Quenched glueball and meson spectrum in the ‘t Hooft limit of lattice Yang-Mills theories

43nd Lattice Conference
College Park, MD
31 July 2026

Andrea Falzetti

in collaboration with:
Mauro Papinutto
Francesco Scardino
Matteo Lombardi

QCD in the ‘t Hooft limit

This is the QCD Euclidean Lagrangian:

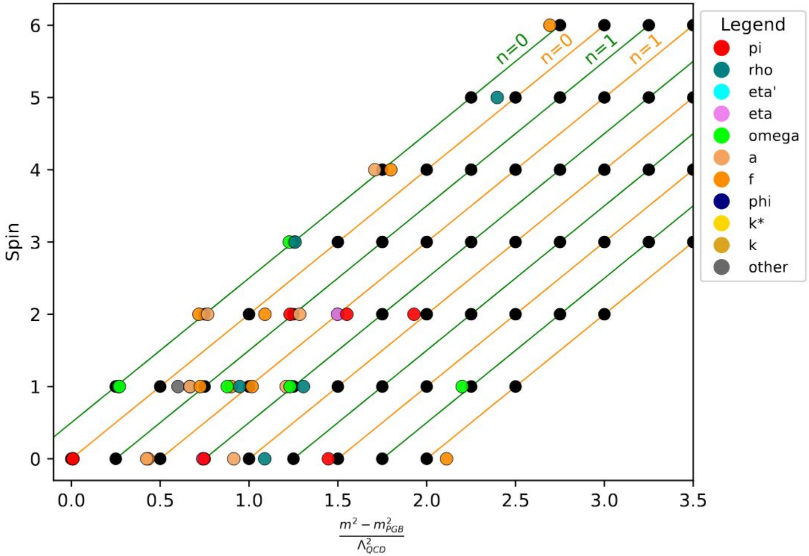
$$\mathcal{L} = \sum_f \overline{\psi^{(f)}}(\not{D} + m^{(f)})\psi^{(f)} + \frac{1}{2g_{YM}^2} \mathrm{Tr}\, F_{\mu\nu} F_{\mu\nu}$$

- It is known that if we send $N_c \to \infty$ keeping $g_{\mathrm{YM}}^2 N_c = g_{\text{'t Hooft}}^2$ constant (planar limit) we are left with a theory of free glueballs and mesons.

- To the leading large N_c order (planar limit) only diagrams with the topology of a sphere survive.
- Moreover in the large N_c expansion glueball and mesons are weakly interacting with coupling $\frac{1}{N_c}$ or $\frac{1}{\sqrt{N_c}}$ respectively

QCD in the 't Hooft limit

There have been several attempts to find an analytic solution to describe the 't Hooft limit of QCD, tackling the problem with different approaches (Strings, topological theories, duality etc.etc.)



We want to use measurements of the spectrum of SU(N) Yang-Mills theory in the 't Hooft limit to investigate the feasibility of a semi-topological approach [2]

[1] Bochicchio, M. **Glueball and Meson Spectrum in Large-*N QCD*** . *Few-Body Syst* **57**, 455–459 (2016)
[2] Bochicchio M., **An asymptotic solution of Large-N QCD, for the glueball and meson spectrum and the collinear S-matrix** AIP Conf.Proc. 1735 (2016), *and* Bochicchio M., *to appear on arXiv*

The lattice setup

- Wilson plaquette action for the gauge group

$$S[U] = \frac{\beta}{N_c} \sum_{\text{plaq.}} \mathrm{Re}\,\mathrm{Tr}\,(1 - U_{\mu\nu}(x))$$

- Heatbath (Cabibbo-Marinari) + overrelaxation updates in a 1:4 mixture
- Periodic Boundary conditions
- Improved actions would make the 2-level sampling less efficient

Our lattices

Tag	$L_0 \times L_i^3$	β	$a\sqrt{\sigma}$
n5lat3	20×14^3	17.43	0.223(26)
n5lat4	20×16^3	17.63	–
n5lat5	24×20^3	18.04	–
n5lat6	32×24^3	18.375	0.148(35)

Tag	$L_0 \times L_i^3$	β	$a\sqrt{\sigma}$
n6lat3	20×14^3	25.32	–
n6lat4	20×16^3	25.55	–
n6lat5	24×20^3	26.22	–
n6lat6	32×24^3	26.71	–

Tag	$L_0 \times L_i^3$	β	$a\sqrt{\sigma}$
n8lat2	16×10^3	44.85	–
n8lat3	20×12^3	45.5	0.217(59)
n8lat4	20×14^3	46.1	0.189(30)
n8lat5	24×16^3	46.7	–
n8lat6	32×20^3	47.75	–

Tag	$L_0 \times L_i^3$	β	$a\sqrt{\sigma}$
n10lat2	16×10^3	70.38	0.314(53)
n10lat3	20×12^3	71.38	0.218(48)
n10lat4	20×14^3	72.4	0.197(30)
n10lat5	24×16^3	73.35	0.173(44)

