

Hadronic vacuum polarization contribution to the τ anomalous magnetic moment

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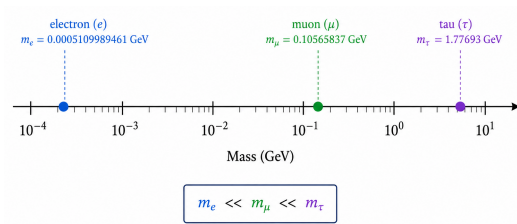
Why a_τ ?

- ▶ Lepton anomalous magnetic moment

$$a_\ell = \frac{g_\ell - 2}{2}$$

- ▶ Contributions from NP at scale Λ_{NP} enters a_ℓ via

$$a_\ell - a_\ell^{\text{SM}} \propto \frac{m_\ell^2}{\Lambda_{\text{NP}}^2}$$



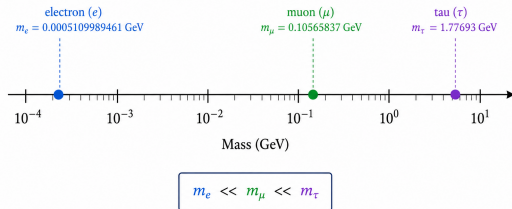
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- ▶ Great theoretical sensitivity but **experimentally challenging**

$$\tau_\tau \approx 2.9 \times 10^{-13} \text{ s}$$

No storage-ring measurements as for a_μ

- ▶ Projected Belle II / FCC-ee sensitivity of $O(10^{-5})$
- ▶ Data-driven estimates based on $e^+e^- \rightarrow \text{hadrons}$ are available

A first-principles determination of a_τ^{HVP} tests HVP consistency and prepares for future measurements

From a_μ to a_τ : probing shorter distances

- ▶ Time Momentum Representation (TMR)

[Bernecker, Meyer 2011; Francis *et al.* 2013]

$$a_\ell^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt K_\ell(t) G^{(\gamma,\gamma)}(t)$$

- ▶ $G^{(\gamma,\gamma)}(t)$: EM current correlator from QCD

→ same dataset as $a_\mu^{\text{HVP,(N)LO}}$, $\Delta\alpha_{\text{had}}(M_Z^2)$

- ▶ $K_\ell(t)$: known kernel function depending on m_ℓ
- ▶ $m_\tau \gg m_\mu$: shifts the relevant Euclidean-time region to shorter distances

From a_μ to a_τ : probing shorter distances

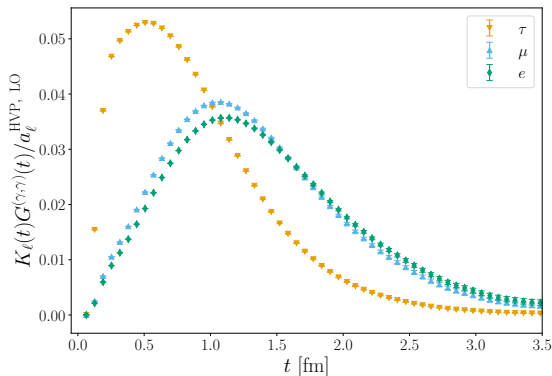
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- ▶ Unit-area normalization isolates the distance sensitivity



See A. Beltran's talk for the latest a_e results

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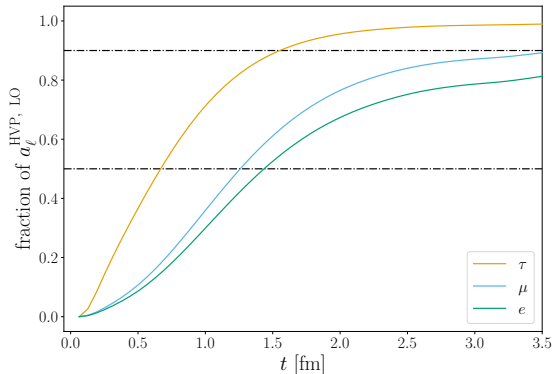
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- ▶ $m_\tau \gg m_\mu$: shifts the relevant Euclidean-time region to shorter distances

