Non-perturbative renormalization of the energy-momentum tensor in $N_f = 3$ Wilson lattice QCD

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Introduction

The energy-momentum tensor (EMT) $T_{\mu\nu}$ contains the conserved currents associated to the Poincaré invariance of QCD. It is a fundamental property of the theory:

- The theory couples to gravity through the EMT
- The hadrons structure can be studied through EMT
- n-point functions of EMT encode thermal properties of the Quark-Gluon Plasma (Equation of State, transport coefficients)

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$\Rightarrow$ EMT not conserved at finite lattice spacing a

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1) Consider continuum Ward identities for the EMT
2) Impose a discrete version of the Ward identities on the lattice renormalized EMT

$\Rightarrow$ Convenient Ward identities can be derived by considering thermal QCD in the presence of shifted boundary conditions [Giusti, Meyer 10-13; Dalla Brida et al. 20]

Continuum Ward identities

We consider thermal QCD with shifted boundary conditions (shift $\boldsymbol{\xi}$), and a twist phase θ_0 for the fermions, along Euclidean time L_0, [Giusti, Meyer 10-13; Dalla Brida et al. 20]

$$A_\mu(x_0 + L_0, \boldsymbol{x}) = A_\mu(x_0, \boldsymbol{x} - L_0\boldsymbol{\xi})$$

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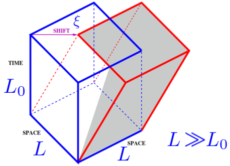

The following Ward identities hold for the traceless components,

$$\langle T_{0k}\rangle_{\boldsymbol{\xi},\theta_0} = -\frac{\partial}{\partial\xi_k} f(L_0, \boldsymbol{\xi}, \theta_0)\,, \quad f(L_0, \boldsymbol{\xi}, \theta_0) = -\frac{1}{L_0 L^3} \ln Z(L_0, \boldsymbol{\xi}, \theta_0)$$

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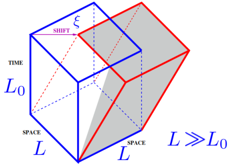

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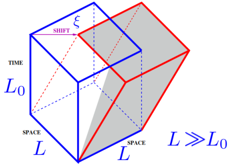

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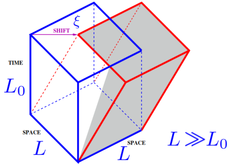

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$\Rightarrow$ Only one-point functions involved: numerically advantageous on the lattice

Renormalization of the lattice EMT

On the lattice, SO(4) broken to the hypercubic group. The continuum non-singlet EMT splits into sextet representation (two fields labelled by G and F),

$$T^{G,\{6\}}_{\mu\nu} = (1-\delta_{\mu\nu})\frac{1}{g_0^2}\left\{F^a_{\mu\alpha}F^a_{\nu\alpha}\right\}\ , \quad T^{F,\{6\}}_{\mu\nu} = (1-\delta_{\mu\nu})\frac{1}{8}\left\{\overline{\psi}\gamma_\mu[\overleftrightarrow{\nabla}^*_\nu + \overleftrightarrow{\nabla}_\nu]\psi + \overline{\psi}\gamma_\nu[\overleftrightarrow{\nabla}^*_\mu + \overleftrightarrow{\nabla}_\mu]\psi\right\}$$

and triplet representation (two fields labelled by G and F),

$$T^{G,\{3\}}_{\mu\nu} = \frac{1}{g_0^2}\left\{F^a_{\mu\alpha}F^a_{\mu\alpha} - F^a_{\nu\alpha}F^a_{\nu\alpha}\right\}, \quad T^{F,\{3\}}_{\mu\nu} = \frac{1}{4}\left\{\overline{\psi}\gamma_\mu\left[\overleftrightarrow{\nabla}^*_\mu + \overleftrightarrow{\nabla}_\mu\right]\psi - \overline{\psi}\gamma_\nu\left[\overleftrightarrow{\nabla}^*_\nu + \overleftrightarrow{\nabla}_\nu\right]\psi\right\}$$

