

$\omega(782)$ and $\phi(1020)$ from Lattice QCD

Haobo Yan (燕浩波)

With Maxim Mai, Marco Garofalo, Ulf-G. Meißner, Chuan Liu, Liuming Liu, Carsten Urbach



北京大学
PEKING UNIVERSITY

History

- Hadron resonances can be accessed on the lattice from scattering its decay particles

e.g. $\Lambda(1405)$ at $SU(3)$ point: Javier Suárez-Sucunza+, arXiv:2605.28447

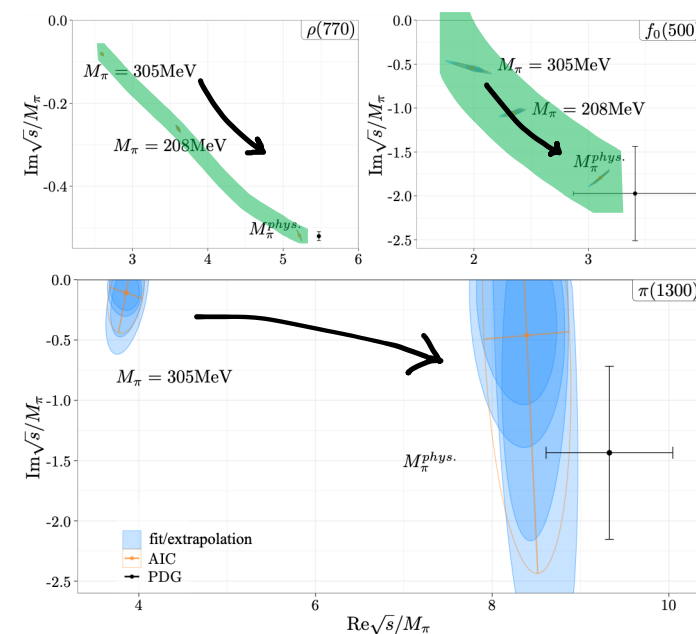
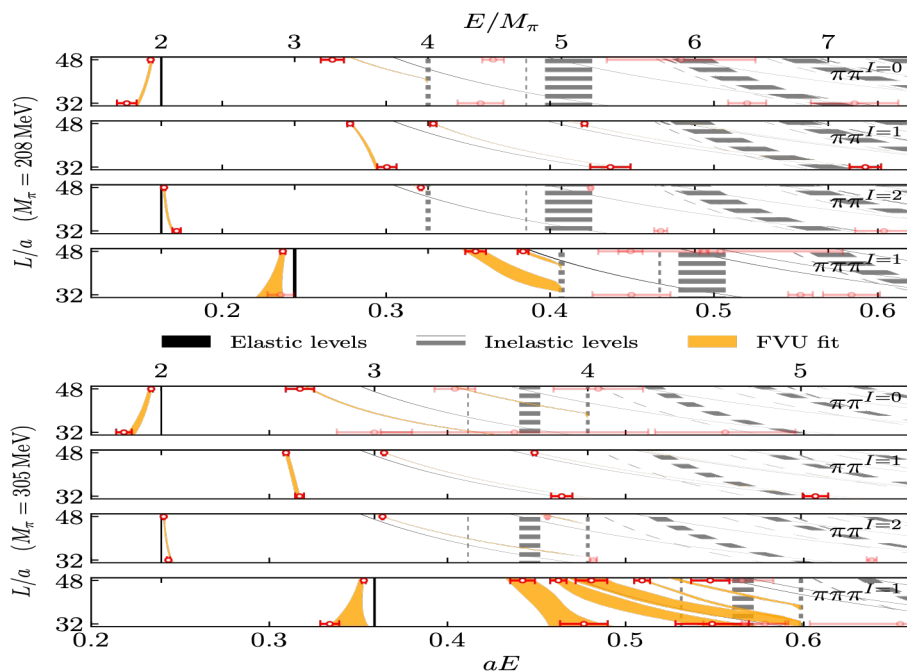
- We have seen many interesting and important two-body resonances

See state-of-art NN : **Miguel Salg** (Wednesday 9:20)

- Three-body example: $\pi(1300) \rightarrow \pi\pi\pi$: it was possible to extract $\pi(1300)$ from $\pi\pi\pi$ scattering

$I = 0$ $\omega(782)$: [agemo] Haobo Yan+, PRL133, 211906 (2024)

$I = 1$ $\pi(1300)$: [agemo] Haobo Yan+, PRL136, 141901 (2026)

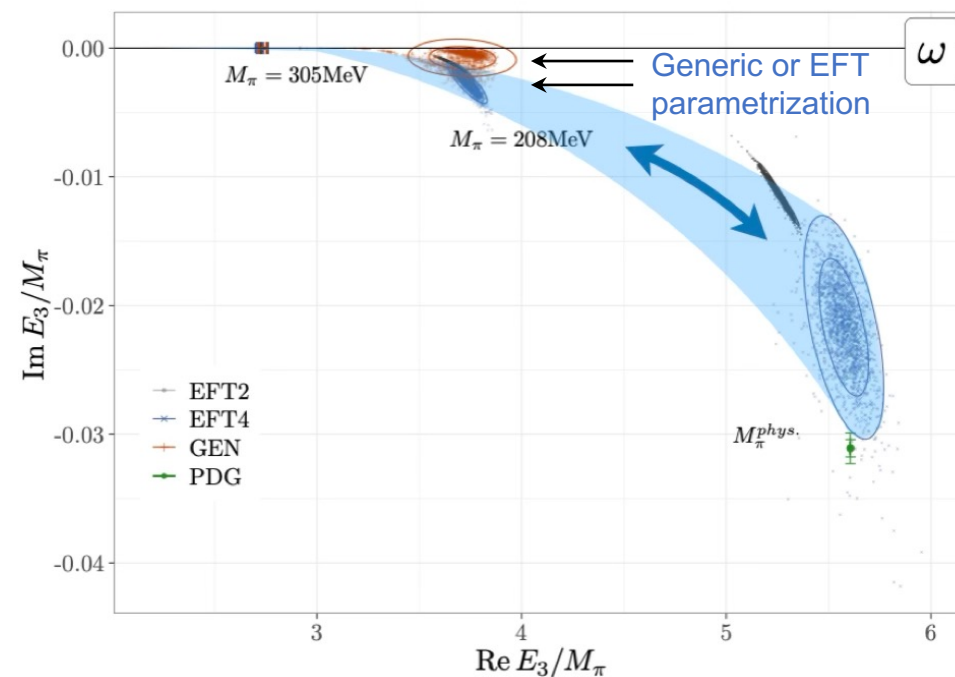
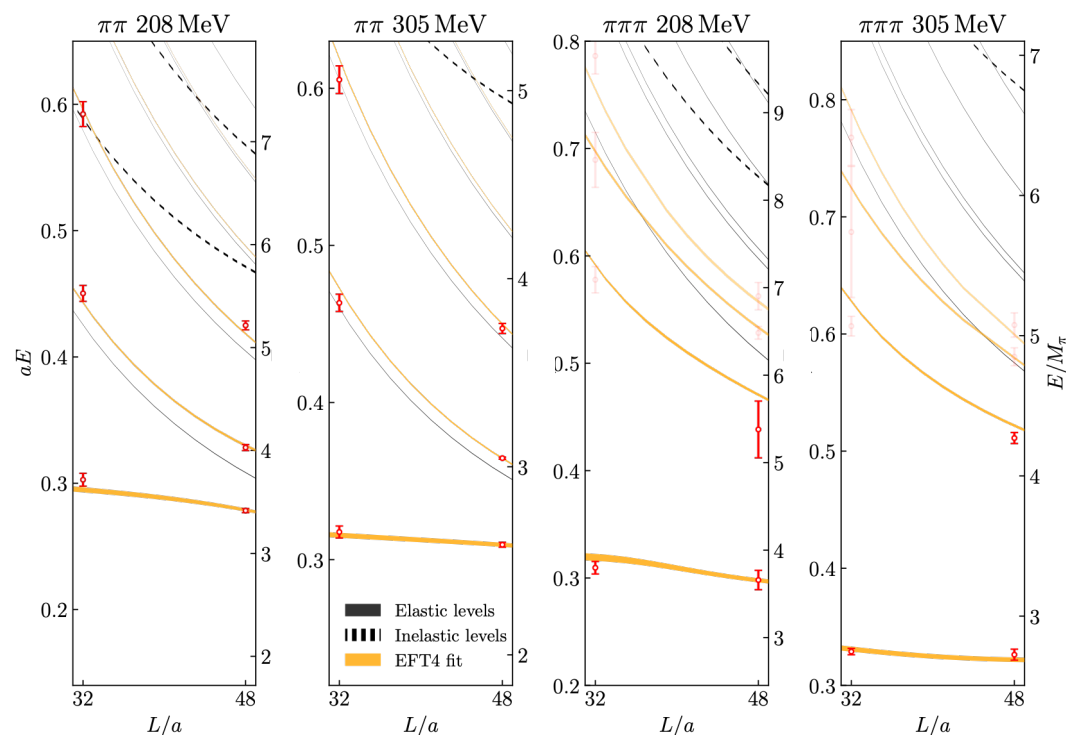


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- Three-body example: $\omega(782) \rightarrow \pi\pi\pi$: it was possible to extract ω from $\pi\pi\pi$ scattering considering only light d.o.f.

