Lattice chiral gauge symmetry via bosonization

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LATTICE 2026@College Park
27/07/2026

- Based on [SO, arXiv:2606.12358]

Introduction

- **Chiral gauge theories** are fundamental to particle physics.
  e.g., the Standard Model and GUTs
- Open problem: **Lattice formulations** of chiral gauge theories
  Challenge: realize chiral **fermions** on the lattice with **exact gauge invariance** in anomaly-free theories.
  - No-go theorem [Nielsen–Ninomiya'81]: "Naive" chiral fermions become vector-like due to doublers
  - Ginsparg–Wilson relation $\{D, \gamma^5\} = aD\gamma^5 D$: successful constructions include
    $U(1)$, [Lüscher'98], $SU(2)_L \times U(1)_Y$, [Kikukawa–Nakayama'00]
- A general lattice construction for non-Abelian chiral gauge theories **remains an open problem**.

Why focus on bosonization?

In **2D**, continuum bosonization motivates a bosonic lattice model: we regularize the bosonized description **instead of** chiral fermions directly.

Fermions $\Longleftrightarrow$ Bosons $\longrightarrow$ Lattice formulation.

Why this helps

- An ultra-local bosonic model realizes chiral symmetry exactly as in the continuum theory, without doublers. cf. [Baig–Chen–Cherman–Neuzil'26]
- The gauge anomaly is encoded directly in the gauge variation of the bosonic action, allowing a direct calculation.

Previous works: 2D $U(1)$ chiral gauge theories

[Berkowitz–Cherman–Jacobson'23], [DeMarco–Lake–Wen'23], [Morikawa–Onoda–Suzuki'24], [Thorngren–Preskill–Fidkowski'26], [Seifnashri'26]......

This work

Can the same strategy be extended to 2D non-Abelian $SU(N)$ chiral gauge theories?

Non-Abelian bosonization and gauge anomalies

Non-Abelian bosonization [Witten'83] gives the following duality: $g \in U(N_f)$

$$\int_{M_2} \mathrm{d}^2 x \sum_{i=1}^{N_f} \bar{\psi}_{L,i} \mathrm{i}\partial\!\!\!/ \psi_{L,i} + \bar{\psi}_{R,i} \mathrm{i}\partial\!\!\!/ \psi_{R,i} \iff \int_{M_2} \mathrm{d}^2 x \, \frac{1}{8\pi} \mathrm{Tr}\{|\partial_\mu g|^2\} + \underbrace{\frac{\mathrm{i}}{12\pi} \int_{M_3} \mathrm{Tr}\{(g^{-1}\mathrm{d}g)^3\}}_{M_2 = \partial M_3,\ \textbf{encodes anomalies}}$$

$$\psi_L \to V_L \psi_L,\ \psi_R \to V_R \psi_R \iff g \to V_L g V_R^{-1}, \quad V_{L/R} = e^{\mathrm{i} \sum_a \alpha^a t^a_{L/R}}$$

For **chiral** $SU(N)$ gauging with $t_L^a \neq t_R^a$, gauge invariance requires a **3D** bulk term:

$$S_{\mathrm{bos}} = \big(\text{terms on } M_2\big) - \mathrm{CS}^L_{M_3}[A] + \mathrm{CS}^R_{M_3}[A], \qquad M_2 = \partial M_3$$

Anomaly inflow: A gauge anomaly obstructs a purely **2D** gauge-invariant formulation and appears as **3D** bulk dependence.

$$\mathrm{Tr}\big(t_L^a t_L^b\big) = \mathrm{Tr}\big(t_R^a t_R^b\big) \quad \Longrightarrow \quad \text{bulk dependence cancels.}$$

Our goal: reproduce this structure at finite lattice spacing

Spectator fermions simplify the lattice problem

Technical difficulty: Directly treating a WZW field with

$$g \mapsto V_L g V_R^{-1}$$

is **not straightforward** in our lattice construction.

Equivalent reformulation: Add **free, gauge-neutral** "spectator" fermions:

$$\Psi_1 = (\psi_L, \psi_R^{\mathrm{sp}}) \Longleftrightarrow g_1, \qquad g_1 \mapsto V_L g_1,$$
$$\Psi_2 = (\psi_L^{\mathrm{sp}}, \psi_R) \Longleftrightarrow g_2, \qquad g_2 \mapsto g_2 V_R^{-1}.$$

Each WZW field is now gauged from **one side only**.