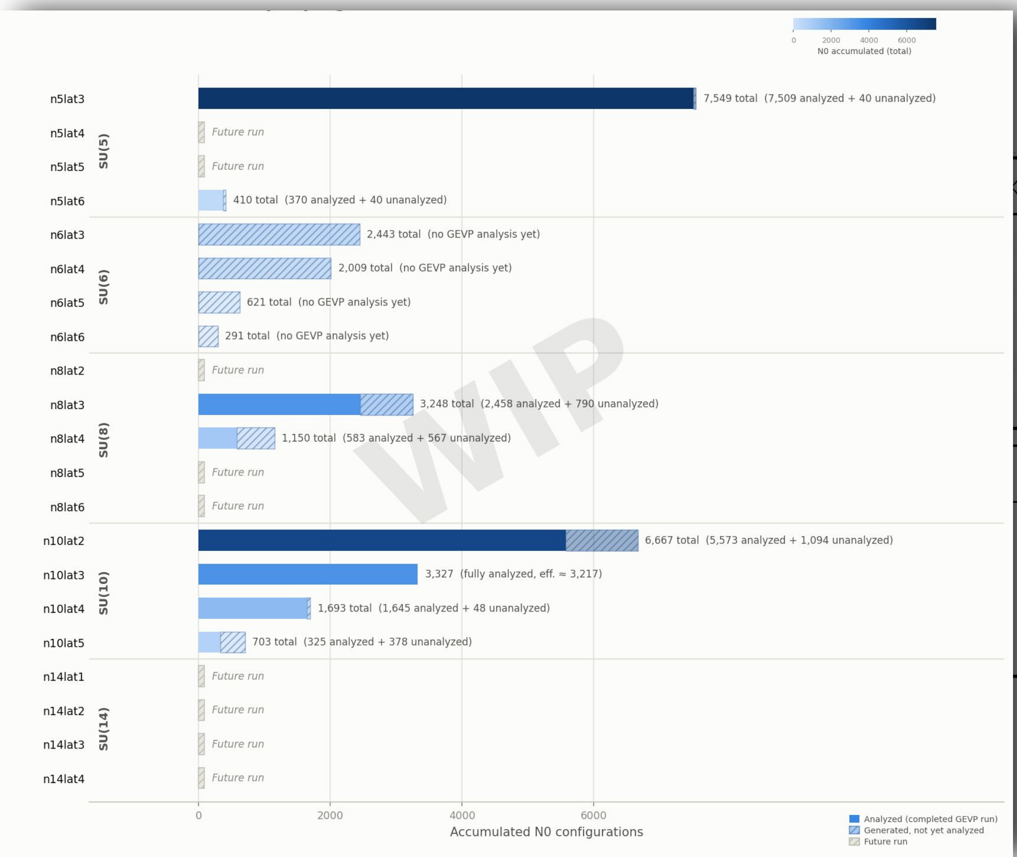
Tag	$L_0 \times L_i^3$	β	$a\sqrt{\sigma}$
n14lat1	16×12^3	137	–
n14lat2	20×12^3	139.2	–
n14lat3	20×14^3	141.85	–
n14lat4	24×20^3	144^*	–

Our lattices

Tag	$L_0 \times L_i^3$	β	
n5lat3	20×14^3	17.43	0.
n5lat4	20×16^3	17.63	
n5lat5	24×20^3	18.04	
n5lat6	32×24^3	18.375	0.

$\times L_i^3$	β	$a\sqrt{\sigma}$
10^3	44.85	–
12^3	45.5	0.217(59)
14^3	46.1	0.189(30)
16^3	46.7	–
20^3	47.75	–

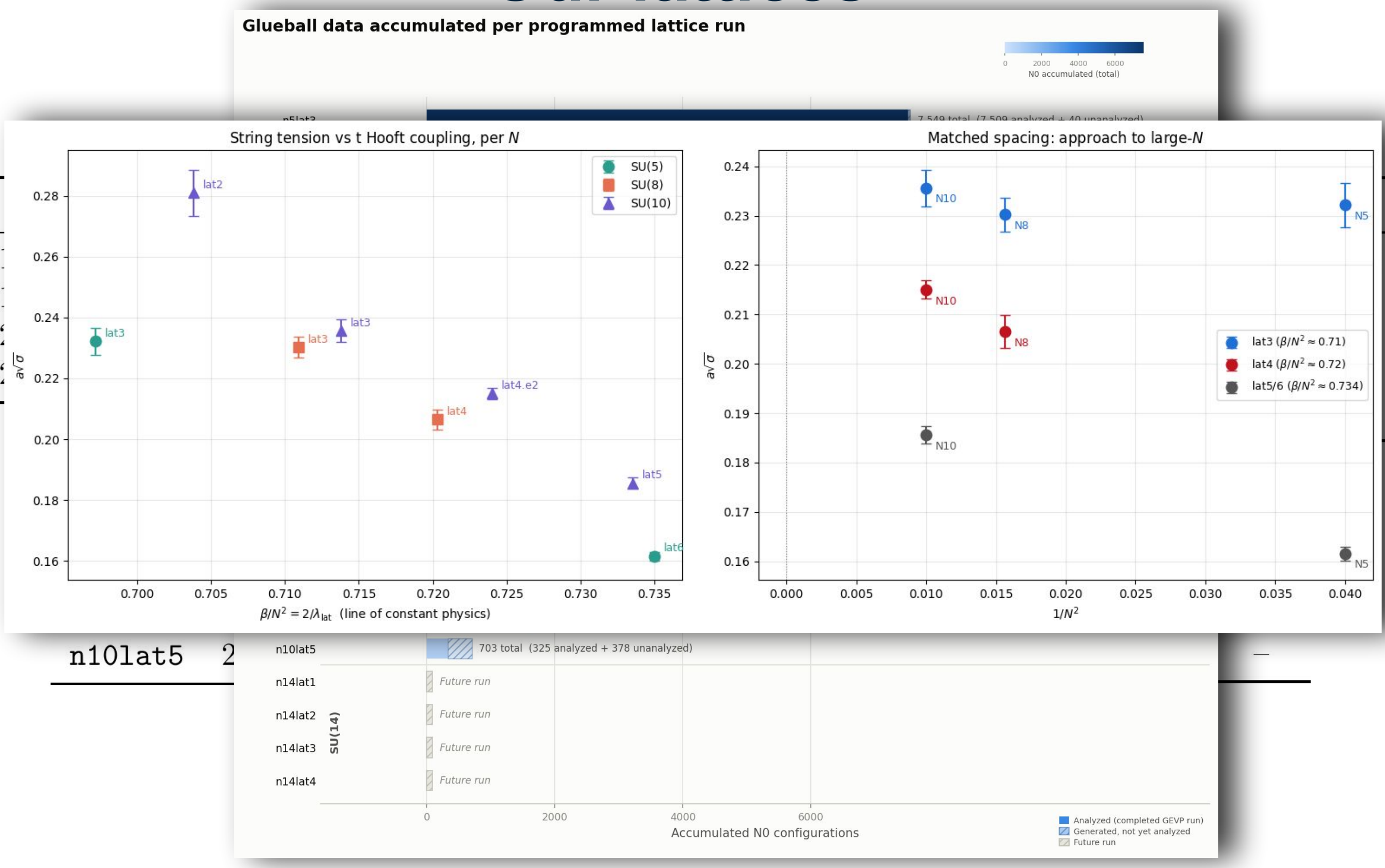
Tag		$a\sqrt{\sigma}$
n10lat2	1	–
n10lat3	2	–
n10lat4	2	–
n10lat5	2	–



