Toward quantum computation of hadronic tensors

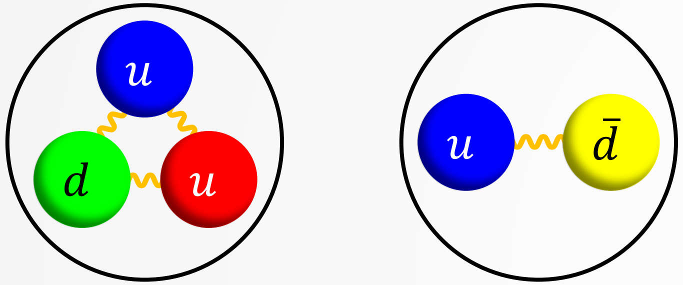
Chung-Chun Hsieh

With Prof. Zohreh Davoudi, Navya Gupta, Jinghong Yang

Introduction

Motivation: hadron structure

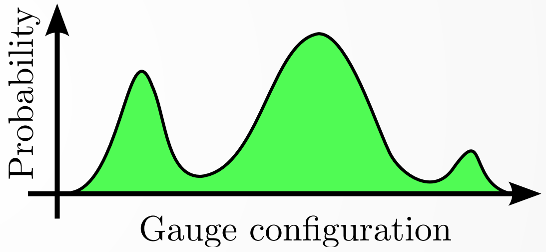
- Gauge theory degrees of freedom → **Hadrons**

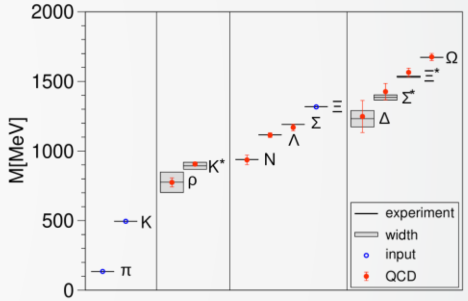


Lattice QCD: Lagrangian formulation

- Euclidean Path integral, success in static quantities

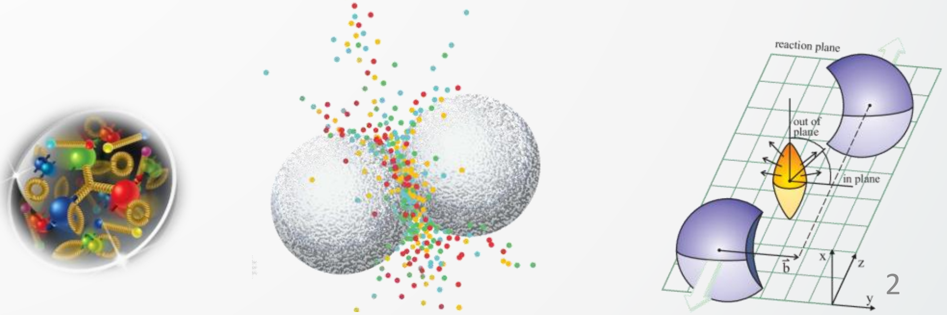
$$\langle \hat{\boldsymbol{O}} \rangle = \frac{1}{Z} \int \mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\mathcal{D}[U]\hat{O}\boldsymbol{e}^{-S_E} \approx \frac{1}{N}\sum_{k=1}^{N} \hat{O}[U^{(k)}]$$





Durr et al. (2008, BMW Collaboration)

- Difficult for real-time dynamics, response function, transports, **hadronic tensors**



PC: MIT, Z. Davoudi; Betz (2009)

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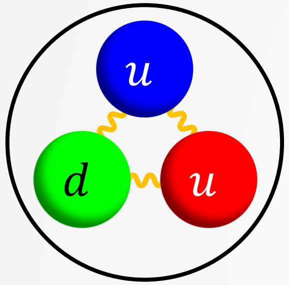
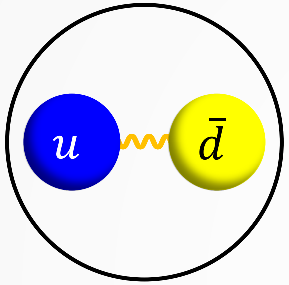
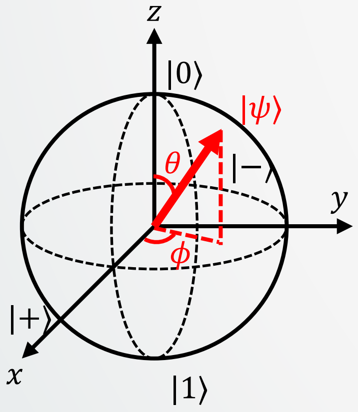




Hamiltonian formulation

$$\langle \hat{\boldsymbol{O}}(\boldsymbol{t}) \rangle = \langle \psi | e^{iHt} \hat{\boldsymbol{O}}(\boldsymbol{0}) e^{-iHt} | \psi \rangle$$



- Exponential growth of Hilbert space → **quantum computing**

$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{-i\phi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

$$|\psi\rangle_1 \otimes |\psi\rangle_2 \otimes \cdots \otimes |\psi\rangle_N \sim 2^N$$

Superposition

Lattice QCD: Lagrangian formulation

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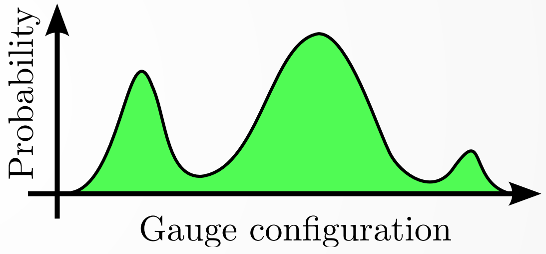

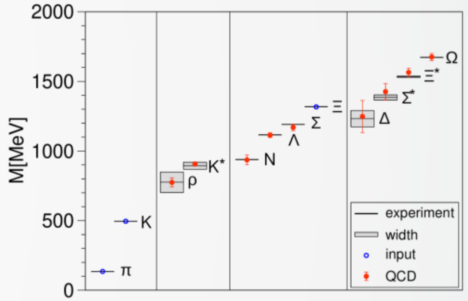

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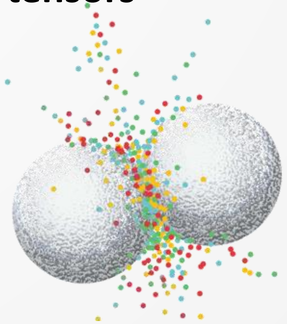

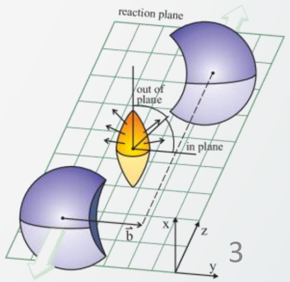

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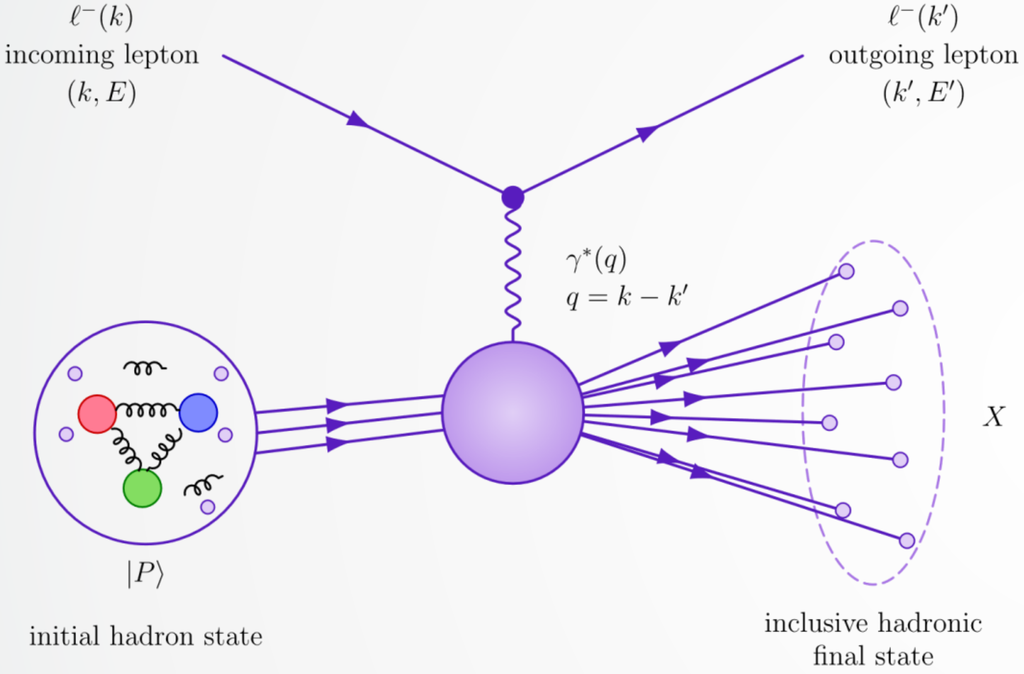
Introduction of hadronic tensors

Deep inelastic scattering



Hadronic tensor

$$W^{\mu\nu}(P,q) = \frac{1}{2\pi}\sum_X (2\pi)^{d+1}\,\delta^{d+1}(P+q-p_X)\,\langle P,S|J^\mu(0)|X\rangle\langle X|J^\nu(0)|P,S\rangle$$

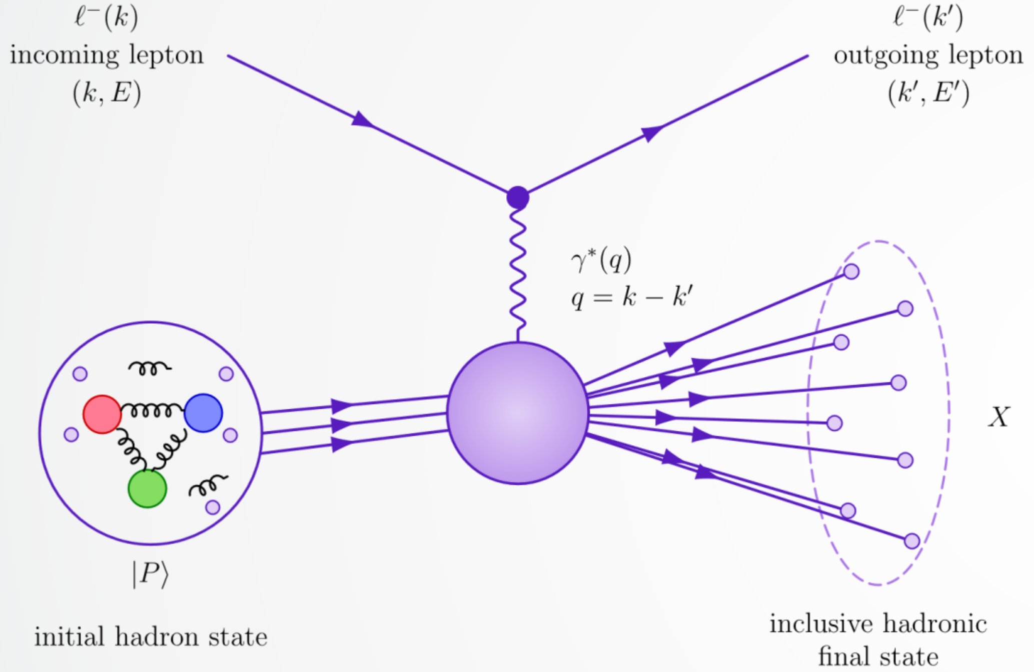
$$= \frac{1}{2\pi}\int d^{d+1}z e^{iqz}\langle P,S|\underbrace{J^\mu(z)}_{e^{iHt}J^\mu(x)e^{-iHt}}J^\nu(0)|P,S\rangle$$

