Recent progress on K→ππ calculation on multiple ensembles

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$K \to \pi\pi$ & Direct CPV

CP odd CP even

$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$

ε' direct CPV

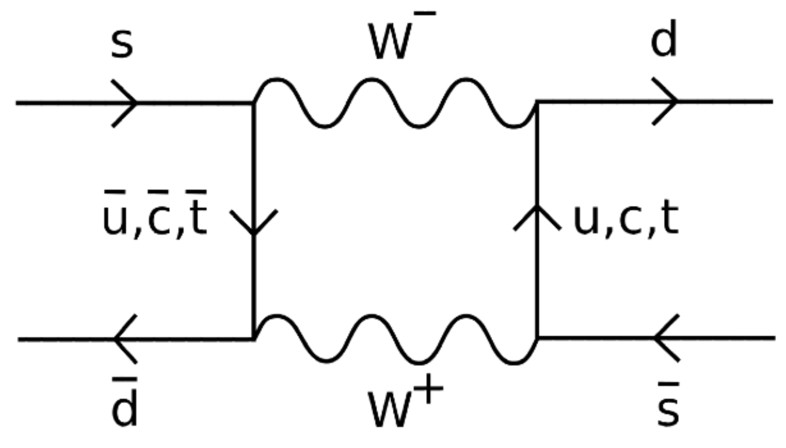
ε indirect CPV

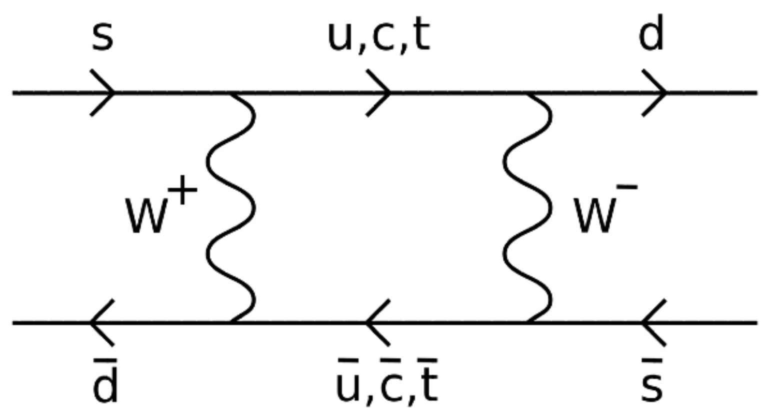
$|\pi\pi\rangle$

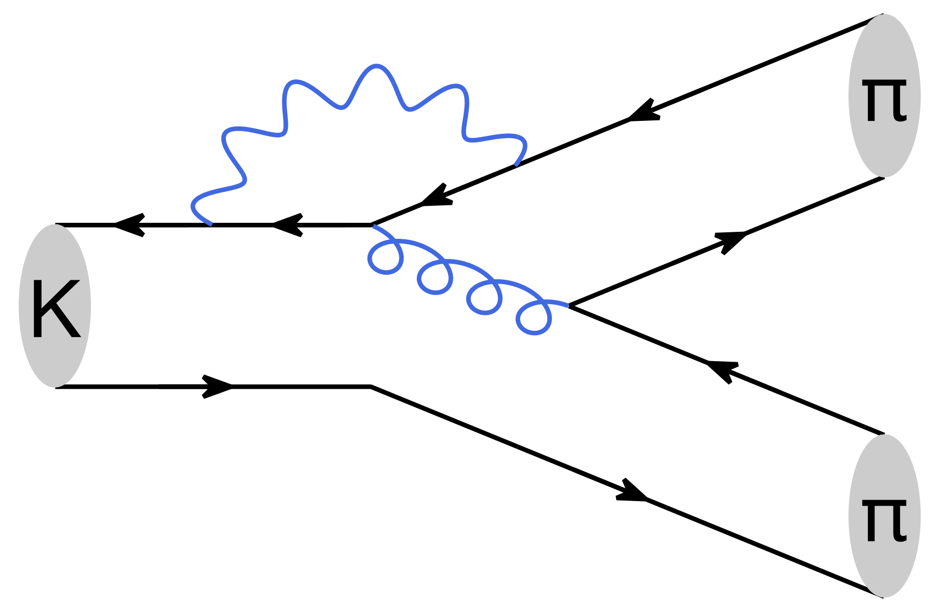
CP even

$$\frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \Big/ \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)} = 1 - 6 \times \mathrm{Re}(\epsilon'/\epsilon)$$

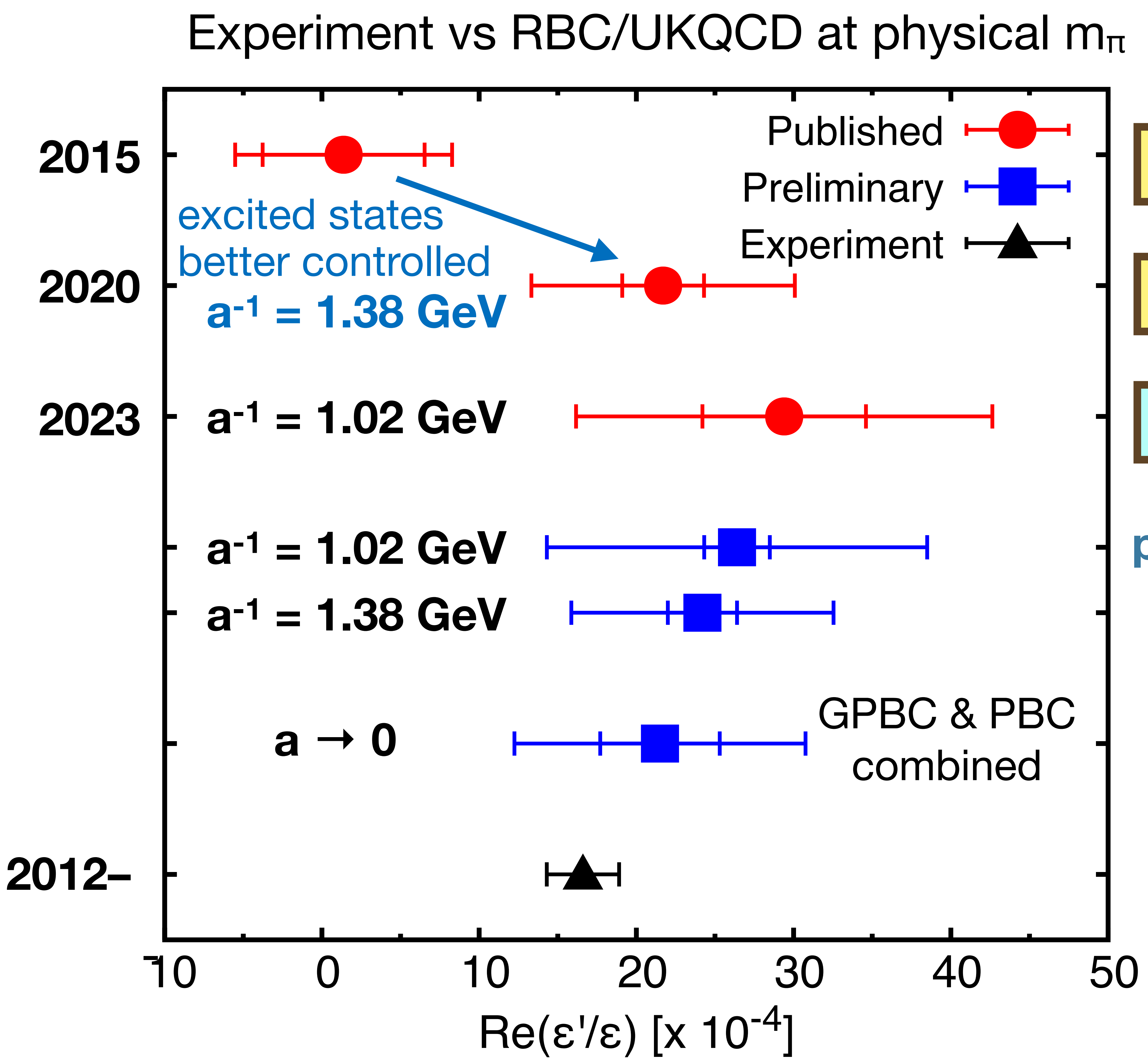
- $|\varepsilon| = 2.228(11) \times 10^{-3}$
- ε' only found in decays
  - $\mathrm{Re}(\varepsilon'/\varepsilon)_{exp} = 1.66(23) \times 10^{-3}$ (KTeV & NA48)
  - Consistent with SM?
  - potential to constrain UT parameter $\bar{\eta}$







Status of ε′ calculation



PRL 115,212001

PRD 102,054509

- G-parity Boundary Conditions (GPBC)
  - $a^{-1} \approx 1.38$ GeV
  - physical kinematics w ground ππ state

PRD 108,094517

preliminary results with multiple lattice spacings

- Periodic Boundary Conditions (PBC)
  - physical kinematics w excited ππ state
  - important for introducing EM/IB effects
  - many ensembles already generated