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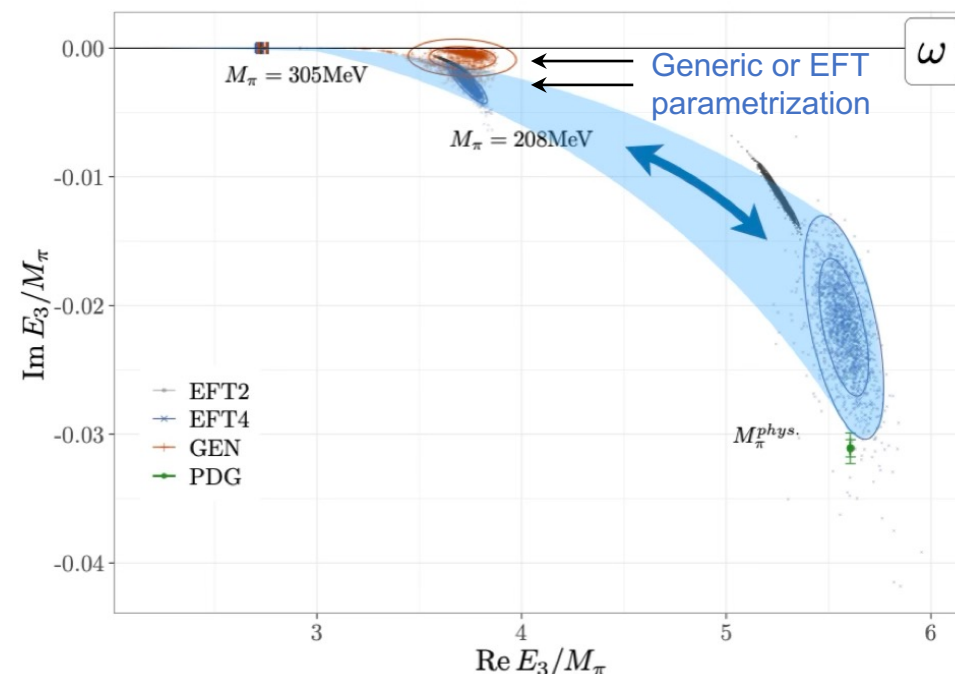
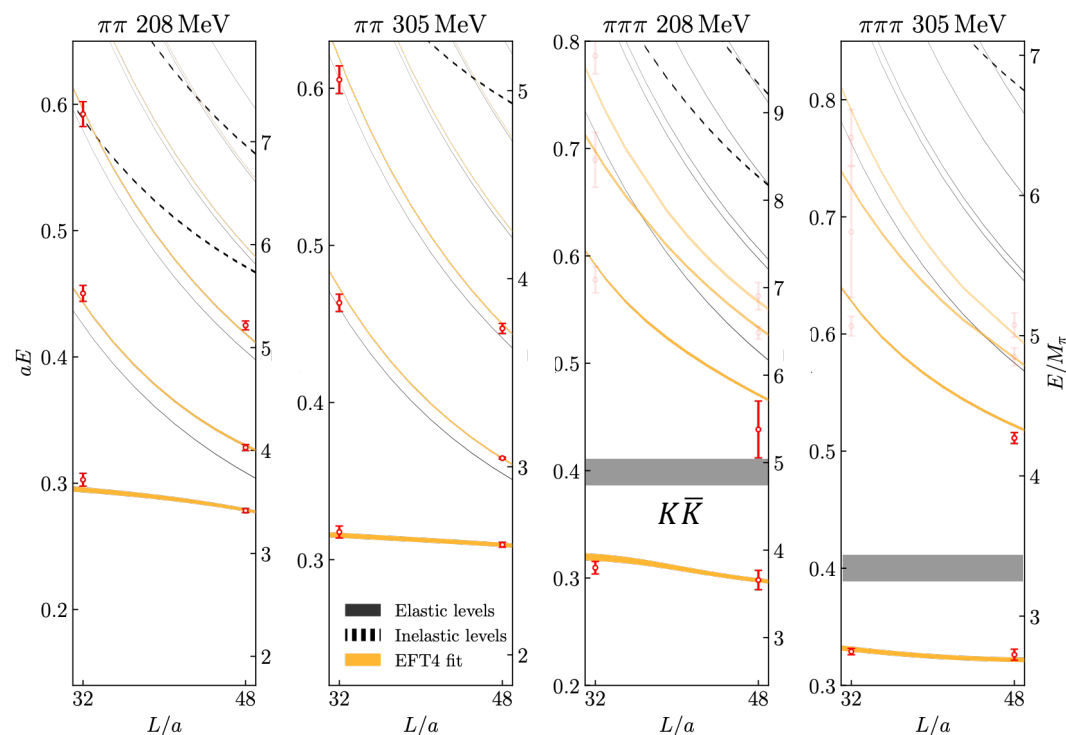


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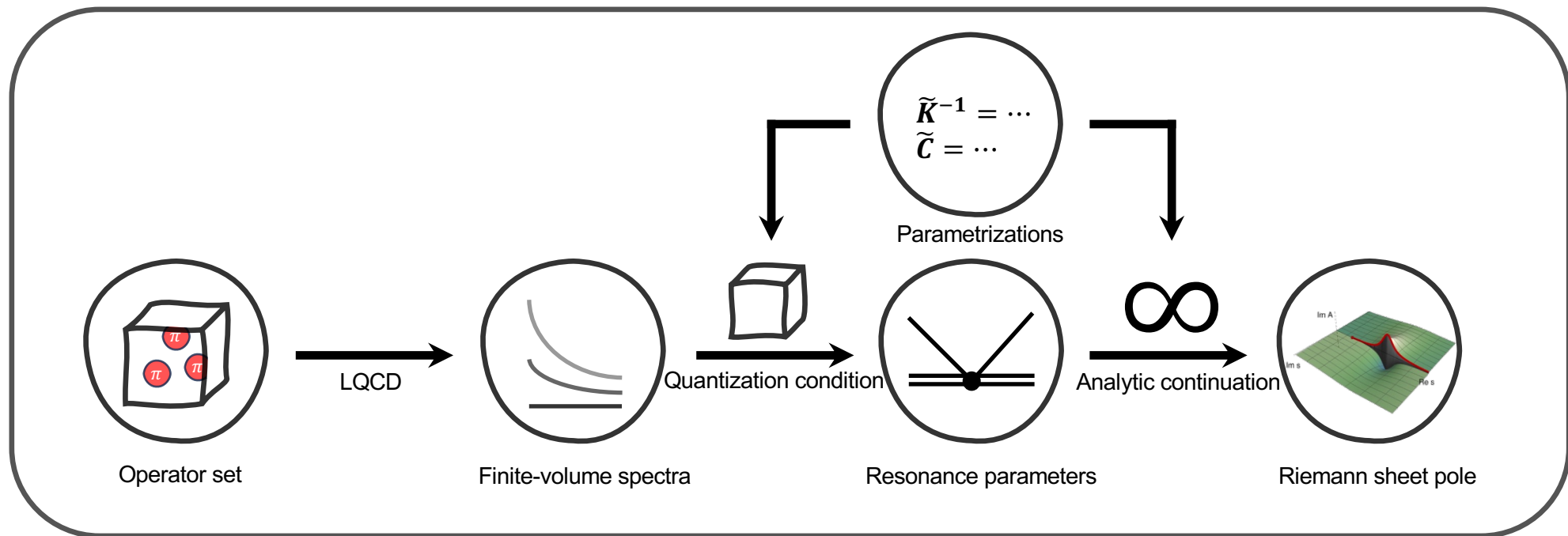
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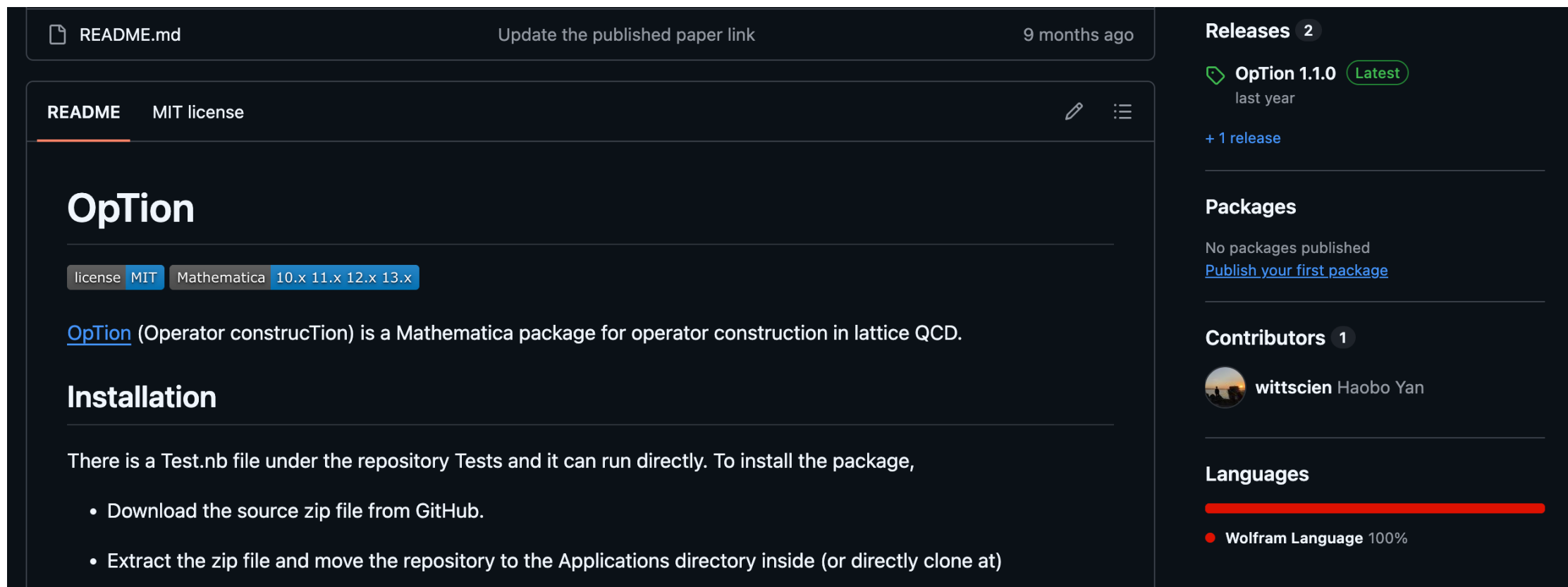


