$\omega(782)$ and $\phi(1020)$ from Lattice QCD

Haobo Yan（燕浩波）

With Maxim Mai, Marco Garofalo, Ulf-G. Meißner, Chuan Liu, Liuming Liu, Carsten Urbach

北京大學
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History

- Hadron resonances can be accessed on the lattice from scattering its decay particles
- We have seen many interesting and important two-body resonances

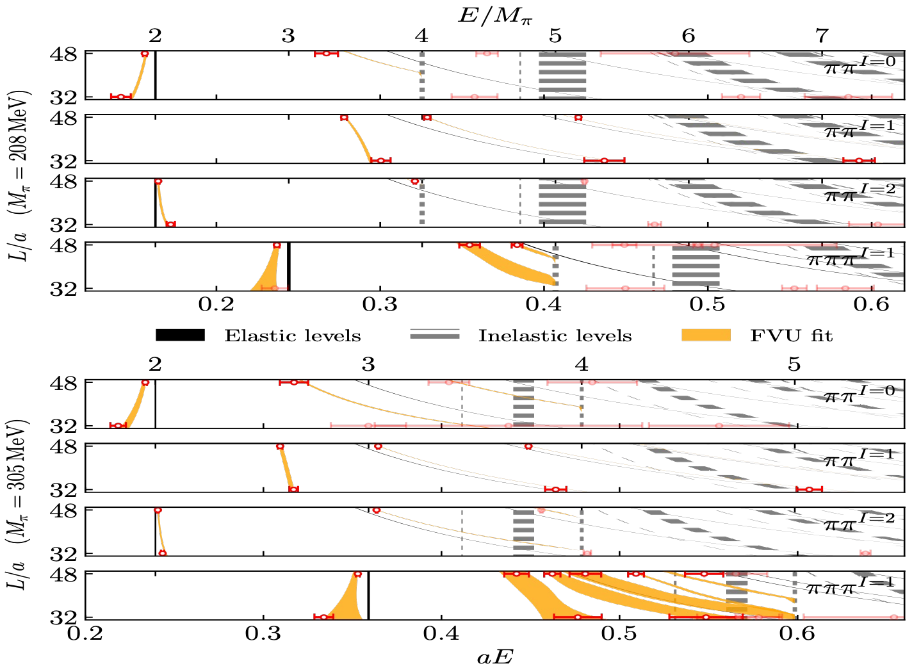
e.g. $\Lambda(1405)$ at $SU(3)$ point: Javier Suárez-Sucunza+, arXiv:2605.28447

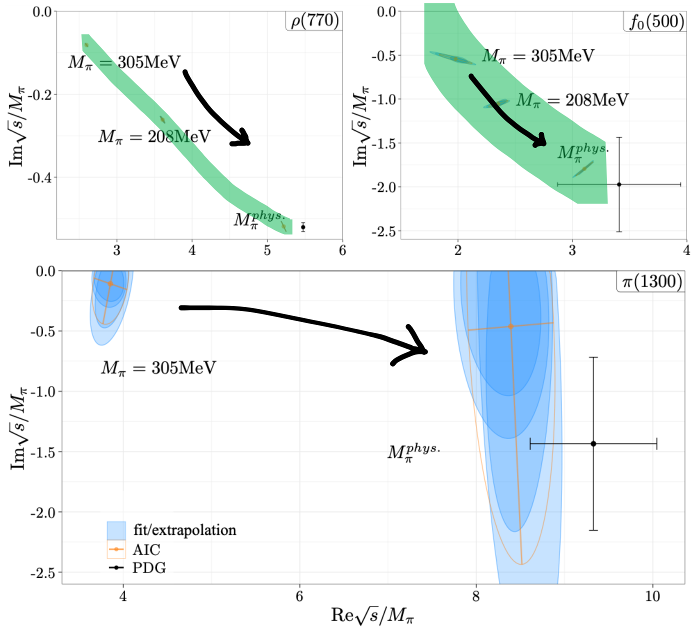
See state-of-art NN: **Miguel Salg** (Wednesday 9:20)

- Three-body example: $\pi(1300) \to \pi\pi\pi$: it was possible to extract $\pi(1300)$ from $\pi\pi\pi$ scattering

$I = 0$ $\omega(782)$: [agemo] Haobo Yan+, PRL133, 211906 (2024)

$I = 1$ $\pi(1300)$: [agemo] Haobo Yan+, PRL136, 141901 (2026)





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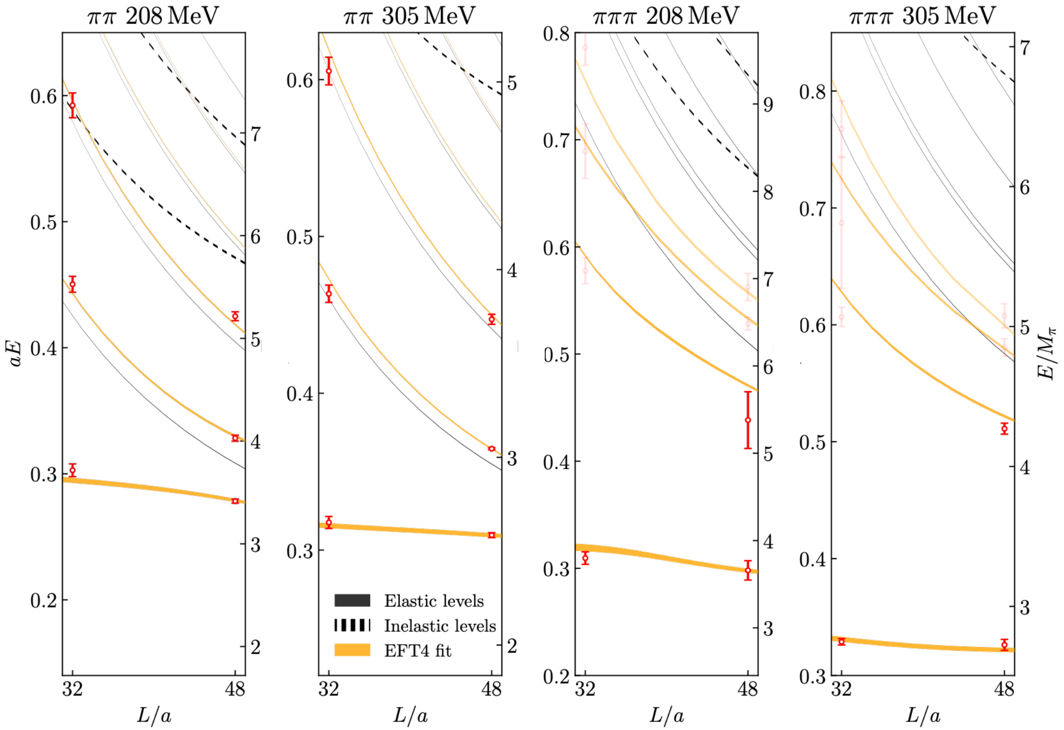

