F_K/F_π and F_{D_s}/F_D from four-flavor Möbius Domain Wall Fermions

Zack B. Hall II *on behalf of : COSMON Collaboration*
NSF MPS - ASCEND Postdoctoral Fellow
Nuclear Science Division
Lawrence Berkeley National Laboratory

LATTICE 2026
JULY 28,2026

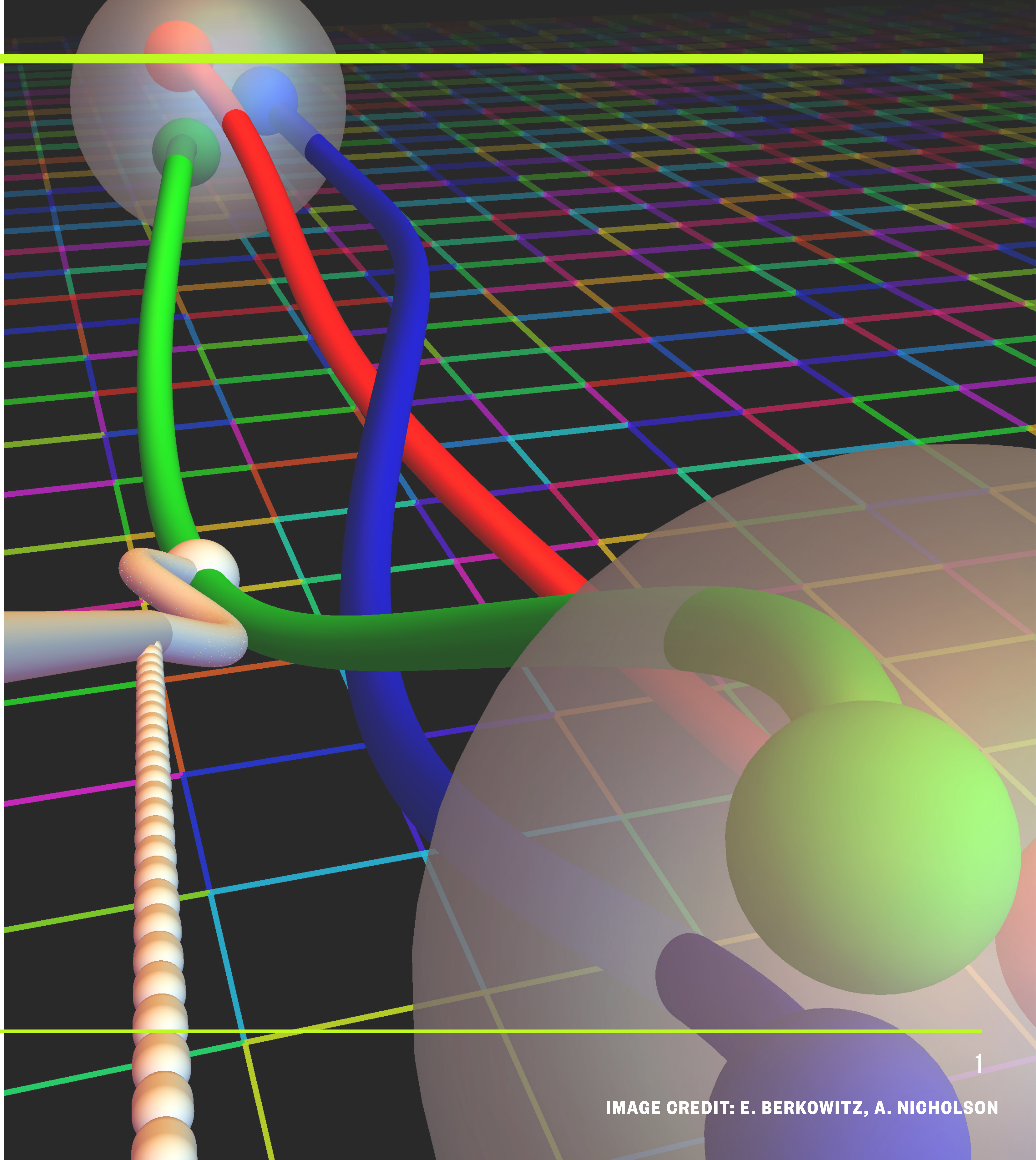


IMAGE CREDIT: E. BERKOWITZ, A. NICHOLSON

Low-Energy Precision Physics

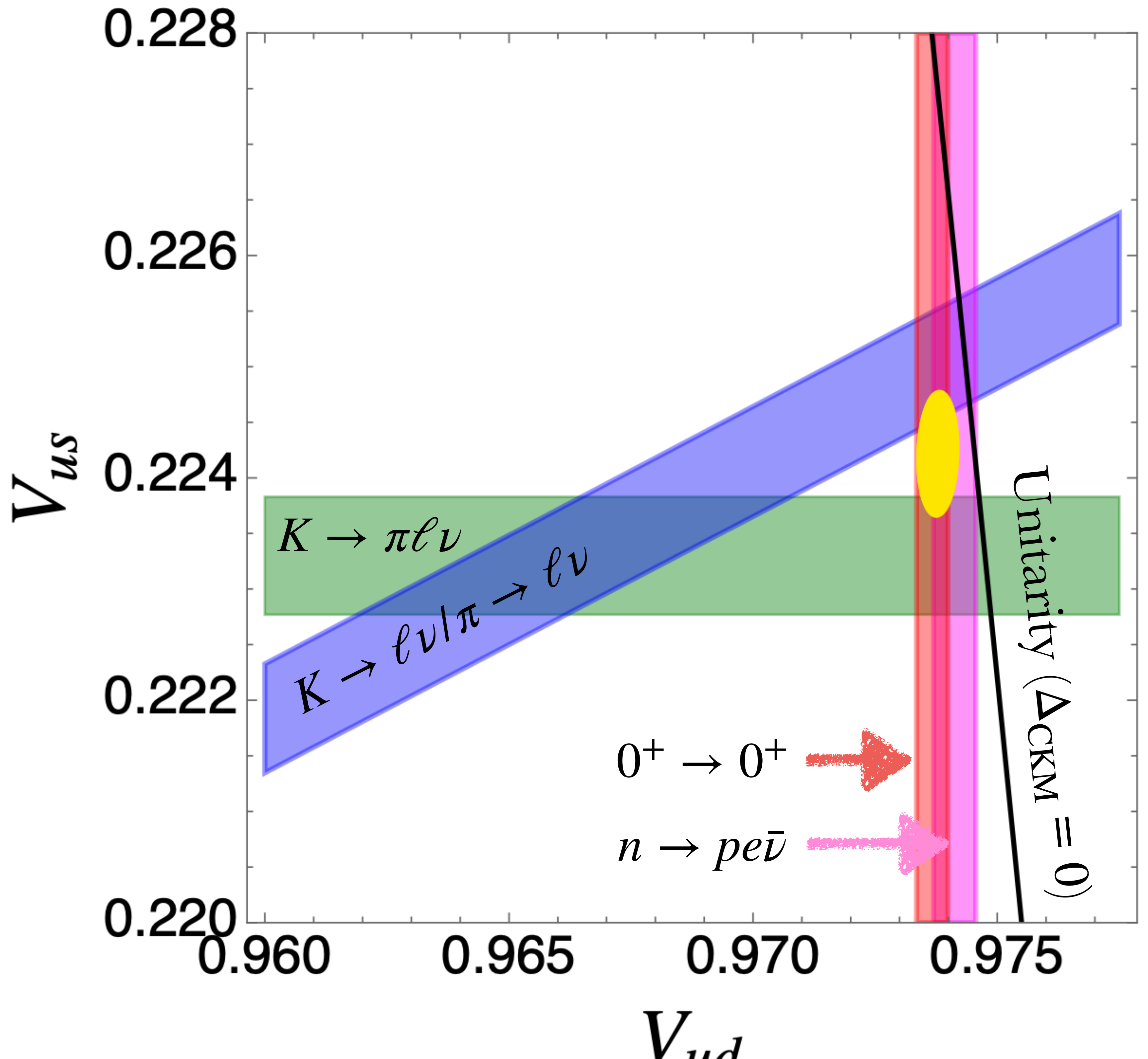
- The top-row Cabbibo-Kabayashi-Maskawa (CKM) matrix unitarity test is one of the most sensitive probes of BSM physics in the low-energy regime.

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{Weak}} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{QCD}$$

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} - 1 = -0.00176(56)$$

$$|V_{ub}|^2 \sim O(10^{-5})$$

- 3σ deficit away from unitarity



- $K_{\ell 2}/\pi_{\ell 2} \sim V_{us}/V_{ud}$, can be calculated with exp and theory
- $K_{\ell 3} \sim V_{us}$, determined from exp & th, full theory underway
- $0^+ \to 0^+ \sim V_{ud}$, most precise but inhibited by NS corrections
- $n \to pe\nu \sim V_{ud}$, becoming a competitive approach

$$V_{us}/V_{ud} = 0.23108(23)_{\text{exp}}(42)_{F_K/F_\pi}(16)_{\text{IB}}[51]_{\text{total}}$$

$$V_{us} = 0.22330(35)_{\text{exp}}(39)_{f_+}(8)_{\text{IB}}[53]_{\text{total}}$$

$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

$$V_{ud}^{n,\text{PDG}} = 0.97441(3)_f(13)_{\Delta_V^R}(82)_\lambda(28)_{\tau_n}[88]_{\text{total}}$$

Phys. Lett. B 838 (2023) 137748, arXiv:2208.11707

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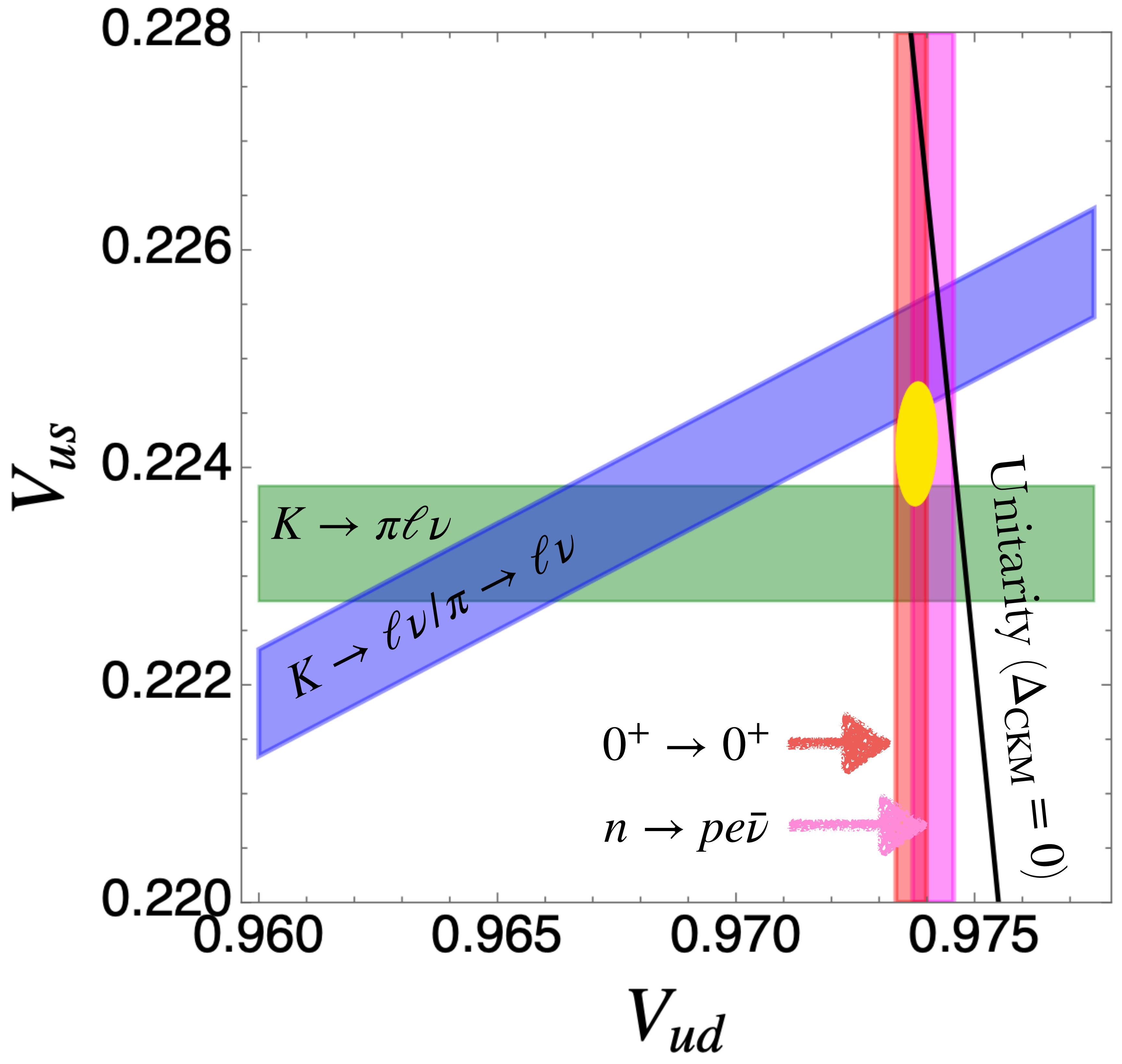