- ▶ Cumulative integral quantifies how rapidly the observable is saturated



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a_τ^{HVP} from Lattice QCD

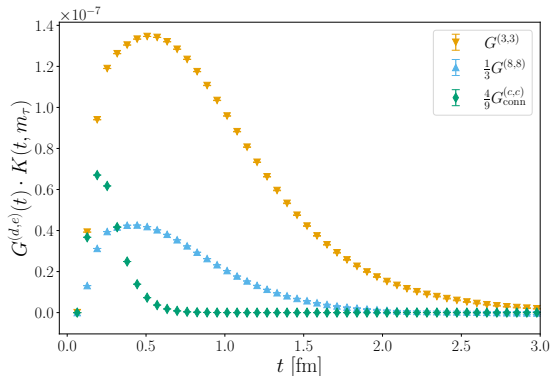
► Time Momentum Representation (TMR)

[Bernecker, Meyer 2011; Francis *et al.* 2013]

► Work in $SU(3)$ -flavour basis $j_k^a = \bar{q}\gamma_k(\lambda_a/2)q$, $a = 3, 8, 0$

$$G_{\mu\nu}^{\gamma\gamma}(x) = G_{\mu\nu}^{33}(x) + \frac{1}{3}G_{\mu\nu}^{88}(x) + \frac{4}{9}G_{\mu\nu}^{cc}(x)$$

$$a_\ell^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt K_\ell(t) G^{(\gamma,\gamma)}(t)$$



a_τ^{HVP} from Lattice QCD

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- ▶ Work in $SU(3)$ -flavour basis $j_k^a = \bar{q}\gamma_k(\lambda_a/2)q$,
 $a = 3, 8, 0$

- ▶ Use windows in the TMR to compute

$$a_\tau^{\text{HVP}} = (a_\tau^{\text{HVP}})^{\text{SD}} + (a_\tau^{\text{HVP}})^{\text{ID}} + (a_\tau^{\text{HVP}})^{\text{LD}}$$

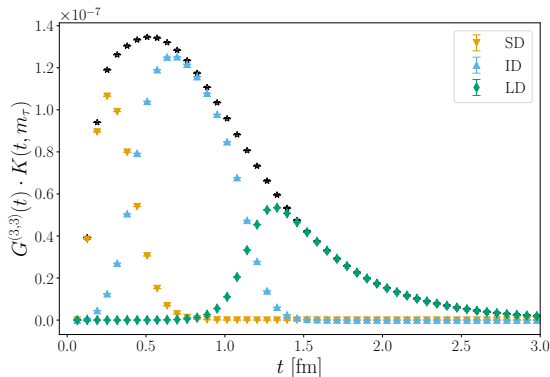
→ SD: bulk of the cutoff effects

→ ID: best-controlled region

→ LD: signal-to-noise ratio

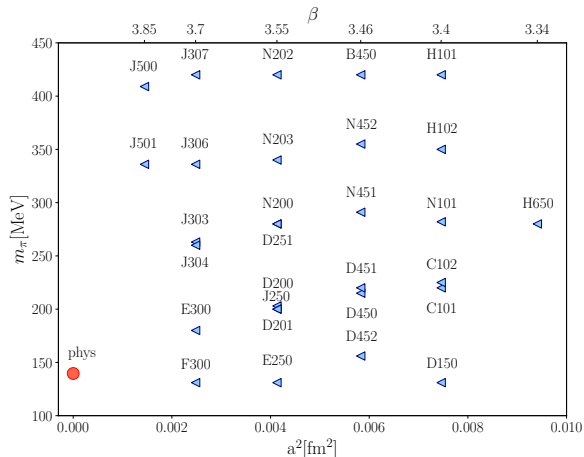
- ▶ Window boundaries adapted to the shorter-distance nature of a_τ

$$a_\ell^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt K_\ell(t) G^{(\gamma,\gamma)}(t)$$



