

Non-perturbative renormalization of the energy-momentum tensor in $N_f = 3$ Wilson lattice QCD

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Coláiste na Tríonóide, Baile Átha Cliath

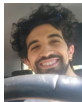
The University of Dublin

Based on

2606.28035 [hep-lat]

In collaboration with

Mattia Dalla Brida, Leonardo Giusti, Mitsuaki Hirasawa, Michele Pepe, Luca Virzì



Lattice 2026

Introduction

The **energy-momentum tensor** (EMT) $T_{\mu\nu}$ contains the conserved currents associated to the Poincaré invariance of QCD. It is a fundamental property of the theory:

- The theory couples to gravity through the EMT
- The hadrons structure can be studied through EMT
- n-point functions of EMT encode thermal properties of the Quark-Gluon Plasma (Equation of State, transport coefficients)

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The lattice breaks continuum spacetime symmetries:

- ⇒ EMT not conserved at finite lattice spacing a
- ⇒ a **finite renormalization** restores the conserved currents when $a \rightarrow 0$

Strategy for renormalization of traceless components of lattice EMT:

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⇒ **Convenient Ward identities can be derived by considering thermal QCD in the presence of shifted boundary conditions**

[Giusti, Meyer 10-13; Dalla Brida et al. 20]

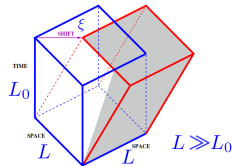
Continuum Ward identities

We consider thermal QCD with shifted boundary conditions (shift ξ), and a twist phase θ_0 for the fermions, along Euclidean time L_0 , [Giusti, Meyer 10-13; Dalla Brida et al. 20]

$$A_\mu(x_0 + L_0, \mathbf{x}) = A_\mu(x_0, \mathbf{x} - L_0 \xi)$$

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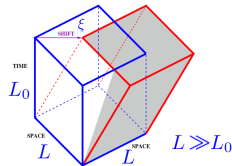
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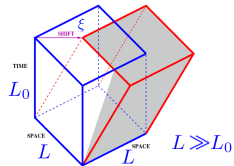
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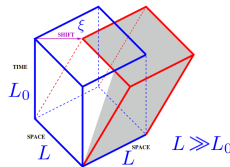
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⇒ Only one-point functions involved: numerically advantageous on the lattice

Renormalization of the lattice EMT

On the lattice, $SO(4)$ broken to the hypercubic group. The continuum non-singlet EMT splits into **sextet representation** (two fields labelled by G and F),

$$T_{\mu\nu}^{G,\{6\}} = (1 - \delta_{\mu\nu}) \frac{1}{g_0^2} \{ F_{\mu\alpha}^a F_{\nu\alpha}^a \} , \quad T_{\mu\nu}^{F,\{6\}} = (1 - \delta_{\mu\nu}) \frac{1}{8} \left\{ \bar{\psi} \gamma_\mu [\vec{\nabla}_\nu^* + \vec{\nabla}_\nu] \psi + \bar{\psi} \gamma_\nu [\vec{\nabla}_\mu^* + \vec{\nabla}_\mu] \psi \right\}$$

and **triplet representation** (two fields labelled by G and F),

$$T_{\mu\nu}^{G,\{3\}} = \frac{1}{g_0^2} \left\{ F_{\mu\alpha}^a F_{\mu\alpha}^a - F_{\nu\alpha}^a F_{\nu\alpha}^a \right\} , \quad T_{\mu\nu}^{F,\{3\}} = \frac{1}{4} \left\{ \bar{\psi} \gamma_\mu [\vec{\nabla}_\mu^* + \vec{\nabla}_\mu] \psi - \bar{\psi} \gamma_\nu [\vec{\nabla}_\nu^* + \vec{\nabla}_\nu] \psi \right\}$$

where $\vec{\nabla}_\mu, \vec{\nabla}_\mu^*$ are lattice covariant derivatives.

The full symmetry is restored only in the continuum: at finite lattice spacing, the EMT renormalizes as

$$T_{\mu\nu}^{R,\{6\}} = Z_G^{\{6\}}(g_0^2) T_{\mu\nu}^{G,\{6\}} + Z_F^{\{6\}}(g_0^2) T_{\mu\nu}^{F,\{6\}} , \quad T_{\mu\nu}^{R,\{3\}} = Z_G^{\{3\}}(g_0^2) T_{\mu\nu}^{G,\{3\}} + Z_F^{\{3\}}(g_0^2) T_{\mu\nu}^{F,\{3\}}$$