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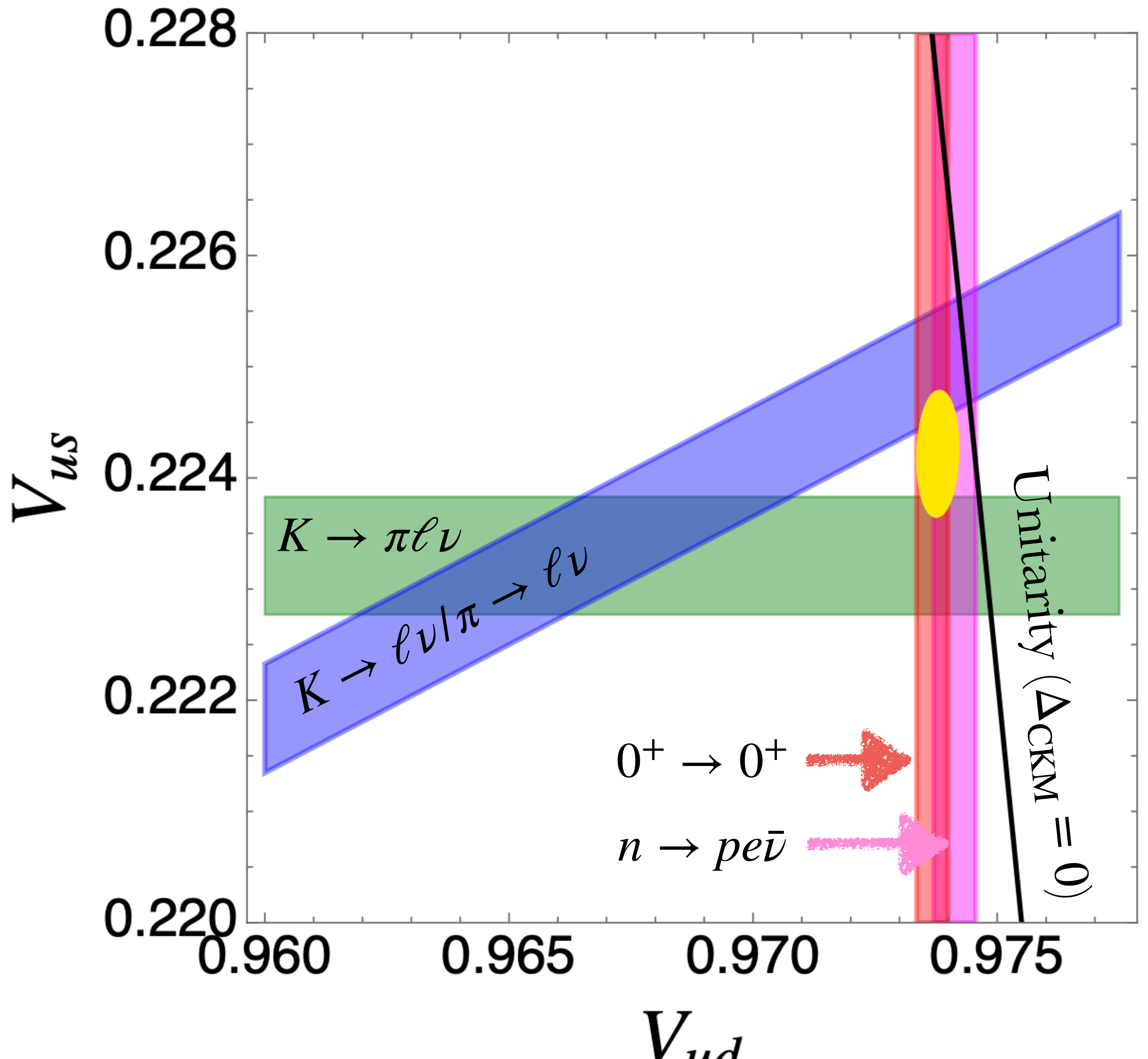

$$\frac{\Gamma\left(K^+ \to \mu\nu[\gamma]\right)}{\Gamma\left(\pi^+ \to \mu\nu[\gamma]\right)} = \frac{|V_{\text{us}}|^2 F_{K^+}^2}{|V_{\text{ud}}|^2 F_{\pi^+}^2} \frac{m_K(1 - m_\mu^2/m_K^2)}{m_\pi(1 - m_\mu^2/m_\pi^2)} \left(1 + \delta_{K/\pi}\right)$$

$$\frac{|V_{us}| F_{K^+}}{|V_{ud}| F_{\pi^+}} = 0.27599(41)$$

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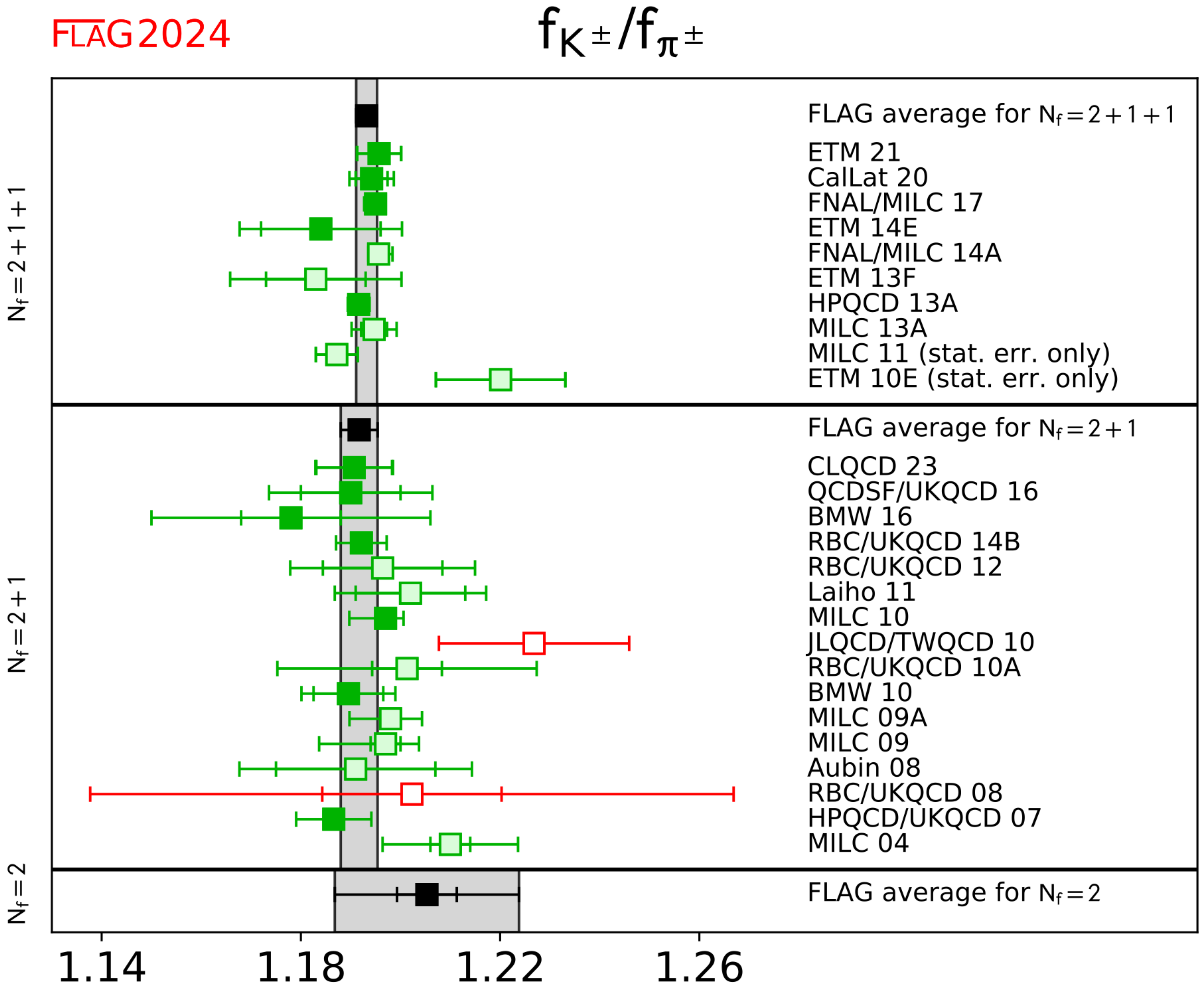
F_K/F_π from Lattice QCD



- **ETM 21**
  Wilson-Clover TwM, three spacings: $a \approx 0.069 - 0.094$ fm,
  two phys: $m_\pi \approx 134 - 346$ MeV $\rightarrow 1.1957(44)(7)$

- **CalLat 20**:
  MDWF on HISQ, four spacings $a \approx 0.06 - 0.15$ fm,
  three phys: $m_\pi \approx 130 - 400$ MeV $\rightarrow 1.1942(32)(31)$

- **FNAL/MILC 17**:
  HISQ, six spacings: $a \approx 0.03 - 0.15$ fm,
  five phys: $m_\pi \approx 129 - 337$ MeV $\rightarrow 1.1950(15)(^{+6}_{-18})$

- **HPQCD 13A**:
  HISQ, three spacings: $a \approx 0.09 - 0.15$ fm,
  three phys: $m_\pi \approx 128 - 306$ MeV $\rightarrow 1.1916(15)(16)$

$$N_f = 2+1+1 \quad F_{K^\pm}/F_{\pi^\pm} = 1.1934(19)$$

- Results from shared HISQ ensembles treated as 100% correlated.

