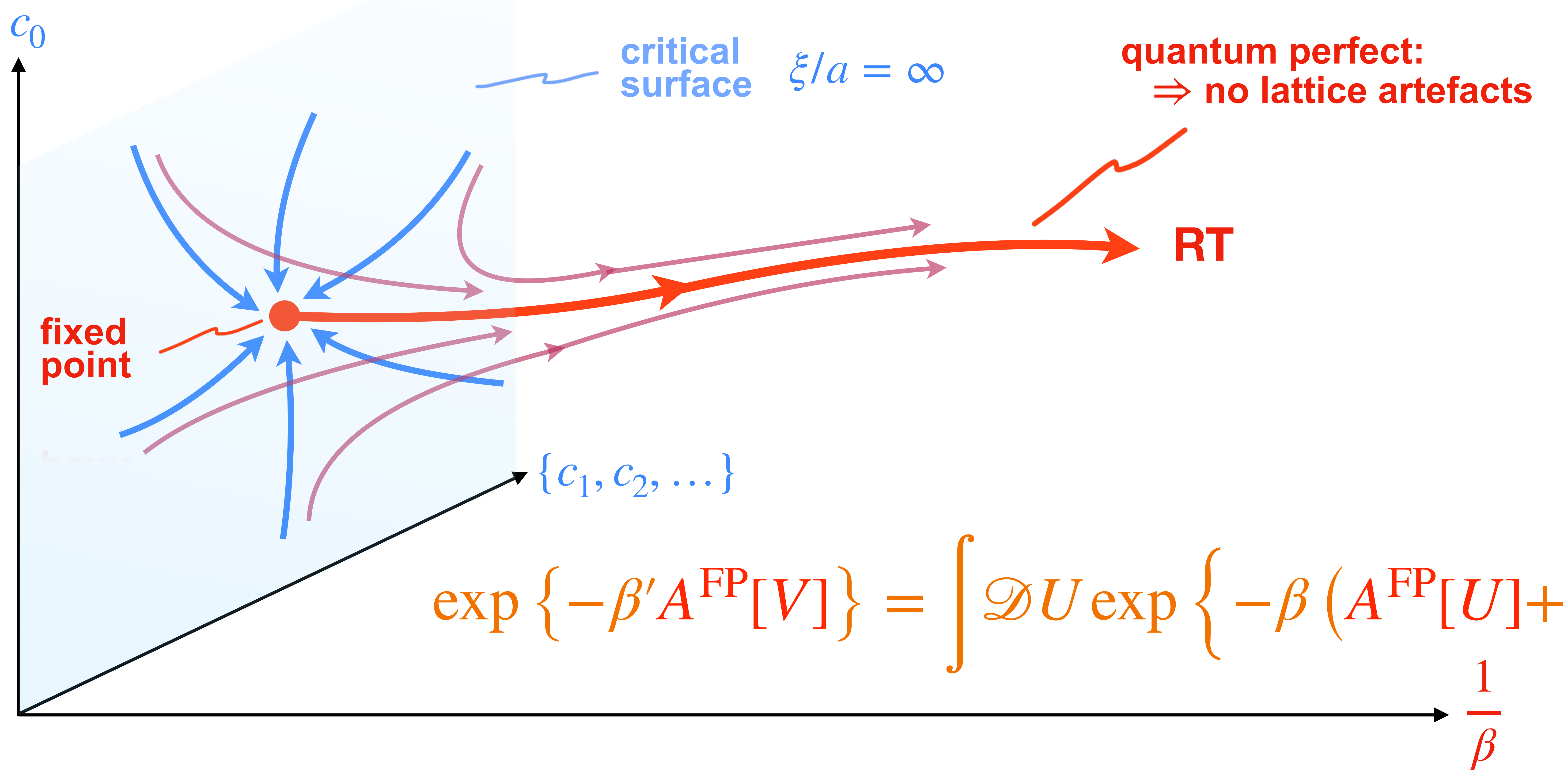

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$\beta \to \infty$

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Classically perfect FP actions

The classical FP action A^{FP} defines an action for all β:

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Perfect lattice action for asymptotically free theories *

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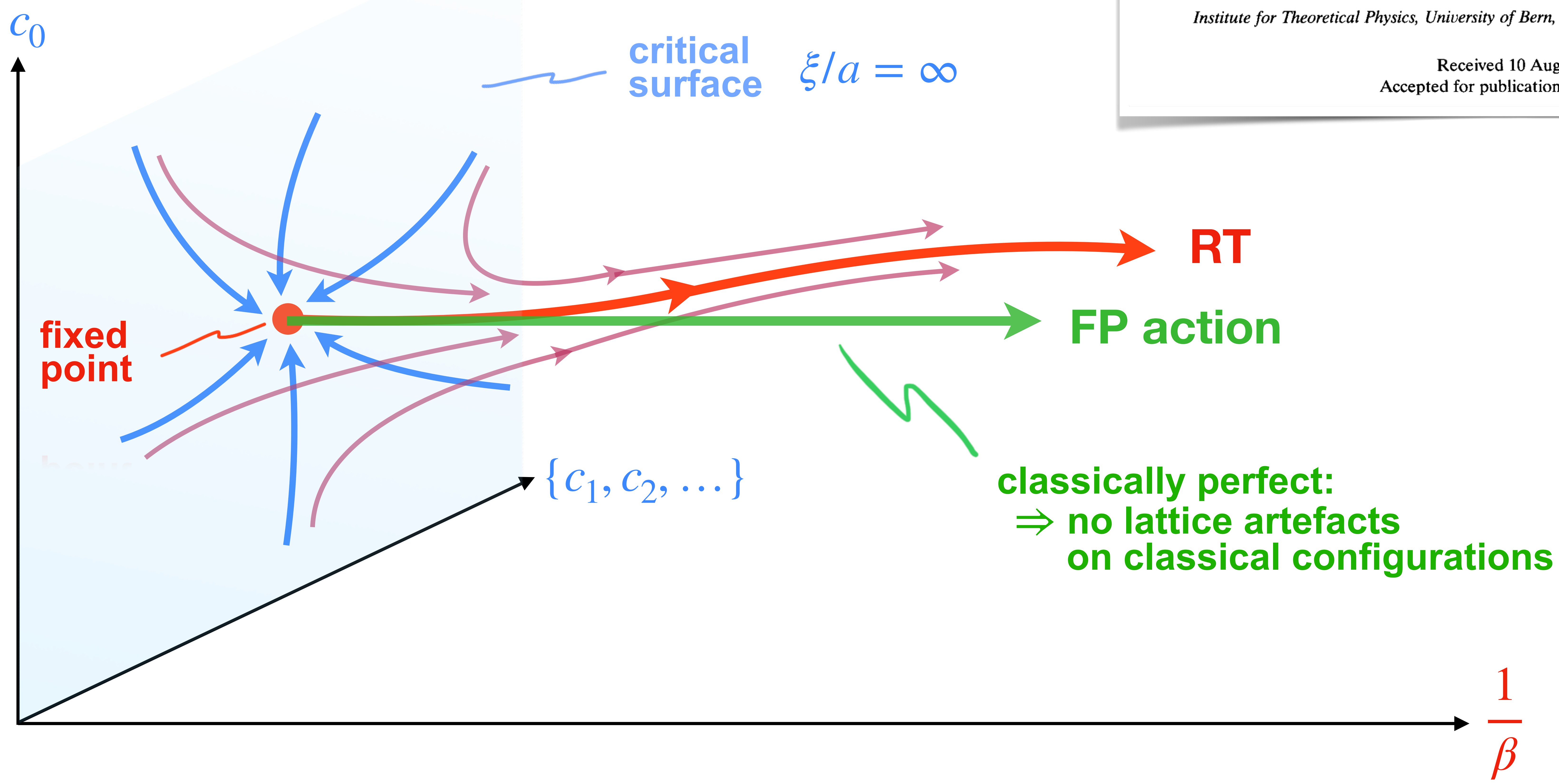

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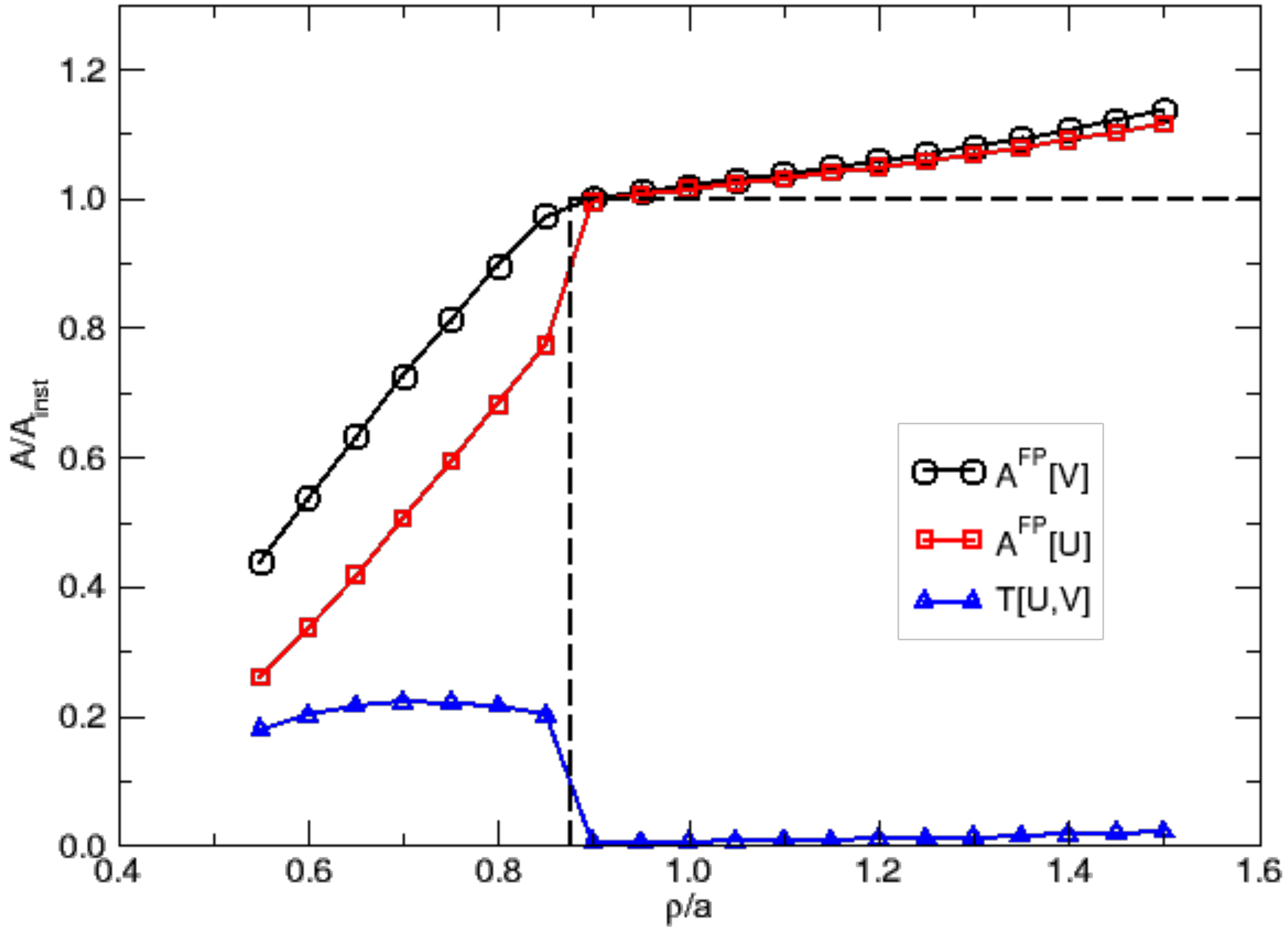
$\Rightarrow A^{\mathrm{FP}}[V]$ has scale invariant instanton solutions



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$$\frac{\delta A^{\mathrm{FP}}[V]}{\delta V} = 0 \quad \Rightarrow \quad \frac{\delta A^{\mathrm{FP}}[U]}{\delta U} = 0$$

$\Rightarrow$ $A^{\mathrm{FP}}[V]$ has scale invariant instanton solutions

- For large $\beta, A^{\mathrm{FP}}[V]$ is very close to $A^{\mathrm{RT}}[V]$:

$\Rightarrow$ lattice artefacts expected to be substantially reduced:

$$\cancel{\mathcal{O}(a^{2n})}, \mathcal{O}(g^2 a^{2n}) \quad n = 1,2,\dots$$

Classically perfect FP gradient flow

Physical reference scales defined through $t^2\langle E\rangle\,|_{t=t_x} = x,\qquad t\frac{d}{dt}\left(t^2\langle E\rangle\right)\Big|_{t=w_x^2} = x$

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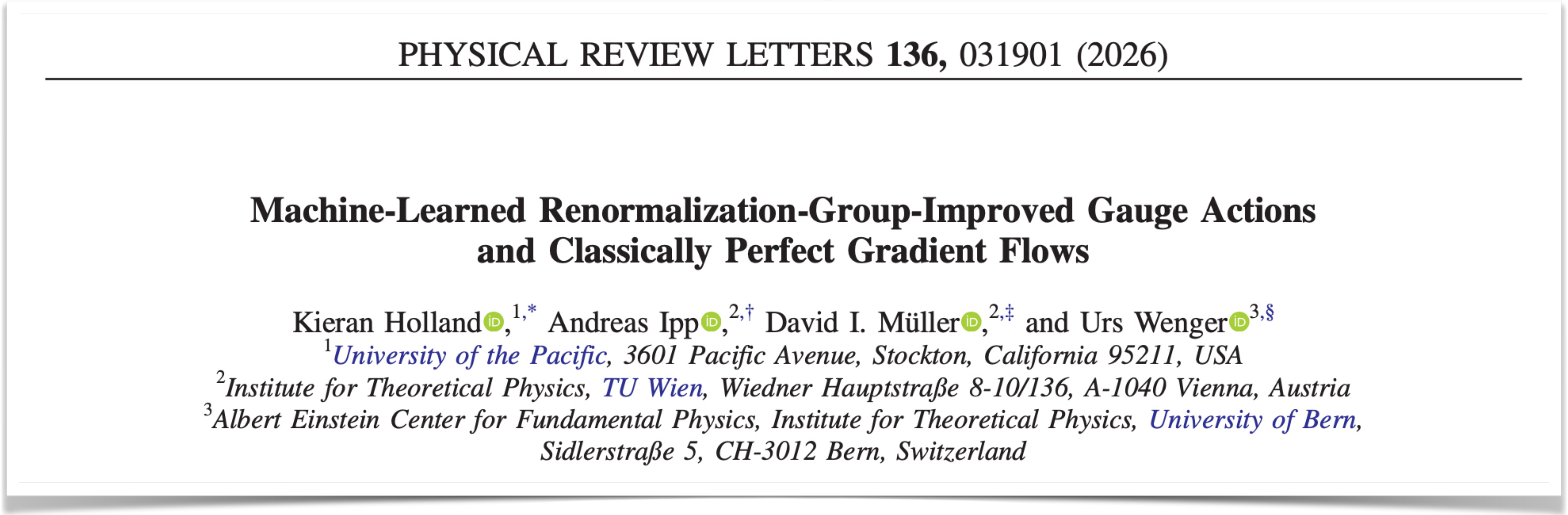
$$t^2\langle E(t)\rangle = \frac{3(N^2-1)g_0^2}{128\pi^2}\left[C(a^2/t) + \mathcal{O}(g_0^2)\right]$$



PHYSICAL REVIEW LETTERS **136,** 031901 (2026)

Machine-Learned Renormalization-Group-Improved Gauge Actions and Classically Perfect Gradient Flows

Kieran Holland,[1,*] Andreas Ipp,[2,†] David I. Müller,[2,‡] and Urs Wenger[3,§]

[1]*University of the Pacific, 3601 Pacific Avenue, Stockton, California 95211, USA*

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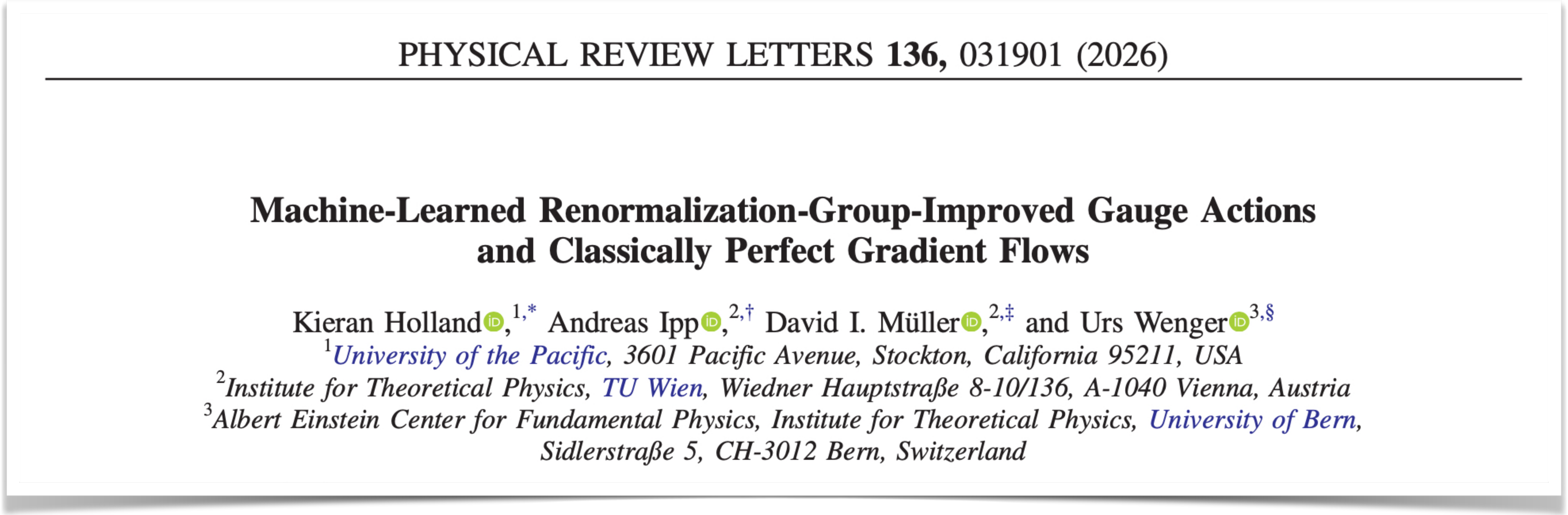
$$= 1$$