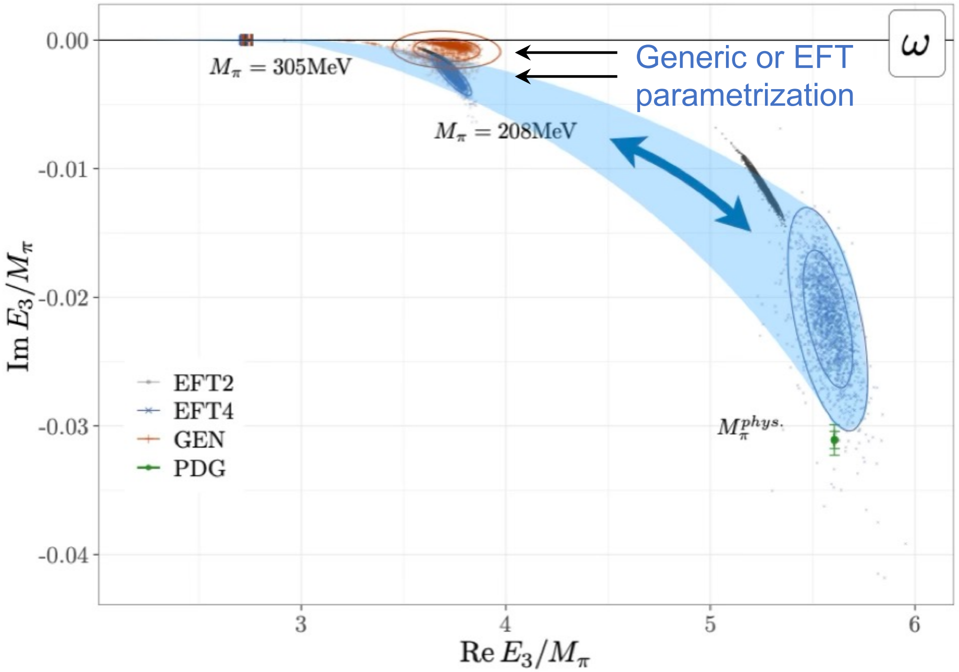

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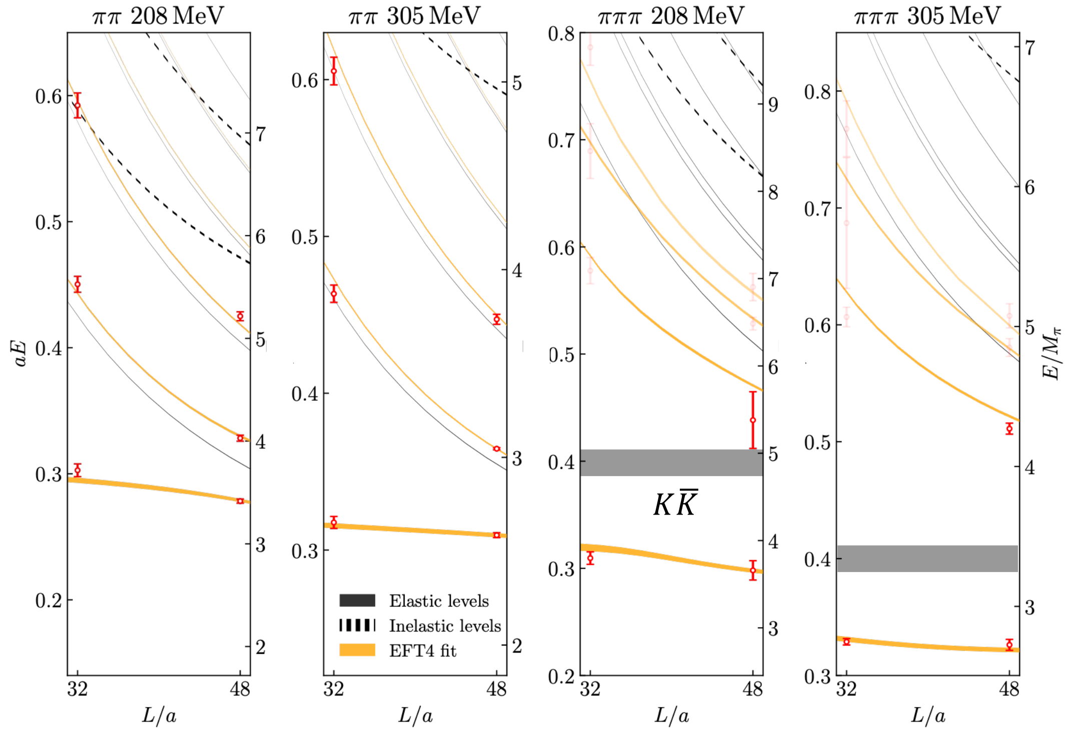
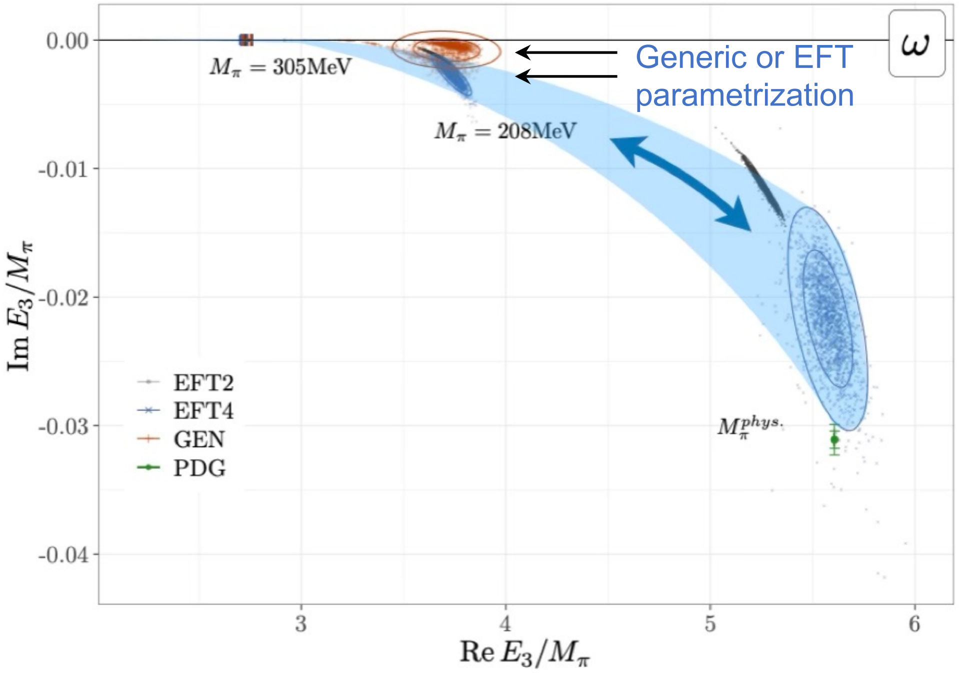
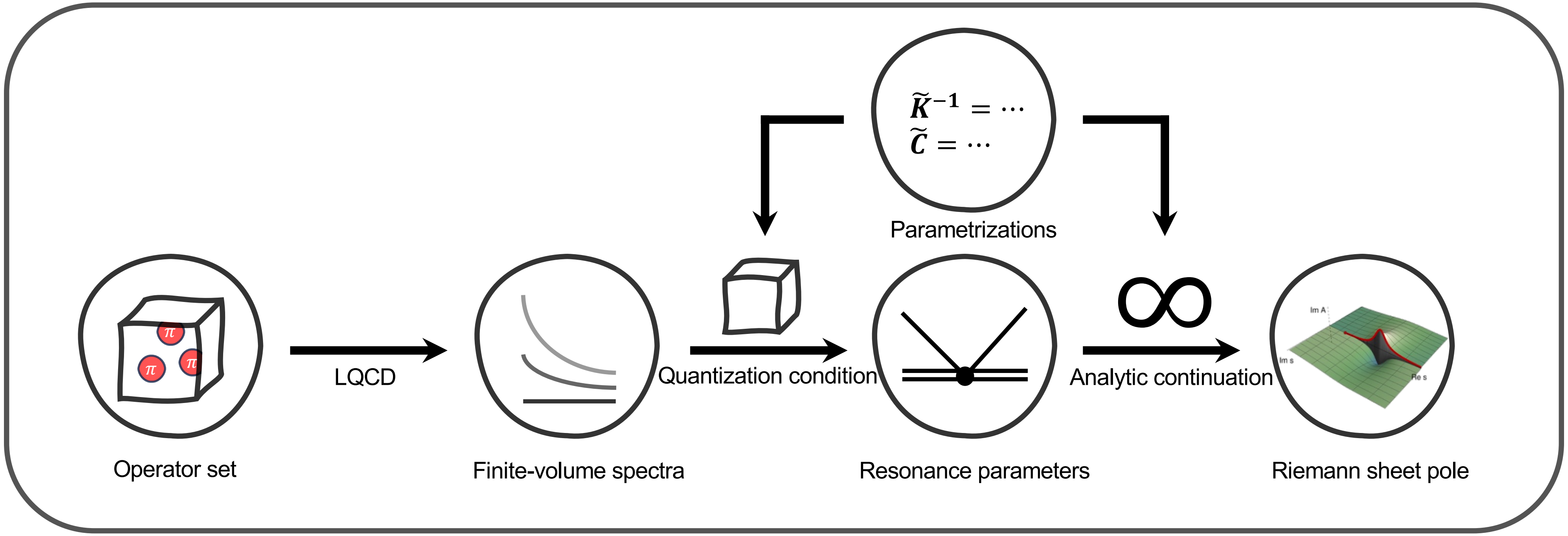