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At fixed g_0^2 and L_0/a , determine $\mathcal{Z}_G^{\{6\}}$, $\mathcal{Z}_F^{\{6\}}$, $\mathcal{Z}_G^{\{3\}}$, $\mathcal{Z}_F^{\{3\}}$ by imposing that the lattice EMT satisfies a discretized version of the continuum Ward identities [\[Dalla Brida et al. 20\]](#)

- Sextet case:

$$\langle T_{0k} \rangle_{\xi, \theta_0} = -\frac{\partial}{\partial \xi_k} f(L_0, \xi, \theta_0)$$

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- $\mathcal{Z}_G^{\{i\}}, \mathcal{Z}_F^{\{i\}}$ depend on L_0, ξ, θ_0 only by discretization effects in a/L_0 that we remove by defining: ($i = 6, 3$)

$$\mathcal{Z}_G^{\{i\}}(g_0^2) = \lim_{1/L_0 \rightarrow 0} \mathcal{Z}_G^{\{i\}}(g_0^2, a/L_0, \xi, \theta_0), \quad \mathcal{Z}_F^{\{i\}}(g_0^2) = \lim_{1/L_0 \rightarrow 0} \mathcal{Z}_F^{\{i\}}(g_0^2, a/L_0, \xi, \theta_0)$$

Computation of the renormalization constants

- Finite-temperature-like setup: $L_0 \times L^3$ lattice, $L/L_0 \gg 1$
- Wilson plaquette gauge action, $N_f = 3$ massless $O(a)$ -improved Wilson quarks
- Bare parameters:
 - $L_0/a = 4$, $6/g_0^2 \in [6.3719, 15.000]$ (8 values)
 - $L_0/a = 6$, $6/g_0^2 \in [6.2735, 15.000]$ (9 values)
 - $L_0/a = 8$, $6/g_0^2 \in [6.4680, 8.7325]$ (8 values)
- Spatial sizes L such that $18 \lesssim L/L_0 \lesssim 70 \rightarrow$ Negligible finite-volume effects
- Boundary conditions along compact direction specified by:
 - shift parameter $\xi = (1, 0, 0)$
 - fermionic phases $\theta_0^A = 0$ or $\theta_0^B = \frac{3\pi}{10}$ ($\theta_0 \in [0, \frac{\pi}{3}]$ periodicity)
- Periodic boundary conditions along spatial directions

[Bresciani et al. 25]

[Roberge, Weiss 86]

Computation of the renormalization constants

At each g_0^2 and L_0/a , $\mathcal{Z}_G^{\{6\}}$ and $\mathcal{Z}_F^{\{6\}}$ are the solutions of the linear system once all the quantities have been determined

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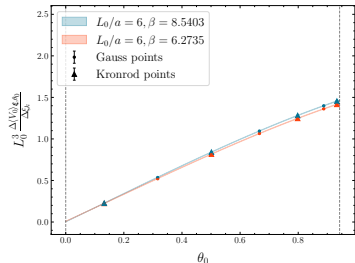
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- $\frac{\Delta}{\Delta\xi_1} f(L_0, \xi, \frac{3\pi}{10}) = \frac{\Delta}{\Delta\xi_1} f(L_0, \xi, 0) - \mathcal{V}_{0,k}^{AB}$

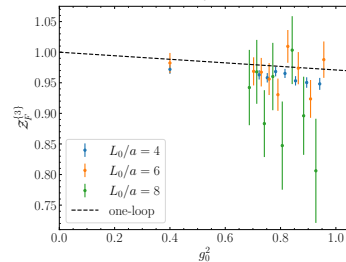
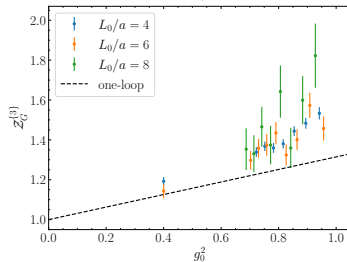
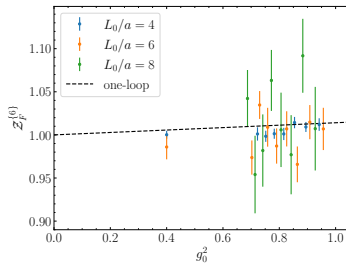
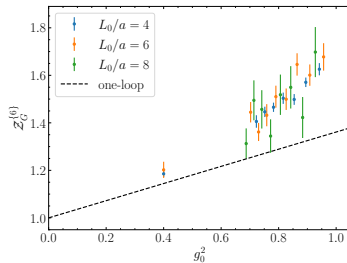
$$\mathcal{V}_{0,k}^{AB} = -\frac{i}{L_0} \int_0^{\frac{3\pi}{10}} d\theta_0 \frac{\Delta}{\Delta\xi_k} \langle V_0 \rangle_{\xi, \theta_0}$$

computed numerically by Gauss quadratures with dedicated Monte Carlo simulations



Computation of the renormalization constants

- $Z_G^{\{3\}}$ and $Z_F^{\{3\}}$ are determined in a very similar way as for the sextet case
- Improvement of data: discretization effects of $O(a^k g_0^0)$ and $O(a^k g_0^2)$, $k \geq 2$, subtracted using perturbation theory
- Residual dependence on L_0/a very small