where $\overleftrightarrow{\nabla}_\mu$, $\overleftrightarrow{\nabla}^*_\mu$ are lattice covariant derivatives.
The full symmetry is restored only in the continuum: at finite lattice spacing, the EMT renormalizes as

$$T^{R,\{6\}}_{\mu\nu} = Z^{\{6\}}_G(g_0^2)\,T^{G,\{6\}}_{\mu\nu} + Z^{\{6\}}_F(g_0^2)\,T^{F,\{6\}}_{\mu\nu}\ , \quad T^{R,\{3\}}_{\mu\nu} = Z^{\{3\}}_G(g_0^2)\,T^{G,\{3\}}_{\mu\nu} + Z^{\{3\}}_F(g_0^2)\,T^{F,\{3\}}_{\mu\nu}$$

Renormalization of the lattice EMT

At fixed g_0^2 and L_0/a, determine $\mathcal{Z}_G^{\{6\}}$, $\mathcal{Z}_F^{\{6\}}$, $\mathcal{Z}_G^{\{3\}}$, $\mathcal{Z}_F^{\{3\}}$ by imposing that the lattice EMT satisfies a discretized version of the continuum Ward identities [Dalla Brida et al. 20]

- Sextet case:

$$\langle T_{0k} \rangle_{\boldsymbol{\xi},\theta_0} = -\frac{\partial}{\partial \xi_k} f(L_0, \boldsymbol{\xi}, \theta_0)$$

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- $\mathcal{Z}_G^{\{i\}}$, $\mathcal{Z}_F^{\{i\}}$ depend on $L_0, \boldsymbol{\xi}, \theta_0$ only by discretization effects in a/L_0 that we remove by defining: $(i = 6, 3)$

$$Z_G^{\{i\}}(g_0^2) = \lim_{1/L_0 \to 0} \mathcal{Z}_G^{\{i\}}(g_0^2, a/L_0, \boldsymbol{\xi}, \theta_0)\,, \quad Z_F^{\{i\}}(g_0^2) = \lim_{1/L_0 \to 0} \mathcal{Z}_F^{\{i\}}(g_0^2, a/L_0, \boldsymbol{\xi}, \theta_0)$$

Computation of the renormalization constants

- Finite-temperature-like setup: $L_0 \times L^3$ lattice, $L/L_0 \gg 1$
- Wilson plaquette gauge action, $N_f = 3$ massless $O(a)$-improved Wilson quarks
- Bare parameters:

  - $L_0/a = 4, \quad 6/g_0^2 \in [6.3719, 15.000]$ (8 values)
  - $L_0/a = 6, \quad 6/g_0^2 \in [6.2735, 15.000]$ (9 values)
  - $L_0/a = 8, \quad 6/g_0^2 \in [6.4680, 8.7325]$ (8 values)

- Spatial sizes L such that $18 \lesssim L/L_0 \lesssim 70 \rightarrow$ Negligible finite-volume effects

[Bresciani et al. 25]

- Boundary conditions along compact direction specified by:
  - shift parameter $\boldsymbol{\xi} = (1, 0, 0)$
  - fermionic phases $\theta_0^A = 0$ or $\theta_0^B = \frac{3\pi}{10}$ $\quad (\theta_0 \in [0, \frac{\pi}{3}]$ periodicity) [Roberge, Weiss 86]
- Periodic boundary conditions along spatial directions