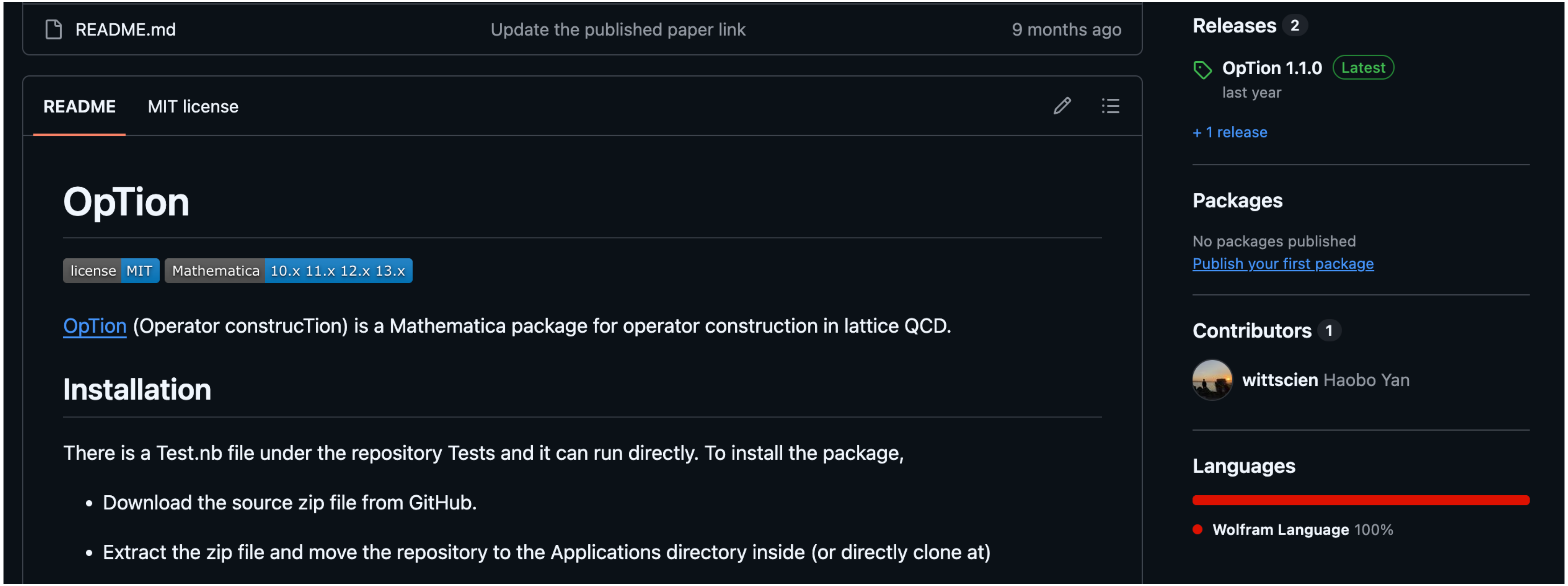
- But ω is not the only one in this region, there could be $\omega - \phi$ mixing, though mostly $\omega \to \pi\pi\pi$ and $\phi \to K\bar{K}$
- Quantization condition should be updated to deal with $2 - 3$ coupled channel

Routine


$\widetilde{K}^{-1} = \cdots$
$\widetilde{C} = \cdots$
Parametrizations
π
LQCD
Quantization condition
Analytic continuation
Operator set
Finite-volume spectra
Resonance parameters
Riemann sheet pole

Operator construction

- **OpTion** is utilized to generate general N-hadron operators with arbitrary momenta



Haobo Yan+, JHEP 10 (2025) 210 (https://github.com/wittscien/OpTion)