Lattice setup - CLS ensembles

- ▶ $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$ -improved Wilson fermions [Lüscher and Schaefer, JHEP 1107 036 - JHEP 1502 043 - 1712.04884 - 2003.13359]
- ▶ Six values of $a \in [0.039, 0.098]$ fm
- ▶ Full range of pion masses $m_\pi \in [130, 420]$ MeV
- ▶ Two different chiral trajectories:
 - $\Phi_4 \propto \text{Tr}(M_q) = \Phi_4^{\text{phys}}$
 - some ensembles on $m_s \approx m_s^{\text{phys}}$



Computational strategy

- ▶ Work in isospin decomposition of the electromagnetic current

$$j_{\mu}^{\text{em}} = \frac{2}{3}\bar{u}\gamma_{\mu}u - \frac{1}{3}\bar{d}\gamma_{\mu}d - \frac{1}{3}\bar{s}\gamma_{\mu}s + \frac{2}{3}\bar{c}\gamma_{\mu}c + \dots = j_{\mu}^{I=1} + j_{\mu}^{I=0} + \frac{2}{3}\bar{c}\gamma_{\mu}c + \dots$$

- ▶ Two discretisations of the vector current, the local (L) and point-split conserved (C)

Set 1: Improvement coefficients from large-volume [1811.08209]

Set 2: Improvement coefficient from Schrödinger Functional setup [1805.07401, 2010.09539]

- ▶ **Finite-volume corrections** via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] pion form factor
 - Correct to $(m_{\pi}L)_{\text{ref}} = 4.29$ for $a \neq 0$ [Borsanyi et al., 2002.12347] [Djukanovic et al., 2411.07696]
 - Correct to $L = \infty$ in the continuum at the physical point

Short distances: controlling cutoff effects

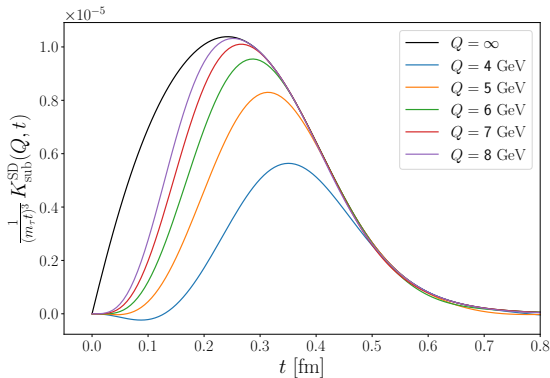
- ▶ Reliable control of SD contribution requires particular attention to $O(a^2 \log(a))$ lattice artefacts
 - removal through pQCD in the high-energy spacelike regime [S. Kuberski *et al.*, 2401.11895]

- ▶ We modify the $K^{\text{SD}}(t)$ QED kernel via

$$K_{\text{sub}}^{\text{SD}}(Q, t) = K^{\text{SD}}(t) - \theta^{\text{SD}}(0) \left(\frac{16\pi m_\tau}{3Q^2} \right)^2 \sin^4 \left(\frac{Qt}{4} \right)$$

