Radiative decays of charmonia

S. Collins, F. Knechtli, **J. Koponen**,
T. Korzec, A. Risch, M. Rodekamp,
J. A. Urrea-Niño, W. Söldner

The 43rd International Symposium
on Lattice Field Theory
University of Maryland, College Park, USA
July 26th - August 1st, 2026

Universität Regensburg

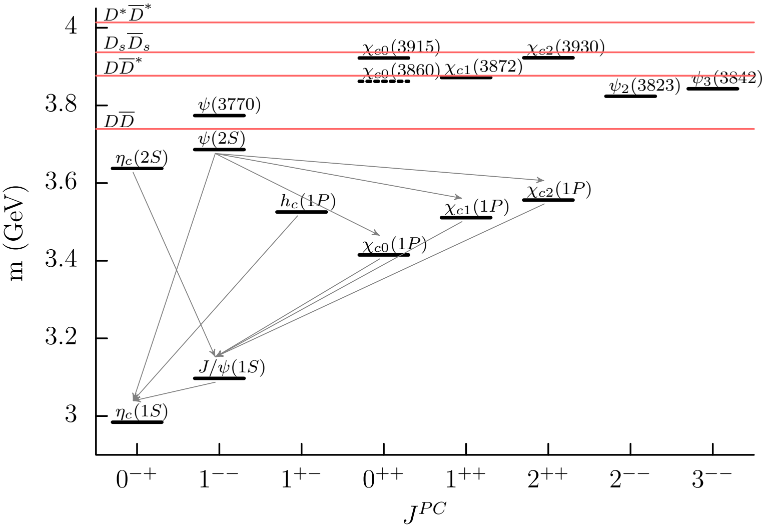
FOR 5269

BERGISCHE UNIVERSITÄT WUPPERTAL

Why charmonium radiative decays?

- Wealth of experimental data
- Decays $\chi_{c1}(3872) \to \psi(2S)\gamma$ and $\chi_{c1}(3872) \to J/\psi\gamma$ measured by Belle, LHCb and BESIII
- Radiative decay rates between the conventional $2S$ and $1S$ levels and $1P$ and $1S$ levels have also been measured
- Theory predictions for the decay rates aid the interpretation of the experimental results
- Electromagnetic (EM) form factors give information on the internal structure
- Start by studying charmonium radiative decays that are well understood (e.g. $J/\psi \to \eta_c\gamma$) to set up the framework
- Later study more exotic $c\bar{c}$ decays

Rich spectrum of charmonium states and $c\bar{c} \to c\bar{c}\gamma$ decays



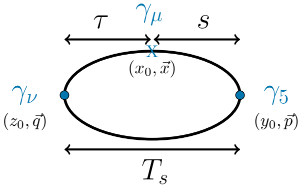
Noisy signal – distillation may help?

- Getting a good signal to noise ratio will be a challenge especially for higher excited states
- In the case of 2-point functions, it has been shown that distillation with optimal profiles helps in extraction of excited states
- Optimized profiles should work well here, hopefully enabling to get a good signal also for the noisier 3-point functions
- Use the building blocks of distillation framework that are already available: eigenvectors, perambulators, elementals
- Calculate generalized perambulators for the current insertion using point sources

3-point functions and distillation

The 3-point function corresponding to the decay $J/\psi \to \eta_c \gamma$ can be written as

$$\mathrm{tr}\Big[\Phi^{[y_0,-\vec{p},-\gamma_5,r_2]}\tau^{[y_0,z_0]}\Phi^{[z_0,\vec{q},\gamma_\nu,r_1]}G^{[x,\mu,z_0,y_0]}\Big]$$



The building blocks of the distillation framework are

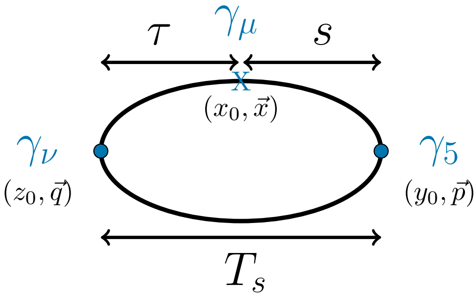
- the elementals

$$\Phi^{[t,\vec{p},\Gamma,r]} = \mathcal{P}^{[t,r]} \odot \phi^{[t,\vec{p}]} \otimes \Gamma, \quad \phi^{[t,\vec{p}]}_{i,j} = \sum_{\vec{x},a} \left(v_i^{[t]}(\vec{x},a)\right)^\dagger \mathrm{e}^{-i\vec{p}\cdot\vec{x}} v_j^{[t]}(\vec{x},a),$$