- But ω is not the only one in this region, there could be $\omega - \phi$ mixing, though mostly $\omega \rightarrow \pi\pi\pi$ and $\phi \rightarrow K\bar{K}$
- Quantization condition should be updated to deal with 2 – 3 coupled channel



Operator construction

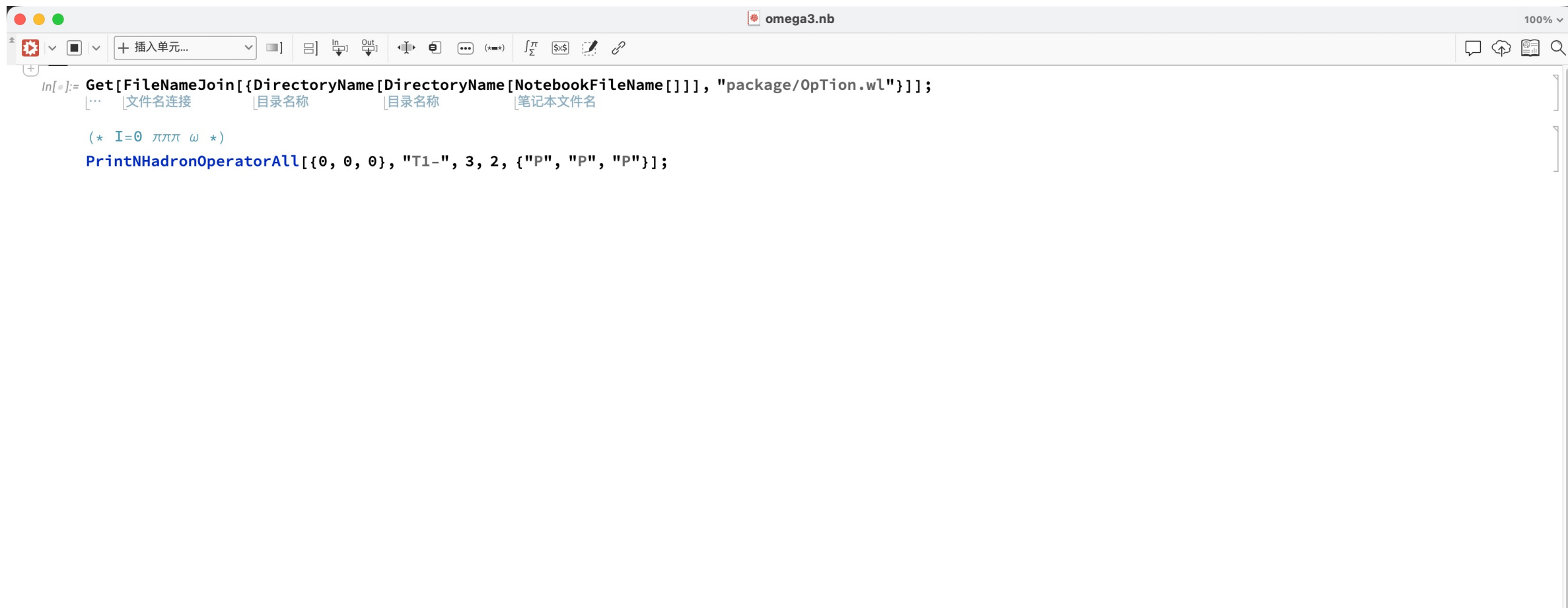
- **OpTion** is utilized to generate general N -hadron operators with arbitrary momenta



The screenshot displays the GitHub repository for **OpTion**. The main content area shows the README, which includes the repository name, license (MIT), supported Mathematica versions (10.x, 11.x, 12.x, 13.x), and a description: "OpTion (Operator construcTion) is a Mathematica package for operator construction in lattice QCD." The installation section provides instructions: "There is a Test.nb file under the repository Tests and it can run directly. To install the package," followed by a list of steps: "Download the source zip file from GitHub" and "Extract the zip file and move the repository to the Applications directory inside (or directly clone at)". The right sidebar contains metadata: "Releases 2" with the latest version "OpTion 1.1.0" (marked "Latest") from "last year"; "Packages" section stating "No packages published" with a link to "Publish your first package"; "Contributors 1" listing "wittscien Haobo Yan"; and "Languages" showing "Wolfram Language" at 100%.

Operator construction

- **OpTion** is utilized to generate general N -hadron operators with arbitrary momenta



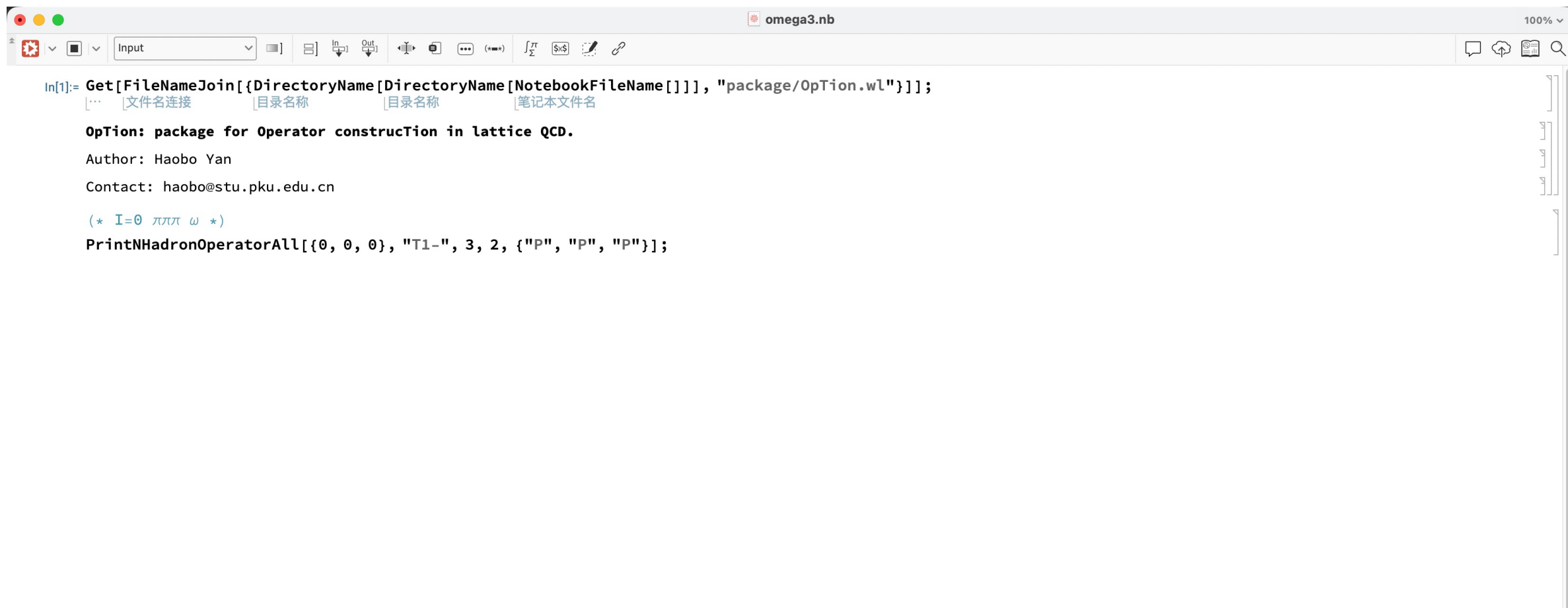
The screenshot shows a Mathematica notebook interface with the title bar 'omega3.nb' and a zoom level of 100%. The notebook contains the following code:

```
In[*]:= Get[FileNameJoin[{DirectoryName[DirectoryName[NotebookFileName[]]], "package/OpTion.wl"}]];
(* I=0  $\pi\pi\pi \omega$  *)
PrintNHadronOperatorAll[{0, 0, 0}, "T1-", 3, 2, {"P", "P", "P"}];
```

Below the code, there are blue annotations: '[...] 文件名连接' under the first argument of Get, '[目录名称] 目录名称' under the second argument of Get, and '[笔记本文件名] 笔记本文件名' under the third argument of Get. The code is wrapped in a comment '(* I=0 $\pi\pi\pi \omega$ *)'.