Computation of the renormalization constants

At each g_0^2 and L_0/a, $\mathcal{Z}_G^{\{6\}}$ and $\mathcal{Z}_F^{\{6\}}$ are the solutions of the linear system once all the quantities have been determined

$$\begin{cases} \boxed{\mathcal{Z}_G^{\{6\}}} \langle T_{01}^{G,\{6\}} \rangle_{\boldsymbol{\xi},0} + \boxed{\mathcal{Z}_F^{\{6\}}} \langle T_{01}^{F,\{6\}} \rangle_{\boldsymbol{\xi},0} = -\frac{\Delta}{\Delta \xi_1} f(L_0, \boldsymbol{\xi}, 0) \\ \boxed{\mathcal{Z}_G^{\{6\}}} \langle T_{01}^{G,\{6\}} \rangle_{\boldsymbol{\xi},\frac{3\pi}{10}} + \boxed{\mathcal{Z}_F^{\{6\}}} \langle T_{01}^{F,\{6\}} \rangle_{\boldsymbol{\xi},\frac{3\pi}{10}} = -\frac{\Delta}{\Delta \xi_1} f(L_0, \boldsymbol{\xi}, \frac{3\pi}{10}) \end{cases}$$

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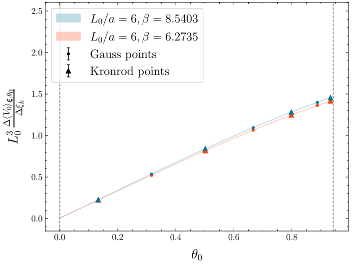
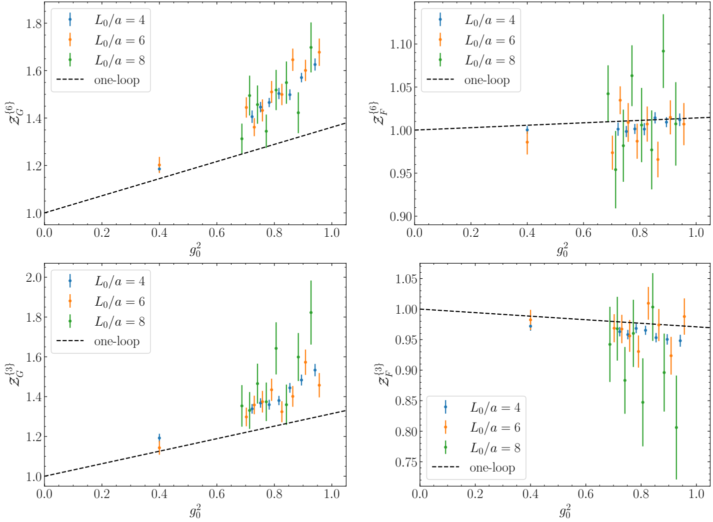
$$\mathcal{V}_{0,k}^{AB} = -\frac{i}{L_0} \int_0^{\frac{3\pi}{10}} d\theta_0 \frac{\Delta}{\Delta \xi_k} \langle V_0 \rangle_{\boldsymbol{\xi},\theta_0}$$

computed numerically by Gauss quadratures with dedicated Monte Carlo simulations



Computation of the renormalization constants

- $\mathcal{Z}_G^{\{3\}}$ and $\mathcal{Z}_F^{\{3\}}$ are determined in a very similar way as for the sextet case
- Improvement of data: discretization effects of $O(a^k g_0^0)$ and $O(a^k g_0^2)$, $k \geq 2$, subtracted using perturbation theory
- Residual dependence on L_0/a very small



$1/L_0 \to 0$ extrapolation

At fixed g_0^2, $\mathcal{Z}_X^{\{i\}}$ depend on $L_0, \boldsymbol{\xi}, \theta_0$ only by irrelevant $O(a^2)$ effects:

$$\mathcal{Z}_X^{\{i\}}(g_0^2, a/L_0) = Z_X^{\{i\}}(g_0^2) + \frac{a}{L_0}\left\{ z_1 \frac{a}{L_0} + z_2\, a\Lambda_{\mathrm{QCD}} + O(a^2) \right\}, \quad X = G, F, \quad i = 6, 3$$