Our lattices

Tag	$L_0 \times$	$a\sqrt{\sigma}$
n5lat3	$20 \times$	—
n5lat4	$20 \times$	0.217(59)
n5lat5	$24 \times$	0.189(30)
n5lat6	$32 \times$	—

n10lat5	2	—

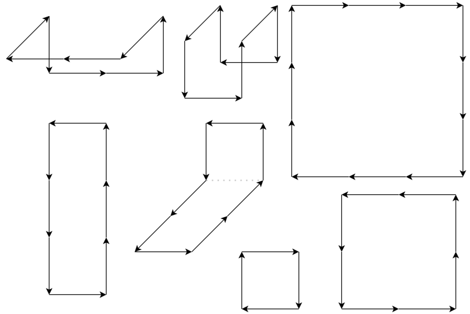


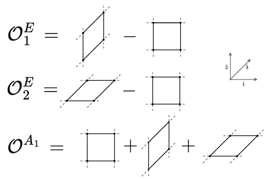
The operators

We are using a basis made of wilson loops of length up to $\ell = 12$ computed at 4 different APE smearing levels of the gauge configurations.

We use subduced representations to produce linear combinations of our loops that transform according to irreducible representations of the enlarged cubic group.

Our operator basis:

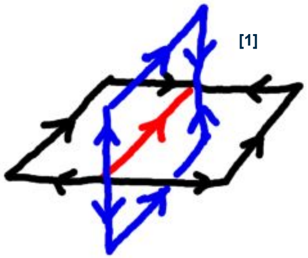




J^{PC}	$N_{\rm ops}$
A_1^{++}	7
A_2^{++}	2
E^{++}	18
T_1^{++}	3
T_2^{++}	6
T_1^{-+}	9
T_2^{-+}	12

×

4 APE smearing levels



[1] G. Korcyl*, S. Cali and P. Korcyl, **Evaluation of SU(3) smearing on FPGA accelerator cards,** *doi.org/10.22323/1.396.0048*

The `cgRed` package

Reorganized reduction scripts into a handy python package to perform O_h reduction for arbitrary loop shapes.

INPUT VALIDATION

- **`isClosed(loop)`** → `bool`
  True if the directed steps sum to zero, i.e. the path closes
- **`isTrivial(loop)`** → `bool`
  True if the loop backtracks on itself (adjacent opposite steps)

OPERATOR CONSTRUCTION

- **`vec(loop, cPar)`** → `[cPar, loop]`
  Label a loop with its C-parity, cPar = ±1
- **`conj(loop)`** → `loop`
  C-conjugation: reverse the order and negate every step
- **`rot(loop, i)`** → `loop`
  Act with the group element i ∈ [0, 47]
- **`vecRot(v, i)`** → `[cPar, loop]`
  Same rotation, applied to a C-parity labelled vector
- **`vecEq(v1, v2)`** → `{0, ±1}`
  Compare up to cyclic symmetry and C-conjugation

BASIS & REPRESENTATION

- **`baseBuild(v)`** → $[v_1, v_2, \ldots]$
  Orbit of v under O_h — a basis of dimension N ≤ 48
- **`repBuild(base)`** → $[M_0 \ldots M_{47}]$
  The 48 real N×N matrices representing the group action

GROUP REDUCTION

- **`repReduceStep(rep, Cindex)`** → `(rep, P)`
  Jordan decomposition against one conjugacy class
- **`repReduce(rep)`** → `(rep, A)`
  Block-diagonalise over all 10 classes; A is the change of basis
- **`findBlocks(rep)`** → $[d_1, d_2, \ldots]$
  Dimensions of the resulting invariant subspaces
- **`diagBlockRep(rep, i_start, size)`** → `rep_block`
  Slice one diagonal block out of the reduced representation
- **`findRep(block, cPar)`** → `(id, mult)`
  Match a block against the character table
- **`findReps(rep, blocks, cPar)`** → `(ids, mults)`
  All irreps and multiplicities, block by block

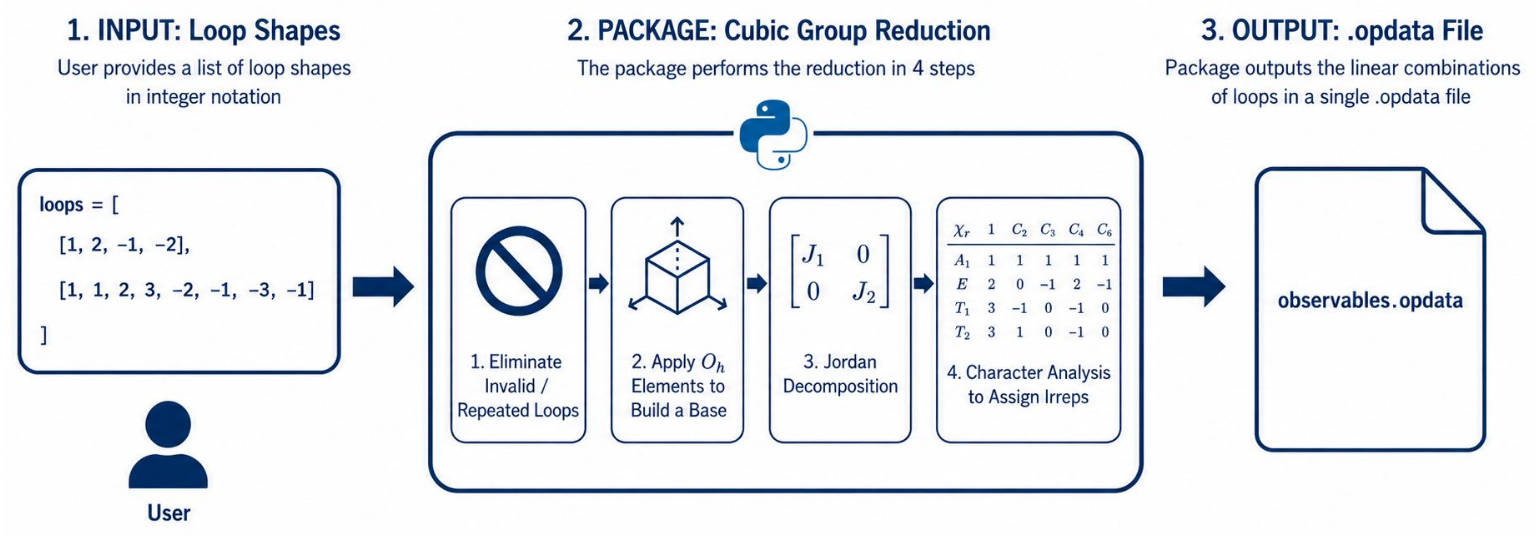
$O_h = O \times \mathbb{Z}_2$ · 48 elements · 10 conjugacy classes · 20 irreps
A1±± A2±± E±± T1±± T2±± (superscripts: parity / C-parity)