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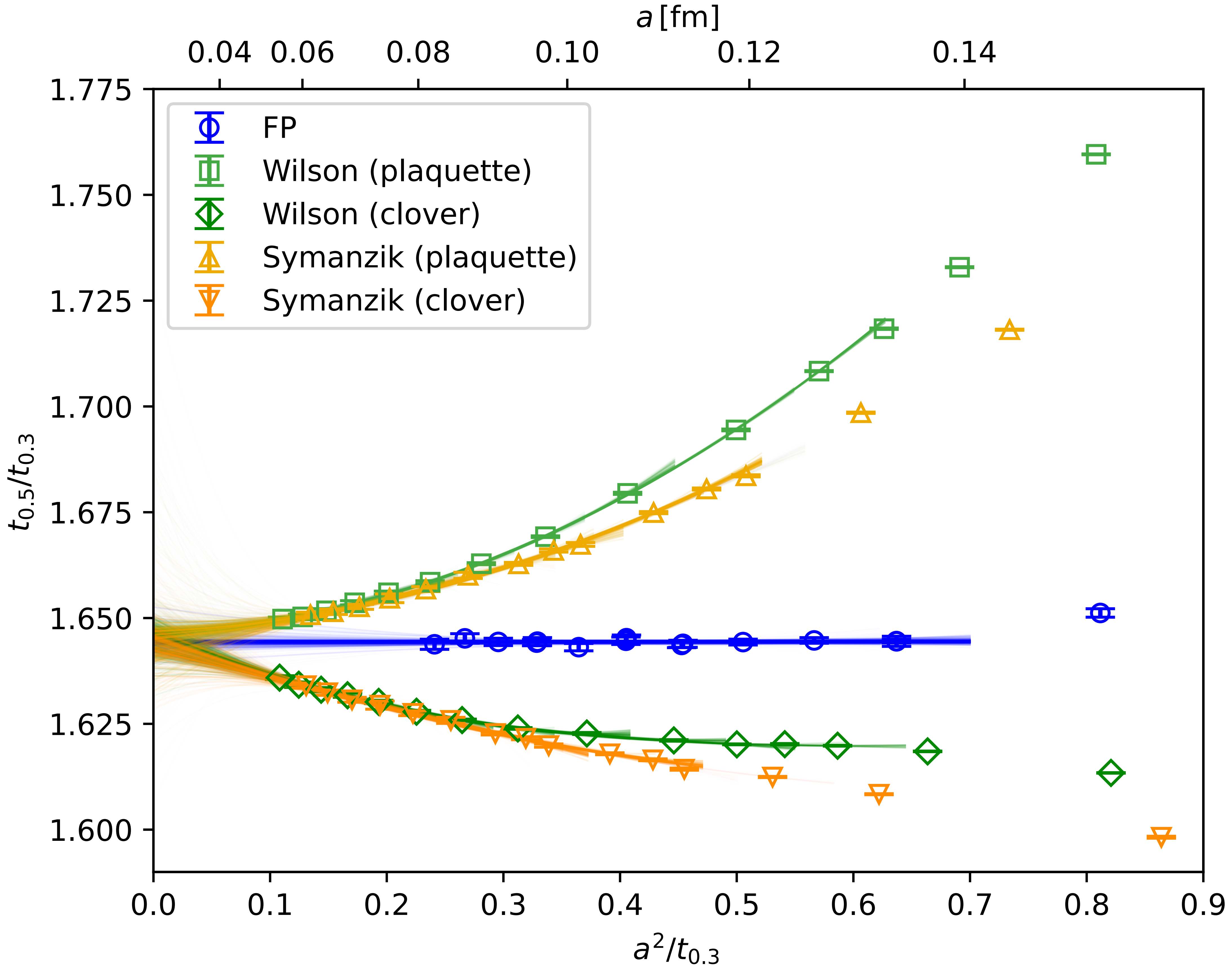

Topology on the lattice

Topological charge defined through $q(x) = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[G_{\mu\nu}(x)G_{\rho\sigma}(x)], \quad Q = \int d^4x\, q(x) \in \mathbb{Z}$

Topological susceptibility at finite flow time is a renormalised physical quantity:

$$\chi_{\mathrm{top}} = \frac{\langle Q^2 \rangle}{L^4}, \quad \langle Q \rangle = 0, \quad [\chi_{\mathrm{top}}] = \mathrm{MeV}^4$$

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- need high statistics for $\langle Q^2 \rangle$
- lattice artifacts associated with Q_{latt}

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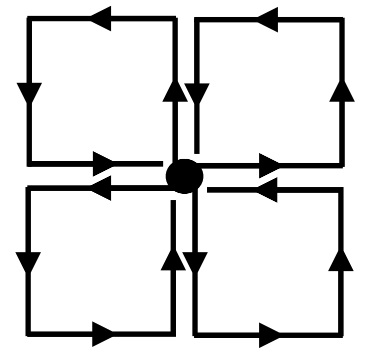
Lattice (clover): $C_{\mu\nu} = \frac{1}{4}\mathrm{Im}\Big($  $\Big)$

$$q'_x = \frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[C_{x,\mu\nu}C_{x,\rho\sigma}]$$

$$= a^4(q(x) + O(a^2)),$$

$$\boxed{Q_{\mathrm{latt}} = \sum_x q'_x \notin \mathbb{Z}}$$

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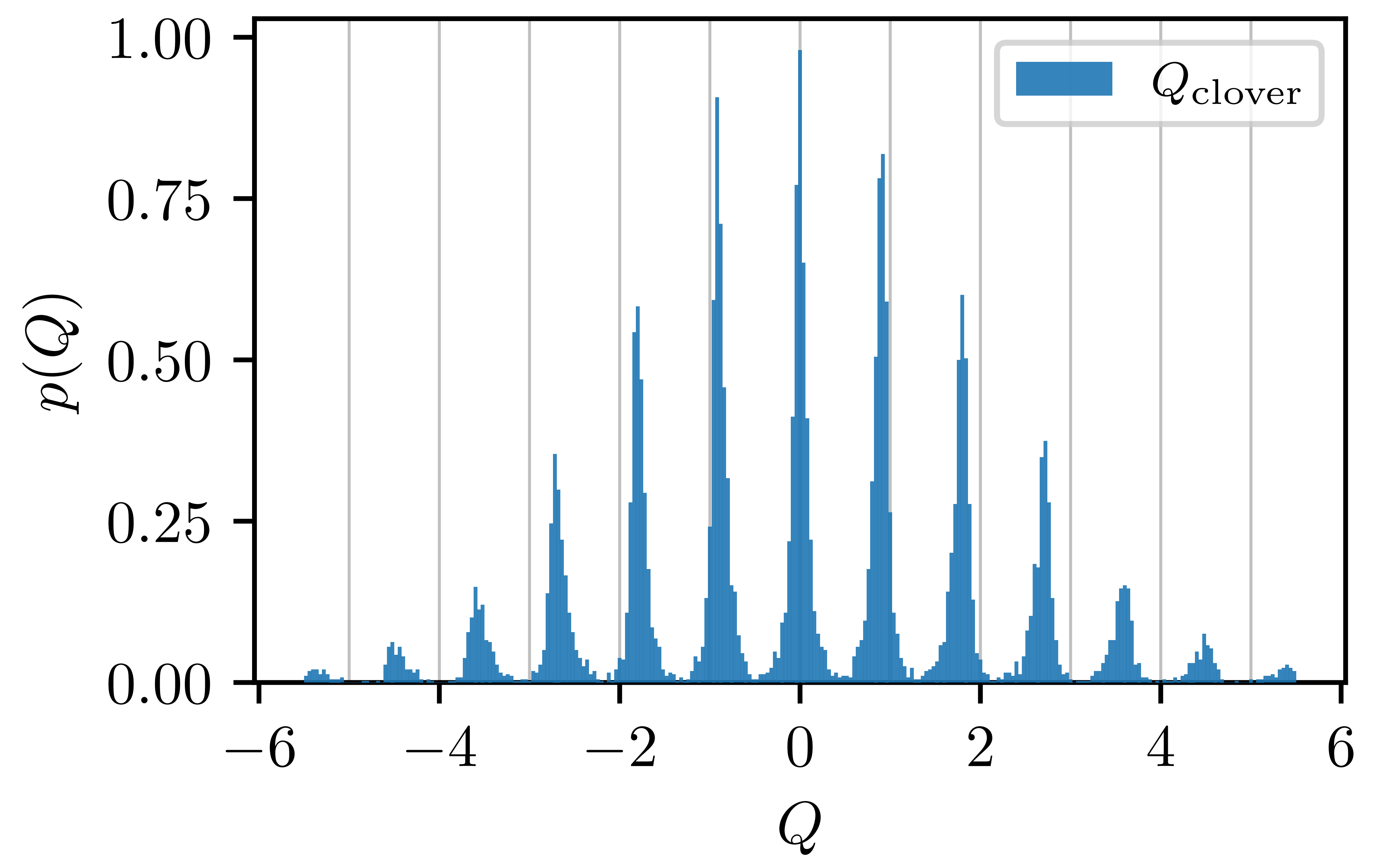
Example:

FP ensemble $\beta = 2.9$ $(a \approx 0.11\ \mathrm{fm})$ on 14^4, Q_{clover} with GF until $t_{0.5}$, 10^5measurements



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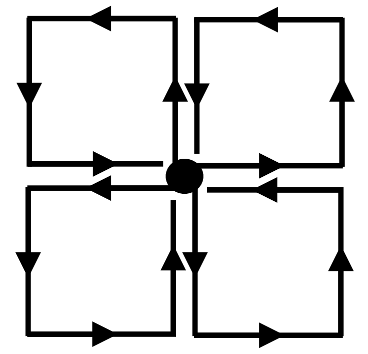

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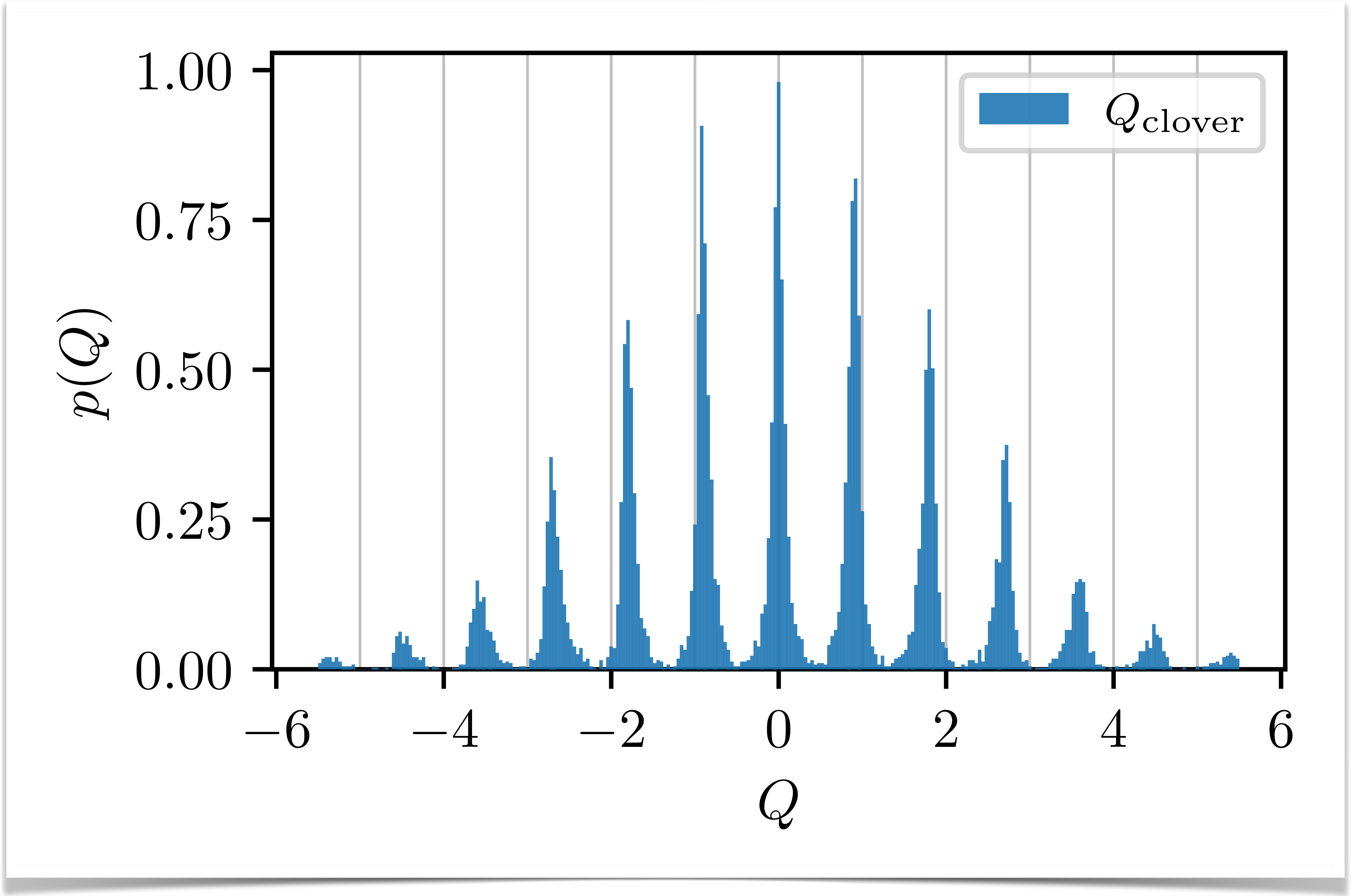
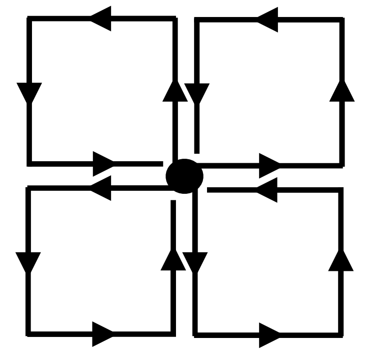


Lattice (clover): $C_{\mu\nu} = \frac{1}{4}\mathrm{Im}\Big($  $\Big)$

Lattice artifacts from Q_{clover}

→ peaks are **shifted** and **smeared**

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[Remember the RGT:

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PHYSICAL REVIEW LETTERS **128,** 032003 (2022)

Lattice Gauge Equivariant Convolutional Neural Networks

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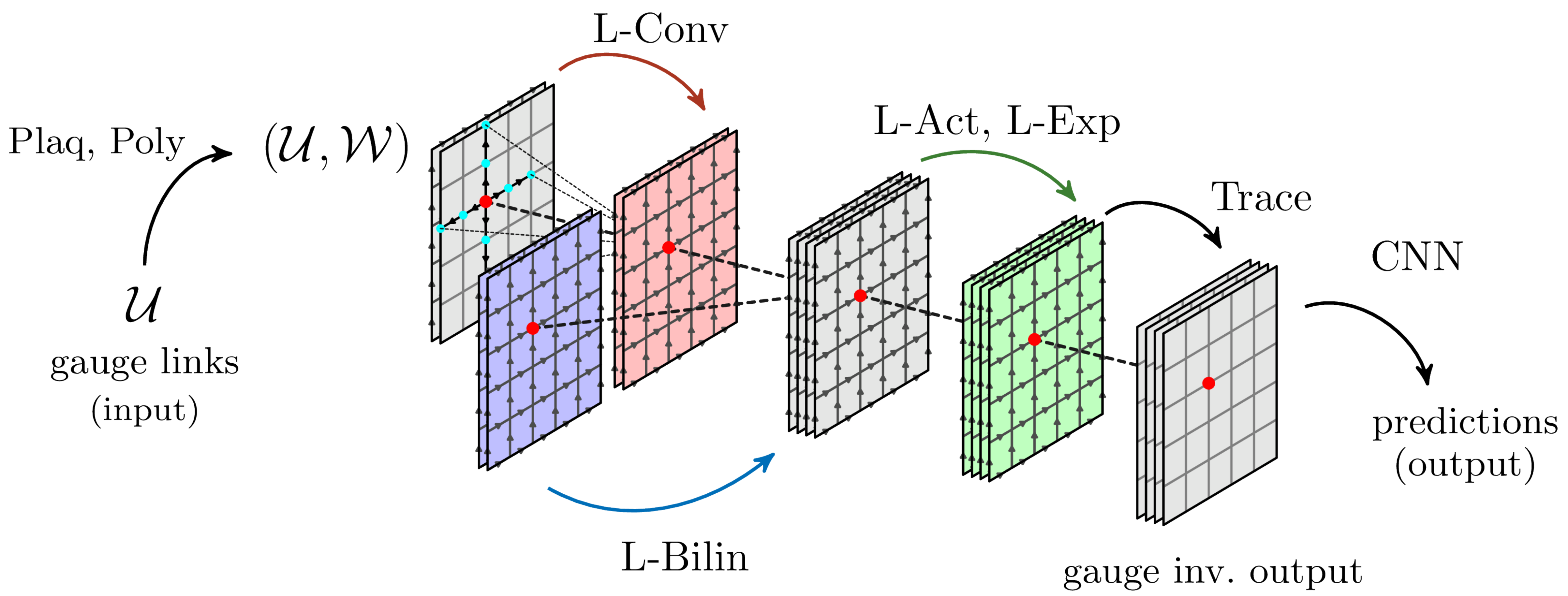