Operator construction

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omega3.nb

```
In[ ]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
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(* I=0 πππ ω *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
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Author: Haobo Yan

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omega3.nb 100%

插入单元...

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$$P_1[e_{x,y}]\,P_2[-e_y]\,P_3[-e_x] - P_1[e_{x,-y}]\,P_2[e_y]\,P_3[-e_x] - P_1[e_{-x,y}]\,P_2[-e_y]\,P_3[e_x] + P_1[e_{-x,-y}]\,P_2[e_y]\,P_3[e_x] - P_1[e_{x,y}]\,P_2[-e_x]\,P_3[-e_y] + P_1[e_{-x,y}]\,P_2[e_x]\,P_3[-e_y] + P_1[e_{x,-y}]\,P_2[-e_x]\,P_3[e_y] - P_1[e_{-x,-y}]\,P_2[e_x]\,P_3[e_y]$$

$$P_1[e_y]\,P_2[e_x]\,P_3[e_{-x,-y}] - P_1[e_x]\,P_2[e_y]\,P_3[e_{-x,-y}] - P_1[-e_y]\,P_2[e_x]\,P_3[e_{-x,y}] + P_1[e_x]\,P_2[-e_y]\,P_3[e_{-x,y}] - P_1[e_y]\,P_2[-e_x]\,P_3[e_{x,-y}] + P_1[-e_x]\,P_2[e_y]\,P_3[e_{x,-y}] + P_1[-e_y]\,P_2[-e_x]\,P_3[e_{x,y}] - P_1[-e_x]\,P_2[-e_y]\,P_3[e_{x,y}]$$

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$$P_1[e_{x,y,z}]\,P_2[-e_y]\,P_3[e_{-x,-z}] - P_1[e_{x,-y,z}]\,P_2[e_y]\,P_3[e_{-x,-z}] + P_1[e_{x,y,-z}]\,P_2[-e_y]\,P_3[e_{-x,z}] - P_1[e_{x,-y,-z}]\,P_2[e_y]\,P_3[e_{-x,z}] - P_1[e_{-x,y,z}]\,P_2[-e_y]\,P_3[e_{x,-z}] + P_1[e_{-x,-y,z}]\,P_2[e_y]\,P_3[e_{x,-z}] - P_1[e_{-x,y,-z}]\,P_2[-e_y]\,P_3[e_{x,z}] + P_1[e_{-x,-y,-z}]\,P_2[e_y]\,P_3[e_{x,z}] - P_1[e_{x,y,z}]\,P_2[-e_x]\,P_3[e_{-y,-z}] + P_1[e_{-x,y,z}]\,P_2[e_x]\,P_3[e_{-y,-z}] - P_1[e_{x,y,-z}]\,P_2[-e_x]\,P_3[e_{-y,z}] + P_1[e_{-x,y,-z}]\,P_2[e_x]\,P_3[e_{-y,z}] + P_1[e_{x,-y,z}]\,P_2[-e_x]\,P_3[e_{y,-z}] - P_1[e_{-x,-y,z}]\,P_2[e_x]\,P_3[e_{y,-z}] + P_1[e_{x,-y,-z}]\,P_2[-e_x]\,P_3[e_{y,z}] - P_1[e_{-x,-y,-z}]\,P_2[e_x]\,P_3[e_{y,z}]$$

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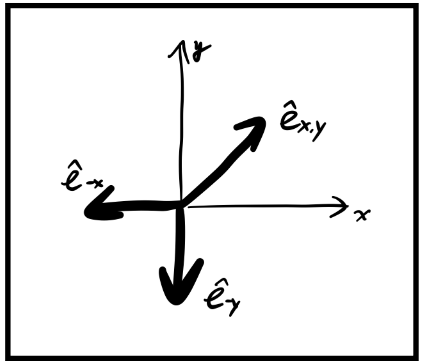

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➢ **OpTion** is utilized to generate general N-hadron operators with arbitrary momenta

See also software by **John Bulava** (Monday 14:40)

See improvement on distillation by **Juan Urrea-Niño** (Monday 14:00)

omega3.nb 100%

+ 插入单元...

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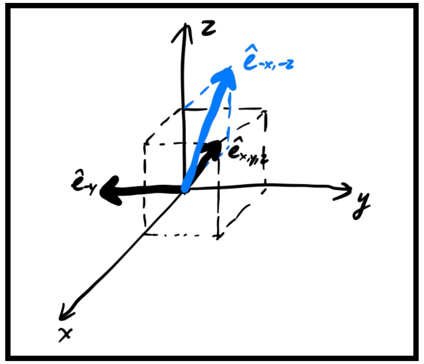


Operator construction

- Isospin projection

$$3 \otimes 3 \otimes 3 = (1 \oplus 3 \oplus 5) \otimes 3 = 1 \oplus 3^3 \oplus 5^2 \oplus 7.$$

$$\begin{cases} (\omega)_{0,0} & = \frac{1}{\sqrt{2}}(\omega^u + \omega^d), \\ (\phi)_{0,0} & = \phi, \\ (\rho\pi)_{0,0} & = -\frac{1}{\sqrt{3}}\left[\rho^+\pi^- + \rho^-\pi^+ + \frac{1}{2}[\rho^u\pi^u - \rho^u\pi^d - \rho^d\pi^u + \rho^d\pi^d]\right], \\ (K\bar{K})_{0,0} & = \frac{1}{\sqrt{2}}\left[K^+K^- + K^0\bar{K}^0\right], \\ (\pi\pi\pi)_{0,0} & = \frac{1}{\sqrt{12}}\left[-\pi^+\pi^u\pi^- + \pi^+\pi^d\pi^- + \pi^u\pi^+\pi^- - \pi^d\pi^+\pi^- + \pi^+\pi^-\pi^u - \pi^+\pi^-\pi^d \right. \\ & \quad \left. -\pi^-\pi^+\pi^u + \pi^-\pi^+\pi^d - \pi^u\pi^-\pi^+ + \pi^d\pi^-\pi^+ + \pi^-\pi^u\pi^+ - \pi^-\pi^d\pi^+\right]. \end{cases}$$