- the perambulators $\tau^{[x_0,y_0]}_{i\alpha,j\beta} = \left(V_i^{[x_0,\alpha]}\right)^\dagger D^{-1} V_j^{[y_0,\alpha]}$

- with eigenvectors $V_i^{[t,\alpha]}(x,\beta,a) = \delta_{\alpha,\beta}\delta_{t,x_0} v_i^{[t]}(\vec{x},a)$.

$$G_{i\alpha,j\beta}^{[x,\mu,z_0,y_0]} = \sum_{\vec{y},\vec{z}} \left(V_i^{[z_0,\alpha]}(z)\right)^\dagger D^{-1}(z,x)\gamma_\mu D^{-1}(x,y)V_j^{[y_0,\beta]}(y)$$

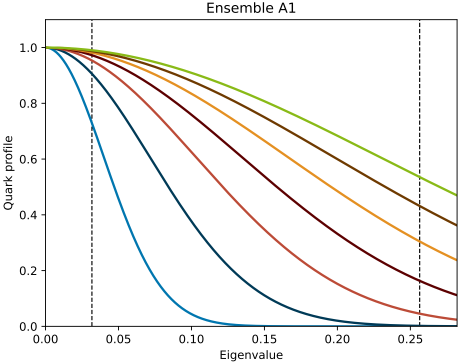


Optimal distillation profiles

- Use seven Gaussian basis profiles

$$\mathcal{P}_{i,j}^{[t,r]} = \delta_{i,j} \mathrm{e}^{-\left(\lambda_i^{[t]}\right)^2 / 2\sigma_s^2}$$

- Solve GEVP to get the optimized profile for a given state
- Optimized profile improves overlap with the desired state
- Construct 3-point functions using the optimized profiles



ArXiv:2205.11564
ArXiv:2603.20178

Lattice set-up

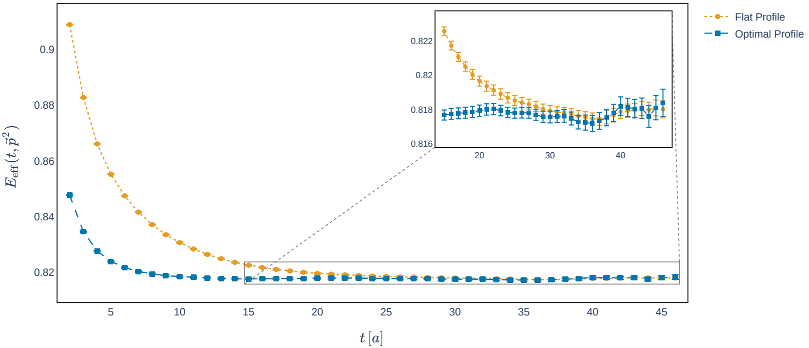
Preliminary results on ensemble A1

- $N_f = 3 + 1$ Wilson quarks with a non-perturbatively determined clover coefficient in a massive $\mathcal{O}(a)$ improvement scheme
- u, d and s quarks are degenerate, with the sum of their masses being equal to its value in nature
- physical charm quark mass
- open boundary conditions in time direction
- lattice spacing $a = 0.052$ fm
- pion mass $m_\pi \approx 420$ MeV
- volume $L^3 \times T = 32^3 \times 96$