FLAG2024: 2411.04268

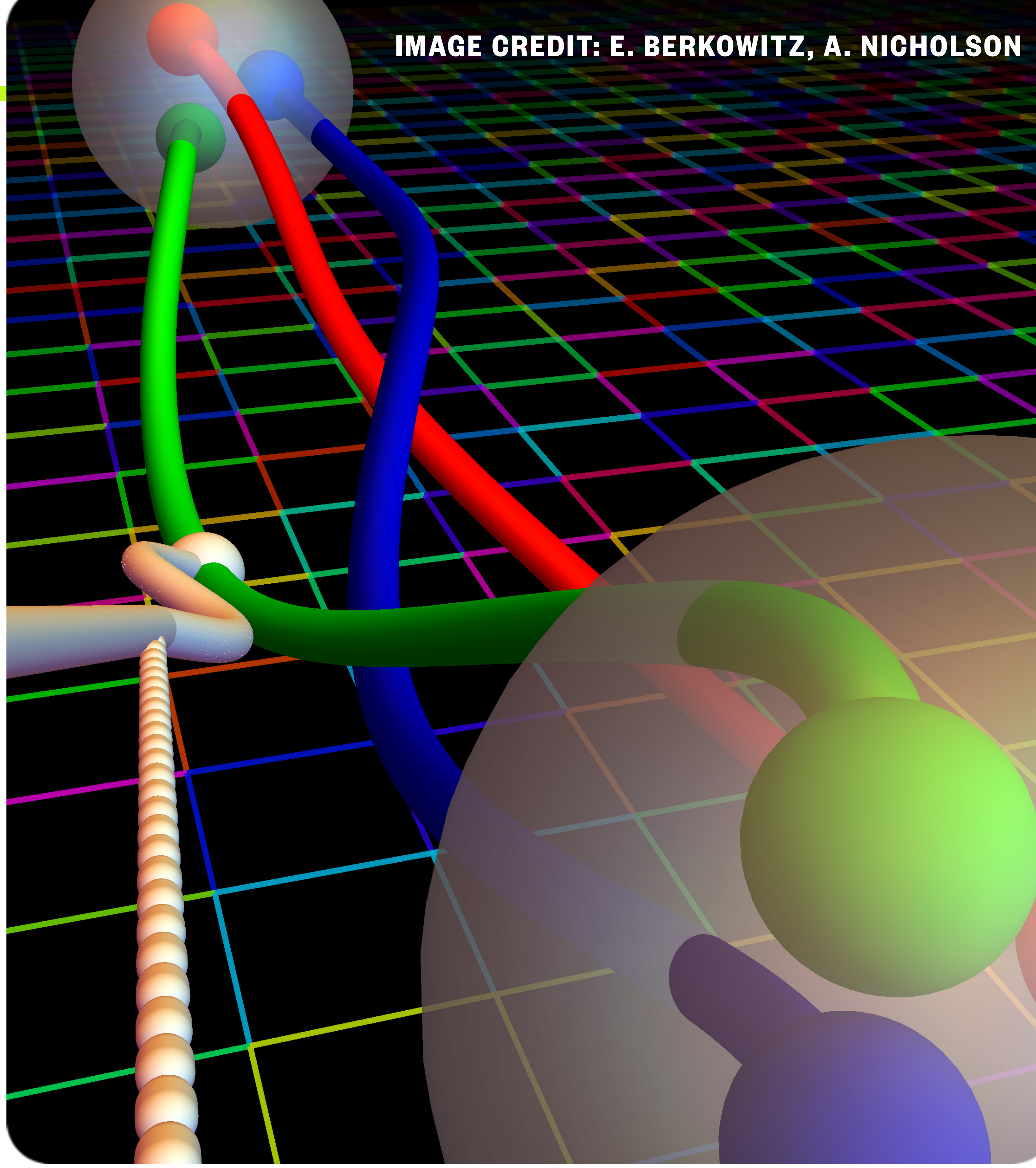
Lattice QCD Setup

- We employ a the Möbius Domain Wall lattice action to describe $N_f = 2 + 1 + 1$ valence and sea quarks.
  - Preserves chiral symmetry (obeys $\{\gamma_5, D\} = aD\gamma_5 D$)
  - Executed with modest computational cost ($\mathcal{O}(100)$ cfgs/ens)
- Form correlation functions from $O_P = \overline{\psi}\gamma_5\psi$, $O_A = \overline{\psi}\gamma_\mu\gamma_5\psi$

$$C_{PP}(t) = (Z_P^0)^2\left(\mathrm{e}^{-E_0 t} + \mathrm{e}^{-E_0(t-T)}\right) \qquad C_{AP}(t) = Z_A^0 Z_P^0\left(\mathrm{e}^{-E_0 t} - \mathrm{e}^{-E_0(t-T)}\right)$$

- Using parameters from simultaneous fits, we can determine the decay constants on each ensemble.

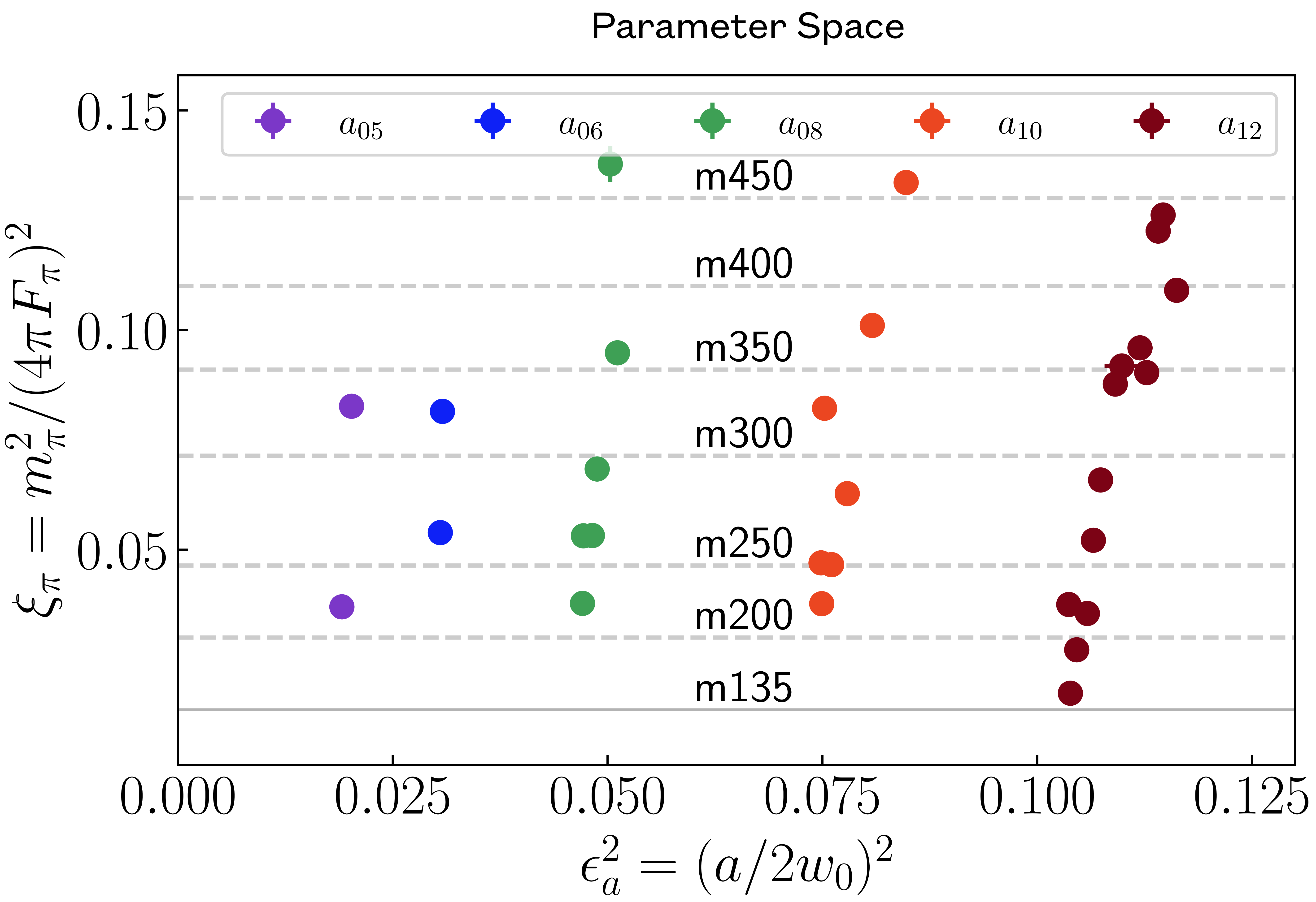
$$F_K = Z_A\sqrt{\frac{Z_{A,K}^0}{m_K}} \qquad F_\pi = Z_A\sqrt{\frac{Z_{A,\pi}^0}{m_\pi}}$$



- Large-scale computational resources are required.

NERSC at Lawrence Berkeley National Lab

Lattice QCD Ensembles



F_K/F_π calculated on 30 ensembles

$\xi_\pi = \frac{m_\pi^2}{(4\pi F_\pi)^2}$

$m_\pi L$

$\epsilon_a = \frac{a}{2w_0}$

Perform simultaneous chiral, infinite volume, continuum extrapolation

$\left(F_K/F_\pi\right)^{\text{phys}}$

This work: 2605.06560 [hep-lat]

Extrapolation Analysis

$$\xi_\pi = \frac{m_\pi^2}{(4\pi F_\pi)^2} \qquad m_\pi L \qquad \epsilon_a = \frac{a}{2w_0}$$

- We use a Bayesian framework to perform a nonlinear fitting strategy.
- Agnostic, data-driven approach to determine the model that best represents the data.
- Pion (Quark) mass dependence parameterized through Chiral Perturbation Theory (χPT) up to $O(\xi_\pi^2)$:

$$\frac{F_K}{F_\pi} = \frac{1 + \delta F_K^{\mathrm{NLO}} + \delta F_K^{\mathrm{N^2LO}} + \cdots}{1 + \delta F_\pi^{\mathrm{NLO}} + \delta F_\pi^{\mathrm{N^2LO}} + \cdots}$$

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$$\frac{F_K}{F_\pi} = 1 + \frac{5}{8}\ell_\pi - \frac{1}{4}\ell_k - \frac{3}{8}\ell_\eta + 4\bar{L}_5(\xi_K - \xi_\pi)$$
$$+ \xi_K^2 F_F\left(\frac{m_\pi^2}{m_K^2}\right) + \hat{K}_1^r\lambda_\pi^2 + \hat{K}_2^r\lambda_\pi\lambda_K$$
$$+ \hat{K}_3^r\lambda_\pi\lambda_\eta + \hat{K}_4^r\lambda_K^2 + \hat{K}_5^r\lambda_K\lambda_\eta + \hat{K}_6^r\lambda_\eta^2$$
$$+ \hat{C}_1^r\lambda_\pi + \hat{C}_2^r\lambda_K + \hat{C}_3^r\lambda_\eta + \hat{C}_4^r\,.$$

$$\frac{F_K}{F_\pi} = \frac{1 + \delta F_K^{\mathrm{NLO}}}{1 + \delta F_\pi^{\mathrm{NLO}}} + \delta F_K^{\mathrm{N^2LO}} - \delta F_\pi^{\mathrm{N^2LO}} + \cdots$$

$$\xi_\pi = \frac{m_\pi^2}{(4\pi F_\pi)^2} \qquad m_\pi L \qquad \epsilon_a = \frac{a}{2w_0}$$

- NLO FV corrections:
  - $$\delta_{\mathrm{FV}}^{\mathrm{NLO}} = 4\xi \sum_{\vec{n}\neq 0} \frac{K_1(m_\pi L|\vec{n}|)}{m_\pi L|\vec{n}|}$$
- Discretization models:
  - $$\delta_a^{\mathrm{N^2LO}} = A_s\epsilon_a^2\left(\xi_K - \xi_\pi\right)$$
  - $$\delta_a^{\mathrm{N^3LO}} = A_{sK}\xi_K\epsilon_a^2\left(\xi_K - \xi_\pi\right) + A_{s\pi}\xi_\pi\epsilon_a^2\left(\xi_K - \xi_\pi\right)$$

https://github.com/gplepage/lsqfit

Phys. Rev. D 97, 091502 (2018) [1711.11328v2]