Real-time correlator

Quantum simulation

Introduction of hadronic tensors

Deep inelastic scattering



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Real-time correlator

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Hadron structure

- DIS: evidence for quark model
- Structure function factorizes into parton distribution functions (PDFs) in DIS limit

$$F_2(x_B, Q^2) = x_B \sum_i \int_{x_B}^1 \frac{dx}{x}\, C_{2,i}\left(\frac{x_B}{x}, \frac{Q^2}{\mu^2}, \alpha_s(\mu)\right) f_i(x,\mu),$$

PDFs: longitudinal momentum content in hadrons

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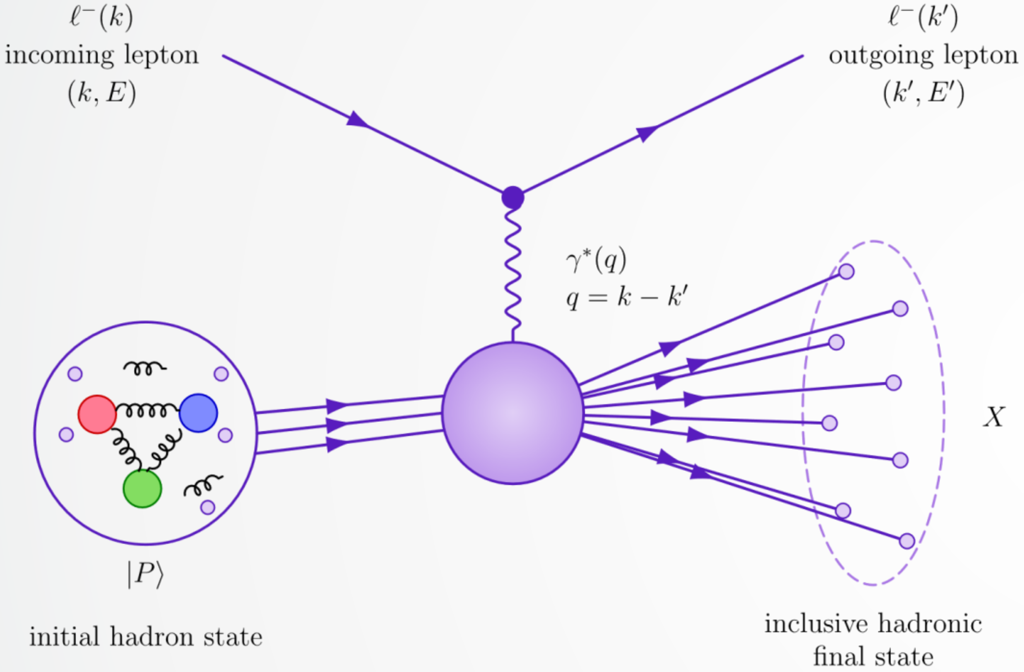

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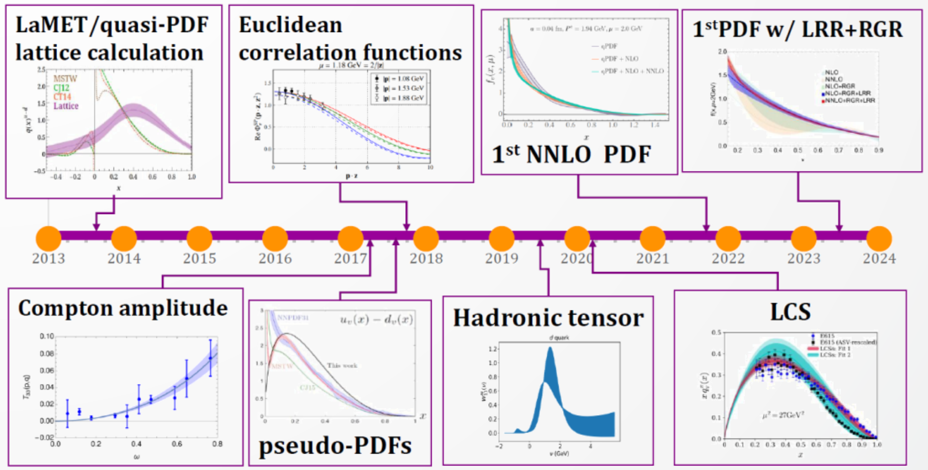
PDFs: longitudinal momentum content in hadrons

- Obtaining PDFs from lattice is difficult & active research area



Ji (2013); Ji (2014); Lin et. al. (2015); Radyushkin (2017); Ma, Qiu (2018); Izubuchi et. al. (2018); Liang et. al. (2020)

PC: Lin (2025)

Outline

- **Introduction**
- **Theoretical formulation**
  - Kogut-Susskind formulation of the Schwinger model
  - Hadronic tensors in the Schwinger model
- **Quantum algorithm**
  - Hadamard-test circuits for real-time correlators
  - State preparation: Hadronic ansatz for interacting creation operator
  - Time evolution: Trotter product formula
- **Results**
  - Trapped-ion emulator & hardware
- **Summary and Outlook**

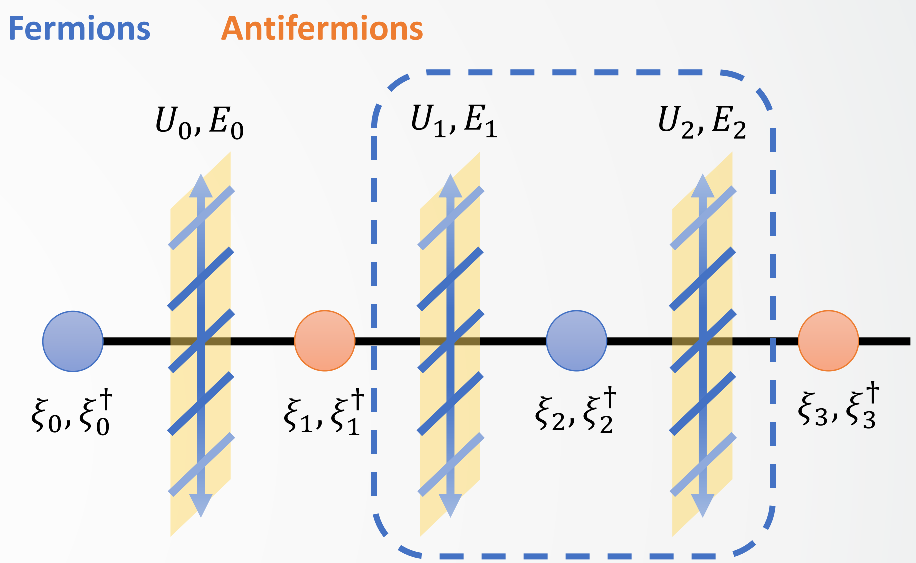
Lattice Schwinger model

Schwinger model

- **Kogut-Susskind Hamiltonian:** 1+1D $U(1)$) with single flavor fermion Kogut, Susskind (1974)

$a = 1$

$$H = \underbrace{\frac{i}{2}\sum_n \left(\xi_n^\dagger \xi_{n+a} U_n - \mathrm{H.c.}\right)}_{H^h} + \underbrace{am_f \sum_n (-1)^{\frac{n}{a}} \xi_n^\dagger \xi_n}_{H^m} + \underbrace{\frac{ag^2}{2}\sum_n E_n^2}_{H^E}$$



Gauss' law: $G_n|\psi\rangle_{phys} = 0 \ \forall n,$

$G_n = E_n - E_{n-1} - Q_n, Q_n = \xi_n^\dagger \xi_n - \frac{1-(-1)^n}{2}$

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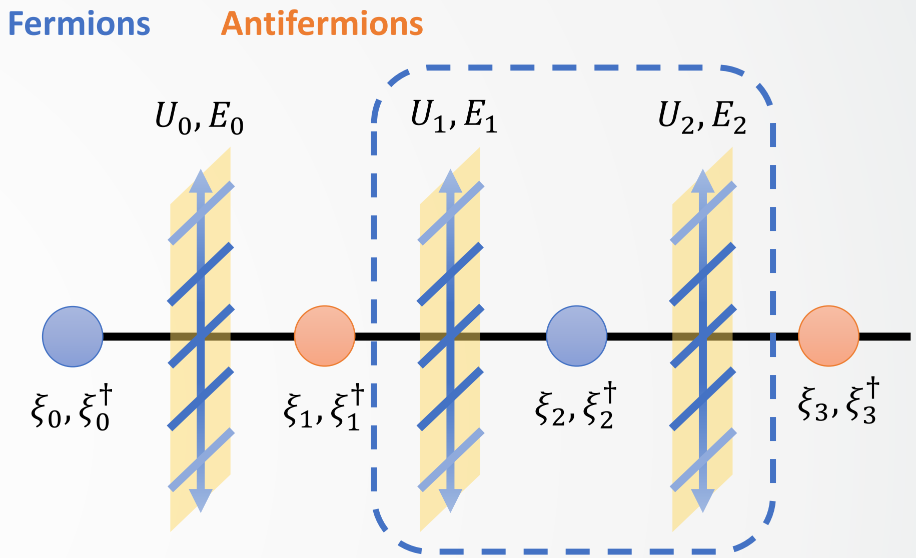
- NISQ simulation: Limited **qubits & entangling gates**
- **Minimal gauge formulation**: remove gauge fields by explicitly solving Gauss' law. In 1+1D PBC, one gauge d.o.f. remains

$$H^h = \frac{1}{4}\sum_{n=0}^{N-2}\left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right) + \frac{\alpha_N}{4}\left(\sigma_{N-1}^- \sigma_0^+ \tilde{U} + \mathrm{h.c.}\right)$$

$$H^m = \frac{m_f}{2}\sum_{n=0}^{N-1}(-1)^{n+1}\sigma_n^z$$

$$H^E = \frac{g^2}{2}\tilde{E}^2 + \frac{g^2}{2}\sum_{n=0}^{N-2}\left\{\tilde{E} + \frac{1}{2}\sum_{k=0}^{n}\left[\sigma_k^z - (-1)^k\right]\right\}^2$$



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Fermions: Jordan-Wigner $\xi_n = \left(\prod_{j<n}(-i\sigma_j^z)\right)\sigma_n^+,\ \xi_n^\dagger = \left(\prod_{j<n}(i\sigma_j^z)\right)\sigma_n^-$

Bosons: binary encoding with $\eta \equiv \lceil \log_2(2\Lambda + 1) \rceil$ qubits

Hadronic tensors of the Schwinger model

DIS hadronic tensors for Schwinger model

- In 1+1D vector hadronic tensor suppressed in DIS limit form LO parton model

$$W^{\mu\nu}(P,q) \sim \frac{m^2}{Q^2} f(x_B)$$

- Intuition: DIS in 1+1D flips quark movement/chirality, forbidden by vector currents

- **Scalar hadronic tensors** $\bar{\psi}(x)\psi(x)$

$$W(P,q) = \frac{1}{2\pi}\int d^2z\, e^{iq\cdot z} \underbrace{\langle P|S(z)S(0)|P\rangle}_{C(x,t)} \sim \left(f(x_B) + \bar{f}(x_B)\right)$$

(DIS limit)