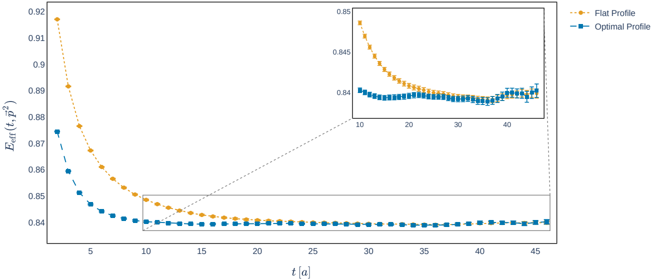
ArXiv:2002.02866

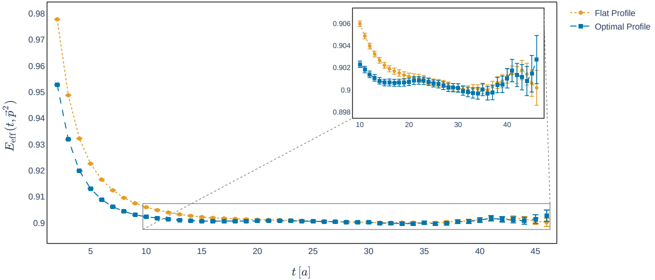
Optimal profiles and 2-point functions

In the case of 2-point functions, optimal profiles have been shown to work very well in isolating the wanted state (e.g. the ground state).



See also the original paper ArXiv:2205.11564

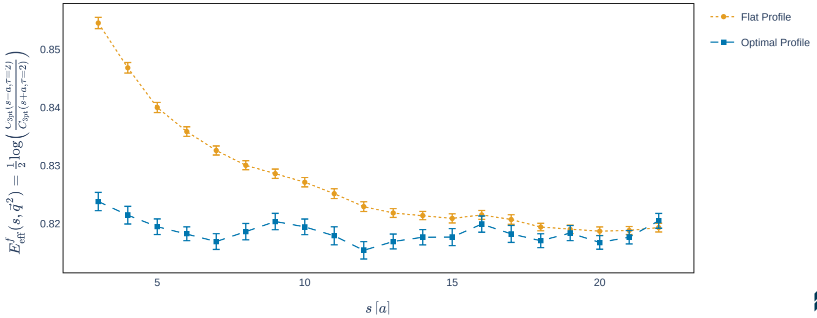

Flat Profile
Optimal Profile
$E_{\mathrm{eff}}(t,\vec{p}^2)$
$t\,[a]$


Flat Profile
Optimal Profile
$E_{\mathrm{eff}}(t,\vec{p}^2)$
$t\,[a]$

Optimal profiles and 3-point functions: $\eta_c \to \eta_c$ at rest

One can define an "effective mass" for the 3-point function:

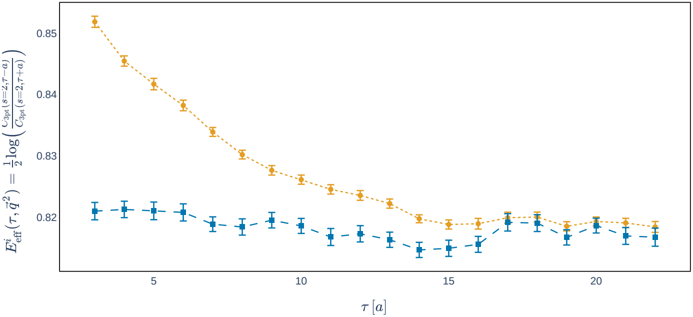
$$E^f_{\text{eff}}(s) = \frac{1}{2}\log\left(\frac{C_{3\text{pt}}(s-a,\tau)}{C_{3\text{pt}}(s+a,\tau)}\right)$$



Optimal profiles and 3-point functions: $\eta_c \to \eta_c$ at rest

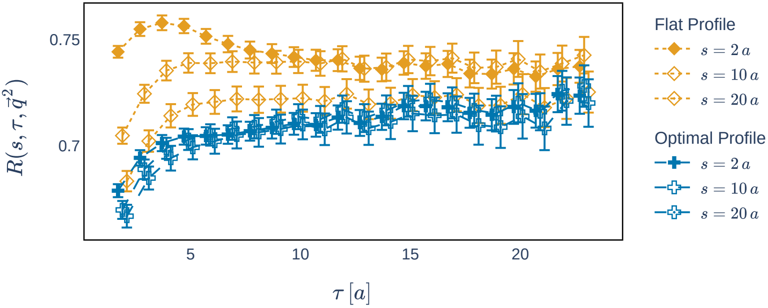
Similarly one can define:

$$E^i_{\text{eff}}(\tau) = \frac{1}{2}\log\left(\frac{C_{3\text{pt}}(s,\tau-a)}{C_{3\text{pt}}(s,\tau+a)}\right)$$