- $\rho\pi$ operators up to 2 mom, $K\overline{K}$ operators up to 4 mom, and one $\pi\pi\pi$ operator

$I=3$: [HadSpec] Maxwell Hansen+, PRL 126, 012001 (2021), Maxim Mai and Micheal Döring, PRL 122, 062503 (2019)…

$I=2$: [HadSpec] Raúl Briceño+, arXiv: 2510.24894, PRL (2026), Feng+, arXiv:2601.16916, PRD (2026)

$I=1$ $\pi(1300)$: [agemo] Haobo Yan+, PRL136 (2026) 14, 141901

$I=1$ $a_1(1260)$: [GWQCD] Maxim Mai+, PRL 127, 222001 (2021)

$I=0$ $\omega(782)$: [agemo] Haobo Yan+, PRL133, 211906 (2024)

Related talks:

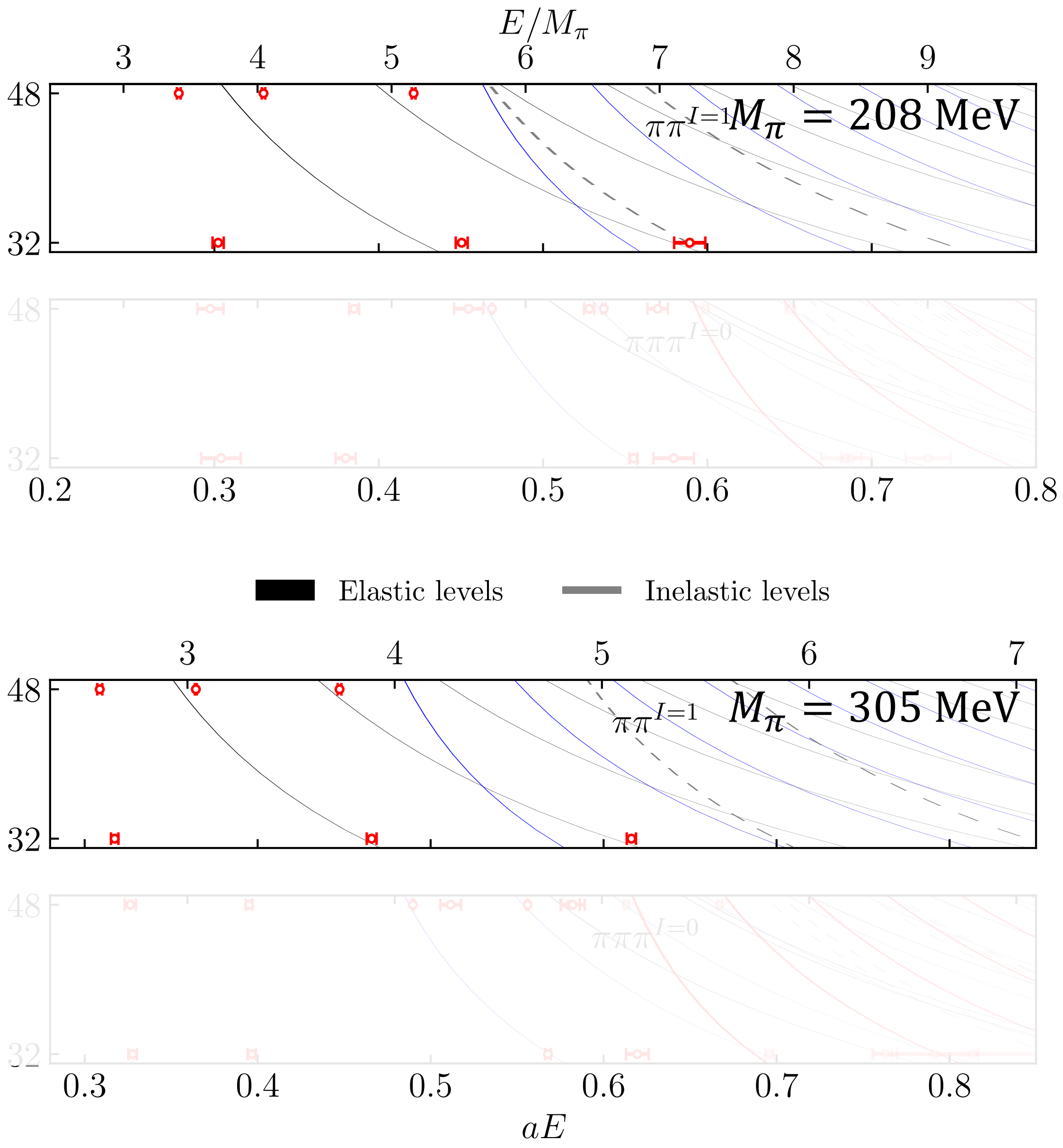
Maxim Mai (Thursday 11:50)

Andrew Jackura (Thursday 14:20)

Max Hansen (Tuesday 16:50)

Juan Jimenez (Tuesday 14:40)

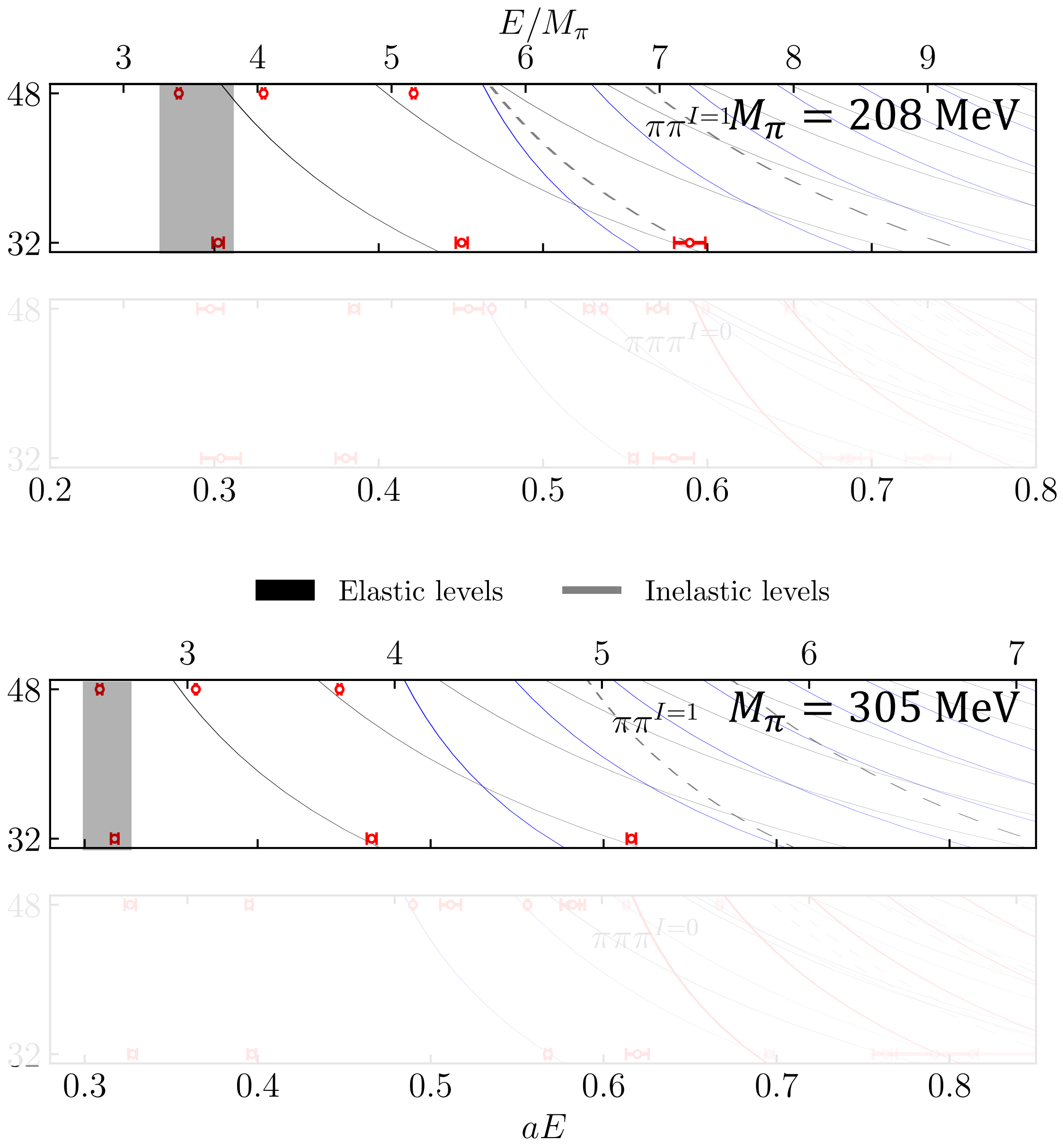
Finite-volume spectra and fit



- CLQCD $N_\mathrm{f} = 2 + 1$ Wilson-Clover ensembles
- $M_\pi = 305$ and 208 MeV and two volumes each

[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

- $I = 1$ $\rho \to \pi\pi$ levels as expected



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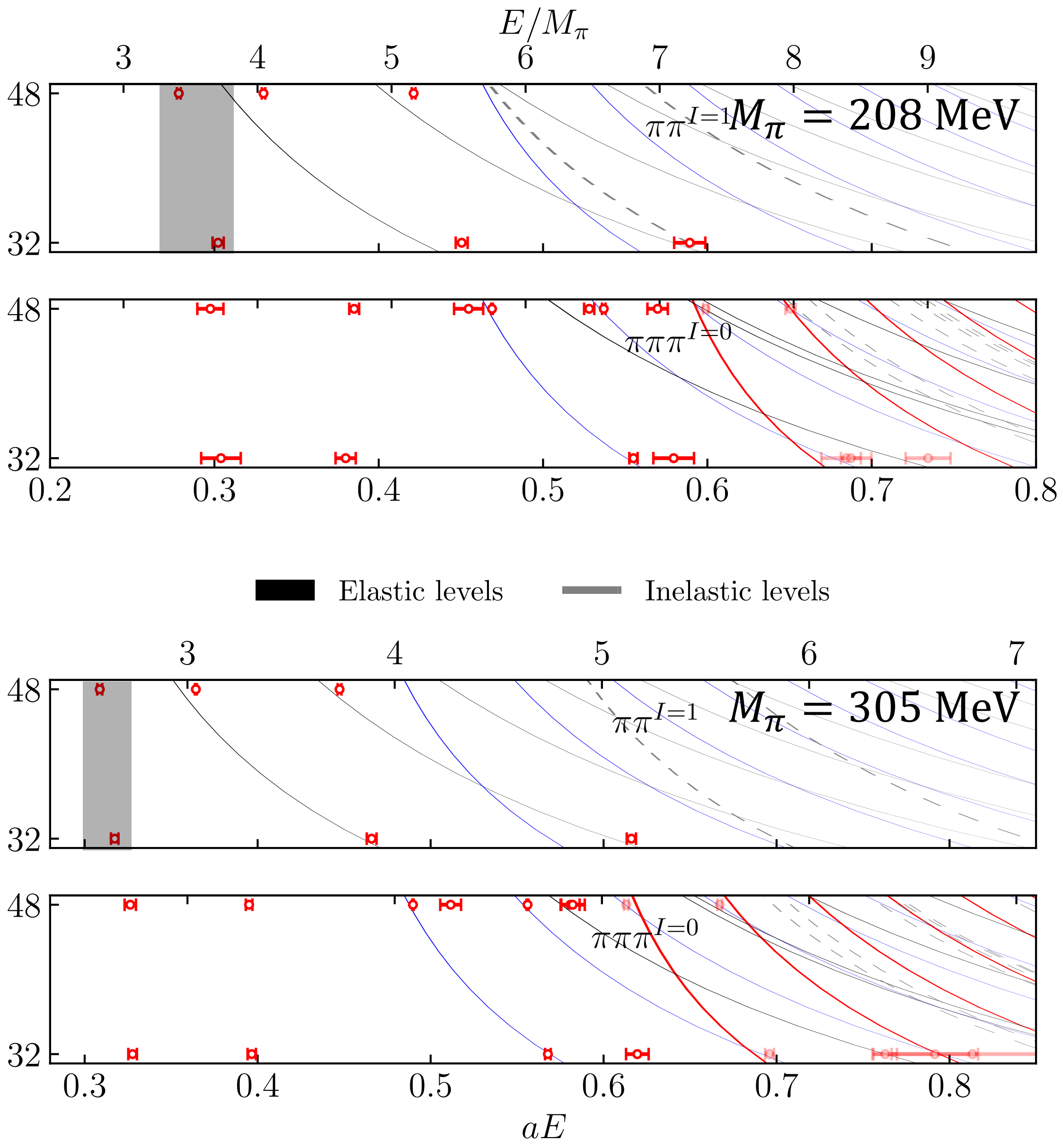
Finite-volume spectra and fit



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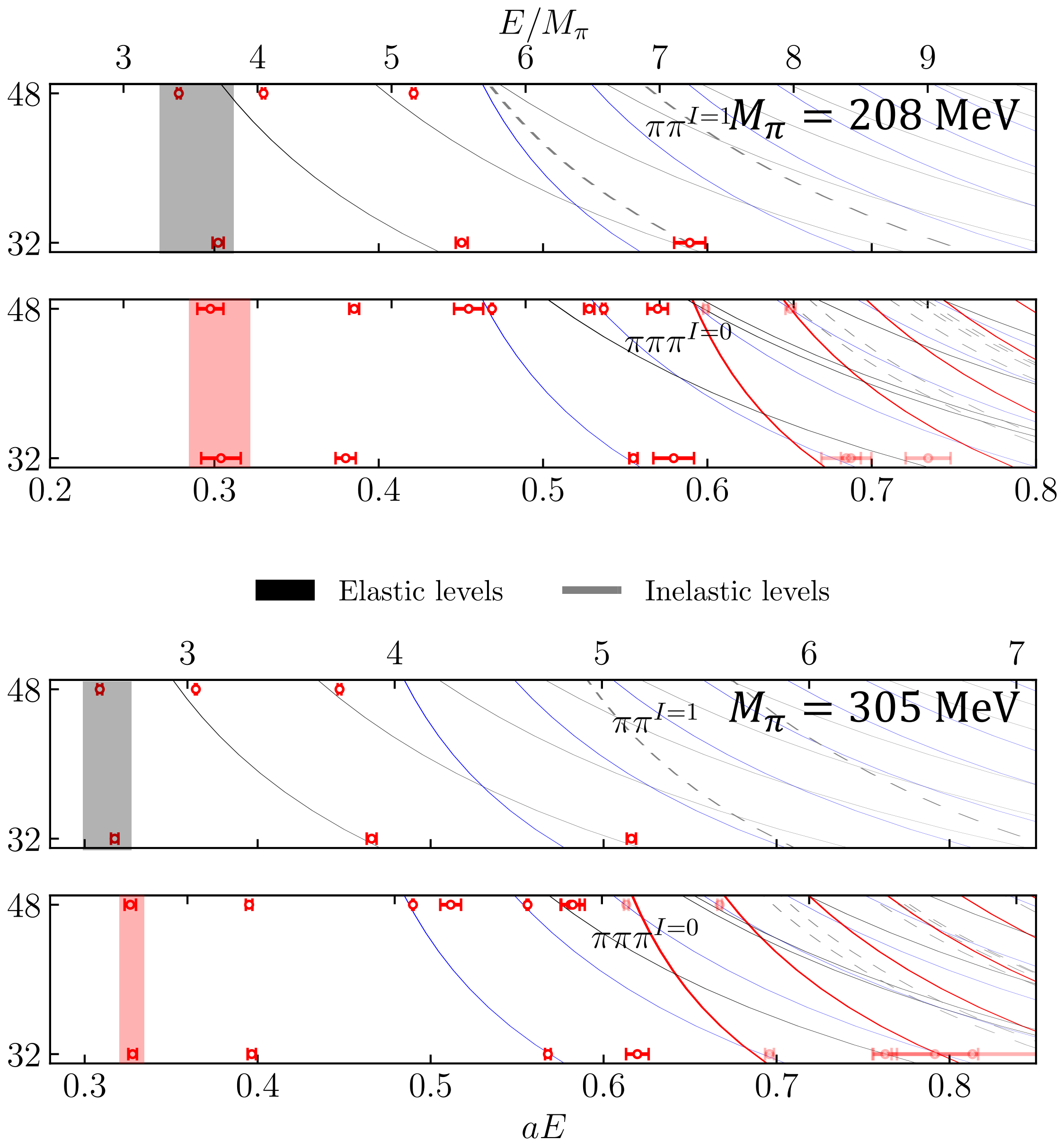
- $I = 1$ $\rho \to \pi\pi$ levels as expected
- Clear indication of ω and ϕ

Finite-volume spectra and fit



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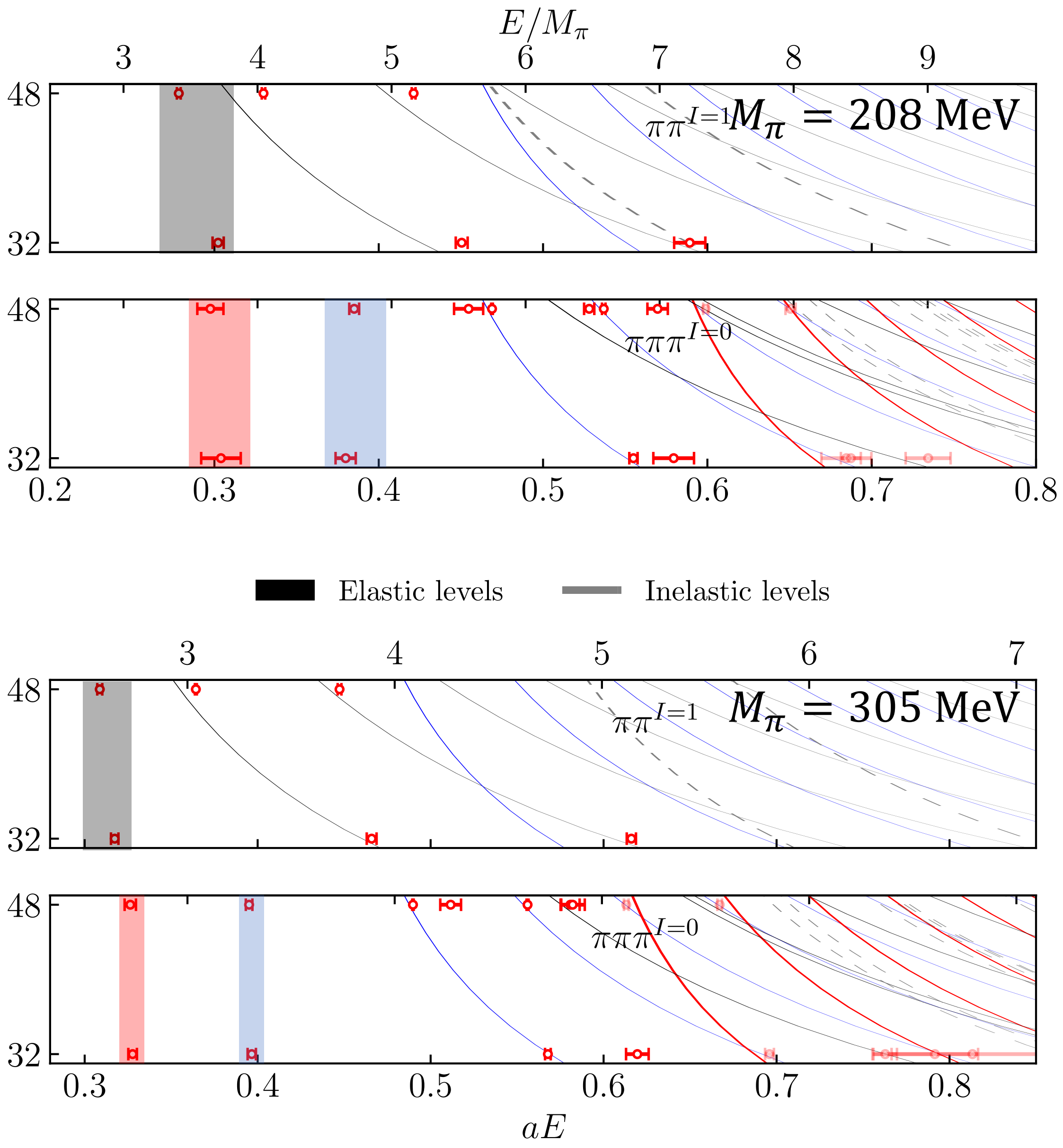

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[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