Operator construction

- **OpTion** is utilized to generate general N -hadron operators with arbitrary momenta



The screenshot shows a Mathematica notebook window titled "omega3.nb" with a 100% zoom level. The interface includes a standard toolbar with icons for file operations, input/output, and mathematical functions. The code in the notebook is as follows:

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(* ... [文件名连接] [目录名称] [目录名称] [笔记本文件名] *)

OpTion: package for Operator construction in lattice QCD.

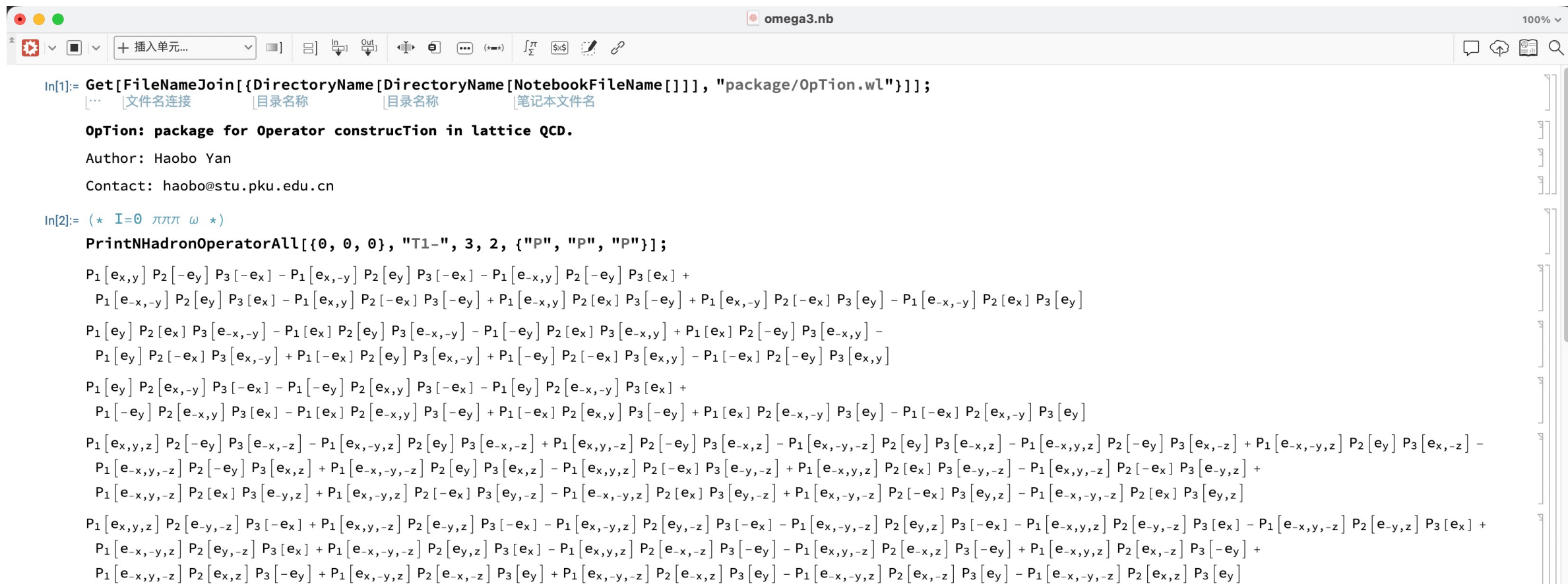
Author: Haobo Yan
Contact: haobo@stu.pku.edu.cn

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Operator construction

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P1[ex,y] P2[-ey] P3[-ex] - P1[ex,-y] P2[ey] P3[-ex] - P1[-ex,y] P2[-ey] P3[ex] +
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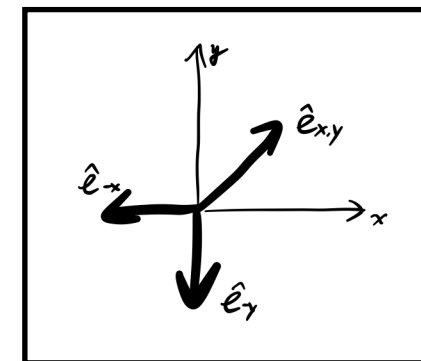
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```



Operator construction

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See also software by **John Bulava** (Monday 14:40)

See improvement on distillation by **Juan Urrea-Niño** (Monday 14:00)

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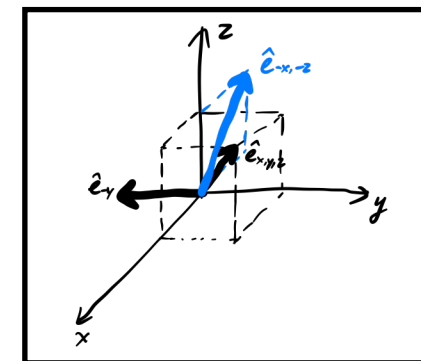
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P1[ey] P2[ex] P3[e-x,-y] - P1[ex] P2[ey] P3[e-x,-y] - P1[-ey] P2[ex] P3[e-x,y] + P1[ex] P2[-ey] P3[e-x,y] -
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```



➤ Isospin projection

$$3 \otimes 3 \otimes 3 = (1 \oplus 3 \oplus 5) \otimes 3 = 1 \oplus 3^3 \oplus 5^2 \oplus 7.$$

$$\left\{ \begin{array}{lcl} (\omega)_{0,0} & = & \frac{1}{\sqrt{2}}(\omega^u + \omega^d), \\ (\phi)_{0,0} & = & \phi, \\ (\rho\pi)_{0,0} & = & -\frac{1}{\sqrt{3}} [\rho^+ \pi^- + \rho^- \pi^+ + \frac{1}{2}[\rho^u \pi^u - \rho^u \pi^d - \rho^d \pi^u + \rho^d \pi^d]], \\ (K\bar{K})_{0,0} & = & \frac{1}{\sqrt{2}} [K^+ K^- + K^0 \bar{K}^0], \\ (\pi\pi\pi)_{0,0} & = & \frac{1}{\sqrt{12}} [-\pi^+ \pi^u \pi^- + \pi^+ \pi^d \pi^- + \pi^u \pi^+ \pi^- - \pi^d \pi^+ \pi^- + \pi^+ \pi^- \pi^u - \pi^+ \pi^- \pi^d \\ & & - \pi^- \pi^+ \pi^u + \pi^- \pi^+ \pi^d - \pi^u \pi^- \pi^+ + \pi^d \pi^- \pi^+ + \pi^- \pi^u \pi^+ - \pi^- \pi^d \pi^+]. \end{array} \right.$$

➤ $\rho\pi$ operators up to 2 mom, $K\bar{K}$ operators up to 4 mom, and one $\pi\pi\pi$ operator

$I = 3$: [HadSpec] Maxwell Hansen+, PRL 126, 012001 (2021), Maxim Mai and Micheal Döring, PRL 122, 062503 (2019)...

$I = 2$: [HadSpec] Raúl Briceño+, arXiv: 2510.24894, PRL (2026), Feng+, arXiv:2601.16916, PRD (2026)

$I = 1 \pi(1300)$: [agemo] Haobo Yan+, PRL136 (2026) 14, 141901

$I = 1 a_1(1260)$: [GWQCD] Maxim Mai+, PRL 127, 222001 (2021)

$I = 0 \omega(782)$: [agemo] Haobo Yan+, PRL133, 211906 (2024)

Related talks:

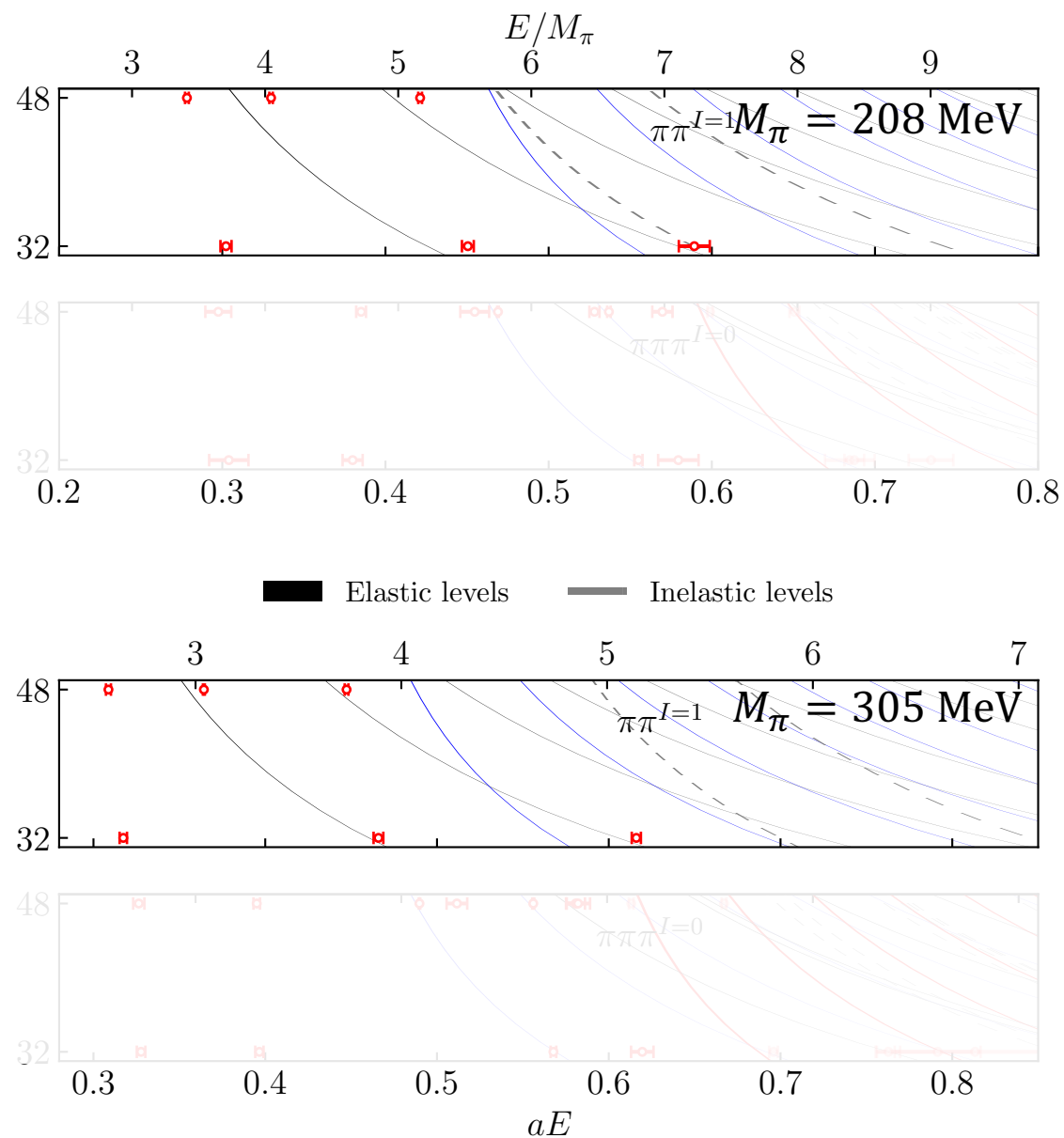
Maxim Mai (Thursday 11:50)

Andrew Jackura (Thursday 14:20)

Max Hansen (Tuesday 16:50)

Juan Jimenez (Tuesday 14:40)

Finite-volume spectra and fit



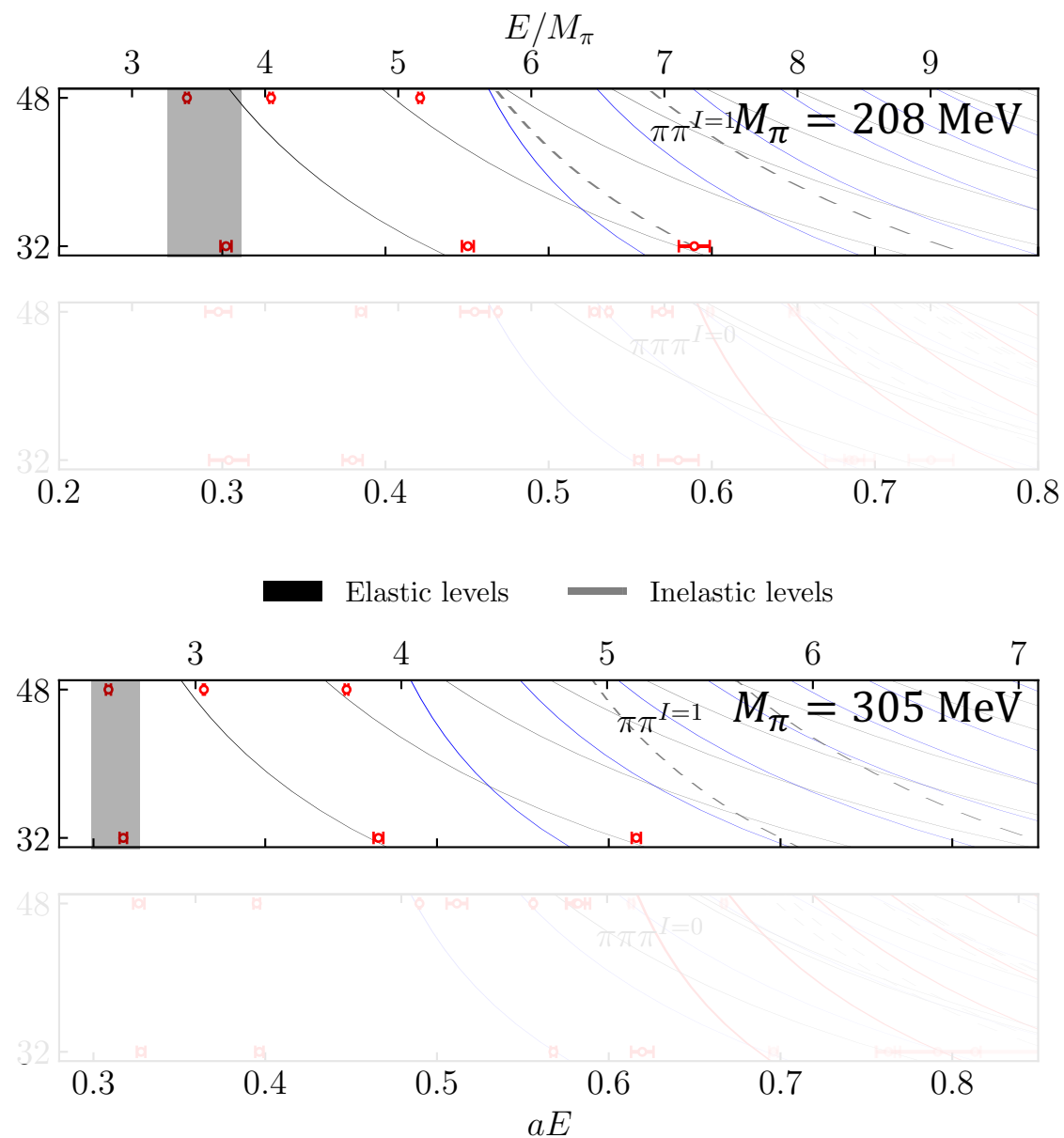
➤ CLQCD $N_f = 2 + 1$ Wilson-Clover ensembles