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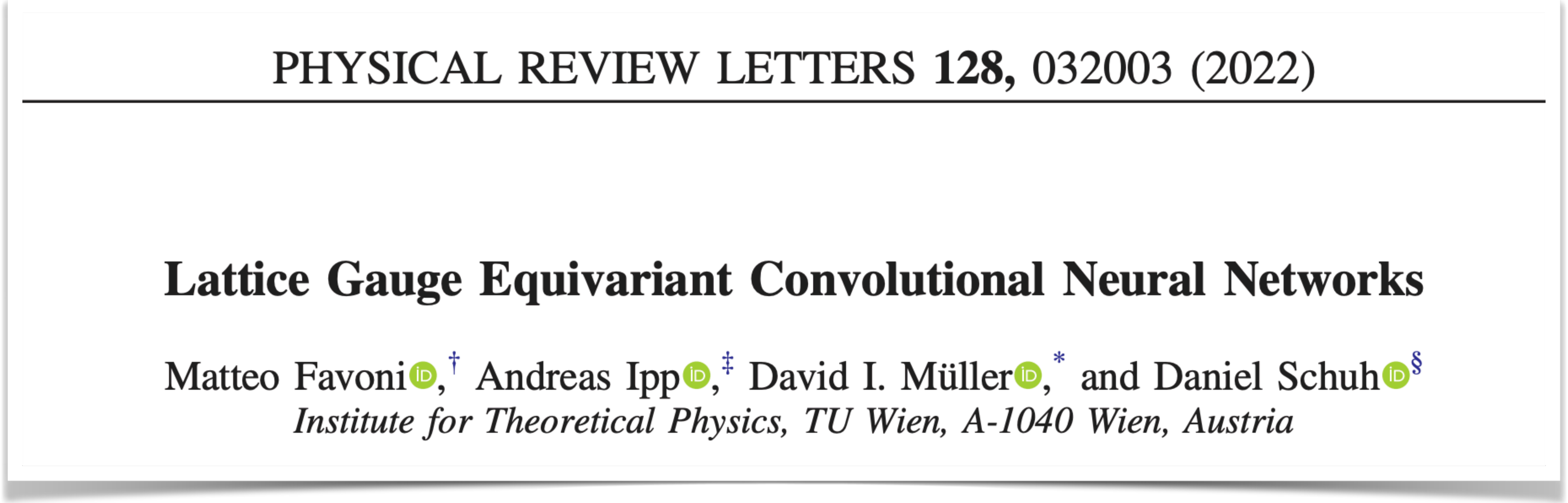
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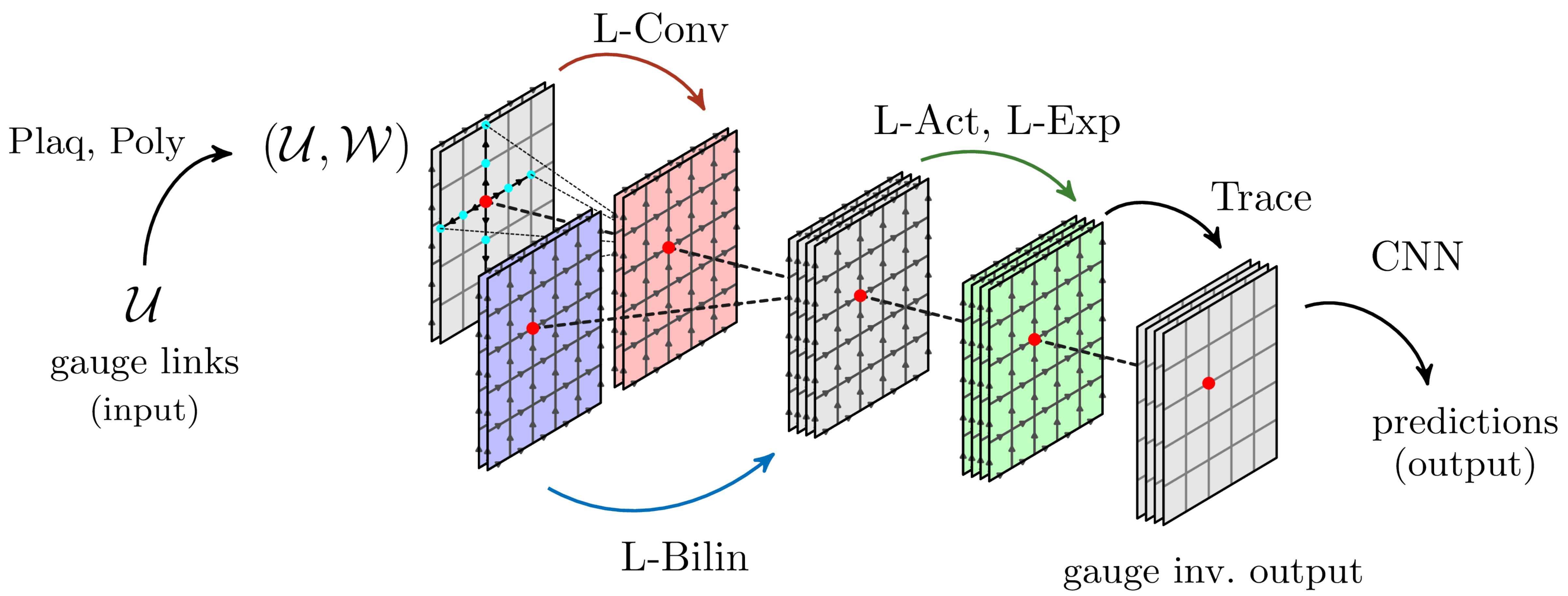

gauge links $U_{x,\mu}$

↓

neural network

↓

gauge inv. output $Q^{\mathrm{FP}}[V]$

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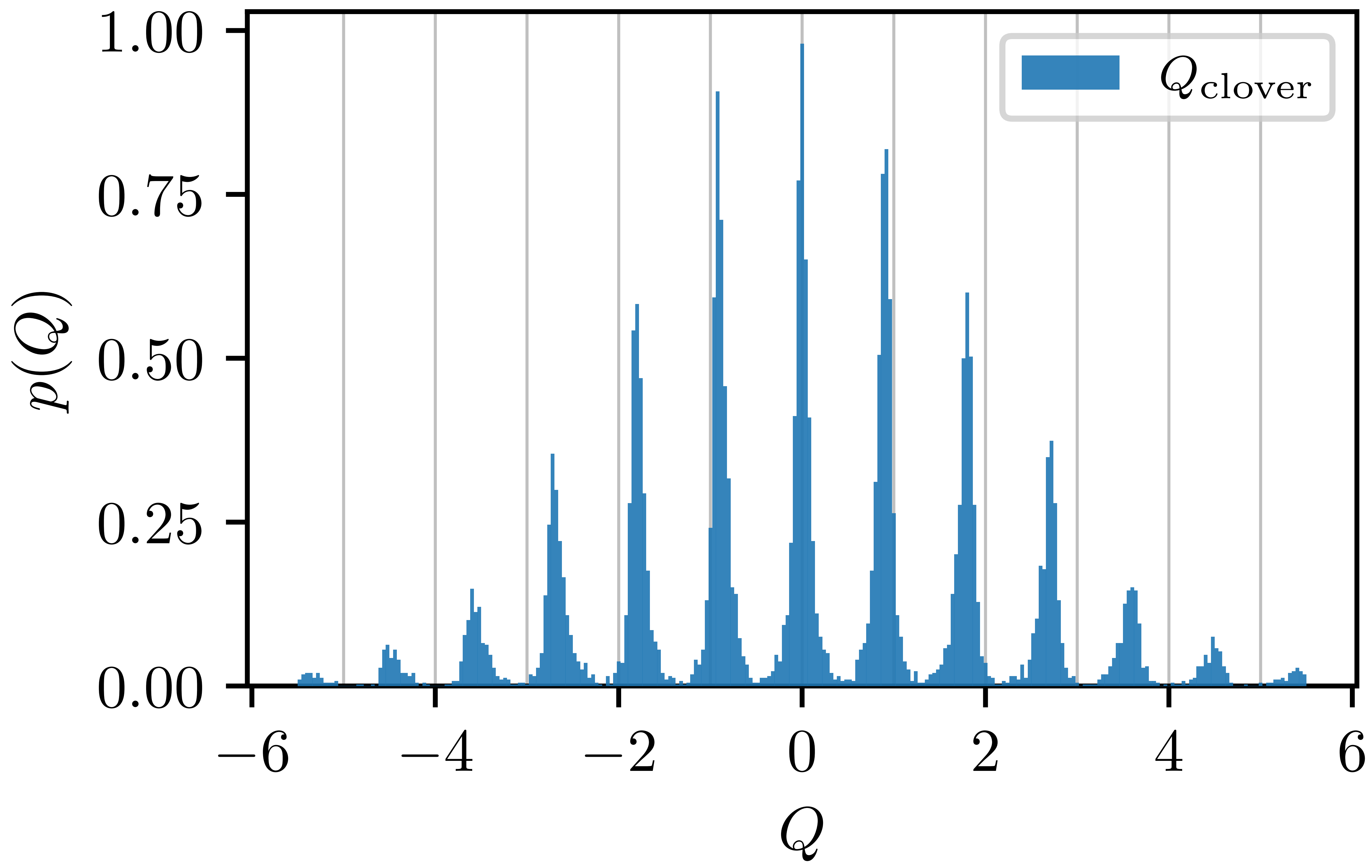

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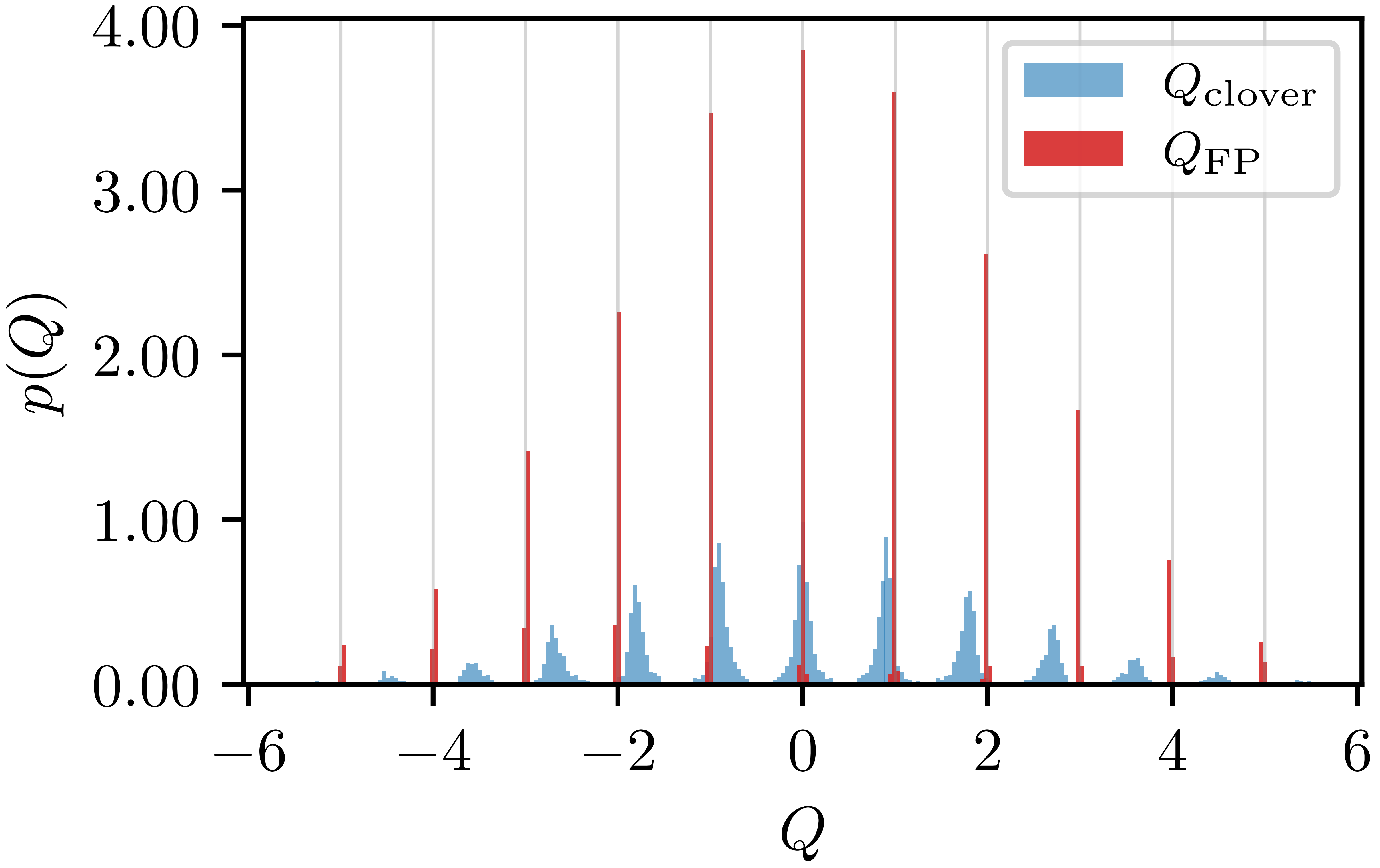

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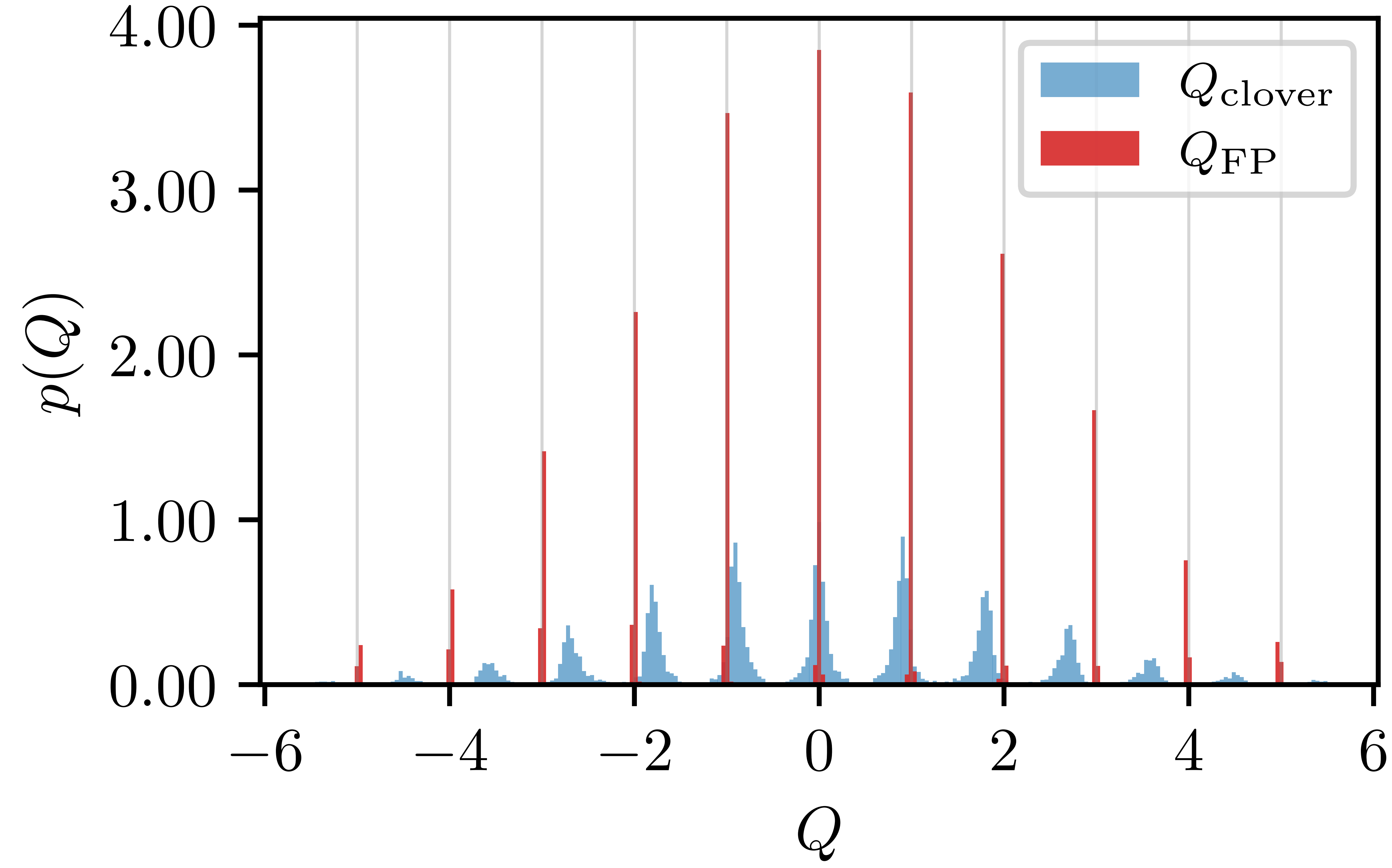

