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Leading-order dynamical QED corrections to thermodynamics of isospin-asymmetric QCD

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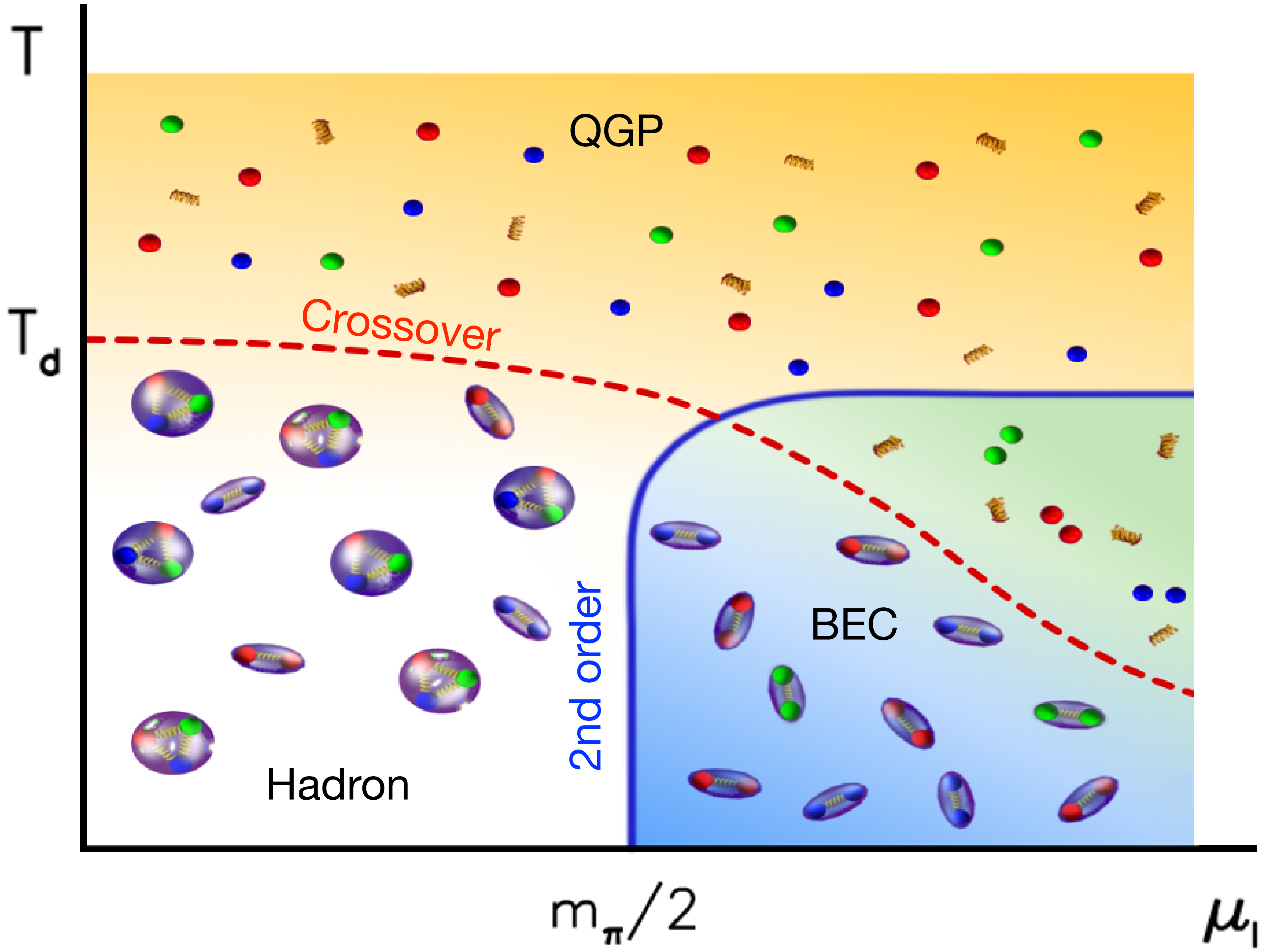
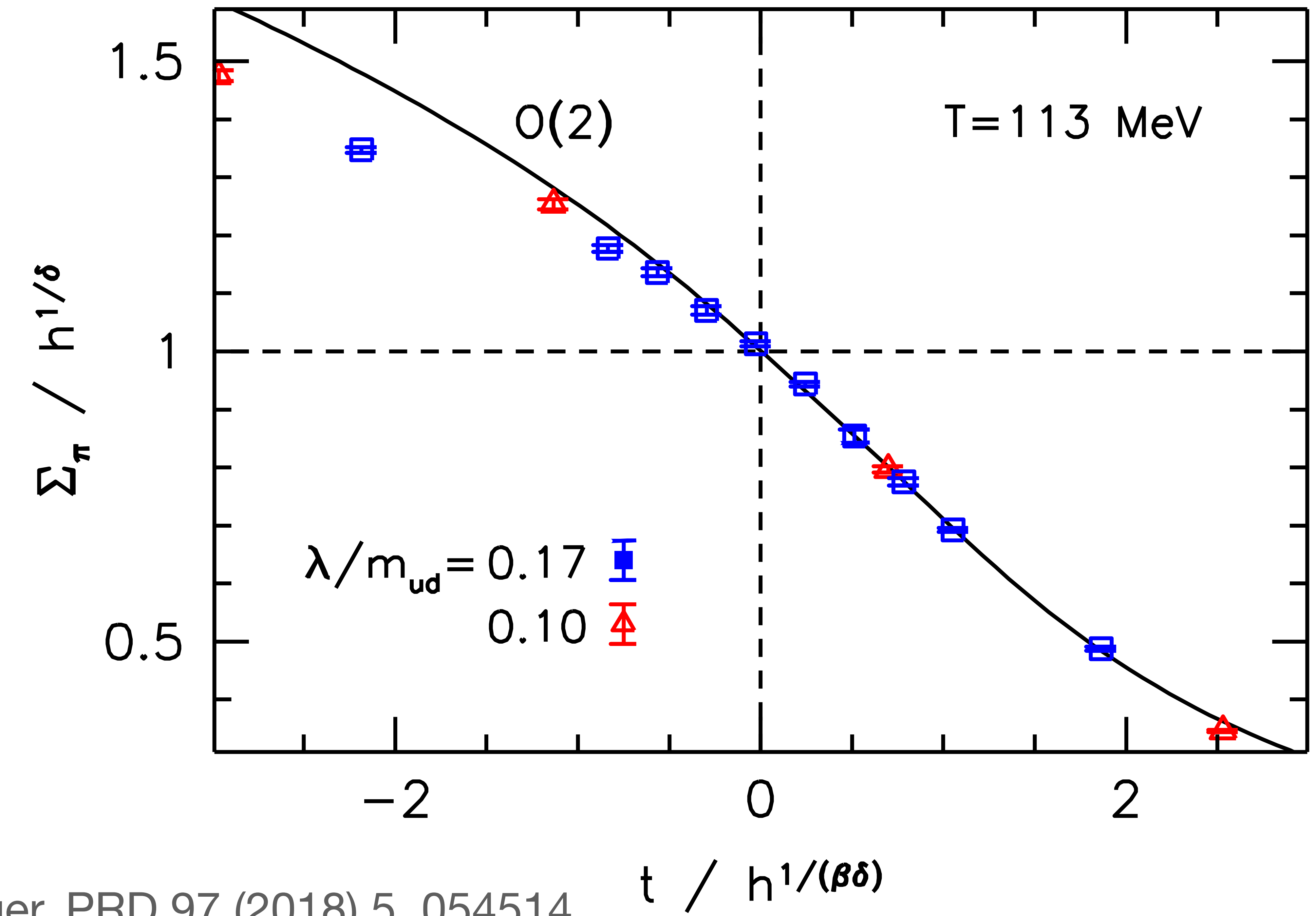
QCD phase transition at nonzero isospin

