Hadronic running of the electromagnetic coupling

Comparisons of perturbative and lattice results above 1GeV

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Background: Electroweak precision

Running of the coupling

- The standard model is very good…
- The electroweak sector sensitive to BSM physics
- $\alpha(m_Z^2)$ is a leading source of error on indirect determinations of M_H and $\sin^2\theta_W^{eff}$ using global EWPD fits

Running of the coupling

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)} \tag{1}$$

$$\Delta\alpha(s) = \Delta\alpha_{\mathrm{lep}}(s) + \Delta\alpha^{(5)}_{\mathrm{had}}(s) + \Delta\alpha_{\mathrm{top}}(s) + \ldots \tag{2}$$

- Thomson limit $\alpha^{-1}(0) = 137.035999177(21)$ [12]
- $\alpha^{-1}(m_Z^2) = 127.930(8)$ [11]
- $\Delta\alpha_{\mathrm{lep}}(m_Z^2) = 314.979(2) \times 10^{-4}$ [14]
- $\Delta\alpha_{\mathrm{top}}(m_Z^2) = -0.720(4) \times 10^{-4}$ [9]
- $\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2) = 275.8(10) \times 10^{-4}$[11]
  $\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2) = 278.21(34)_{\mathrm{lat}}(35)_{\mathrm{pQCD}} \times 10^{-4}$ [5]

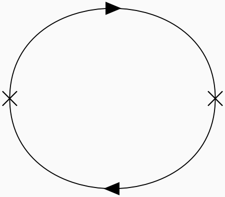
Desired precision of FCC-ee $\Delta\alpha^{(5)}_{\mathrm{had}}(m_Z^2) = 0.3 \times 10^{-4} \rightarrow < 10^{-5}$[2]

The hadronic component (timelike momenta)

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = 4\pi\alpha \text{Re}\bar{\Pi}(q^2), \quad \bar{\Pi}(q^2) = \Pi(q^2) - \Pi(0) \tag{3}$$

$$(q_\mu q_\nu - g_{\mu\nu} q^2)\Pi(q^2) = i \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0)\rangle \tag{4}$$

$$j_\mu = \sum_{q=u,d,s,c,b} \bar{\psi}_q \gamma_\mu \psi_q \tag{5}$$



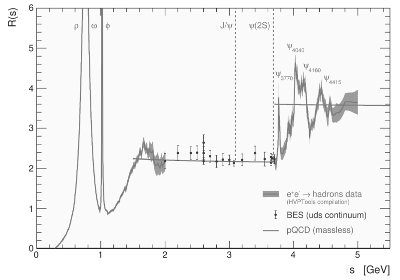
Adler function

HVP is a smooth function in Euclidean spacetime $Q^2 = -q^2$

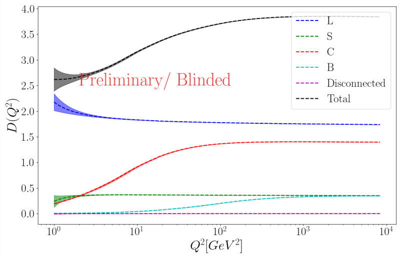
Adler function finite $\forall Q$ and tends to constant

$$D(Q^2) = 12\pi^2 Q^2 \frac{d\Pi(Q^2)}{dQ^2} \tag{6}$$

$$\Delta\alpha_{had}^{(5)}(Q^2) = \frac{\alpha}{3\pi}\int_0^Q dQ'^2 \frac{D(Q'^2)}{Q'^2} \tag{7}$$



(a) Figure from [6]



(b)

Euclidean split method

Low momenta : Lattice good, perturbation less good
High momenta : Lattice less good, perturbation very good

$$\begin{aligned}\Delta\alpha^{(5)}(m_z^2) = \Delta\alpha^{(5)}(-Q_0^2)_{\mathrm{LQCD}} + [\Delta\alpha^{(5)}(-m_z^2) - \Delta\alpha^{(5)}(-Q_0^2)]_{\mathrm{pQCD}} \\ +[\Delta\alpha^{(5)}(m_z^2) - \Delta\alpha^{(5)}(-m_z^2)]_{\mathrm{pQCD}}\end{aligned} \tag{8}$$

Final term analytically continues back to the real world,
$[\Delta\alpha^{(5)}(m_z^2) - \Delta\alpha^{(5)}(-m_z^2)]_{\mathrm{pQCD}} = 0.000045(2)$ [8]

Discrete Adler function

Use a telescopic sum to separate scales, defining the discrete Adler function,

$$\Delta\Pi(Q^2) = \hat{\Pi}(Q^2) - \hat{\Pi}(Q^2/2) \tag{9}$$

$$\hat{\Pi}(Q^2) = \sum_i \Delta\Pi(2^i) + \hat{\Pi}(1) \tag{10}$$

Calculate $\hat{\Pi}(1)$ then compute discrete Adler function $\Delta\Pi(2^i)$ for $i = 1, 2, 3, 4...$ to find $\Pi(Q^2)$ at desired momenta.
Different intervals have different systematics.