- ▶ Subtracted contribution restored with pQCD through the Adler function

$$b(Q^2) = \left(\frac{4m_\tau \alpha}{Q} \right)^2 [\Pi(Q^2) - \Pi(Q^2/4)]$$



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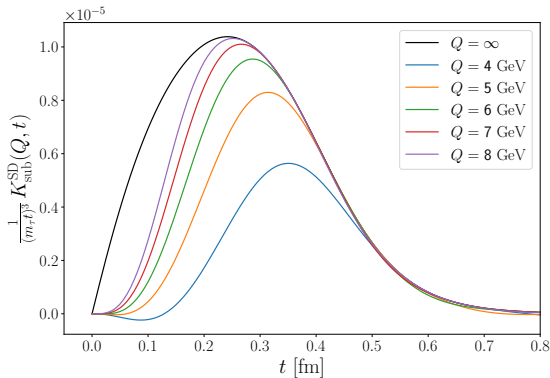
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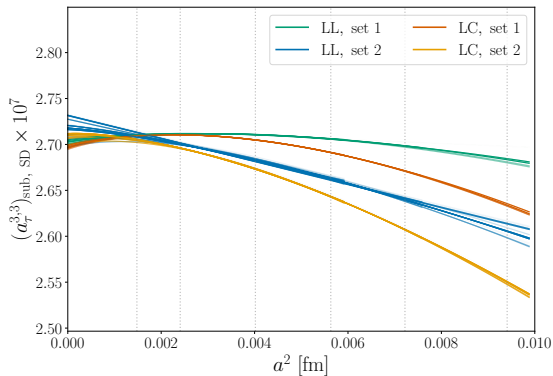
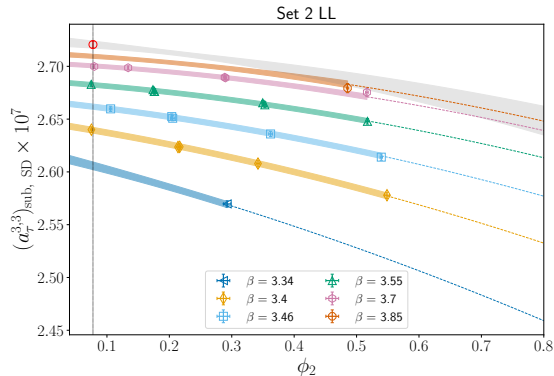
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- ▶ Use tree-level improvement to reduce cutoff effects



$$\mathcal{O}(a) \rightarrow \mathcal{O}(a) \frac{\mathcal{O}^{\text{tl}}(0)}{\mathcal{O}^{\text{tl}}(a)}$$

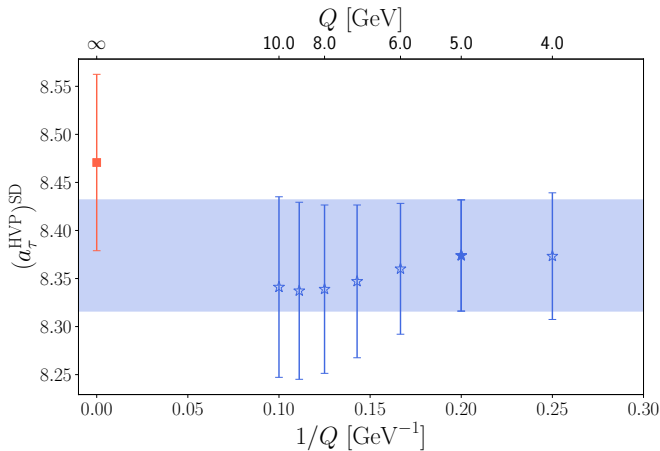
$(a_\tau^{\text{HVP}})^{\text{SD}}$ in the isovector channel



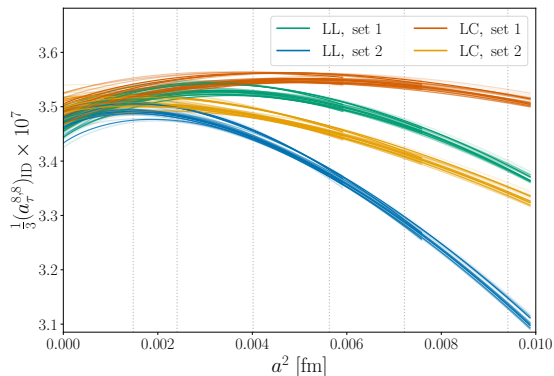
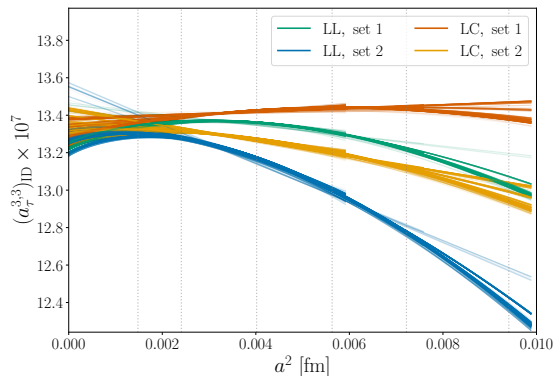
- ▶ Dependence of $(a_\tau^{(3,3)})^{\text{SD}}$ on $\Phi_2 = 8t_0 m_\pi^2$ and a^2 : mild chiral dependence, strong cutoff effects
- ▶ Each line represents a fit in the model average: four sets of data differ by $O(a^2)$
- ▶ $\sqrt{t_0}$ from a combination of f_π and f_K [A. Bussone *et al.*, 2510.20450]