We include in our definition of the renormalization constants a $1/L_0 \to 0$ extrapolation at fixed g_0^2 to remove those $O(a^2)$ effects. This is performed by a global fit of the data with general fit form

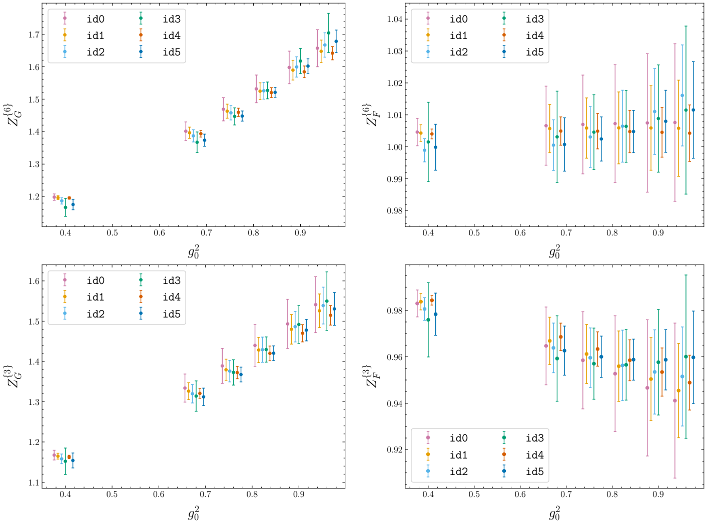
$$\mathcal{Z}_X^{\{i\}}(g_0^2, a/L_0) = Z_X^{\{i\}}(g_0^2) + \frac{a}{L_0}\left(d_{11} g_0^4 + d_{12} g_0^6\right) + \left(\frac{a}{L_0}\right)^2 \left(d_{21} g_0^4 + d_{22} g_0^6\right),$$

$$\text{where} \quad Z_X^{\{i\}}(g_0^2) = 1 + Z_X^{\{i\}(1)} g_0^2 + c_1 g_0^4 + c_2 g_0^6$$

- Perturbative result to one-loop imposed in $Z_X^{\{i\}}(g_0^2)$
- Linear term in a/L_0 present in principle, though suppressed by $a\Lambda_{\mathrm{QCD}} \lesssim 0.001$
- Coefficients c_i, d_{ij} to be determined from the fit

$1/L_0 \to 0$ extrapolation

- Several fits attempted:
  - w/wo $L_0/a = 4$
  - different terms included in fit function
- All fits are in agreement and have good χ^2/dof
- In particular, linear term in a/L_0 is not resolved within statistical errors
- Final procedures include $L_0/a = 4, 6, 8$ data and discretization effects $\propto (a/L_0)^2 g_0^4$ in fit functions



$1/L_0 \to 0$ extrapolation

Final parametrizations:

$$Z_G^{\{6\}}(g_0^2) = 1 + Z_G^{\{6\}(1)} g_0^2 + c_1 g_0^4 + c_2 g_0^6$$

$$Z_F^{\{6\}}(g_0^2) = 1 + Z_F^{\{6\}(1)} g_0^2 + c_1 g_0^4$$

$$Z_G^{\{3\}}(g_0^2) = 1 + Z_G^{\{3\}(1)} g_0^2 + c_1 g_0^4 + c_2 g_0^6$$

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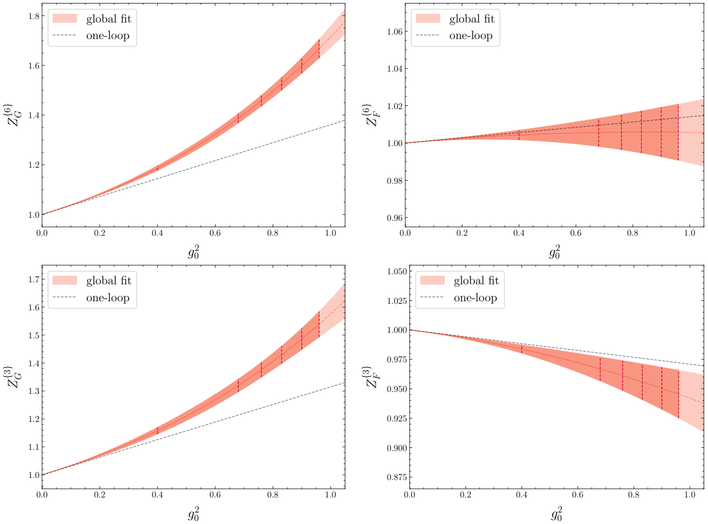
- Precision at percent-level for $0.4 \le g_0^2 \le 0.96$
- $Z_G^{\{6\}}$ and $Z_G^{\{3\}}$ deviate significantly from one-loop result
- $Z_F^{\{6\}}$ and $Z_F^{\{3\}}$ agree with one-loop result



Conclusions

- First non-perturbative definition of the EMT in QCD by computing the renormalization constants for the traceless components
- Renormalization factors determined for $N_f = 3$ Wilson lattice QCD by imposing the lattice EMT to satisfy a discretized version of continuum Ward identities
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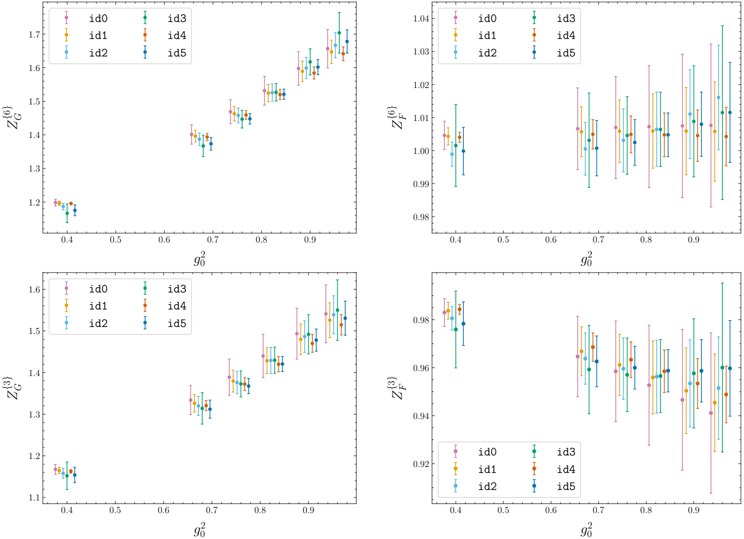
- These results are ready-to-use to study QCD with three flavours at very high temperatures from EMT correlation functions (e.g. transport coefficients)

BACKUP

$1/L_0 \to 0$ extrapolation

	id0	id1	id2
L_0/a	4,6,8	4,6,8	4,6,8
g_0^4	x	x	x
g_0^6			x
ag_0^4	x		
$a^2g_0^4$		x	x
$a^2g_0^6$			

	id3	id4	id5
L_0/a	4,6,8	6,8	6,8
g_0^4	x	x	x
g_0^6	x		x
ag_0^4			
$a^2g_0^4$	x		
$a^2g_0^6$	x		



Non-perturbative renormalization constants

g_0^2	$Z_G^{\{6\}}$	$Z_F^{\{6\}}$	$Z_G^{\{3\}}$	$Z_F^{\{3\}}$
0.40	1.187(10)	1.0043(26)	1.158(12)	0.984(4)
0.68	1.387(19)	1.006(8)	1.320(23)	0.967(10)
0.76	1.458(22)	1.006(9)	1.376(27)	0.961(13)
0.83	1.526(26)	1.006(11)	1.43(3)	0.956(15)
0.90	1.60(3)	1.006(13)	1.49(4)	0.950(18)
0.96	1.67(4)	1.006(15)	1.54(5)	0.945(20)