Time momentum Representation

$$\hat{\Pi}(Q^2) = \Pi(Q^2) - \Pi(0) = 2a \sum_t k_0(t, Q^2) \mathrm{Re} C(t) \tag{11}$$

$$C(t) = \frac{1}{3} \sum_{i=1}^{3} C_{ii}(t) = \frac{1}{3} \sum_{i=1}^{3} \int d^3x \langle j_i(t,x) j_i(0) \rangle \tag{12}$$

$$k_0(t, Q^2) = \frac{\cos(Qt) - 1}{Q^2} + \frac{t^2}{2} \tag{13}$$

$$\hat{\Pi}(Q^2) = \hat{\Pi}^{(l)}(Q^2) + \hat{\Pi}^{(s)}(Q^2) + \hat{\Pi}^{(c)}(Q^2) + \hat{\Pi}^{(b)}(Q^2) + \hat{\Pi}^{(disc)}(Q^2) \tag{14}$$

Use different discretisations of j_i and k_0 to estimate systematics

Blinding

- Lattice and perturbative analysis are currently separately blinded
- All perturbative results are rescaled by a random number between $\pm 3\%$
- Lattice results shown are from Ref. [10] (2023) and are not blinded

Perturbative calculation

Running parameters

- $\overline{MS}$ scheme
- Take reference values for parameters and run to relevant scale μ
- (De)couple at the mass scale, $m_q(m_q)$
- Two independent implementations compared with RunDec [13] and Adlerpy [7]

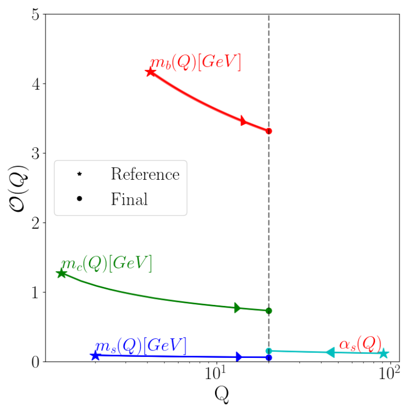


Figure 2: Running of parameters from reference scale to Q

Adler function

$$
\begin{aligned}
D_{\text{tot}}(Q) = D^{(\text{l})}(Q) + D^{(\text{s})}(Q) + D^{(\text{c})}(Q) + D^{(\text{b})}(Q) \\
+ D^{(\text{dis})}(Q) + D_{\text{QED}}(Q) + D_{\text{cond}}(Q)
\end{aligned}
\tag{15}
$$

$$
\hat{\Pi}(Q_2^2) - \hat{\Pi}(Q_1^2) = \frac{1}{12\pi^2} \int_{\log Q_1^2}^{\log Q_2^2} d\left(\log Q^2\right) D(Q^2). \tag{16}
$$

Errors:

- Systematic errors: truncation in α_s, heavy quark interpolation
- Parameter variation

Results

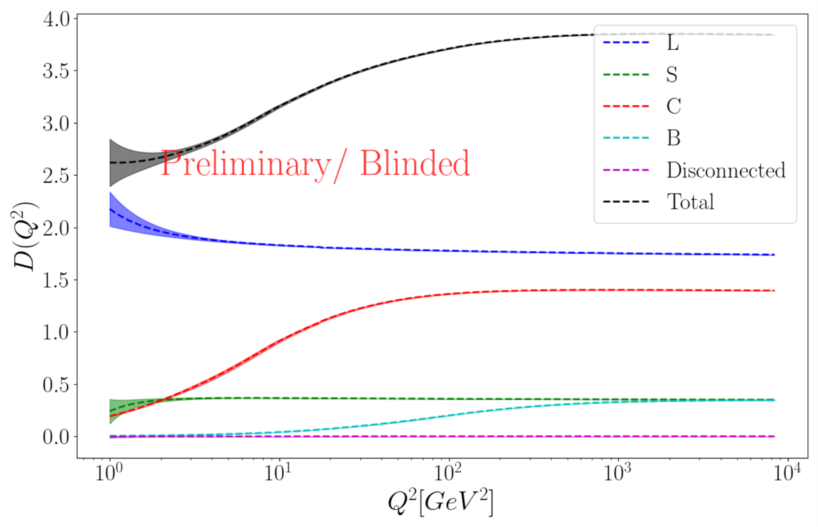


Figure 3: Perturbative results for Adler function.