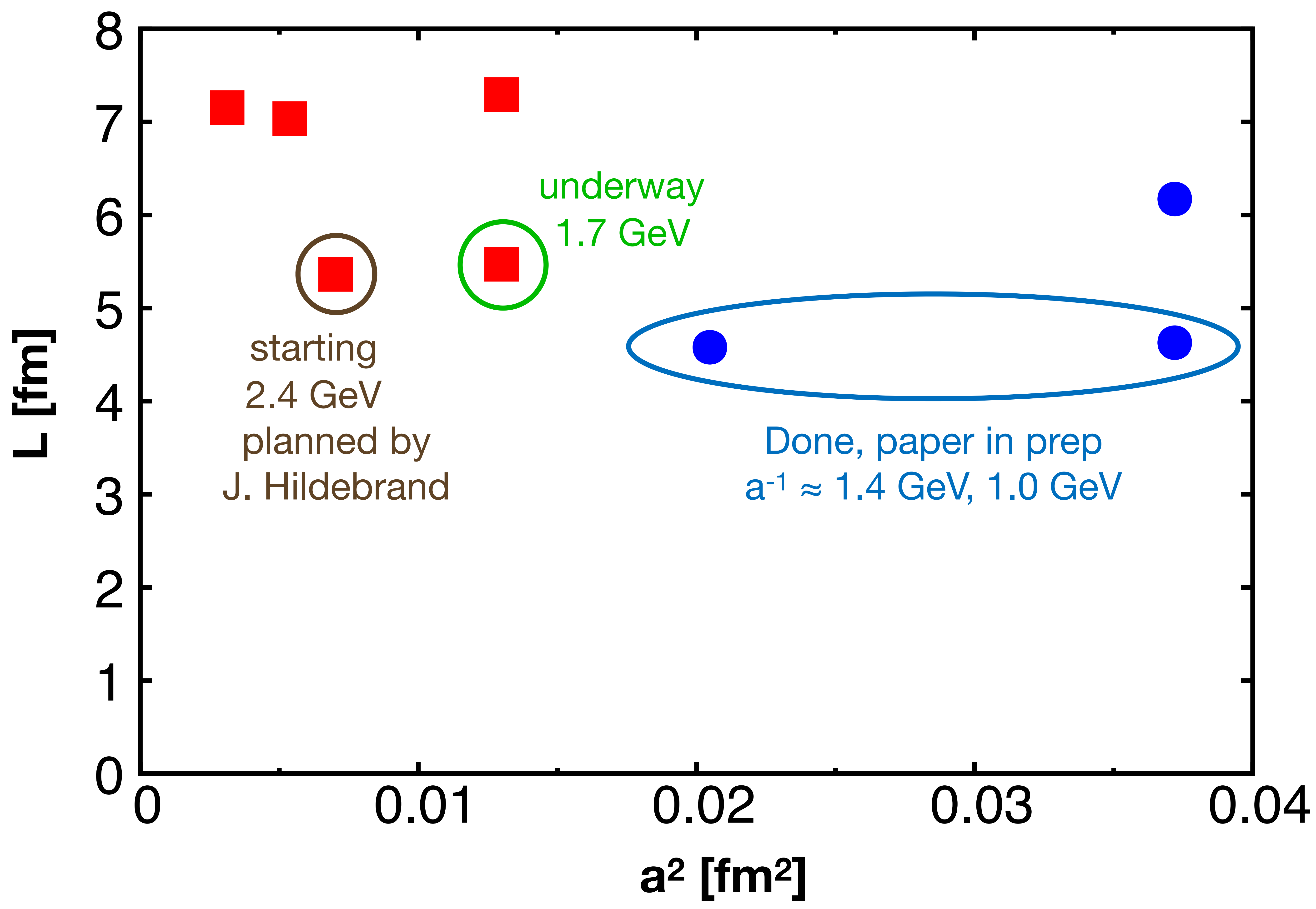
Lattice ensembles

- 2+1 Möbius DWF
- 2 families of gauge action

  ■ **Iwasaki**

  ● **Iwasaki + DSDR**

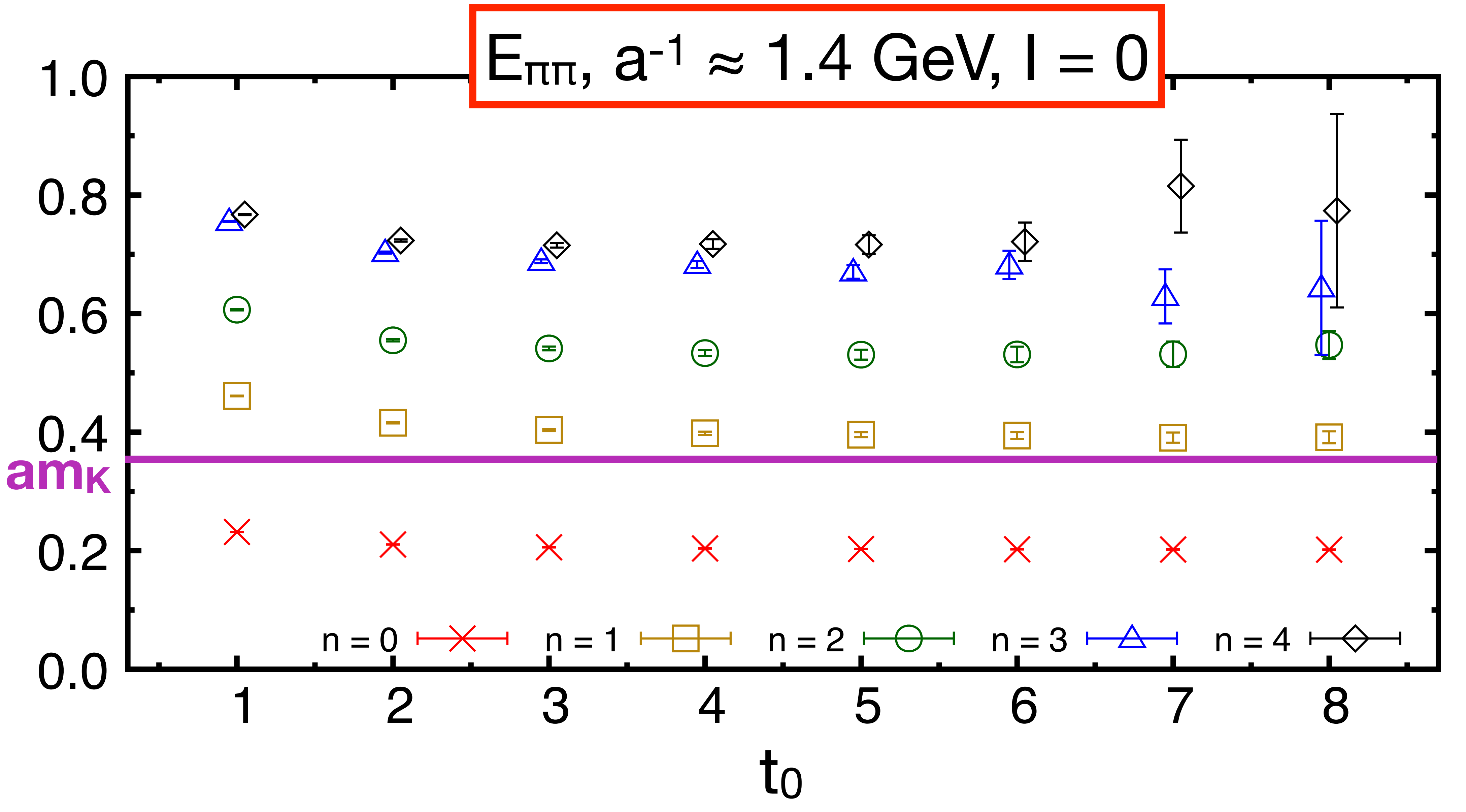
RBC/UKQCD 2+1 lattice ensembles at physical m_π

