Hadronic vacuum polarization contribution to the τ anomalous magnetic moment

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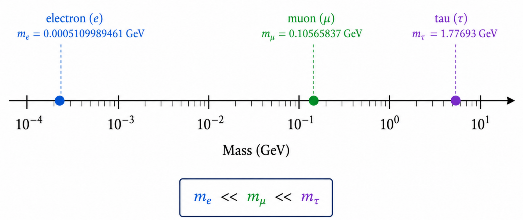
Why a_τ?

- Lepton anomalous magnetic moment

$$a_\ell = \frac{g_\ell - 2}{2}$$

- Contributions from NP at scale $\Lambda_{\rm NP}$ enters a_ℓ via

$$a_\ell - a_\ell^{\rm SM} \propto \frac{m_\ell^2}{\Lambda_{\rm NP}^2}$$



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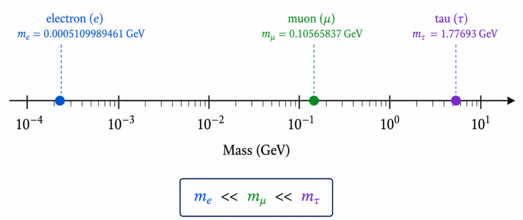

- Great theoretical sensitivity but experimentally challenging

$$\tau_\tau \approx 2.9 \times 10^{-13}\ \mathrm{s}$$

No storage-ring measurements as for a_μ

- Projected Belle II / FCC-ee sensitivity of $O(10^{-5})$

- Data-driven estimates based on $e^+e^- \to \mathrm{hadrons}$ are available

A first-principles determination of a_τ^{HVP} tests HVP consistency and prepares for future measurements

From a_μ to a_τ: probing shorter distances

- Time Momentum Representation (TMR)
  [Bernecker, Meyer 2011; Francis *et al.* 2013]

$$a_\ell^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}t K_\ell(t) G^{(\gamma,\gamma)}(t)$$

- $G^{(\gamma,\gamma)}(t)$: EM current correlator from QCD

  $\rightarrow$ same dataset as $a_\mu^{\mathrm{HVP,(N)LO}}$, $\Delta\alpha_{\mathrm{had}}(M_Z^2)$

- $K_\ell(t)$: known kernel function depending on m_ℓ

- $m_\tau \gg m_\mu$: shifts the relevant Euclidean-time region to shorter distances

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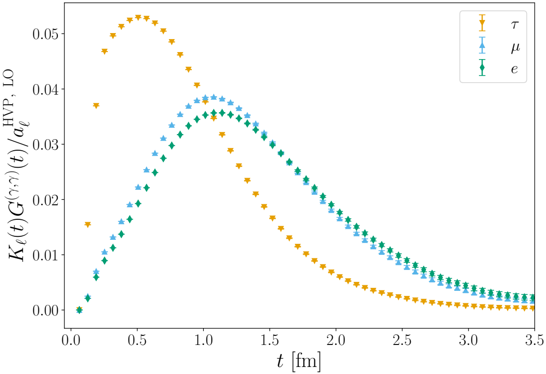
- Unit-area normalization isolates the distance sensitivity



See A. Beltran's talk for the latest a_e results

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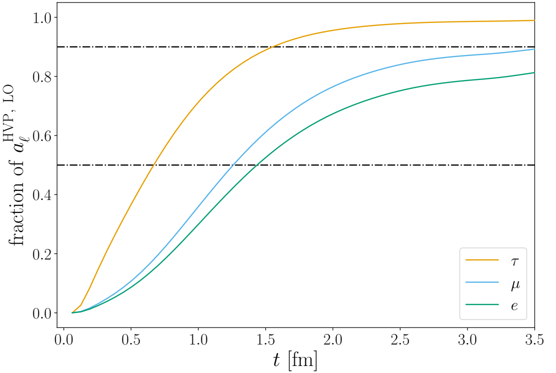
- Cumulative integral quantifies how rapidly the observable is saturated



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a_τ^{HVP} from Lattice QCD

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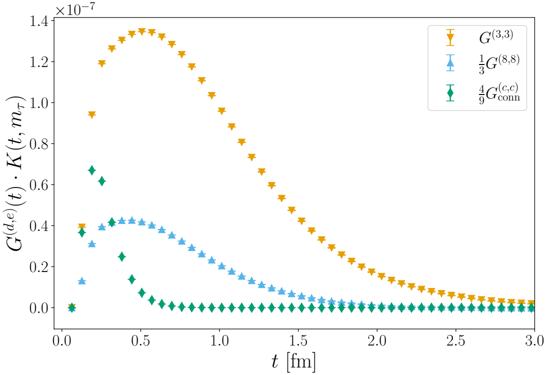
- Work in $SU(3)$-flavour basis $j_k^a = \bar{q}\gamma_k(\lambda_a/2)q$, $a = 3, 8, 0$

$$G_{\mu\nu}^{\gamma\gamma}(x) = G_{\mu\nu}^{33}(x) + \frac{1}{3}G_{\mu\nu}^{88}(x) + \frac{4}{9}G_{\mu\nu}^{cc}(x)$$