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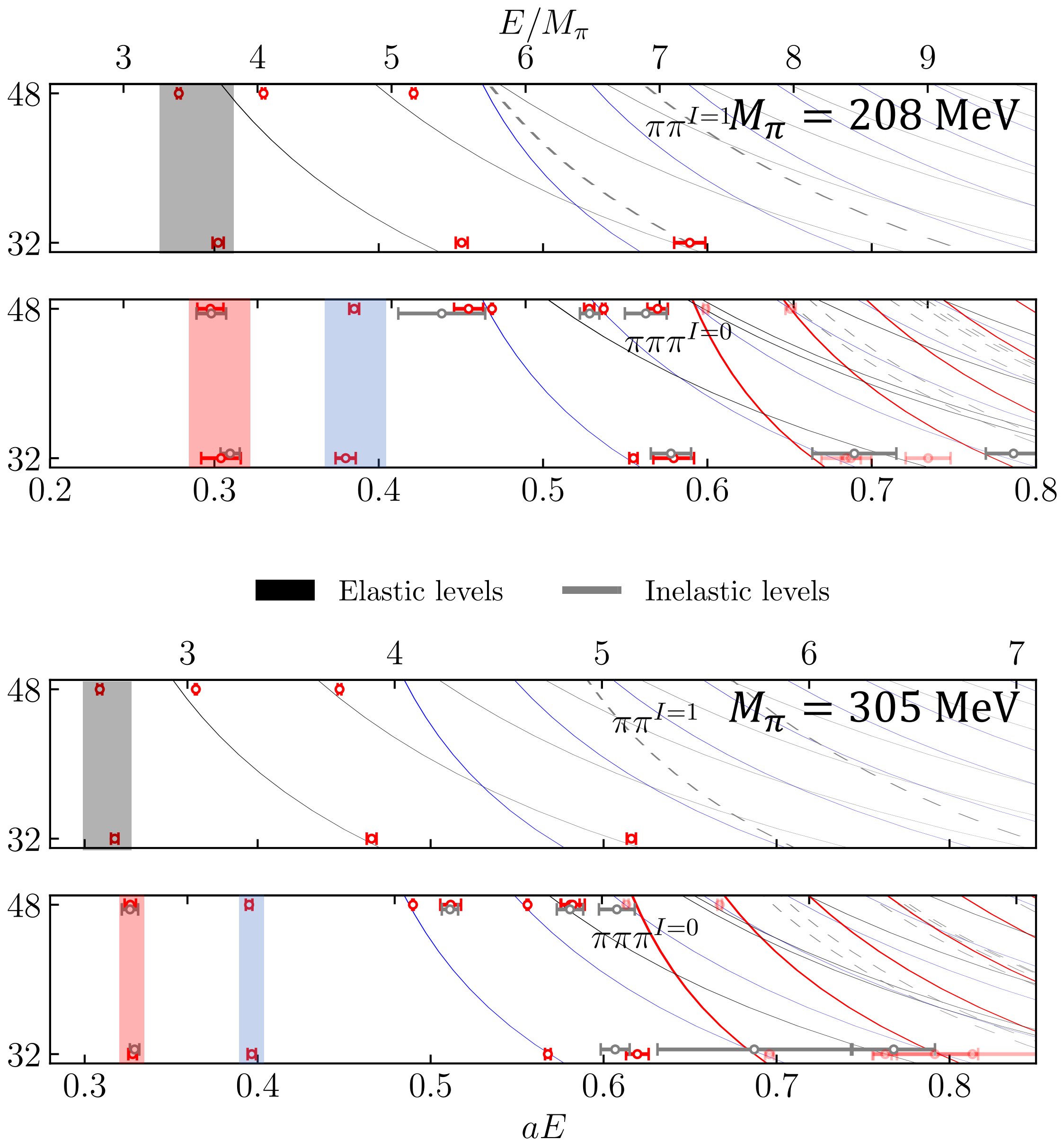
Finite-volume spectra and fit



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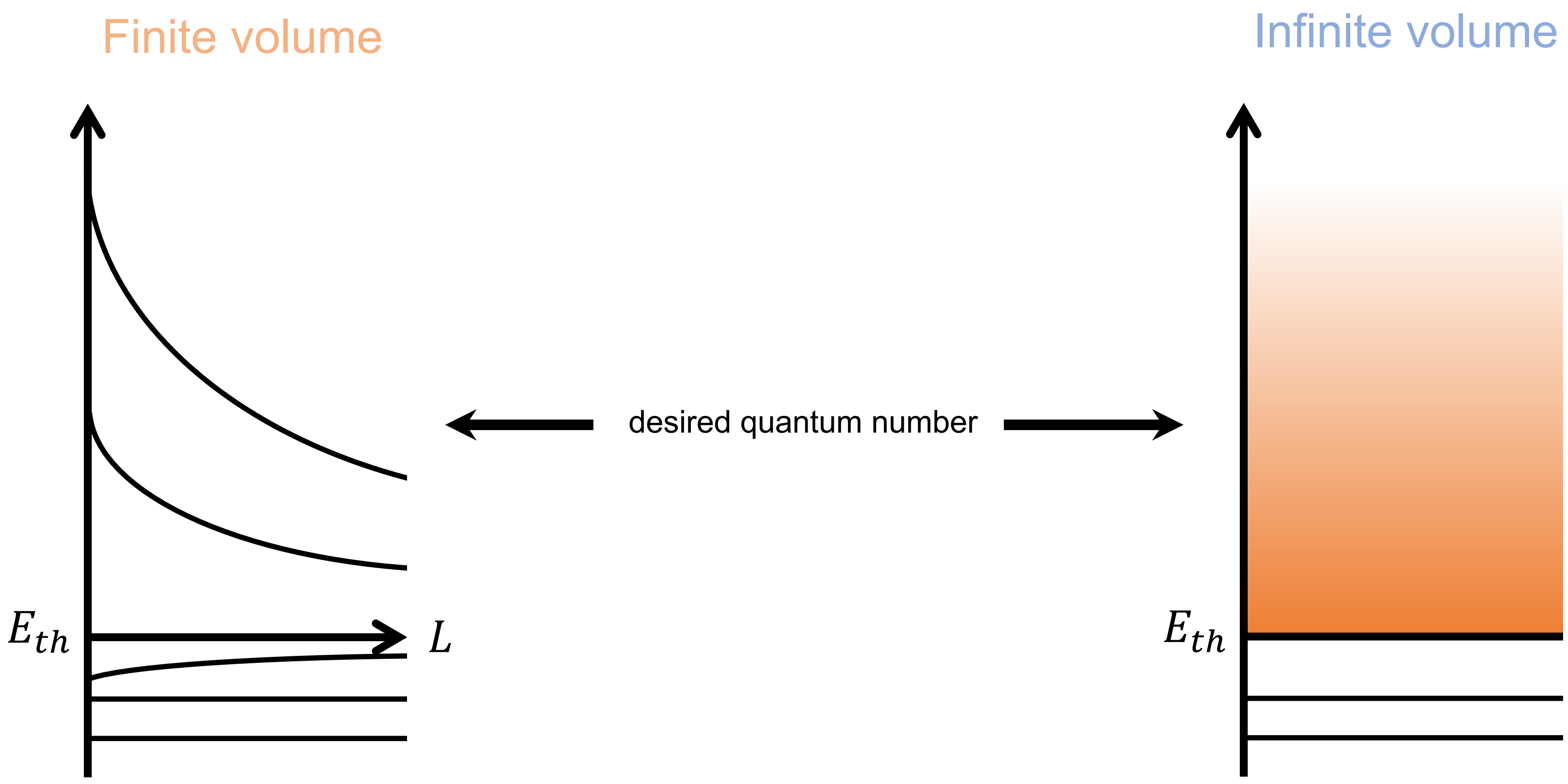
[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

- $I = 1$ $\rho \to \pi\pi$ levels as expected
- Clear indication of ω and ϕ
- Additional levels compared to light-only operators

Routine



- To extract the infinite-volume information from finite-volume spectra

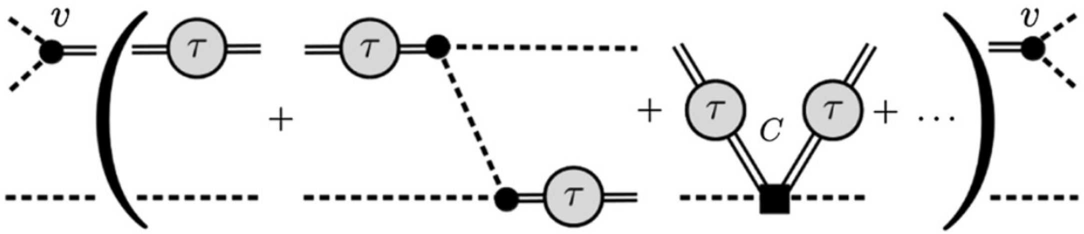
Quantization condition: $f\big(\mathcal{M}(E_n),\ \mathcal{F}(E_n, L)\big) = 0$

- Quantization condition accounts for the polynomial finite-volume effects

Quantization condition from unitarity

- In FVU (**F**inite-**V**olume **U**nitarity): Unitarity ⇒ integral equation of the three-body amplitude

$T^{\infty} =$



$$\xrightarrow{\int \to \sum} \quad T^L = \infty \quad \longleftrightarrow \quad E_n$$

Maxim Mai and Micheal Döring, PRL 122, 062503 (2019)

- This work: extend FVU to the two-three-body coupled systems

$$\left\langle \chi_\Gamma \middle| \begin{pmatrix} \tau_\rho \cdot E_{L,\rho} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \tau_\rho & 0 \\ 0 & 1 \end{pmatrix} \frac{\mathbb{1}}{\mathbb{1} - (B+C)\hat{\tau}} (B+C) \begin{pmatrix} \tau_\rho & 0 \\ 0 & 1 \end{pmatrix} \middle| \chi_\Gamma \right\rangle = \infty$$

See a more complete introduction to FVU by **Maxim Mai** (Thursday 11:50)