- Quantum simulation of hadronic tensors & PDFs

Ikeda et. al. (2025); Zhou et. al. (2026);
Bañuls et. al. (2026); Chen, Chen, Meher (2025);
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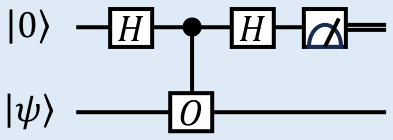
Hadamard test



$$\frac{1 + Re(\langle\psi|O|\psi\rangle)}{2}$$

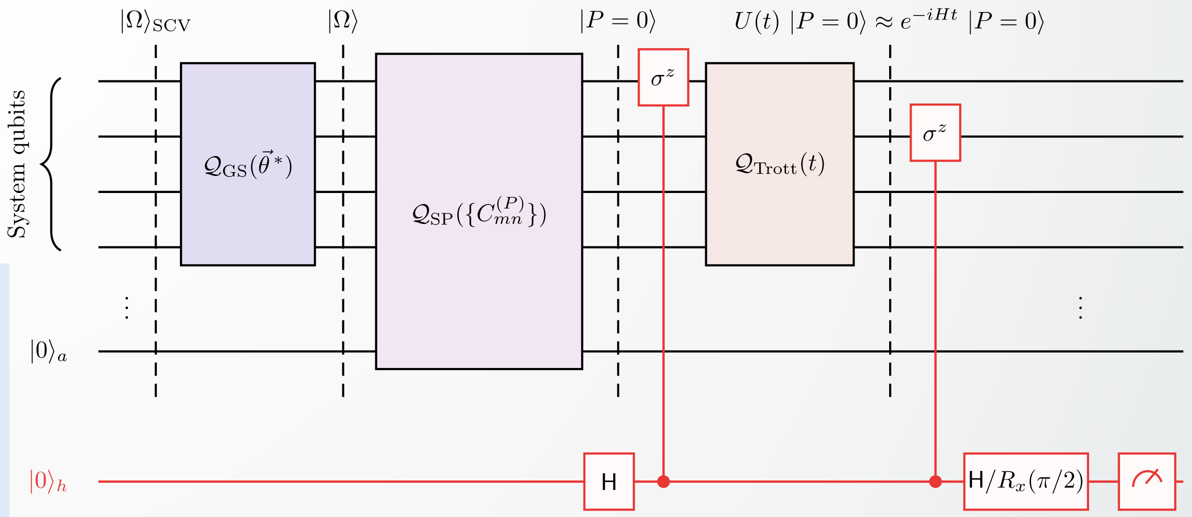
Quantum algorithm for real-time correlators

Lamm, Lawrence, Yamauchi (2020); Mueller, Tarasov, Venugopalan (2020); Li et. al. (2022); Barata et. al. (2025); Wang et. al. (2025); Burbano et. al. (2026); Chiew et. al. (2026); Kemper et. al. (2026)

$$C(x,t) = \langle P|\, S(x,t)\, S(0,0)\,|P\rangle = \frac{1}{4}\sum_{\substack{i\in\{n,n+1\}\\ j\in\{0,1\}}} (-1)^{i+j}\langle P|\,\sigma_i^Z(t)\,\sigma_j^Z(0)\,|P\rangle$$

Hadronic state preparation

Time evolution



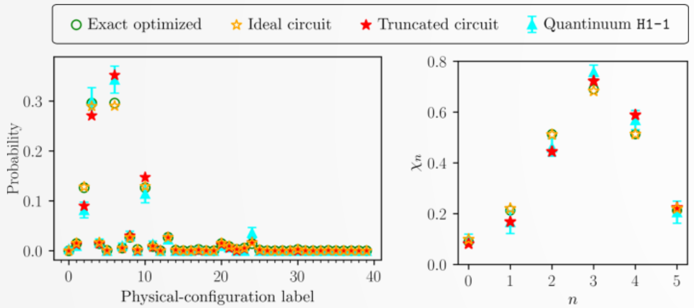
Only current operator controlled

Hadronic tensors of the Schwinger model

- Quantum circuits from hadron scattering, already demonstrated on trapped-ion computers

State preparation:

Z. Davoudi, CCH, and S. V. Kadam, Quantum 8, 1520 (2024), arXiv:2402.00840



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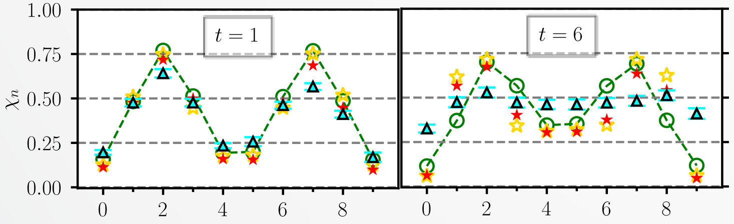

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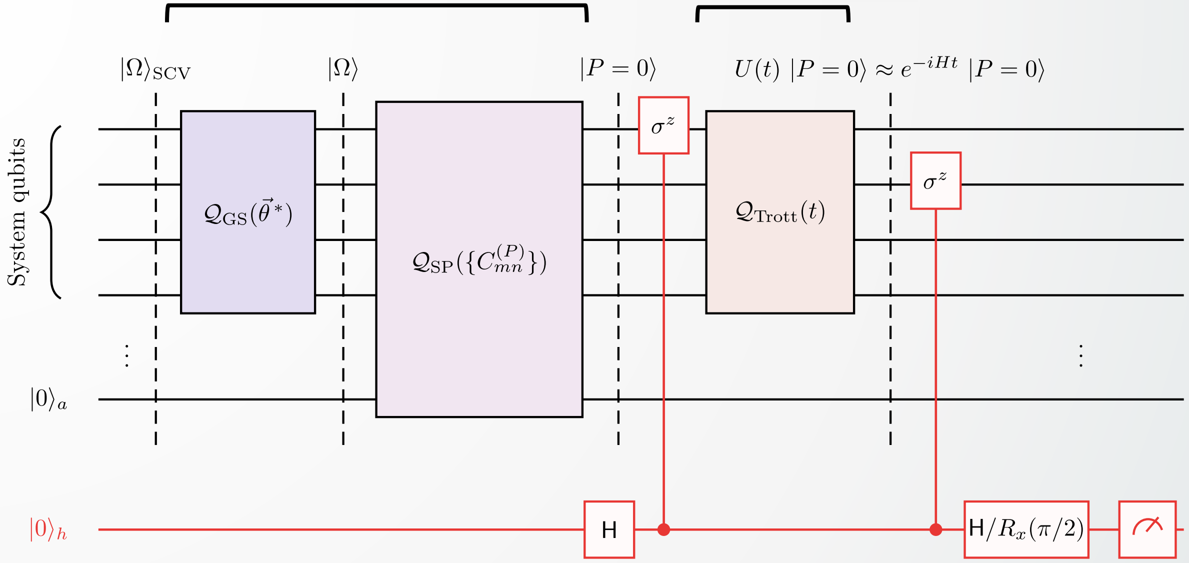

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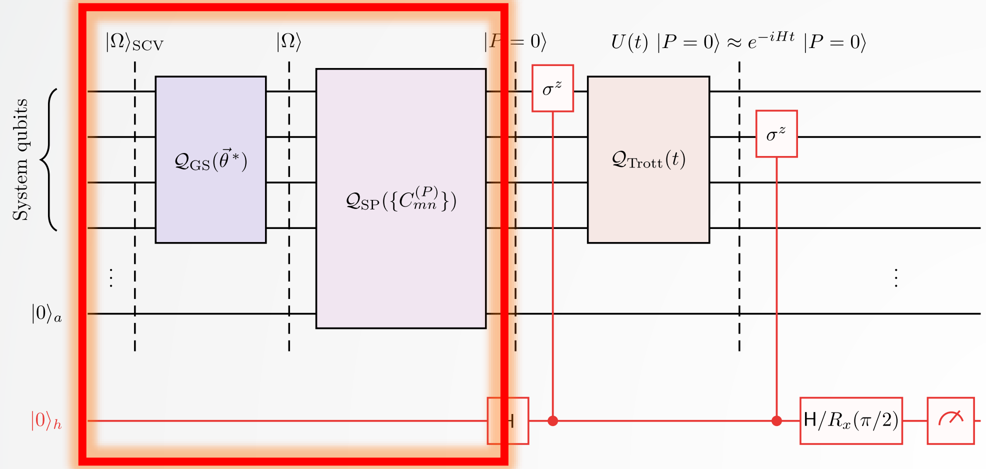

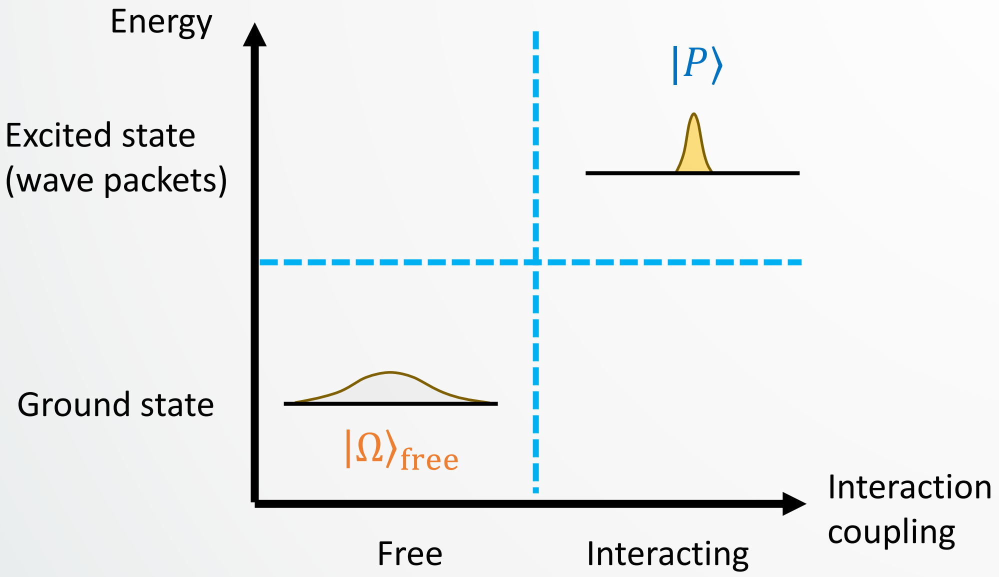

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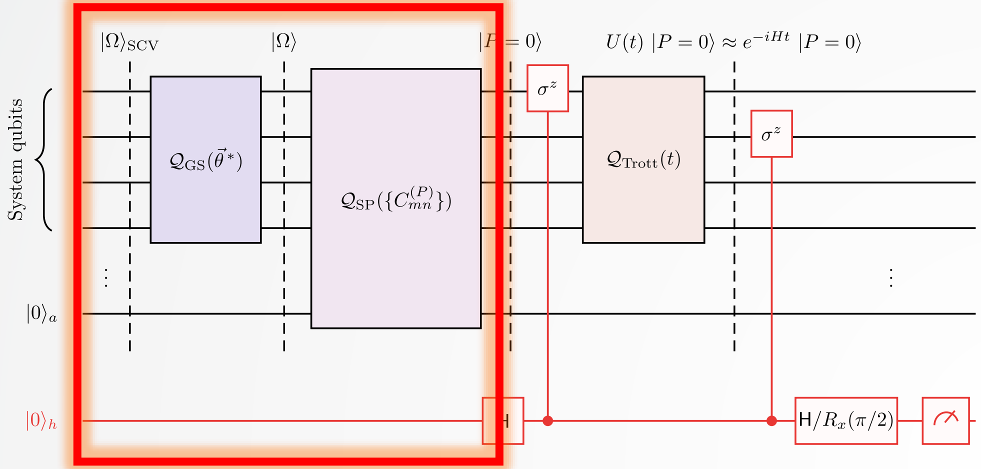
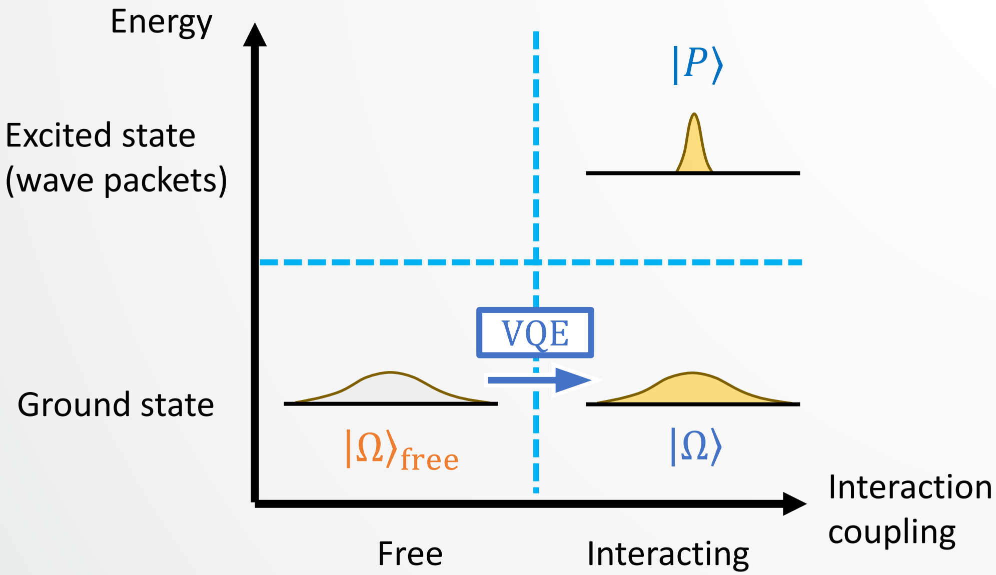
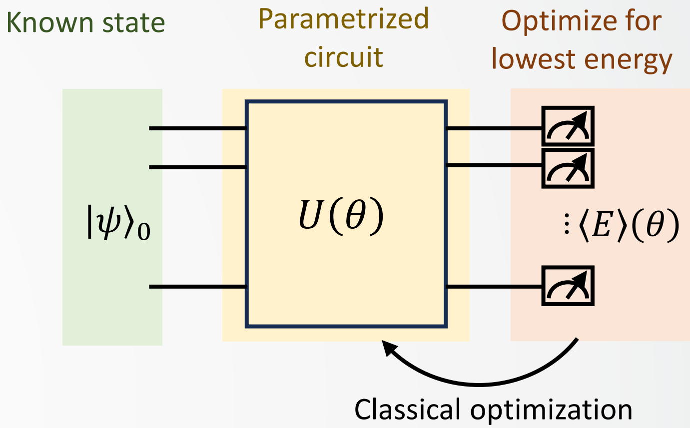