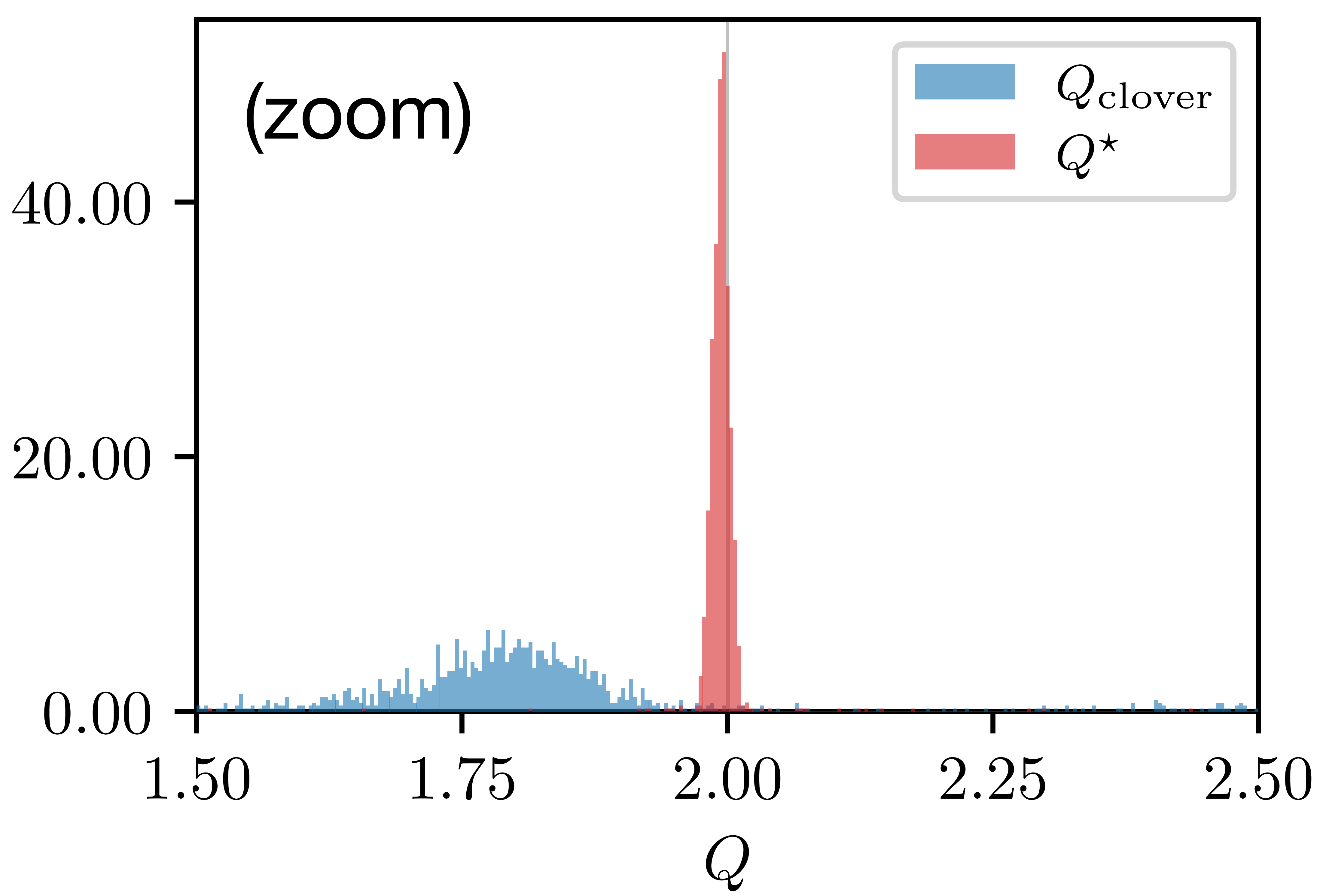

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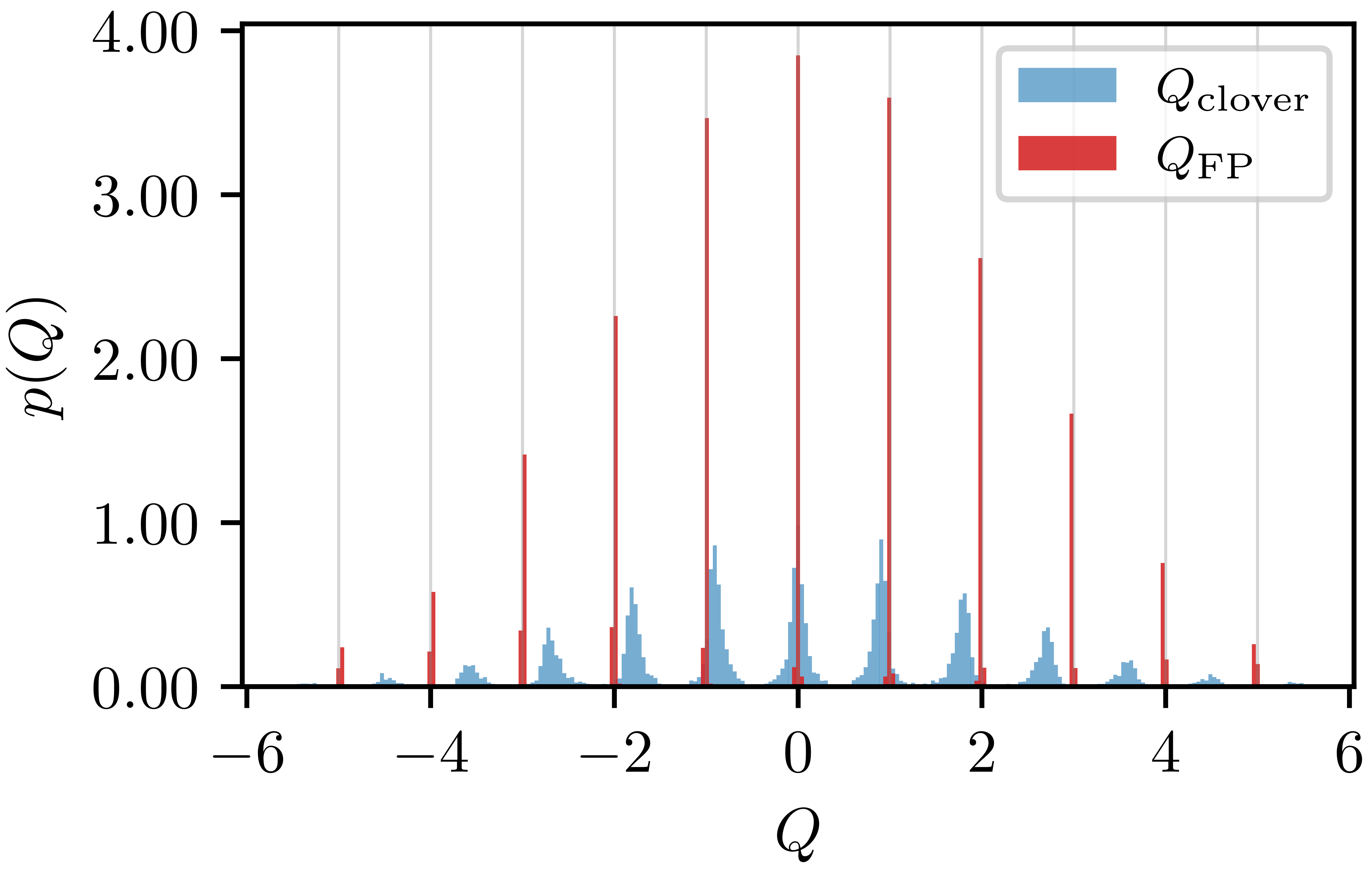


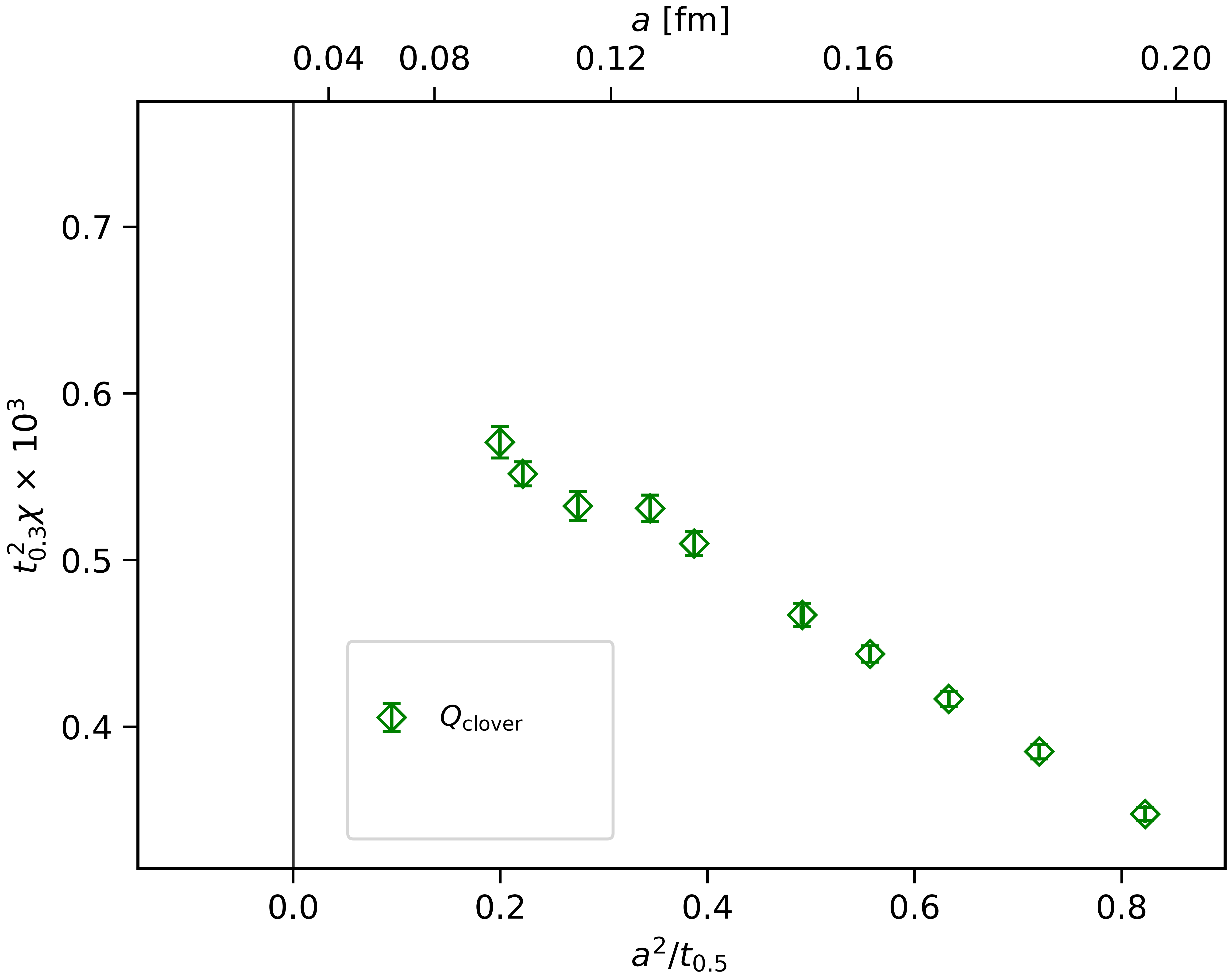
Q^{FP} are close to integer

Distribution is very sharp

Machine-learned Q^{FP} is almost continuum-like!

Continuum extrapolations for χ_{top}

Q_{clover} on FP gauge ensembles:



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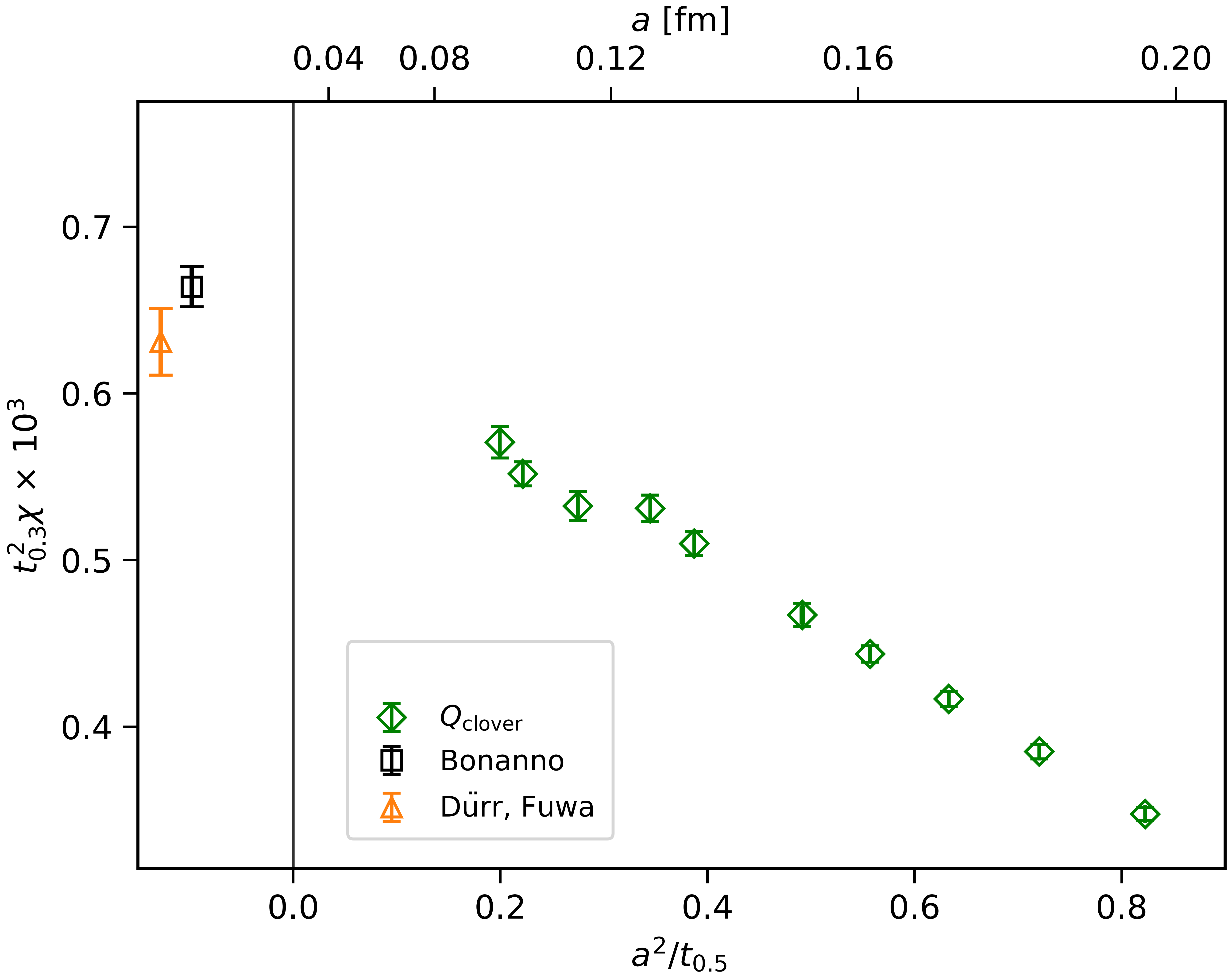

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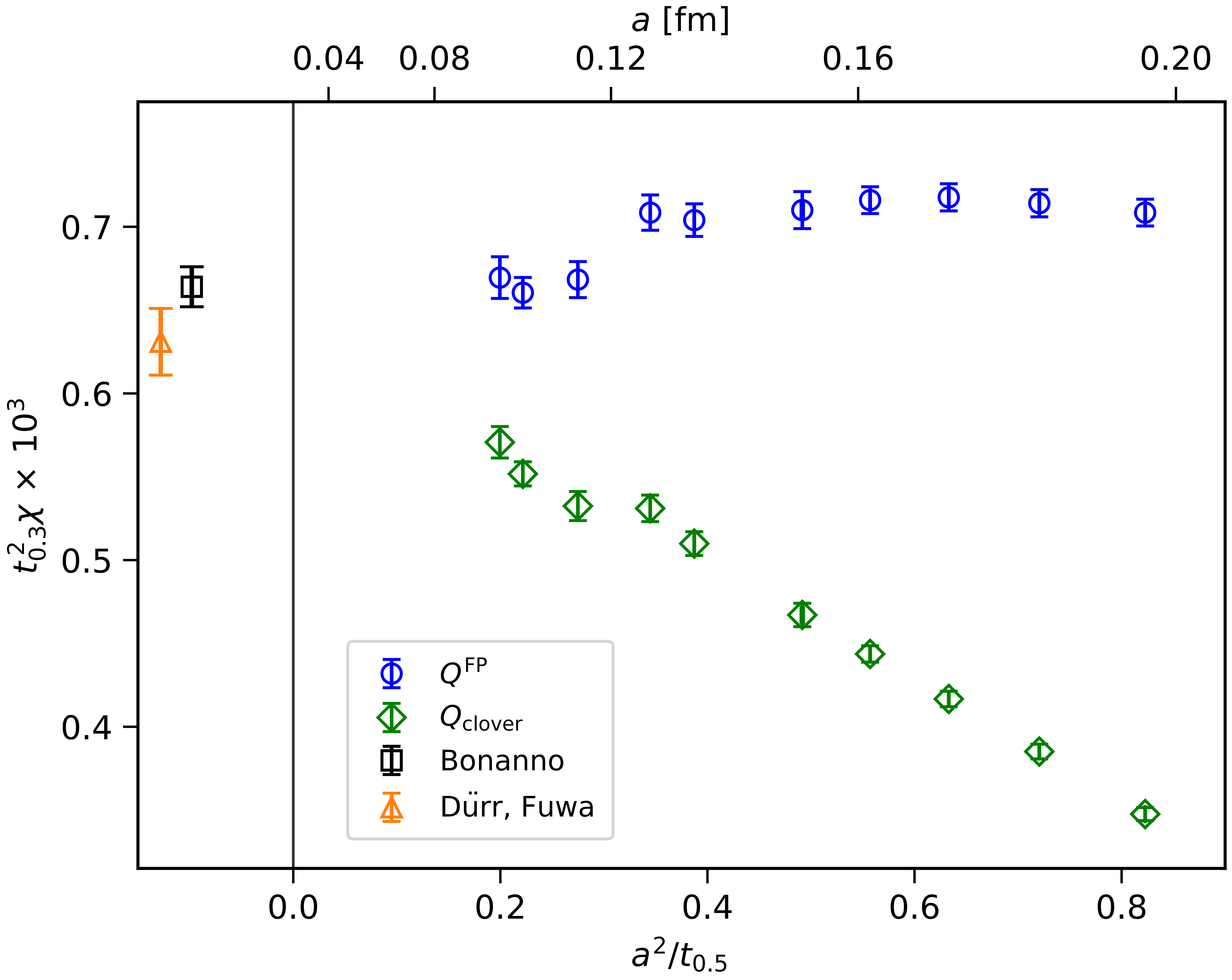
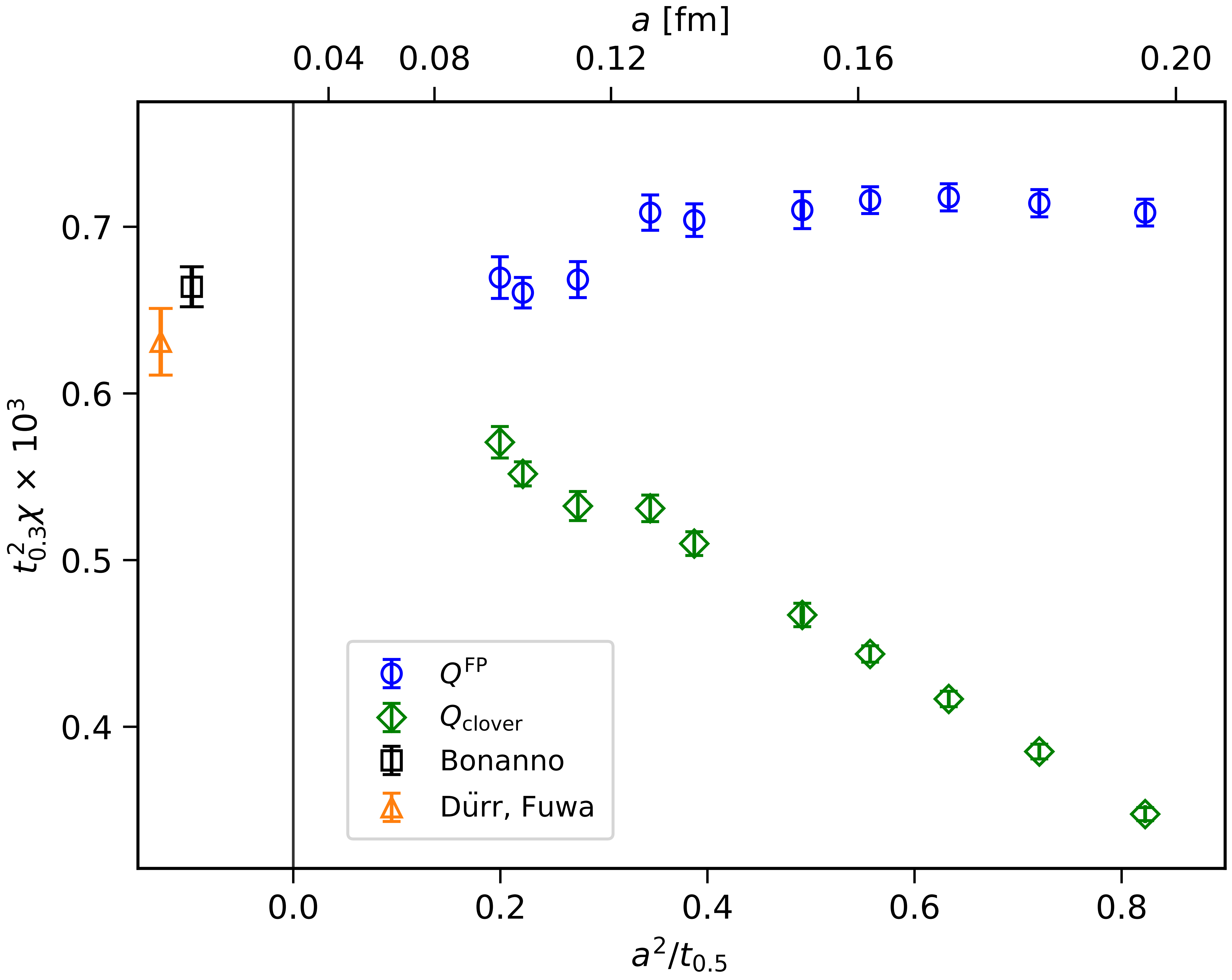