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- Work in $SU(3)$-flavour basis $j_k^a = \bar{q}\gamma_k(\lambda_a/2)q$, $a = 3, 8, 0$

- Use windows in the TMR to compute

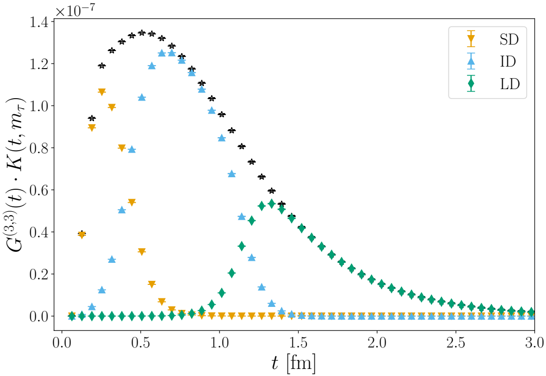
$$a_\tau^{\mathrm{HVP}} = (a_\tau^{\mathrm{HVP}})^{\mathrm{SD}} + (a_\tau^{\mathrm{HVP}})^{\mathrm{ID}} + (a_\tau^{\mathrm{HVP}})^{\mathrm{LD}}$$

  → SD: bulk of the cutoff effects

  → ID: best-controlled region

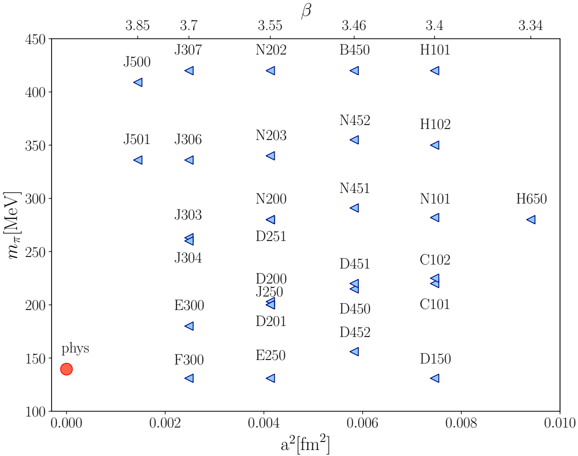
  → LD: signal-to-noise ratio

- Window boundaries adapted to the shorter-distance nature of a_τ



Lattice setup - CLS ensembles

- $N_f = 2+1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions [Lüscher and Schaefer, JHEP 1107 036 - JHEP 1502 043 - 1712.04884 - 2003.13359]
- Six values of $a \in [0.039, 0.098]$ fm
- Full range of pion masses $m_\pi \in [130, 420]$ MeV
- Two different chiral trajectories:
  - $\rightarrow$ $\Phi_4 \propto \mathrm{Tr}(M_q) = \Phi_4^{\text{phys}}$
  - $\rightarrow$ some ensembles on $m_s \approx m_s^{\text{phys}}$



Computational strategy

- Work in isospin decomposition of the electromagnetic current

$$j_\mu^{\mathrm{em}} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \ldots = j_\mu^{I=1} + j_\mu^{I=0} + \frac{2}{3}\bar{c}\gamma_\mu c + \ldots$$

- Two discretisations of the vector current, the local (L) and point-split conserved (C)

Set 1: Improvement coefficients from large-volume [1811.08209]

Set 2: Improvement coefficient from Scrödinger Functional setup [1805.07401, 2010.09539]

- Finite-volume corrections via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
  - → Correct to $(m_\pi L)_{\mathrm{ref}} = 4.29$ for $a \neq 0$ [Borsanyi et al., 2002.12347] [Djukanovic et al., 2411.07696]
  - → Correct to $L = \infty$ in the continuum at the physical point