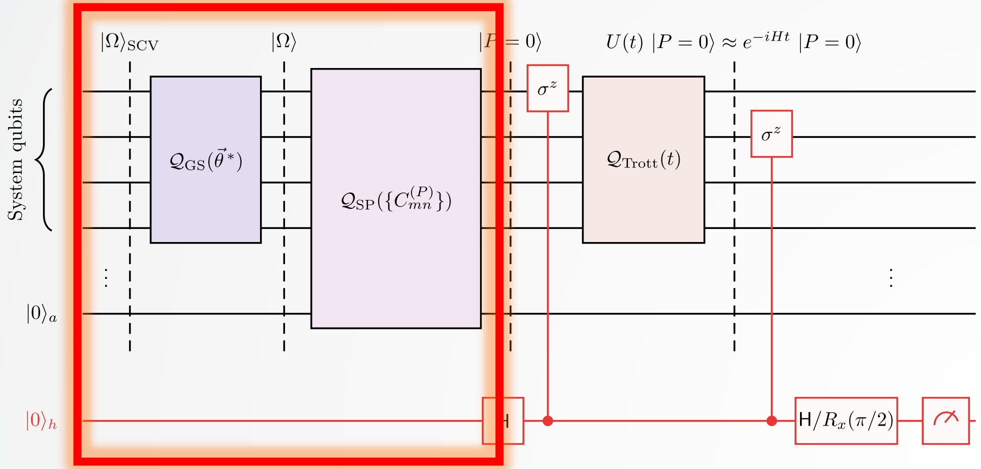
Variational quantum eigensolver (VQE)

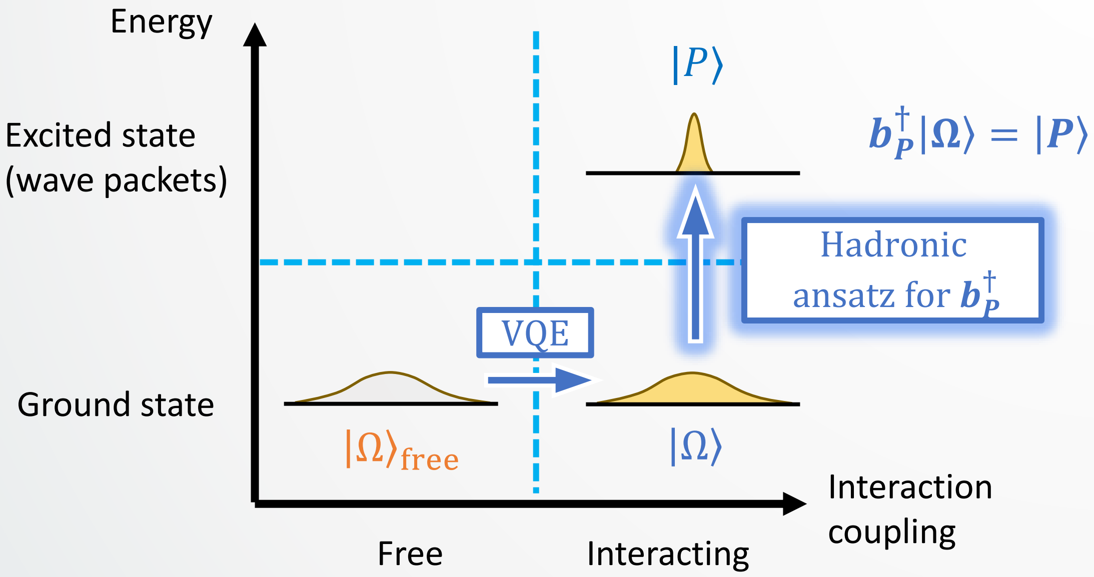
Peruzo et. al. (2014);
Mclean et. al. (2016)



- NISQ friendly
- Variational principle for ground state

State preparation





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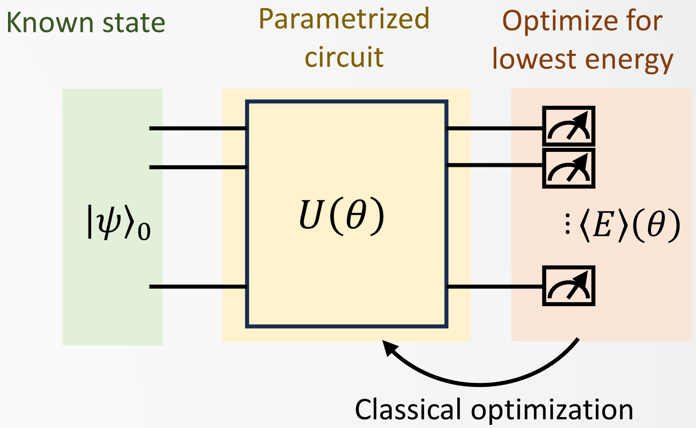

- NISQ friendly
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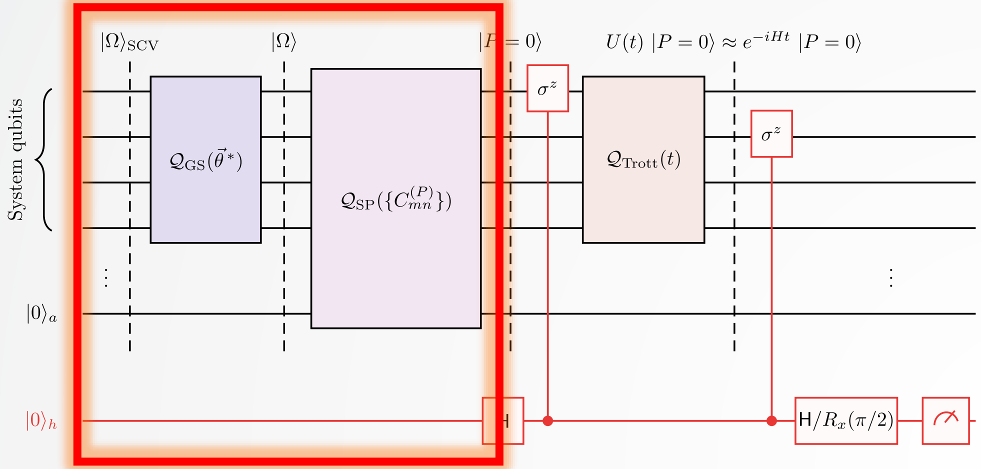
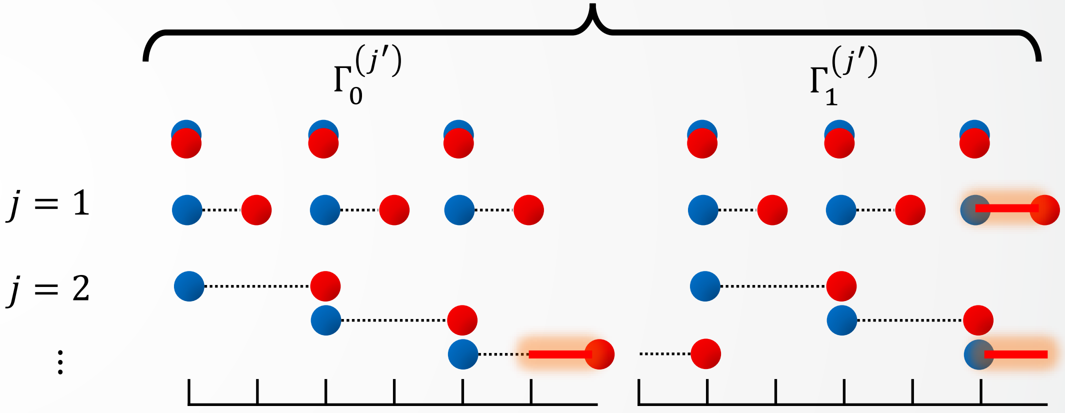
Hadronic ansatz

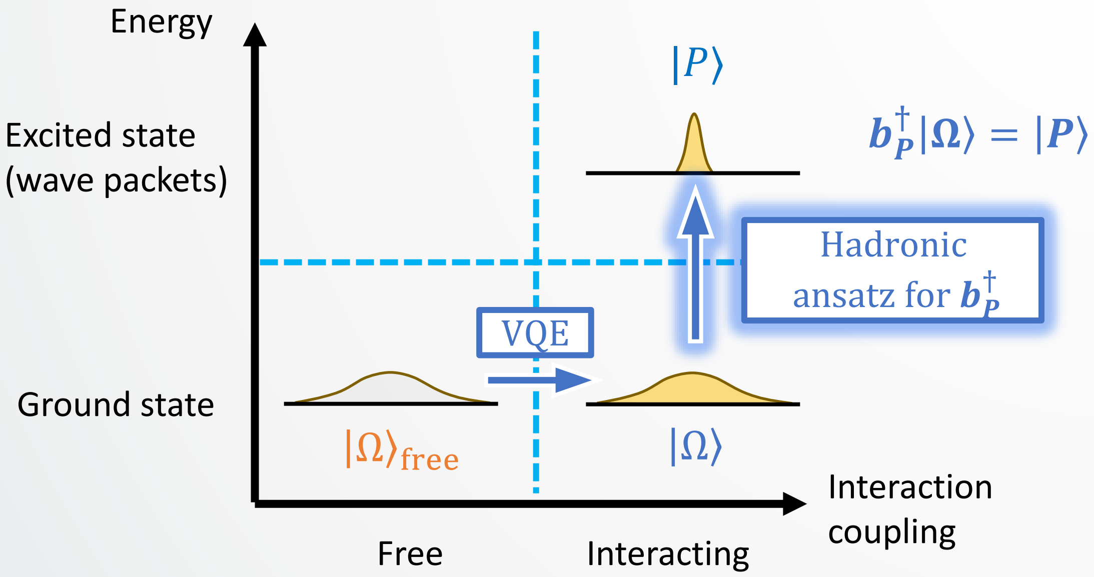
- Gauge invariant bound state excitations
- **definite quantum number**
- Improved implementation according to physical importance