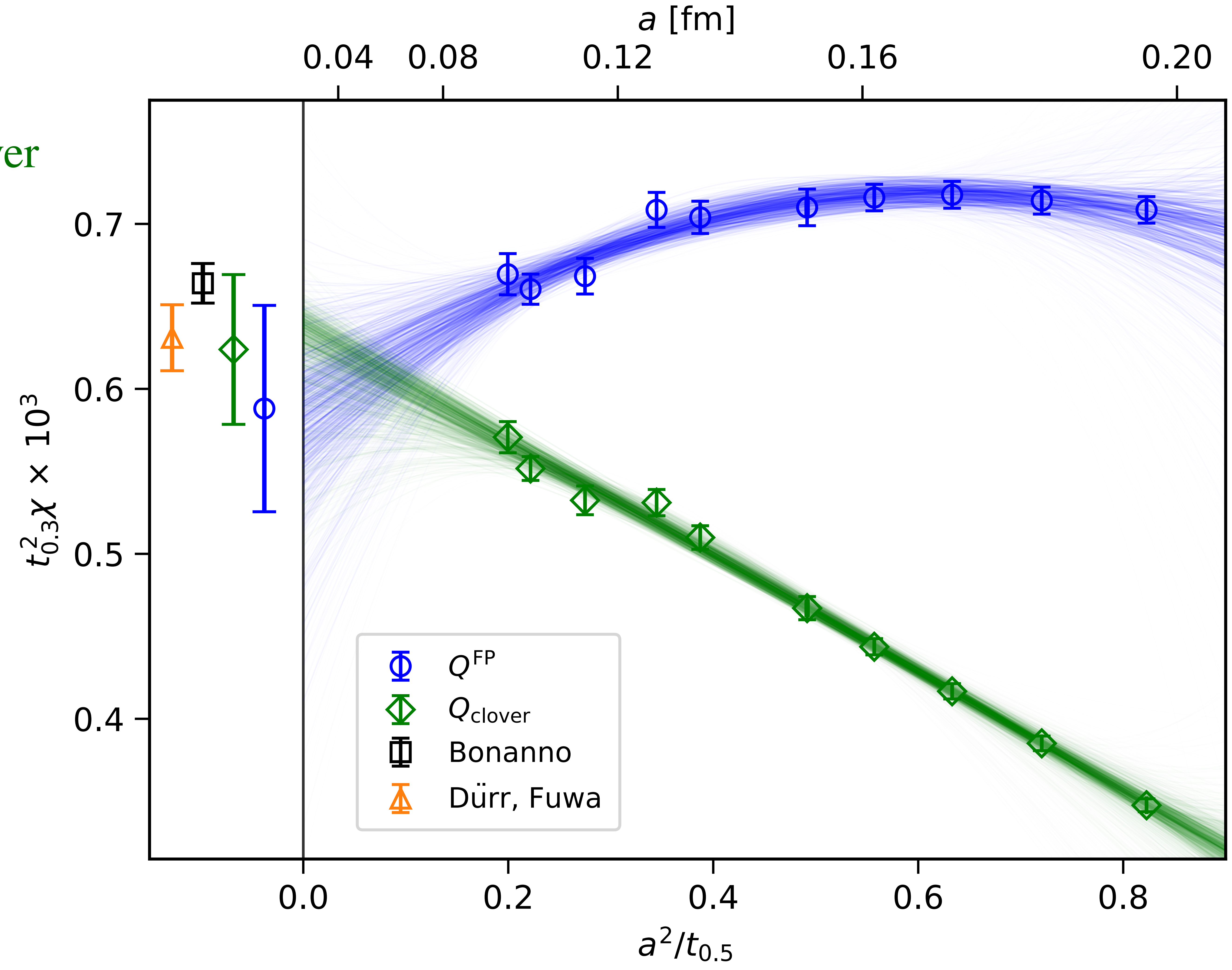
Continuum extrapolations for χ_{top}

Q^{FP} much flatter than Q_{clover}!



Continuum extrapolations for χ_{top}

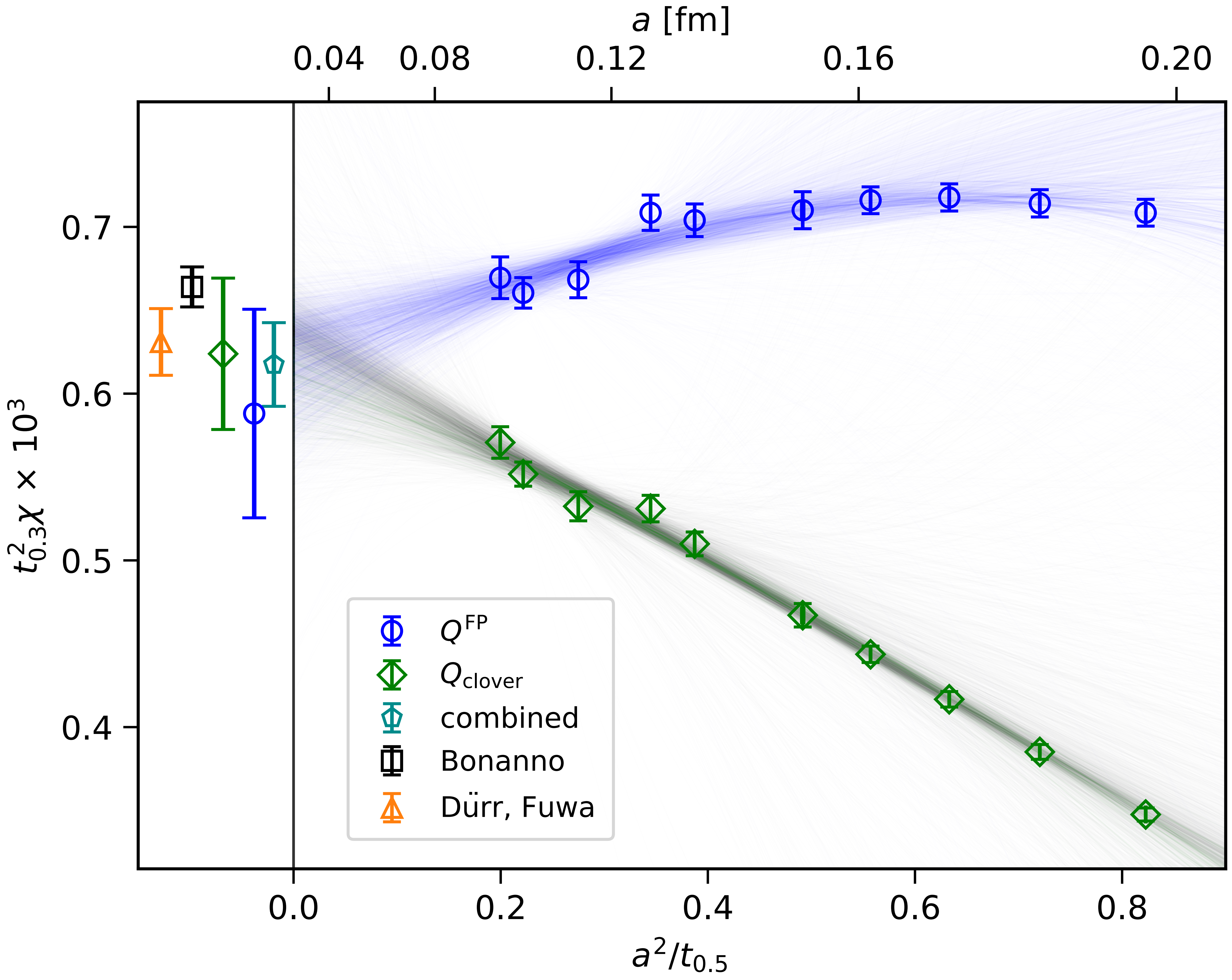
Continuum limits for Q^{FP} and Q_{clover}



Continuum extrapolations for χ_{top}

Continuum limit combined:

$$t_{0.3}^2\,\chi = 0.618(25)\cdot 10^{-3}$$



Continuum extrapolations for χ_{top}

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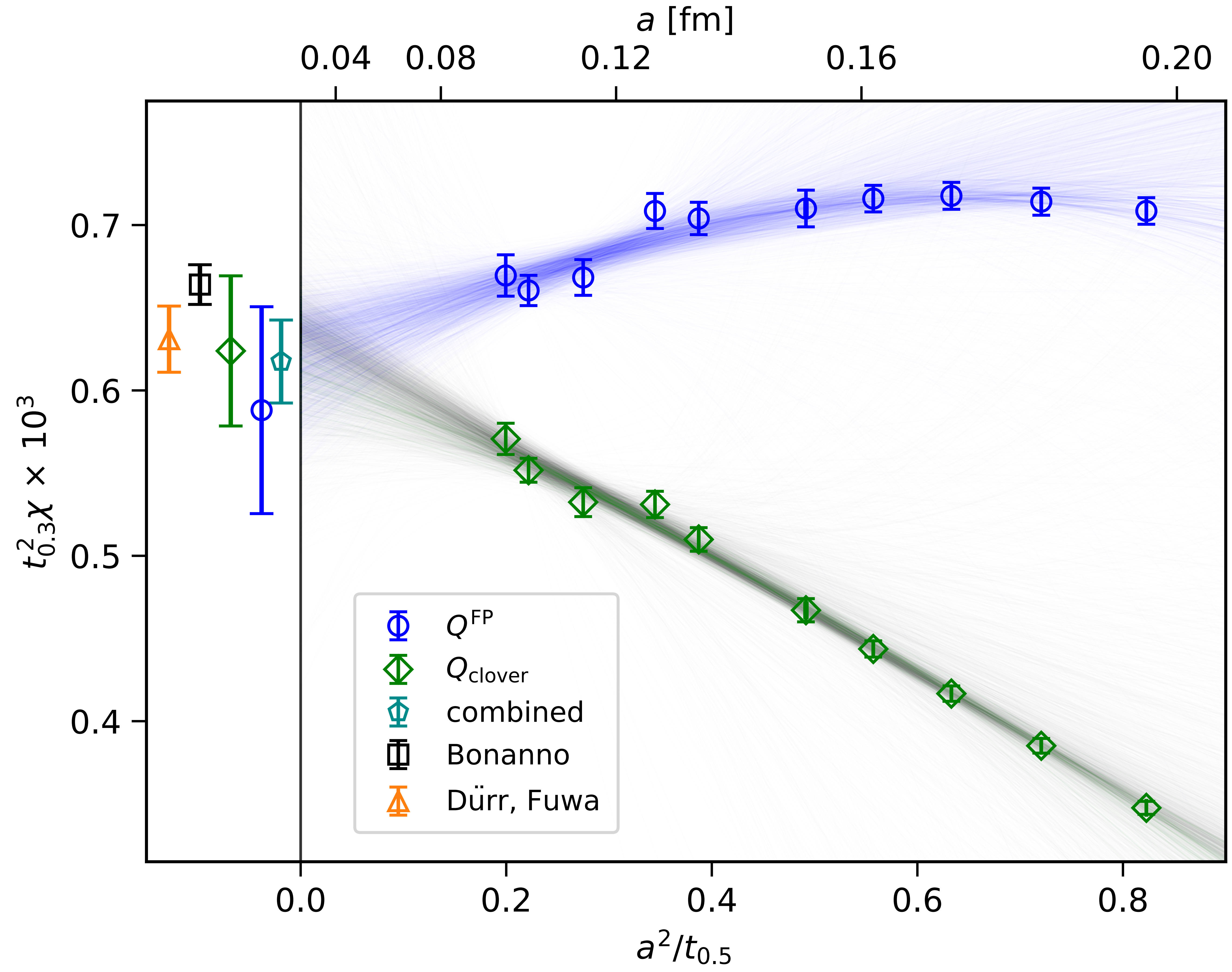
Recent values from literature:

Bonanno [arXiv:2510.08006]

$$t_{0.3}^2\,\chi = 0.664(12)\cdot 10^{-3}$$

Dürr, Fuwa [arXiv:2501.08217]

$$t_{0.3}^2\,\chi = 0.631(20)\cdot 10^{-1}$$



Thermodynamics with FP actions

FP action simulations on $L_s^3 \times 2$ lattices ($L_t = 2 \rightarrow a \approx 0.3\,\mathrm{fm}$):

$\Rightarrow$ reaching large aspect ratios L_s/L_t is easy!

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First determine β_c using three different definitions:

Polyakov loop susceptibility:

$$\chi_L \propto \langle L^2 \rangle - \langle L \rangle^2$$

$\Rightarrow$ peaks at $\beta_c^{\chi_{\max}}$

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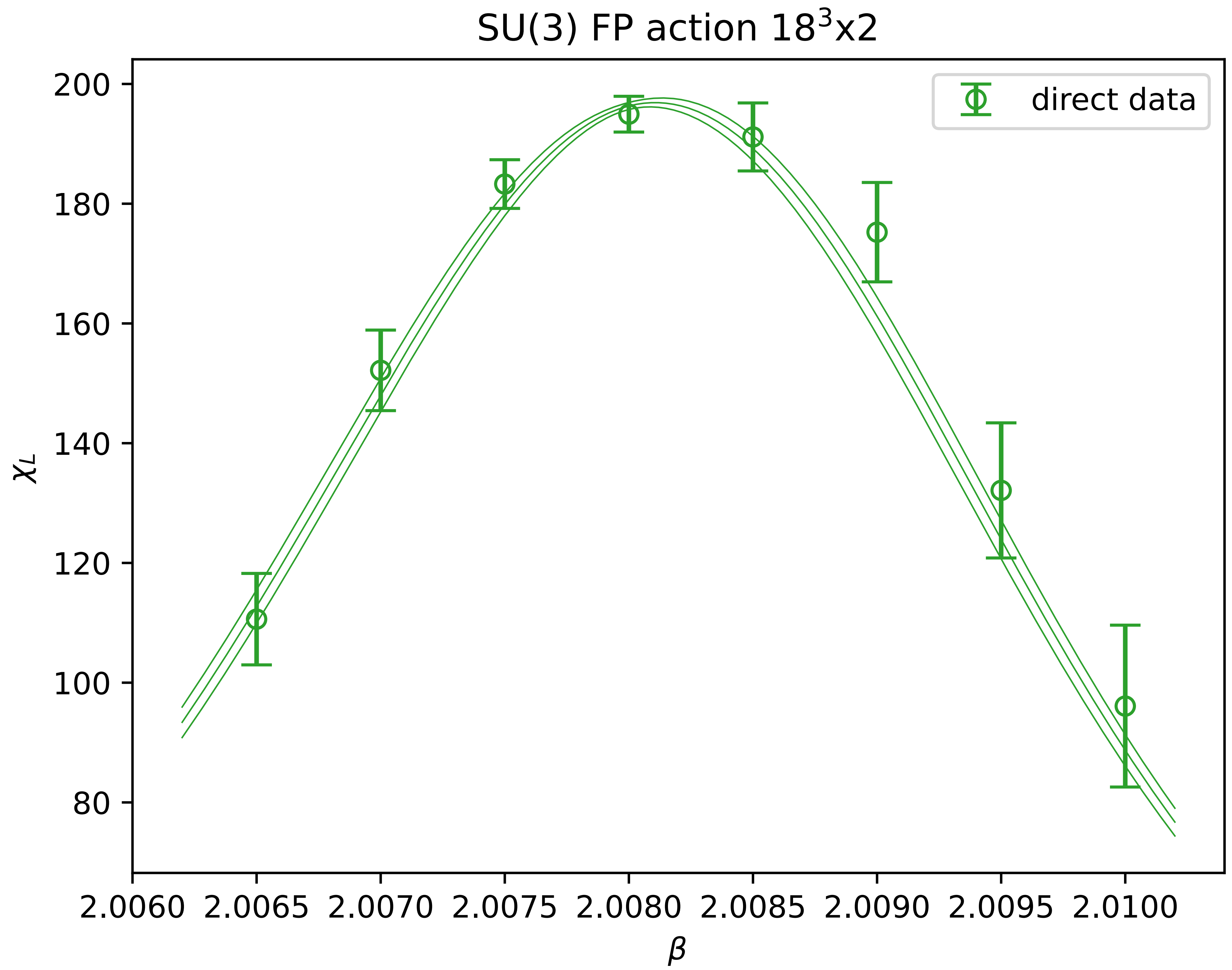
First determine β_c using three different definitions (with multi canonical reweighting):