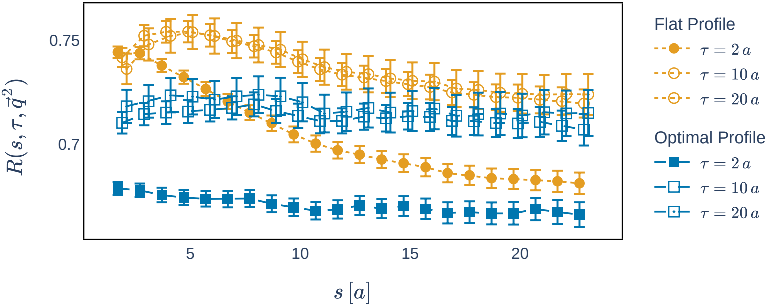
Ratios: $\langle \eta_c(\vec{p}') | \gamma^0 | \eta_c(\vec{0}) \rangle$, $\vec{p}' = (0, 0, 2)$ as an example

$$R(s,\tau) = \frac{C^{fi}_{3pt}(s,\tau)}{\sqrt{C^{f}_{2pt}(2s)C^{i}_{2pt}(2\tau)}} = \widetilde{A}_{00} + \text{excited states}$$



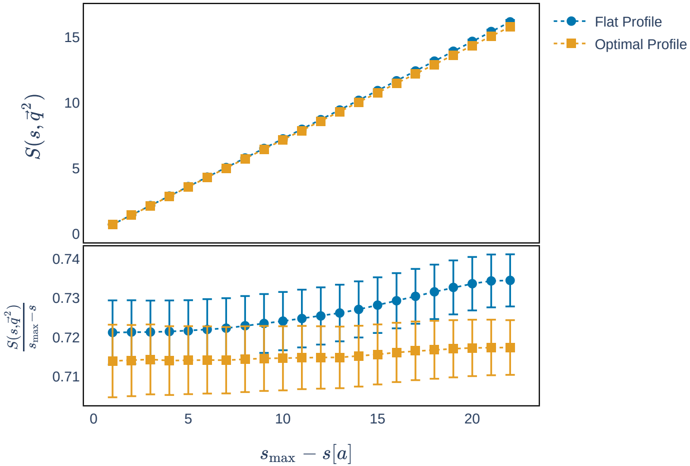
Ratios: $\langle \eta_c(\vec{p}') | \gamma^0 | \eta_c(\vec{0}) \rangle$, $\vec{p}' = (0,0,2)$ as an example

$$R(s,\tau) = \frac{C^{fi}_{3pt}(s,\tau)}{\sqrt{C^f_{2pt}(2s)C^i_{2pt}(2\tau)}} = \widetilde{A}_{00} + \text{excited states}$$



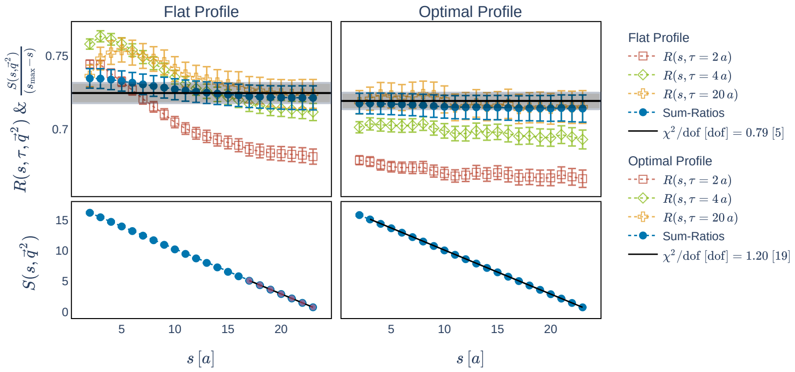
Summation method

$$S(s,\vec{q}^2) = \frac{1}{N_\tau} \sum_{\tau=\tau_{\min}}^{\tau_{\max}} \left[\sum_{s',\tau'=s,\tau}^{s_{\max},\tau_{\max}} \frac{R(s',\tau')}{\tau_{\max}-\tau'} \right] = \widetilde{A}_{00}\,(s_{\max}-s) + \ldots$$