Short distances: controlling cutoff effects

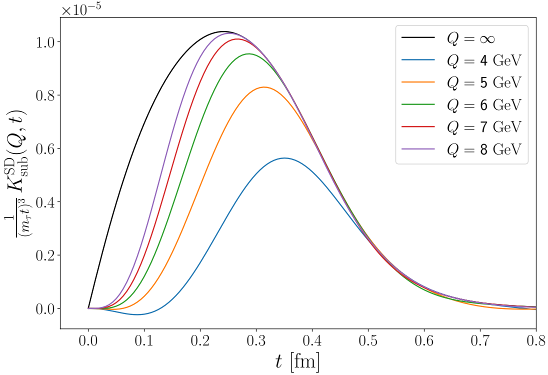
- Reliable control of SD contribution requires particular attention to $O(a^2 \log(a))$ lattice artefacts
  - $\rightarrow$ removal through pQCD in the high-energy spacelike regime [S. Kuberski *et al.*, 2401.11895]

- We modify the $K^{\mathrm{SD}}(t)$ QED kernel via

$$K^{\mathrm{SD}}_{\mathrm{sub}}(Q,t) = K^{\mathrm{SD}}(t) - \theta^{\mathrm{SD}}(0)\left(\frac{16\pi m_\tau}{3Q^2}\right)^2 \sin^4\left(\frac{Qt}{4}\right)$$

- Subtracted contribution restored with pQCD through the Adler function

$$b(Q^2) = \left(\frac{4m_\tau \alpha}{Q}\right)^2 \left[\Pi(Q^2) - \Pi(Q^2/4)\right]$$



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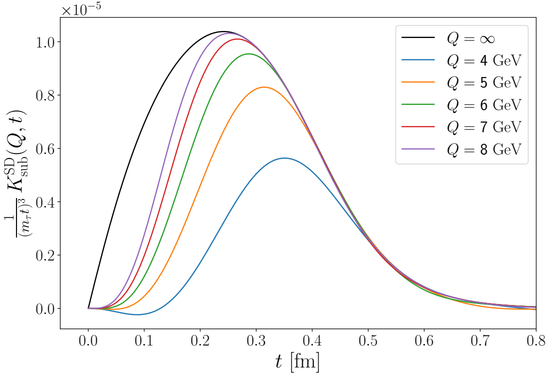
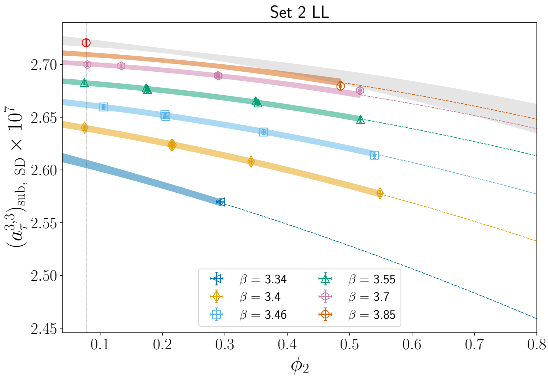
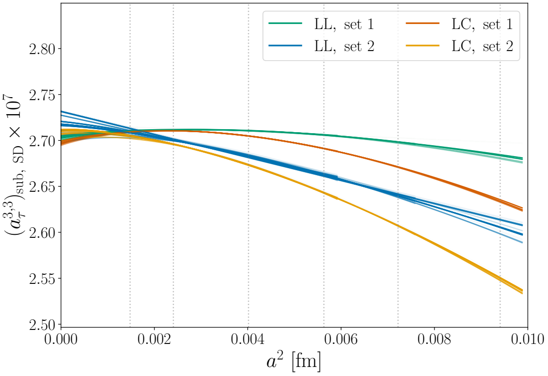


- Use tree-level improvement to reduce cutoff effects

$$\mathcal{O}(a) \rightarrow \mathcal{O}(a)\frac{\mathcal{O}^{\rm tl}(0)}{\mathcal{O}^{\rm tl}(a)}$$