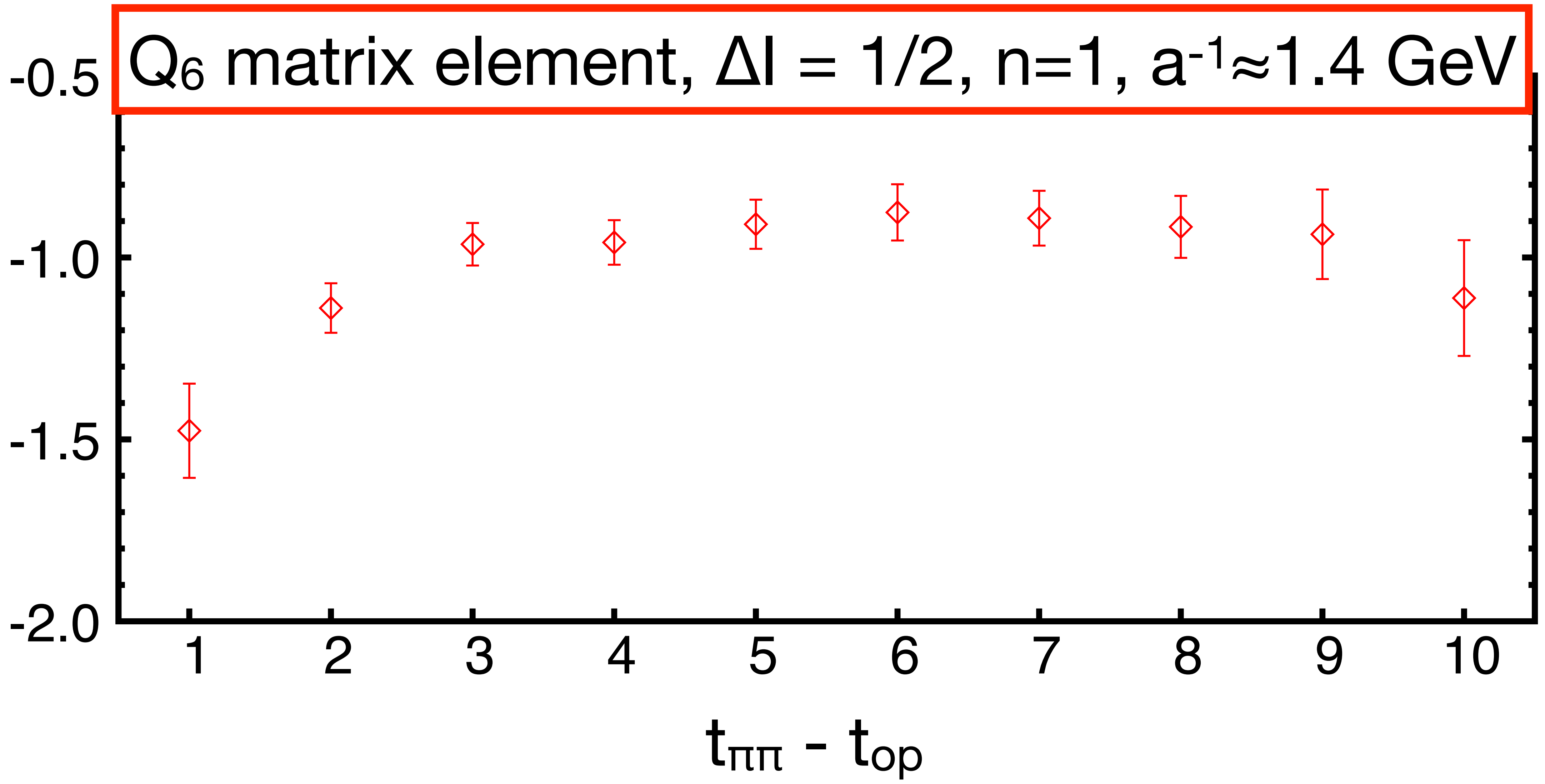


Variational method

Lüscher-Wolff, NPB339,222(1990)

- Solving GEVP (Generalized Eigenvalue Problem)
  - $C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0)$
  - $O'_n = \sum_a v^*_{n,a} O_a$ couples mostly with n-th state
  - $\lambda_n(t,t_0) = e^{-E_n(t-t_0)}$
- ππ operators used in this work:
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$
  - $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$
  - $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$
  - $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$
  - $\sigma \sim \bar{u}u + \bar{d}d$: controls $f_0(500)$ resonance
  - KK ~ $\overline{K}K + K^+K^-$: for checking further excited-state contamination





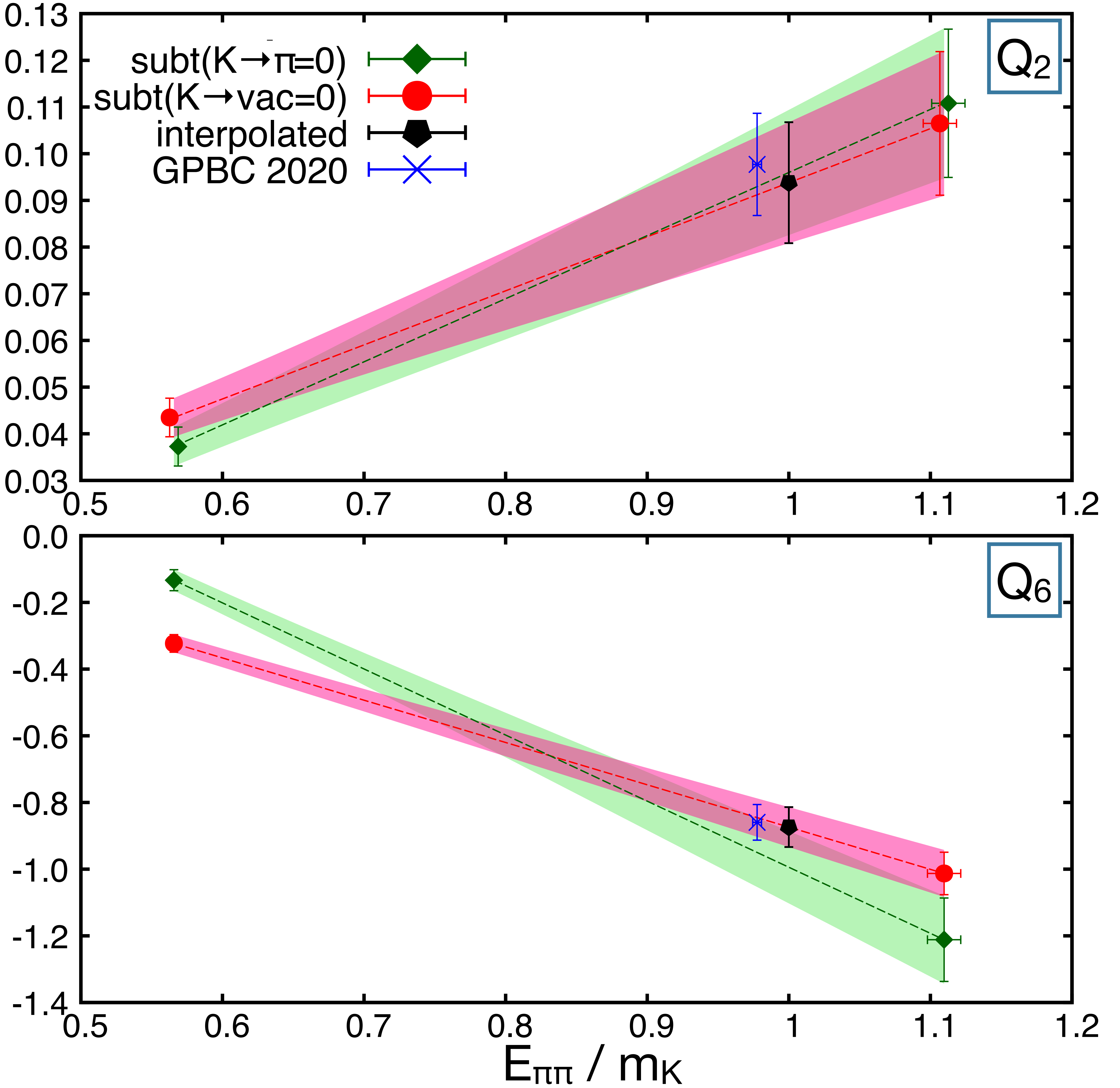
Interpolation with multiple subtraction schemes

Q_i : power divergent ~ $\dfrac{m_d - m_s}{a^2}\bar{s}\gamma_5 d$

$\rightarrow Q_i^{\text{subt}} = Q_i - \alpha_i \underline{\bar{s}\gamma_5 d}$

AWI kills its ME on shell but …

- important for statistical precision
- makes interpolation more accurate



- 2 subtraction schemes

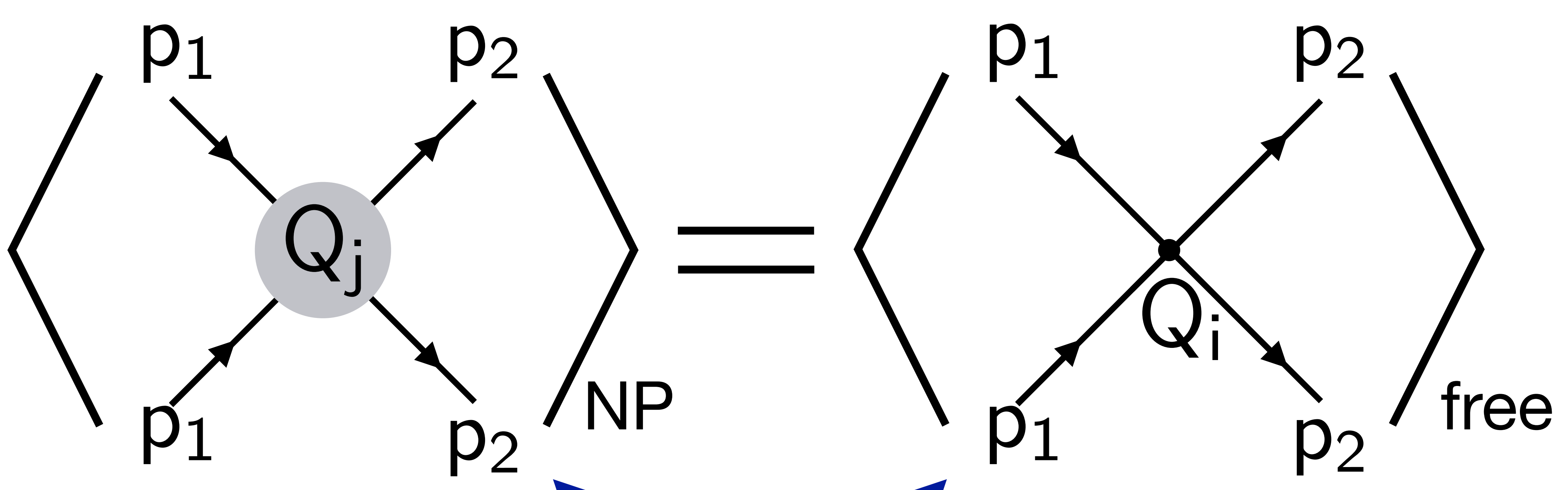
  $\langle 0|Q_i^{\text{subt}}|K\rangle = 0$ - primary scheme