See FVU extension to two-body system with left-hand cut by **Ivan Vujmilovic** (Friday 14:00)

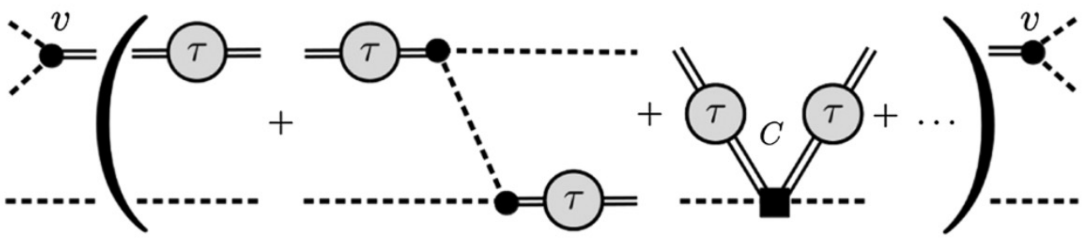
See extension to the Roper channel N(1440) by **Jinzi Wu** on Friday at Fri(1440)

State-of-the-art formalisms

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

See Mai PRL (2019) for quantitative tests of smooth and hard cutoffs

Quantization condition from unitarity

$T^{\infty} =$  $\xrightarrow{\int \to \sum}$ $T^{L} = \infty \longleftrightarrow E_n$

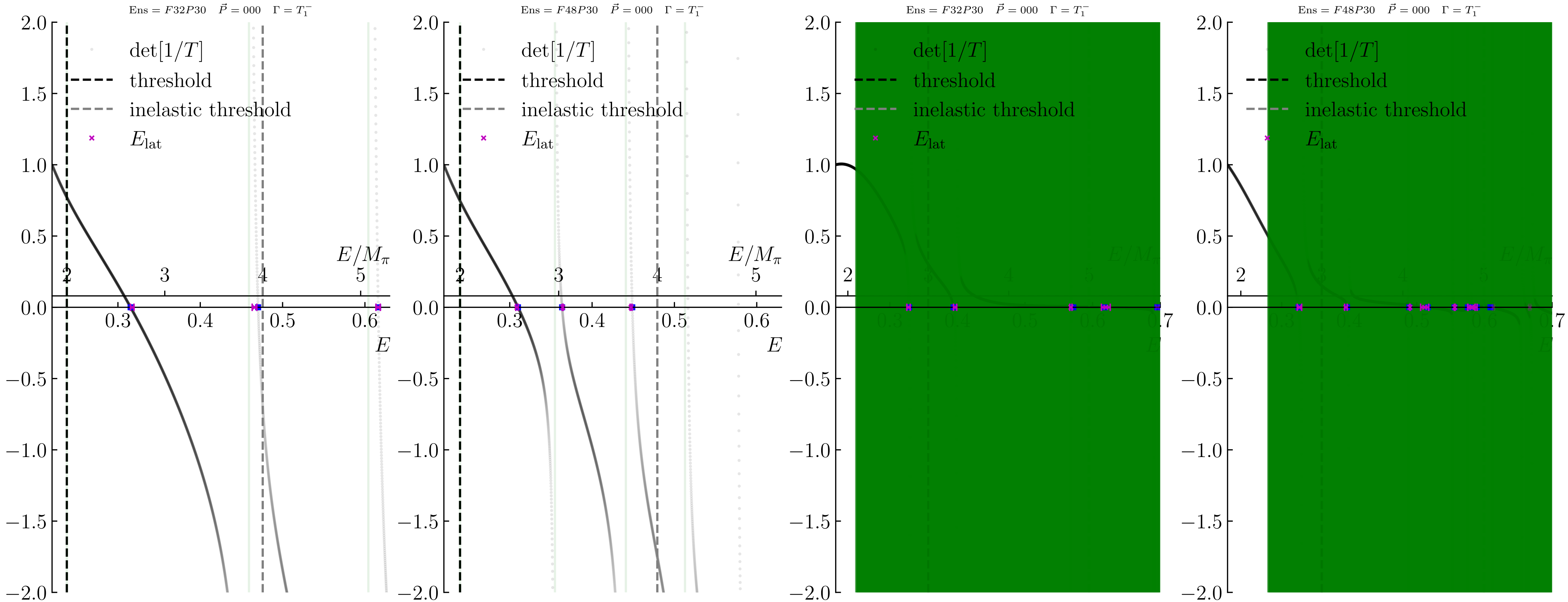
- In the angular mom basis, three-body $C = \begin{bmatrix} p' \frac{c_{11}^{\omega}}{s - M_{\omega}^2} p & p' i c_{12}^{0} p \\ p' i c_{12}^{0} p & p' \frac{c_{22}^{\phi}}{s - M_{\phi}^2} p \end{bmatrix}$

two-body $\begin{cases} \widetilde{K}_{\rho}^{-1} = a_0 + a_1 \sigma \\ \widetilde{K}_{K}^{-1} = \frac{s - 4E_K^2}{2} \left(\frac{4E_K^2}{s} \right) \end{cases}$

- Spectator mom cutoff $= [1, 1, 1]$

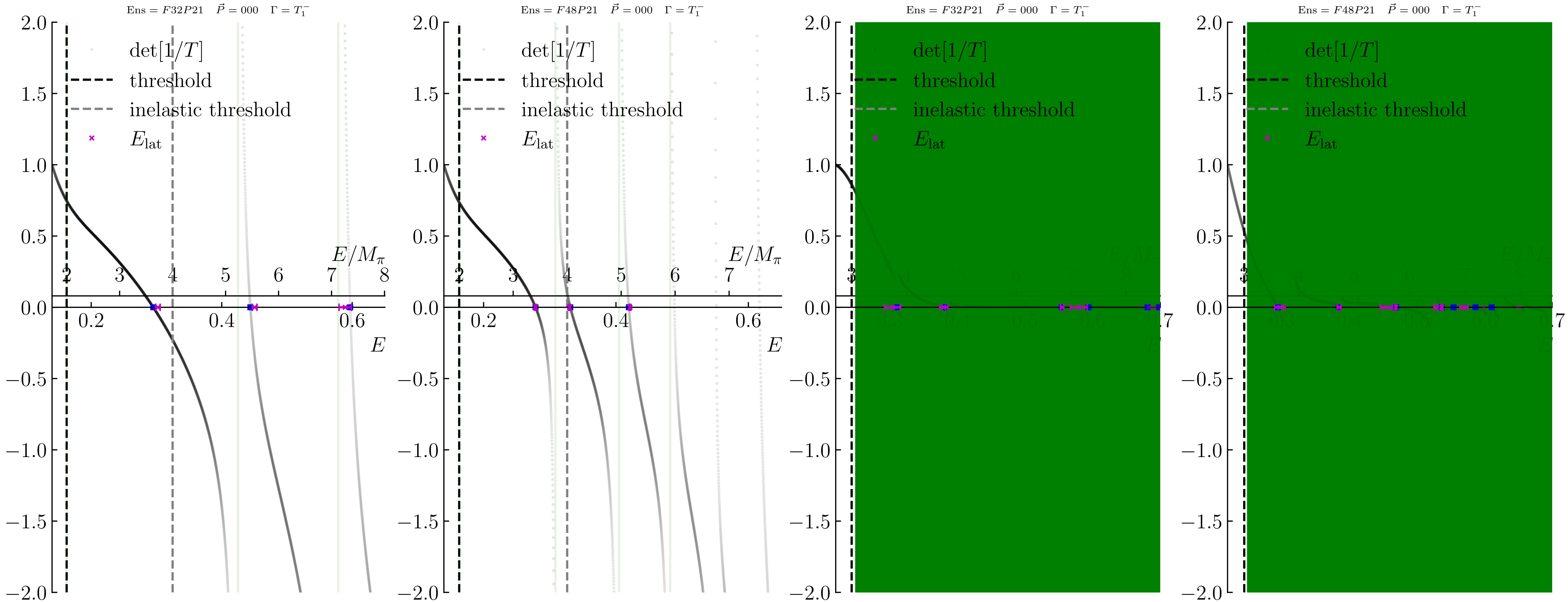
Quantization condition from unitarity

- $\chi^2/d.o.f. \approx 1.6$



Quantization condition from unitarity

➤ $\chi^2/d.o.f. \approx 1.9$



Summary

- Including strangeness produces more levels, and they are low in the spectra
- Extension of FVU to coupled 2-3 channels
- We are working on extracting ω and ϕ simultaneously from coupled-channel scattering

- Radial excitations of kaons
- Finally Roper in the long future
- Matrix element with three-body hadronic state

- I made my thesis available on 2 weeks ago: arXiv:2607.12508

Thank you!