OUTPUT

- **`saveOparFile(loop, cPar, fname)`** → `.opar`
  One operator, one C-parity — openQCD input format
- **`saveOpdataFile(loops, fname, names)`** → `.opdata`
  Whole operator set, both C-parities, as JSON
- **`plot_loop(loop)`** → `figure`
  3-D render of the loop path on the lattice
- **`produceLoopShapes(lmax, fname)`** → `file`
  Enumerate every independent loop shape up to length lmax

The `cgRed` package

Reorganized reduction scripts into a handy python package to perform O_h reduction for arbitrary loop shapes.

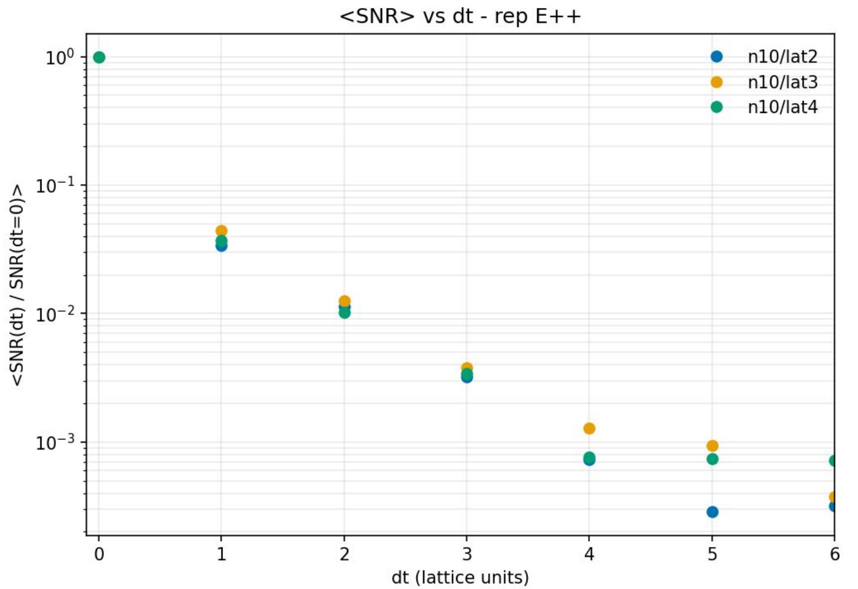


The operators

We are using a basis made of wilson loops of length up to $\ell = 12$ computed at 4 different APE smearing levels of the gauge configurations.

2 point correlators

$$
\begin{aligned}
C_{ij}(\Delta t) &= \langle \mathcal{O}_i(t_0)\mathcal{O}_j(t_0+\Delta t)\rangle \\
&= \sum_{n=1} \langle 0|\, \mathcal{O}_i(t_0)\, |n\rangle \langle n|\, \mathcal{O}_j(t_0+\Delta t)\, |0\rangle = \\
&= \sum_{n=1} 2e^{-am_n\cdot\frac{L_t}{2}} \langle 0|\, \mathcal{O}_i(0)\, |n\rangle \langle n|\, \mathcal{O}_j(0)\, |n\rangle \cosh\left(am_n\left(\Delta t - \frac{L_t}{2}\right)\right)
\end{aligned}
$$



GEVP

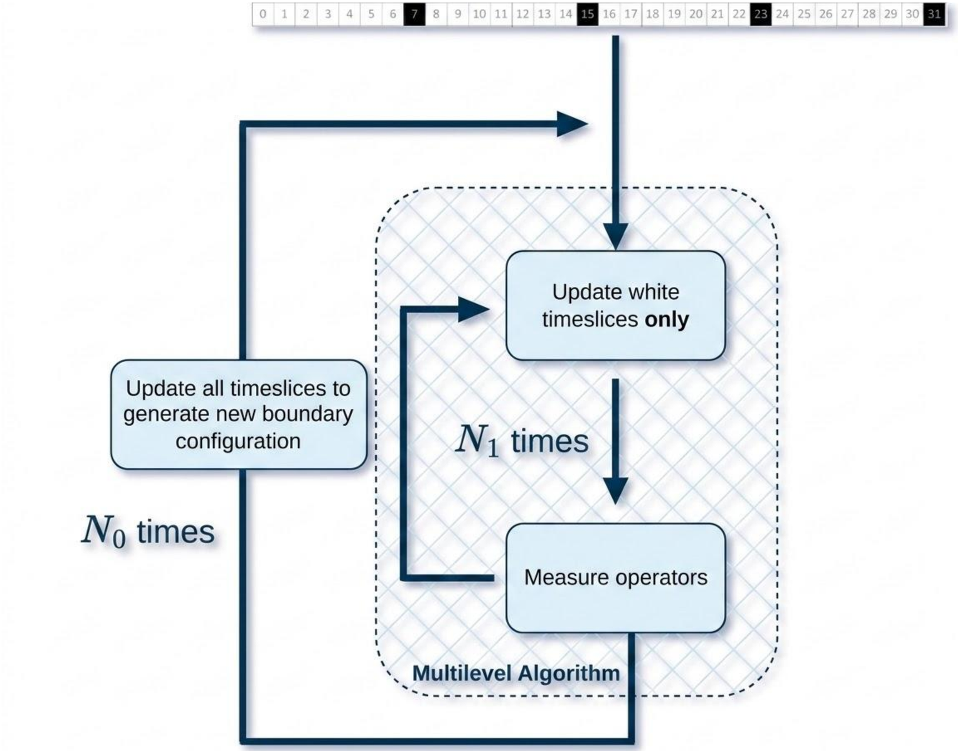
$$C(t)\psi_b = \lambda_b(t,t_0)C(t_0)\psi_b \longrightarrow \lambda_b(t,t_0) = e^{-am_b(t-t_0)}$$

$$a\cdot m_b = \log\left(\frac{\lambda_b(t,t_0)}{\lambda_b(t+1,t_0)}\right)$$

The Multilevel algorithm [1]

Because of the severe statistical noise affecting glueball correlators in the large time separation limit we have implemented a multilevel algorithm.