State preparation



$$b_P^\dagger = \frac{1}{\mathcal{N}} \underbrace{\sum_j}_{\text{order}} \underbrace{\sum_{i\in\{0,1\}}}_{\text{staggered}} \underbrace{\sum_{m,n\in\Gamma_i^{(j)}}}_{\text{site pairs}} e^{-\alpha_i^{(j),P}|m-n|^2}\, \bar{C}_{m,n}^P \underbrace{\widetilde{\mathcal{M}}_{m,n}}_{\xi_m^\dagger \xi_n \left(\xi_{N-1}^\dagger \xi_0 U\right)}$$





Hadronic ansatz

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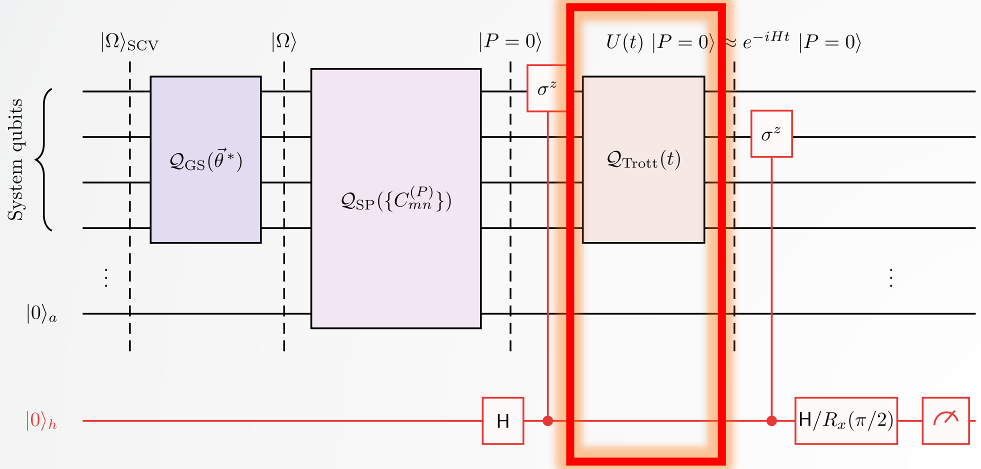
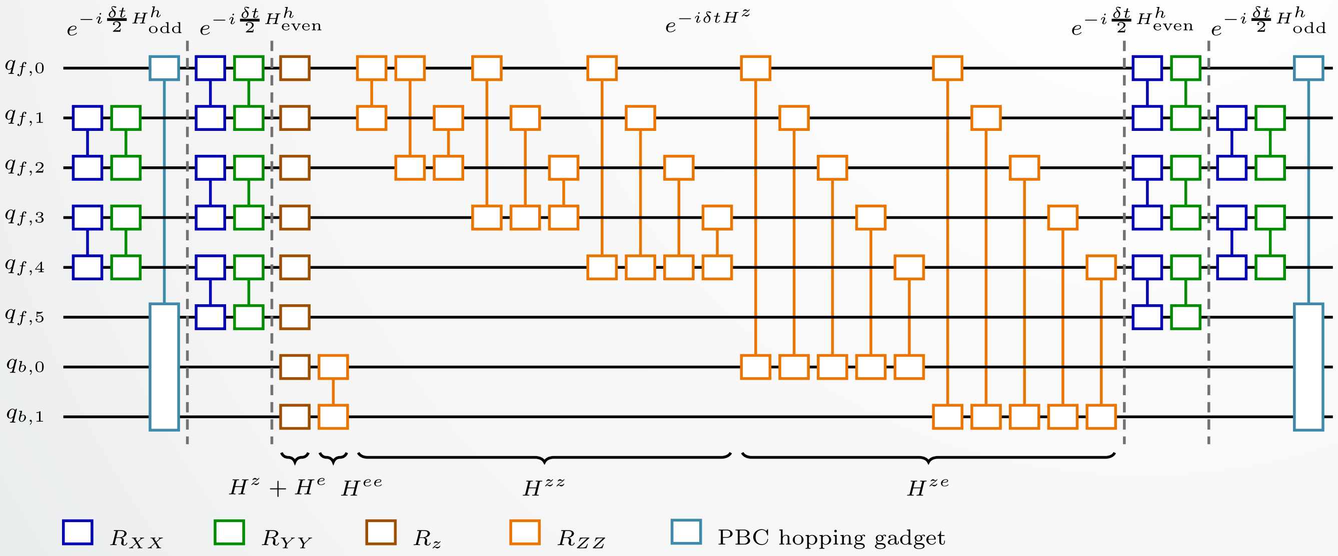
- **definite quantum number**
- Improved implementation according to physical importance

Hadronic state $|k=0\rangle$ fidelity ~ 0.97. For detailed performance, see Davoudi, Hsieh, Kadam (2025)

Time evolution



Second order Trotter, N_T steps



Hamiltonian

- N qubits for fermion, $\eta = [\log_2 2\Lambda + 1]$ qubits for the single bosonic mode, 2 ancilla for state prep and Hadamard test
- $\widetilde{U}$ approximated with quantum Fourier transform

$$H = H^h + H^z + H^{zz} + H^e + H^{ee} + H^{ze}$$

$$H^h = \frac{1}{4}\sum_{n=0}^{N-2}\left(\sigma^{\mathrm{x}}_n\sigma^{\mathrm{x}}_{n+1} + \sigma^{\mathrm{y}}_n\sigma^{\mathrm{y}}_{n+1}\right) + \frac{1}{2}\left(\sigma^-_{N-1}\sigma^+_0\,\widetilde{U} + \mathrm{h.c.}\right)$$

$$H^z = \sum_{n=0}^{N-1}\left[\frac{m_f}{2}(-1)^{n+1} + \frac{g^2}{2}\left(-\frac{1}{2}\left\lfloor\frac{N-n}{2}\right\rfloor - (N-1-n)\,c_\Lambda\right)\right]\sigma^{\mathrm{Z}}_n$$

$$H^e = -\frac{g^2}{2}N\left(c_\Lambda + \frac{1}{4}\right)\sum_{\tilde{n}=0}^{\eta-1} 2^{\tilde{n}}\tilde{\sigma}^{\mathrm{Z}}_{\tilde{n}}$$

$$H^{zz} = \sum_{n=0}^{N-2}\sum_{m=0}^{n-1}\frac{g^2}{4}(N-1-n)\;\sigma^{\mathrm{Z}}_m\sigma^{\mathrm{Z}}_n$$

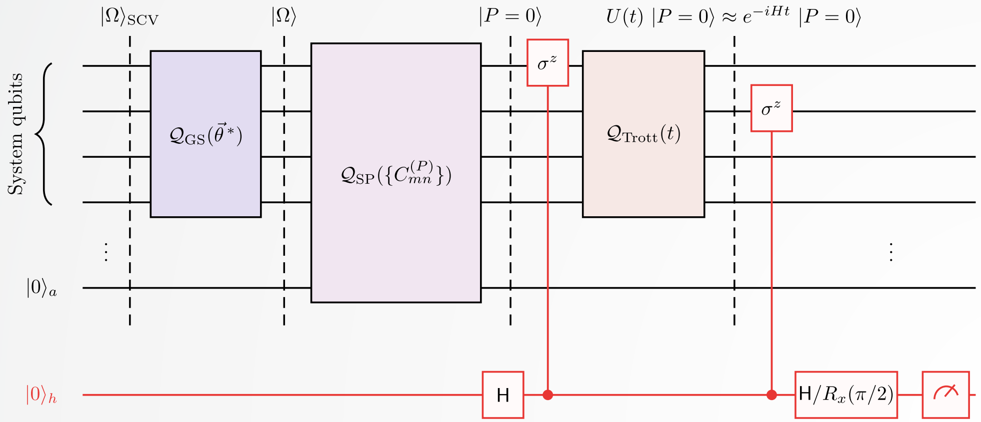
$$H^{ee} = \frac{g^2 N}{2}\sum_{\tilde{n}=0}^{\eta-1}\sum_{\tilde{m}=0}^{\tilde{n}-1} 2^{\tilde{m}+\tilde{n}-1}\tilde{\sigma}^{\mathrm{Z}}_{\tilde{m}}\tilde{\sigma}^{\mathrm{Z}}_{\tilde{n}}$$

$$H^{ze} = \frac{g^2}{2}\sum_{n=0}^{N-2}\sum_{\tilde{n}=0}^{\eta-1}(N-1-n)\cdot 2^{\tilde{n}-1}\,\sigma^{\mathrm{Z}}_n\tilde{\sigma}^{\mathrm{Z}}_{\tilde{n}}$$

Cross terms from quadratic electric field

$$c_\Lambda = -\Lambda + \frac{1}{2}(2^\eta - 1)$$

Time evolution



Resource

	$\mathcal{Q}_{GS}$	$\mathcal{Q}_{SP}$	$\mathcal{Q}_{Trott}$
Entangling gates per Trotter layer	$2N+\eta^2+\eta+2$	$20N+3\eta-4$	$\frac{1}{2}N^2+\frac{3}{2}N+N\eta+\frac{3}{2}\eta^2-\frac{1}{2}\eta+3$

- Quadratic dependence from fermionic formulation
- State preparation with $j=1$ ansatz
- Few Trotter steps already require $\sim$1000 two-qubit gates

Hamiltonian

- N qubits for fermion, $\eta = [\log_2 2\Lambda + 1]$ qubits for the single bosonic mode, 2 ancilla for state prep and Hadamard test
- $\widetilde{U}$ approximated with quantum Fourier transform

$$H = H^h + H^z + H^{zz} + H^e + H^{ee} + H^{ze}$$

$$H^h = \frac{1}{4}\sum_{n=0}^{N-2}\left(\sigma_n^{\mathrm{x}}\sigma_{n+1}^{\mathrm{x}} + \sigma_n^{\mathrm{y}}\sigma_{n+1}^{\mathrm{y}}\right) + \frac{1}{2}\left(\sigma_{N-1}^{-}\sigma_0^{+}\,\widetilde{U} + \mathrm{h.c.}\right)$$

$$H^z = \sum_{n=0}^{N-1}\left[\frac{m_f}{2}(-1)^{n+1} + \frac{g^2}{2}\left(-\frac{1}{2}\left\lceil\frac{N-n}{2}\right\rceil - (N-1-n)\,c_\Lambda\right)\right]\sigma_n^{\mathrm{Z}}$$

$$H^e = -\frac{g^2}{2}N\left(c_\Lambda + \frac{1}{4}\right)\sum_{\tilde{n}=0}^{\eta-1} 2^{\tilde{n}}\tilde{\sigma}_{\tilde{n}}^{\mathrm{Z}}$$

$$H^{zz} = \sum_{n=0}^{N-2}\sum_{m=0}^{n-1}\frac{g^2}{4}(N-1-n)\;\sigma_m^{\mathrm{Z}}\sigma_n^{\mathrm{Z}}$$

$$H^{ee} = \frac{g^2 N}{2}\sum_{\tilde{n}=0}^{\eta-1}\sum_{\tilde{m}=0}^{\tilde{n}-1} 2^{\tilde{m}+\tilde{n}-1}\tilde{\sigma}_{\tilde{m}}^{\mathrm{Z}}\tilde{\sigma}_{\tilde{n}}^{\mathrm{Z}}$$