Full result for $(a_\tau^{\text{HVP}})^{\text{SD}}$

- ▶ Any dependence on the subtraction scale Q is removed after combining lattice and pQCD
- ▶ Noticeable shift when $a^2 \log(a)$ effects are not subtracted ($Q \rightarrow \infty$)
- ▶ Final uncertainty dominated by systematics involved in the continuum extrapolation
- ▶ Central values are blinded!

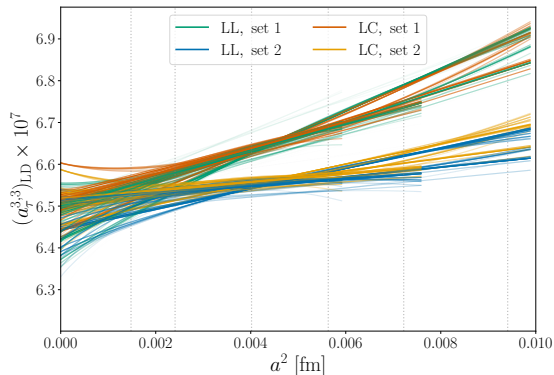
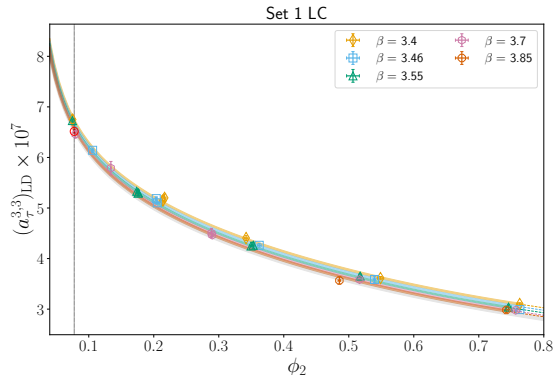


$(a_\tau^{\text{HVP}})^{\text{ID}}$ in the isovector and isoscalar channels



- ▶ a^2 dependence of $(a_\tau)^{\text{ID}}$ in the isovector (*left*) and isoscalar (*right*) channels
- ▶ Intermediate distances account for the bulk of a_τ^{HVP}
- ▶ Higher-order cutoff effects remain relevant in $(a_\tau)^{\text{ID}}$

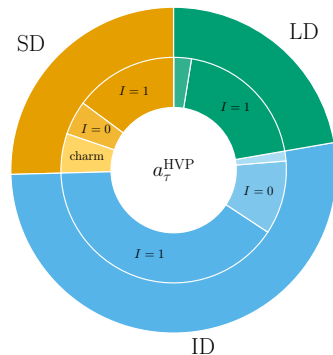
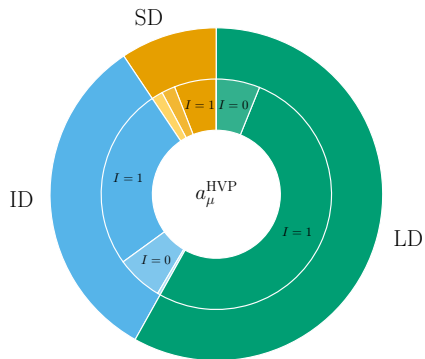
Long distances: from dominant in a_μ to subleading in a_τ



- ▶ Low-mode averaging and spectral reconstruction of the $\pi\pi$ contribution improve signal at large distances
- ▶ Chiral dependence well constrained across the range of pion masses
- ▶ Strong chiral dependence observed in the LD contribution

From a_μ to a_τ : where does the weight move?