  $\langle \pi|Q_i^{\text{subt}}|K\rangle = 0$
  - just for systematic error estimation
  - leaves power divergence on CP-odd part a little at finite L_s

RI/SMOM schemes

Aoki et al, PRD78,054510 (2007)

$$\frac{Z_{ij}^{\mathrm{RI/SMOM}}(\mu)}{Z_q(\mu)^2}$$



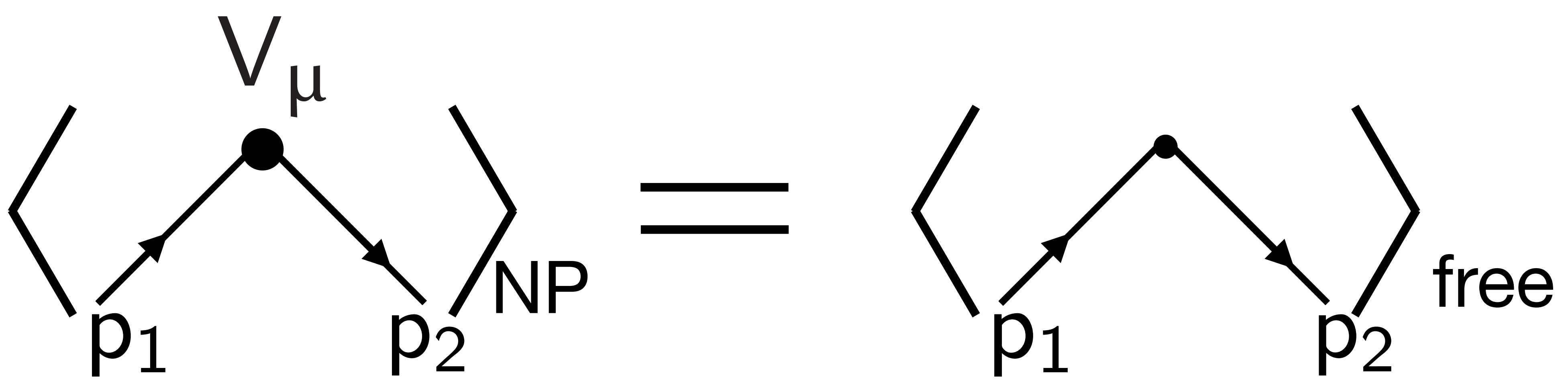
Amputated vertex functions

$$\mu^2 = p_1^2 = p_2^2 = (p_1 - p_2)^2$$

operator carries momentum with "symmetric" magnitude

$Z_q(\mu)$

$$\frac{Z_V}{Z_q(\mu)}$$

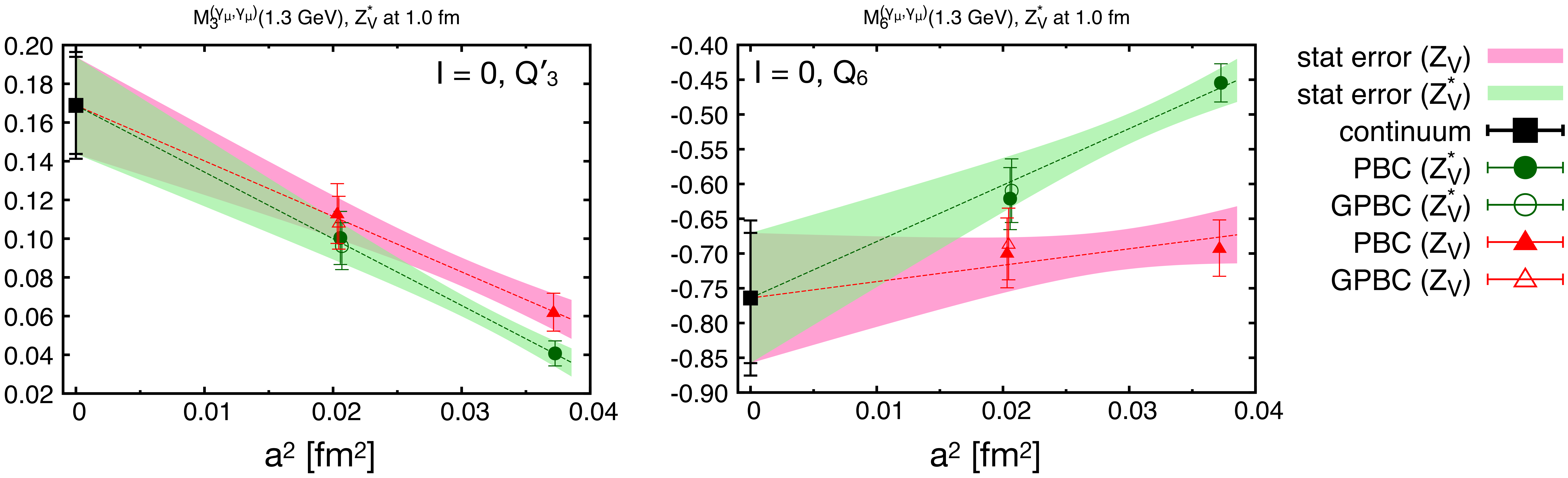


$$Z_V = \frac{\langle \pi | V_i^{\mathrm{conserved}} | \pi \rangle}{\langle \pi | V_i | \pi \rangle}$$

$$Z_V^* = \frac{\langle V_i^{\mathrm{convserved}}(1.0\ \mathrm{fm}) V_i(0)^\dagger \rangle}{\langle V_i(1.0\ \mathrm{fm}) V_i(0)^\dagger \rangle}$$

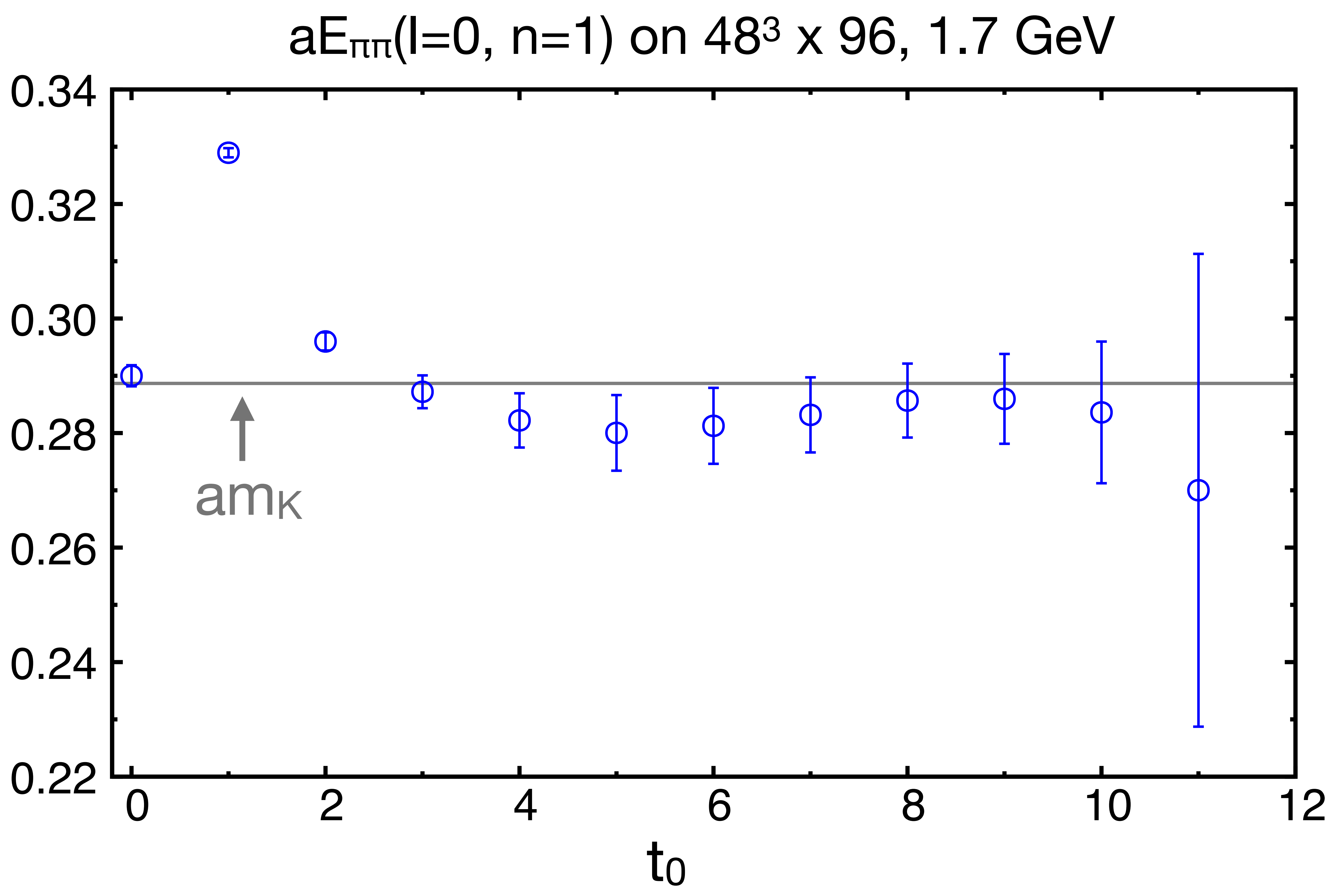
Matrix elements renormalized by Z_V and Z_V^* should have the same $a \to 0$ limit