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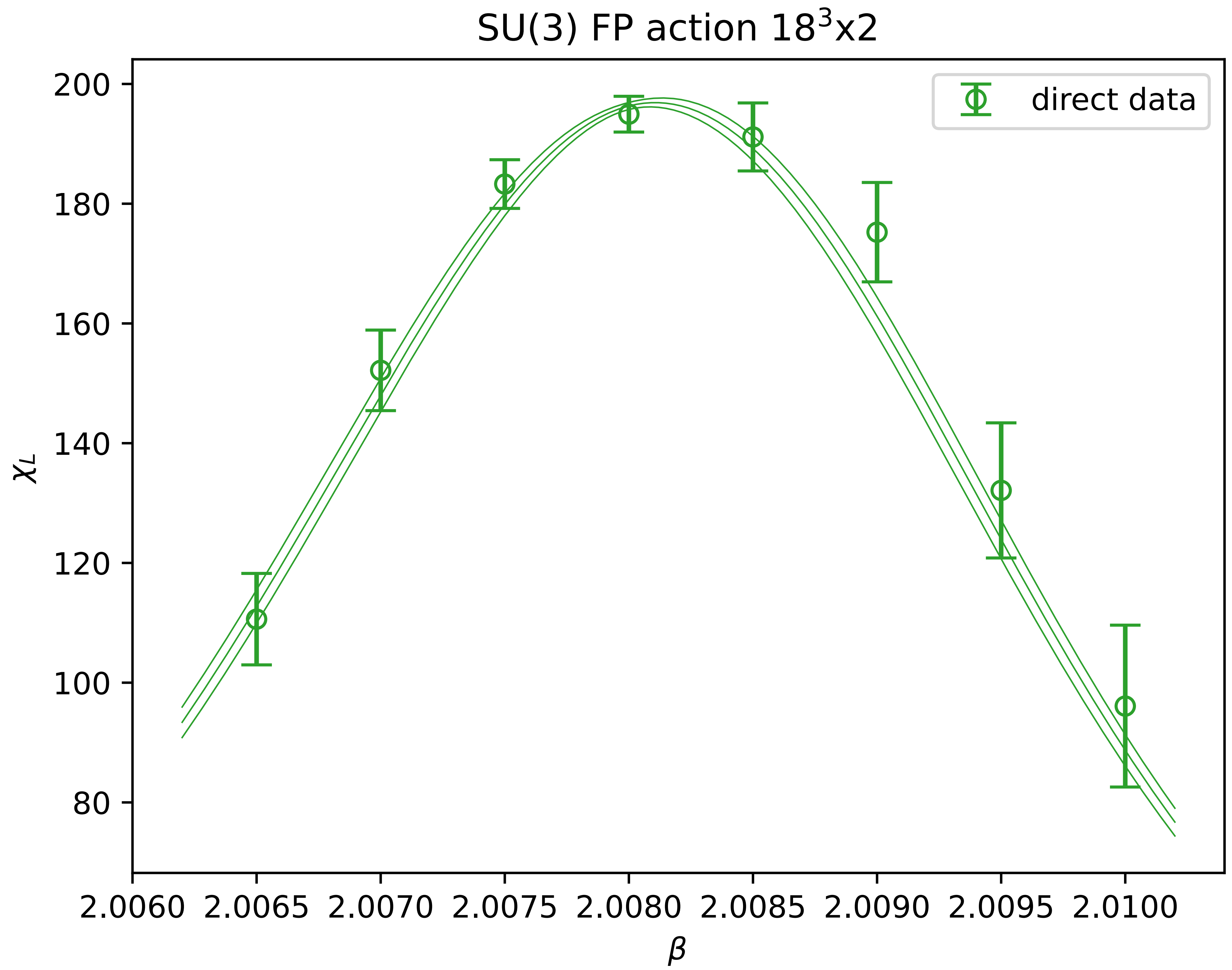

Specific heat:

$$C \propto \langle s^2 \rangle - \langle s \rangle^2$$

$\Rightarrow$ peaks at $\beta_c^{C_{\max}}$

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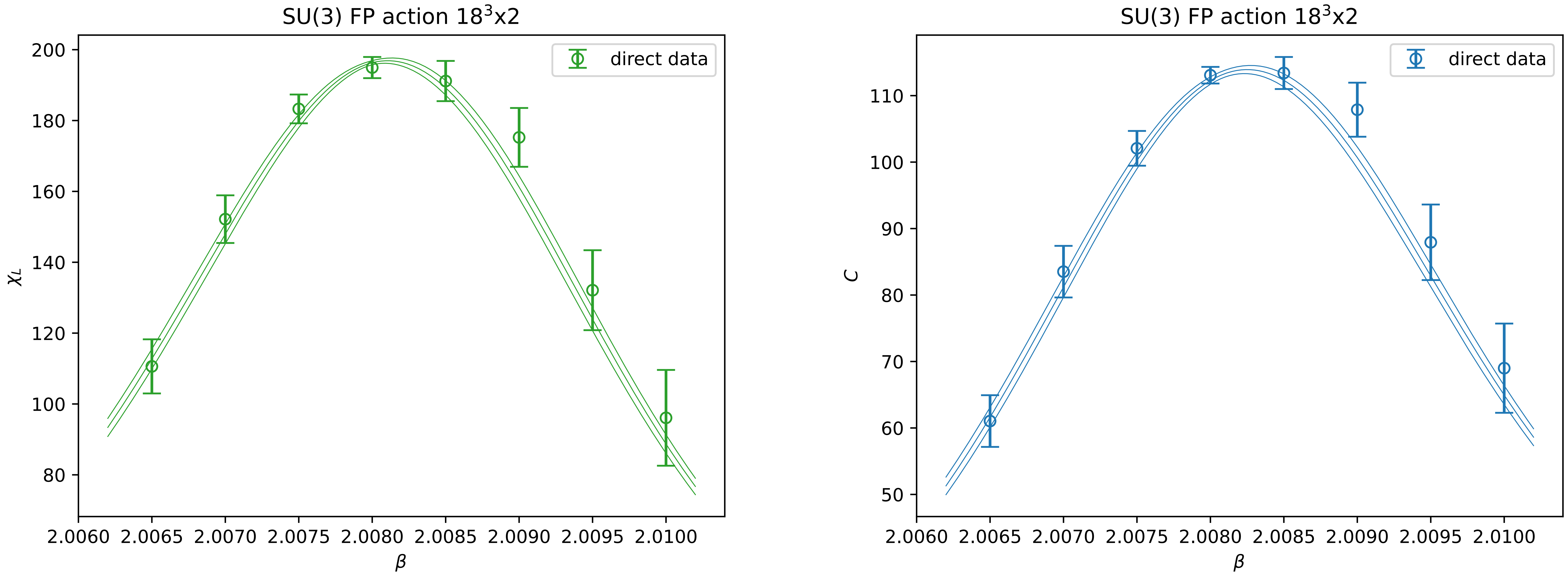

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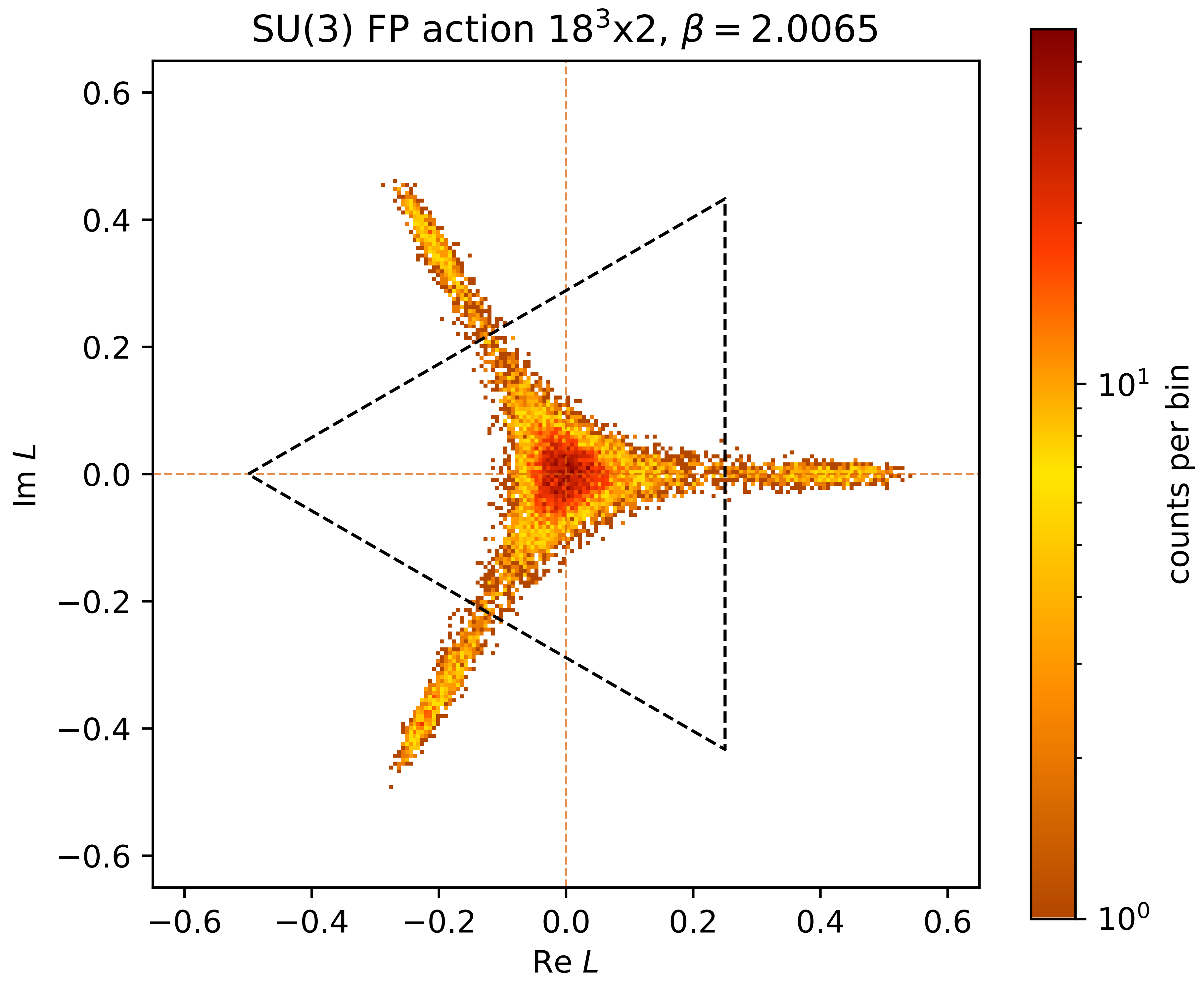

ratio of weights:

$$R = \frac{3w_c - w_d}{3w_c + w_d}$$

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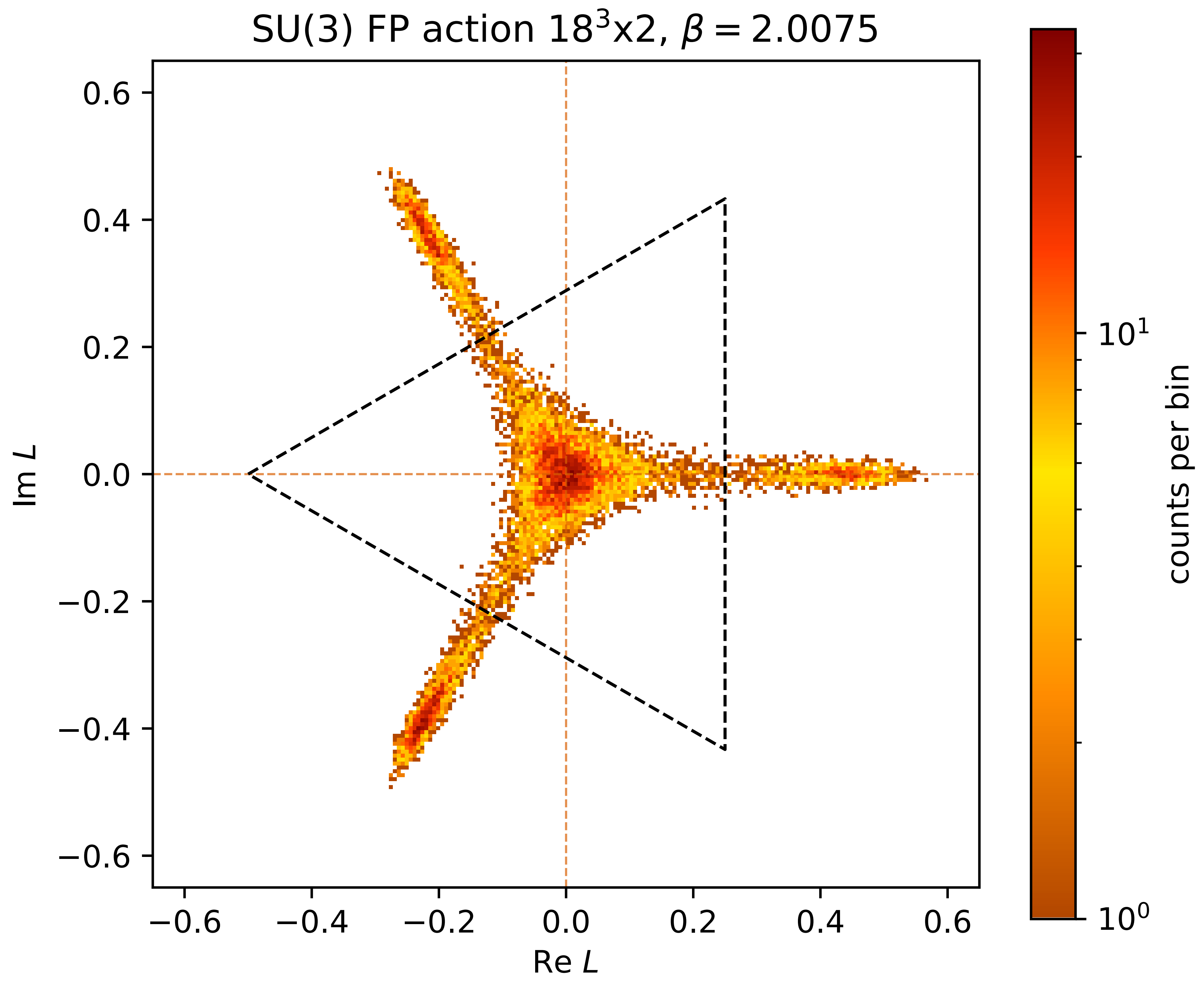

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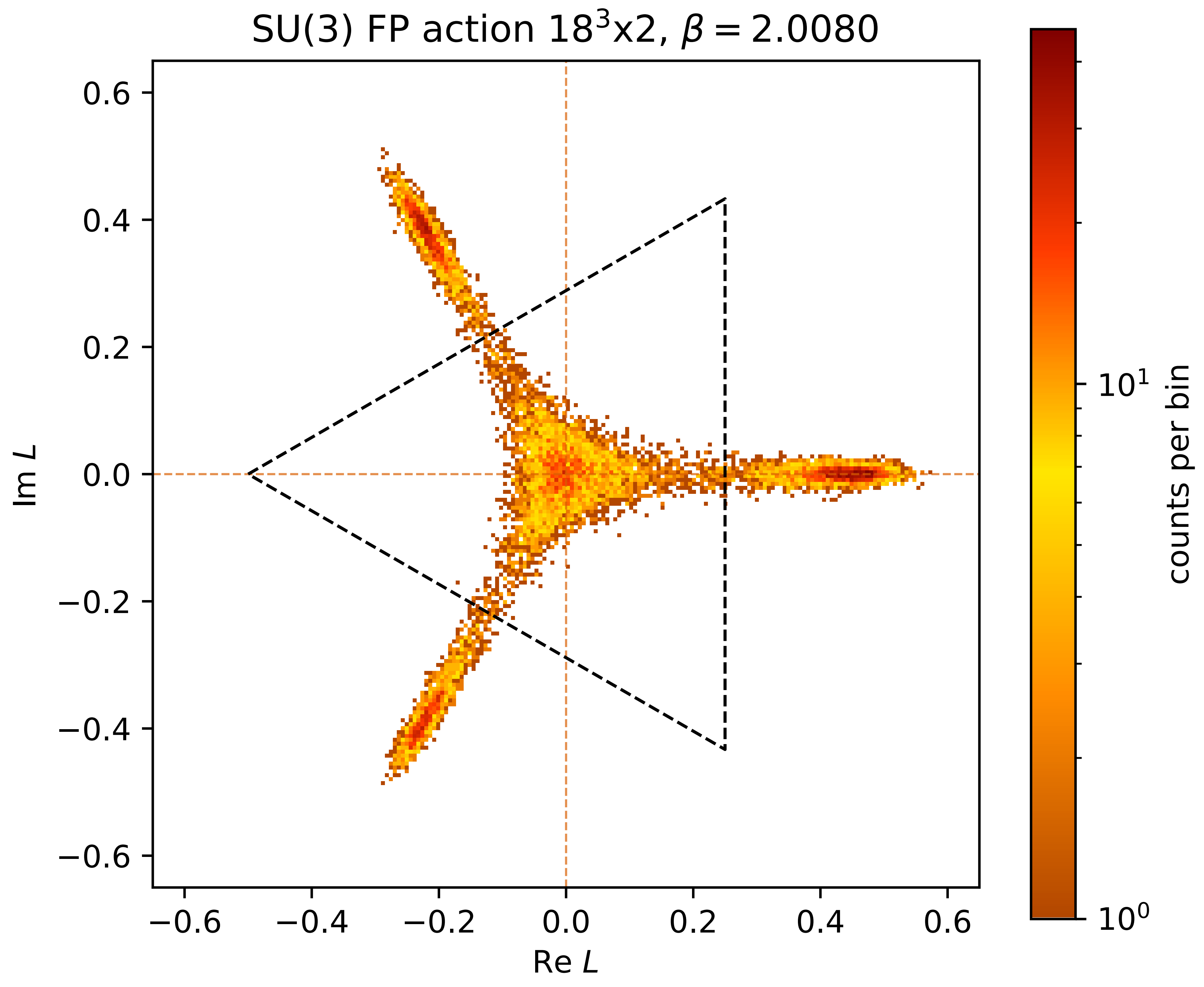