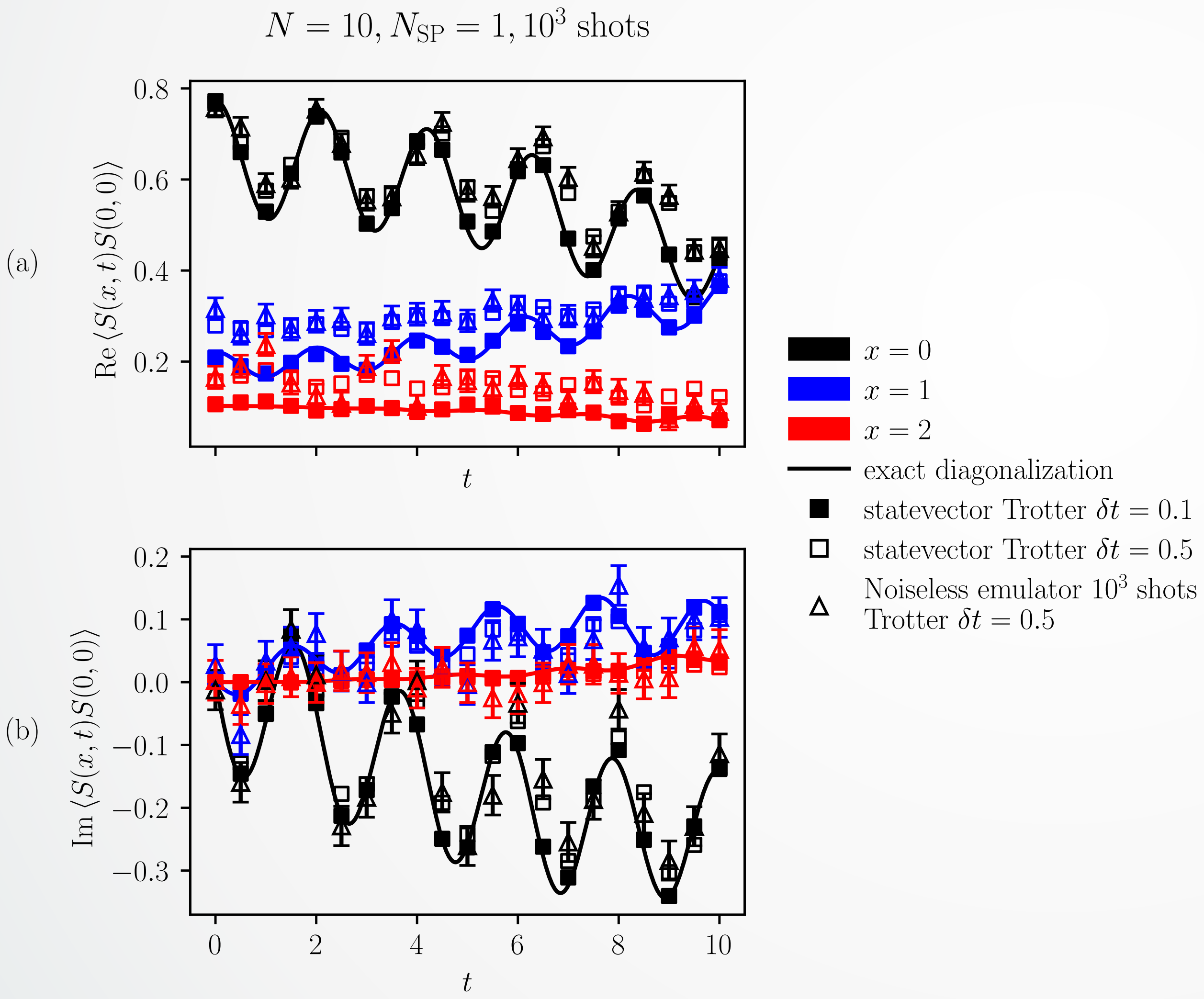
$$H^{ze} = \frac{g^2}{2}\sum_{n=0}^{N-2}\sum_{\tilde{n}=0}^{\eta-1}(N-1-n)\cdot 2^{\tilde{n}-1}\;\sigma_n^{\mathrm{Z}}\tilde{\sigma}_{\tilde{n}}^{\mathrm{Z}}$$

Cross terms from quadratic electric field

$$c_\Lambda = -\Lambda + \frac{1}{2}(2^\eta - 1)$$

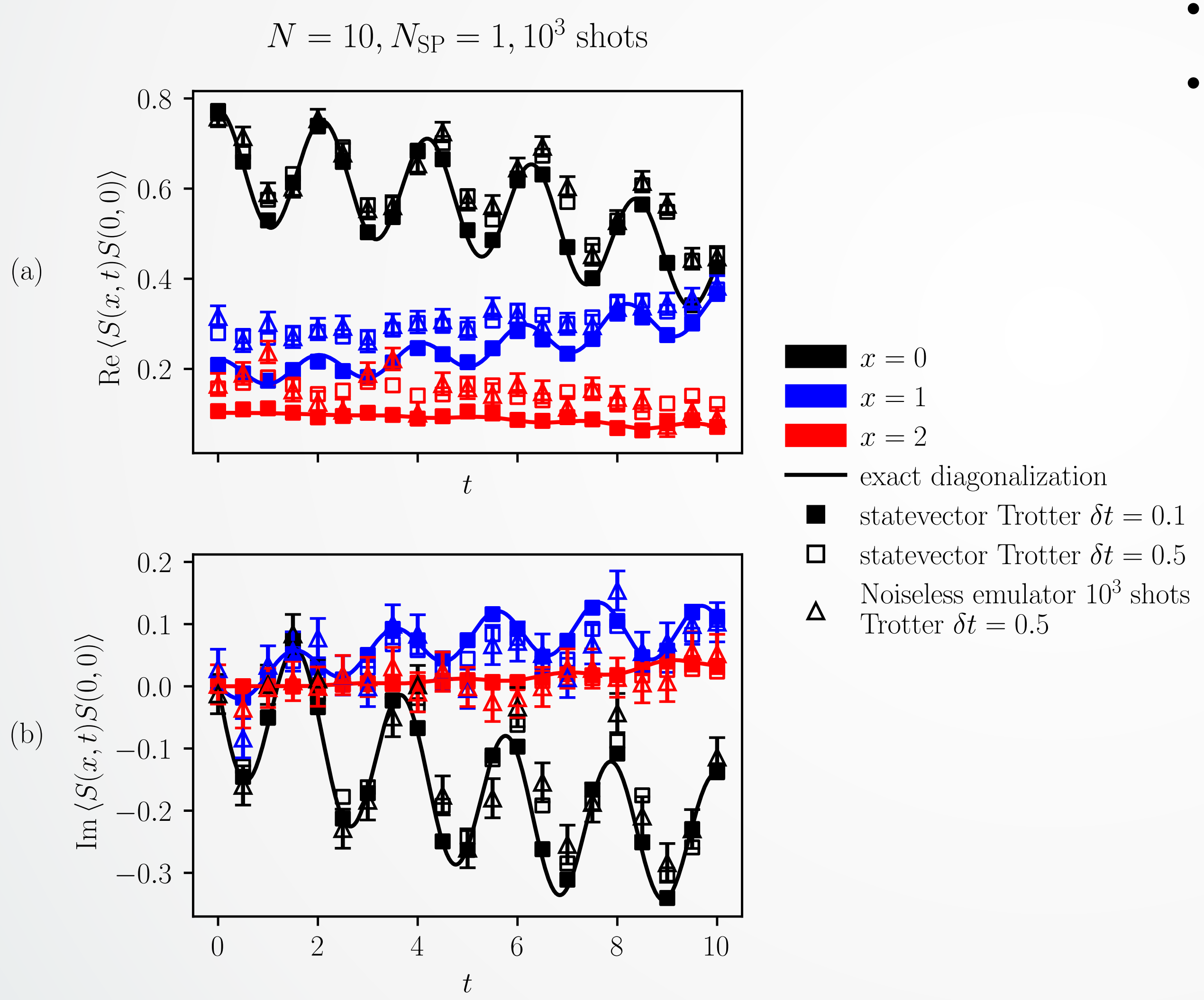
Results

Noiseless simulation



Results

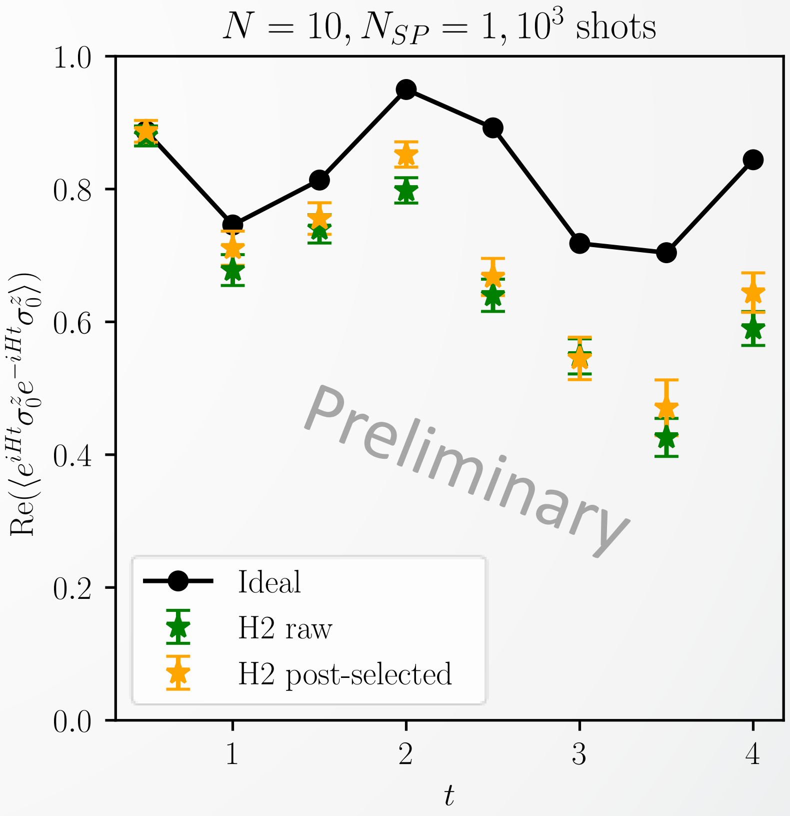
Noiseless simulation



Hardware

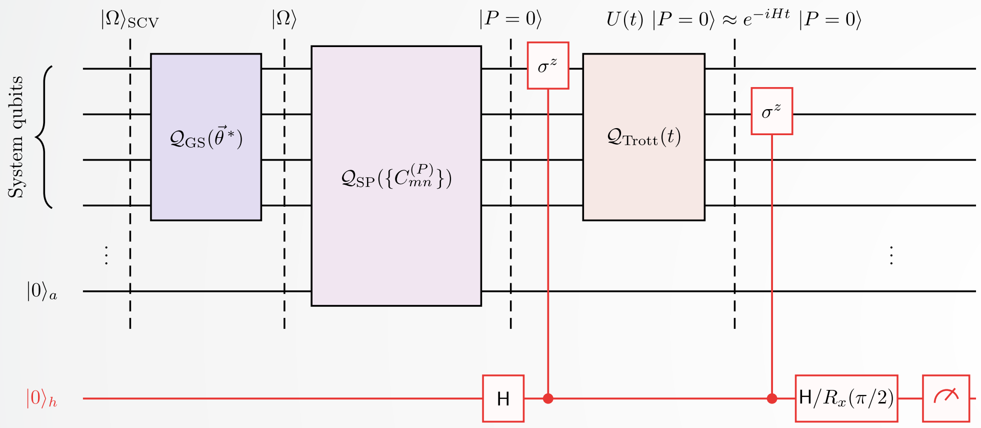
QUANTINUUM

- Quantinuum H2 machine
- Post selection with Gauss law (total charge) & ancilla



Summary and outlook

Summary



- Near-term quantum algorithm for real-time correlators to obtain hadronic tensors
- Hadronic state preparation + Trotter time evolution
- Circuit realized on trapped-ion hardware

Outlook

- Near term:
  - Hardware error mitigation
  - Efficient circuitization
  - Improved ansatz
  - **Uncertainties from finite volume, truncation…**
- Far term:
  - Larger spacetime volume for Fourier transform
  - Weak coupling regime, DIS limit
  - Non-Abelian gauge theories
  - Higher spacetime dimensions



Thanks for listening!

