Noise reduction for disconnected diagrams in η correlation functions

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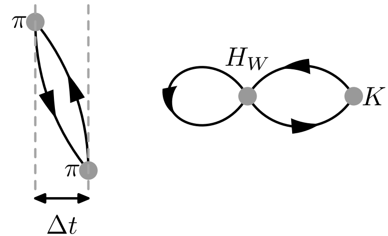
Outline

- Introduction: disconnected diagrams in $K \to \pi\pi$
- Introduction: disconnected diagrams and the noise problem
- Noise reduction via truncated configurations
- Properties of the method
- Numerical results (η meson)
- Summary and outlook

Introduction: disconnected diagrams in $K \to \pi\pi$

- $K \to \pi\pi$ decay is key to understanding direct CP violation (ε'/ε).
- The $\Delta I = 1/2$ rule and ε'/ε require both connected and disconnected diagrams.
- RBC/UKQCD:
  Mon 4:30 PM, Jonas Hildebrand;
  Mon 4:50 PM, Masaaki Tomii.
- Statistical noise of the disconnected part does not decrease as the distance between the two disconnected parts increases. But the signal does decrease.
- Noise from disconnected diagrams dominates in the large time region.

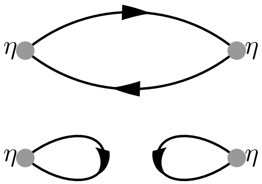


$\langle \pi\pi | H_W | K \rangle_{\mathrm{disc}}$

- Introduce time separation Δt between the two pion operators to reduce noise. The separation forces the quarks in the $\pi\pi$ bubble to propagate some non-zero distance ($> \Delta t$) to suppress the size and the fluctuation of the $\pi\pi$ bubble.
  RBC/UKQCD Phys. Rev. D 104 (2021) 11, 114506
- Here, we try to develop this idea further.

Introduction: disconnected diagrams

- Another example: η meson, operator $\eta_s(t) = i\bar{s}(t)\gamma_5 s(t)$ with Coulomb gauge fixed wall source.



- Correlator has connected and disconnected parts:

$$\langle \mathcal{T}\eta_s(t_A)\eta_s(t_B)\rangle = \Big\langle \mathrm{Tr}\big(\gamma_5 S_s(t_A, t_B)\gamma_5 S_s(t_B, t_A)\big) - \mathrm{Tr}\big(\gamma_5 S_s(t_A, t_A)\big)\mathrm{Tr}\big(\gamma_5 S_s(t_B, t_B)\big)\Big\rangle$$

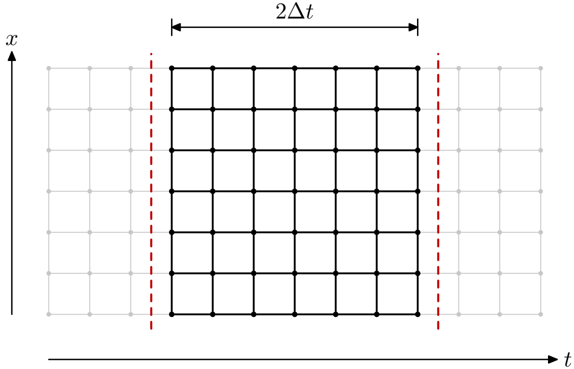
- We cannot easily introduce a Δt parameter to force the propagator in the disconnected diagram to propagate.
- Ideally, we are interested in the quark loop propagating some distance and then coming back.
- However, a lot of noise results from the quark only propagating within a short distance.
- Try to subtract this part of the contribution legally.

Truncated configurations

- Truncated gauge configuration:

$$U_\mu^{t_1,t_2}(\vec{x},t) = \begin{cases} U_\mu(\vec{x},t) & \text{if } t_1 \le t \le t_2 \\ 0 & \text{otherwise} \end{cases}$$

- The truncated gauge field has open boundary condition. The gauge links outside the range are 0, not 1 (corresponding to free field).
- Truncated propagator: $S_q^{t_1,t_2}(x,y) = D_q^{-1}[U^{t_1,t_2}]_{x,y}$ only depends on the truncated gauge field.
- The truncated propagator $D_q^{-1}[U^{t_1,t_2}]_{x,y}$ can be viewed as a valid gluonic operator spanning $[t_1,t_2]$.



Method: noise-reduced operator

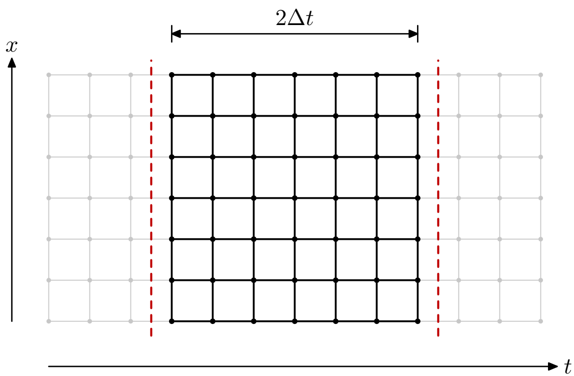
- Define an η meson operator based on the truncated disconnected diagram:

$$\eta_s^{\mathrm{disc},\Delta t}(t) = -i\,\mathrm{Tr}\left(\gamma_5 S_s^{t-\Delta t, t+\Delta t}(t,t)\right) \tag{1}$$

- A valid, *gluonic*, η interpolating operator spanning $[t-\Delta t, t+\Delta t]$.
- Noise-reduced (NR) operator:

$$\eta_s^{\mathrm{NR},\Delta t}(t) = \eta_s(t) - \eta_s^{\mathrm{disc},\Delta t}(t) \tag{2}$$

- Subtraction reduces disconnected-diagram noise; Δt is tunable.