➤ $M_\pi = 305$ and 208 MeV and two volumes each

[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

➤ $I = 1$ $\rho \rightarrow \pi\pi$ levels as expected

Finite-volume spectra and fit



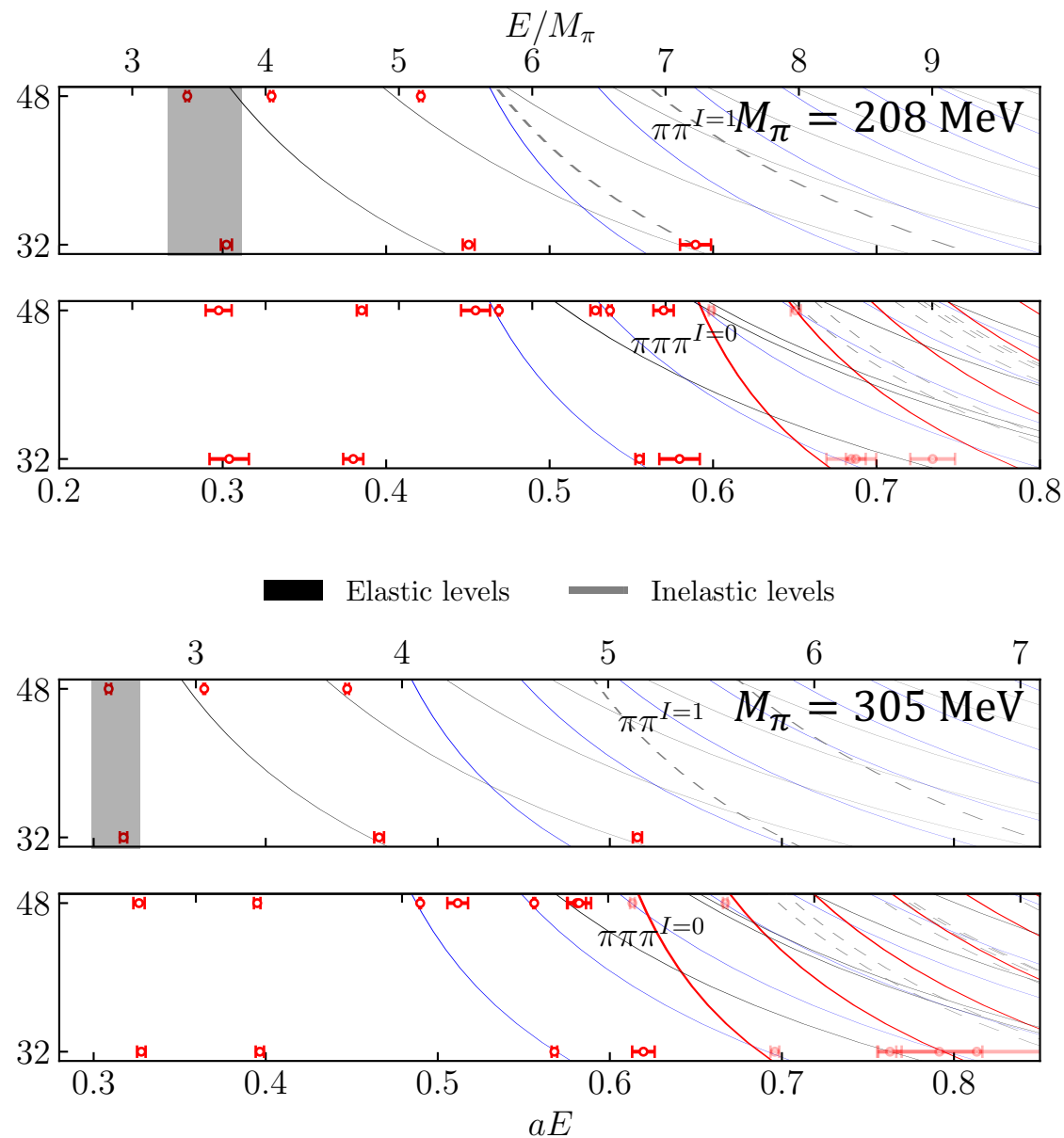
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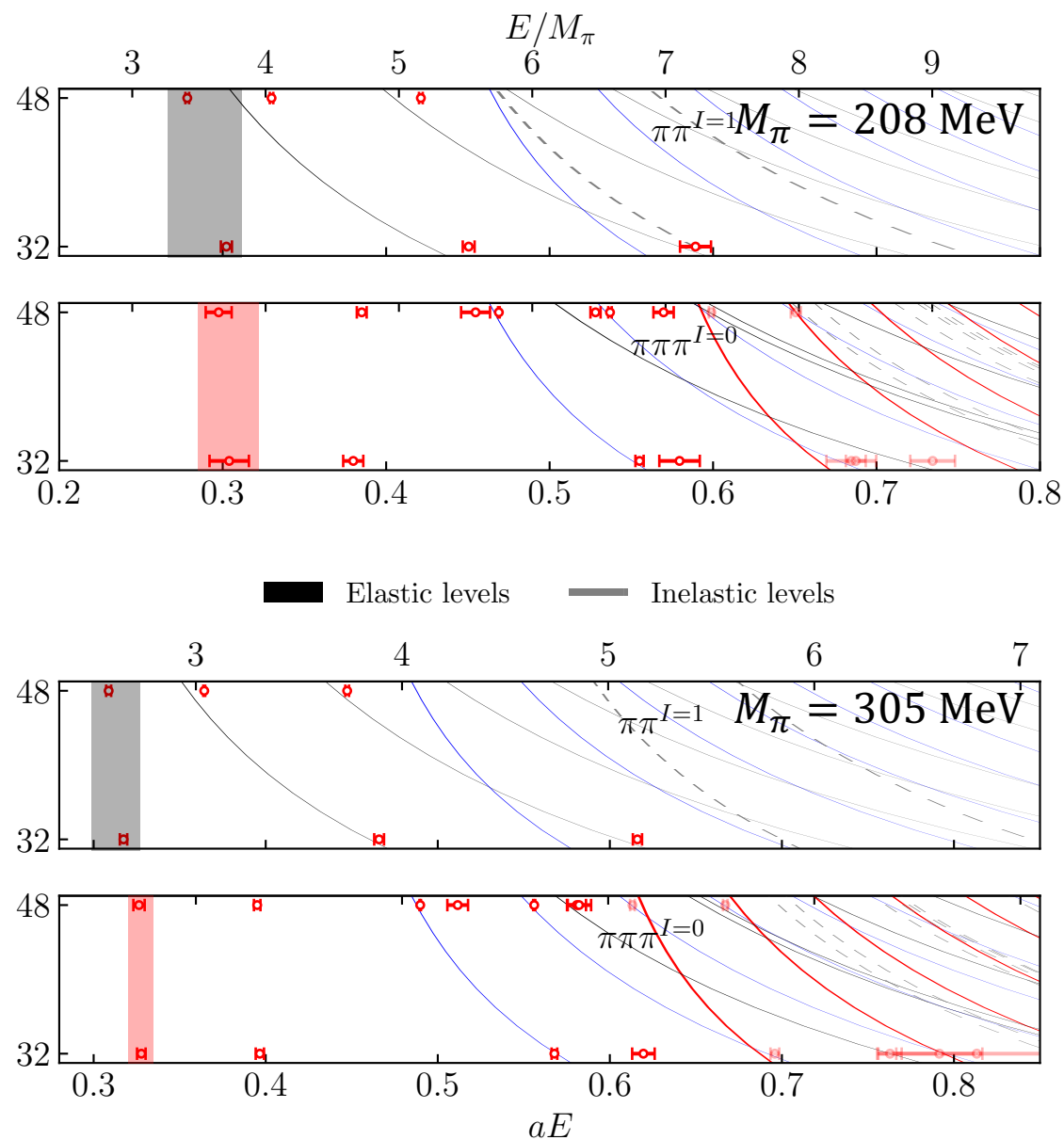


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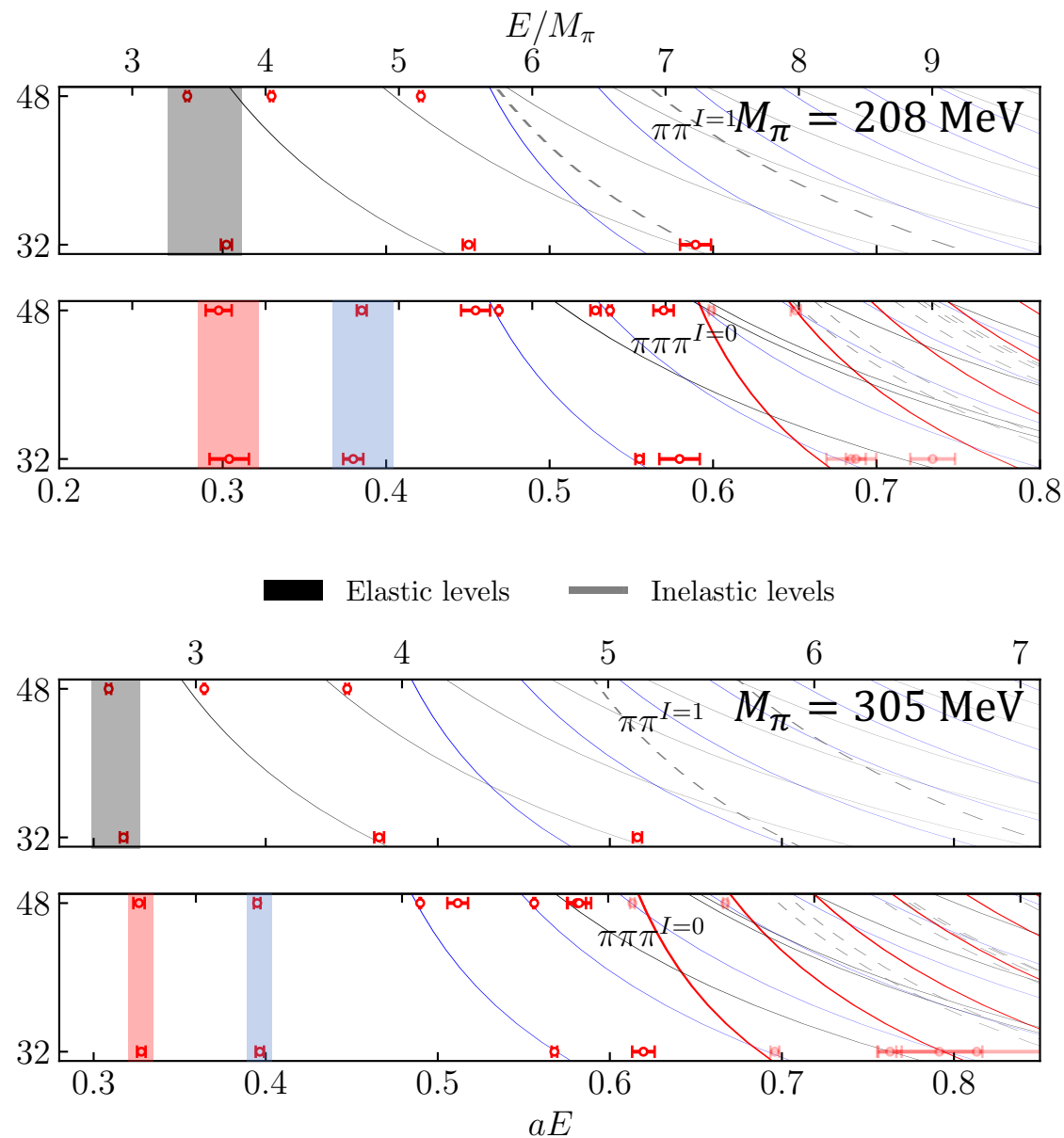
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Finite-volume spectra and fit