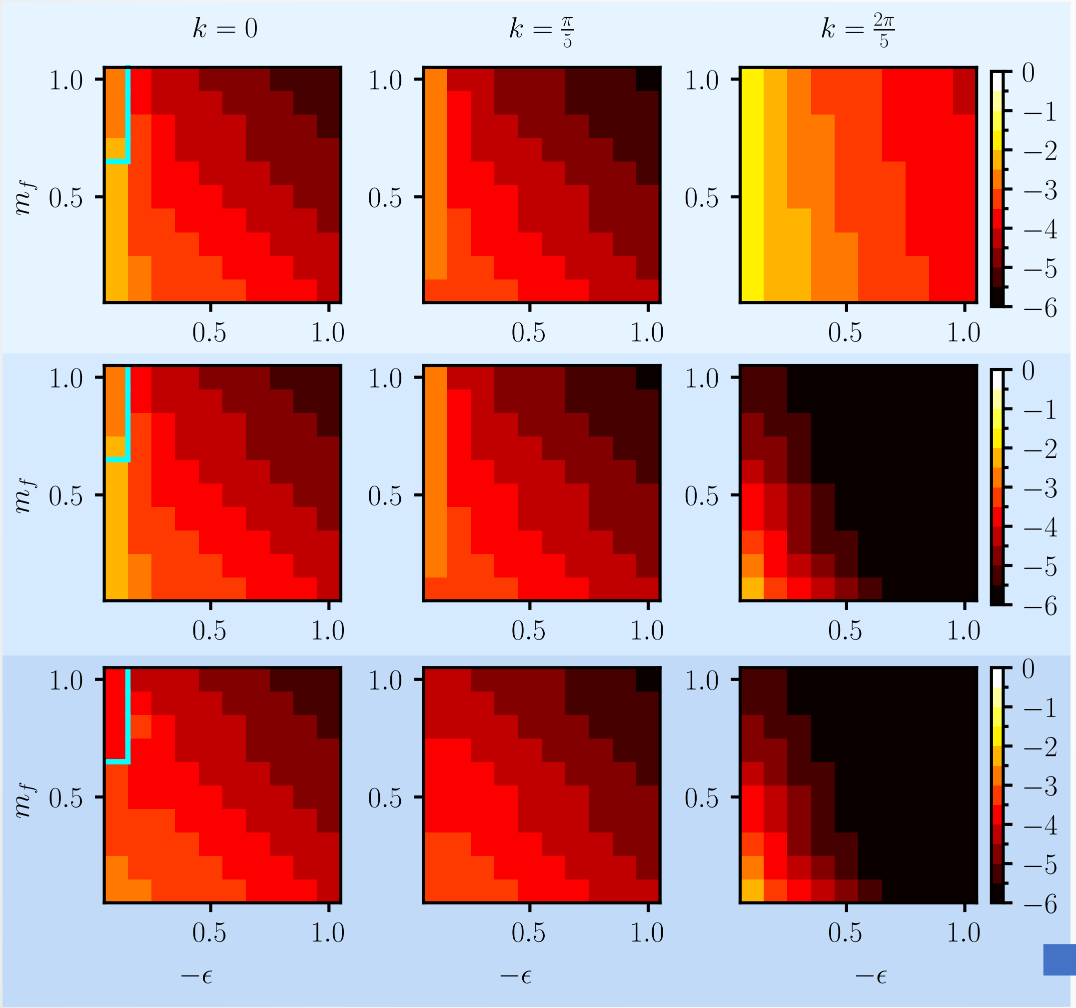
Backups

Numerical verification in Z_2 LGT

$$F = \left|{}_{\mathrm{Op}}\langle k|k\rangle_{\mathrm{Exact}}\right|^2$$

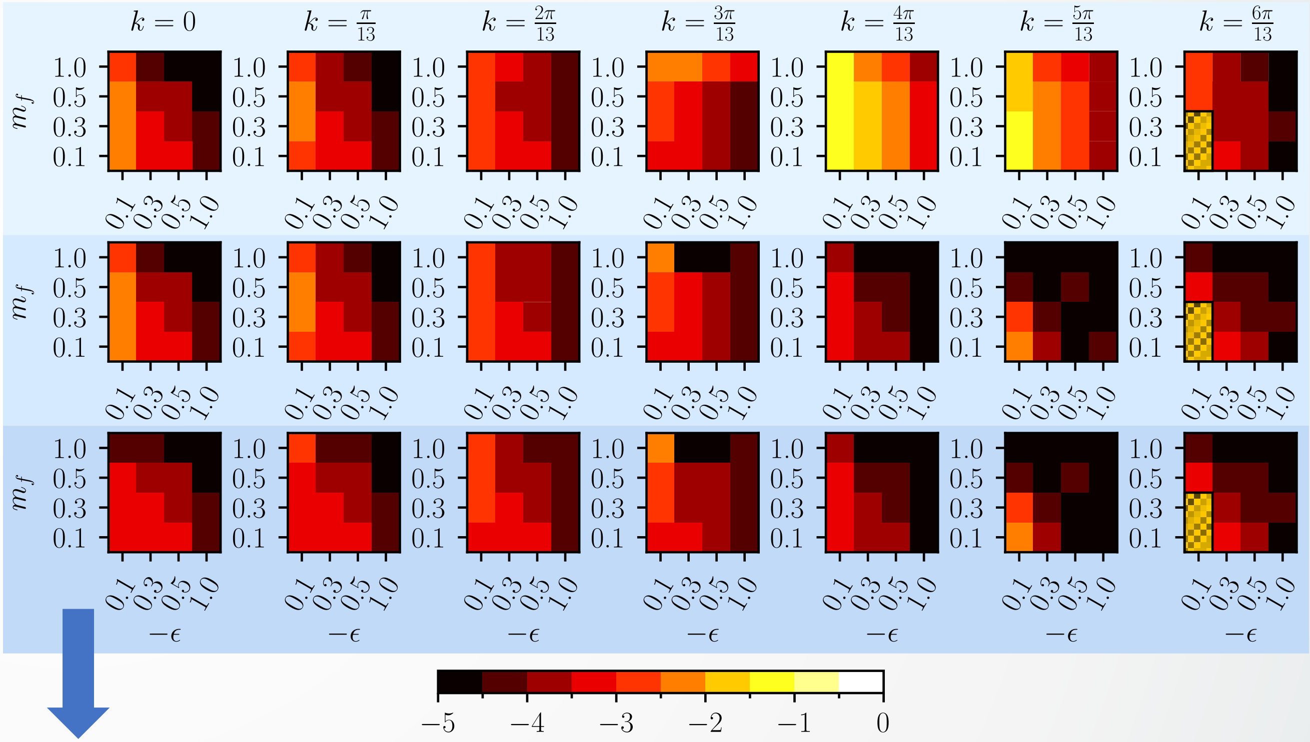
$N = 10$

$\log_{10}(1-F)$



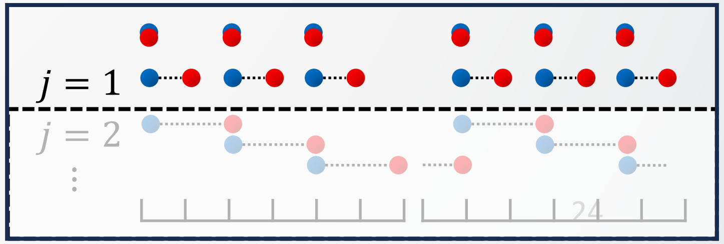
$N = 26$

$\log_{10}(1-F)$



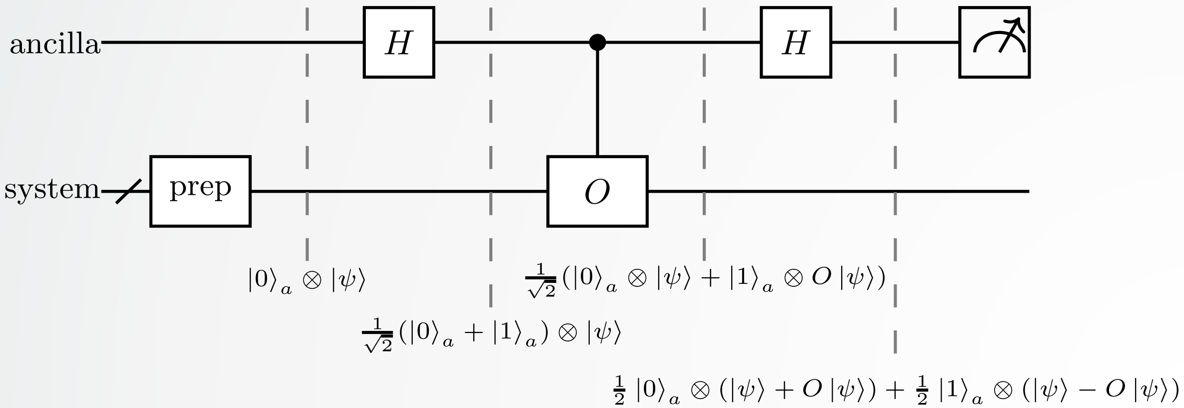
MPS

Exact diagonalization

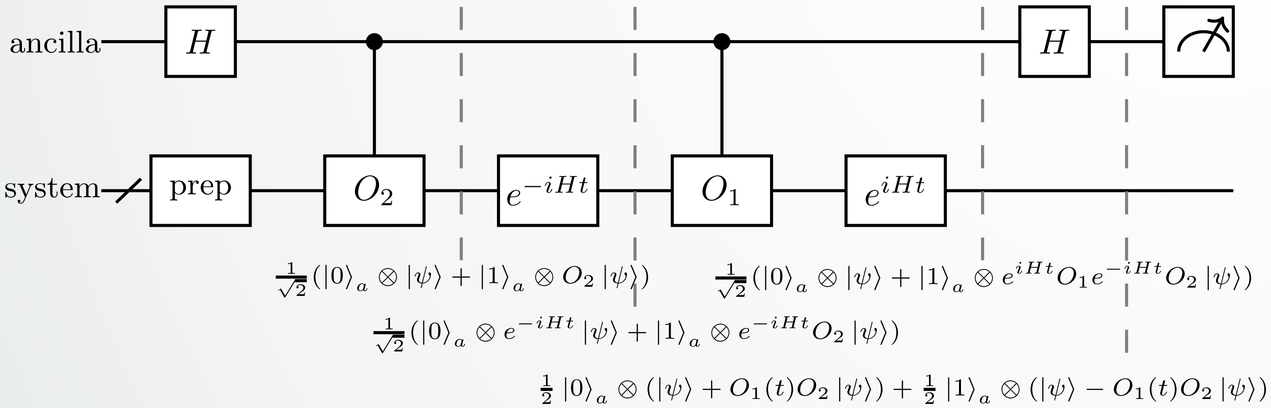


Measuring observables

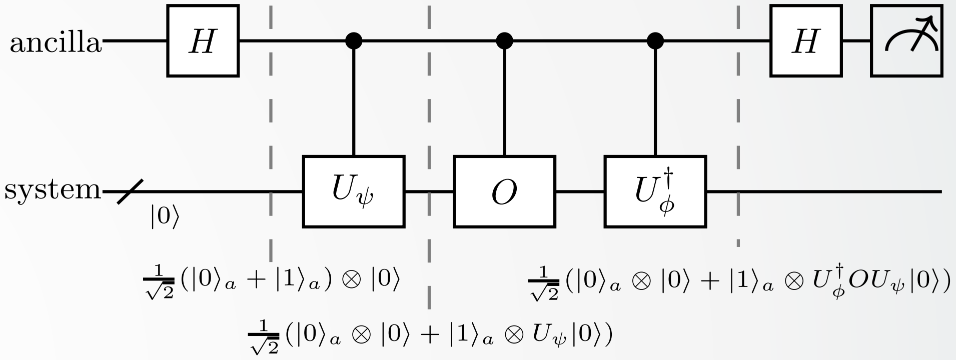
(a) Measuring $\langle\psi|O|\psi\rangle$