We use the locality of the Wilson action to turn our montecarlo averages into nested averages over different boundary configurations.



[1] *Meyer H.* **Locality and Statistical Error Reduction on Correlation Functions,** arXiv:hep-lat/0209145

The Multilevel algorithm [1]

When we are measuring correlators of operators at time slices that reside in different regions of the lattice the path integral will factorize into an average over boundary configurations.

Example:

$$\begin{cases} t_0 = 3a \rightarrow t_0 \in R_1 \\ t_1 = 10a \rightarrow t_1 \in R_2 \end{cases}$$

$$\langle \mathcal{O}_i(t_0)\mathcal{O}_j(t_1)\rangle = \int \mathcal{D}U e^{-S[U]}\mathcal{O}_i(t_0)\mathcal{O}_j(t_1) =$$

$$= \int \mathcal{D}U_{\partial R} e^{-S_{\partial R}[U_{\partial R}]}\left(\int \mathcal{D}U_{R_1} e^{-S_{R_1}[U_{R_1}]}\mathcal{O}_i(t_0)\right)\left(\int \mathcal{D}U_{R_2} e^{-S_{R_2}[U_{R_2}]}\mathcal{O}_j(t_1)\right) =$$

$$= \left\langle \langle \mathcal{O}_i(t_0)\rangle_{R_1} \langle \mathcal{O}_j(t_1)\rangle_{R_2} \right\rangle_{\partial R}$$

This greatly reduces the error on the correlator, especially in the large time-separation region, which is also the most critical for reliable extraction of the glueball masses.

[1] *Meyer H.* **Locality and Statistical Error Reduction on Correlation Functions,** arXiv:hep-lat/0209145

The Multilevel algorithm

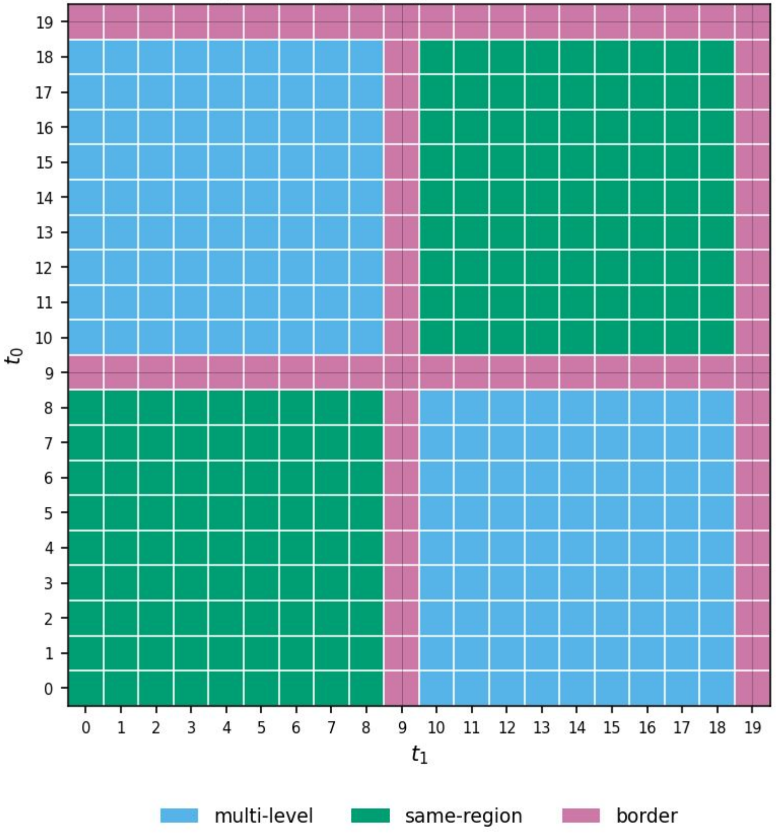
We build $C_{ij}(t_0, t_1)$ for each (t_0, t_1)pair using multilevel only in pairs residing in two different dynamical regions.

dt averaging

$$C_{ij}(t_0, t_1) \longrightarrow C_{ij}(\Delta t)$$

We use optimal averaging along the lines of [1]. This also provides a convenient sanity check that the algorithm works by keeping track of the resulting weights in the average.

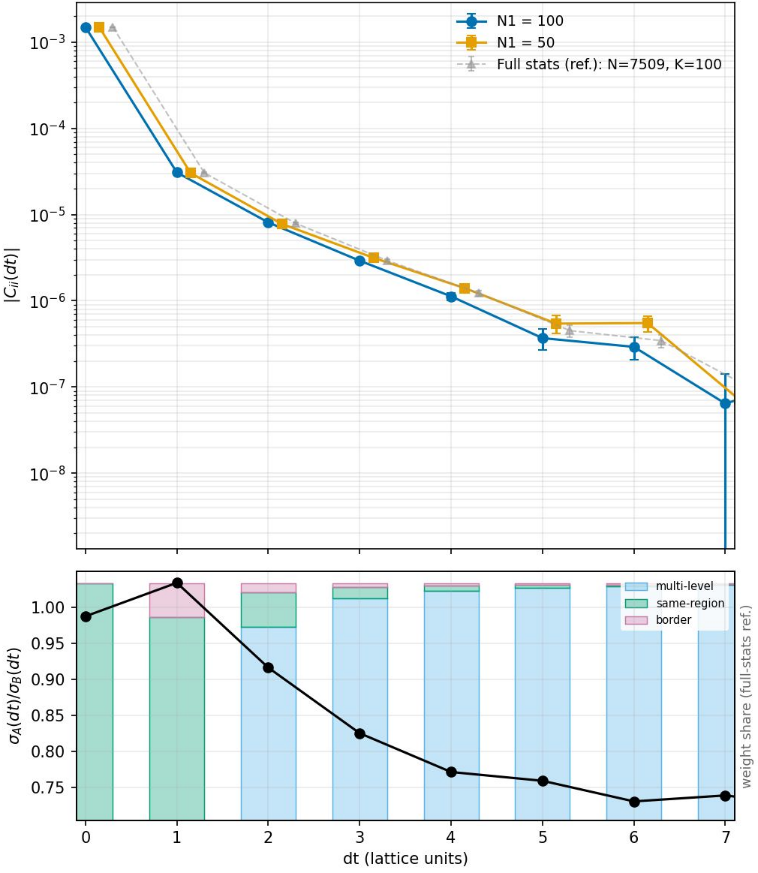
Example L = 20, delta = 10



[1] *Barca, Knechtli, Peardon, Schaefer, Urrea-Niño*, **Performance of two-level sampling for the glueball spectrum in pure gauge theory** *(2023)* *arxiv:2312.11372*