Results

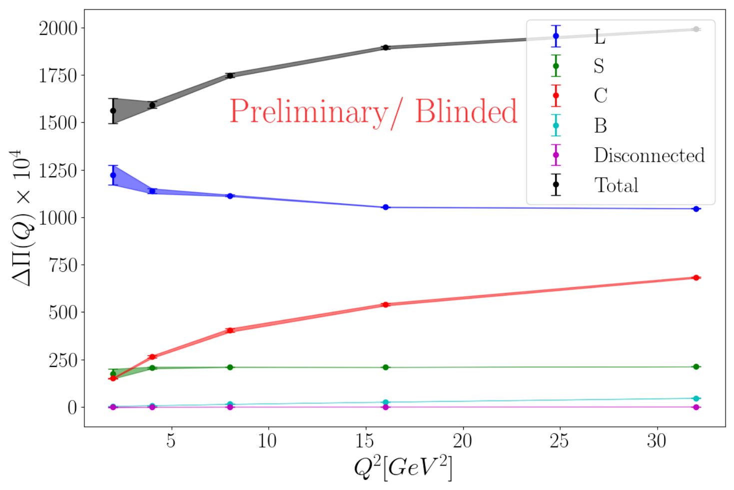


Figure 4: Perturbative results for
$\Delta\Pi(Q^2) = \hat{\Pi}(Q^2) - \hat{\Pi}(Q^2/2) = \frac{1}{12\pi^2}\int_{\log Q^2/2}^{\log Q^2} d\left(\log Q^2\right) D(Q^2)$

Results

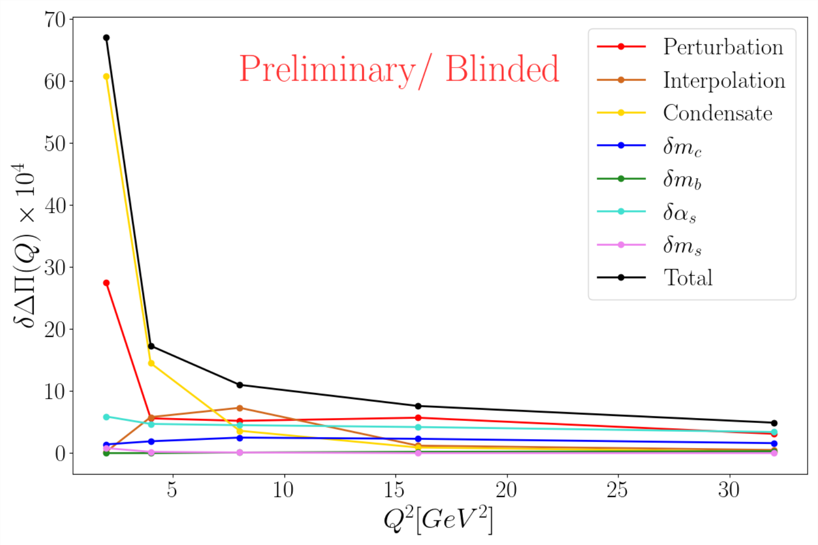


Figure 5: Error contributions on perturbative results for $\Delta\Pi(Q^2) = \hat{\Pi}(Q^2) - \hat{\Pi}(Q^2/2)$