QCD phase diagram in $T - \mu_I$ plane



$O(2)$ scaling of order parameter



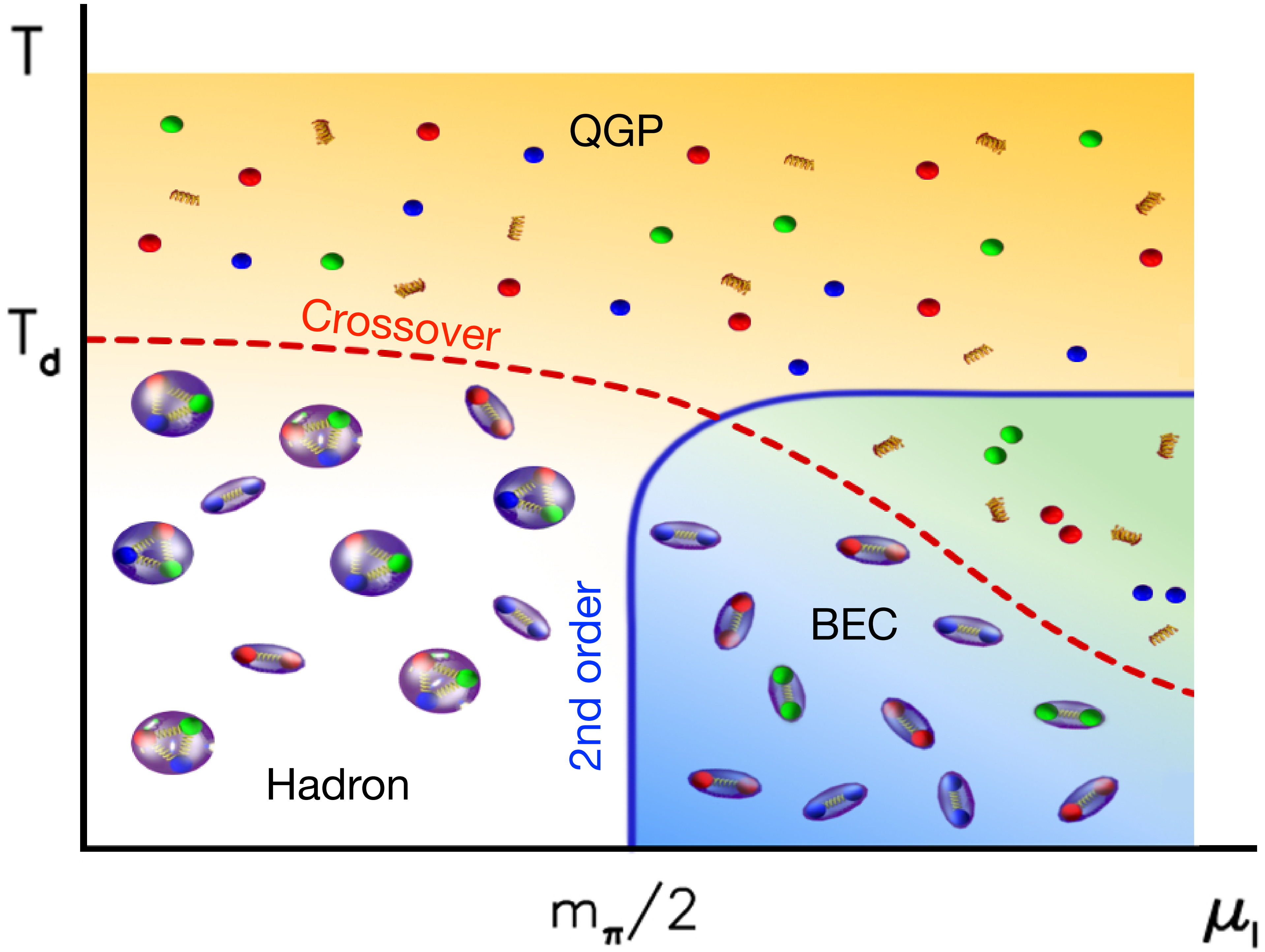
B. Brandt, G. Endrodi, S. Schmalzbauer, PRD 97 (2018) 5, 054514

— Sign-problem-free simulations at $\mu_I \neq 0$ + Pionic λ driving spontaneous breaking at finite V

— Second order $O(2)$ transition at onset of charged-pion BEC

QCD phase transition at nonzero isospin

QCD phase diagram in $T - \mu_I$ plane



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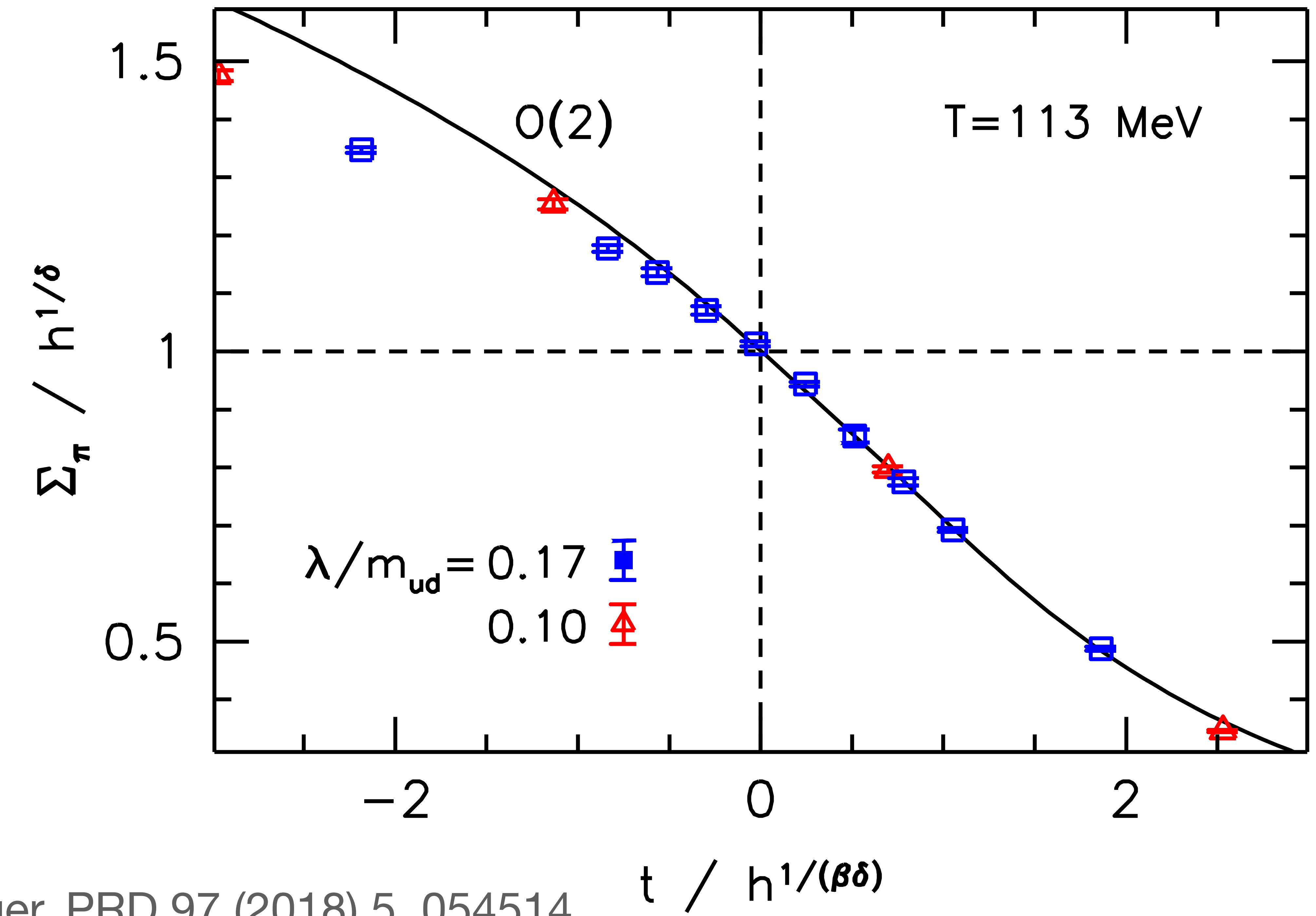

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What if dynamical photons are included in isospin-asymmetric QCD?