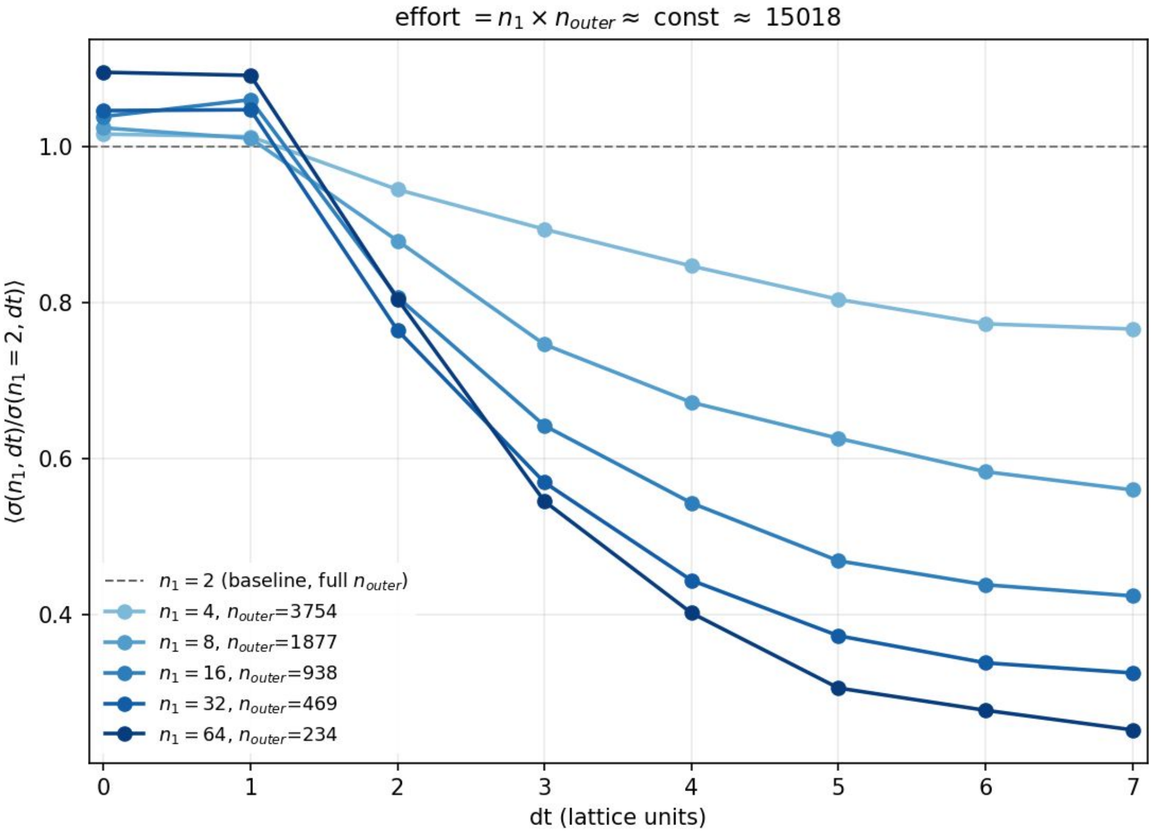
The Multilevel algorithm

n5/lat3 - A1++



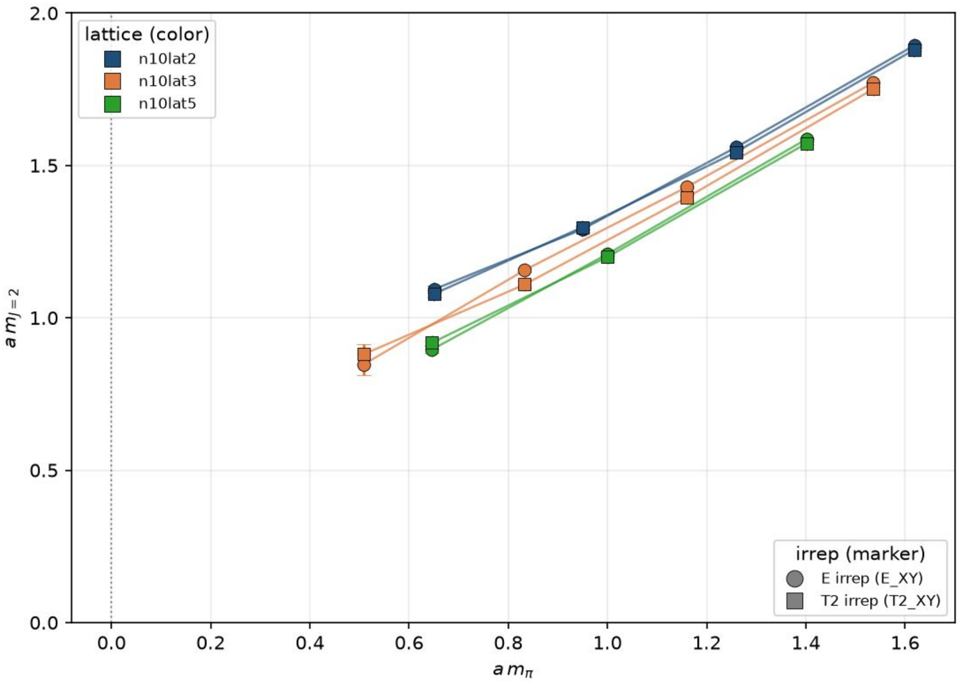
- After dt-reduction this is what we are left with.
- At equal effort the maximum improvement on the errorbars scales like $\sqrt{N_1}$
- Equal effort comparison (*i.e.* constant $N_0 \times N_1$ shows how the theoretical efficiency is reached when the statistical share of multilevel contributions to $C(\Delta t)$saturates