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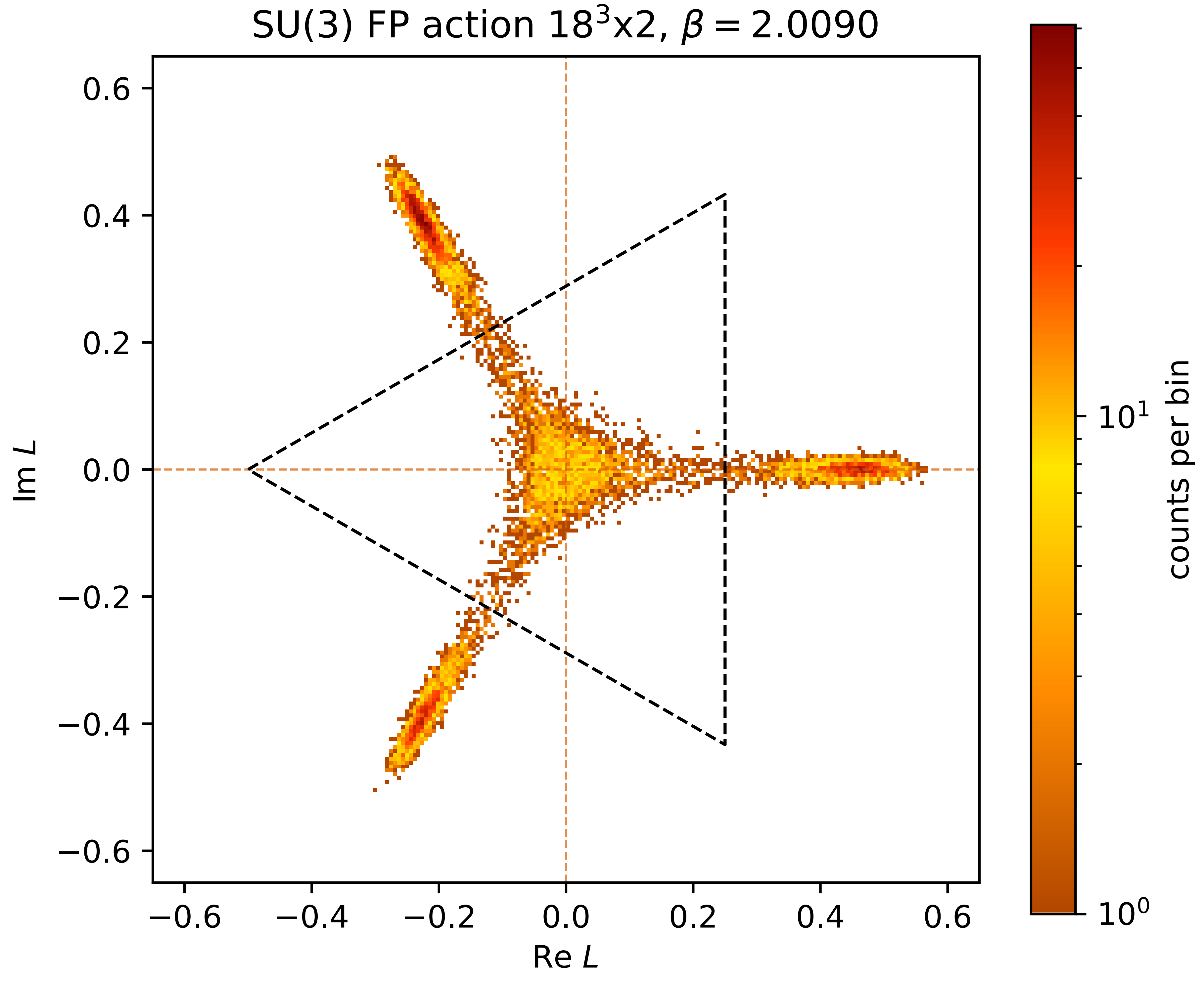

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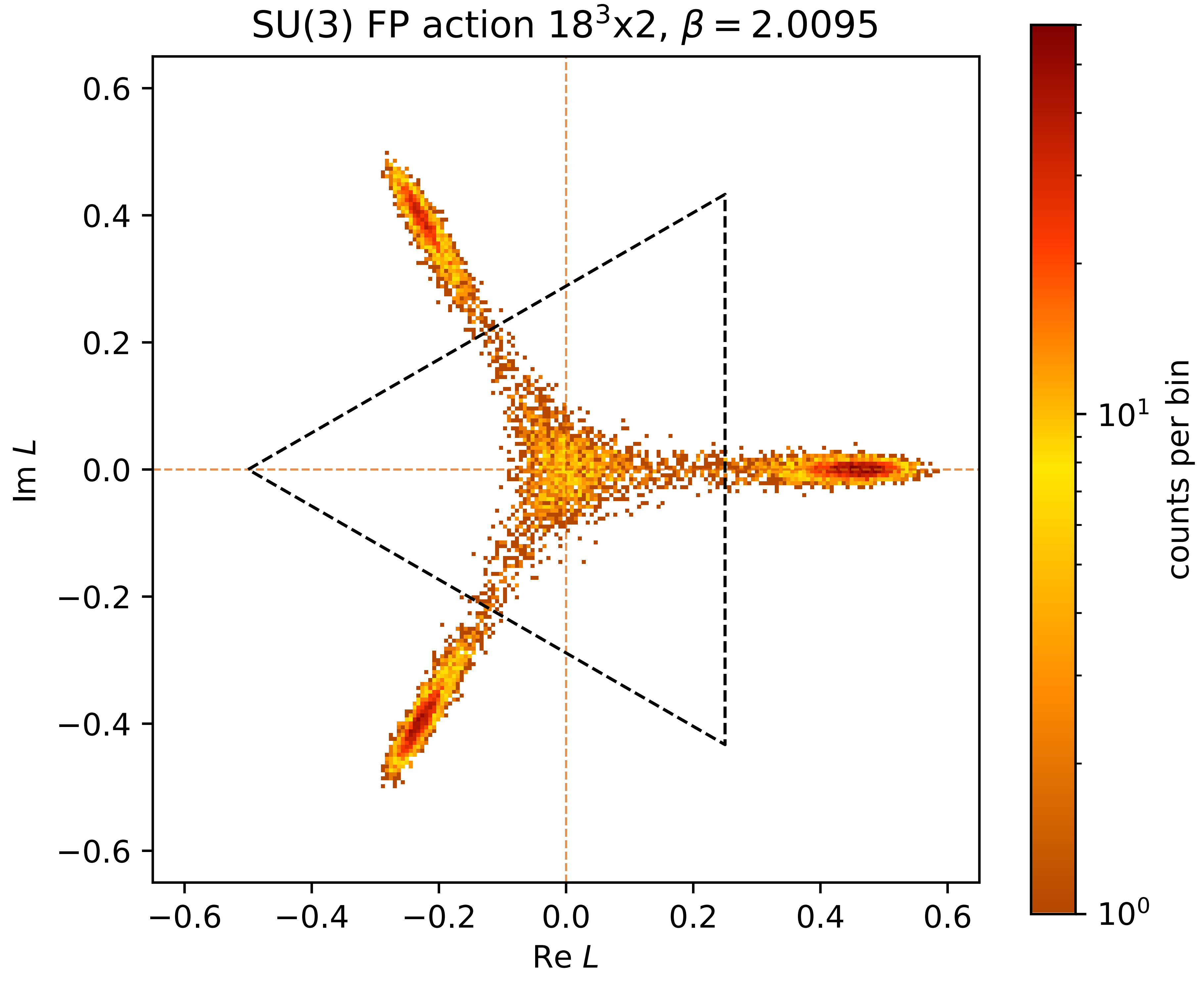

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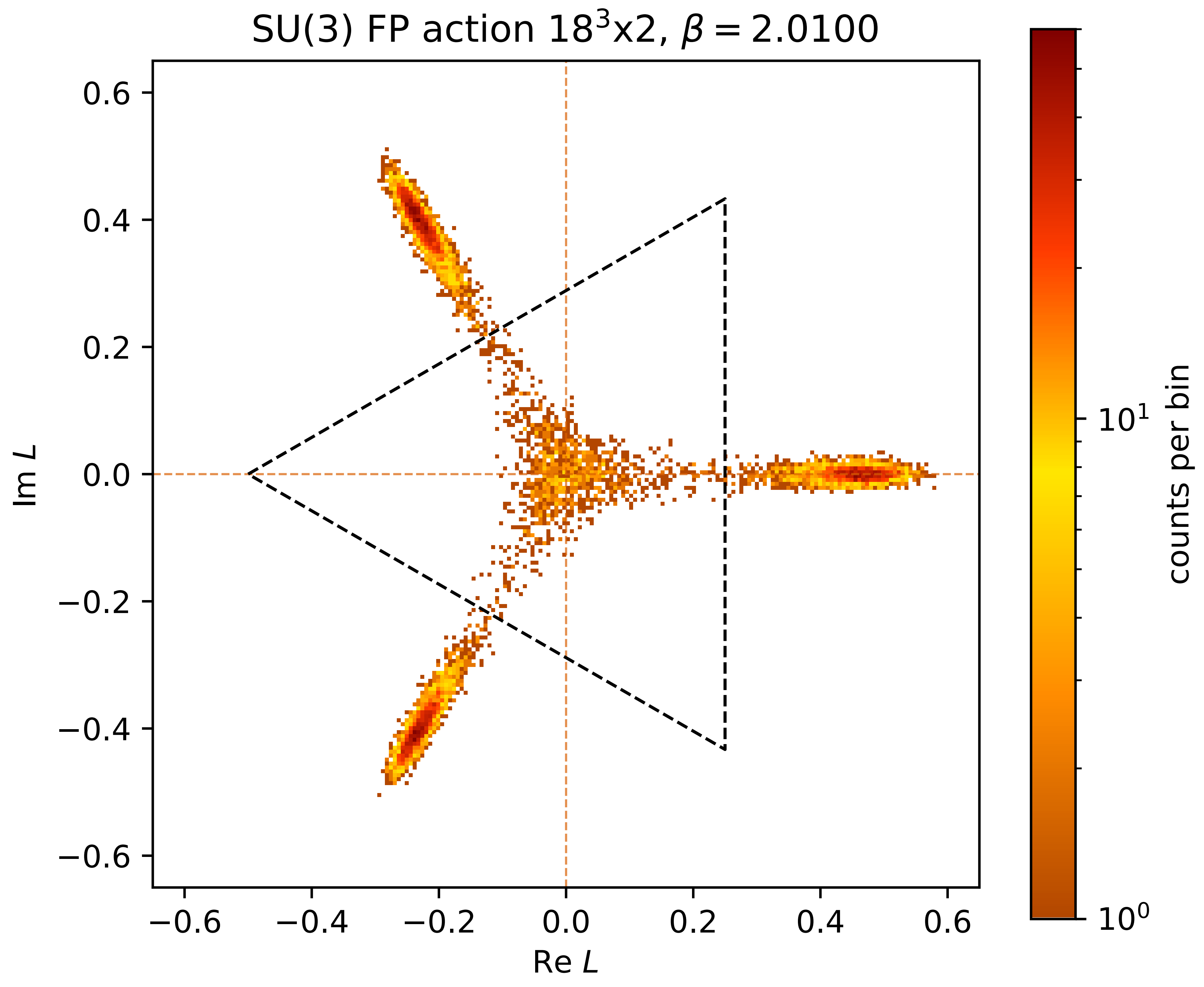

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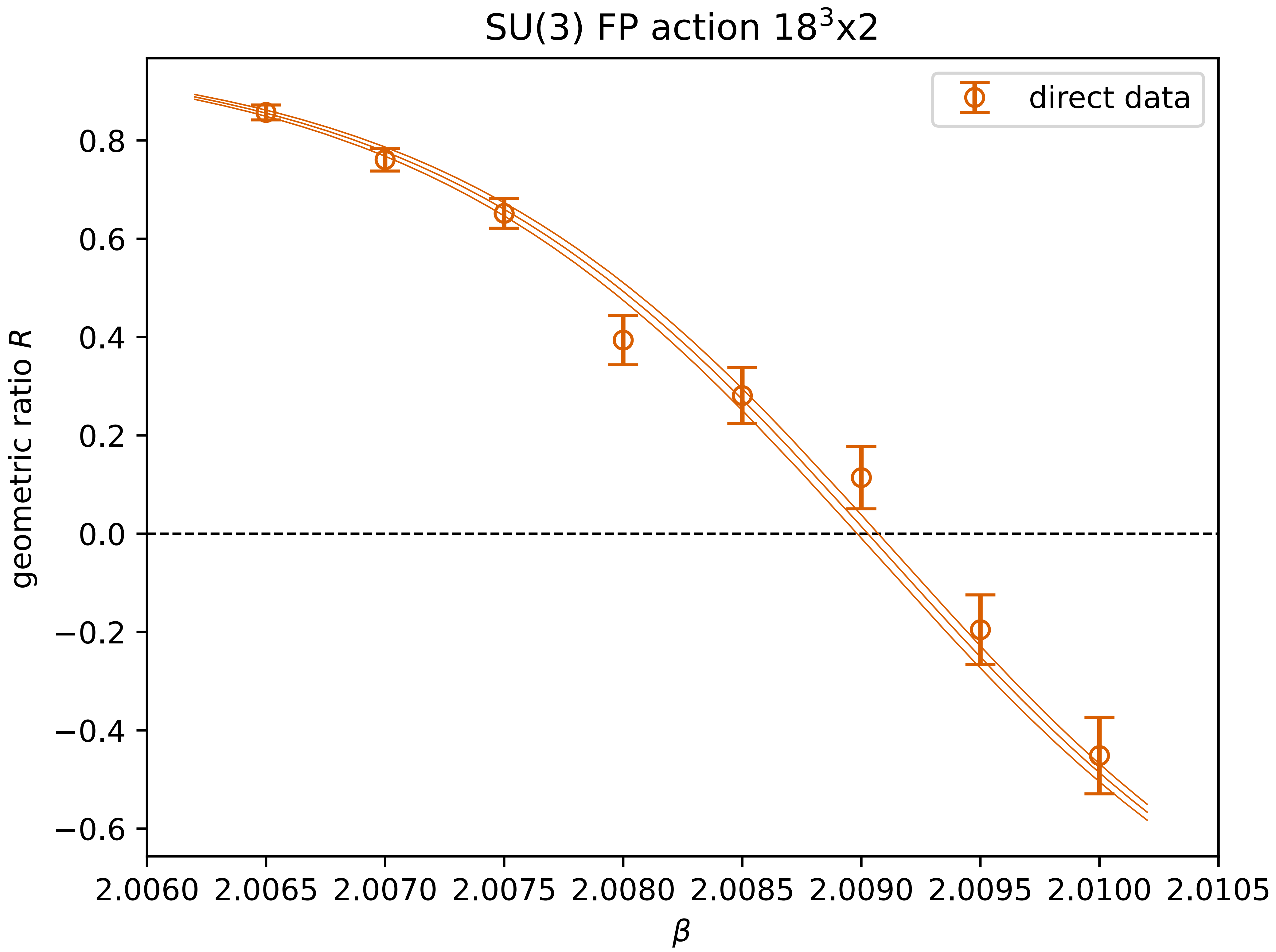

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$\Rightarrow$ crosses zero at β_c^R