1st order transition driven by dynamic photons

- Charged pion $\langle \pi^{\pm} \rangle$ coupled to dynamic photons :

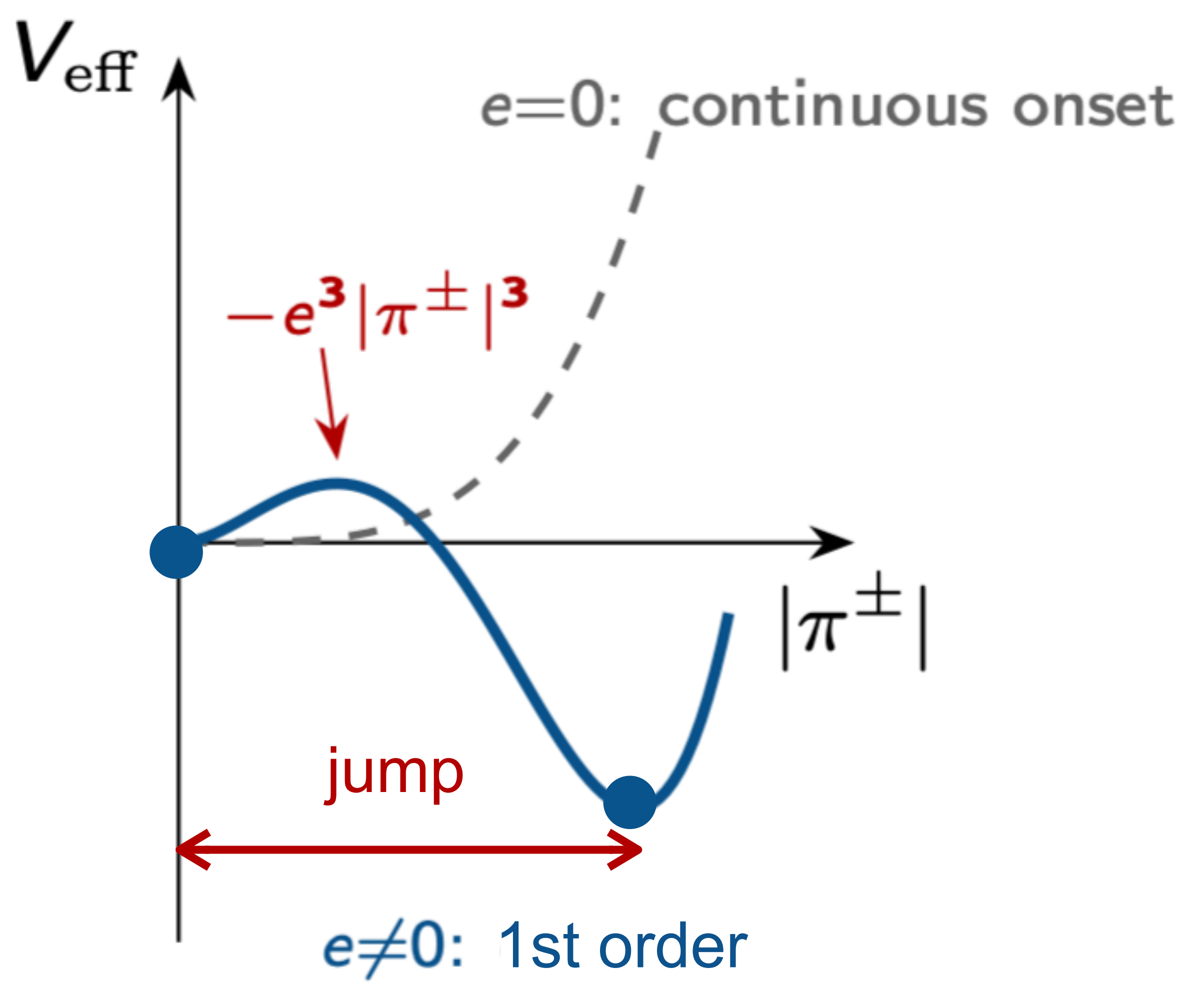
$$F[\pi^{\pm}, A^{EM}] \sim |D_\mu \pi^{\pm}|^2, \text{with } D_\mu = \partial_\mu - ieA_\mu^{EM}$$

- Cubic term appears after gauge fluctuations integrated :

Halperin, Lubensky, Ma, PRL 32 (1974) 292

$$V_{\text{eff}}(\pi^{\pm}) \supset -e^3 |\pi^{\pm}|^3$$

I. Giannakis, et.al., PRL 93 (2004) 232301



— e=0: single minimum slides off zero continuously, no barrier ⇒ 2nd order

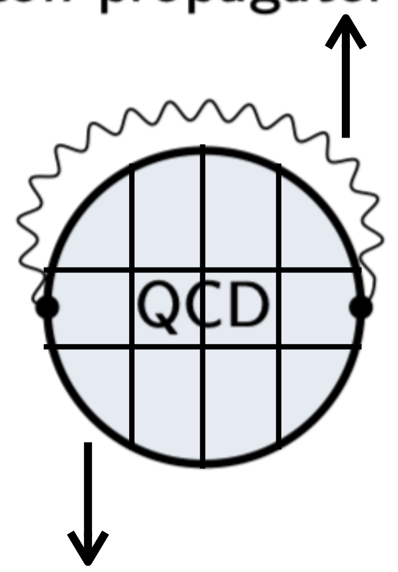
— e≠0: barrier and rival minimum appear ⇒ 1st order

Strategy: expansion in e around QCD at $\mu_I \neq 0$

☹️ Complex action for QCD+QED at $\mu_I \neq 0$: unequal charges $q_u \neq q_d$ break $u \leftrightarrow d$ pairing

😃 Taylor expand the free-energy density :

$$f(\mu_I, e) = f(\mu_I, 0) + \frac{e^2}{2} \underbrace{\left.\frac{\partial^2 f}{\partial e^2}\right|_{e=0}}_{\text{(the pineapple)}} + \mathcal{O}(e^4)$$



The 'pineapple': leading order dynamical QED effect, as photon propagator $\otimes$ vector current-current correlator

$$= \left.\frac{\partial^2 f}{\partial e^2}\right|_{e=0} = \sum_x \Delta_{\mu\mu}(x) \left\langle j_\mu(x) j_\mu(0) \right\rangle = \frac{1}{V_4} \sum_{k \in \mathrm{BZ} \cap \mathrm{QED}_{L\,/\,TL}} \frac{\Pi_{\mu\mu}(k)}{\hat{k}^2}$$

(Feynman-gauged photon, lattice momentum $\hat{k}$)

Simulation details

- Isospin-asymmetric staggered lattices :
  - Fixed volume $N_s^3 \times N_\tau = 24^3 \times 6$; fixed $\beta_g = 3.45$; at physical point $m_s/m_l \approx 28.15$
  - Scan: $a\mu_I \in \{0.025,\ 0.075,\ 0.1,\ 0.105,\ 0.125,\ 0.15\}$, $a\lambda \in \{0.0006,\ 0.001,\ 0.0025,\ 0.0075\}$

- Full (2+1)-f current-current correlator : connected + disconnected + tadpole
  - Low mode projection is used for speeding up the CG inversions
  - Measurements with different types of source :