Physical Point

model	expansion form	include $\ln(m_\pi)$ @N^{2}LO?	include N^{3}LO $\epsilon_a^4(\xi_K - \xi_\pi)$?	include N^{3}LO $\epsilon_a^2(\xi_K - \xi_\pi)\xi_P$?	chi2/dof	Q	logGBF	weight	F_K/F_π
nnlo_ratio_ct	Eq. (B2)	✗	✗	✗	1.219	0.190	123.409	0.205	1.1989(17)
nnlo_ratio_ct_a4	Eq. (B2)	✗	✓	✗	1.201	0.207	123.393	0.202	1.2007(30)
nnnlo_ratio_ct_a4	Eq. (B2)	✗	✓	✓	1.167	0.242	123.049	0.143	1.1989(39)
nnnlo_ratio_ct	Eq. (B2)	✗	✗	✗	1.194	0.214	122.912	0.125	1.1971(33)
nnlo_ct_a4	Eq. (B1)	✗	✓	✗	1.207	0.201	122.701	0.101	1.1980(28)
nnlo_ct	Eq. (B1)	✗	✗	✗	1.235	0.176	122.572	0.089	1.1959(16)
nnnlo_ct_a4	Eq. (B1)	✗	✓	✗	1.175	0.234	122.306	0.068	1.1970(37)
nnnlo_ct	Eq. (B1)	✗	✗	✓	1.209	0.200	122.058	0.053	1.1950(32)
nnnlo_a4	Eq. (B1)	✓	✓	✗	1.270	0.147	119.476	0.004	1.2006(39)
nnnlo	Eq. (B1)	✓	✓	✓	1.300	0.126	119.278	0.003	1.1988(34)
nnnlo_ratio_a4	Eq. (B2)	✓	✓	✗	1.335	0.104	118.798	0.002	1.2002(39)
nnnlo_ratio	Eq. (B2)	✓	✓	✓	1.362	0.089	118.652	0.002	1.1985(34)
nnlo_a4	Eq. (B1)	✓	✓	✗	1.353	0.094	118.638	0.002	1.2029(32)
nnlo	Eq. (B1)	✓	✗	✗	1.375	0.083	118.561	0.002	1.2011(23)
nnlo_ratio_a4	Eq. (B2)	✓	✓	✗	1.444	0.055	117.446	0.001	1.2027(32)
nnlo_ratio	Eq. (B2)	✓	✗	✗	1.463	0.049	117.398	0.001	1.2010(23)
Bayes average									1.1984(29)(17)

This work: 2605.06560 [hep-lat]

Physical Point

- Chiral-continuum-IV fits for 16 models over all 30 ensembles.

$$\frac{F_K}{F_\pi} = 1.1984(23)^s(07)^\chi(17)^a(17)^M[34]^{\text{total}}$$

statistical	0.19%
chiral	0.06%
continuum	0.14%
model	0.14%
total	0.28%

$$\left(\frac{F_{K^\pm}}{F_{\pi^\pm}}\right)_{\text{FLAG}} = 1.1934(19)$$

Cirigliano et. al Phys. Lett. B 700, 7–10 (2011),arXiv:1102.0563

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CalLat Collaboration Phys. Rev. D 102, 034507 (2020), arXiv:2005.04795

Physical Point - Strong Isospin Breaking

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$$\frac{F_K}{F_\pi} = 1.1984(23)^s(07)^\chi(17)^a(17)^M[34]^{\text{total}}$$

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$$\left(\frac{F_{K^\pm}}{F_{\pi^\pm}}\right)_{\text{FLAG}} = 1.1934(19)$$

$$\delta F^{\text{iso}}_{K-\pi} = -\frac{1}{4}(\ell_{\hat{K}^+} - \ell_{\bar{K}}) + 4\bar{L}_5(\xi_{\hat{K}^+} - \xi_{\bar{K}}) + \frac{1}{4}(\xi_{K^0} - \xi_{K^\pm})\frac{\ell_\eta - \ell_{\pi^0}}{\xi_\eta - \xi_{\pi^0}}$$

$$\delta F^{\text{iso}'}_{K-\pi} = -\frac{1}{6}\frac{\xi_{K^0} - \xi_{K^\pm}}{\xi_\eta - \xi_{\pi^0}}\left[4\left(\frac{F_K}{F_\pi} - 1\right) + \xi_{\bar{\pi}}\ln\left(\frac{\xi_{\bar{K}}}{\xi_{\bar{\pi}}}\right) - \xi_{\bar{K}} + \xi_{\bar{\pi}}\right]$$

$$\ell_P = \xi_P\lambda_P = \frac{m_P^2}{(4\pi F_\pi)^2}\ln\frac{m_P^2}{\mu^2}$$

- $\delta F^{\text{iso}}_{K-\pi} \sim$ NLO SU(3) χPT, $\delta F^{\text{iso}'}_{K-\pi}$ replacing $\bar{L}_5$ with F_K/F_π in the isospin limit.
- Using the "Edinburgh Consensus":

$$0.5 \times \delta_{\text{SU}(2)} = -0.00216(54)$$

Cirigliano et. al Phys. Lett. B 700, 7–10 (2011),arXiv:1102.0563

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Extrapolation Results

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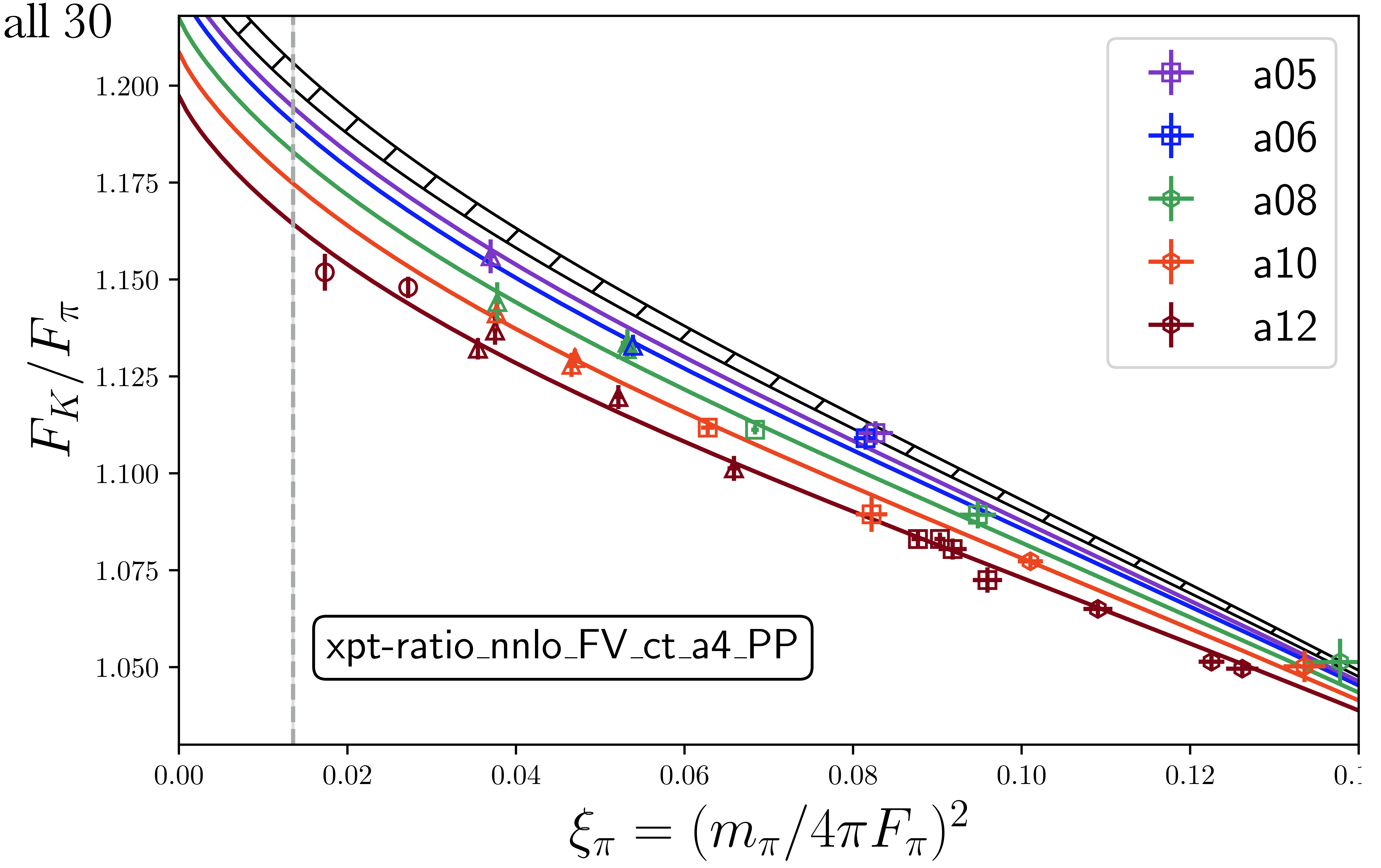

Extrapolation Results

$$\delta_a^{\mathrm{N^2LO}} = A_s \epsilon_a^2 \left(\xi_K - \xi_\pi\right)$$

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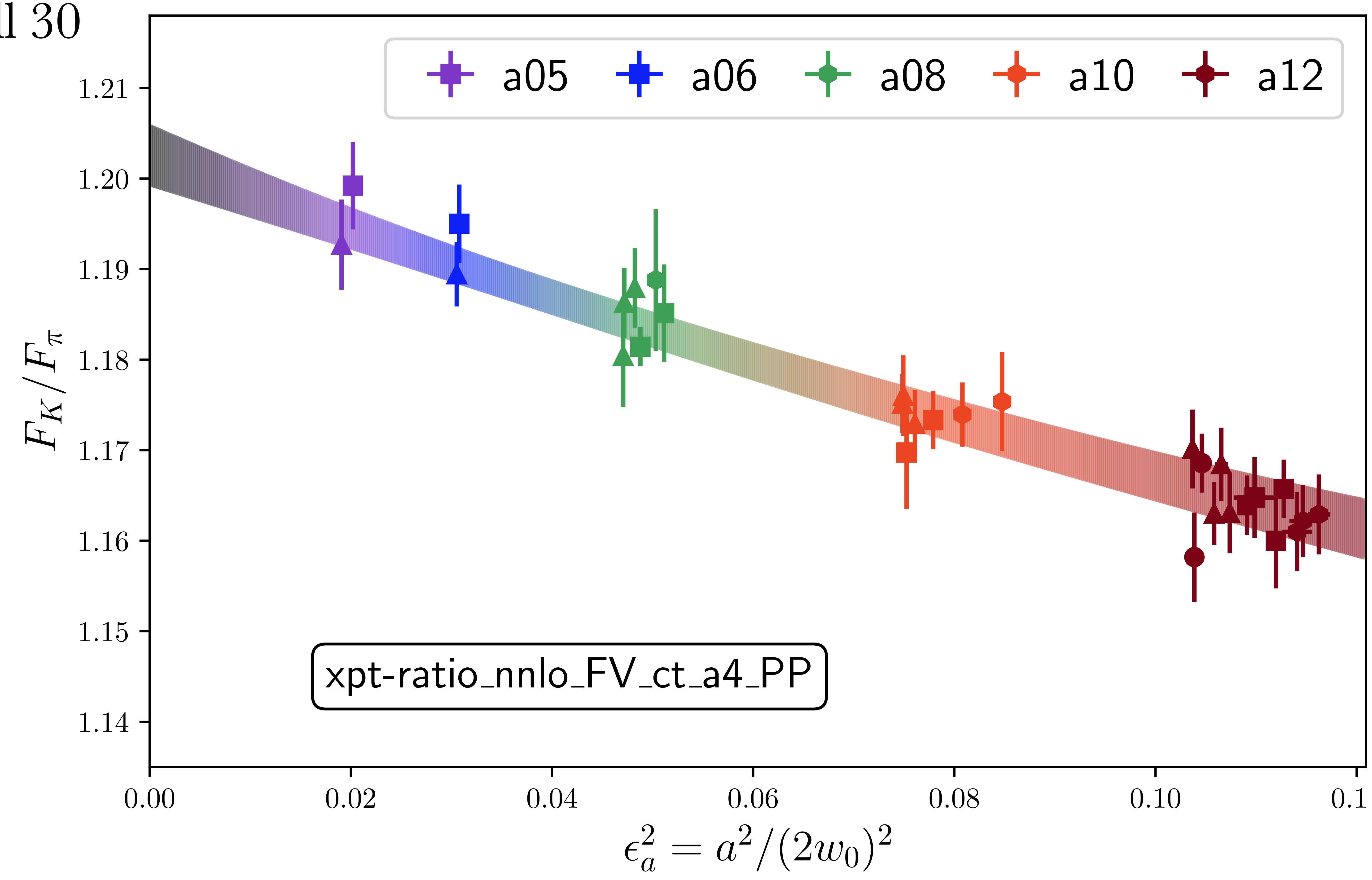