The Multilevel algorithm



n5/lat3 - A1++

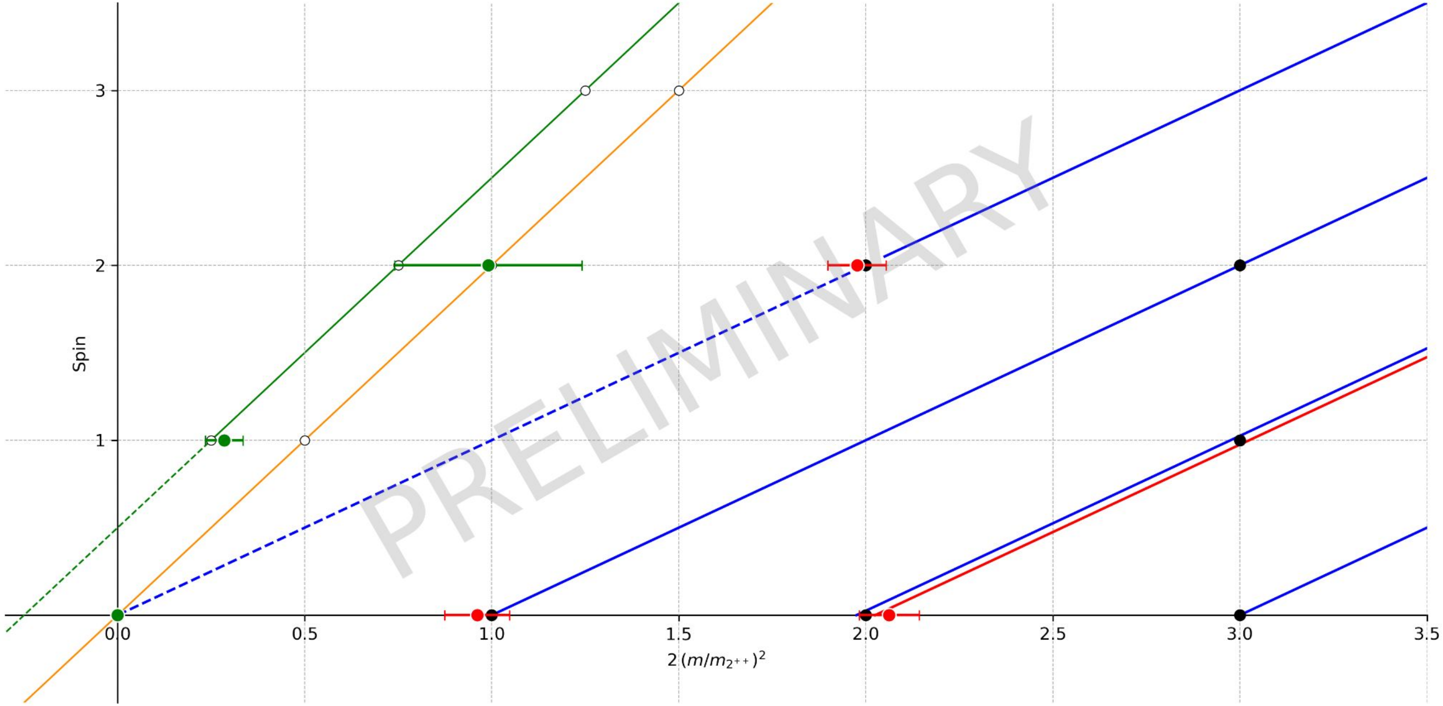
Meson observables



- We are also interested in the reconstruction of the leading meson trajectory above S = 1.
- We use quark bilinears with the insertion of the covariant derivative as interpolators. And measure independently the rho and pi mass for consistency.
- Preliminary runs have been performed for SU(10) at an earlier snapshot of the data on 3 of the 4 lattices

Tag	κ values
n10lat2	0.13, 0.14, 0.148, 0.155
n10lat3	0.13, 0.14, 0.148, 0.155
n10lat5	0.13, 0.14, 0.148

Spectrum results (preiminary)



Trajectories and points are based on STFT predictions [1]