Point source : full 4D correlator $C^{4D}_{\mu\mu} = \langle j_\mu(x, y, z, \tau)\ j_\mu(0) \rangle$

Novel planar source : $C^{\text{2D}}_{\mu\mu}(\Delta z, \Delta\tau) = \sum_{\vec{w}_\perp} C^{4D}_{\mu\mu}(\vec{w}_\perp, \Delta z, \Delta\tau)$

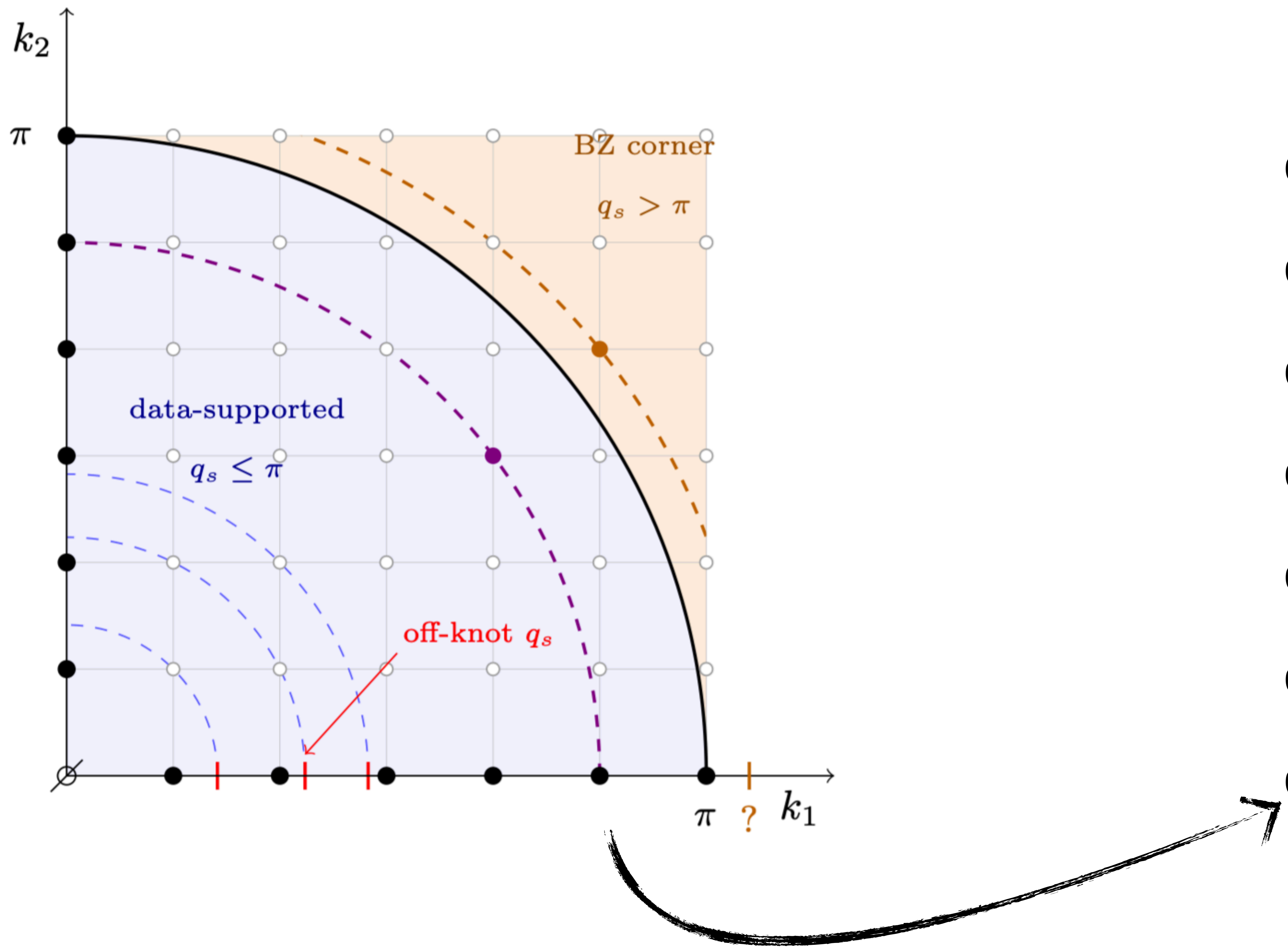
Zero in-plane-momentum projection

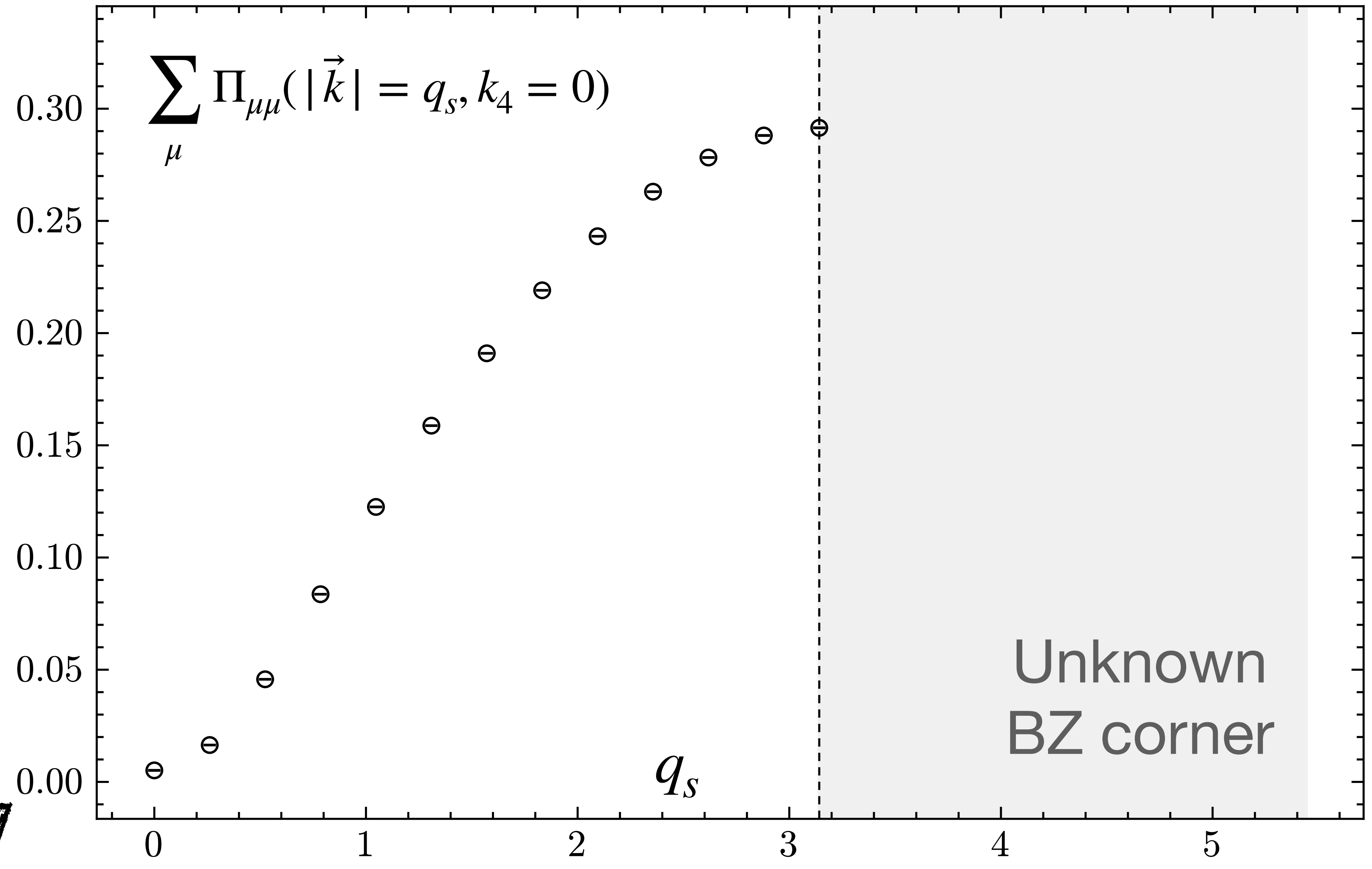
$$\widetilde{C}^{\text{2D}}_{\mu\mu}(k_3, k_4) = \sum_{x,y} e^{-i\sum_{n=1}^{4} k_n x_n} C^{4D}_{\mu\mu}(x, y, z, \tau)$$

with $k_n = (0,\ 0,\ k_3,\ k_4)$

From $C^{2D}(z,\tau)$ to Brillouin-zone sum

$$= \frac{1}{V_4} \sum_{k \in \mathrm{BZ} \cap \mathrm{QED}_{L\,/\,TL}} \frac{\sum_\mu \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2}$$