a → 0 of matrix elements



- Globally constrained continuum extrapolation with multiple determinations of $Z_q(\mu)$ from Z_V & Z_V^*
- GPBC 2020 results included
- Lattices are coarse → systematic error from continuum limit ~20%

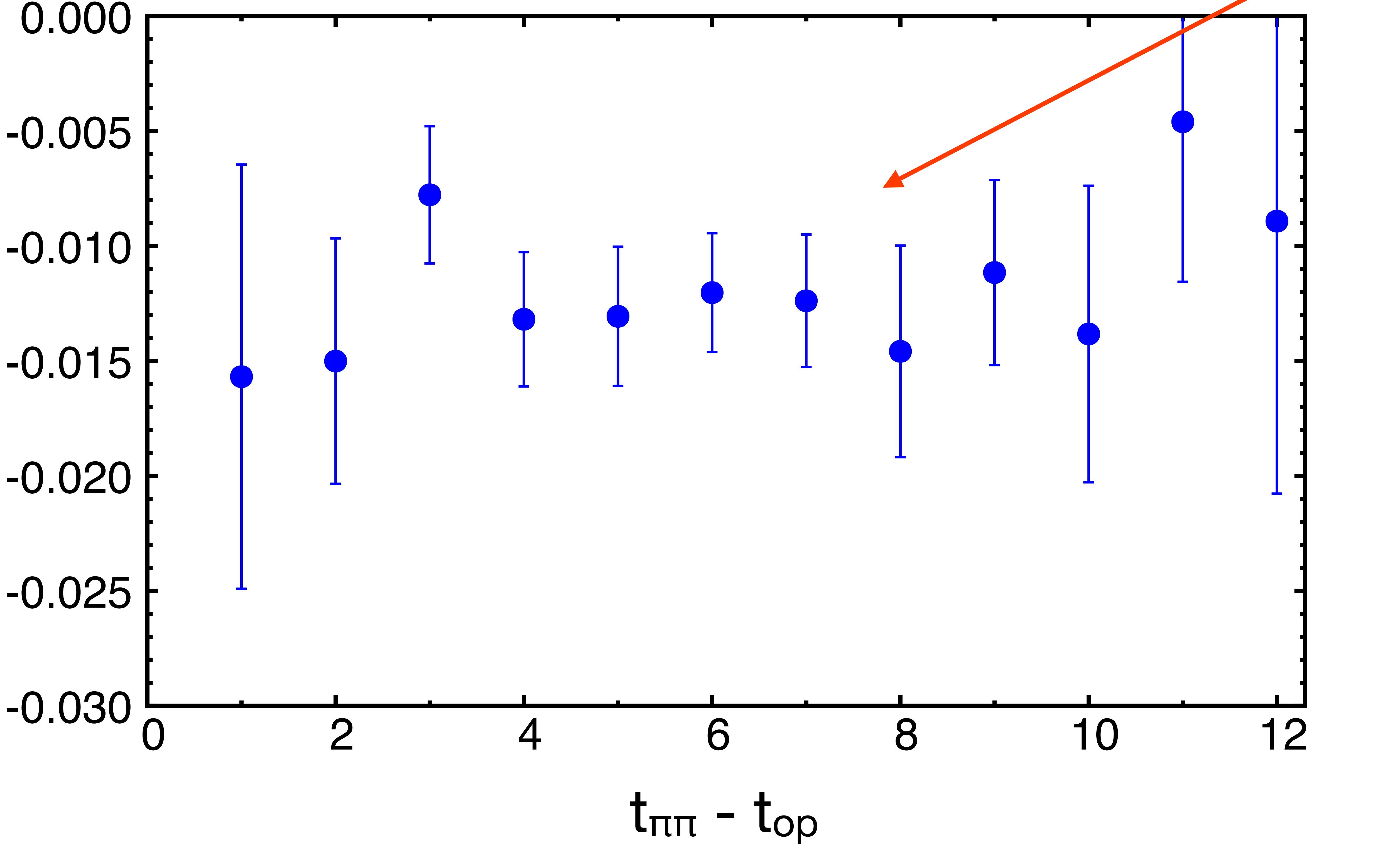
Prospect for finer lattice calculations



- New ensembles realize closer-to-physical kinematics with the 1st excited ππ state in the rest frame
- More direct calculation of on-shell matrix elements possible

Statistics concerning

$\Delta I = 1/2$, $n = 1$ Q_6 matrix element
on $48^3 \times 96$, $a^{-1} \approx 1.7$ GeV

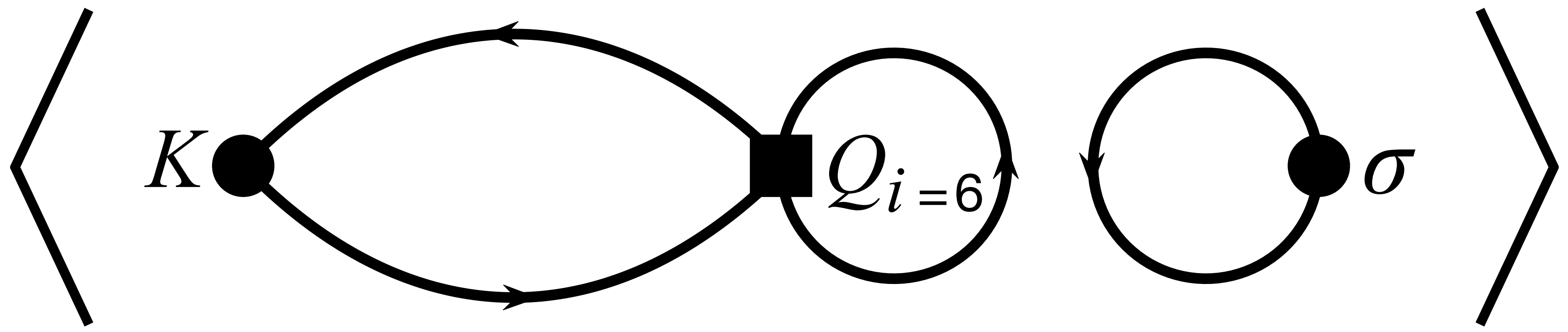


- 116 configs
  (cf. ~470 configs on coarser lattices)
- precision per configuration slightly worse
- only ~180 configurations available with 2,000 low modes