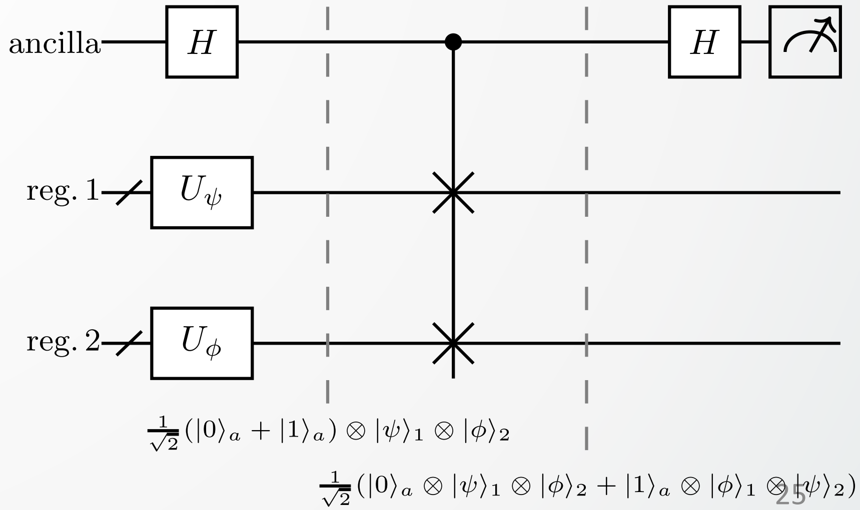
(b) Measuring $\langle\psi|O_1(t)O_2(0)|\psi\rangle$



(a) Measuring $\langle\phi|O|\psi\rangle$



(b) SWAP test for $|\langle\phi|\psi\rangle|^2$



$\mathcal{Q}_{WP}$: wave-packet circuits

Z. Davoudi, A. F. Shaw, J. R. Stryker, Quantum 7, 1213 (2023)

$$b_\Psi^\dagger = \sum_{m,n} C_{m,n}^{(j)} \widetilde{\mathcal{M}}_{m,n} \quad \text{non-unitary}$$

- Wave packet creation: **Ancilla encoding**

$$\boxed{b_\Psi|\Omega\rangle \approx 0} \qquad \boxed{[b_\Psi, b_\Psi^\dagger] \approx \mathbb{I}}$$

$$\Theta_\Psi \equiv b_\Psi^\dagger \otimes |1\rangle\langle 0|_a + b_\Psi \otimes |0\rangle\langle 1|_a$$

$$e^{-i\frac{\pi}{2}\Theta_\Psi}|\Omega\rangle \otimes |0\rangle_a \approx -i|\Psi\rangle \otimes |1\rangle_a$$

- Second wave packet:

$$e^{-i\frac{\pi}{2}\Theta_{\Psi_2}}|\Psi_1\rangle \otimes |0_{a'}\rangle$$
$$\approx -i|\Psi_1, \Psi_2\rangle \otimes |1_{a'}\rangle$$

$$\boxed{(\Psi_1|\Psi_2) = \sum_k \Psi_1^*(k)\Psi_2(k) \approx 0}$$

$$\cancel{+(\Psi_1|\Psi_2)\cos(\pi/\sqrt{2})\,|\Psi_2\rangle \otimes |0_{a'}\rangle}$$
$$\cancel{-i(\Psi_1|\Psi_2)\left[\frac{1}{\sqrt{2}}\sin(\pi/\sqrt{2}) - 1\right]|\Psi_2, \Psi_2\rangle \otimes |1_{a'}\rangle}$$

S. P. Jordan, K. S. M. Lee, and J. Preskill, Science 336, 1130 (2012)

- **Singular Value Decomposition circuit**
  - Find a basis that diagonalizes

$$\Theta \equiv b^\dagger \otimes |1\rangle\langle 0|_a + b \otimes |0\rangle\langle 1|_a$$

$$\downarrow$$

$$\boxed{b^2 = {b^\dagger}^2 = 0,} \qquad b = VSW^\dagger$$

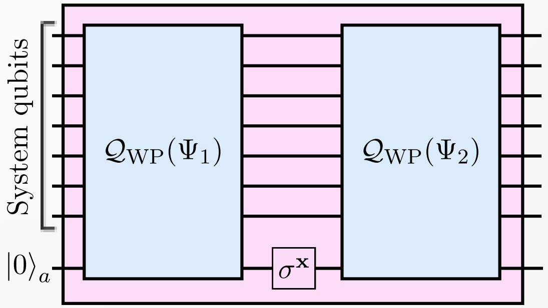
Suppose the SVD for b_Ψ is easy to obtain

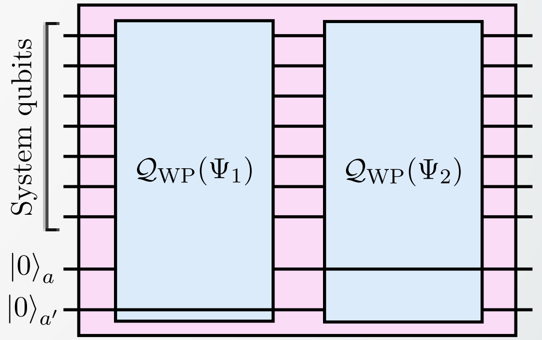
$$\Theta = \mathcal{U}^\dagger D \mathcal{U}$$

$$\begin{cases} \mathcal{U} = (\mathrm{Had}_a)(V^\dagger \otimes |0\rangle\langle 0|_a + W^\dagger \otimes |1\rangle\langle 1|_a) \\ D = S \otimes \sigma_a^z \end{cases}$$

$$\boldsymbol{e^{-i\frac{\pi}{2}\Theta_{m,n}} = \mathcal{U}_{m,n}^\dagger e^{-i\frac{\pi}{2}D_{m,n}} \mathcal{U}_{m,n}}$$

1 or 2 ancilla



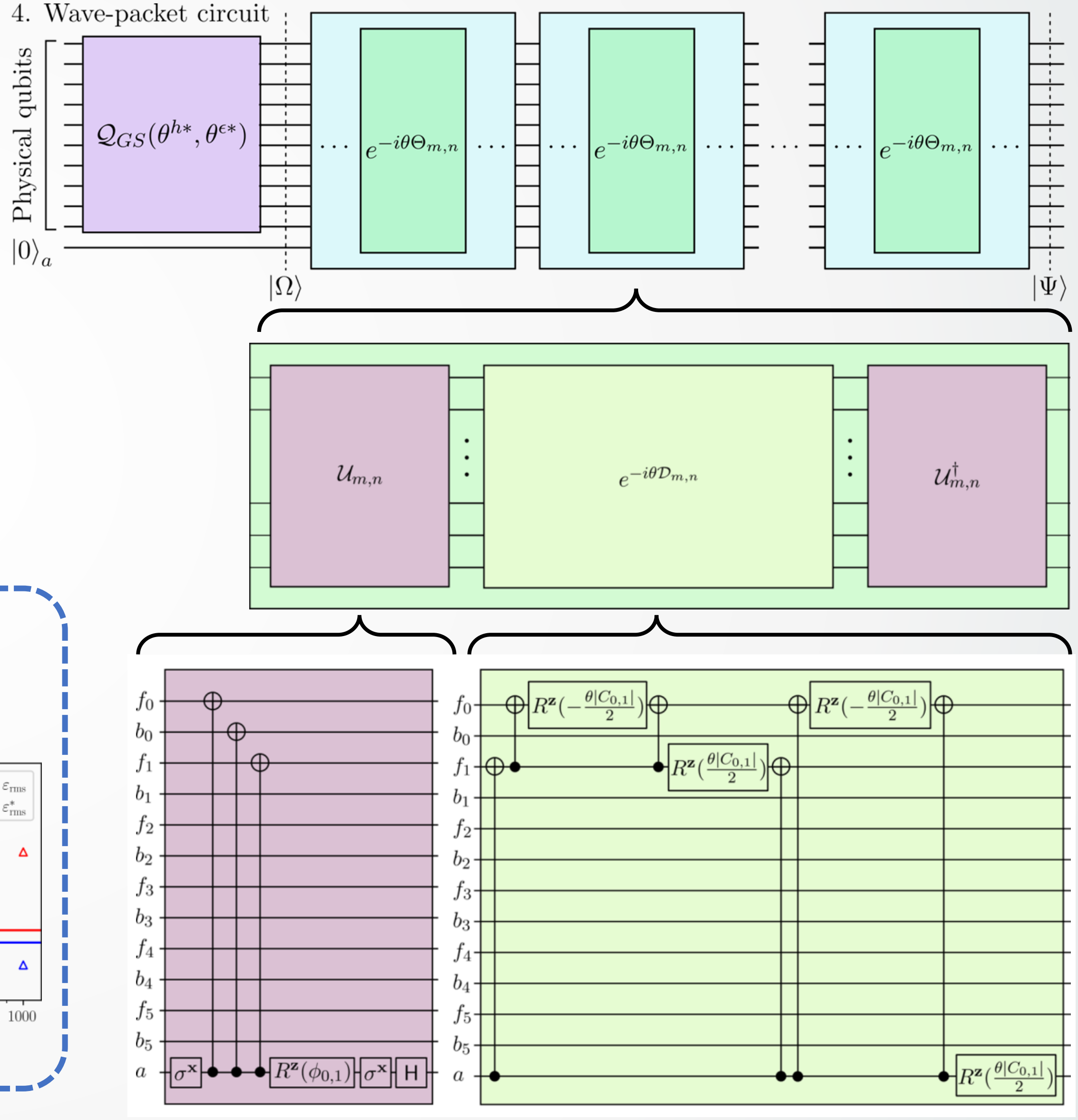


Circuit and results

Circuitization

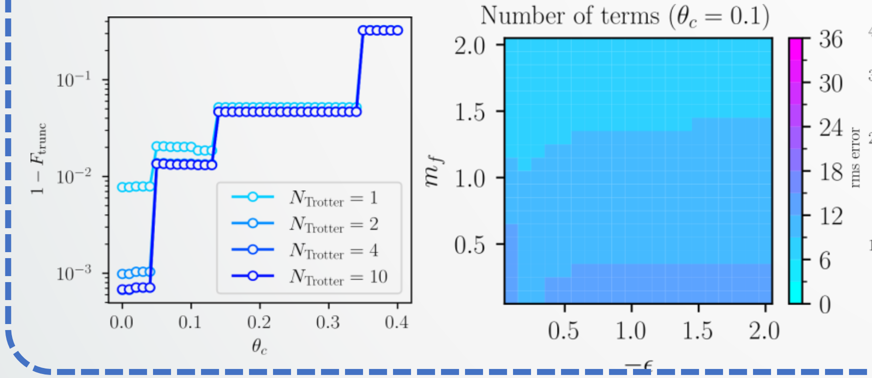
- Non-unitary operator $b_\Psi^\dagger$
  - **Ancilla** encoding — Jordan, Lee, Preskill (2012)
- Non-commuting summands in $b_\Psi^\dagger$
  - Product formula
- Complicated multi-spin operator
  - Choose better basis using Singular Value Decomposition (**SVD**) circuit — Davoudi, Shaw, Stryker (2023)

$$b_\Psi^\dagger = \sum_{m,n} C_{m,n} \widetilde{\mathcal{M}}_{m,n}$$

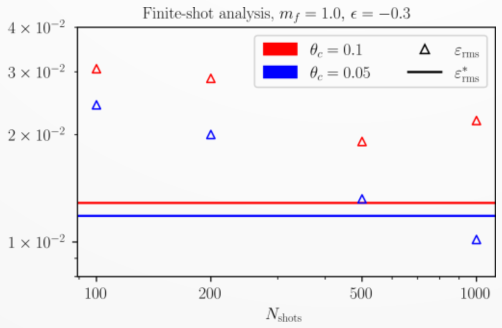


NISQ implementation

Trotter error & angle truncation



Shot analysis



SVD circuits