disclaimer

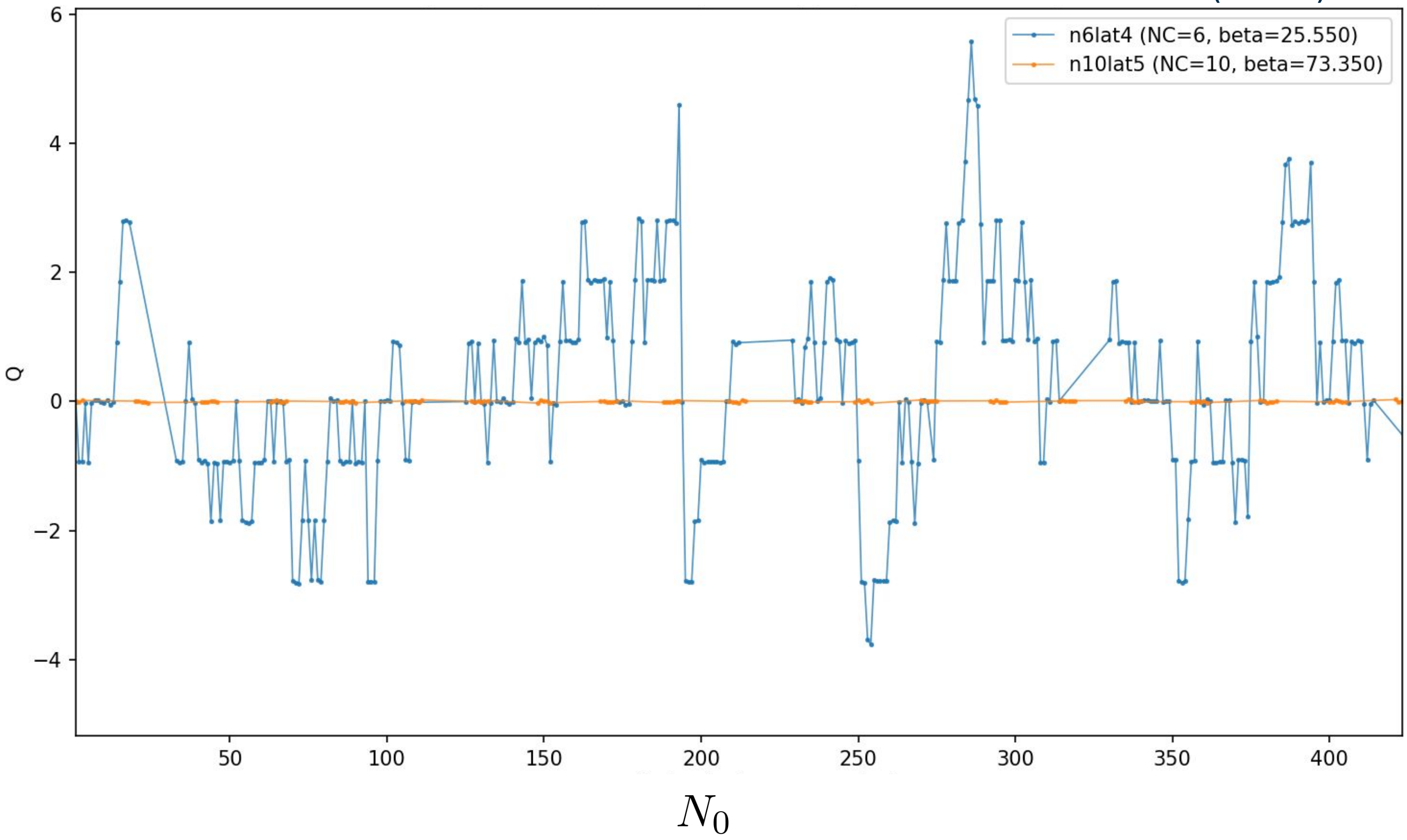
Still not ‘t Hooft extracted but SU(10)

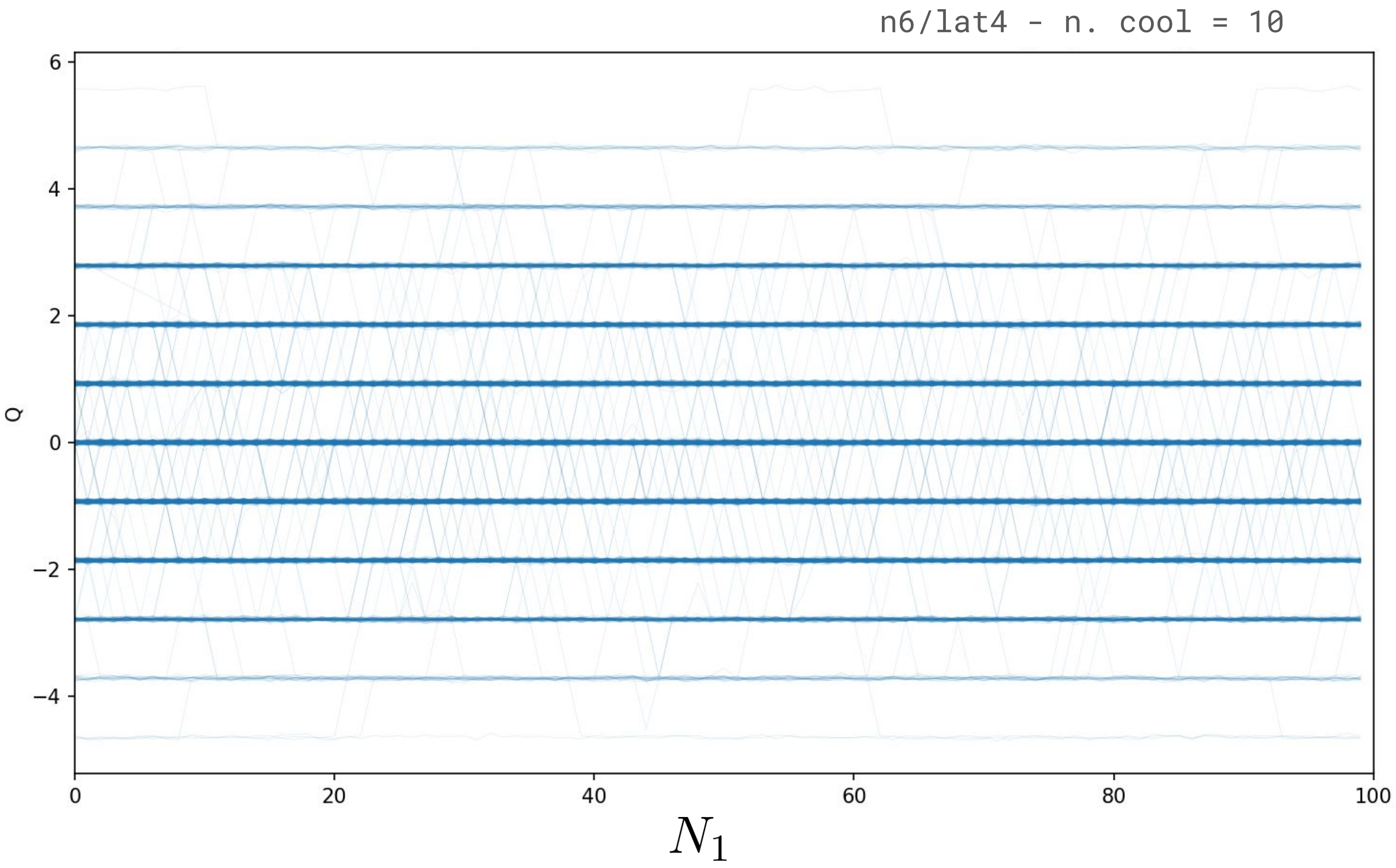
[1] Bochicchio M., **An asymptotic solution of Large-N QCD, for the glueball and meson spectrum and the collinear S-matrix** AIP Conf.Proc. 1735 (2016), *and* Bochicchio M., *to appear on arXiv*

Thank you for your kind attention!

Backup slides

n. cool (CabMar) =10





Along the "inner" markov chain we observe Q autocorrelation comparable to the "outer" markov chain