Summary of the analysis

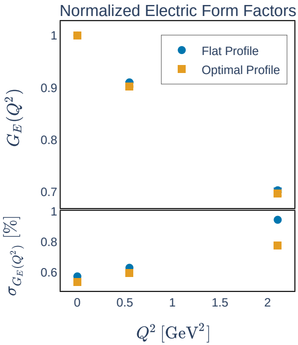
$$R(s,\tau) = \frac{C_{3pt}^{f\,i}(s,\tau)}{\sqrt{C_{2pt}^{f}(2s)C_{2pt}^{i}(2\tau)}} \quad \wedge \quad S(s,\vec{q}^2) = \frac{1}{N_\tau}\sum_{\tau=\tau_{\min}}^{\tau_{\max}}\left[\sum_{s',\tau'=s,\tau}^{s_{\max},\tau_{\max}} \frac{R(s',\tau')}{\tau_{\max}-\tau'}\right]$$

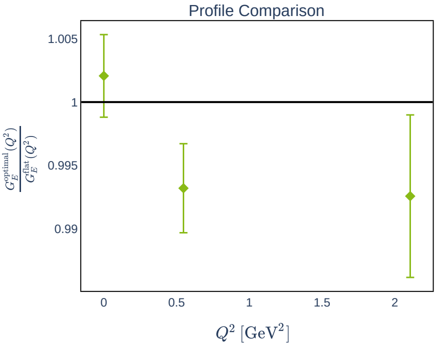


η_c form factor

Only a single form factor (charge form factor)

$$\langle \eta_c(\vec{p}')|j^\mu|\eta_c(\vec{p})\rangle = (p+p')^\mu G^E_{\eta_c}(Q^2); \quad Q^2 = -(E-E')^2 + |\vec{p}-\vec{p}'|^2$$





Can compare with earlier work, e.g. ArXiv:2301.08213

Conclusions

- Rich spectrum of charmonium states (including glue-rich states) and radiative decays
- Only some of the decays (e.q. $J/\psi \to \eta_c \gamma$) are well understood — plenty to study
- Experimental data from Belle, LHCb, BESIII, CLEO-c
- Lattice QCD predictions can aid the interpretation of the experimental results
- Distillation framework is well suited for the task

TO DO list

- More Q^2 points, extract form factors from the matrix elements
- Fit Q^2 dependence
- Look at decays that have noisier signal to really test the method
- More ensembles (lattice spacing and finite volume dependence)
- ...

THANK YOU!

Backup slides

Generalized perambulators

$$G_{i\alpha,j\beta}^{[x,\mu,z_0,y_0]} = \sum_{\vec{y},\vec{z}} \left(V_i^{[z_0,\alpha]}(z)\right)^\dagger D^{-1}(z,x)\gamma_\mu D^{-1}(x,y) V_j^{[y_0,\beta]}(y)$$

1. Compute D^{-1} for fixed x, at the cost of 12 inversions.
2. For every i, α and z_0, compute (no inversions)

$$\left(\eta_i^{[z_0,\alpha]}(x)\right)^\dagger = \sum_{\vec{z}} \left(V_i^{[z_0,\alpha]}(z)\right)^\dagger D^{-1}(z,x)$$