➤ CLQCD $N_f = 2 + 1$ Wilson-Clover ensembles

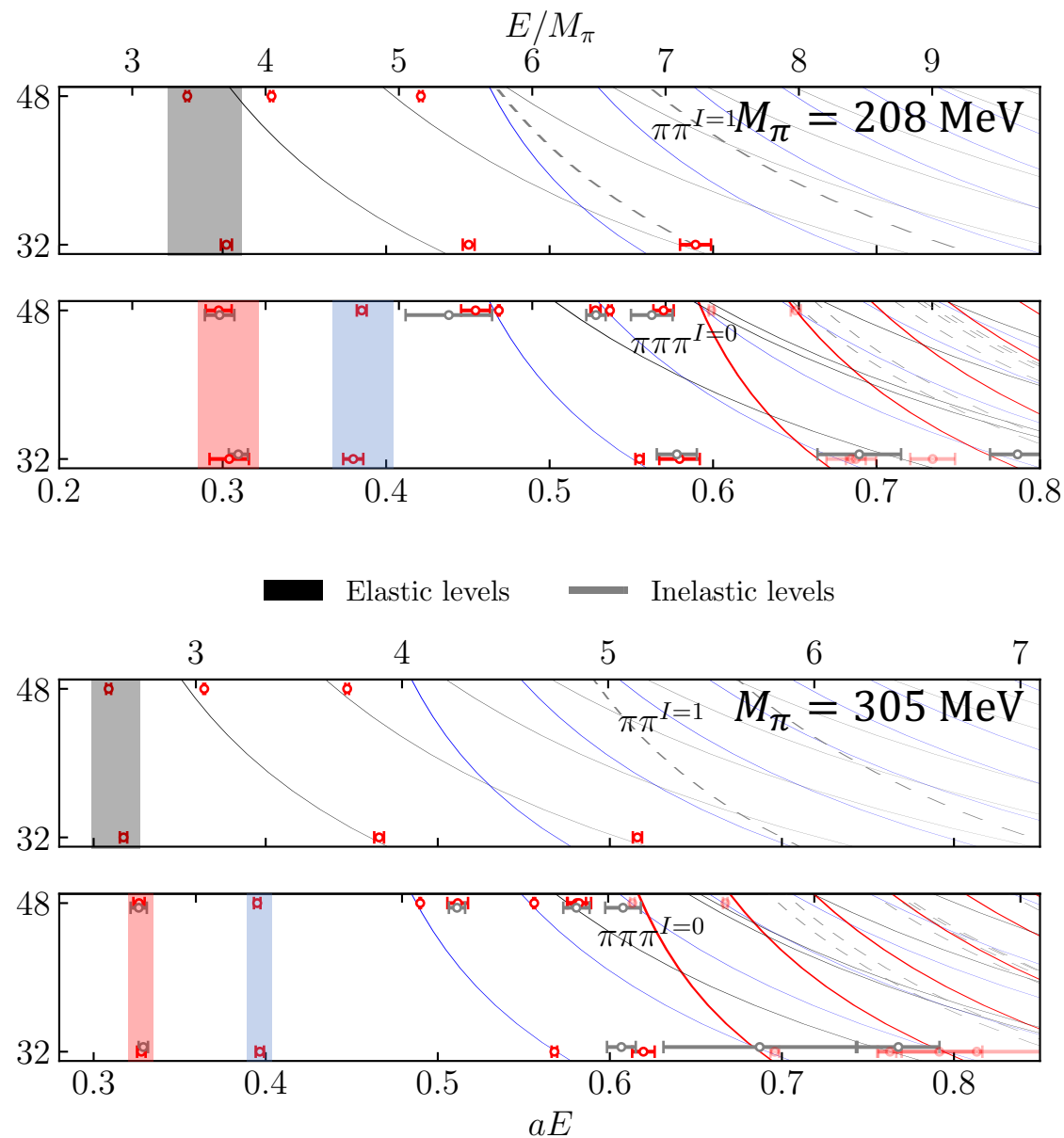
➤ $M_\pi = 305$ and 208 MeV and two volumes each

[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

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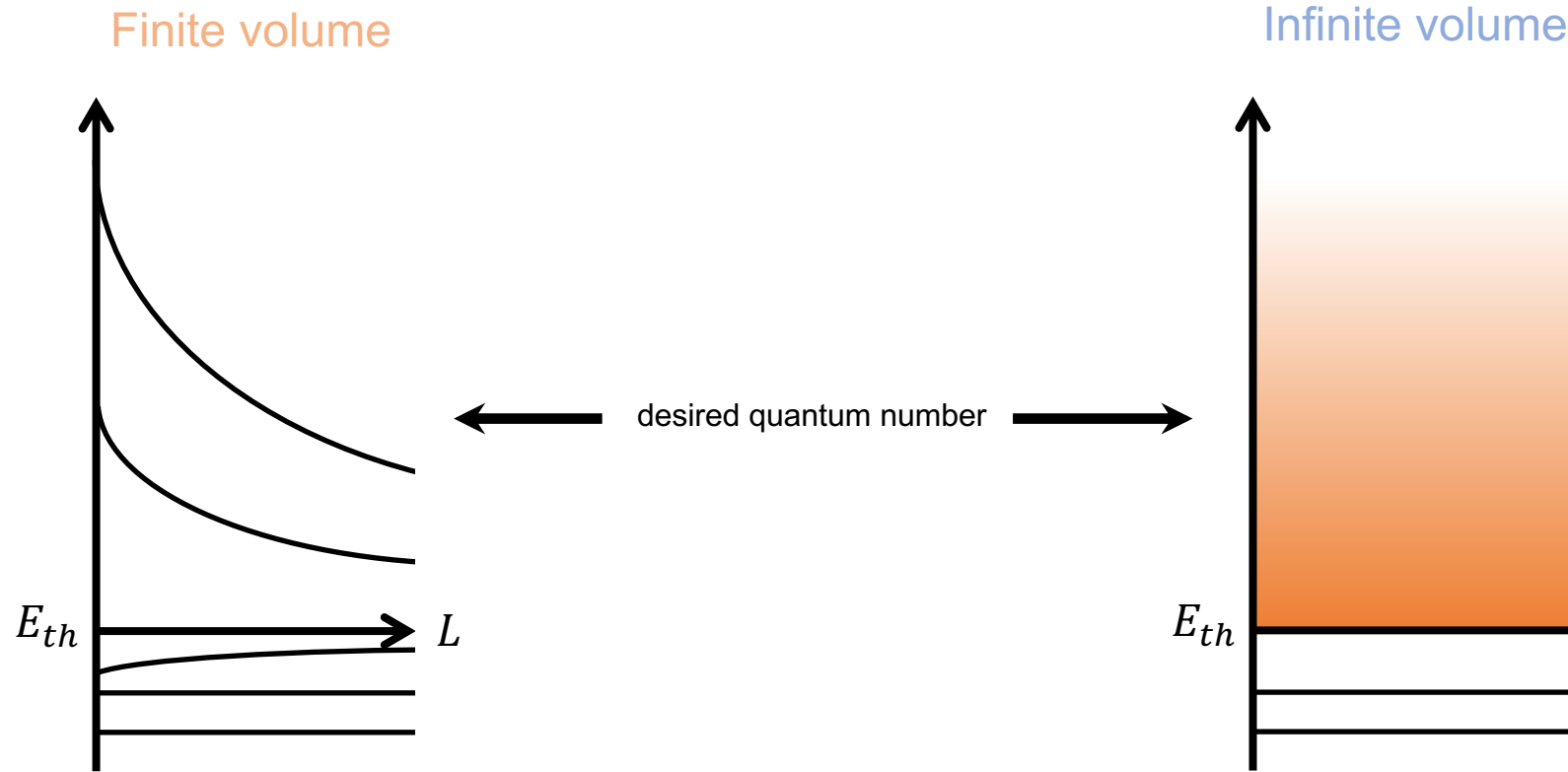
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[CLQCD] Zhicheng Hu+ PRD 109, 054507 (2024)