$(a_\tau^{\mathrm{HVP}})^{\mathrm{SD}}$ in the isovector channel

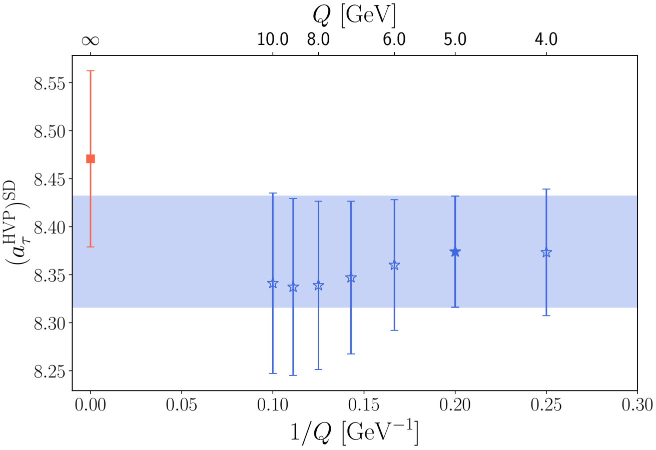




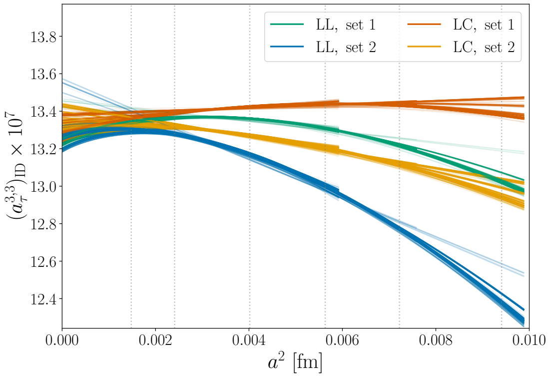
- Dependence of $(a_\tau^{(3,3)})^{\mathrm{SD}}$ on $\Phi_2 = 8t_0 m_\pi^2$ and a^2: mild chiral dependence, strong cutoff effects
- Each line represents a fit in the model average: four sets of data differ by $O(a^2)$
- $\sqrt{t_0}$ from a combination of f_π and f_K [A. Bussone *et al.*, 2510.20450]

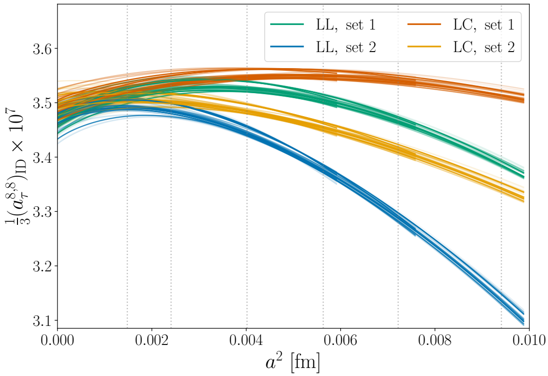
Full result for $(a_\tau^{\mathrm{HVP}})^{\mathrm{SD}}$

- Any dependence on the subtraction scale Q is removed after combining lattice and pQCD
- Noticeable shift when $a^2 \log(a)$ effects are not subtracted ($Q \to \infty$)
- Final uncertainty dominated by systematics involved in the continuum extrapolation
- Central values are blinded!



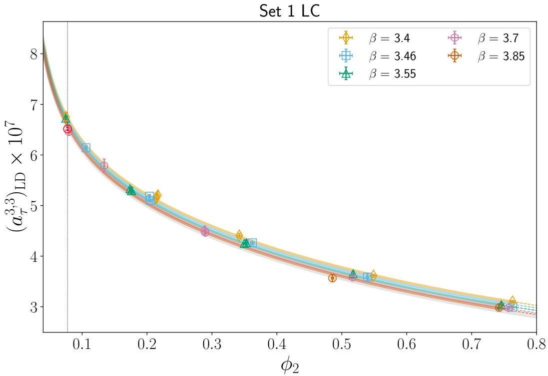
$(a_\tau^{\mathrm{HVP}})^{\mathrm{ID}}$ in the isovector and isoscalar channels

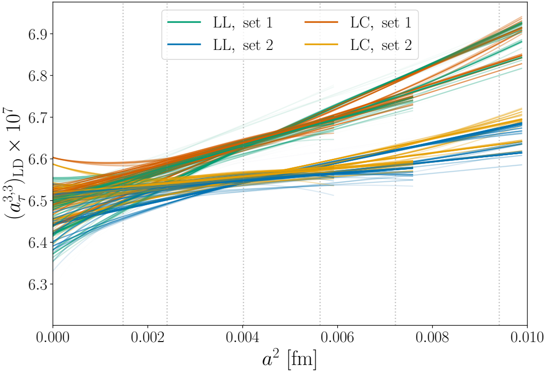




- a^2 dependence of $(a_\tau)^{\mathrm{ID}}$ in the isovector (*left*) and isoscalar (*right*) channels
- Intermediate distances account for the bulk of a_τ^{HVP}
- Higher-order cutoff effects remain relevant in $(a_\tau)^{\mathrm{ID}}$