3. For every i, α and z_0, compute (no inversions)

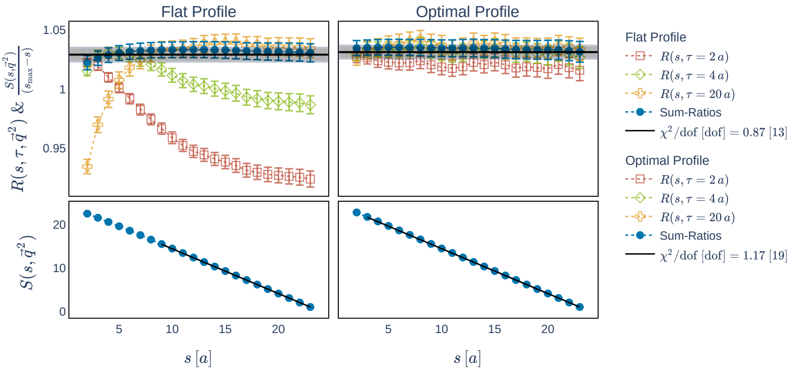
$$\left(\xi_i^{[z_0,\alpha]}(x)\right)^\dagger = \sum_{\vec{z}} \left(V_i^{[z_0,\alpha]}(z)\right)^\dagger \gamma_5 D^{-1}(z,x)\gamma_5$$

4. Combine to get the generalized perambulator

$$G_{i\alpha,j\beta}^{[x,\mu,z_0,y_0]} = \left(\eta_i^{[z_0,\alpha]}\right)^\dagger \gamma_\mu \xi_j^{[y_0,\beta]}$$

Renormalising the current

As we are using the local operators γ_μ rather than the conserved current, we need to calculate Z_V and renormalise it



η_c form factor

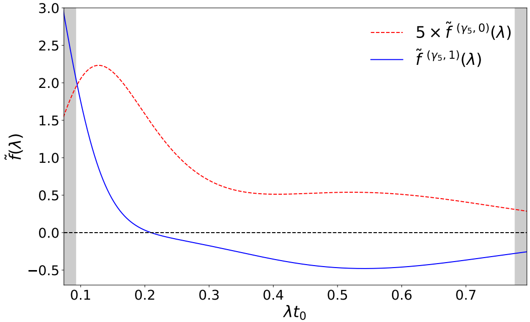
- The matrix element of a pseudoscalar meson, e.g. η_c, has a decomposition involving a single form factor:

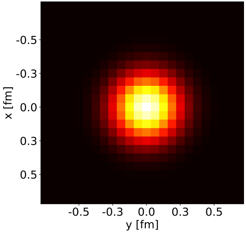
$$\langle \eta_c(\vec{p}')|j^\mu|\eta_c(\vec{p})\rangle = (p+p')^\mu F_{\eta_c}(Q^2)$$

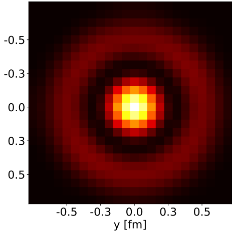
- The form factor only depend on the virtuality of the photon, $Q^2 = -(E-E')^2 + |\vec{p}-\vec{p}'|^2$

- $\langle r^2\rangle = -6 \left.\dfrac{\mathrm{d}F(Q^2)}{\mathrm{d}Q^2}\right|_{Q^2=0}$, the charge radius, can be determined from the Q^2 dependence of the FF at $Q^2=0$

Optimal profiles: ArXiv 2205.11564







J/ψ form factors

- A vector meson has three radiative form factors and the relevant matrix element can be written as

$$
\begin{aligned}
&\langle J/\psi(\lambda', \vec{p}')|j^\mu|J/\psi(\lambda, \vec{p})\rangle = \\
&\quad - (p+p')^\mu \left[\epsilon^*(\lambda', \vec{p}') \cdot \epsilon(\lambda, \vec{p})\right] G_1(Q^2) \\
&\quad + \left\{\epsilon^\mu(\lambda, \vec{p})\left[\epsilon^*(\lambda', \vec{p}') \cdot p\right] + \epsilon^{\mu *}(\lambda', \vec{p}')\left[\epsilon(\lambda, \vec{p}) \cdot p'\right]\right\} G_2(Q^2) \\
&\quad - (p+p')^\mu \left[\epsilon^*(\lambda', \vec{p}') \cdot p\right]\left[\epsilon(\lambda, \vec{p}) \cdot p'\right] \frac{1}{2m^2} G_3(Q^2)
\end{aligned}
$$

  where λ is the helicity and ϵ is the polarization vector

- There is no unique decomposition of this matrix element in terms of form factors: any linearly independent set of form factors can be used

- The chosen set is often the charge E_0, magnetic-dipole M_1 and electric quadrupole E_2 form factors