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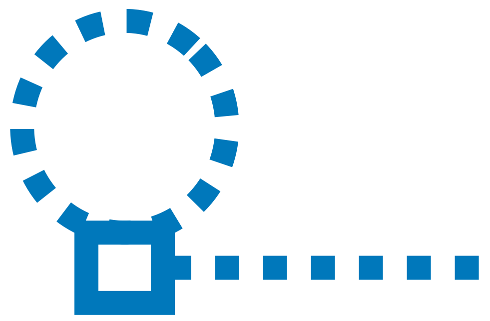

$$\rightarrow \frac{m_P^2}{(4\pi F_\pi)^2}\left[\ln\frac{m_P^2}{(4\pi F_\pi)^2} + 4\sum_{\vec{n}\neq 0}\frac{K_1(m_\pi L|\vec{n}|)}{m_\pi L|\vec{n}|}\right]$$

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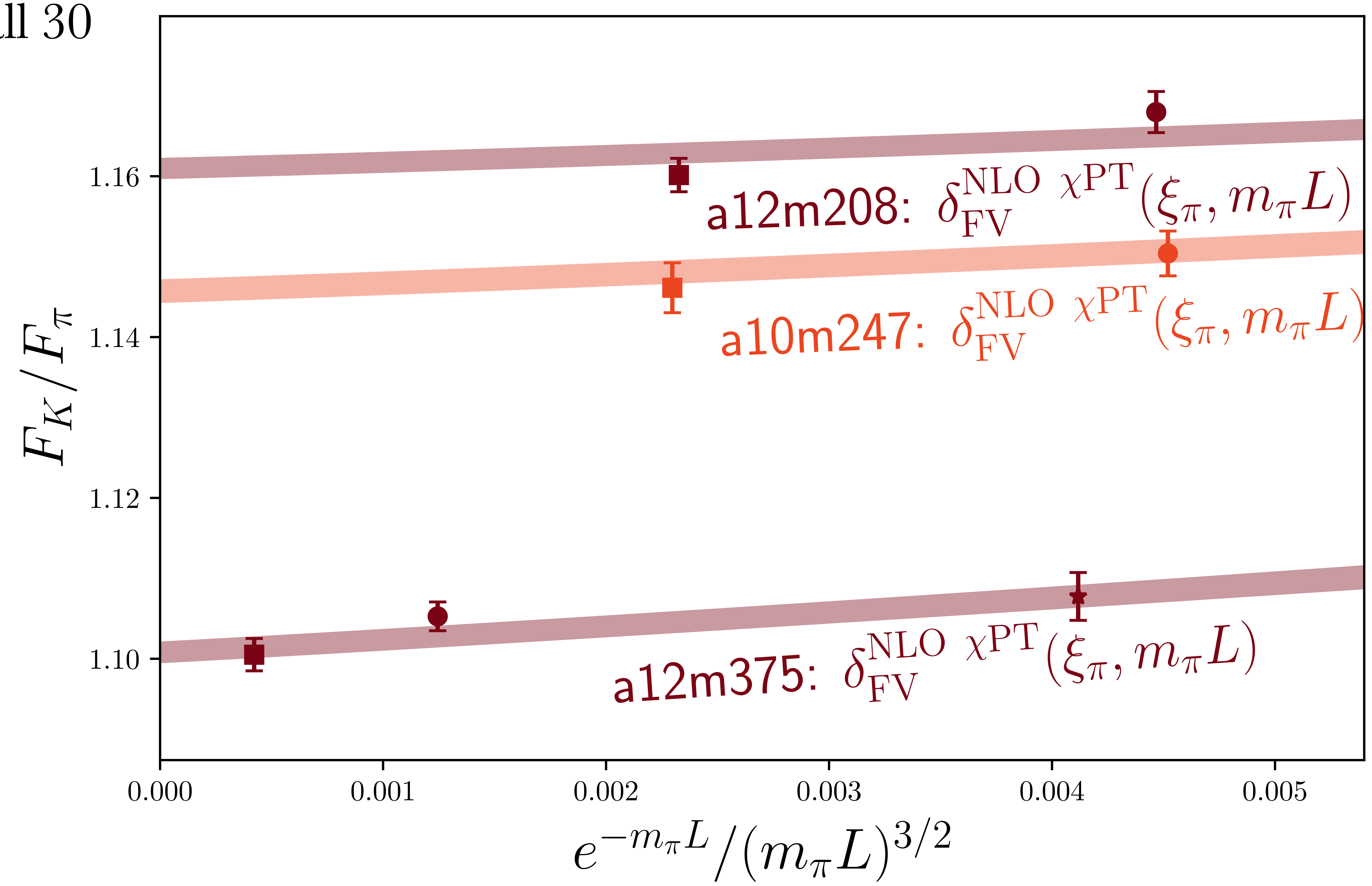

RE: Low-Energy Precision Physics

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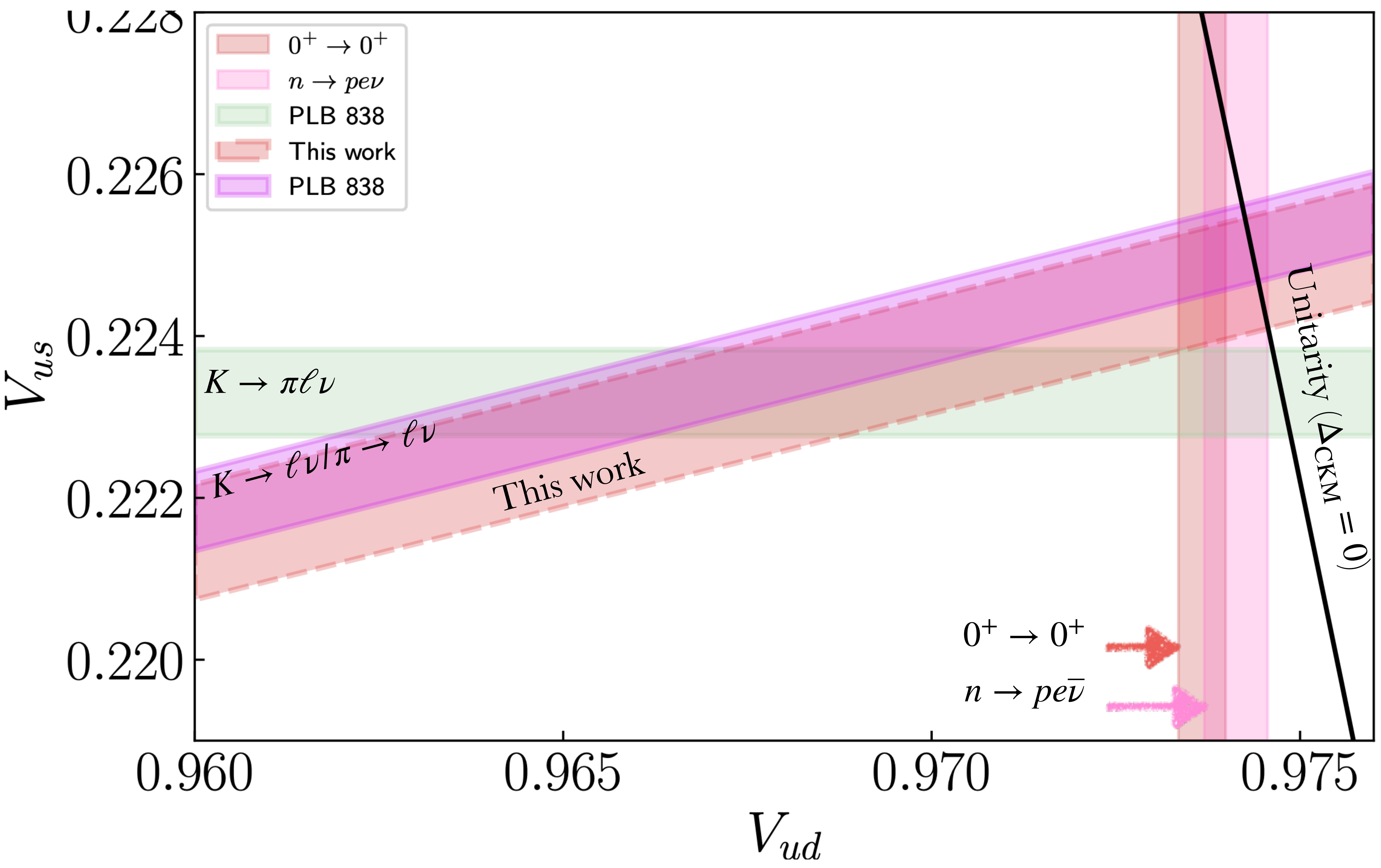
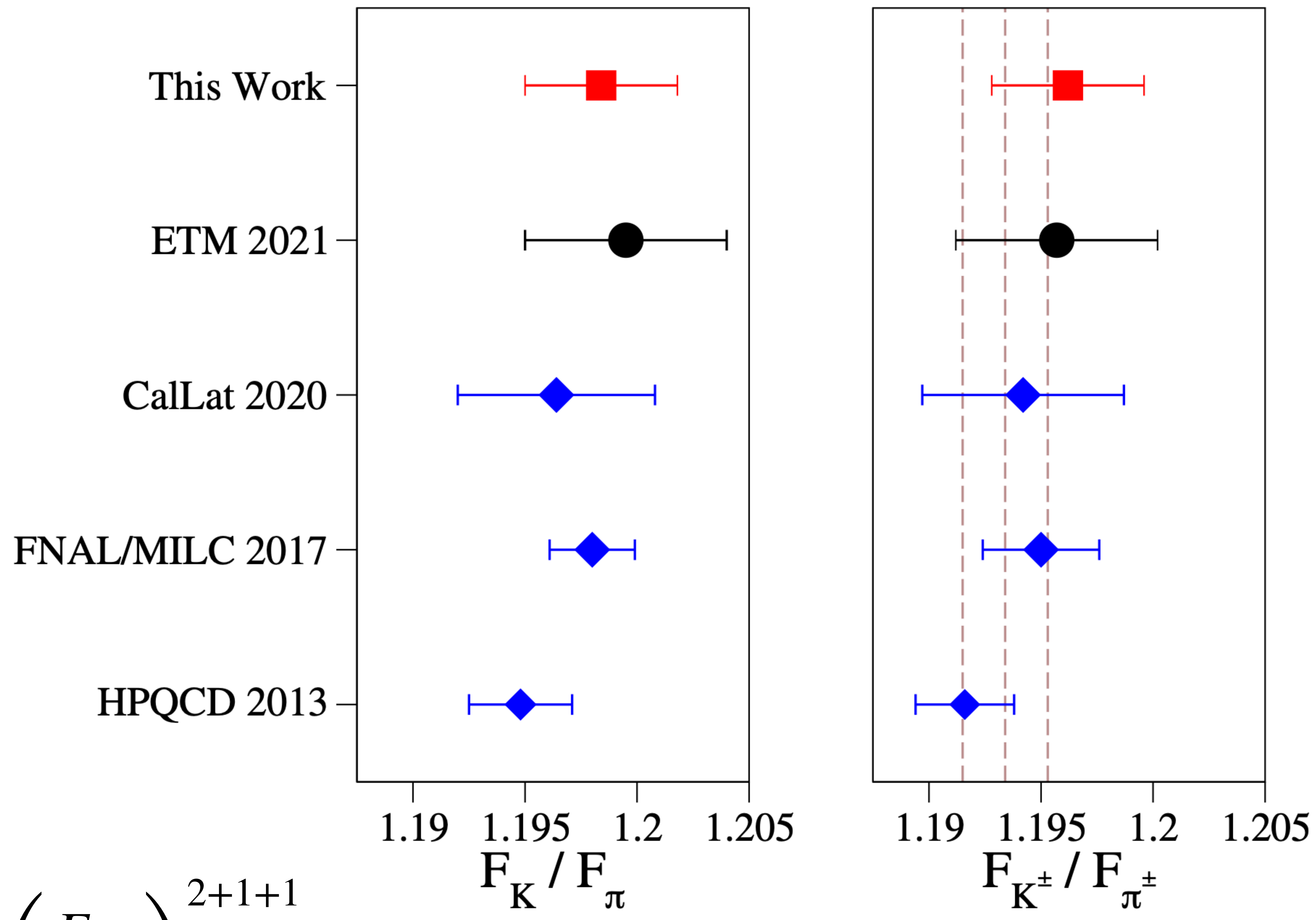


Summary & Outlook



$$\left(\frac{F_{K^\pm}}{F_{\pi^\pm}}\right)^{2+1+1}_{\text{FLAG}} = 1.1934(19)$$

$$\frac{F_{K^\pm}}{F_{\pi^\pm}} = 1.1962(34) \qquad \frac{|V_{us}|}{|V_{ud}|} = 0.23068(74)$$

- First implementation of 4-flavor Domain Wall fermions, determining F_K/F_π at 0.28% uncertainty.
- Results overlap within $\sim 1\sigma$ for the most recent determinations and with FLAG.
- Currently generating other ensembles at the physical point → further reduce FLAG uncertainty.
- Exploring meson matrix elements and baryons.
  - V_{us} limited by $f_+(0)$, a single calculation dominates the four flavor FLAG average.