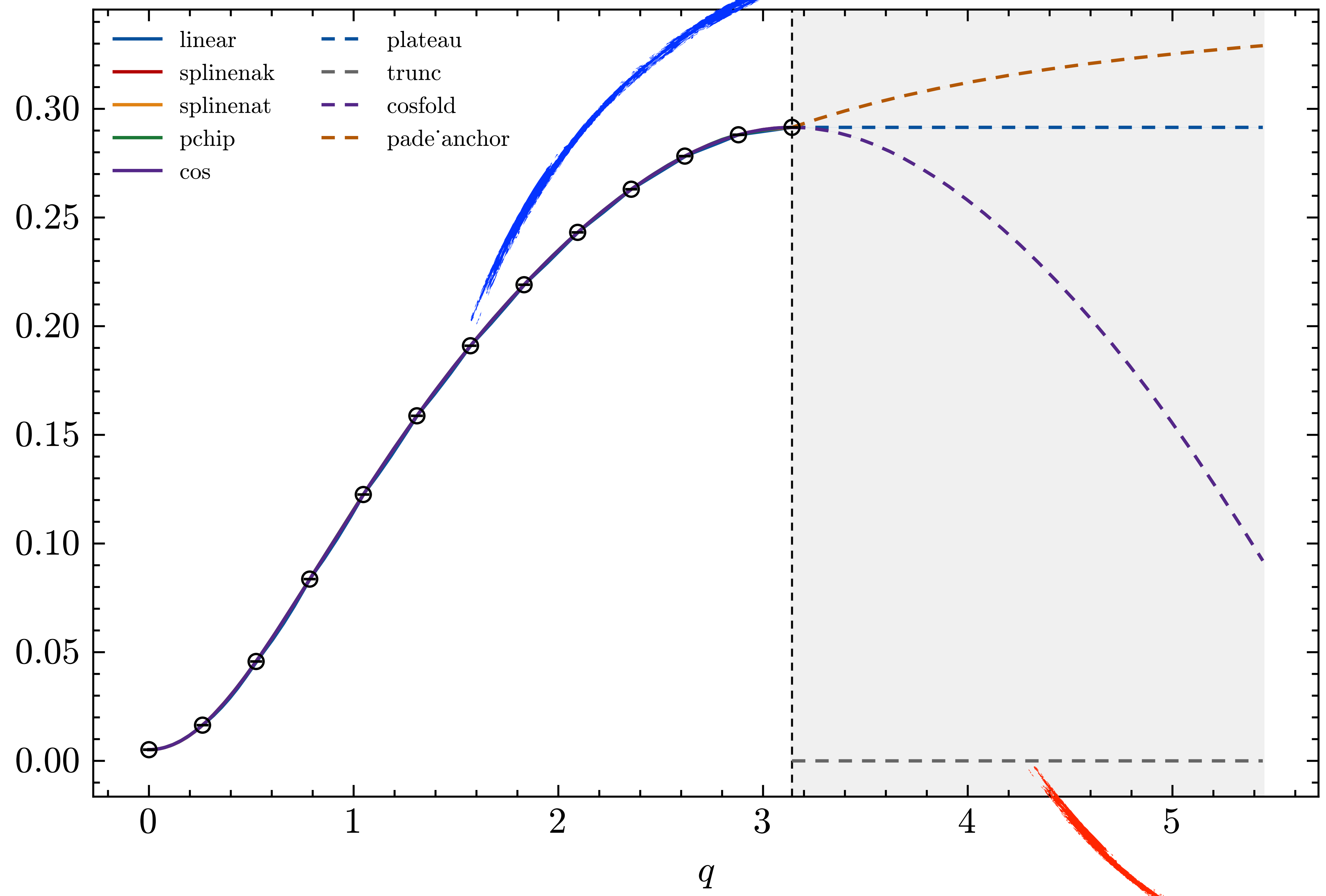


$$q_s = \frac{2\pi}{N_s}\sqrt{\sum_{i=1}^{3} m_i^2}\text{, with only one of } m_i \in \{\frac{-N_s}{2}+1, \cdots, \frac{N_s}{2}\} \text{ and other zero}$$

☹️ 2D planar correlator fixes $\Pi_{\mu\mu}(k)$ exactly only on the axis slice $k = \{(0,0,k_3,k_4), (0,k_2,0,k_4), (k_1,0,0,k_4)\}$

From $C^{2D}(z, \tau)$ to Brillouin-zone sum

Reconstruction of $\Pi(\vec{k}, k_4)$ via $O(3)$ spatial symm.



- Interior $|\vec{k}| \leq \pi$

interpolate in q^2 via ansatz { linear, spline, PCHIP, cosine poly. }

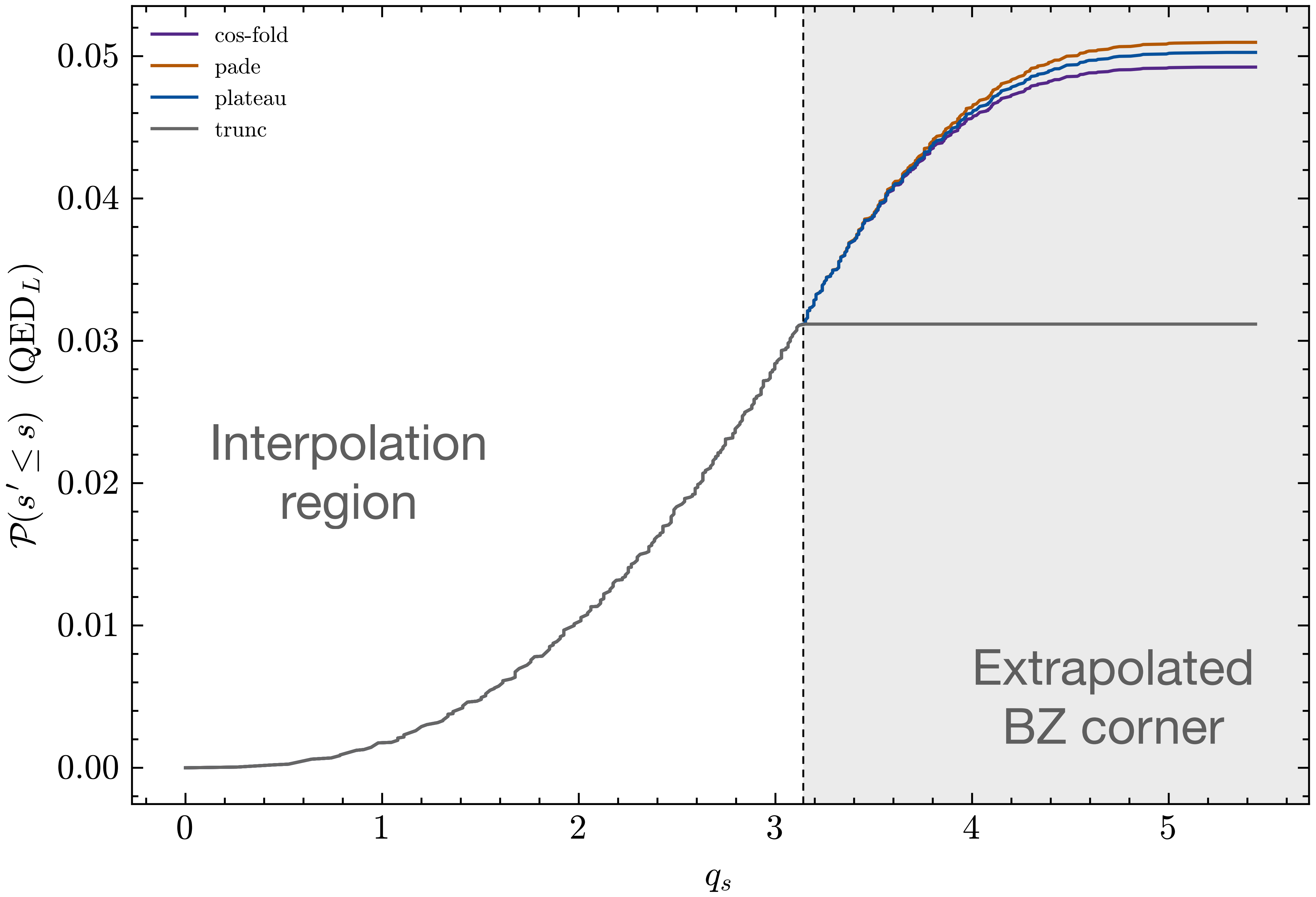
- BZ corner $\pi < |\vec{k}| \leq \sqrt{3}\pi$