The original sector is **unchanged**

$$Z_{\mathrm{orig+sp}}[A] = Z_{\mathrm{orig}}[A] Z_{\mathrm{sp}}, \qquad \langle \mathcal{O} \rangle_{\mathrm{orig+sp}} = \langle \mathcal{O} \rangle_{\mathrm{orig}},$$

for any $\mathcal{O}[\psi_L, \psi_R, A]$ without spectator fields. We construct a lattice counterpart of the **spectator-added version of bosonized model**.

Lattice setup and our lattice action

Use a 3D cubic lattice with a 2D boundary: e.g.

$$\Lambda_3 = T^2_{\mathrm{lat}} \times \mathbb{Z}_{\geq 0}, \qquad \Lambda_2 = \partial\Lambda_3 = T^2_{\mathrm{lat}} \times \{0\}.$$

- $U_{nm} = e^{\mathrm{i}\sum_a A^a_{nm} t^a} \in SU(N)$ on links, $U^{L/R}_{nm} := e^{\mathrm{i}\sum_a A^a_{nm} t^a_{L/R}}$,
- $g_1(n), g_2(n) \in U(N_f)$ on sites.

The basic **gauge-invariant dressed links** are ($g_1 \to V_L g_1$, $g_2 \to g_2 V_R^{-1}$)

$$X^L_{nm} := g_1(n)^\dagger U^L_{nm} g_1(m), \qquad X^R_{nm} := g_2(n) U^R_{nm} g_2(m)^\dagger.$$

Impose smoothness and admissibility conditions: $\|1 - X^{L/R}_{nm}\| < \varepsilon$, $\|1 - U_p\| < \delta$,

$$S_{\mathrm{lat}} = S^{\Lambda_2}_{\mathrm{kin}}[g_1, g_2, U] + \mathrm{i} \sum_{c \subset \Lambda_3} \Big(k^L_c[g_1, U] - k^R_c[g_2^\dagger, U] \Big).$$

$S^{\Lambda_2}_{\mathrm{kin}}$: boundary kinetic terms; $k^{L/R}_c$: bulk WZ/CS-type cube terms.

Constructing the cube term $k_c^{L/R}$

- Use a Seiberg-type $3D$ lattice Chern–Simons term for each cube:

  $$k_c[U] = \frac{1}{12\pi}\int_c \mathrm{Tr}\big[(S_c\,\mathrm{d}(S_c)^{-1})^3\big] + \frac{1}{4\pi}\sum_{p\subset\partial c}\int_p \mathrm{Tr}\big[(P_p)^{-1}\mathrm{d}P_p(S_c)^{-1}\mathrm{d}S_c\big]$$

  $S_c(x;U)$ and $P_p(x;U)$ interpolate links from a vertex n_0 to $x \in c$, including fractional powers $(U_{nm})^{\xi}$. [Seiberg'84], [Lüscher'81]

  $k_c[U] \sim A\mathrm{d}A + \frac{2\mathrm{i}}{3}(A)^3 \sim CS[A]$ in the continuum, as required for anomaly inflow. cf. [Laursen'92]

- **Replace** each link variables U_{nm} in k_c with a gauge-invariant dressed links $X_{nm}^{L/R}$: $U_{nm} \to X_{nm}^{L/R}$, $S_c(x;U) \to S_c(x;X^{L/R})$

  The smoothness (admissibility) conditions make the fractional powers $(X_{nm}^{L/R})^{\xi}$ well defined, avoiding the $(-1)^{\xi}$ ambiguity.

  Thus, $k_c^L[g_1,U]$ and $k_c^R[g_2^{\dagger},U]$ are defined by $k_c[X^{L/R}]$.