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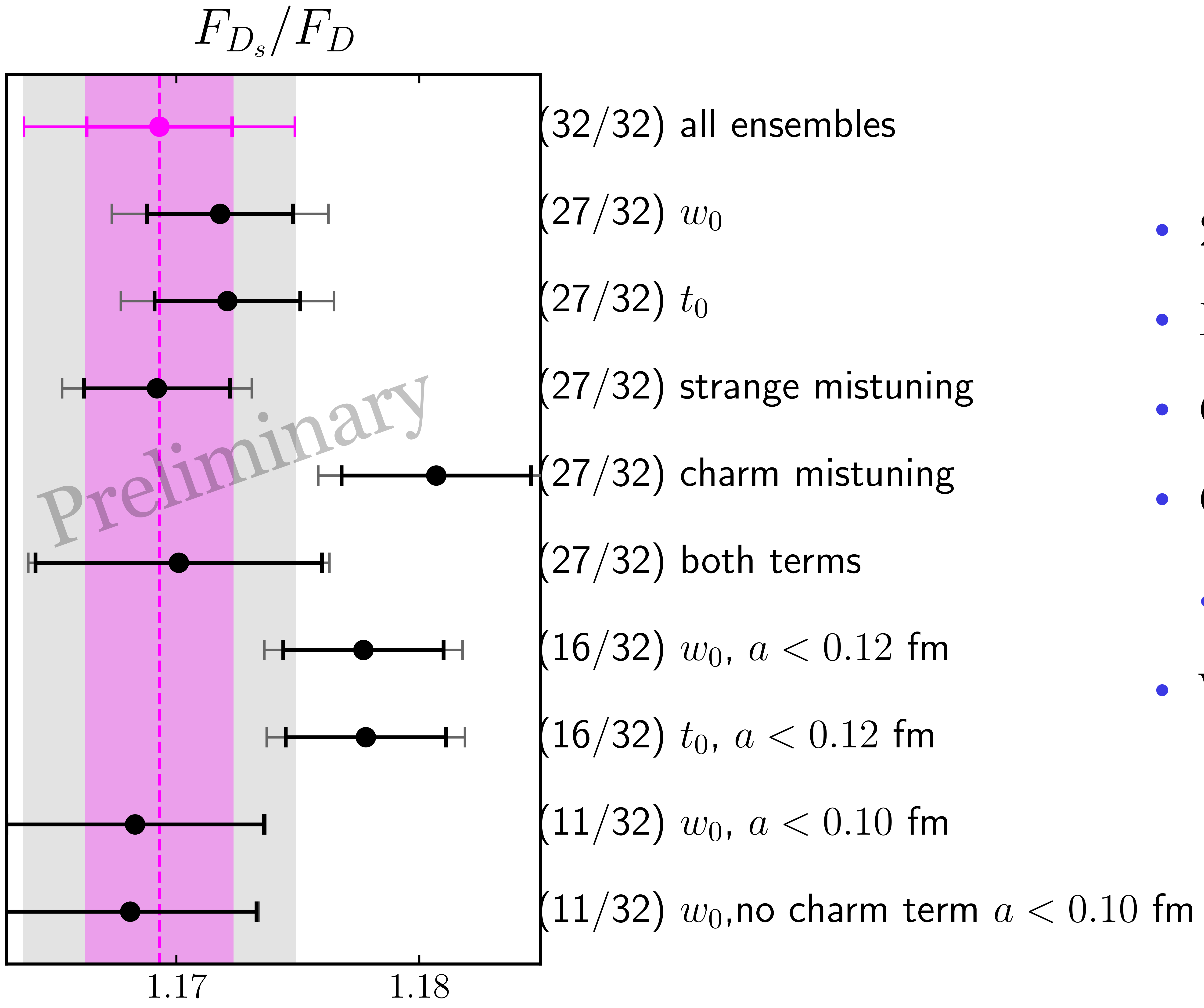
ETM 21: PRD 104 074520
CalLat 20: PRD 102 034507
FNAL/MILC 17: PRD 98 074512
HPQCD 14: PRD 88 074504

Towards the Charm Sector



- 2 new ensembles - a06m228, a10m247L
- Exploring fits based on HMχPT.
- Checking outliers in data.
- Choosing how to parameterize charm dependence:
  - $\delta_c = A_c(\xi_c - \xi_c^{\text{phys}})$?
- What's the impact without coarsest ensembles?

Stay tuned!

Acknowledgements

Collaborators

André Walker-Loud	LBNL/UC Berkeley
Bigeng Wang	UK/LBNL
Dimitra Pefkou	UC Berkeley/LBNL
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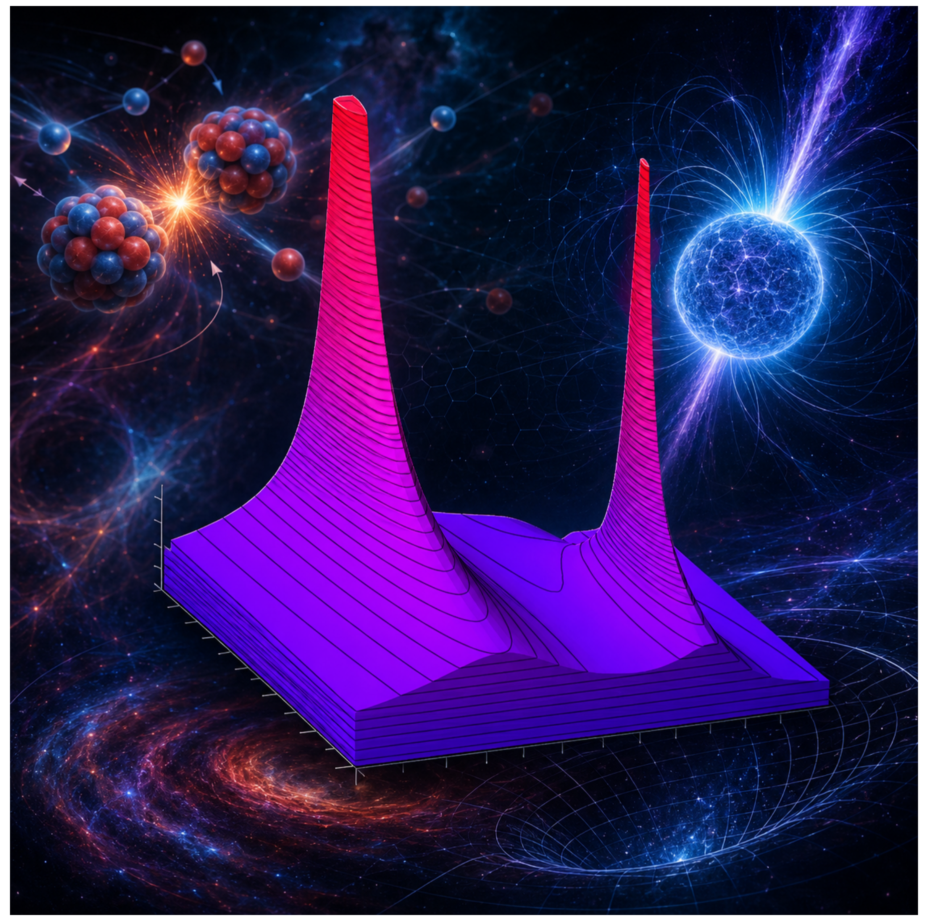
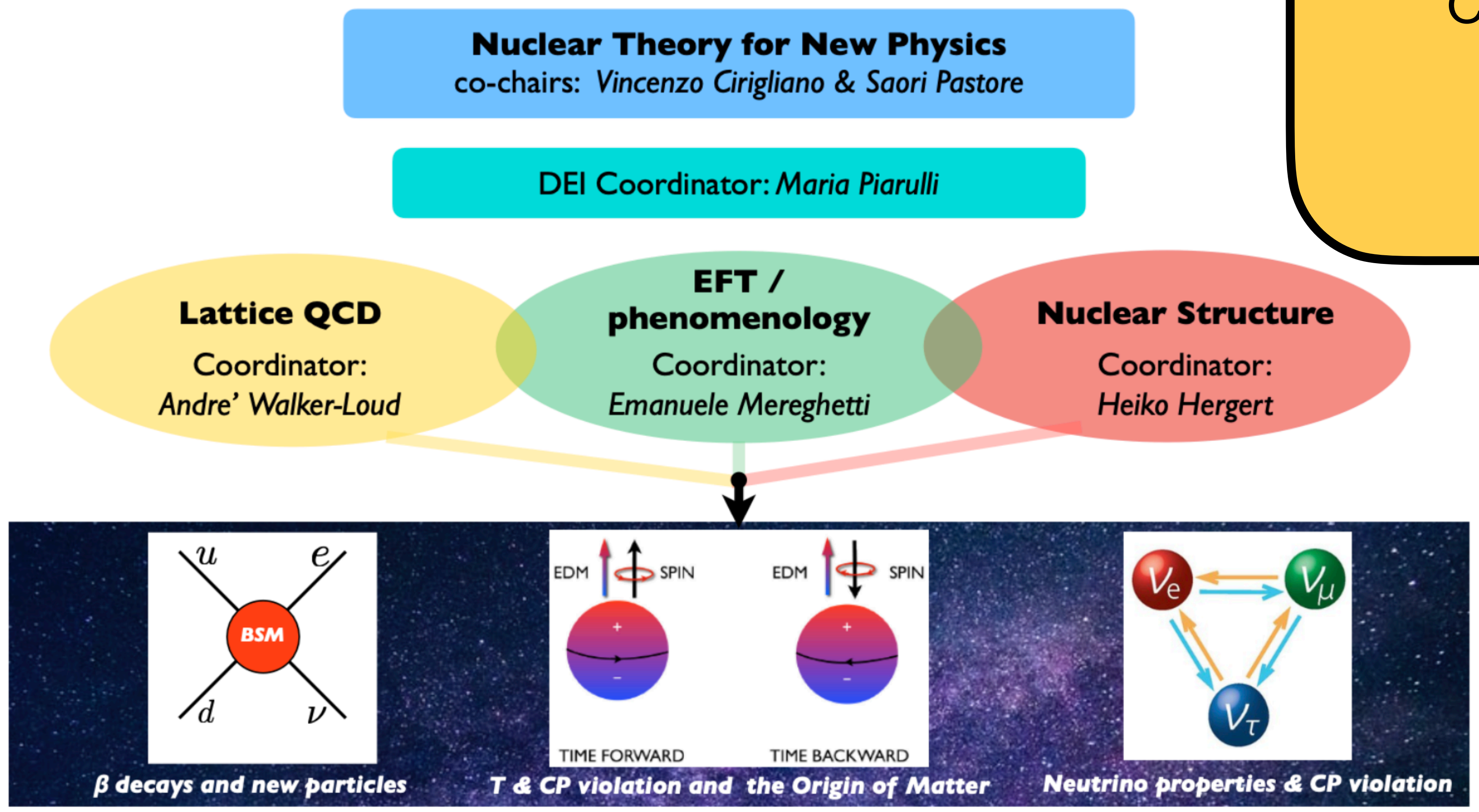
Thomas Richardson	UC Berkeley/LBNL
Amy Nicholson	UNC Chapel Hill
Jamie Hudspith	Carnegie Mellon
Fernando Romero-Lopez	University of Bern
Henry Monge-Camacho	ORNL
Colin Morningstar	Carnegie Mellon
Andrew Hanlon	Kent State
Miguel Salg	University of Bern
Pavlos Vranas	LLNL