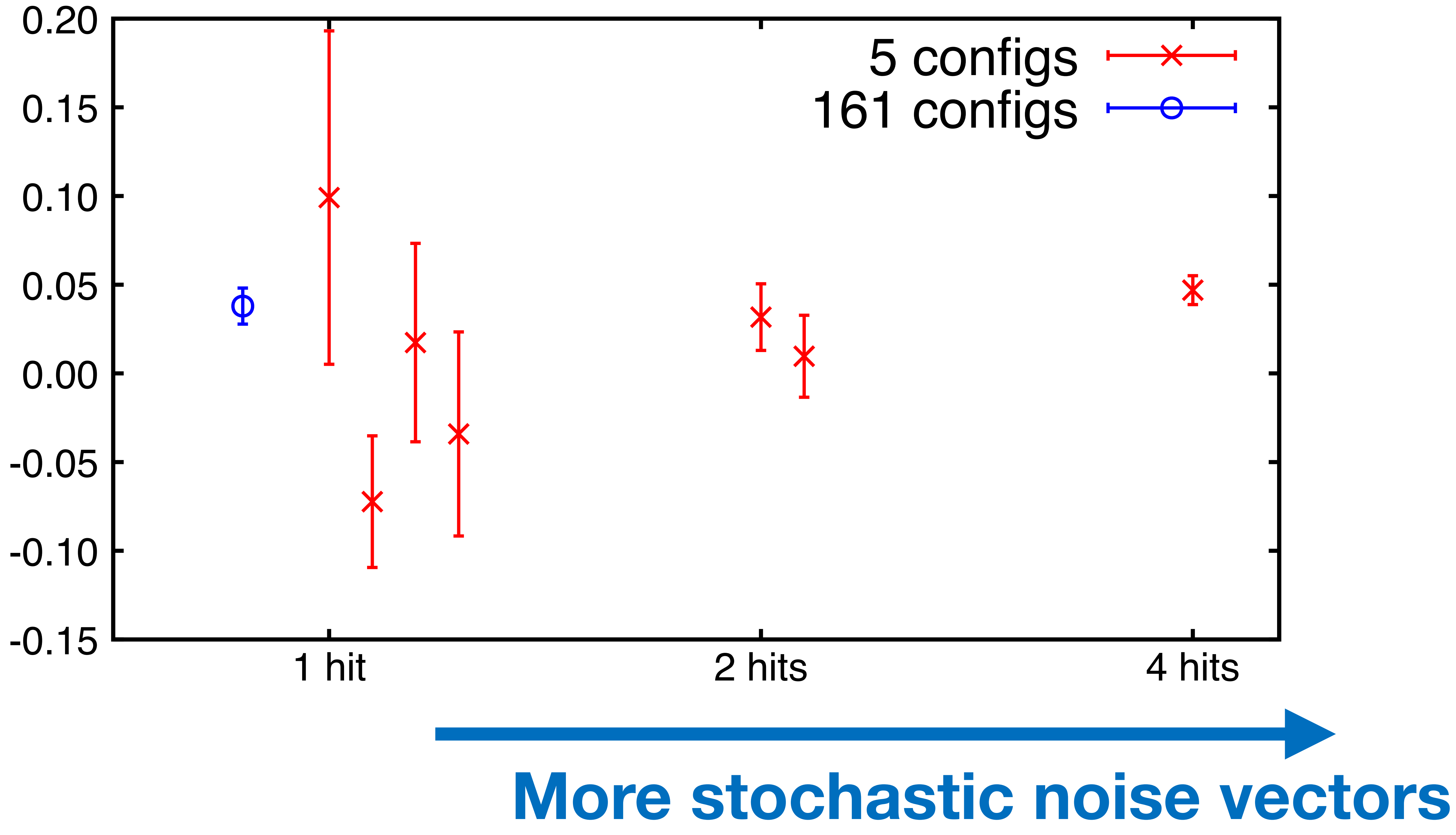
Increasing stochastic noise vectors

- All-to-all quark propagators
  - 2,000 low modes for ud
  - spin-color-time dilution for high modes (4x3x96) x N_{hit} CG inversions
- Significant reduction of statistical error with more high modes observed

$\langle O_\sigma(22) Q_6^{subt}(16) O_K(0)^\dagger \rangle_{disconn.}$



:dominant error source for $\mathrm{Im}A_0$



Significant error sources

* percentages are preliminary

Statistical error (12-15% on ReA_0, ImA_0)	• will be significantly reduced by more stochastic high modes
Finite lattice spacing/ continuum limit (~20% on ReA_0, ImA_0)	• finer-lattice calculations being pursued - 1.7 GeV, 2.4 GeV, …
Step scaling / perturbative $\mathrm{SMOM} \to \overline{\mathrm{MS}}$ matching (~14% on ReA_0)	• step scaling being implemented to higher scales (μ_h: 4 GeV → 8 GeV) and its continuum limit planned [J. Hildebrand's talk] • NNLO matching could also improve
Charm-loop effect (~12% on ReA_0, ImA_0)	• sol1: directly introduce charm quark on $a^{-1} \approx 2.35$ GeV (and finer) • sol2: nonperturbatively match 3-flavor MEs to 4-flavor ones • sol3: NNLO for 3-flavor Wilson coefficients
EM/IB effects (~25% on ε'/ε)	• framework has been established ‣ Christ et al, PRD106, 014508 (2022) / Christ, Lundstrum 2606.26336 • exploratory lattice calculation planned

Summary

- Finishing up Iwasaki+DSDR data analysis
  - Consistency seen on 1.38 GeV ensemble b/w PBC & GPBC
  - Careful study on various systematic errors
  - Multiple lattice ensembles and 1st attempt at $a \to 0$
- Towards calculations on Iwasaki ensembles
  - Promising pathway to reducing the systematic error from $a \to 0$
  - Statistical errors to be reduced by more high modes
  - Work also continues to reduce other systematic errors

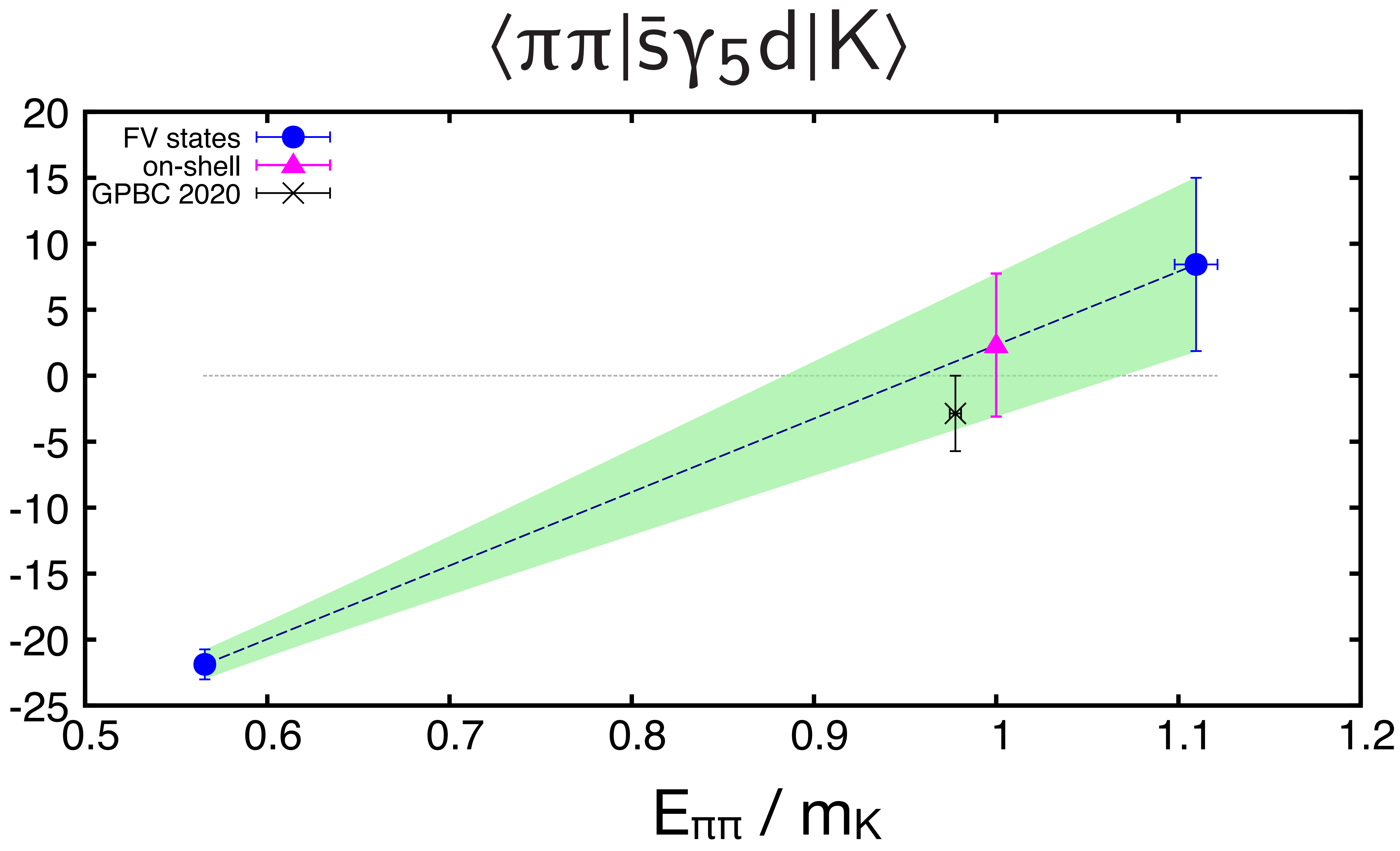
Interpolation of $\bar{s}\gamma_5 d$ matrix element

- Axial Ward identity

$$(m_s + m_d)\bar{s}\gamma_5 d = \partial_\mu \bar{s}\gamma_\mu\gamma_5 d$$

$$\langle\pi\pi|\bar{s}\gamma_5 d|K\rangle \sim (E_K - E_{\pi\pi})\langle\pi\pi|\bar{s}\gamma_4\gamma_5 d|K\rangle$$