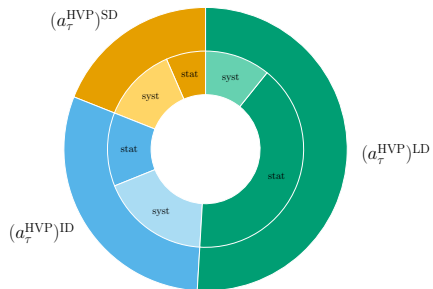
- ▶ **Outer ring:** SD/ID/LD decomposition of a_ℓ^{HVP} for $\ell = \mu, \tau$
- ▶ **Inner ring:** ($I = 1$), ($I = 0$) and charm-connected contributions within each window



The tau shifts the HVP weight from long distances toward the intermediate- and short-distance regions

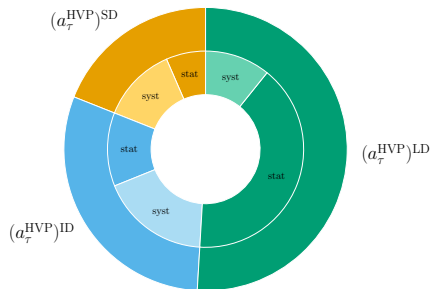
Benchmarking precision before unblinding

- ▶ Contribution to the variance of a_τ^{HVP}

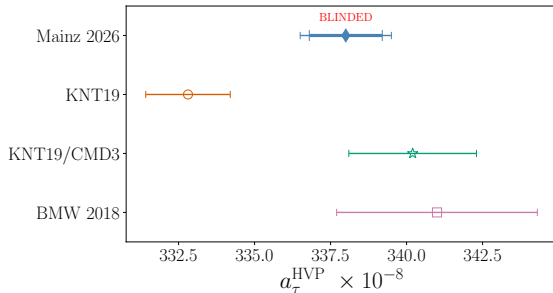


Benchmarking precision before unblinding

- ▶ Contribution to the variance of a_τ^{HVP}



- ▶ a_τ^{HVP} central values are still blinded!



- ▶ LQCD achieves a precision on a_τ^{HVP} comparable to data-driven determinations

Conclusions

Summary

- ▶ We presented an ongoing LQCD determination of a_τ^{HVP} on CLS ensembles
- ▶ Compared to a_μ , the τ kernel shifts the relevant support to shorter Euclidean times
 - continuum extrapolation are hard to control
- ▶ Central value still blinded, achieved lattice precision comparable to data-driven evaluations
- ▶ Independent, first-principles benchmark for a_τ^{HVP}

Outlook

- ▶ Isospin-breaking corrections calculation (subleading for a_τ^{HVP})
- ▶ Unblind!

Thank You!

Related works of the Mainz group:

- ▶ A. Beltran, Wed 9:20
- ▶ D. Erb, Wed 11:10
- ▶ S. Lahrtz, Wed 11:30
- ▶ D. Albadea, Wed 11:50
- ▶ A. de Santis, Wed 9:00



FVC and noise reduction strategies

- ▶ **Finite-volume corrections** via spacelike [Hansen and Patella, 1904.10010, 2004.03935] and timelike [Meyer, 1105.1892][Lellouch and Lüscher, hep-lat/0003023] **pion form factor**
 - Correct to $(m_\pi L)_{\text{ref}} = 4.29$ for $a \neq 0$ [Borsanyi et al., 2002.12347] [Djukanovic et al., 2411.07696]
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- ▶ **Low-Mode Averaging (LMA)**
 - Improved estimator for the light-connected correlation function
 - LMA is used for all ensembles with $m_\pi < 280$ MeV

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- ▶ **Low-Mode Averaging** (LMA)
- ▶ **Bounding Method** in isovector and isoscalar channels

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 - ▶ **Low-Mode Averaging** (LMA)
 - ▶ **Bounding Method** in isovector and isoscalar channels
 - ▶ **Spectral reconstruction** of the $\pi\pi$ contribution in isovector channel
[Djukanovic et al., 2411.07696][Nolan Miller @ Lattice24]
-
- ▶ $N_{\pi\pi} = 4$ states used to reconstruct the correlator
 - ▶ Solves signal-to-noise problem beyond $t = 2.5$ fm
 - ▶ Increase the precision by a factor of 2 on physical point ensemble E250: 0.2% for $\bar{\Pi}^{(3,3)}$