Isolating the genuine bulk contribution

$$S_{\rm lat} = S^{\Lambda_2}_{\rm kin}[g_1, g_2, U] + \underbrace{\mathrm{i} \sum_{c\subset\Lambda_3} \left(k^L_c[g_1, U] - k^R_c[g_2^\dagger, U] \right)}_{\text{bulk } \Lambda_3 \text{ dependence} \rightarrow \text{ anomaly}} .$$

Introduce a new interpolation $S^L_{I,c}(x; U^L)$ and an extension $g^c_1(x)$ so that (I labels the choice of interpolation)

$$S^L_c(x; X^L) = g_1(n_0)^\dagger \quad \underbrace{S^L_{I,c}(x; U^L)}_{\text{depends only on } U^L} \quad g^c_1(x),$$

The bulk contribution then decomposes schematically as

$$\sum_{c\subset\Lambda_3} k^L_c[g_1, U] \sim \underbrace{\frac{\mathrm{i}}{12\pi} \sum_c \int_c \mathrm{Tr}\left\{ (g^{c\dagger}_1 \mathrm{d} g^c_1)^3 \right\}}_{\text{WZ term}} + \underbrace{\sum_c k_c[U]}_{\text{pure-gauge bulk term}} \quad \mod 2\pi.$$

By its topological nature, the WZ term depends only on the boundary variables.

Anomaly cancellation

By explicit calculations, we have

$$\exists \mathcal{A}^a \in \mathbb{C}, \quad (S_I^{L/R})^{-1} \mathrm{d} S_I^{L/R} = \mathcal{A}^a t^a_{L/R} \qquad \text{(same } A^a\text{)},$$

Hence,

$$\exists \mathcal{B}^{ab} \in \mathbb{C}, \quad \sum_c k_c[U^{L/R}] = \sum_{a,b} \mathcal{B}^{ab} \mathrm{tr}\left\{ t^a_{L/R} t^b_{L/R} \right\},$$

$$\underbrace{\mathrm{i} \sum_{c \subset \Lambda_3} \left(k^L_c[g_1, U] - k^R_c[g_2^\dagger, U] \right)}_{\textbf{bulk } \Lambda_3 \text{ dependence} \rightarrow \textbf{ anomaly}}$$

$$= \textbf{(boundary } \Lambda_2 \textbf{ terms)} + \sum_{a,b} \mathcal{B}^{ab} \left(\mathrm{tr}\left\{ t^a_L t^b_L \right\} - \mathrm{tr}\left\{ t^a_R t^b_R \right\} \right)$$

Result: If $\mathrm{tr}\left\{ t^a_L t^b_L \right\} = \mathrm{tr}\left\{ t^a_R t^b_R \right\}$, the bulk dependence—or equivalently, the anomaly—cancels exactly even before taking the infinite-volume and continuum limits.

Summary

- Bosonization has provided explicit 2D $U(1)$ lattice chiral gauge theories; we extend this strategy to non-Abelian $SU(N)$.
- Spectator fermions give a simple, explicit lattice formulation.
- At finite lattice spacing, our framework reproduces **the continuum gauge-anomaly structure**.

However, showing that the lattice model flows to the desired bosonized theory remains **an open problem**.

Future directions

- Numerical simulations (possibly using tensor networks)
- Applications to higher dimensions

Backup: How to extract original observables

Non-Abelian bosonization gives

$$M_1(g_1)_{ij} = -\mathrm{i}\psi_{L,i}\psi^{\mathrm{sp}}_{R,j}, \qquad M_2(g_2)_{ij} = -\mathrm{i}\psi^{\mathrm{sp}}_{L,i}\psi_{R,j}.$$

The spectator sector is free and decoupled, so the correlators factorize:

$$\langle \mathcal{O}(\psi\psi^{\mathrm{sp}}, A)\rangle = \langle \mathcal{O}_{\mathrm{sp}}(\psi^{\mathrm{sp}})\rangle \langle \mathcal{O}_{\mathrm{original}}(\psi, A)\rangle.$$

Therefore, away from zeros of the spectator correlator,

$$\frac{\langle \mathcal{O}_{\mathrm{bosonic}}[g_1, g_2, U]\rangle_{\mathrm{lattice}}}{\langle \mathcal{O}_{\mathrm{sp}}(\psi^{\mathrm{sp}})\rangle_{\mathrm{lattice}}} \xrightarrow{a\to 0} \langle \mathcal{O}(\psi, A)\rangle.$$

This step assumes that the lattice model flows to the level-1 gauged WZW point.