extrapolate in q^2 via ansatz { plateau, cos-fold, Pade, drop-corner}

$$= \frac{1}{V_4} \sum_{k \in \mathrm{BZ} \cap \mathrm{QED}_{L\,/\,TL}} \frac{\sum_\mu \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2}$$ can be constructed completely!

Result: cumulative contribution to pineapple

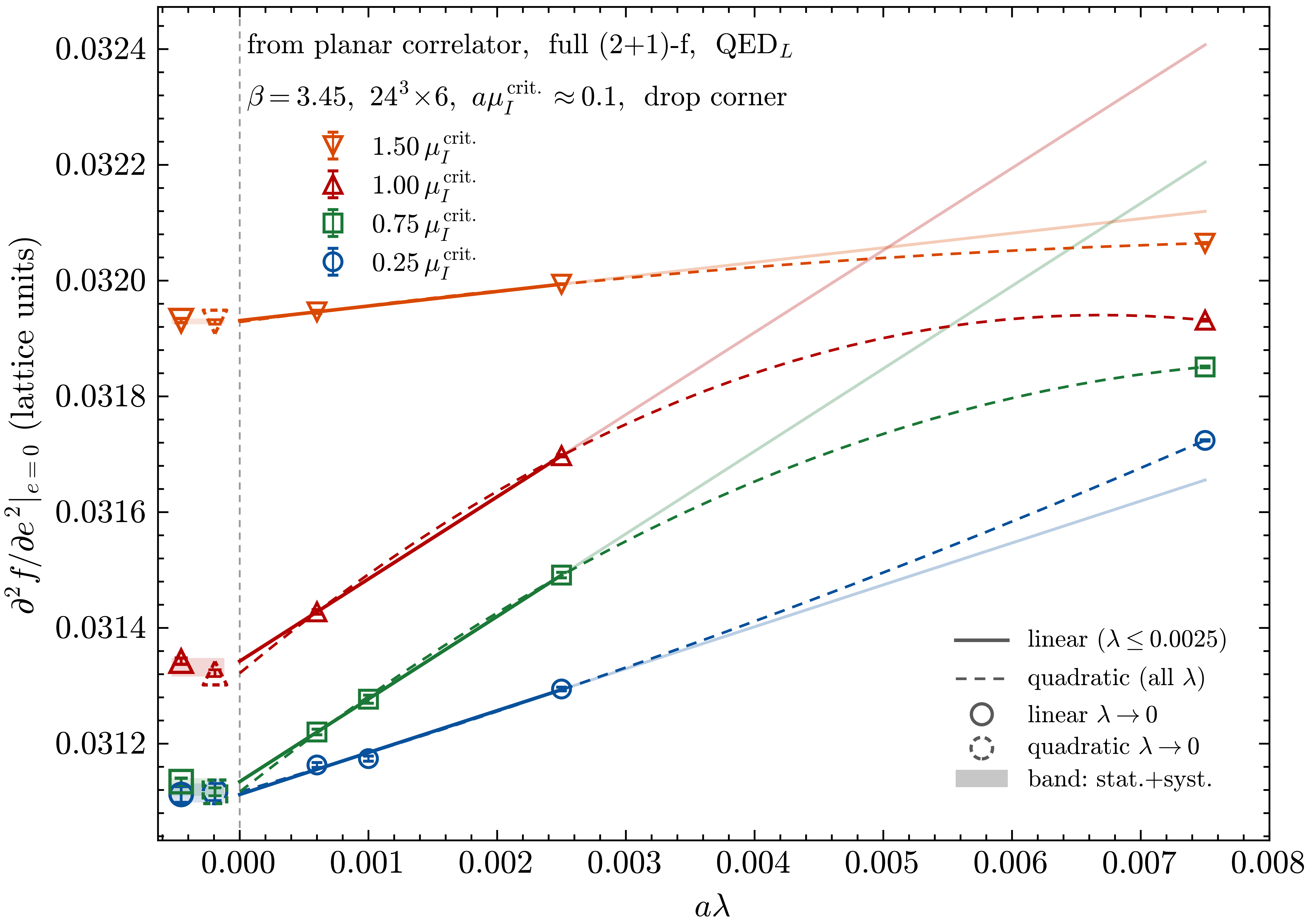
Accumulate momentum sum ordered by q_s : $P(q_s' \le q_s) = \frac{1}{V_4} \sum_{k_4, |\vec{k}'| \le q_s} \frac{\sum_\mu \Pi_{\mu\mu}(\vec{k}, k_4)}{\hat{k}^2}$



- Interior $|\vec{k}| \le \pi$: different interp. ansatz almost coincide — correlator data well shape the curve profile
- BZ corner $\pi < |\vec{k}| \le \sqrt{3}\pi$: contributes about 35%; different schemes split; UV-sensitive piece