Results

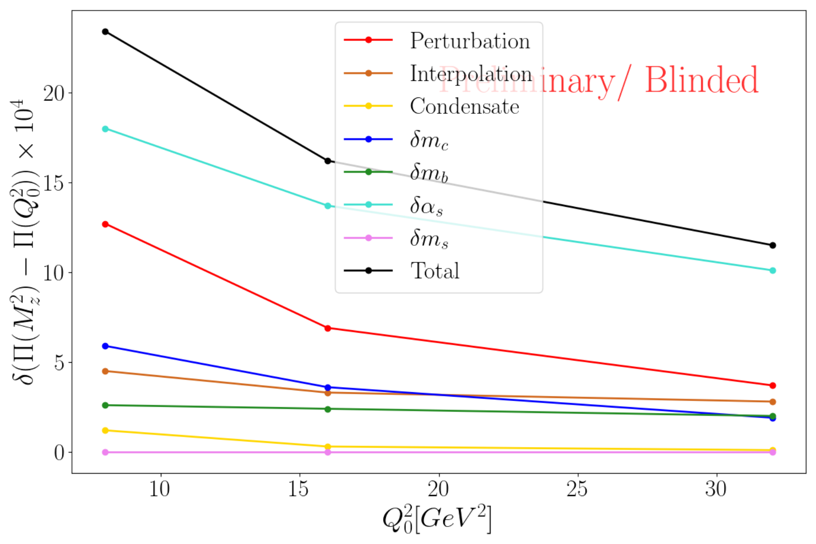


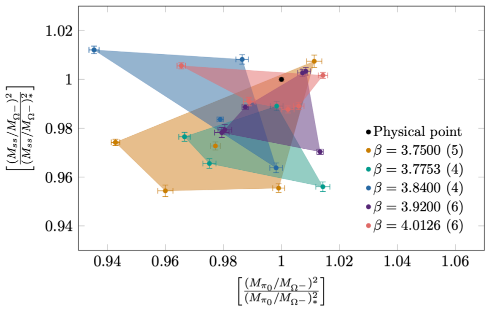
Figure 6: Error contributions on perturbative results for running to m_z

Lattice calculation

Lattice setup

In Ref. [10] (2023)

- 4Stout ensembles
- 2+1+1 flavours
- 6fm box
  - 5 spacings 0.118fm - 0.0640fm (now additional spacings: 0.048fm[3] and 0.036fm)
  - Physical point: $\{aM_{\Omega^-}, \frac{M_{\pi_0}}{M_{\Omega^-}}, \frac{M_{ss}}{M_{\Omega^-}}, m_c/m_s = 11.85, \Delta M^2, e\}$
- Ensembles bracket physical point
- 4HEX ensembles used for finite volume corrections on the light

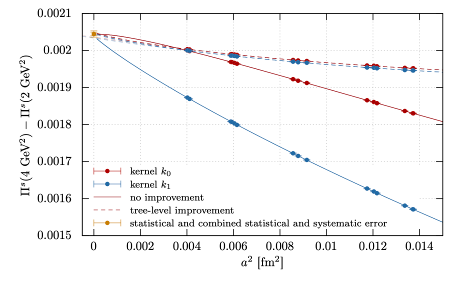


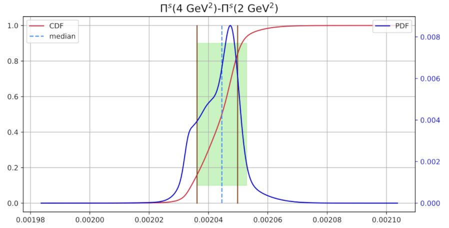
Fitting

Combine fits with combination of flat and Akaike information criterion (AIC) weighting

- Choice of kernel and improvements
- Tree-level improvement
- Vary physical masses
- Mass extraction ranges
- Finite volume corrections

Figures taken from [10] (2023), continuum limit for Π^s





Results

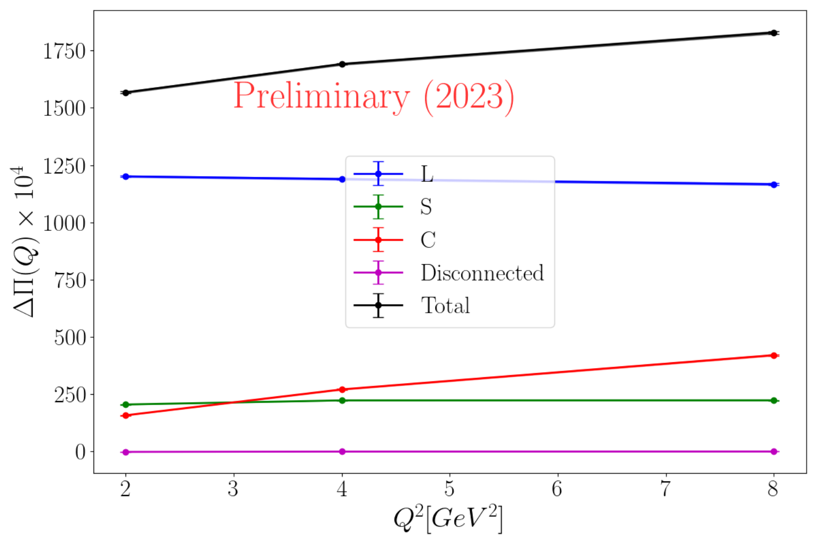


Figure 8: Results taken from from [10] (2023)

Next steps and outlook

Next steps

Once we are happy with both the perturbative and lattice calculations...