Transition form factors: $J/\psi \to \eta_c\gamma$ $\quad (J^{PC} = 1^{--} \to J^{PC} = 0^{-+})$

- The simplest physical process to study is the $J/\psi \to \eta_c\gamma$ radiative transition

- For a vector meson to pseudoscalar meson transition the kinematic decomposition is

$$\langle \eta_c(\vec{p}')|j^\mu|J/\psi(\lambda,\vec{p})\rangle = \epsilon^{\mu\nu\rho\sigma} p'_\nu p_\rho \epsilon_\sigma(\lambda,\vec{p}) \frac{2}{m_{j/\psi} + m_{\eta_c}} F_{J/\psi\eta_c}(Q^2)$$

- A single form factor, but again the helicity λ and the polarization vector $\epsilon_\sigma(\lambda,\vec{p})$ enter

- The value of the form factor at $Q^2 = 0$ can be related to the radiative partial width

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{64\alpha}{27} \frac{|\vec{q}|^3}{(m_{j/\psi} + m_{\eta_c})^2} \left|F_{J/\psi\eta_c}(0)\right|^2,$$

where $|\vec{q}|^2 = (m^2_{J/\psi} - m^2_{\eta_c})^2/(4m^2_{J/\psi})$

Transition form factors: $\chi_{c0} \to J/\psi\gamma$ $\quad (J^{PC} = 0^{++} \to J^{PC} = 1^{--})$

- The kinematic decomposition for the $\chi_{c0} \to J/\psi\gamma$ transition is more complicated, with multiple form factors

$$\langle \chi_{c0}(\vec{p}')|j^\mu|J/\psi(\lambda,\vec{p})\rangle =$$
$$\Omega^{-1}(Q^2)\Bigg(E_1(Q^2)\left[\Omega(Q^2)\epsilon^\mu(\vec{p},\lambda) - (\epsilon(\vec{p},\lambda)\cdot p')\left(p^\mu(p\cdot p') - m^2_{J/\psi}p'^\mu\right)\right]$$
$$+ \frac{C_1(Q^2)}{\sqrt{Q^2}} m_{J/\psi}\left(\epsilon(\vec{p},\lambda)\cdot p'\right)\left[(p\cdot p')(p+p')^\mu - m^2_{\chi_{c0}}p^\mu - m^2_{j/\psi}p'^\mu\right]\Bigg)$$

where $\Omega(Q^2) = (p\cdot p')^2 - m^2_{J/\psi}m^2_{\chi_{c0}}$

- Two form factors: the electric dipole $E_1(Q^2)$ and the longitudinal electric dipole $C_1(Q^2)$

- Only E_1 will contribute to the partial width

$$\Gamma(\chi_{c0} \to J/\psi\gamma) = \frac{16\alpha}{9}\frac{|\vec{q}|}{m^2_{\chi_{c0}}}\left|E_1(0)\right|^2,$$

where $\vec{q}$ is the momentum of the γ in the χ_{c0} rest frame

Transition form factors: $\chi_{c2} \to J/\psi\gamma$ $\quad (J^{PC} = 2^{++} \to J^{PC} = 1^{--})$

- The kinematic decomposition for the $\chi_{c2} \to J/\psi\gamma$ transition (and similarly for $\psi_2 \to \chi_{c1}\gamma$) reads

$$\langle \chi_{c2}(\lambda', \vec{p}')|j^\mu|J/\psi(\lambda, \vec{p})\rangle = a_1^\mu E_1(Q^2) + a_2^\mu M_2(Q^2) + a_3^\mu E_3(Q^2) + a_4^\mu C_1(Q^2) + a_5^\mu C_2(Q^2)$$

- There are five form factors: electric dipole E_1, magnetic quadrupole M_2, electric octupole $E3$, Coulomb dipole C_1 and Coulomb quadrupole C_2

- Only E_1, M_2 and E_3 will contribute to the partial width

$$\Gamma(\chi_{c2} \to J/\psi\gamma) = \frac{16\alpha}{45} \frac{|\vec{q}|}{m^2_{\chi_{c2}}} \left(|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2\right),$$

where $\vec{q}$ is the momentum of the γ in the χ_{c2} rest frame