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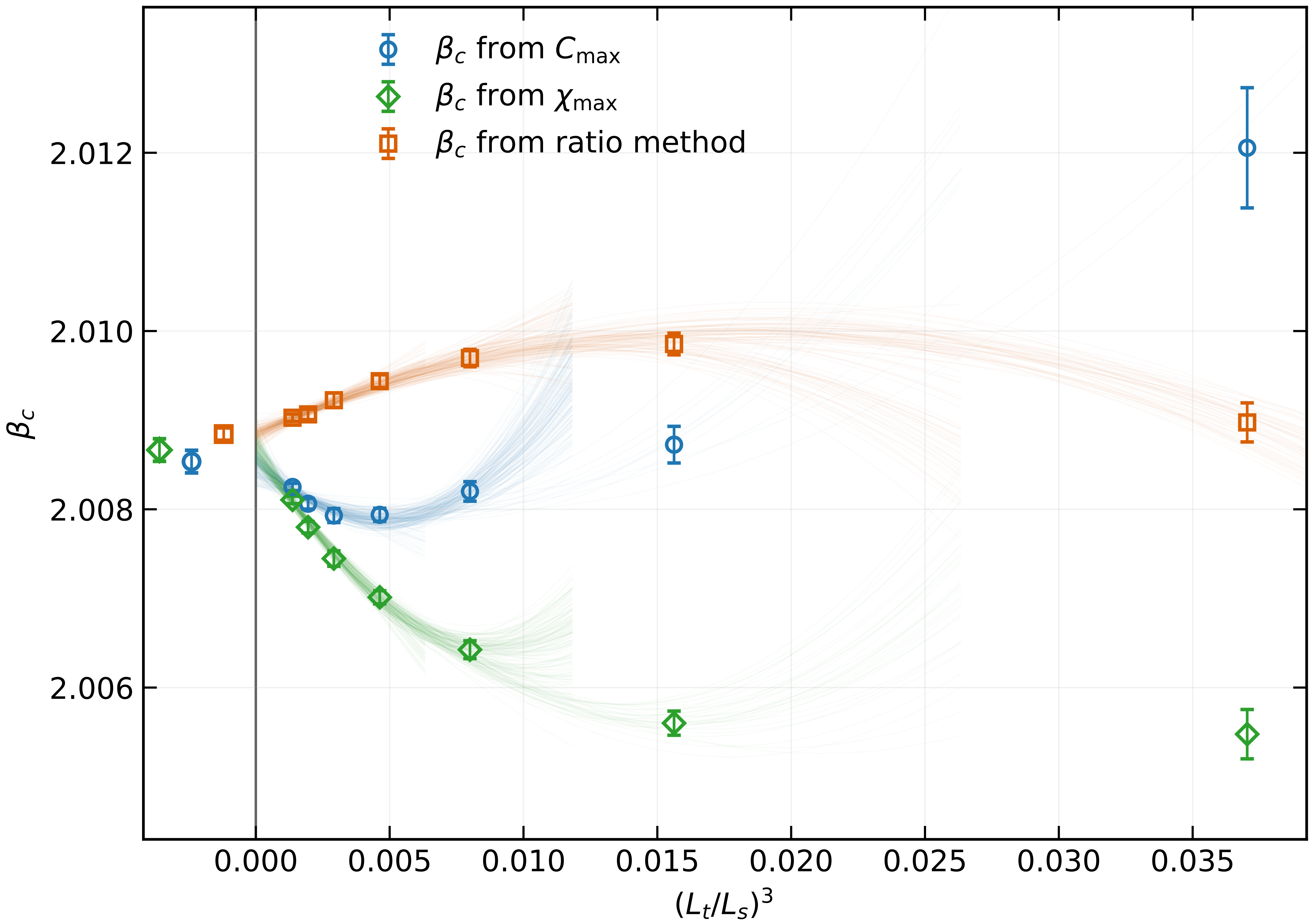
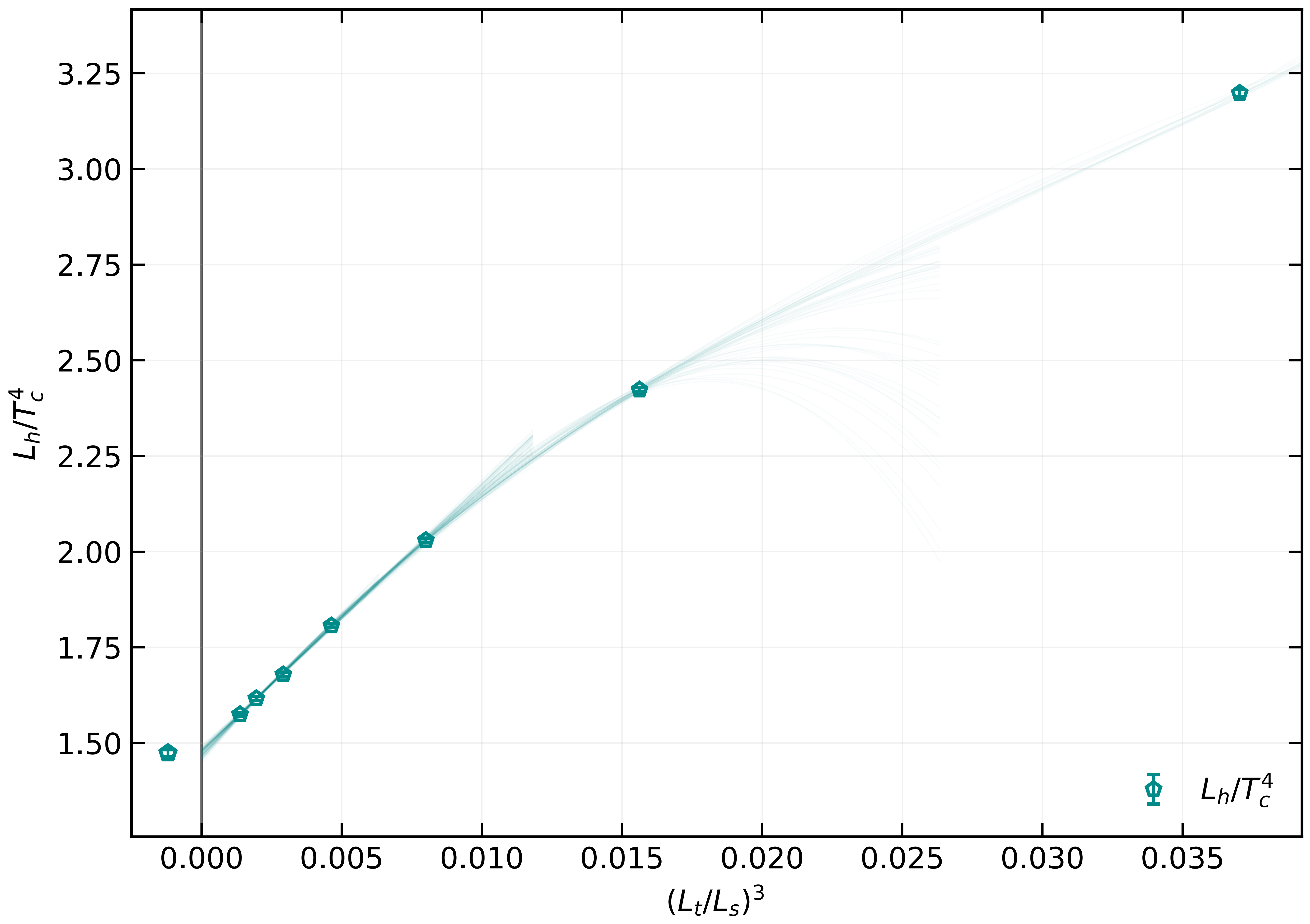


- consistent results in the thermodynamic limit
- large aspect ratios $L_s/L_t > 7$ are necessary
- here, up to $L_s/L_t = 9$

Thermodynamics with FP actions

FP action simulations on $L_s^3 \times 2$ lattices ($L_t = 2 \rightarrow a \approx 0.3\,\mathrm{fm}$):

- latent heat L_h/T_c^4 from C_{max} (bare)



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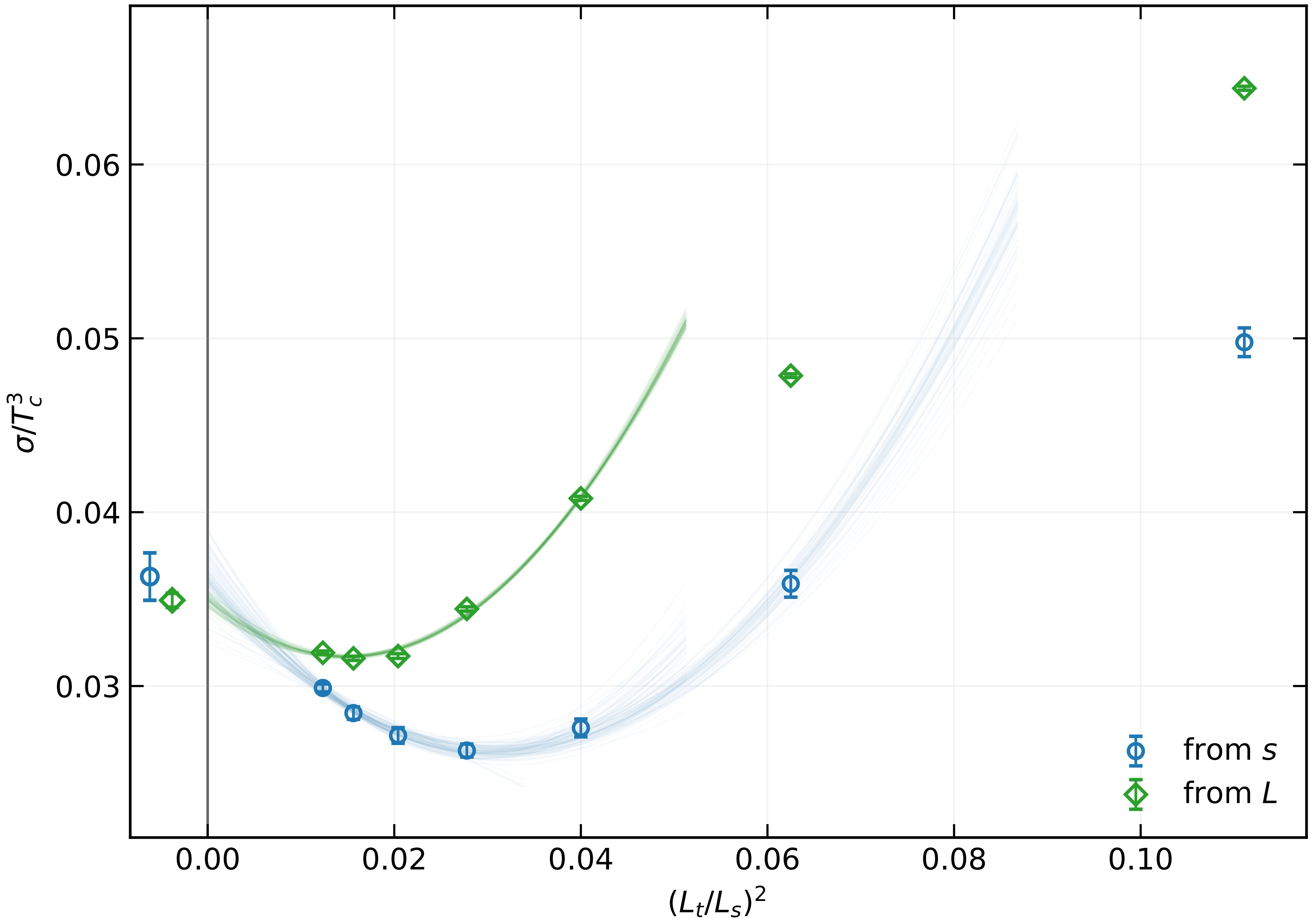

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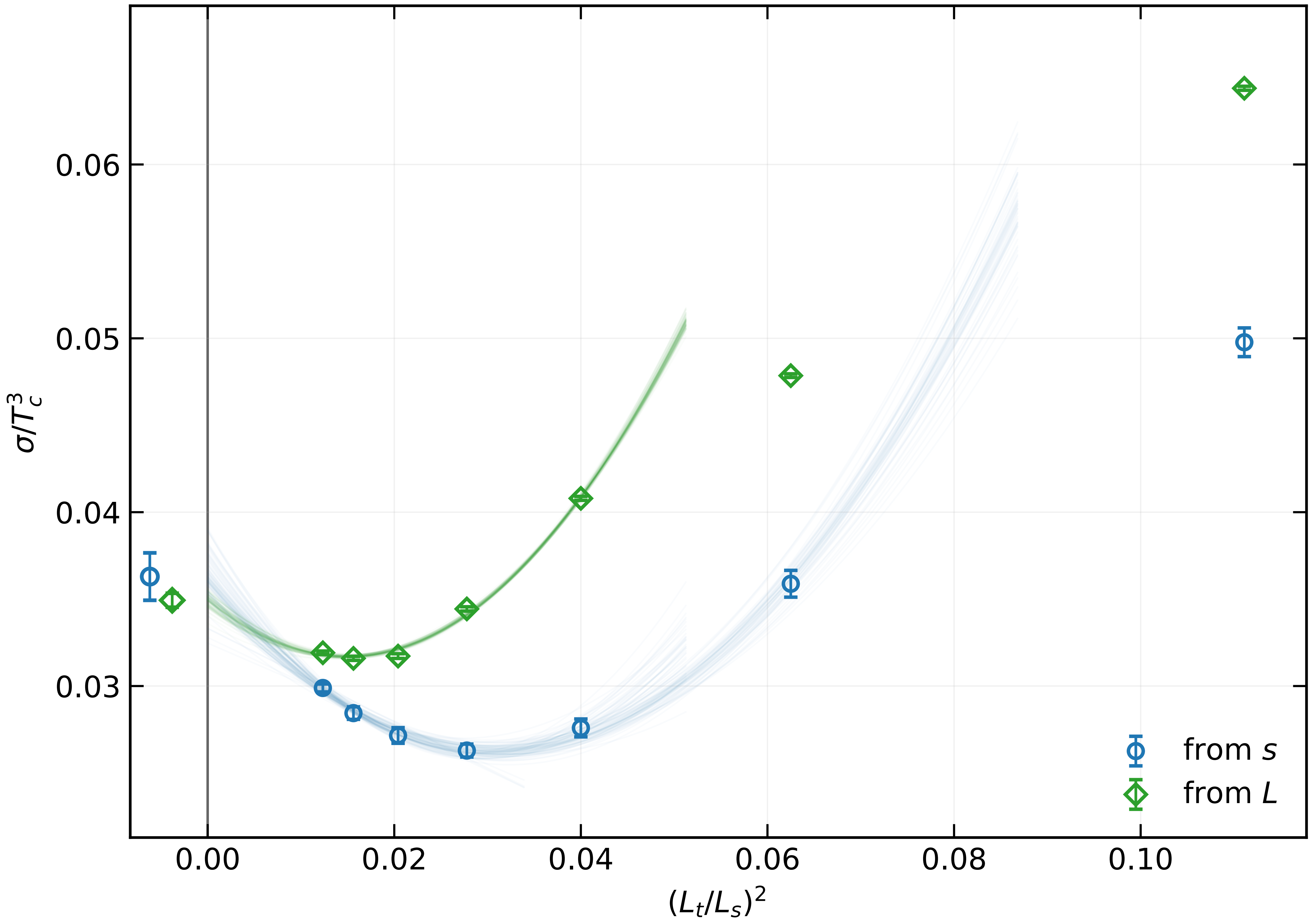
- latent heat L_h/T_c^4 from $C_{\max}$ (bare)
- interface tension σ/T_c^3 from obs. distributions
- consistency only due to large aspect ratios



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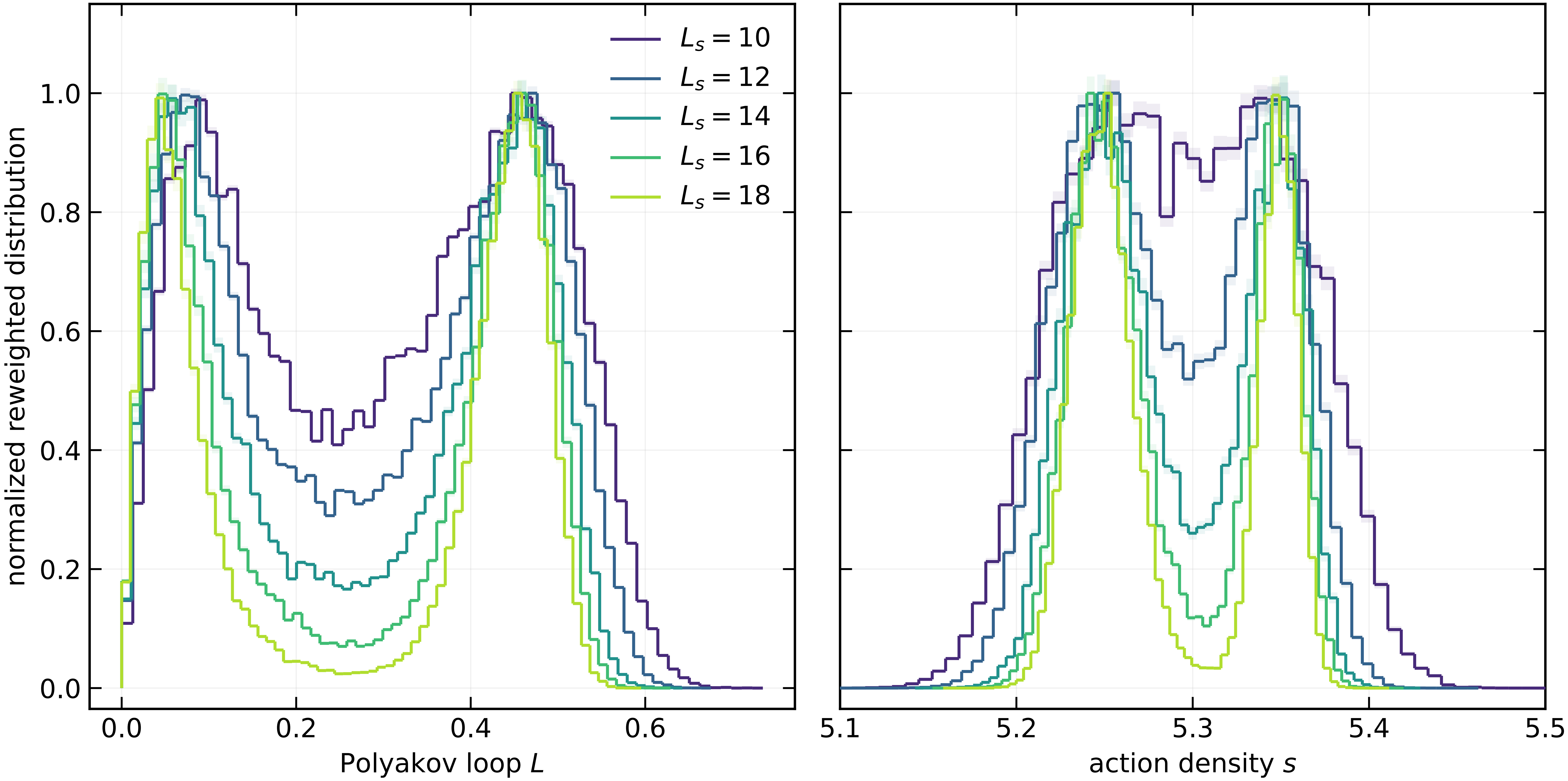

Machine-learned FP action: Conclusions

Availability of lattice convolutional neural network (L-CNN) is crucial for:

- machine learning the FP action
- derivatives for the HMC and gradient flow
- machine learning the FP topological charge

classically perfect !

close to integer !

The ML FP action in action:

- topological susceptibility from coarse lattices ✓
- thermodynamics on coarse lattices with large aspect ratios ✓
- avoids critical slowing down and topological freezing ✓

Thank you!