$1/L_0 \rightarrow 0$ extrapolation

At fixed g_0^2 , $\mathcal{Z}_X^{\{i\}}$ depend on L_0, ξ, θ_0 only by irrelevant $O(a^2)$ effects:

$$\mathcal{Z}_X^{\{i\}}(g_0^2, a/L_0) = Z_X^{\{i\}}(g_0^2) + \frac{a}{L_0} \left\{ z_1 \frac{a}{L_0} + z_2 a \Lambda_{\text{QCD}} + O(a^2) \right\}, \quad X = G, F, \quad i = 6, 3$$

We include in our definition of the renormalization constants a $1/L_0 \rightarrow 0$ extrapolation at fixed g_0^2 to remove those $O(a^2)$ effects. This is performed by a global fit of the data with general fit form

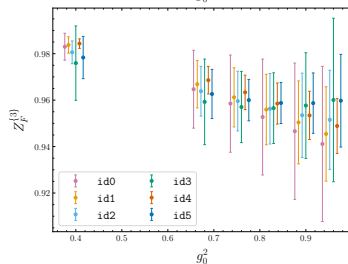
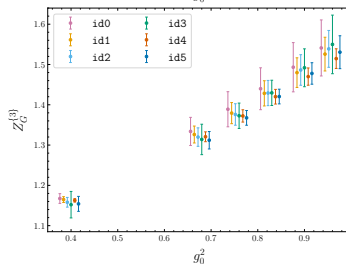
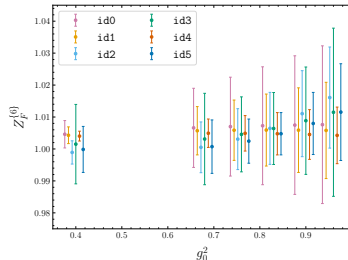
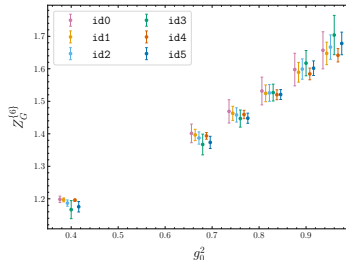
$$\mathcal{Z}_X^{\{i\}}(g_0^2, a/L_0) = Z_X^{\{i\}}(g_0^2) + \frac{a}{L_0} (d_{11}g_0^4 + d_{12}g_0^6) + \left(\frac{a}{L_0} \right)^2 (d_{21}g_0^4 + d_{22}g_0^6),$$

$$\text{where } Z_X^{\{i\}}(g_0^2) = 1 + Z_X^{\{i\}(1)}g_0^2 + c_1g_0^4 + c_2g_0^6$$

- Perturbative result to one-loop imposed in $Z_X^{\{i\}}(g_0^2)$
- Linear term in a/L_0 present in principle, though suppressed by $a\Lambda_{\text{QCD}} \lesssim 0.001$
- Coefficients c_i, d_{ij} to be determined from the fit

$1/L_0 \rightarrow 0$ extrapolation

- Several fits attempted:
 - w/wo $L_0/a = 4$
 - different terms included in fit function
- All fits are in agreement and have good χ^2/dof
- In particular, linear term in a/L_0 is not resolved within statistical errors
- Final procedures include $L_0/a = 4, 6, 8$ data and discretization effects $\propto (a/L_0)^2 g_0^4$ in fit functions



$1/L_0 \rightarrow 0$ extrapolation

Final parametrizations:

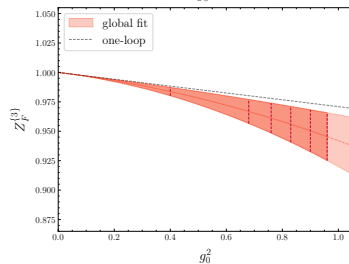
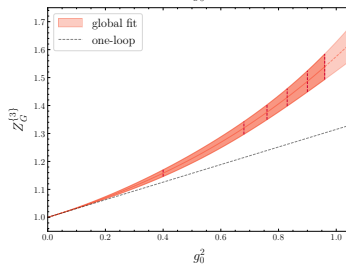
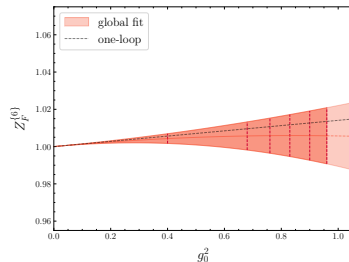
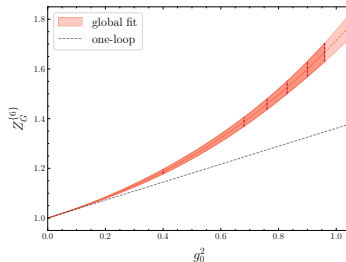
$$Z_G^{\{6\}}(g_0^2) = 1 + Z_G^{\{6\}(1)} g_0^2 + c_1 g_0^4 + c_2 g_0^6$$

$$Z_F^{\{6\}}(g_0^2) = 1 + Z_F^{\{6\}(1)} g_0^2 + c_1 g_0^4$$

$$Z_G^{\{3\}}(g_0^2) = 1 + Z_G^{\{3\}(1)} g_0^2 + c_1 g_0^4 + c_2 g_0^6$$

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- Precision at percent-level for $0.4 \leq g_0^2 \leq 0.96$
- $Z_G^{\{6\}}$ and $Z_G^{\{3\}}$ deviate significantly from one-loop result
- $Z_F^{\{6\}}$ and $Z_F^{\{3\}}$ agree with one-loop result



Conclusions

- First non-perturbative definition of the EMT in QCD by computing the renormalization constants for the traceless components
- Renormalization factors determined for $N_f = 3$ Wilson lattice QCD by imposing the lattice EMT to satisfy a discretized version of continuum Ward identities
- Thermal QCD with shifted boundary conditions and a twist phase for fermions unlocks Ward identities that can be efficiently enforced numerically on the lattice

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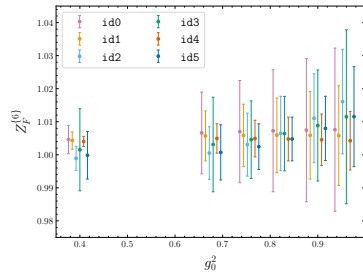
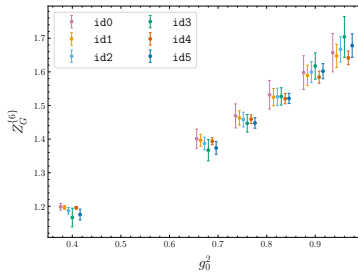
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- These results are ready-to-use to study QCD with three flavours at very high temperatures from EMT correlation functions (e.g. transport coefficients)

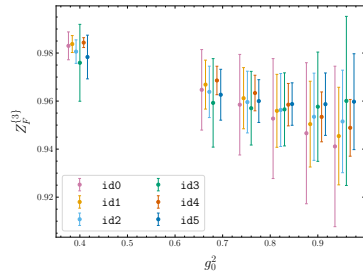
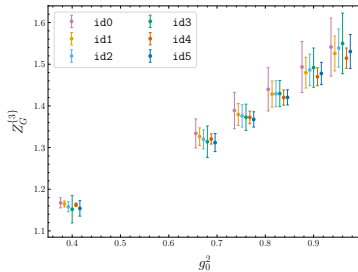
BACKUP

$1/L_0 \rightarrow 0$ extrapolation

	id0	id1	id2
L_0/a	4,6,8	4,6,8	4,6,8
g_0^4	x	x	x
g_0^6			x
ag_0^4	x		
$a^2g_0^4$		x	x
$a^2g_0^6$			



	id3	id4	id5
L_0/a	4,6,8	6,8	6,8
g_0^4	x	x	x
g_0^6	x		x
ag_0^4			
$a^2g_0^4$	x		
$a^2g_0^6$	x		



Non-perturbative renormalization constants

g_0^2	$Z_G^{\{6\}}$	$Z_F^{\{6\}}$	$Z_G^{\{3\}}$	$Z_F^{\{3\}}$
0.40	1.187(10)	1.0043(26)	1.158(12)	0.984(4)
0.68	1.387(19)	1.006(8)	1.320(23)	0.967(10)
0.76	1.458(22)	1.006(9)	1.376(27)	0.961(13)
0.83	1.526(26)	1.006(11)	1.43(3)	0.956(15)
0.90	1.60(3)	1.006(13)	1.49(4)	0.950(18)
0.96	1.67(4)	1.006(15)	1.54(5)	0.945(20)