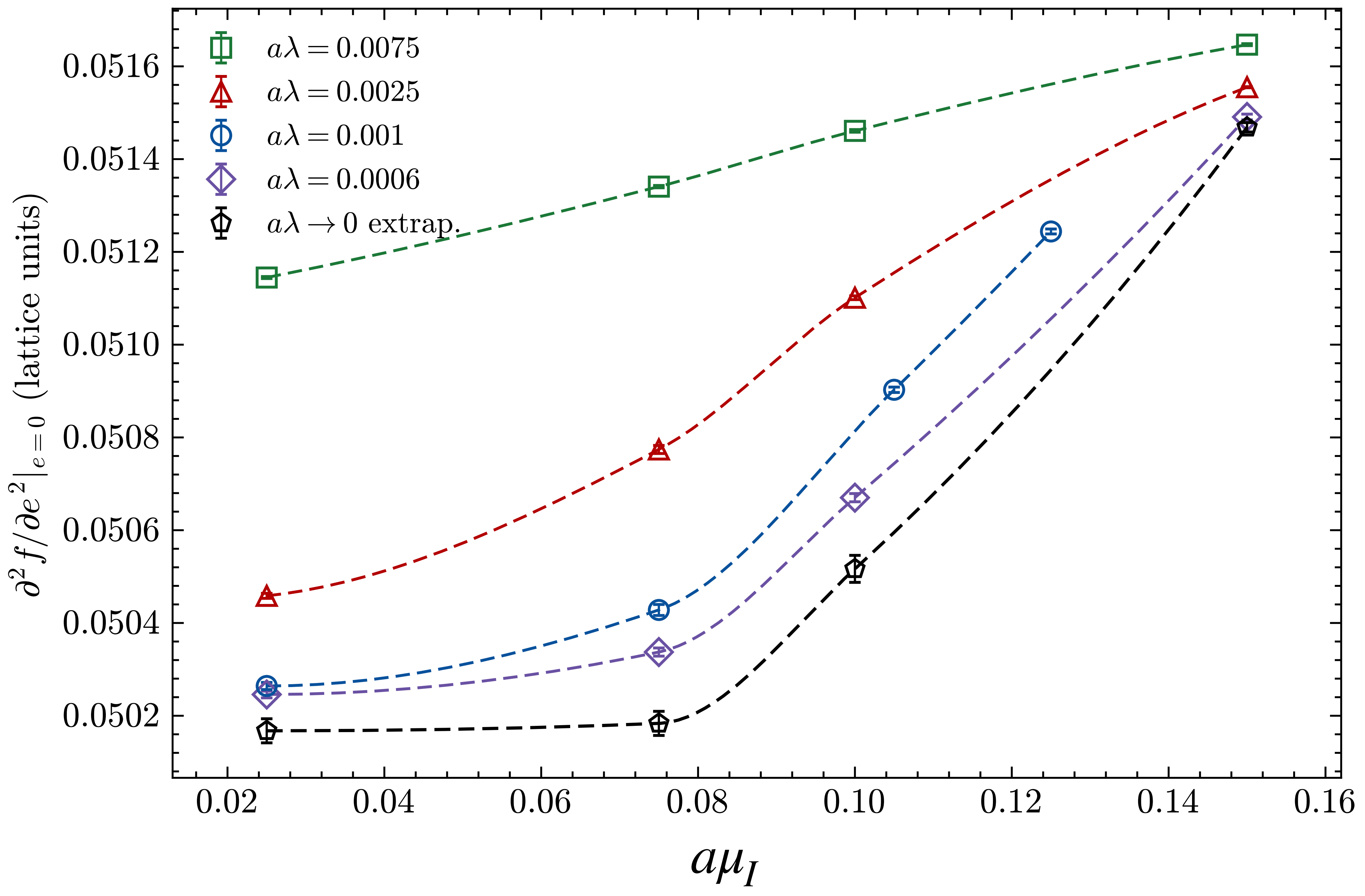
Result: $\lambda \to 0$ extrapolation

$\lambda \to 0$ extrapolation via linear and quadric form

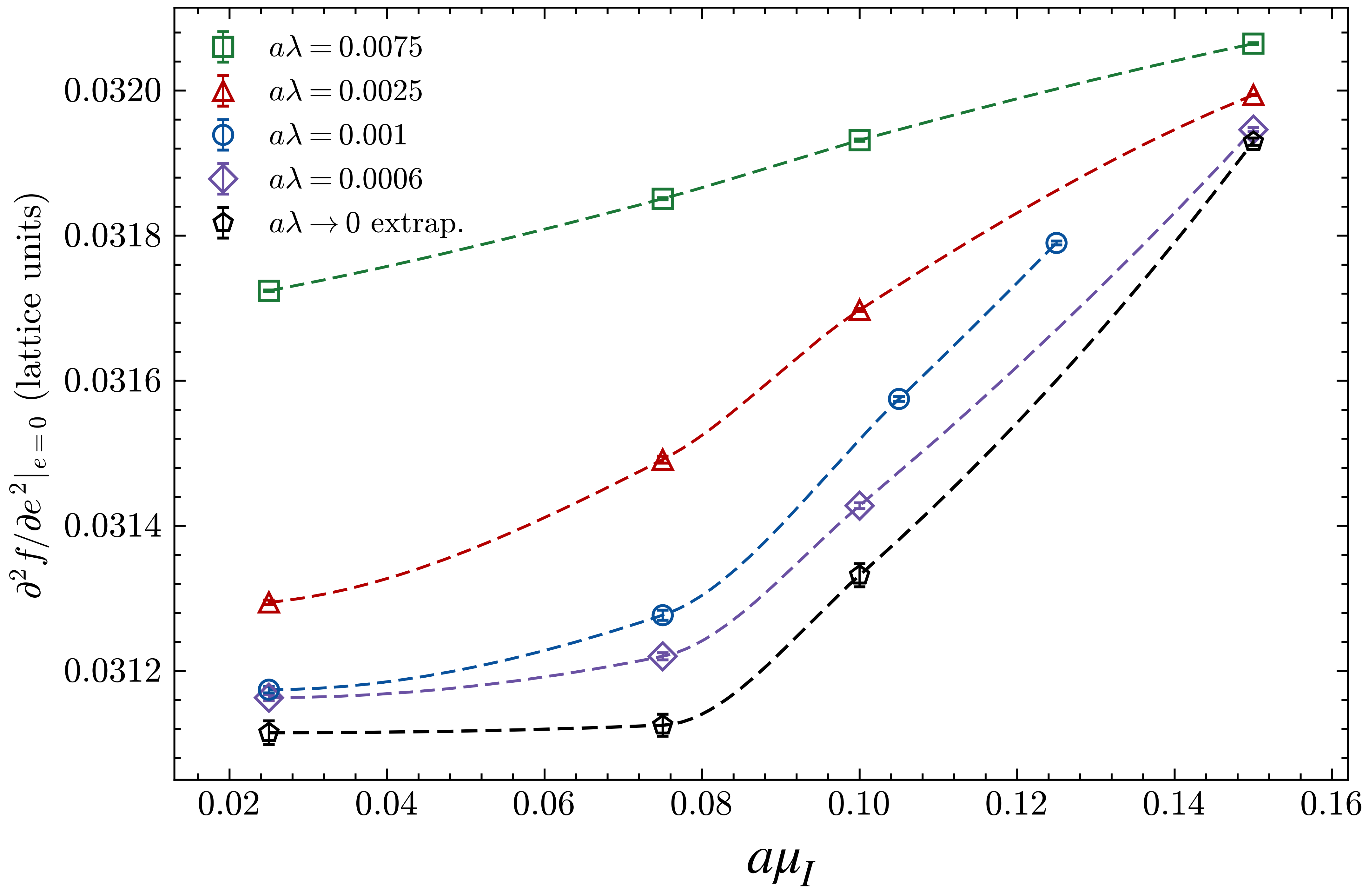


Result: pineapple across (λ, μ_I) grid and $\lambda \to 0$

Plateau extrap. for BZ corner



Drop-corner for BZ corner



- Same trend in μ_I and $\lambda \to 0$ extrap. irrespective of treatments for BZ corner
- Silver Blaze plateau appears after $\lambda \to 0$ extrapolation: flat in $a\mu_I$ below $a\mu_I^{\text{crit.}} \approx 0.1$
- Abrupt jump at onset of BEC $\mu_I^{\text{crit.}}$

Summary & outlook

- [x] 

is calculated from 2D planar current-current correlator

- [x] μ_I independence before onset of BEC, while jump at transition region — 1st order indication