- In-depth comparison of lattice and perturbation
- Full unblinding and determine $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$

Conclusion

- Future electroweak precision experiments require improved precision for the calculation of $\alpha(m_z^2)$
- Leading source of error in running from the Thomson limit to m_z is from the hadronic component $\Delta\alpha_{\mathrm{had}}^{(5)}(m_z^2)$
- A hybrid approach of lattice and perturbation theory can precisely determine $\Delta\alpha_{\mathrm{had}}^{(5)}(m_z^2)$

Conclusion

- Future electroweak precision experiments require improved precision for the calculation of $\alpha(m_z^2)$
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- A hybrid approach of lattice and perturbation theory can precisely determine $\Delta\alpha_{\text{had}}^{(5)}(m_z^2)$
- Thanks for listening

Backup slides

Precision electro-weak fitting

Electroweak mixing angle, θ_W^{eff}, can be precisely determined in experiment

$$\sin^2\theta_W^{eff} = 0.23153 \pm 0.00016 \tag{17}$$

and from electroweak fitting

$$\begin{aligned} \sin^2\theta_W^{eff} = 0.231488 \pm & \quad 0.000029_{m_{top}} \pm 0.000015_{m_Z} \pm 0.000035_{\alpha(m_z)} \\ \pm & \quad 0.000010_{\alpha_s} \pm 0.000001_{m_H} \pm 0.000047_{\text{theory}} \\ = 0.23149 \pm & \quad 0.000070_{tot} \end{aligned} \tag{18}$$

Values from [1]

$\hat{\Pi}(1)$

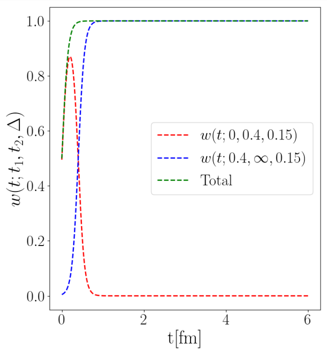
Can breakdown the light and disconnected components of $\hat{\Pi}(1)$ further into short and long-distance windows,

$$w(t;t_1,t_2,\Delta) = \Theta(t;t_1,\Delta) - \Theta(t;t_2,\Delta)$$
$$\Theta(t,t',\Delta) = \frac{1}{2}\left(1+\tanh\frac{(t-t)}{\Delta}\right)$$

$$\hat{\Pi}_{t_1-t_2} = \int dt w(t;t_1,t_2,\Delta)C(t)k(t,Q^2) \tag{19}$$

$$\hat{\Pi}(1) = \hat{\Pi}_{00-0.4} + \hat{\Pi}_{0.4-\infty} \tag{20}$$

Have separate dominant systematics at different distances



Fitting

$$[\mathcal{O}(a,Q^2)] = A + BX_l + CX_s + DX_{\delta_m} + Ee_s^2 + Fe_se_v + Ge_v^2 \tag{21}$$

$$X_l = \left[\frac{(M_{ll}/M_{\Omega}^{-})^2}{(M_{ll}/M_{\Omega}^{-})^2_*}\right] - 1; \quad X_s = \left[\frac{(M_{ss}/M_{\Omega^-})^2}{(M_{ss}/M_{\Omega^-})^2_*}\right] - 1 \tag{22}$$

$$A = A_0 + A_2(a^2\alpha_s^n(1/a)) + A_{2l}a^2 log(a^2) + A_4(a^2\alpha_s^n(1/a))^2 \tag{23}$$

$$B, C, F, G : f_0 + f_2a^2; \tag{24}$$

$$D, E : f_0 + f_2a^2 + f_lX_l + f_sX_s; \tag{25}$$

- A continuum extrapolation
- B, C interpolation to the physical point
- D strong isospin breaking
- E,F,G QED isospin

Fit variation

Combine fits with combination of flat and Akaike information criterion (AIC) weighting

- Fit functions
- Cut ensembles
- Vary physical masses
- Mass extraction ranges
- Choice of kernel and improvements
- Finite volume corrections

Physical point

Lattice parameters: $\{\beta, m_l, m_s, m_c, L/a\}$
Define the physical point through:
$\{aM_{\Omega^-}, \frac{M_{ll}}{M_{\Omega^-}}, \frac{M_{ss}}{M_{\Omega^-}}, m_c/m_s = 11.85, \Delta M^2, e\}$
$M^*_{\Omega^-} = 1672.45 MeV$ [11]
$M^*_{ll} = 135 MeV$
$M^*_{ss} = 689.89(28)(40)[49] MeV$ [4]
$\Delta M^2 = (M^2_{dd} - M^2_{uu}) = 13170(320)(270)[420] MeV^2 2$ [4]

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