➤ $I = 1 \rho \rightarrow \pi\pi$ levels as expected

➤ Clear indication of ω and ϕ

➤ Additional levels compared to light-only operators



- To extract the **infinite-volume** information from **finite-volume** spectra

$$\text{Quantization condition: } f(\mathcal{M}(E_n), \mathcal{F}(E_n, L)) = 0$$

- Quantization condition accounts for the polynomial finite-volume effects

Quantization condition from unitarity

- In FVU (**F**inite-**V**olume **U**nitariness): Unitarity $\Rightarrow$ integral equation of the three-body amplitude

$$T^\infty = \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots \right) \xrightarrow{\int \rightarrow \sum} T^L = \infty \longleftrightarrow E_n$$

Maxim Mai and Micheal Döring, PRL 122, 062503 (2019)

- This work: extend FVU to the two-three-body coupled systems

$$\left\langle \chi_\Gamma \left| \begin{pmatrix} \tau_\rho \cdot E_{L,\rho} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} \tau_\rho & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{1 - (B + C)\hat{\tau}} (B + C) \begin{pmatrix} \tau_\rho & 0 \\ 0 & 1 \end{pmatrix} \right| \chi_\Gamma \right\rangle = \infty$$

See a more complete introduction to FVU by **Maxim Mai** (Thursday 11:50)

See FVU extension to two-body system with left-hand cut by **Ivan Vujmilovic** (Friday 14:00)

See extension to the Roper channel N(1440) by **Jinzi Wu** on Friday at Fri(1440)

State-of-the-art formalisms

- FVU [Mai and Döring, 2017]
- RFT [Hansen and Sharpe, 2014]
- NREFT [Hammer, Pang, and Rusetsky, 2017]

See Mai PRL (2019) for quantitative tests of smooth and hard cutoffs

Quantization condition from unitarity

$$T^\infty = \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots \right) \xrightarrow{\int \rightarrow \sum} T^L = \infty \longleftrightarrow E_n$$

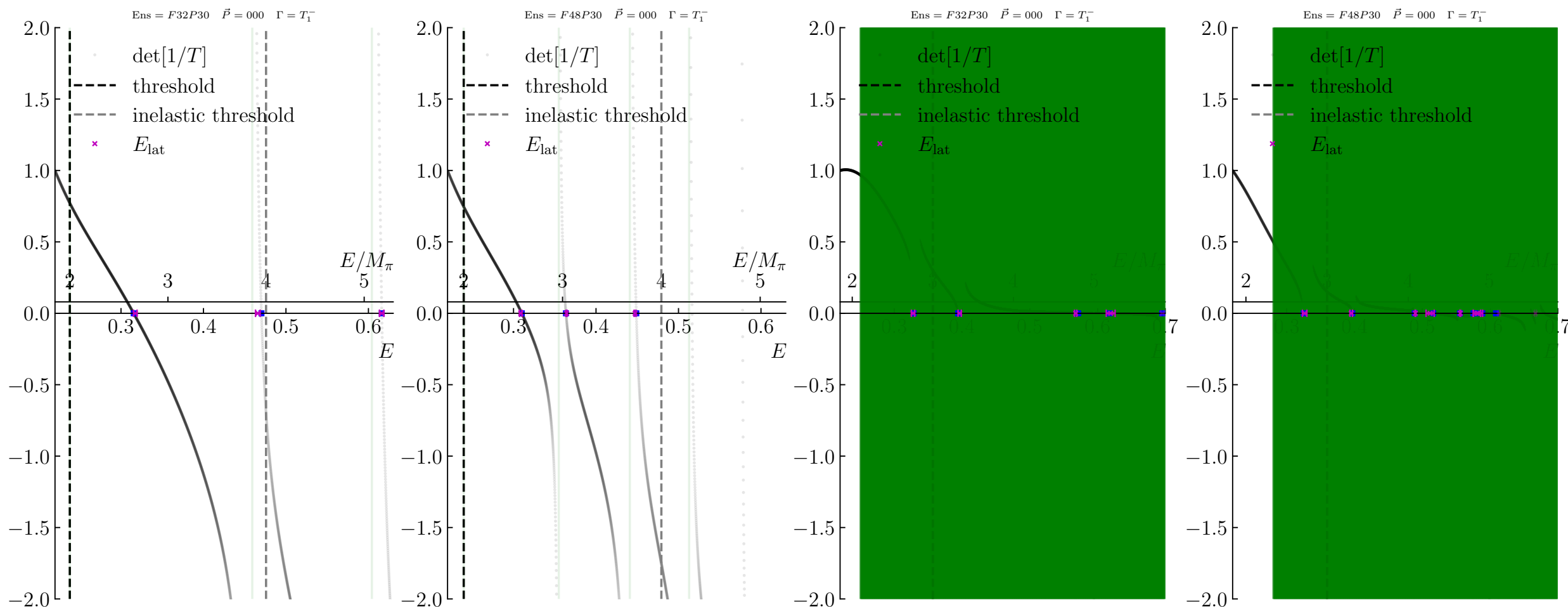
➤ In the angular mom basis, three-body $C = \begin{bmatrix} p' \frac{c_{11}^\omega}{s-M_\omega^2} p & p' i c_{12}^0 p \\ p' i c_{12}^0 p & p' \frac{c_{22}^\phi}{s-M_\phi^2} p \end{bmatrix}$

two-body $\begin{cases} \tilde{K}_\rho^{-1} = a_0 + a_1 \sigma \\ \tilde{K}_K^{-1} = \frac{s-4E_K^2}{2} \left(\frac{4E_K^2}{s} \right) \end{cases}$

➤ Spectator mom cutoff = $[1, 1, 1]$

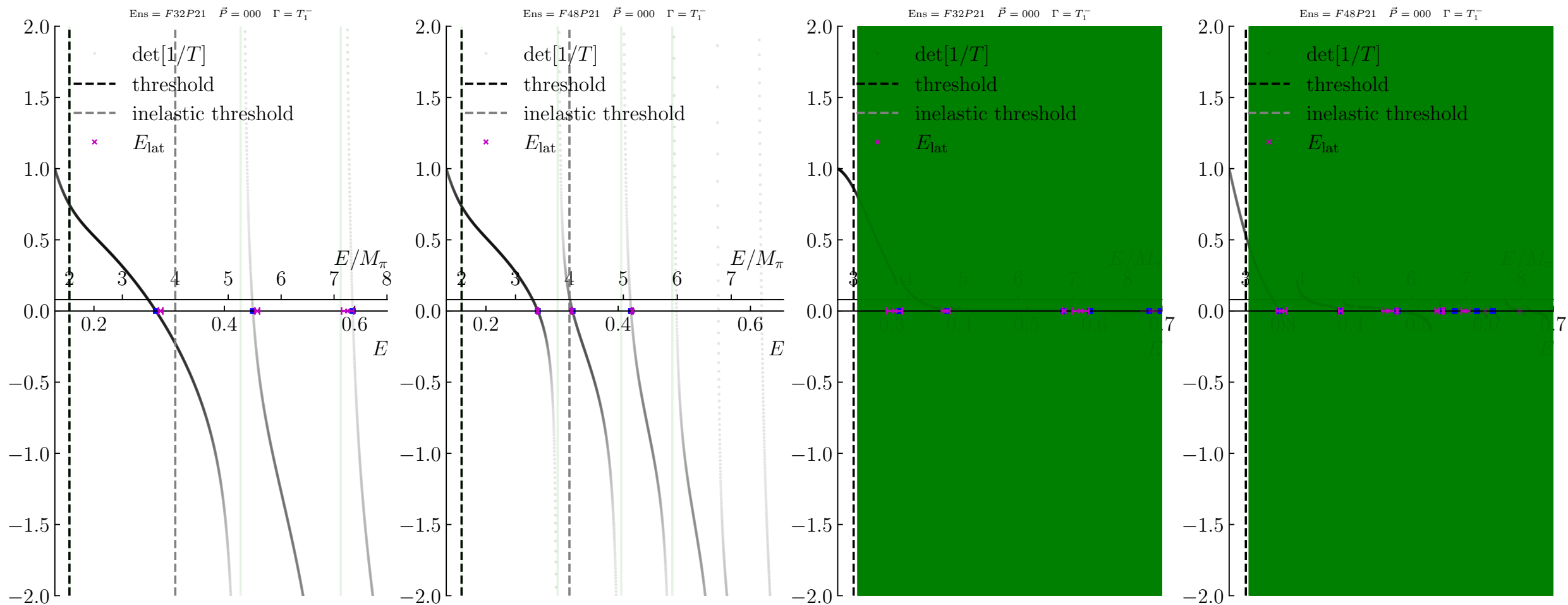
Quantization condition from unitarity

➤ $\chi^2/d.o.f. \approx 1.6$



Quantization condition from unitarity

➤ $\chi^2/d.o.f. \approx 1.9$



Summary

- Including strangeness produces more levels, and they are low in the spectra
- Extension of FVU to coupled 2-3 channels
- We are working on extracting ω and ϕ simultaneously from coupled-channel scattering
- Radial excitations of kaons
- Finally Roper in the long future
- Matrix element with three-body hadronic state
- I made my thesis available on 2 weeks ago: [arXiv:2607.12508](https://arxiv.org/abs/2607.12508)

Thank you!