- Correlation function with the noise-reduced (NR) operator:

$$\begin{aligned}\langle \eta_s^{\mathrm{NR},\Delta \mathrm{t}}(t_A)\eta_s^{\mathrm{NR},\Delta \mathrm{t}}(t_B)\rangle =& \Big\langle \mathrm{Tr}\big(\gamma_5 S_s(t_A,t_B)\gamma_5 S_s(t_B,t_A)\big) \\ &- \mathrm{Tr}\big(\gamma_5 S_s(t_A,t_A) - \gamma_5 S_s^{t_A-\Delta t, t_A+\Delta t}(t_A,t_A)\big) \\ &\times \mathrm{Tr}\big(\gamma_5 S_s(t_B,t_B) - \gamma_5 S_s^{t_B-\Delta t, t_B+\Delta t}(t_B,t_B)\big)\Big\rangle\end{aligned}$$

- Connected diagram contribution is not affected at all.
- Size and fluctuations of the disconnected diagrams are significantly reduced.
- Effective time separation reduced by $2\Delta t$, but reduced statistical noise compensates.
- Special case: $\Delta t \to +\infty$ is equivalent to dropping disconnected diagrams.
  - The choice of $\Delta t \to +\infty$ is not valid. Since when $\Delta t \to +\infty$, the two operators overlap, and $t_{\mathrm{sep}} < 0$.
- Another way to look at this method is that: we introduce additional gluonic operators to better interpolate to the desired quantum state.
- The operator $\eta_s^{\mathrm{disc},\Delta \mathrm{t}}(t)$ and $\eta_s^{\mathrm{NR},\Delta \mathrm{t}}(t)$ are also studied in the context of the two-level sampling algorithm by L. Barca, J. Crawford, J. Finkenrath, S. Schaefer.
  See slides 31, 32 of the CERN talk.

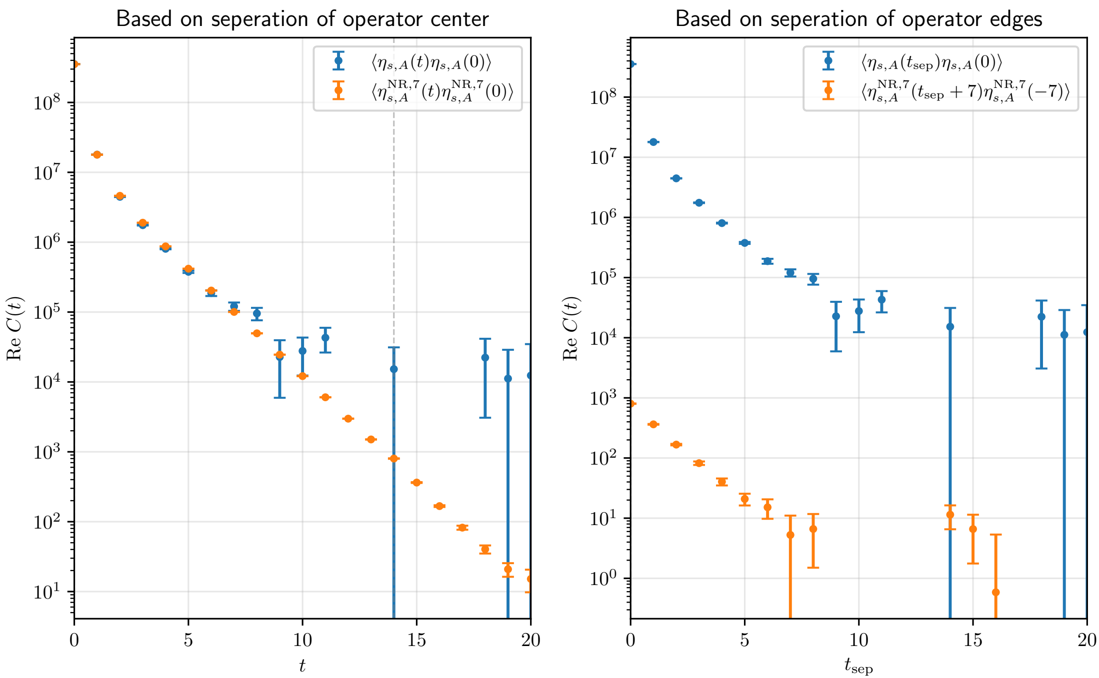
Numerical results: lattice setup

- RBC/UKQCD 24ID Mobiüs domain wall fermion ensemble.
  $24^3 \times 64$, $a^{-1} = 1.023$ GeV, $M_\pi = 143$ MeV, $M_K = 516$ MeV.
- Measured on 113 configurations, calculated Coulomb gauge fixed wall source propagators for all time slices.
- Fermionic η operators:

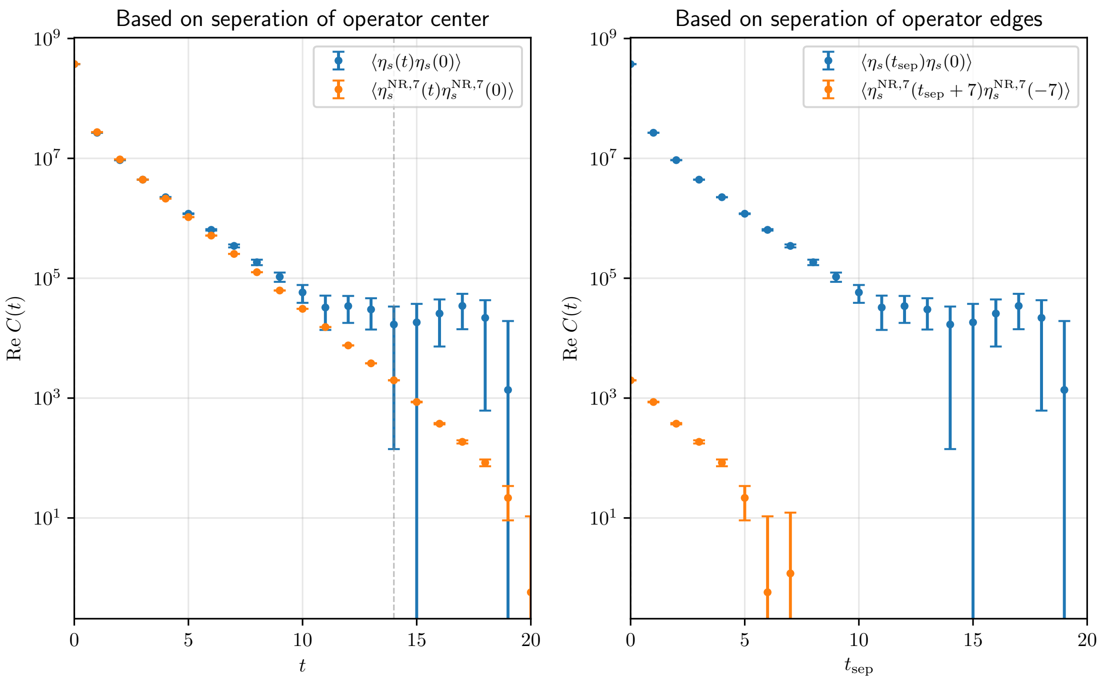
$$\eta_s = i\bar{s}\gamma_5 s \qquad \eta = i(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s)/2$$
$$\eta_{s,A} = i\bar{s}\gamma_t\gamma_5 s \quad \eta_A = i(\bar{u}\gamma_t\gamma_5 u + \bar{d}\gamma_t\gamma_5 d - 2\bar{s}\gamma_t\gamma_5 s)/2$$

- Here, we only construct noise-reduced ($\eta_s^{\mathrm{NR},\Delta t}$, $\eta_{s,A}^{\mathrm{NR},\Delta t}$) operators for strange quark propagators.
- $\Delta t = 1, 3, 5, 7$ in lattice units ($a = 0.193$ fm).

$$\eta_{s,A} = i\bar{s}\gamma_t\gamma_5 s \tag{3}$$



$$\eta_s = i\bar{s}\gamma_5 s \tag{4}$$



- Minimum time separation included in the fit:
  $t_{\text{sep,min}} = 3$ for fermionic and $\Delta t = 1$ noise reduced operators;
  $t_{\text{sep,min}} = 1$ for $\Delta t = 3, 5, 7$ noise reduced operators;
  $t_{\text{sep,min}} = 2$ for correlation functions between the above two kinds of operators.
- Maximum time difference is $t_{\text{max}} = 20$.

Fit	Pure fermionic	With noise reduced (NR) operators
Operators	4	12
Data points	288	2592
Parameters	10	26
χ^2/dof	0.76 ± 1.08	1.07 ± 0.78
E_0 (GeV)	0.517(60)	0.549(34)
E_1 (GeV)	0.911(120)	0.828(47)

- Noise reduced (NR) operators include noise-reduced operators with $\Delta t = 1, 3, 5, 7$.

Summary and outlook

- Presented a noise reduction technique for disconnected diagrams.
- Uses truncated gauge configurations to subtract a controlled estimate of the disconnected contribution.
- Demonstrated on the η meson with $\eta_s(t) = i\bar{s}(t)\gamma_5 s(t)$.
- This method should work for disconnected diagrams that have a lot of short-distance noise but not much short-distance signal.
- Outlook: apply to other observables with disconnected diagrams, optimize Δt (one can also use different Δt on the left and right side of the operator), include even more operators.
- A totally different application of the truncated gauge field propagator:
  Mon 4:10 PM, Christoph Lehner.

Thank You!