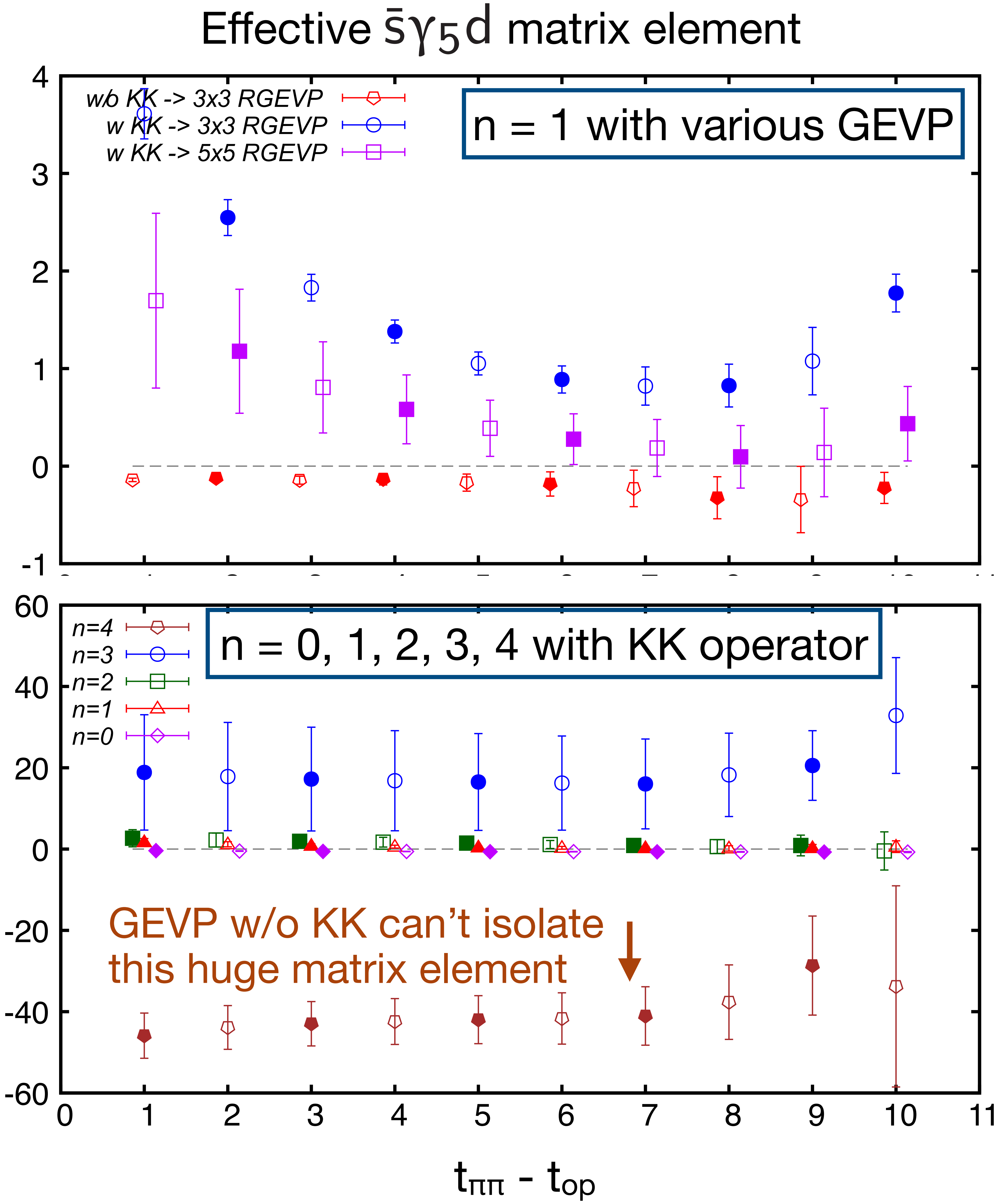
$$\rightarrow 0 \ \text{(on-shell)}$$

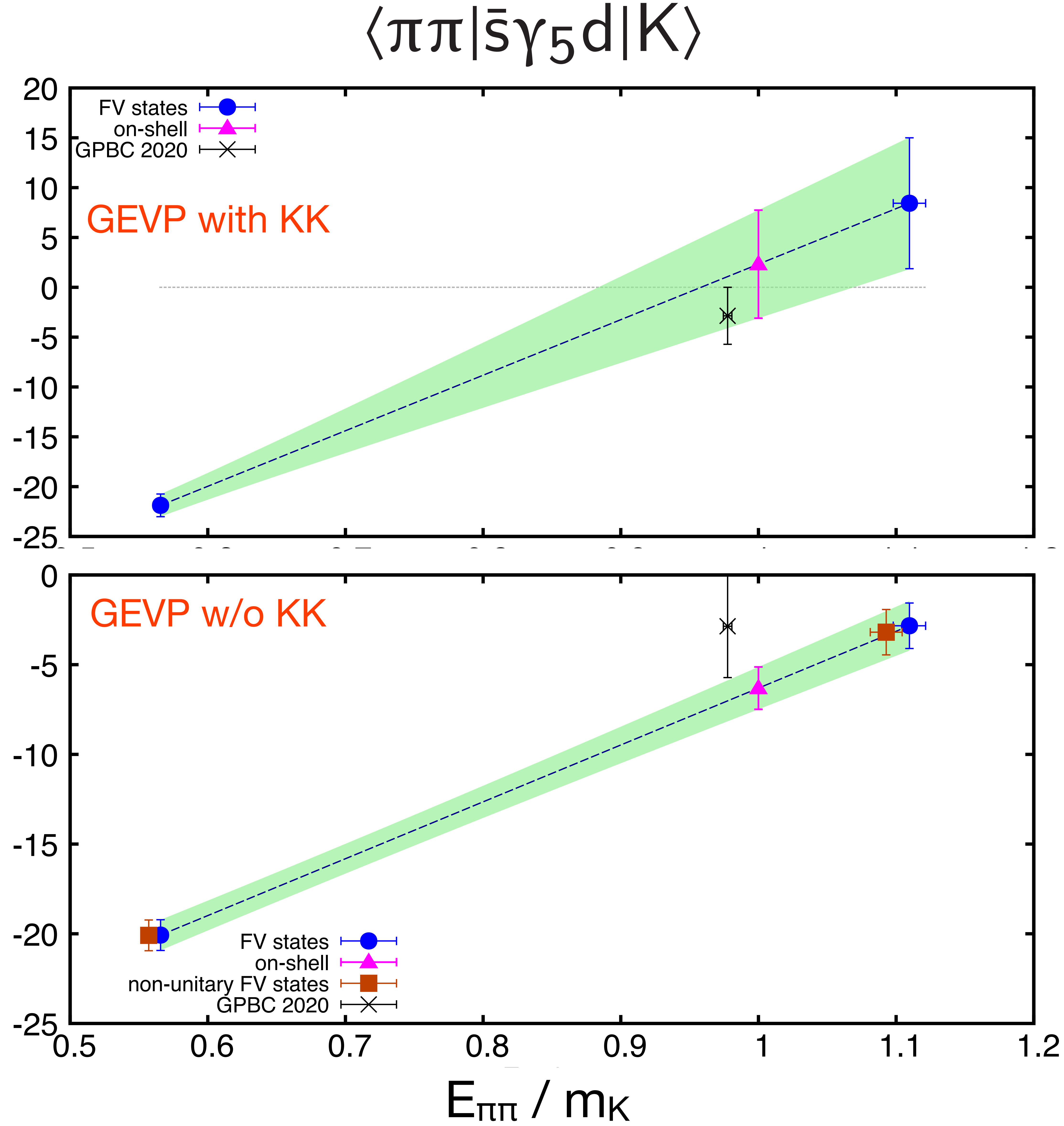


Linear interpolation of $\bar{s}\gamma_5 d$ matrix element certainly consistent with zero