Long distances: from dominant in a_μ to subleading in a_τ

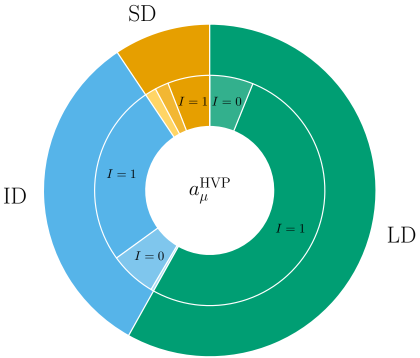


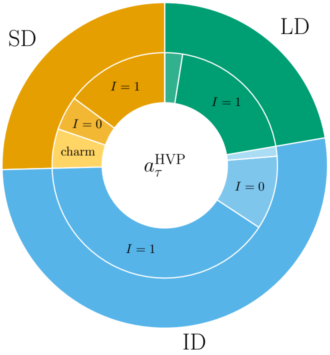


- Low-mode averaging and spectral reconstruction of the $\pi\pi$ contribution improve signal at large distances
- Chiral dependence well constrained across the range of pion masses
- Strong chiral dependence observed in the LD contribution

From a_μ to a_τ: where does the weight move?

- **Outer ring:** SD/ID/LD decomposition of a_ℓ^{HVP} for $\ell = \mu,\, \tau$
- **Inner ring:** $(I = 1)$, $(I = 0)$ and charm-connected contributions within each window

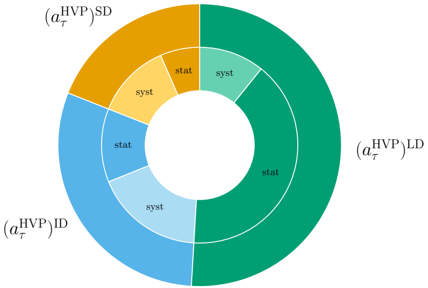




The tau shifts the HVP weight from long distances toward the intermediate- and short-distance regions

Benchmarking precision before unblinding

- Contribution to the variance of a_τ^{HVP}



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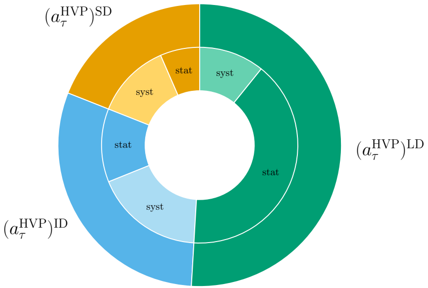
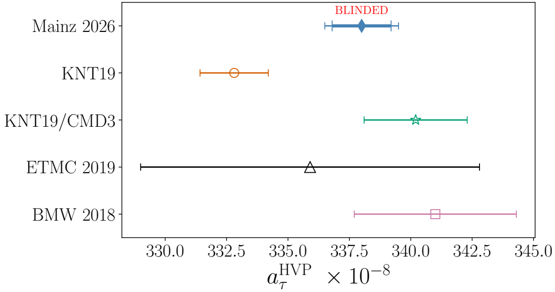


- a_τ^{HVP} central values are still blinded!



- LQCD achieves a precision on a_τ^{HVP} comparable to data-driven determinations

Conclusions

Summary

- We presented an ongoing LQCD determination of a_τ^{HVP} on CLS ensembles
- Compared to a_μ, the τ kernel shifts the relevant support to shorter Euclidean times

  $\rightarrow$ continuum extrapolation are hard to control
- Central value still blinded, achieved lattice precision comparable to data-driven evaluations
- Independent, first-principles benchmark for a_τ^{HVP}

Outlook

- Isospin-breaking corrections calculation (subleading for a_τ^{HVP})
- Unblind!

Thank You!

Related works of the Mainz group:

- A. Beltran, Wed 9:20
- D. Erb, Wed 11:10
- S. Lahrtz, Wed 11:30
- D. Albandea, Wed 11:50
- A. de Santis, Wed 9:00

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FVC and noise reduction strategies

- Finite-volume corrections via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
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- Low-Mode Averaging (LMA)
  - $\rightarrow$ Improved estimator for the light-connected correlation function
  - $\rightarrow$ LMA is used for all ensembles with $m_\pi < 280\ \mathrm{MeV}$

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- Low-Mode Averaging (LMA)
- Bounding Method in isovector and isoscalar channels
- Spectral reconstruction of the $\pi\pi$ contribution in isovector channel [Djukanovic et al., 2411.07696][Nolan Miller @ Lattice24]

- $N_{\pi\pi} = 4$ states used to reconstruct the correlator
- Solves signal-to-noise problem beyond $t = 2.5$ fm
- Increase the precision by a factor of 2 on physical point ensemble E250: 0.2% for $\bar{\Pi}^{(3,3)}$