Connecting the Standard Model to Nuclear Physics



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Extra Slides

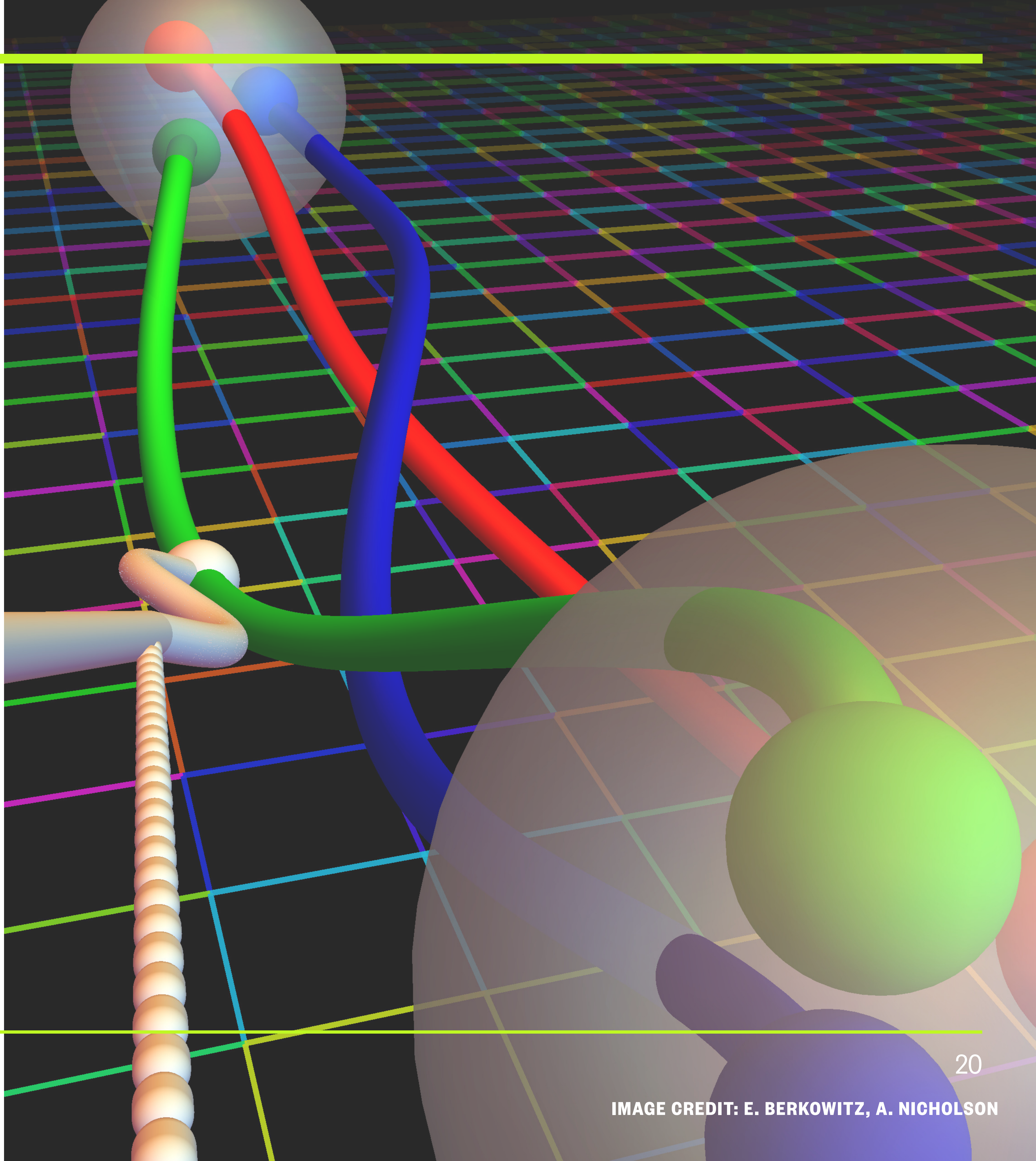


IMAGE CREDIT: E. BERKOWITZ, A. NICHOLSON

Extrapolation Analysis

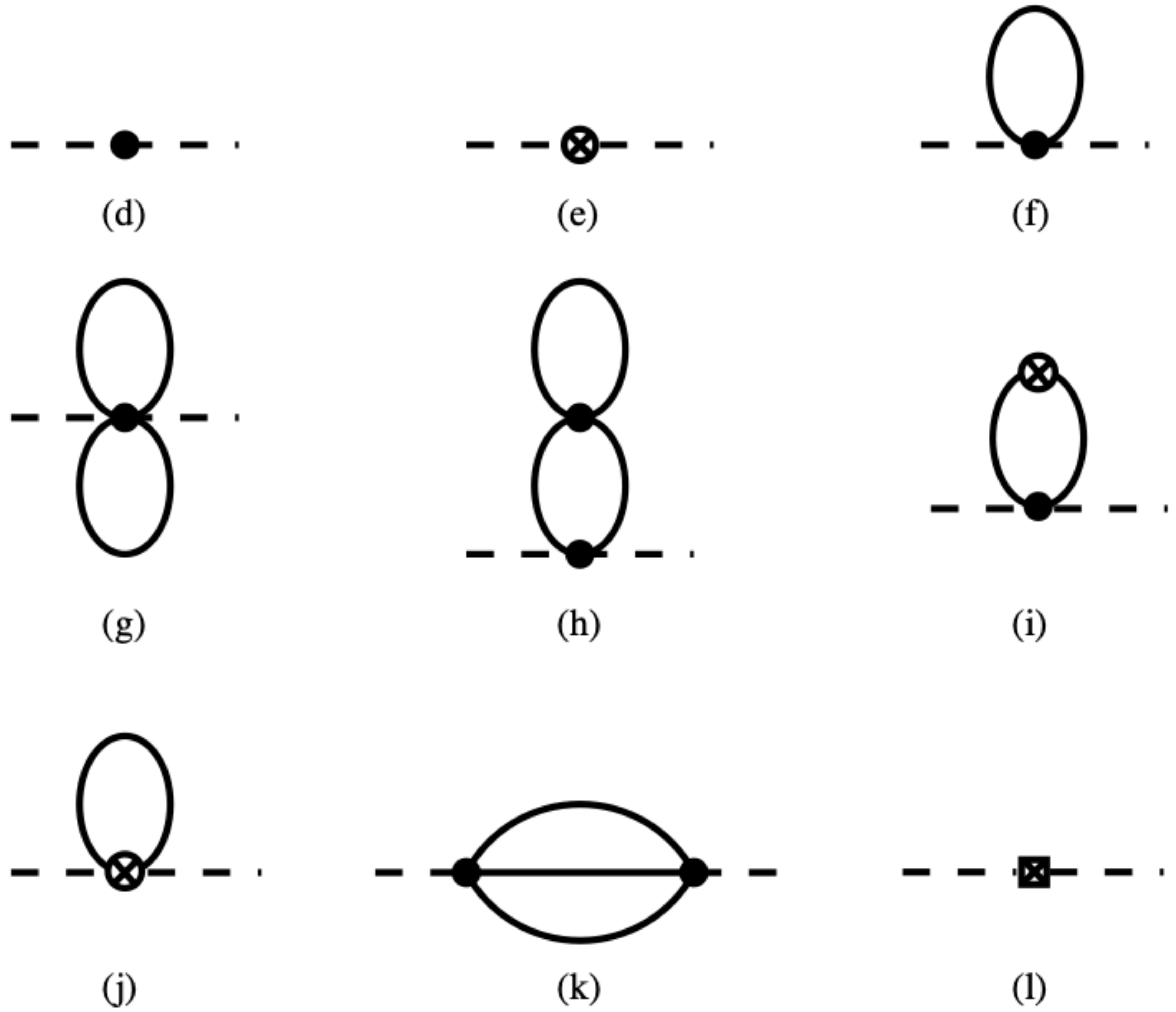
- We use a Bayesian framework to perform a nonlinear fitting strategy.
- Agnostic, data-driven approach to determine the model that best represents the data.
- Pion (Quark) mass dependence parameterized through Chiral Perturbation Theory (χPT) up to $O(\xi_\pi^2)$:

$$\xi_\pi = \frac{m_\pi^2}{(4\pi F_\pi)^2} \qquad m_\pi L \qquad \epsilon_a = \frac{a}{2w_0}$$

$$\frac{F_K}{F_\pi} = \frac{1 + \delta F_K^{\mathrm{NLO}} + \delta F_K^{\mathrm{N^2LO}} + \cdots}{1 + \delta F_\pi^{\mathrm{NLO}} + \delta F_\pi^{\mathrm{N^2LO}} + \cdots}$$

$$= 1 + \delta F_K^{\mathrm{NLO}} - \delta F_\pi^{\mathrm{NLO}} + \delta F_K^{\mathrm{N^2LO}} - \delta F_\pi^{\mathrm{N^2LO}}$$

$$-\delta F_K^{\mathrm{NLO}} \delta F_\pi^{\mathrm{NLO}} + (\delta F_\pi^{\mathrm{NLO}})^2 + \cdots$$