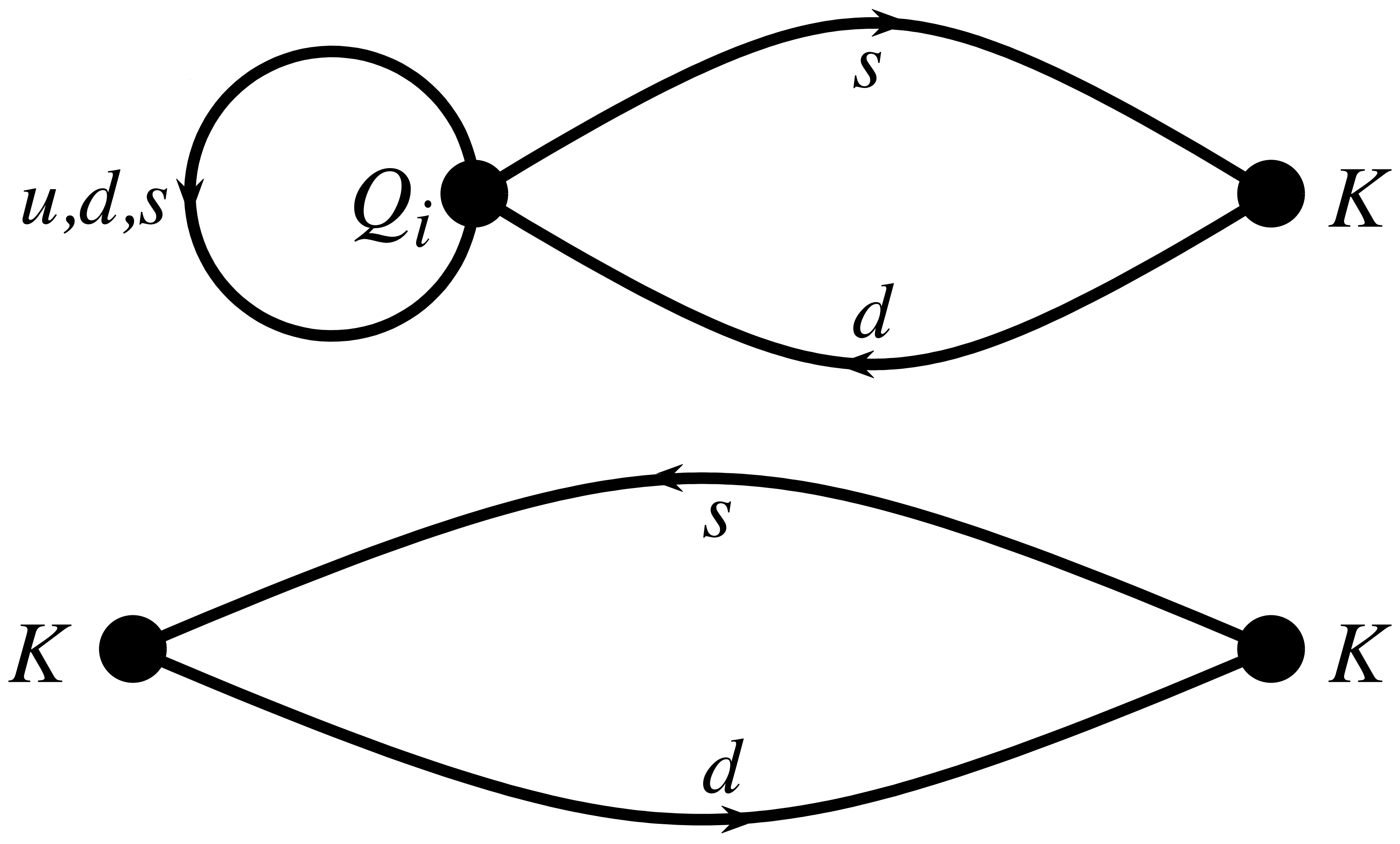
→ Linear interpolation may give accurate on-shell matrix elements compared to statistical uncertainly

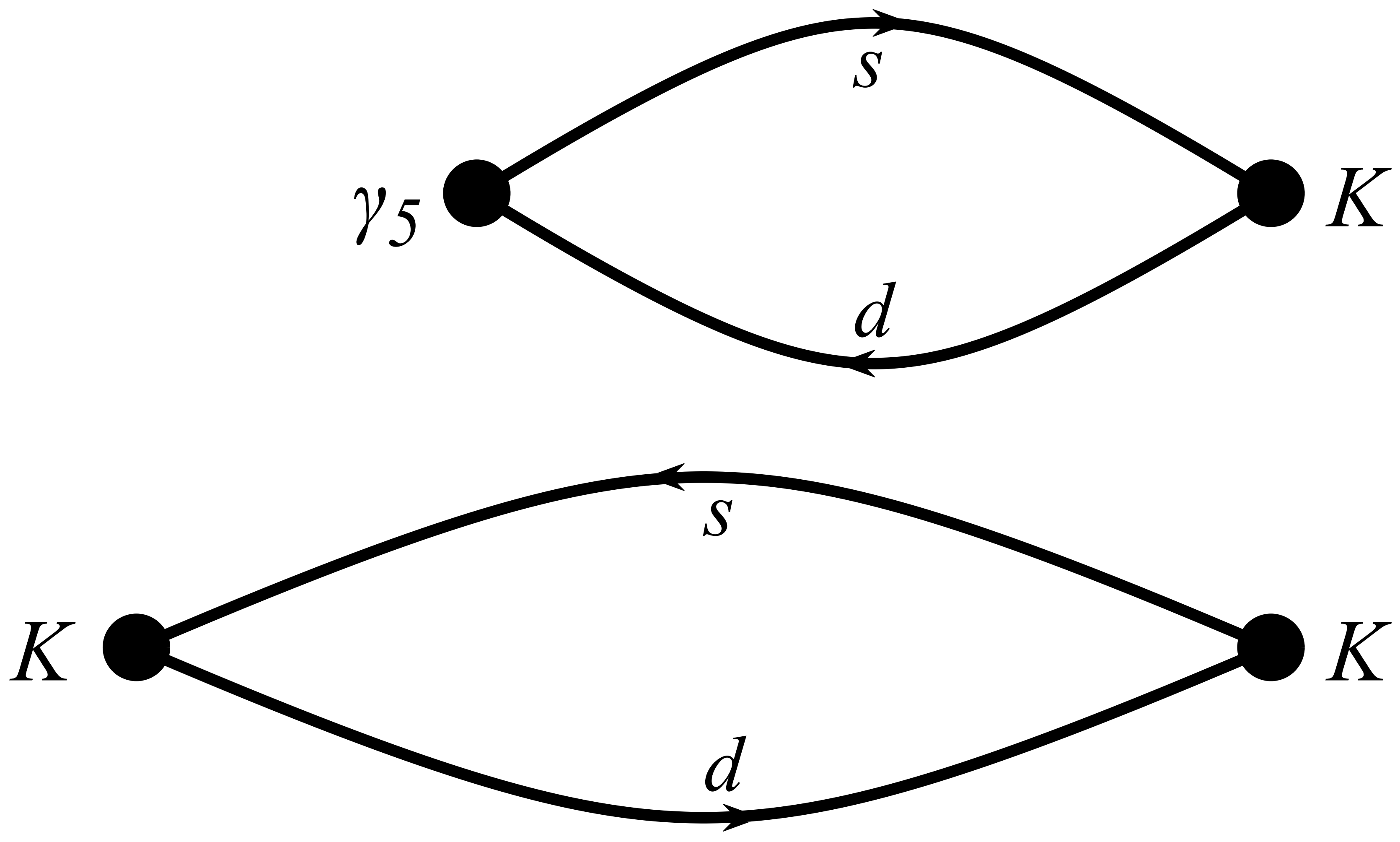
$\bar{s}\gamma_5 d$ matrix element w & w/o KK



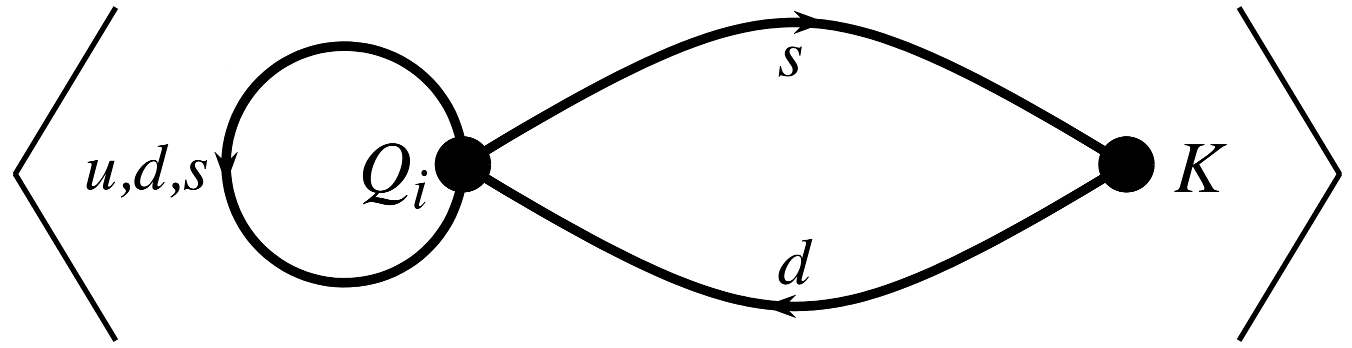


K → KK





suppressed if



= 0

Not suppressed

$\Delta I = 1/2$ operators mix with $\bar{s}\gamma_5 d$

Q_i : power divergent ~ $\dfrac{m_d - m_s}{a^2}\bar{s}\gamma_5 d + \dfrac{m_s + m_d}{a^2}\bar{s}d$

$$\rightarrow Q_i^{\mathrm{subt}} = Q_i - \alpha_i \bar{s}\gamma_5 d - \alpha_i \frac{m_s + m_d}{m_d - m_s}\bar{s}d$$

- $K \rightarrow$ vac subtraction scheme

$$\langle 0|Q_i^{\mathrm{subt}}|K\rangle = 0 \Rightarrow \alpha_i = \frac{\langle 0|Q_i|K\rangle}{\langle 0|\bar{s}\gamma_5 d|K\rangle}$$

- $K \rightarrow \pi$ subtraction scheme

$$\langle \pi|Q_i^{\mathrm{subt}}|K\rangle = 0 \Rightarrow \alpha_i = \frac{m_d - m_s}{m_s + m_d}\frac{\langle \pi|Q_i|K\rangle}{\langle \pi|\bar{s}d|K\rangle}$$

power-divergent part is mutual at infinite L_s

interpolation of Z_V^* / Z_V

10 % discrepancy in the continuum limit →

Z of 4-quark operator ~ Z_V^2

→ 20% error also expected