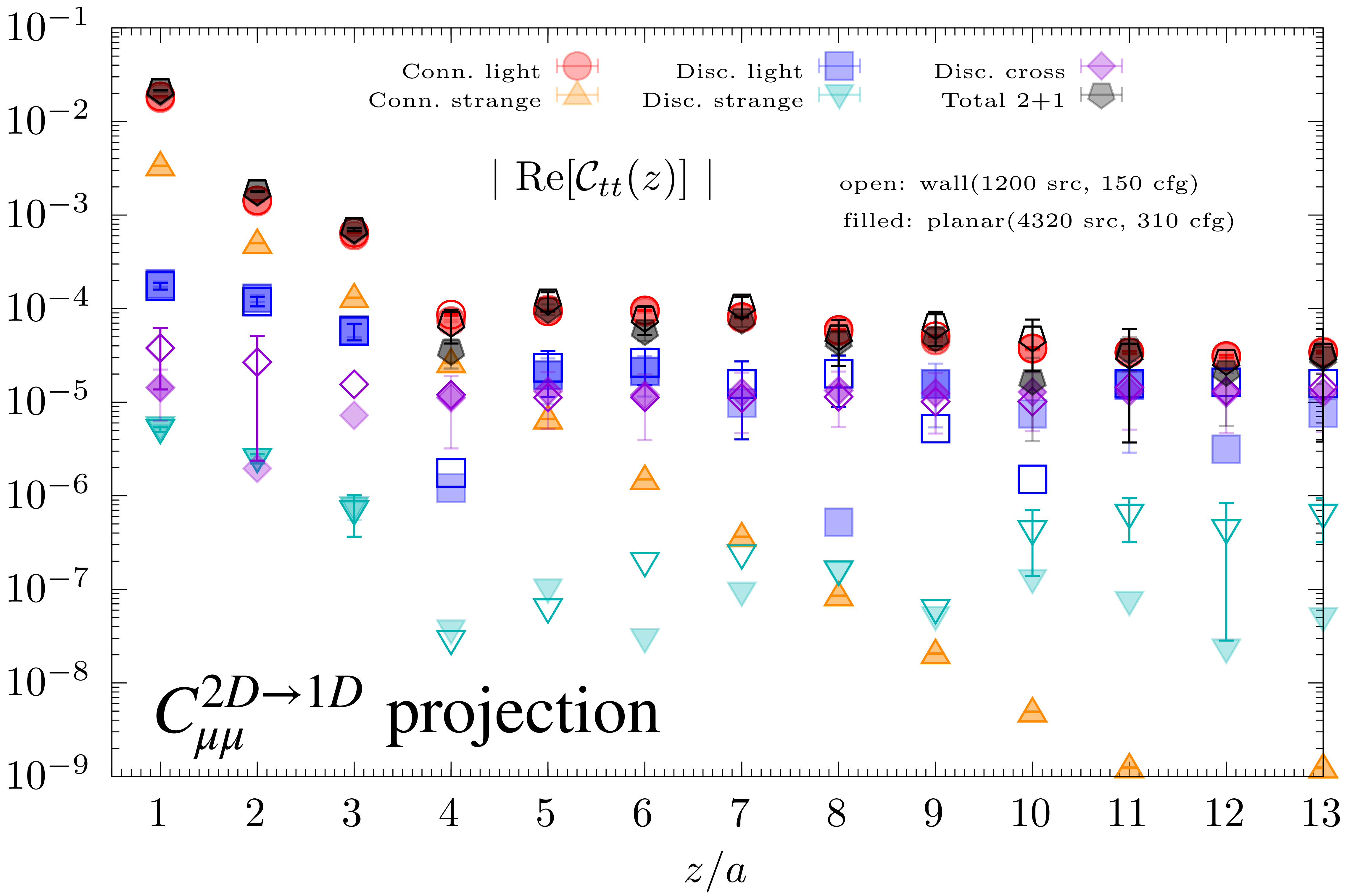
- [ ] To deal with UV divergence by subtracting $\mu_I = 0$ baseline

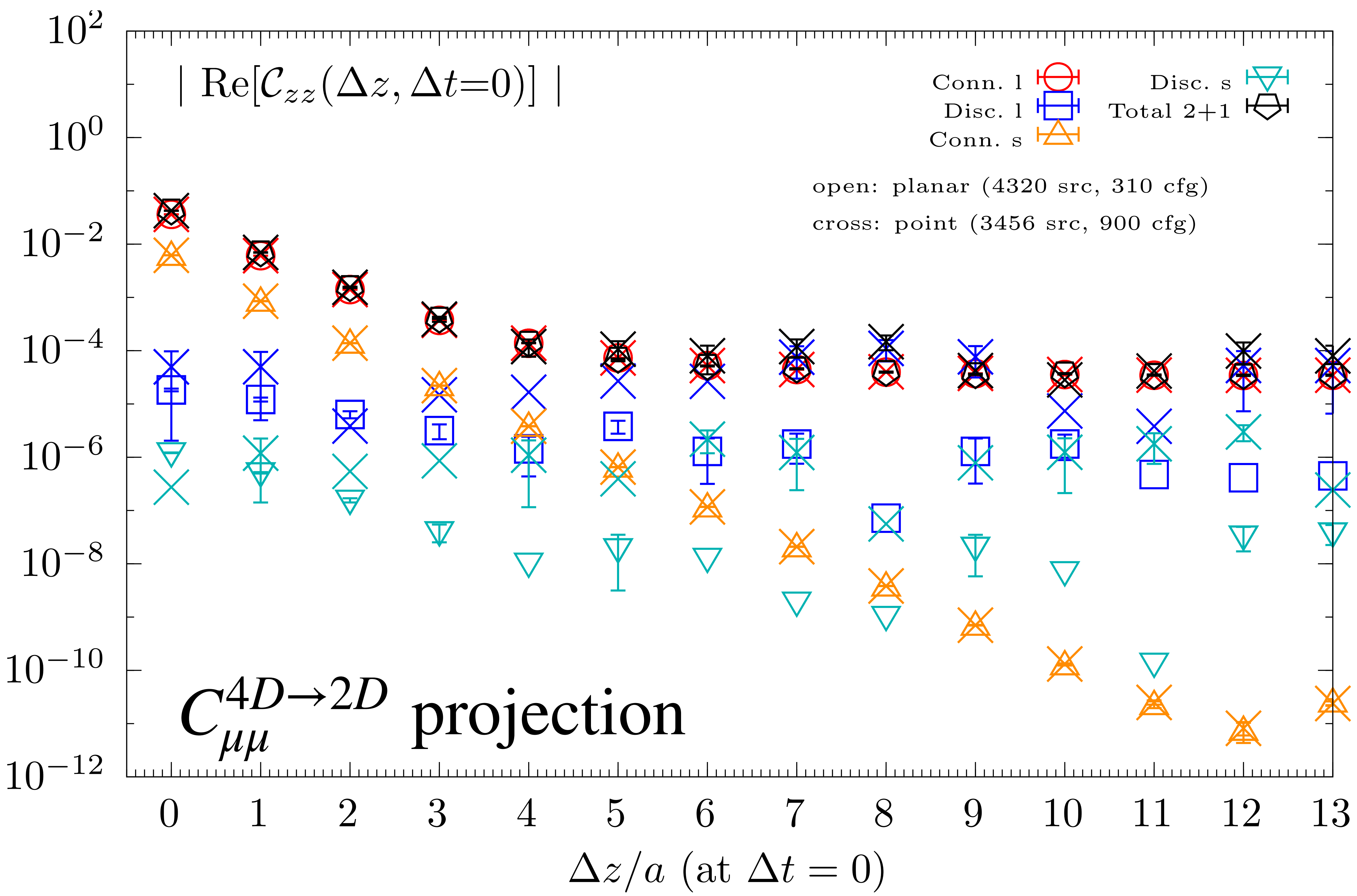
- [ ] Even with a point source, using $\mathcal{O}(10^3)$ noisy estimators gives good S/N, w/o the necessity of corner-extrap.

Backup

Validation: point v.s. planar v.s. wall source

Three independent estimators of the same correlator projection : wall (open), planar (filled), point (cross)





The new planar source works well, channel-by-channel agreement in 1D and 2D projections