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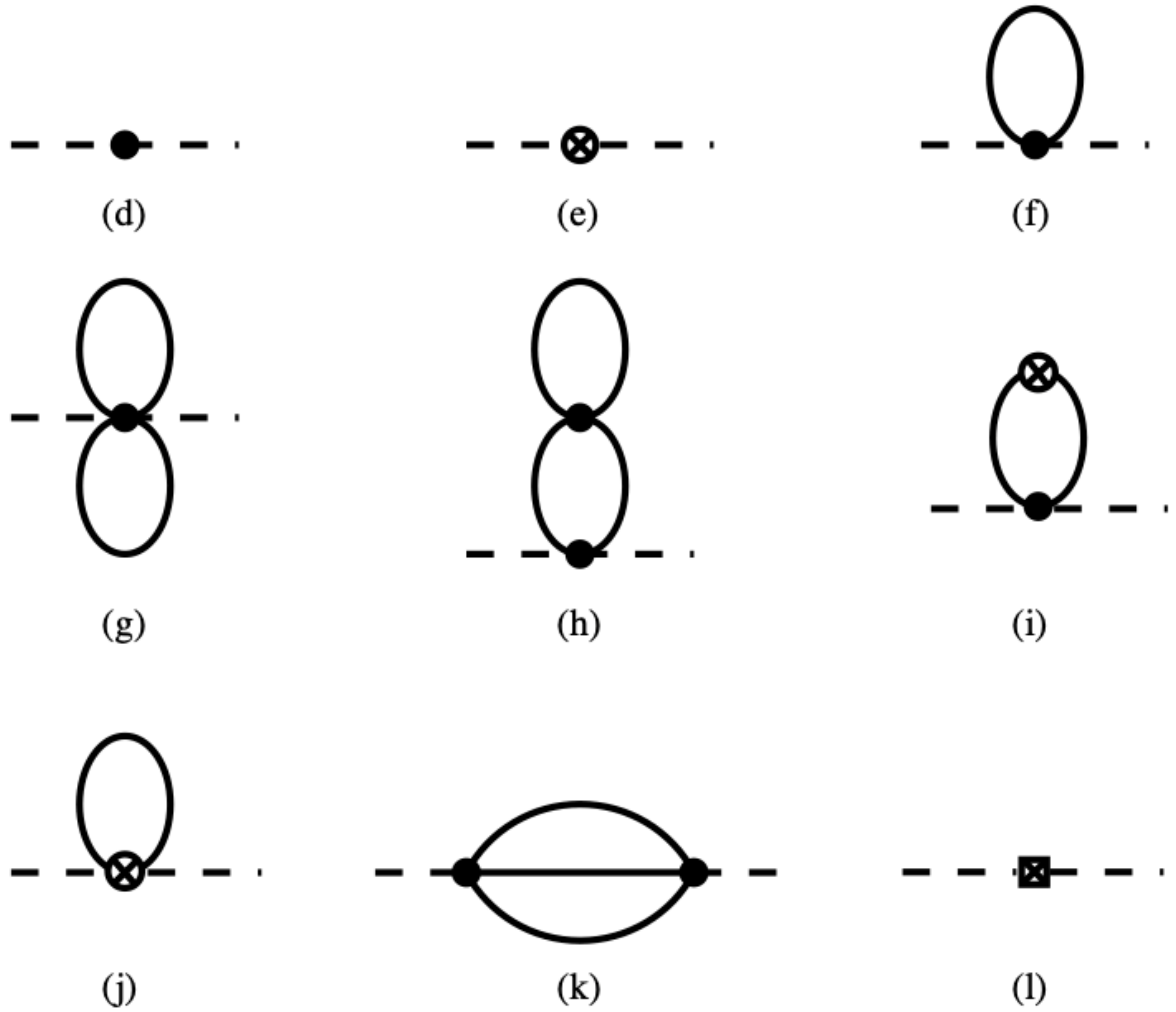

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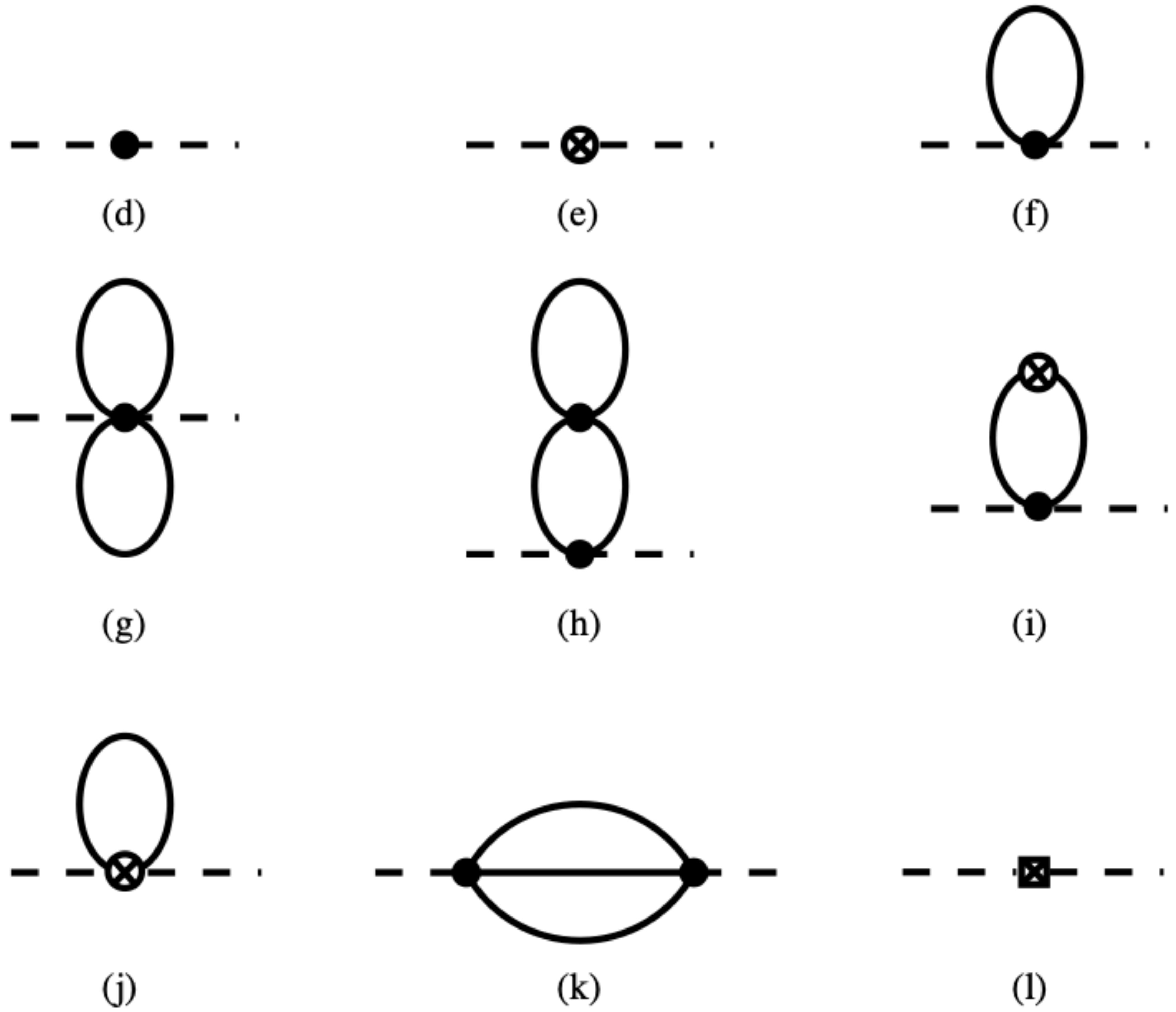
$$\begin{aligned}\frac{F_K}{F_\pi} &= 1 + \frac{5}{8}\ell_\pi - \frac{1}{4}\ell_k - \frac{3}{8}\ell_\eta + 4\bar{L}_5(\xi_K - \xi_\pi) \\ &+ \xi_K^2 F_F\left(\frac{m_\pi^2}{m_K^2}\right) + \hat{K}_1^r\lambda_\pi^2 + \hat{K}_2^r\lambda_\pi\lambda_K \\ &+ \hat{K}_3^r\lambda_\pi\lambda_\eta + \hat{K}_4^r\lambda_K^2 + \hat{K}_5^r\lambda_K\lambda_\eta + \hat{K}_6^r\lambda_\eta^2 \\ &+ \hat{C}_1^r\lambda_\pi + \hat{C}_2^r\lambda_K + \hat{C}_3^r\lambda_\eta + \hat{C}_4^r\,.\end{aligned}$$

$$\ell_P = \xi_P\lambda_P = \frac{m_P^2}{(4\pi F_\pi)^2}\ln\frac{m_P^2}{\mu^2}$$



RE: Low-Energy Precision Physics

	Ref.	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$
$N_f = 2+1+1$		0.22483(61)	0.97439(14)
$N_f = 2+1$		0.22490(54)	0.97436(12)

$$(V_{us}/V_{ud})^{2+1} = 0.23082(55)$$

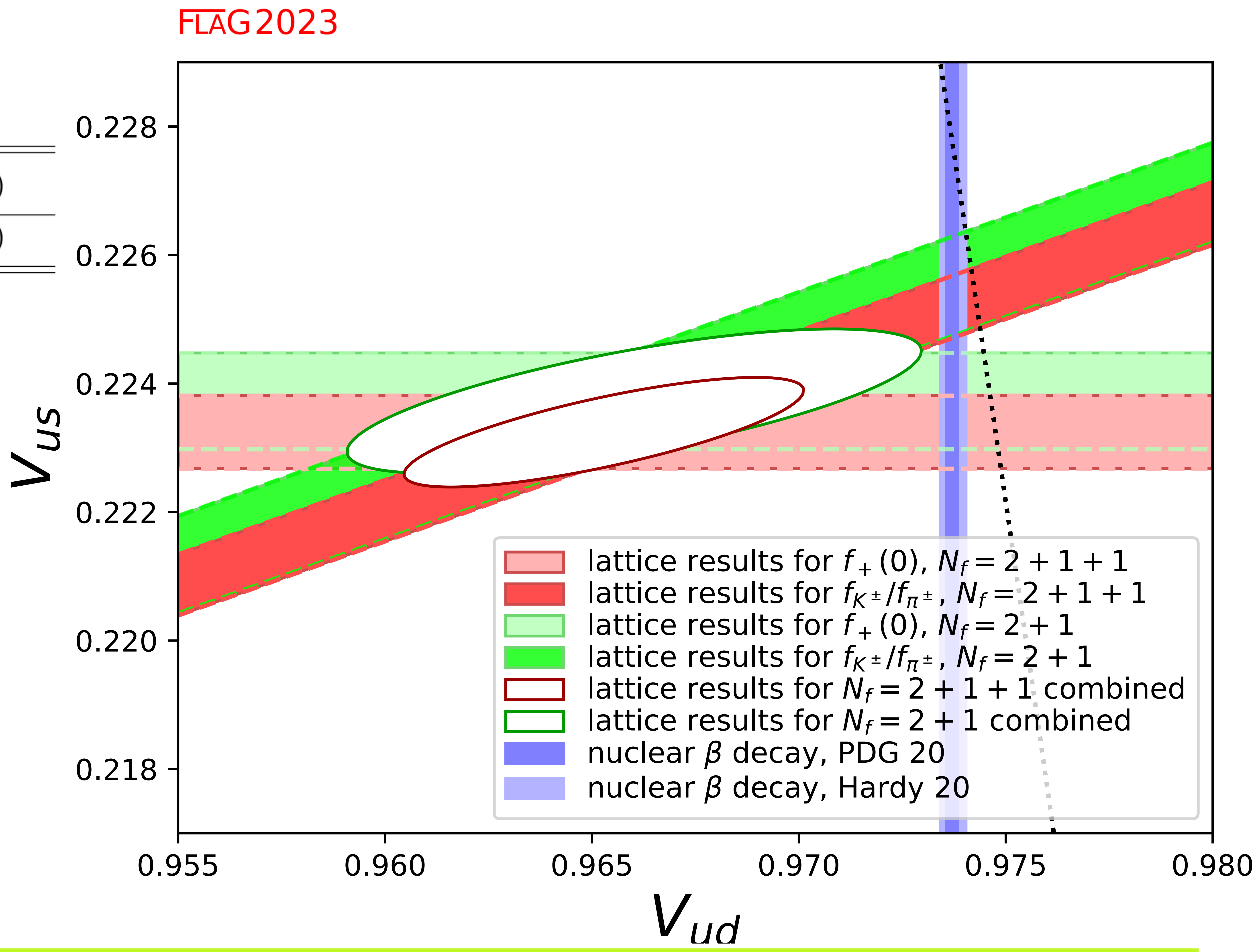
$$(V_{us}/V_{ud})^{2+1+1} = 0.23074(63)$$

$$(\lvert V_{\mathrm{us}}\rvert / \lvert V_{\mathrm{ud}}\rvert)^{\mathrm{PDG}} = 0.23130(54)$$

$$(\lvert V_{\mathrm{us}}\rvert)^{\mathrm{PDG}} = 0.22530(54)$$

$$\frac{F_{K^\pm}}{F_{\pi^\pm}} = 1.1962(34)$$

$$\frac{\lvert V_{\mathrm{us}}\rvert}{\lvert V_{\mathrm{ud}}\rvert} = 0.23068(74)$$



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